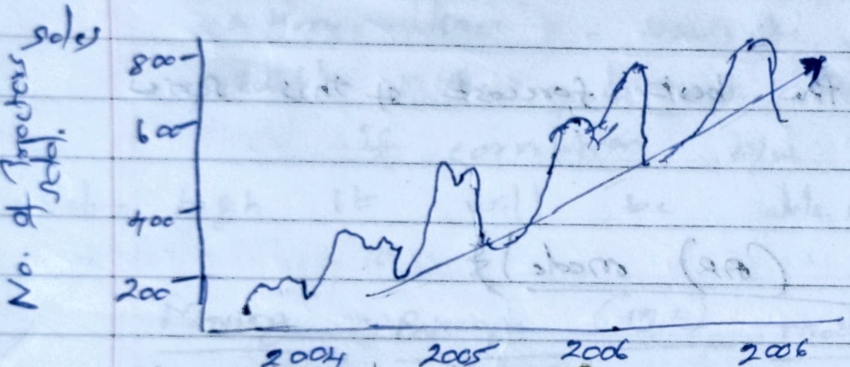
* Time series : is a set of observations on the values that a variable takes at different times.

Example : sales trend, stock market prices, weather forecast

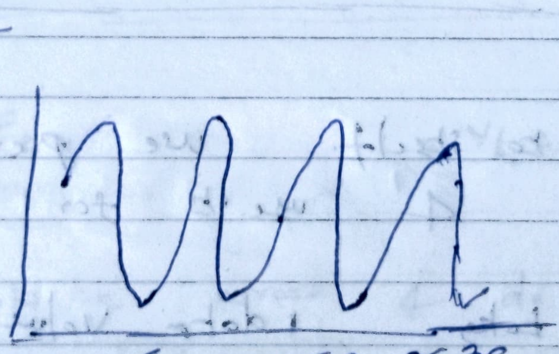
Time series Data patterns



* Trend - There is an upward trend in the figure. over the period of time the trend value goes up & sales increases

- There is an upward & downward. The sales in beginning gradually picks up reaches peak & there is downwards

Seasonality



Cyclic - It follows a cycle. Very fixed cycle. Sine series.

Straight time series

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Random - white noise is completely random there is no relation b/w past & future data. we cannot do time series analysis.

White Noise

* A series purely random in nature is called as white noise

* Mean = 0, constant variance & uncorrelated.

Average is the best forecast of this series

Auto Regressive (AR) model

* Y_t depends only of past values. Y_{t-1}, Y_{t-2}, Y_{t-3} etc

$$Y_t = f(Y_{t-1}, Y_{t-2}, Y_{t-3})$$

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \beta_3 Y_{t-3} \ldots$$

Regress to itself. we previous value of the data & use it for regression.

AR(1) . takes 1 data value

$$Y_{June} = \beta_0 + \underbrace{\beta_1 Y_{may}}_{\text{multiplying factor}}$$

June month, sales is dependent on May month sales

constant

Linear Equation.

$$y = ax + b$$

Value prediction using the previous month.

β_0 - intercept

β_1 - slope

if $p=1$ AR(1)

$$AR(2) \Rightarrow Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 Y_{t-2}$$

simple regression on a single variable by taking a time line - uses the past time values to predict the current time values.

if correlation b/w past values & current values high it will be able to predict better.

<u>Moving Average (MA) model</u>

Y_t depends only on random error terms

$$Y_t = f(\varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2}, \varepsilon_{t-3} \dots)$$

or

$$Y_t = \beta + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \theta_3 \varepsilon_{t-3} + \dots$$

$Y_{t-1} \xrightarrow{\text{gives}} \varepsilon_{t-1}$

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \varepsilon_t$$

ε_t ↓ Error

calculating with errors

Take the errors & do it with Y_t.

& use this function to fit the line.

