

<9

previous video).

eg -10 mod 2

2) congruent modulo

Two integers a and b are said to be congruent modulo n if

$$(a \bmod n) = (b \bmod n)$$

This is written as:

$$a \equiv (b \bmod n) \quad \text{or} \quad b \equiv (a \bmod n)$$

the
keys.

Properties

(i)

sd,

→ n divides $(a-b)$

$$(a-b)$$

$$b \equiv a \pmod{n}$$

(ii)

$$\pmod{n}$$

(iii)

$$\pmod{n}$$

$$(a \bmod n) = (b \bmod n)$$

This is written as:

$$a \equiv (b \bmod n) \quad \text{or} \quad b \equiv (a \bmod n)$$

the
keys.

e.g. $73 \equiv 4 \pmod{23}$ means $73 \bmod 23 = 4 \bmod 23$

Properties of congruence

- (i) $a \equiv b \pmod{n}$ if $n \mid (a - b)$ → divides $(a - b)$
- (ii) $a \equiv b \pmod{n}$ implies $b \equiv a \pmod{n}$
- (iii) if $a \equiv b \pmod{n}$ and $b \equiv c \pmod{n}$
then, $a \equiv c \pmod{n}$

unge

$$\times 19 = \\ 5(69)$$

Set of residues or residue classes modulo n

$$\mathbb{Z}_n = \{ 0, 1, 2, \dots, (n-1) \}$$

eg $\mathbb{Z}_6 = \{ 0, 1, 2, 3, 4, 5 \}$

each integer in \mathbb{Z}_n represents
a residue class.

$(\text{mod } 2)$ mod 2

d 2

Euler's Totient function

- It is represented using phi as $\phi(n)$ and may also be called Euler's phi function.
- Euler's totient fn is defined as the
[no. of] [+ve integers] less than n that
are coprime to n.

$$n \geq 1$$

$$\phi(5) = \{1, 2, 3, 4\}$$

$$\phi(6) = \{1, 5\}$$

no. of elements in these sets
is the totient fn.

Note → Two integers a, b are said to be
relatively prime, if they have no common integer factor

no. of +ve integers less than n that
are coprime to n .

$$n \geq 1$$

$$\phi(1) = 1$$

$$\phi(5) = \{1, 2, 3, 4\} = 4$$

$$\begin{matrix} 1 \\ 0 \\ 2 \\ \times \\ 3 \\ \times \\ 4 \\ \Sigma \end{matrix}$$

$$\phi(6) = \{1, 5\} = 2$$

no. of elements in these sets
is the totient fⁿ.

Note \rightarrow Two integers a, b are said to be
relatively prime, mutually prime or
coprime if the only +ve integer | factor
that divides both of them is 1.

Now, when $n \rightarrow \text{prime}$ $\phi(n) = n - 1$

eg $\phi(5) = 4$ // we have seen the eg. above

$$\phi(23) = 23 - 1$$

$$= 22$$

Now, when $n \rightarrow \text{prime}$ $\phi(n) = n - 1$

eg $\phi(5) = 4$ // we have seen the eg. above

$$\begin{aligned}\phi(23) &= 23 - 1 \\ &= 22\end{aligned}$$

Also, $\phi(a * b) = \phi(a) * \phi(b)$ // a and b
should be coprime

eg $\phi(35) = \phi(7) * \phi(5)$
 $= 6 * 4 = 24$

eg $\phi(12) = \phi(3) * \phi(4) = 2 * 2 = 4$
 $\phi(15) = \phi(3) * \phi(5) = 2 * 4 = 8$

like and share :)

{ 1, 2, 4, 5, 7, 8 } { 11, 13, 14 }

Euler's Theorem

→ also called Fermat-Euler theorem or Euler's totient theorem
(alc to wikipedia)

Euler's theorem states that if x and n are coprime positive integers, then

$$x^{\phi(n)} \equiv 1 \pmod{n}$$

where $\phi(n) \rightarrow$ Euler's totient τ^n .

Note → It is a generalized version of Fermat's Theorem.

eg let $x = 11, n = 10$ both are coprime
∴ we can represent them as

where $\phi(n) \rightarrow$ Euler's totient function
 $x^{\phi(n)} \bmod n = 1 \bmod n$

Note → It is a generalized version of Fermat's
Theorem.

e.g.

let $x = 11, n = 10$

both are coprime
∴ we can represent them as

$$11^{\phi(10)} \equiv 1 \bmod 10$$

$$11^4 \equiv 1 \bmod 10$$

$$14641 \equiv 1 \bmod 10 \quad \text{which is true}$$

$$\phi(10) = \phi(2) * \phi(5)$$

$$= 1 * 4$$

$$= 4$$

Note →

$$x^{\phi(n) \cdot a} \equiv 1 \bmod n$$

$$11^8 = 214,358,881$$

$$\boxed{11^4 \equiv 1 \pmod{10}}$$

$$\boxed{14641 \equiv 1 \pmod{10}} \quad \text{which is true}$$

$$\begin{aligned}\phi(10) &= \phi(2) * \phi(5) \\ &= 1 * 4 \\ &= 4\end{aligned}$$

$$\phi(a * b) = \phi(a)^{\phi(b)}$$

$$\phi(5) = 4$$

$$\boxed{x^{\phi(n) \cdot a} \equiv 1 \pmod{n}}$$

$11^8 = 214,358,881$

i.e.

$$\begin{aligned}11^{4*2} &\equiv 1 \pmod{10} \\ 11^{40} &\equiv 1 \pmod{10}\end{aligned}$$

i.e. any multiple
of $\phi(n)$ will
give the same
result.

Note →

Solve by Eulers Theorem

Ours

$$\underline{4^{99} \text{ Mod } 35}$$

$$x = 4, n = 35$$

by eulers theorem,

$$4^{\phi(35)} \equiv 1 \pmod{35}$$

$$\boxed{4^{24} \equiv 1 \pmod{35}} \quad \text{--- (1)}$$

$$\begin{aligned}\phi(35) &= \phi(7) \cdot \phi(5) \\ &= 6 * 4\end{aligned}$$

$$\phi(35) = 24$$

$$4^{99} \rightarrow 4^{24(4)} \cdot 4^3$$

$$\begin{aligned}\therefore 4^{99} \pmod{35} &= 4^{24 \times 4 + 3} \pmod{35} \\ &= (4^{24})^4 \cdot 4^3 \pmod{35} \\ &= (4^{24})^4 \pmod{35} \cdot 4^3 \pmod{35}\end{aligned}$$

$$\therefore (a \times b) \pmod{n} \equiv (a \pmod{n})(b \pmod{n})$$

$$4^3 \pmod{35}$$

Fermat's Theorem / Fermat's Little Theorem

→ especial case of euler theorem
if n is prime and x is a true integer not divisible by n then $\boxed{x^{n-1} \equiv 1 \pmod{n}}$

$$\boxed{\phi(n)=n-1}$$

$n \rightarrow$ prime no.

x is not divisible by n

Also,

$x, n \rightarrow$ coprime

eg $x=3, n=5$

$$3^{5-1} = 3^4 = 81$$

$$\boxed{\therefore 81 \equiv 1 \pmod{5}}$$

euler theorem

$$x^{\phi(n)} \equiv 1 \pmod{n}$$

$$x^{n-1} \equiv 1 \pmod{n}$$

} fermat's theorem

Lagrange theorem

$n \rightarrow$ prime no.
 x is not divisible by n

$\phi(n) = n - 1$
Also,
 $x, n \rightarrow$ coprime

$$81/5 = 14.2$$

eg

$$x = 3, n = 5$$

$$3^{5-1} = 3^4 = 81$$

$$\therefore 81 \equiv 1 \pmod{5}$$

euler theorem

$$x^{\phi(n)} \equiv 1 \pmod{n}$$

$$x^{n-1} \equiv 1 \pmod{n}$$

} fermat's theorem

Another form of fermat's theorem

$$x^n \equiv x \pmod{n}$$

in books

$$a^p \equiv a \pmod{p}$$

eg

$$x = 3, n = 5$$

$$x^n = 3^5 = 243 \equiv 3 \pmod{5}$$

(mod 5)

remainder

$a_1 = 1, a_3 = 3$
 $m_1 = 7, m_3 = 11$
give to one

CHINESE REMAINDER THEOREM

Chinese Remainder theorem states that there always exists an "x" that satisfies the given congruence.

$$x \equiv \text{rem}[0] \pmod{\text{num}[0]}$$

$$x \equiv \text{rem}[1] \pmod{\text{num}[1]}$$

...

and $(\text{num}[0], \text{num}[1], \dots, \text{num}[m-1])$ must be coprime to each other one another.

e.g.

$$x \equiv 1 \pmod{5}$$

$$x \equiv 3 \pmod{7}$$

\rightarrow 5 and 7 are coprime

$$\begin{aligned}m_1 &= 77 \\b_1 &= 55 \\3 &= 35\end{aligned}$$

$$(\text{mod } m_1) = 1$$

$$x \equiv \text{rem}[1] \pmod{\text{num}[1]}$$

and $(\text{num}[0], \text{num}[1], \dots, \text{num}[m-1])$ must be coprime to ~~each other~~ one another.

eg.

$$x \equiv 1 \pmod{5}$$

$$x \equiv 3 \pmod{7} \rightarrow 5 \text{ and } 7 \text{ are coprime}$$

we have to find this $x = 31$

eg

$$x \equiv 2 \pmod{3}$$

$$x \equiv 3 \pmod{4}$$

$$x \equiv 1 \pmod{5}$$

$$\gcd(3, 4) = \gcd(4, 5)$$

$$= \gcd(3, 5) = 1$$

Then only x exists

have to find
value such
that
of value

$$\text{here } x = 11$$

Explaining Chinese Remainder Theorem

if

$$x \equiv a_1 \pmod{m_1}$$

$$x \equiv a_2 \pmod{m_2}$$

$$x \equiv a_3 \pmod{m_3}$$

$$\begin{array}{r} 77 \\ \times 5 \\ \hline 3 \end{array}$$

(i) $\gcd(m_1, m_2) = \gcd(m_2, m_3) = \gcd(m_3, m_1) = 1$
ie all coprime

(ii) $x = (M_1 \times_1 a_1 + M_2 \times_2 a_2 + M_3 \times_3 a_3 + \dots + M_n \times_n a_n) \pmod{M}$

$$M = m_1 * m_2 * m_3 * \dots * m_n$$

$$M_i^o = \frac{M}{m_i} \quad \text{eg} \quad M_1 = \frac{M}{m_1} = \frac{m_1 * m_2 * m_3}{m_1} = m_2 * m_3$$

$$\therefore \boxed{M_1 = m_2 * m_3}$$

Similarly $M_2 = m_1 * m_3$

" " $M_3 = m_1 * m_2$

$$M_i^{\circ} = \frac{M}{m_i}$$

eg $M_1 = \frac{M}{m_1} = \frac{m_1 m_2 m_3}{m_1} = m_2 m_3$

$$M_1 = m_2 m_3$$

Similarly $\rightarrow M_2 = m_1 m_3 = \frac{M}{m_2} = m_1 \cancel{m_2} m_3 = m_1 m_3$
 " $M_3 = m_1 m_2$

To calculate X_i°   multiplicative inverse of 

$$M_i^{\circ} X_i^{\circ} \equiv 1 \pmod{m_i}$$

eg $M_1 X_1 \equiv 1 \pmod{m_1}$

~~books~~/books.

Numerical Chinese Remainder Theorem

e.g

$$x \equiv 1 \pmod{5}$$

$$x \equiv 1 \pmod{7}$$

$$x \equiv 3 \pmod{11}$$

$$x \equiv a_i \pmod{m_i}$$

$$\text{so } a_1 = 1, a_2 = 1, a_3 = 3$$

$$m_1 = 5, m_2 = 7, m_3 = 11$$

Solu → Since 5, 7 and 11 all are relatively prime to one another. So, we can find x .

i.e $\gcd(5, 7) = \gcd(7, 11) = \gcd(11, 5) = 1$

$$M = m_1 * m_2 * m_3 = 5 * 7 * 11 = 385$$

$$\boxed{M = 385}$$

$$M_1 = \frac{M}{m_1} = m_2 m_3 = 7 * 11 = 77 \quad \left. \right\} \quad M_1 = 77$$

$$M_2 = \frac{M}{m_2} = m_1 m_3 = 5 * 11 = 55 \quad \left. \right\} \quad M_2 = 55$$

$$M_3 = \frac{M}{m_3} = m_1 m_2 = 5 * 7 = 35 \quad \left. \right\} \quad M_3 = 35$$

$$\begin{aligned} M_1 &= \frac{M}{m_1} = m_2 m_3 = 7 * 11 = 77 \\ M_2 &= M_1 m_3 = 5 * 11 = 55 \\ M_3 &= M_1 m_2 = 5 * 7 = 35 \end{aligned} \quad \left. \begin{array}{l} M_1 = 77 \\ M_2 = 55 \\ M_3 = 35 \end{array} \right\}$$

Now we will calculate x_1 value.

$$M_1 x_1 \equiv 1 \pmod{m_1} \text{ ie } M_1 x_1 \pmod{m_1} = 1$$

$$77 \cdot x_1 \pmod{5} = 1$$

$$2 \cdot x_1 \pmod{5} = 1$$

$$\boxed{\therefore x_1 = 3}$$

Similarly $M_2 x_2 \equiv 1 \pmod{m_2}$

$$55 \cdot x_2 \pmod{7} = 1$$

$$6 \cdot x_2 \pmod{7} = 1$$

$$\boxed{x_2 = 6}$$

$\left. \begin{array}{l} \text{we have to find} \\ \text{a value such} \\ \text{that} \\ 6 \cdot x_2 \text{ at value} \\ (7 k \text{ multiple} + 1) \text{ ho} \end{array} \right\}$

$$\therefore \boxed{x_3 = 6}$$

Now,

$$\begin{array}{ll} a_1 = a_2 = 1 & a_3 = 3 \\ m_1 = 5 & m_2 = 7 \\ M_1 = 77 & M_2 = 55 \\ x_1 = 3 & x_2 = 6 \end{array}$$

$$\begin{array}{ll} m_3 = 11 & \\ M_3 = 35 & \\ x_3 = 6 & \end{array}$$
$$M = 385$$

$$x = (M_1 x_1 a_1 + M_2 x_2 a_2 + M_3 x_3 a_3) \bmod M$$

$$x = (77(3)(1) + 55(6)(1) + 35(6)(3)) \bmod (385)$$

$$(231 + 330 + 630) \bmod 385$$

$$x = 1191 \bmod 385$$

$$\boxed{x = 36}$$

If u dont have calculator

$$1191 - 385 = 806$$

THE EUCLIDEAN / EUCLID'S ALGORITHM

used for determining 'GCD' of two positive integers
(generally large nos.)

$$\rightarrow \boxed{\begin{aligned} \gcd(a,b) &= \gcd(b, a \bmod b) \\ \gcd(a,0) &= a \end{aligned}}$$

[eg.]

$$\gcd(1025, 35) \text{ i.e } \gcd(a, b)$$

$$= \gcd(b, a \cdot 1 \cdot b) = \gcd(35, 10)$$

$$= \gcd(10, 5)$$

$$= \gcd(5, 0)$$

$$\begin{aligned} 600 &= 2^3 \cdot 5^2 \cdot 3^1 \\ 45 &= 2^0 \cdot 5^1 \cdot 3^2 \\ \text{GCD (HCF)} &\rightarrow 2^0 \cdot 5^1 \cdot 3^1 \\ &= 15 \end{aligned}$$

$$1025 = 35 \times 29 + 10$$

$$35 = 10 \times 3 + 5$$

$$\frac{18}{6 \cdot 0} \quad 0$$

$$\frac{35}{x \cdot 29} \quad 10 \cdot 5$$

$$= \gcd(b, a \cdot 1 \cdot b) = \gcd(35, 10)$$

$$= \gcd(10, 5)$$

$$= \gcd(5, 0)$$

$$= 5$$

Answer

$$45 = \frac{2^0 \cdot 5^1 \cdot 3^2}{2^0 \cdot 5^1 \cdot 3^1}$$

$$\text{GCD} / \text{HCF} \rightarrow = \boxed{15}$$

$$1025 = 35 \cdot 29 + 10$$

$$35 = 10 \cdot 3 + 5$$

$$10 = 5 \cdot 2 + 0$$

$\therefore 5 = \text{Ans}$

eg

$$\gcd(11, 7) \equiv \gcd(a, b)$$

$$\gcd(b, a \cdot 1 \cdot b) = \gcd(7, 11 \cdot 1 \cdot 7)$$

$$= \gcd(7, 4)$$

$$= \gcd(4 \pmod 4) = \gcd(4, 3)$$

$$= \gcd(3, 4 \cdot 1 \cdot 3) = \gcd(3, 1)$$

$$= \gcd(1, 3 \cdot 1 \cdot 1)$$

$$(1, 0)$$

GCD of 2740 and 1760

q	r_1	r_2	r
1	2740	1760	980
1	1760	980	780
1	980	780	200
3	780	200	180
1	200	180	20
9	180	20	0
	20	0	

$$r = r_1 \% r_2$$

$$\begin{array}{r} 1760 \\ \times 2 \\ \hline 3520 \end{array}$$

we will calculate the remainder r as $r_1 \% r_2$

Then replace

r_1 by r_2
r_2 by r

Continue until $r_2 = 0$. At this time, we stop
and $\text{GCD}(a, b) = r_1$ value.

EXTENDED EUCLIDEAN ALGO

Ques find the GCD of (161, 28) and the value of "s" and "t".

Given two integers \underline{a} and \underline{b} we often need to find 2 integers, s and t such that

$$s*a + t*b = \gcd(a, b)$$

So, extended euclidean algo can calculate $\underline{\gcd(a, b)}$ and the value of \underline{s} and \underline{t} .

cofft. of Bezout's identity
(in Number theory).

EXTENDED EUCLIDEAN ALGO

Ques
soln

find the GCD of $(161, 28)$ and the value of "s" and "t".
 a, b

$$s = s_1 - q s_2$$

$$t = t_1 - q t_2$$

q	r_1	r_2	r	s_1	s_2	s	t_1	t_2	t
5	161	28	21	1	0	1	0	1	-5
1	28	21	7	0	1	-1	1	-5	6
3	21	7	0	1	-1	4	-5	6	-23
7	0			<u>-1</u>	<u>4</u>		<u>6</u>	<u>-23</u>	<u>-5 - 18</u>
				<u>s</u>			<u>t</u>		=

$$\text{GCD} = 7$$

$$s = -1$$

$$t = 6$$

$$(-1) \times 161 + 6 \times 28 = 7$$

EXAMPLE 2.10

Given $a = 17$ and $b = 0$, find $\gcd(a, b)$ and the values of s and t .

SOLUTION We use a table to follow the algorithm.

q	r_1	r_2	r	s_1	s_2	s	t_1	t_2	t
	17	0		1	0		0	1	

Note that we need no calculation for q , r , and s . The first value of r_2 meets our termination condition. $\gcd(17, 0) = 17$, $s = 1$, and $t = 0$. This indicates why we should initialize s_1 to 1 and t_1 to 0. The answer

$$(1 \times 17) + (0 \times 0) = 17$$

EXAMPLE 2.11

Given $a = 0$ and $b = 45$, find $\gcd(a, b)$ and the values of s and t .

SOLUTION We use a table to follow the algorithm.

q	r_1	r_2	r
0	0	45	

MULTIPLICATIVE INVERSE

in CRYPTOGRAPHY

Ques Find the multiplicative Inverse of 11 in \mathbb{Z}_{26} .

Solu → Before solving

check, M·I possible or not

if $\boxed{\gcd(x, n) = 1}$ possible

$\gcd(11, 26) = 1$ true ∴ possible

q_1	r_1	r_2	r	t_1	t_2	t
<u>26</u>	<u>11</u>					$T = t_1 - q_1 t_2$

Solve until $r_2 \neq 0$

When $r_2 = 0$ value of t_1 will be M.I.