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ARMA model

Combines AR and MA

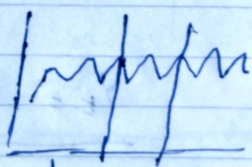
$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \beta_3 Y_{t-3} \dots$$
$$\varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \theta_3 \varepsilon_{t-3}$$

Stationarity of Time series

* A series is said to be "strictly stationary", if the mean, variance & covariance is constant over period of time or time invariant

* A series which is not stationary can be made stationary after differencing.

* After differencing once, the series is called as integrated of order 1 and denoted by $I(1)$. In general $I(d)$



only stationary we can perform AR, MA ...

sales	
250	NA
300	50
280	-20
300	20

$\nwarrow I(1)$ Integrated order of 1

when u differentiate the stationary attribute will be nullified.

Why stationarity

* Most models assume stationarity of data. In other words, standard techniques are invalid if data is non-stationary.

* Autocorrelation may result due to non-stationarity

* Autoregression results in spurious regression.

Time series is non-stationary, we cannot apply any model (MA, AR) & results in spurious regression

** make data stationary - by simply differencing it, Integrated order of 1
Differencing once.

ARIMA model P d q
ARIMA: AutoRegressive Integrated Moving Average
combining all three methods.

It has three patterns (p, d, q)
ARIMA (0, 0, 1) - only applying MA
(1, 1, 1) - Auto regression using one value prior to it AR(1)
I(1) - one difference

Pvalues 0.2

0,0,0
* Most successful model in time series Analysis.
0 1 0 non stationary to stationary
0 0 1 MA with q value of 1 using on error.
2 0 0

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Autocorrelation

* Autocorrelation is the similarity between observations as a function of the time lag between them.

Correlation varies b/w -1 to +1

-vely correlation — strongly +ve correlated

x increases
y increases

zero - no correlation.

correlate a variable to itself

x_t is correlated with x_{t-1}

	x_t	x_{t-1}
Jan	250	NA
	300	250
	280	300
	300	280
	350	300
		350

- How much is this month sale related to past month sale.

correlation

$x \simeq x = 1$

x_t x_{t-1}

0.7 +ve correlate.

Akaike Information Criteria (AIC)

The AIC is defined by a simple equation from the sum-of-squares & number of degrees of freedom of the two models

$$\Delta AIC = N \times \ln\left(\frac{SS_2}{SS_1}\right) + 2\Delta DF$$

↓ Mean squared error
very minimum..

Moving Averages

Week	Sales	3 Week MA	Error	MAD |Error|	Error²	|%.E|
1	39					
2	44					
3	40					
4	45	= 39+44+40/3 = 41	4	4	16	8.89
5	38	= 44+40+45/3 = 43	-5	5	25	13.16
6	43	= 40+45+38/3 = 41	2	2	4	4.65
7	39	= 45+38+43/3 = 42	-3	3	9	7.69
8		38+43+39/3 = 40		14/4	54	34.39

MAD - 3.5

MSE 13.5

MAPE 8.60%

Error = Forecast value - Actual value

45 - 41 = 4

38 - 43 -5

43 - 41 = 2

39 - 42 = -3

|Error| - Errors & Deviation are used interchangeably. Two bars around Error here denote absolute value

MAE (Mean Absolute Error) - We simply average these absolute errors. First we sum up the absolute errors & then divide the total by 4

MAD = 4+5+2+3/4 = 14/4 = 3.5

Mean Absolute Deviation

Error² = 16+25+4+9/4 = 13.5

%. Mean Squared Error (MSE)

|%. Error| - Mean Absolute percent error

The absolute percent error is a measure of the error as a percentage of actual value

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MAPE = |Error| / Actual × 100%

Mean Absolute percentage error.