$\boxed{\mathbb{Z}_n} \rightarrow$ set of residues $\mathbb{Z}_n = \{0, 1, 2, \dots, (n-1)\}$

e.g. $\mathbb{Z}_2 = \{0, 1\}$

$\mathbb{Z}_5 = \{0, 1, 2, 3, 4\}$

$\mathbb{Z}_{20} = \{0, 1, 2, \dots, 19\}$

If $\boxed{\text{gcd}(x, n) = 1}$ possible
 $\text{gcd}(11, 26) = 1$ true. \therefore possible

q_1	r_1	r_2	r	t_1	t_2	t
2	<u>26</u>	<u>11</u>	4	0	1	-2
2	11	4	3	1	-2	5
1	4	3	1	-2	5	$5 - 3(-2)$
3	3	1	0	5	-7	
	1	0		-7	26	

$T = t_1 - 2t_2$

solve until $r_2 \neq 0$

when $r_2 = 0$ value of t_1 will be M.I.

Multiplicative inverse of a number in Cryptography

2.00

Solu → Before solving check, if M.I possible or not

$\text{gcd}(x, n) = 1$ possible
 $\text{gcd}(11, 26) = 1$ true ∴ possible

q_1	r_1	r_2	r	t_1	t_2	t
2	26	11	4	0	1	-2
2	11	4	3	-1	-2	5
1	4	3	1	-2	5	-7
3	3	1	0	5	-7	26
	1	0		-7	26	

$T = t_1 - q_1 t_2$

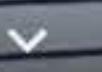
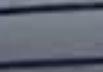
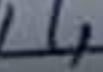
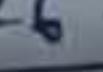
$0 - 2(1)$
 $5 - 3(-7)$

solve until $r_2 \neq 0$

when $r_2 = 0$ value of t_1 will be M.I.

$$\begin{array}{r} -7 \\ + 26 \\ \hline 19 \end{array}$$

$$\mathbb{Z}_{26} = \{0, 1, 2, \dots, 25\}$$



multiplicative inverse of

11 in \mathbb{Z}_{26}

$$11 * (M^{-1}) \equiv 1 \pmod{26}$$

$$11 * 19 \equiv 1 \pmod{26}$$

$$209 \equiv 1 \pmod{26}$$

Ques Find the multiplicative Inverse of
11 \rightarrow Before solving
check M.I possible or not
if $\frac{\text{gcd}(x, n) = 1}{\text{gcd}(11, 26) = 1}$ possible

q_1	r_1	r_2	r	t_1	t
2	26	11	4	0	1
2	11	4	3	-1	-2
1	4	3	1	1	5
3	3	1	0	0	

AUTHENTICATION FUNCTIONS

Authentication → verifying the user's identity

Raman → John

An authenticator must be there to authenticate the message.

Types of Authentication | Types of fn to produce authentication

- (i) ^{Message} Encryption (ciphertext act as authenticator)
- (ii) MAC (message authentication code)
 - we will have some authentication fn and we apply them on the plaintext along with the key which produces a fixed length code called MAC

message

fixed length code (MAC)

Authenticator

used to

Types of Authentication / Types of fn to produce authentication

- (i) Message Encryption (ciphertext act as authenticator)
- (ii) MAC (message authentication code)
 - we will have some authentication fn and we apply them on the plaintext along with the key which produces a fixed length code called MAC

1Mb ↴
1Kb ↴

$$C(M, K) \stackrel{\text{message}}{\uparrow} \neq \text{fixed length code (MAC)} \stackrel{\text{key}}{\downarrow}$$

authentication fn

This will act as an authenticator here.

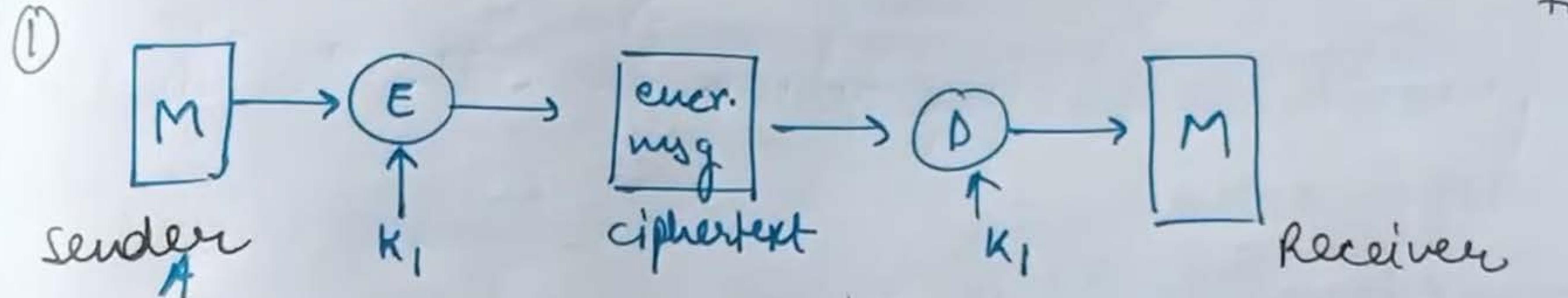
(iii) Hash functions (H)

$$H(M) \stackrel{\text{msg}}{\rightarrow} = \text{fixed length code (Hash code 'h')}$$

act as an

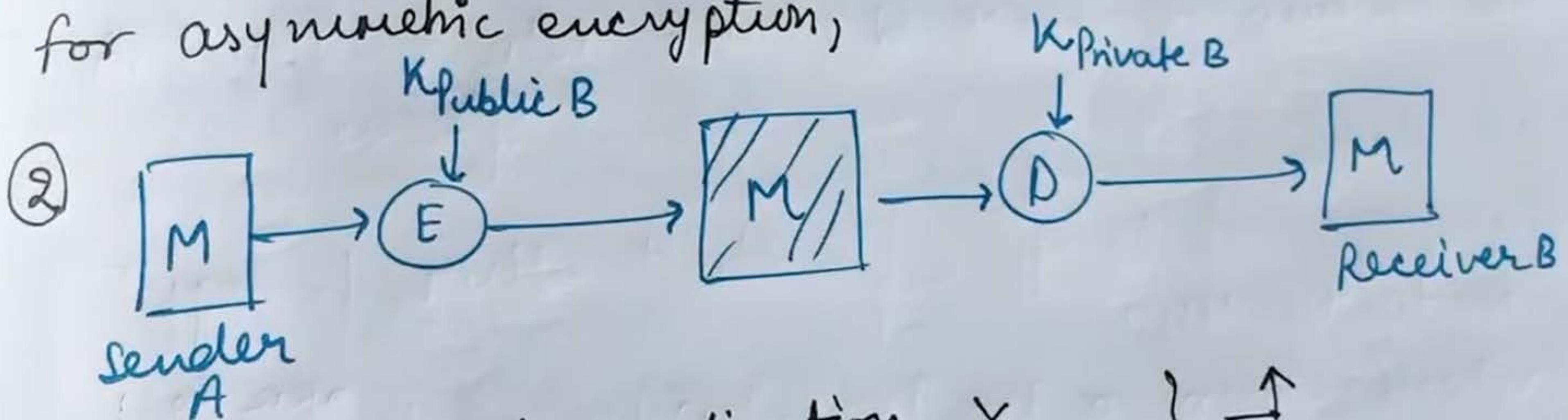
UA

1. Message encryption \rightarrow ciphertext is an authenticator

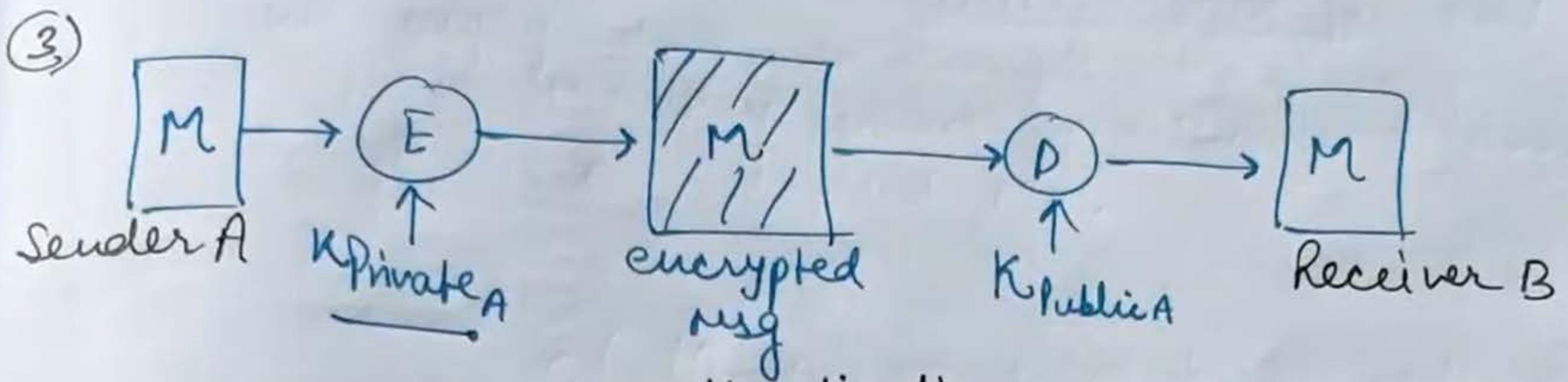


\rightarrow Key K_1 shared only b/w Sender & Receiver only.

for asymmetric encryption,



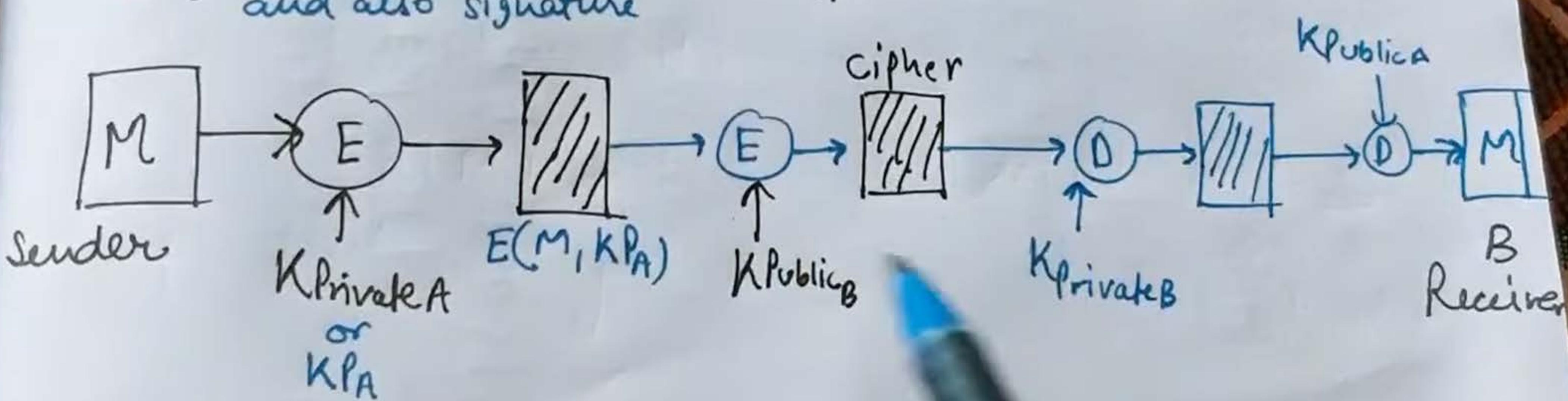
Authentication X } \uparrow
 confidentiality ✓



authentication ✓

confidentiality ✗

④ To get both, use dual encryption & decryption
and also signature



ION

the 5 principles of security.
authenticity of the msg is vimp.
must be there to authenticate

authentication

Message Authentication Code (MAC)

MAC (message authentication code)

- We will use a secret key to generate a small fixed size ^{1 block} of data called MAC or cryptographic checksum.
- It is then appended with the message.
- The communicating parties will share a secret common key which will be used to create the MAC

Let
A → sender
B → receiver

when A sends a msg to B, it calculates the MAC as a fn of the message and the key.

$$\boxed{\text{MAC} = C(K, M)}$$

where

M = input message

C = mac function

of the msg is up.
be there to authenticate

authentication

ipher text

Cede (MAC)

value

small fixed size of data called MAC or
cryptographic checksum.

- It is then appended with the message.
 - The communicating parties will share a secret common key.
- 2kb → 2 kb which will be used to create the MAC

Let A → sender
B → receiver

when A sends a msg to B, it calculates the
MAC as a fn of the message and the key.

$$\boxed{\text{MAC} = C(K, M)}$$

where

M = input message

C = MAC function

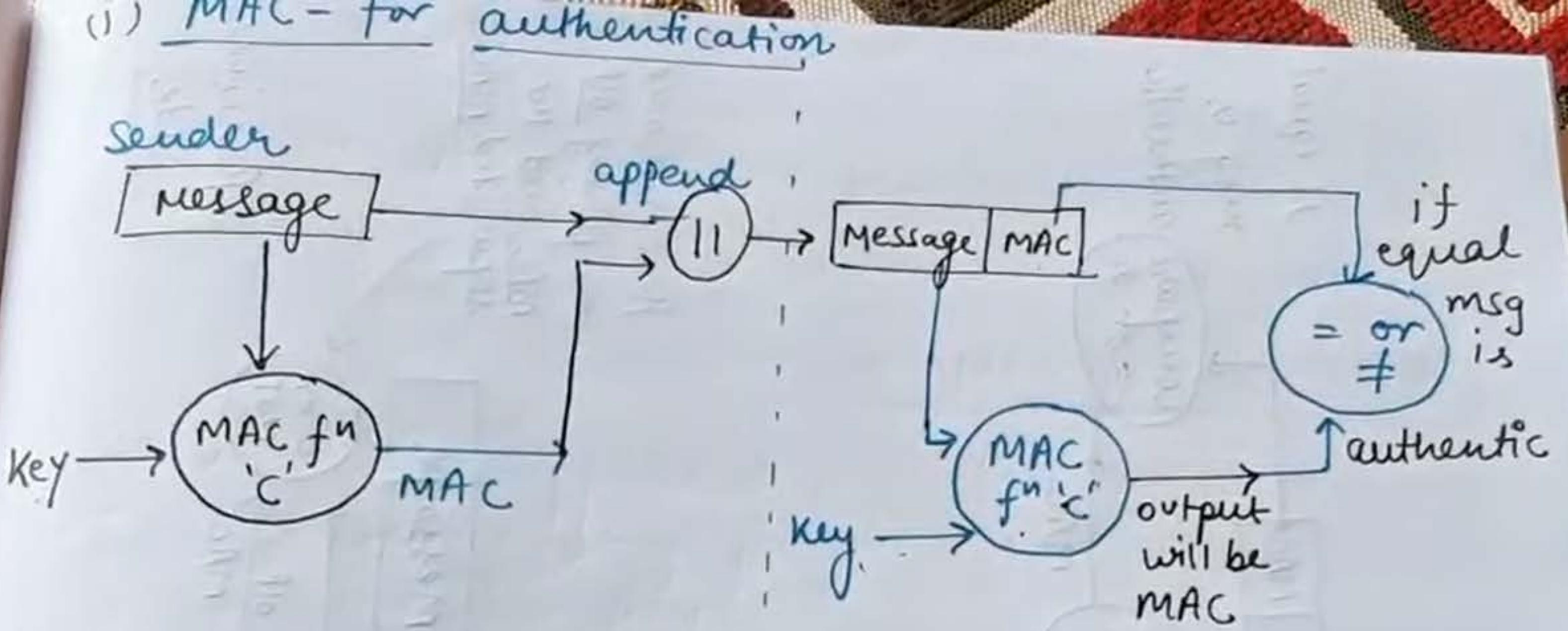
K = shared secret key

der & receiver

key
mac fn C.

AC
is will

the msg
as
again



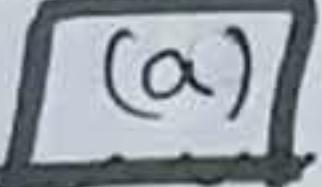
We separate the msg and the MAC

only authentication is achieved

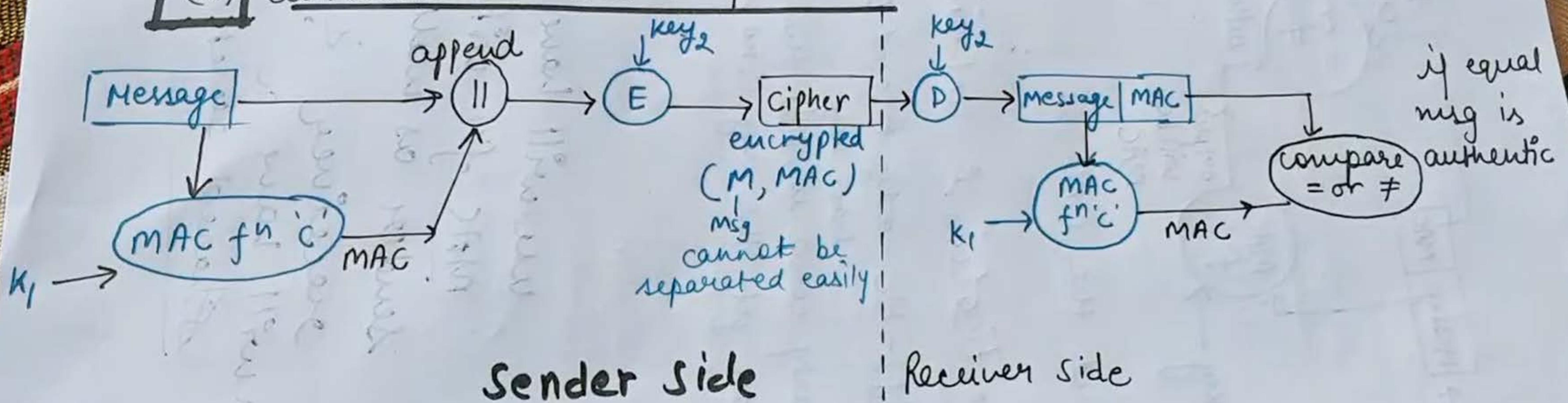
provides (i) confidentiality - b/c only A and B have key

(ii) if 3rd party come in, b/w them he can get the msg ∴ no security

2) MAC - for authentication & confidentiality



(a) authentication tied to plain text



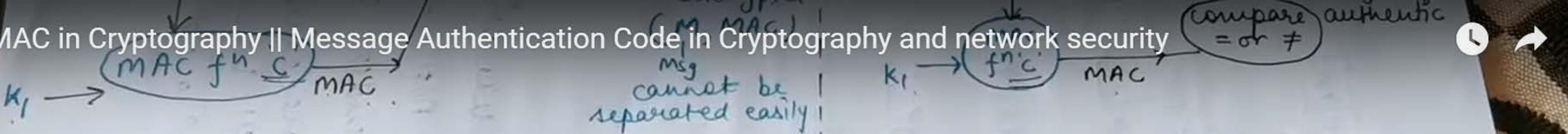
Sender & Receiver both have the MAC fn

and key₂

so plain text tied to cipher text

Here we are comparing of MAC and the

MAC in Cryptography II Message Authentication Code in Cryptography and network security