= 4/45 = 8.89 %
5/38 = 13.16 %
4/43 = 4.65 %
3/39 = 7.69 %

sum of Abs = 34.39 %

MAPE = 8.60 %

Forecasting

Exponential Smoothing

objectives

Calculate forecasts using exponential smoothing
calculate mean squared error, MSE

Week	A_t Sales	F_t Forecast	
1	39	39	$F_1 = A_1$
2	44	39	$F_2 = 0.2 \times (39) + 0.8 \times (39)$ $F_2 = A_1$
3	40	40	$F_3 = 0.2 \times 44 + 0.8 \times 39$
4	45	40	$F_4 = 0.2 \times 40 + 0.8 \times 40.0$
5	38	41	$F_5 = 0.2 \times 45 + 0.8 \times 40$
6	43	40.40	$F_6 = 0.2 \times 38 + 0.8 \times 41$
7	39	40.92	$F_7 = 0.2 \times 43 + 0.8 (40.40)$
8		40.54	$F_8 =$

$$F_{t+1} = F_t + \alpha (A_t - F_t)$$

where α is smoothing constant

$$0 \leq \alpha \leq 1$$

$$F_{t+1} = \alpha A_t + (1-\alpha) F_t$$

Smoothing Forecast $\alpha = 0.2$

$1-\alpha = 0.8$

$F_{t+1} = 0.2(A_t) + 0.8(F_t)$

Week	Sales	Forecast	Error	$Error^2$
1	39			
2	44	39.00	5.00	25.00
3	40	40.00	0.00	0.00
4	45	40.00	5.00	25.00
5	38	41.00	-3.00	9.00
6	43	40.40	2.60	6.76
7	39	40.92	-1.92	3.69

MSE = 11.58

Add the squared Error & divide by 6

MSE = 11.58

Weighted Moving Average

Time Series data from 7 weeks of Sales

Week	Sales
1	39
2	44
3	40
4	45
5	38
6	43
7	39
8	

Example-1

Forecast sales using 4-week weighted moving averages with weights 0.4, 0.3, 0.2 & 0.1

** It places more weight on the most recent data

Soln

Week	Sales	4WMA
1	39	
2	44	
3	40	
4	45	
5	38	42.7
6	43	41.1
7	39	41.6
8		40.6

$F_5 = 0.4 \times 45 + 0.3 \times 40 + 0.2 \times 44 + 0.1(39)$.

$F_6 = 0.4(38) + 0.3(45) + 0.2(40) + 0.1(44)$

$F_7 = 0.4(43) + 0.3(38) + 0.2(45) + 0.1(40)$

$F_8 = 0.4(39) + 0.3(43) + 0.2(38) + 0.1(45)$

MAD ← Mean Absolute Deviation.

Week	Sales	4WMA	|Error|
1	39		
2	44		
3	40		
4	45		
5	38	42.7	4.7
6	43	41.1	1.9
7	39	41.6	2.6

$MAD = (4.7 + 1.9 + 2.6)/3$

$= 3.07$

Weighted Moving Averages

Weights: 3 & 2
3+2=5

Week	Sales	2WMA
1	39	
2	44	
3	40	
4	45	
5	38	
6	43	
7	39	
8		

Soln We multiply the sales value by the weights as we did before, but in this case, we also divide by the total weight which is 5

Week	Sales	2WMA	
1	39		
2	44		
3	40	42.0	$F_3 = \frac{3(44) + 2(39)}{5}$
4	45	41.6	$F_4 = 3(40) + 2(44) / 5$
5	38	43.0	$F_5 = 3(45) + 2(40) / 5$
6	43	40.8	$F_6 = 3(38) + 2(45) / 5$
7	39	41.0	$F_7 = 3(43) + 2(38) / 5$
8		40.6	$F_8 = 3(39) + 2(43) / 5$

|Error|

Week	Sales	2WMA	|Error|
1	39		
2	44		
3	40	42.0	2.0
4	45	41.6	3.4
5	38	43.0	5.0
6	43	40.8	2.2
7	39	41.0	2.0

On averaging these five values, we obtain Mean Absolute deviation value of 2.92

Comparing forecast Errors

Method	MAD
4-Week Weighted Average (Weights .4, .3, .2, .1)	3.07
2-Week Weighted Average (Weights 3 4 2)	2.92

Since the MAD is an error measure, smaller MADs produce better smoothing of the data

∴, using MAD, the 2-week weighted average method produced a better forecast.