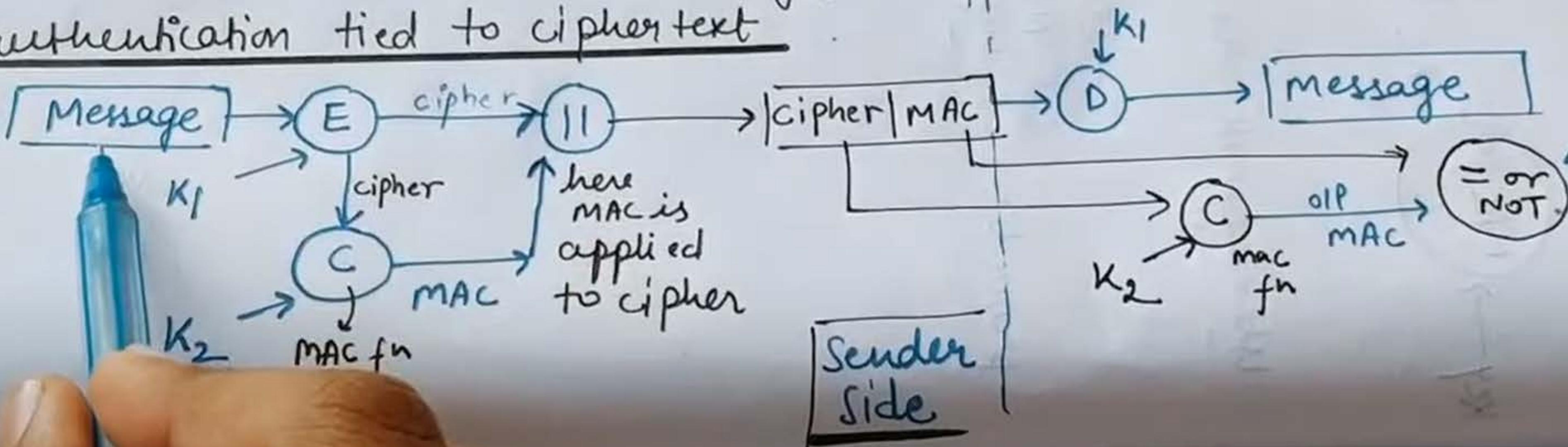


compare
= or ≠
authentic

Sender Side

Sender & Receiver both have the MAC fn and key K_2

b) authentication tied to cipher text



Here we are comparing oIP MAC and the separated MAC

Receive Side

achieved?

How

I give you
Rs

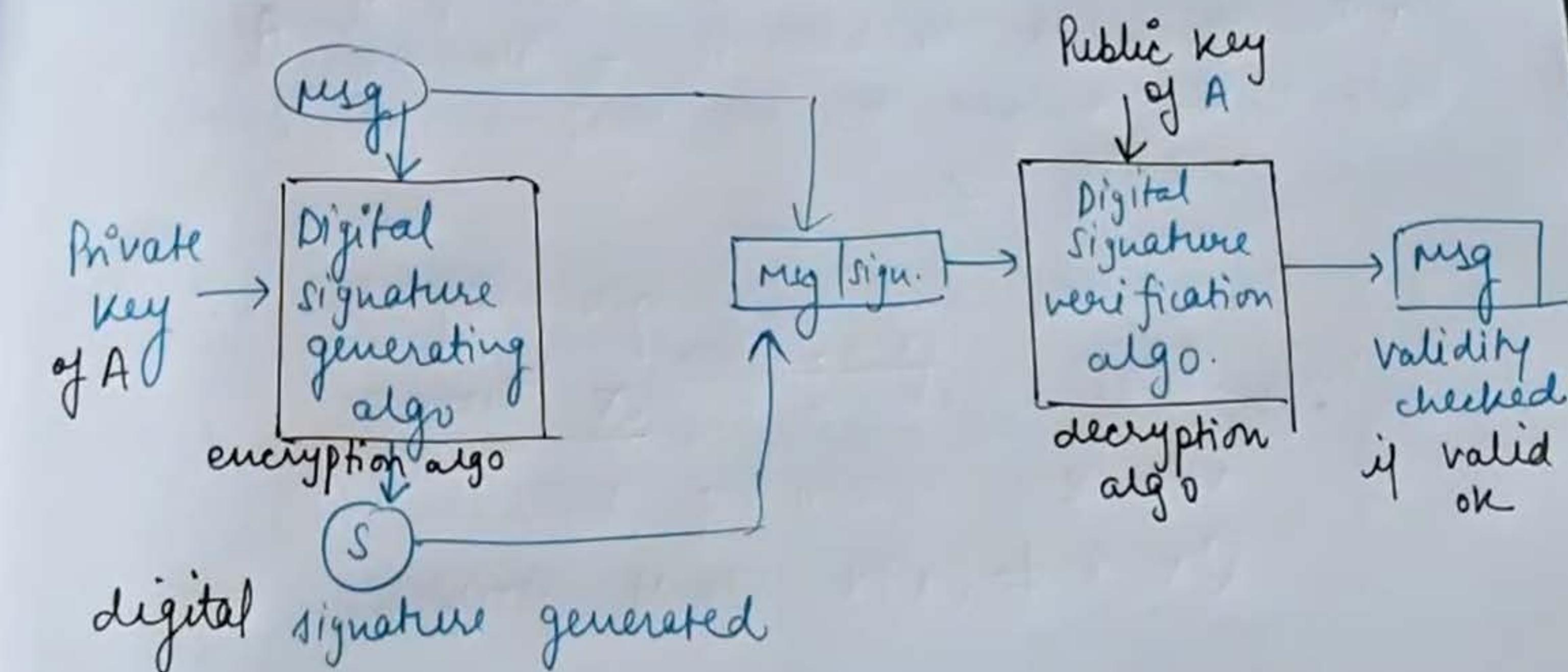
members



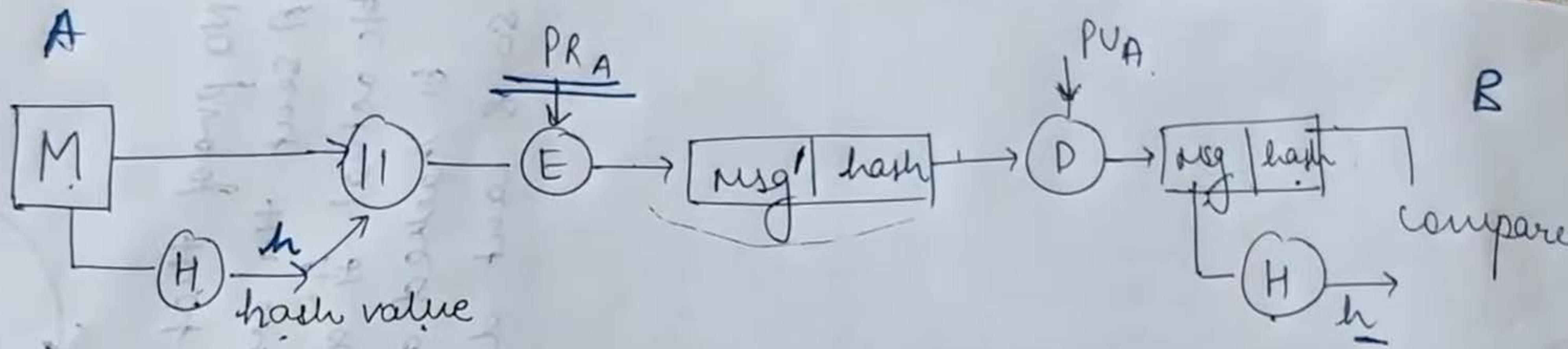
B

Digital signature

- rising role in e-commerce, online transaction, etc.
- based on asymmetric key cryptography
 - encryption → private key
 - decryption → public key
- used for authentication & non repudiation & msg integrity
- not used for confidentiality



code
 use.
 decryption \rightarrow private key
 decryption \rightarrow public key
 \rightarrow used for msg
authentication & non repudiation & msg
 \rightarrow not used for confidentiality integrity



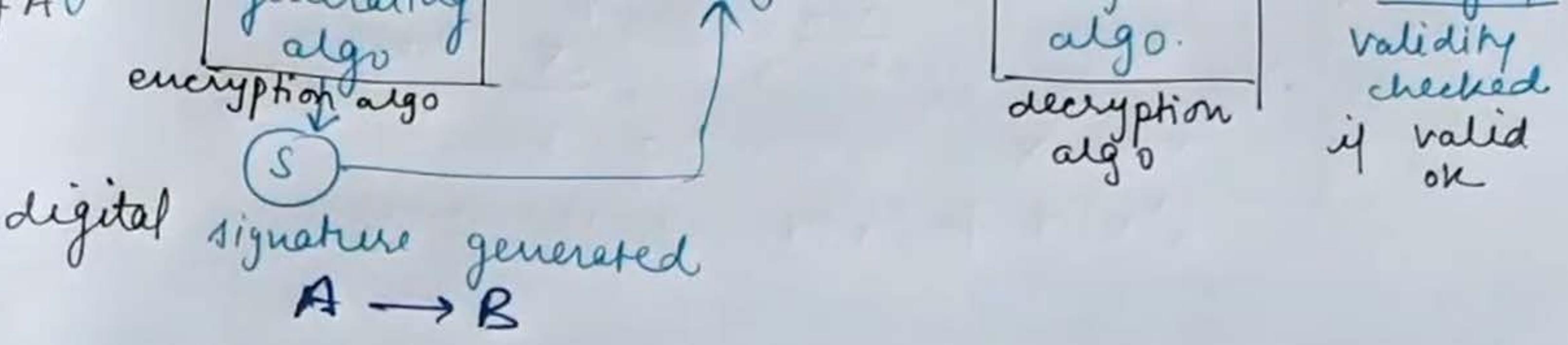
$H \rightarrow$ hash fn.

$h \rightarrow$ hash code

$\text{II} \rightarrow$ append

general concept of digital
signature

Note \rightarrow The signature must use some info. unique
to the sender to prevent both forgery



→ also provides msg integrity
blk if msg changed then at receiver
side, we will not get the exact msg.

achieved using Hashing
concept using msg digest |
hash values

Note → When we sign a document digitally,
= we send the signature as a separate
document.
Sender sends & does → msg & signature.

Non repudiation
achieved by using a trusted 3rd party

| Pg 351

Pg 352

confidentiality

Digital Signature

- signature must use some info unique to the sender, to prevent forgery & denial.
- It must be easy to produce digital signatures.
- " " " " to verify & recognize " " .
- we need (i) key generation algo → to generate private key
(ii) Signing Algo $M \rightarrow P$ and Private key , $O \rightarrow P \rightarrow$ Digital sign
(iii) verifying algo → using public key & sign.

3.4 Confidentiality

A digital signature does not provide confidential communication. If confidentiality is required, the message and the signature must be encrypted using either a secret-key or public-key cryptosystem. Figure 13.5 shows how this extra level can be added to a simple digital signature scheme.

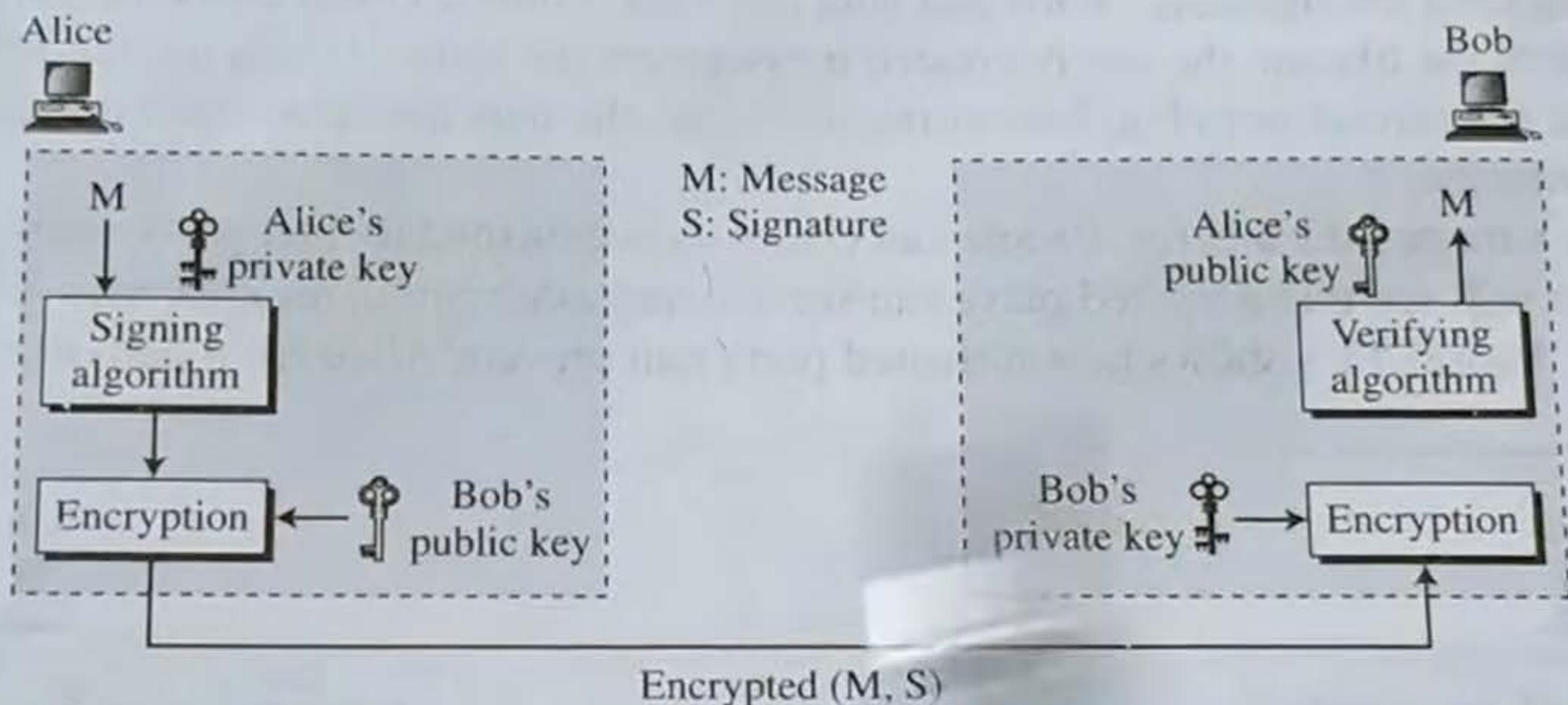


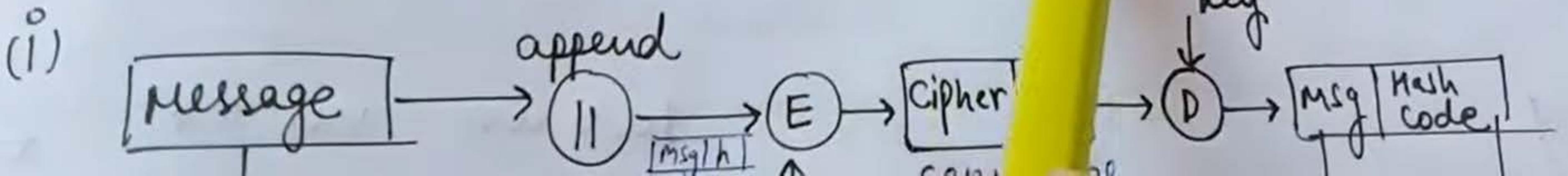
Fig. 13.5 Adding confidentiality to a digital signature scheme

We have shown asymmetric-key encryption options. We emphasize that two keys are used at each end. Encryption/decryption can also be done with

(iii) HASH FUNCTIONS

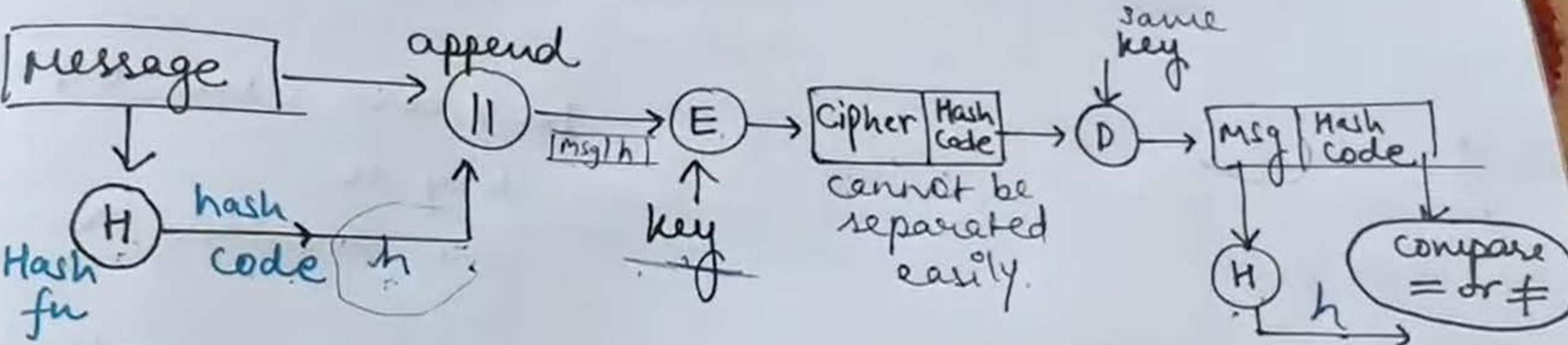
They are also called **Compression Functions**.

There are dif methods to provide authentication in dif situations



There are dif methods to provide authentication
in dif situations

(i)



authentication + confidentiality

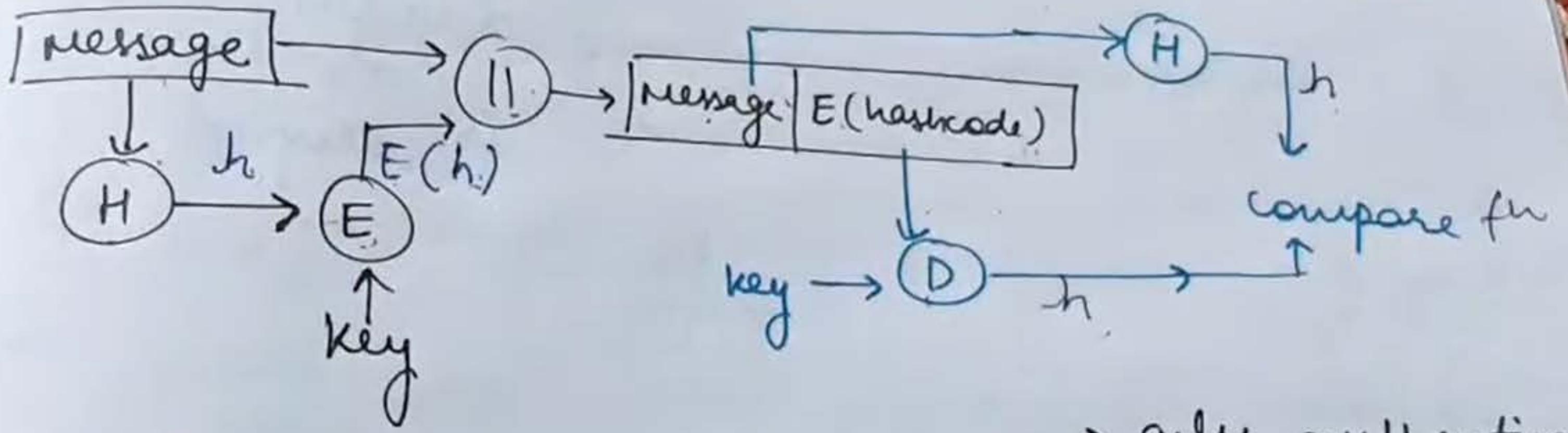
if both hashcodes
equal in the end

maintained b/c
msg was encrypted
before sending

b/c only A & B share the
secret key, the msg must have
come from A & has not been
altered.

12AH

method2 -



- only authentication
- No confidentiality

only Hash code is encrypted, using symmetric encryption.

If you need only authentication & no confidentiality
so we can use it b/c
> your msg is not private messages, the
processing time will be less b/c

we are not encrypting the message. We
are only encrypting the hash code
encrypted hash code

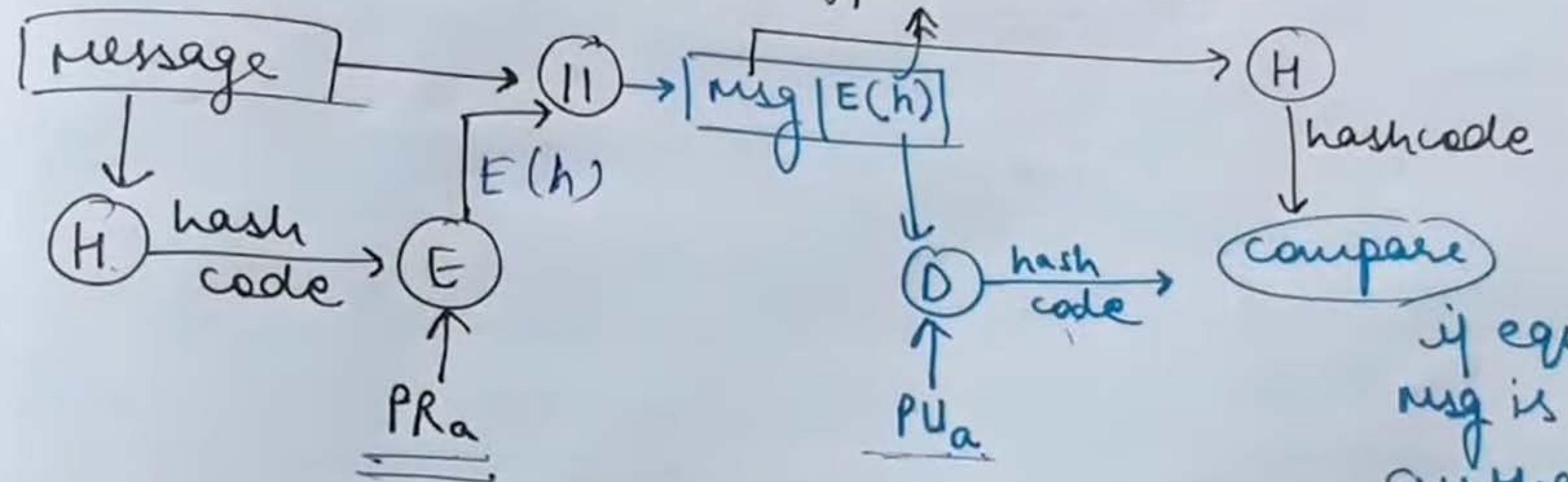
method3

11F

Hash Functions in Cryptography

2.00

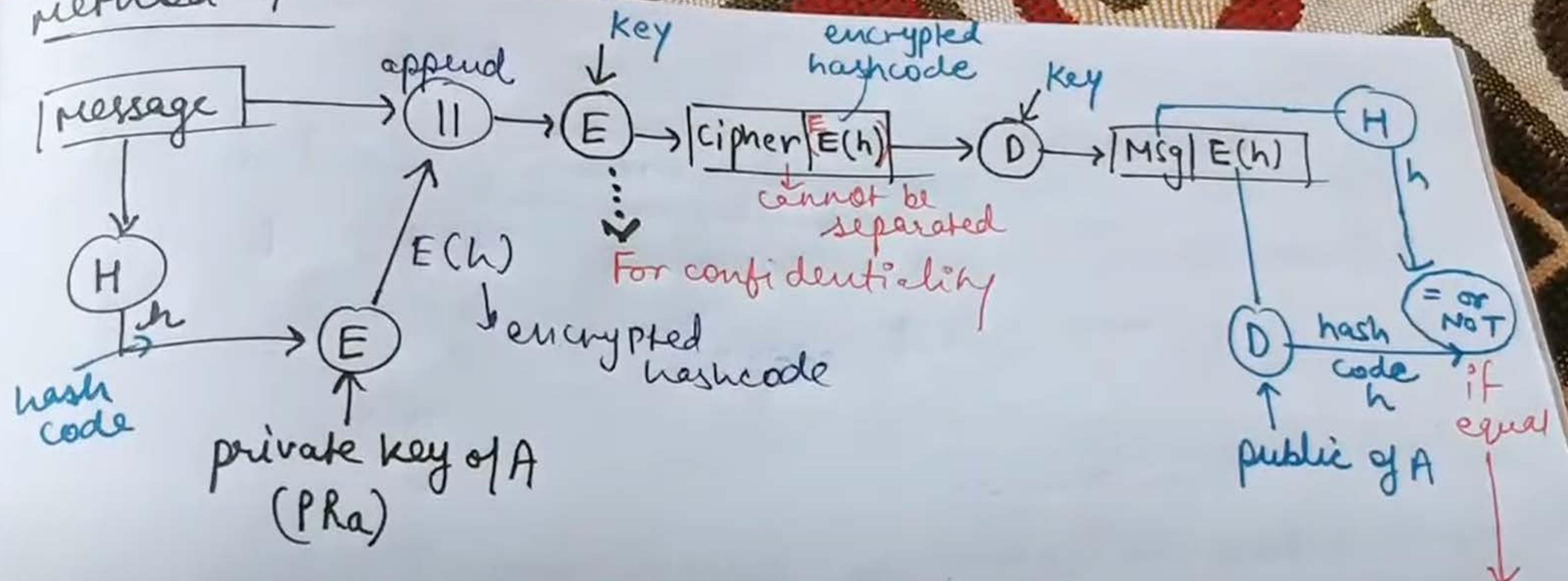
method 3



- no confidentiality
- only authentication
- processing time will be less as the msg is not encrypted.

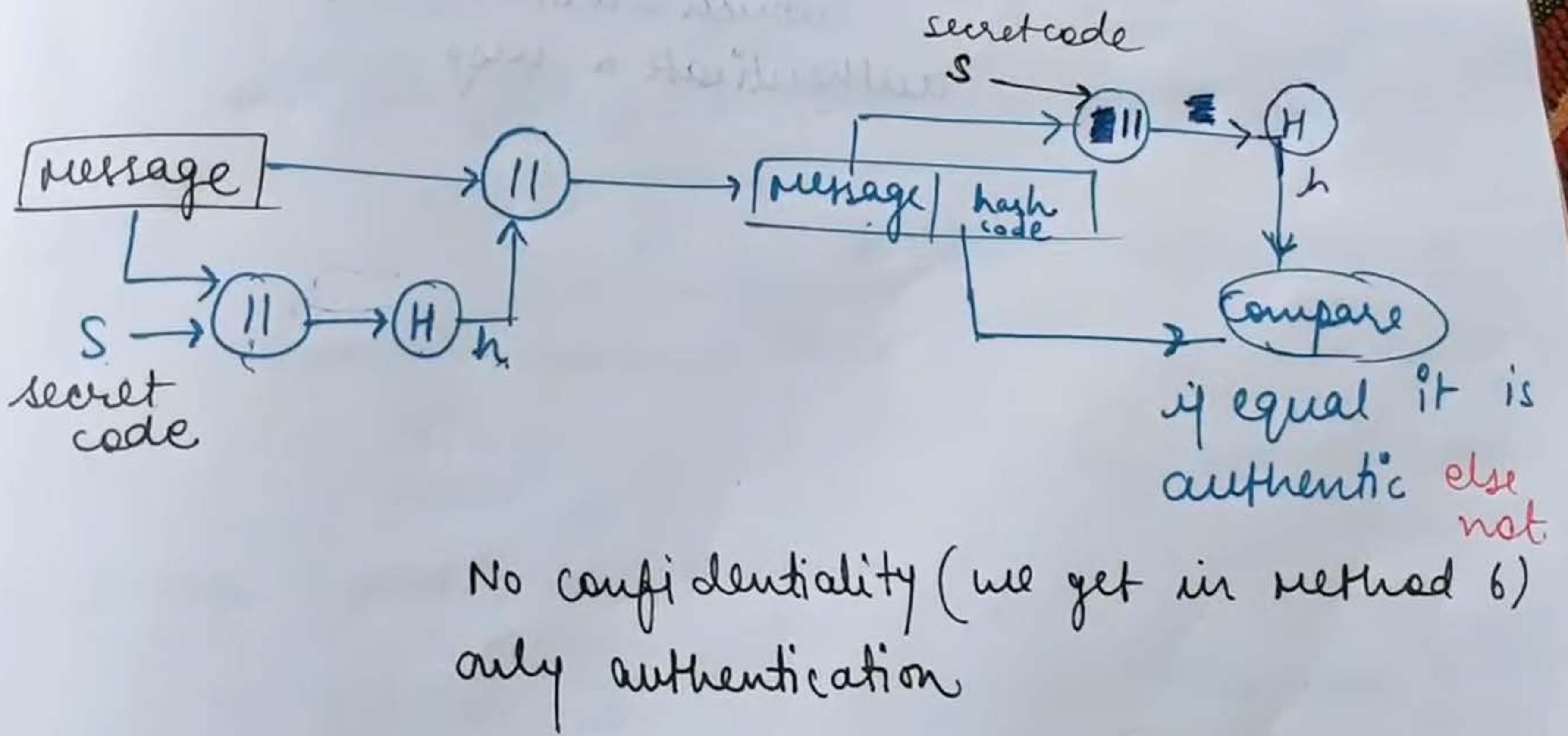
(Same concept as above but using Asymmetric key crypto)

method - 4



used when we need confidentiality + authentication
using Symmetric key

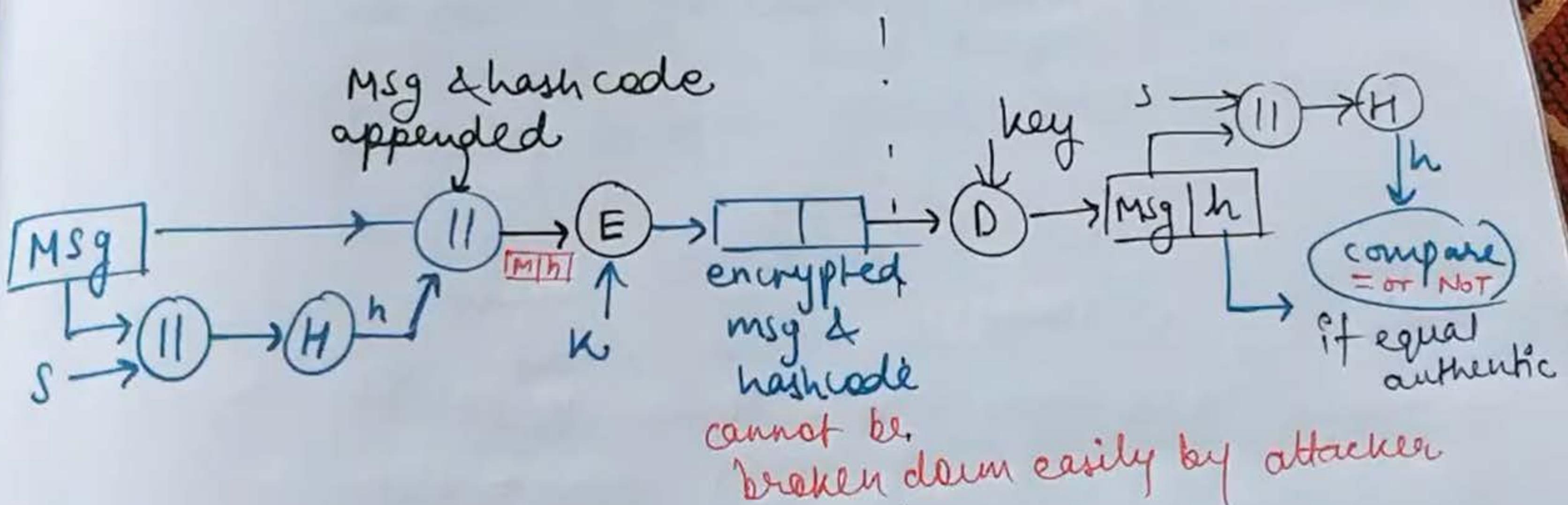
method - 5 Sender & Receiver will have a secret code 's' and it must be kept secret



method - 6

confidentiality can be added to the prev. approach by encrypting the entire msg.

Take the msg & append with the secret code 's'. Then apply Hash fn. It gives 'h'. Now append 'h' & msg. Now encrypt using key 'k'. we get encrypted(msg+h). Now, it will be sent to receiver side



In receiver side we will use decryption algo &
(msg+hashcode)

STEGANOGRAPHY

Basic idea → information hide / covered writing.

It is the practice of concealing messages / file / image (i.e., any type of information) within another file, message or image/video.

Note → Later, we will extract it at its destination.

Y TUO

It is derived from Greek words
steganos meaning covered or concealed.
& graphia which means writing.

IMP

* Steganography is different from cryptography but, using both together can improve security of the protected data/info. and prevent the detection of the secret communication.

In cryptography, we make the data unreadable (by encryption)

In steganography, we are hiding the existence of data.

CHAPTER 2

→ hiding the existence of
data.

Various forms of Steganography are :

- 1) Text Steganography
- 2) Audio "
- 3) Video "
- 4) Images " (hiding the data in the img. file)

∴ in short steganography can be used to
hide/conceal any type of digital content
(including text, img, video, audio).

Elliptic Curve Cryptography

ECC

- It is asymmetric / public key cryptosystem.
- It provides equal security with smaller key size (as ^{eg:} compared to RSA) as compared to non ECC algos.
ie small key size and high security
- It makes use of Elliptic curves.
- Elliptic curves are defined by some mathematical functions - cubic fm.
eg
$$y^2 = x^3 + ax + b$$
 // equation of degree 3

YASDE

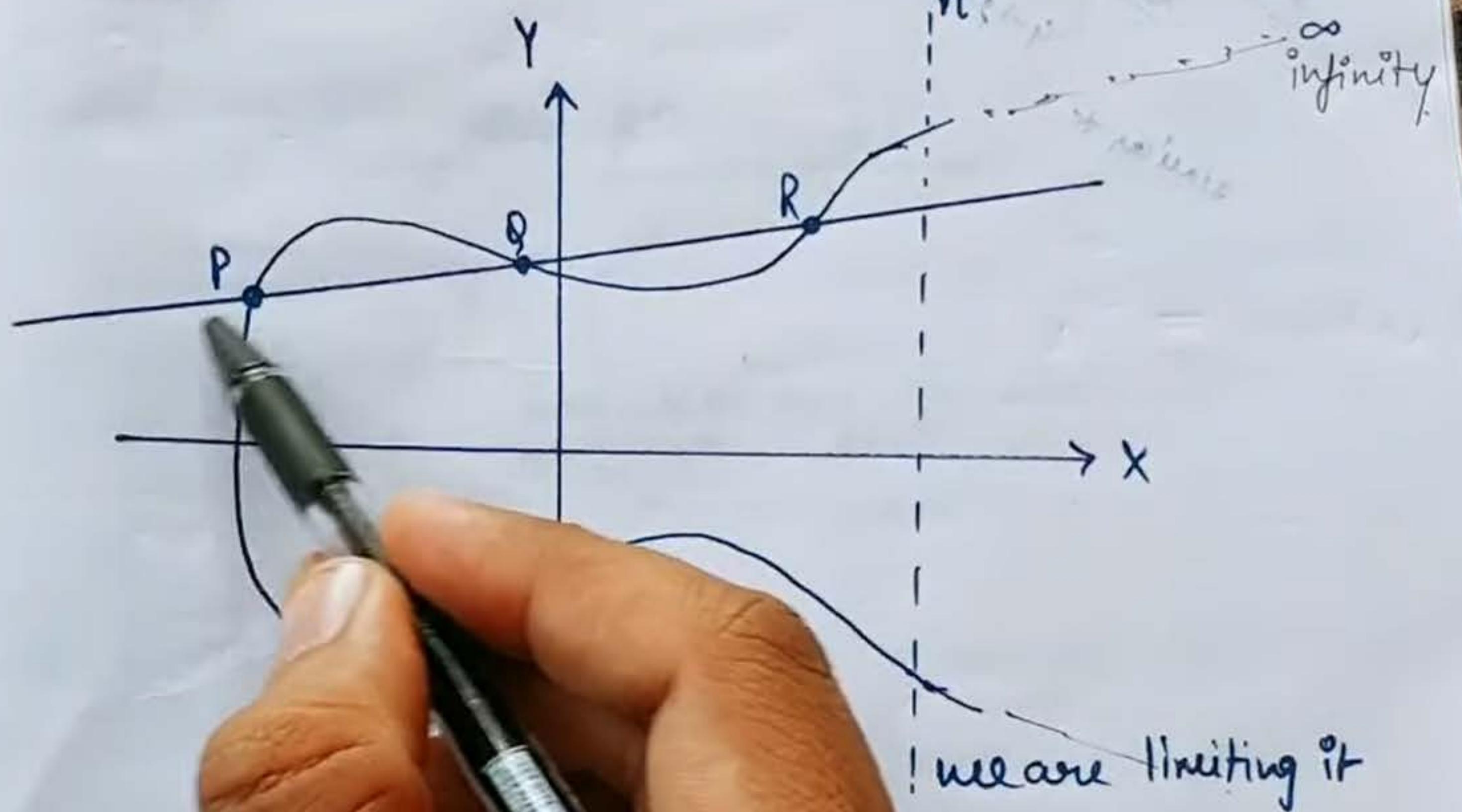
→ These curves are defined by some mathematical functions - cubic fun.

e.g.

$$y^2 = x^3 + ax + b$$

// equation of degree 3

DECR

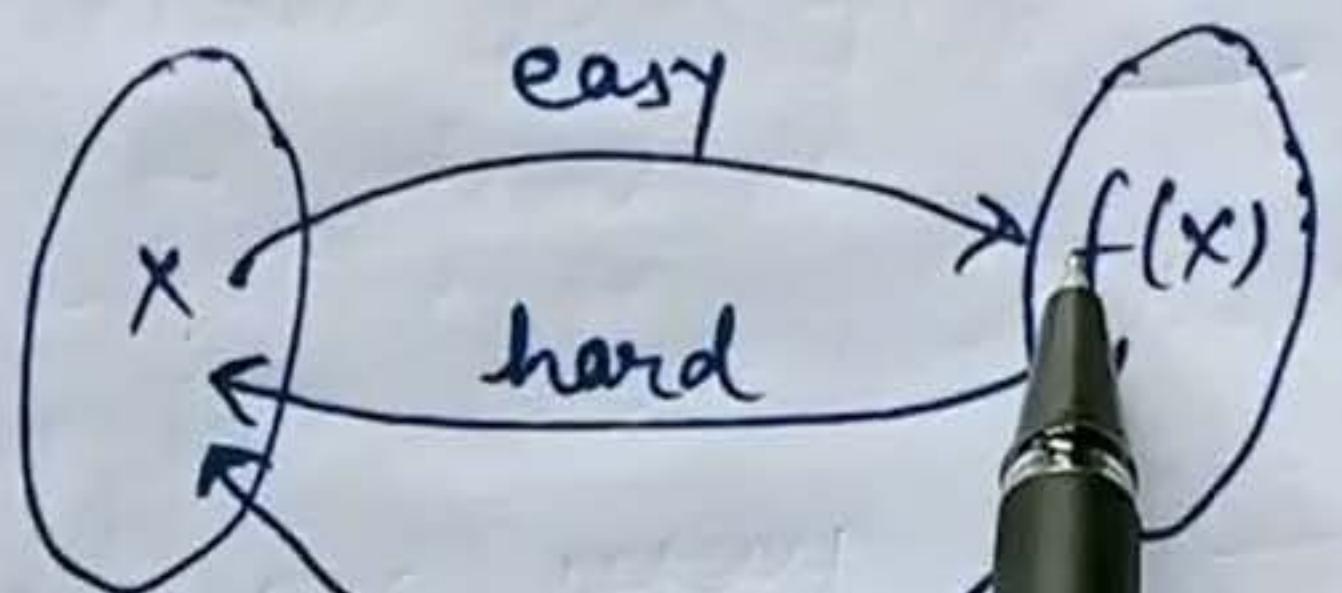


→ Symmetric

→ If we draw
3 points

touch a max of

A Trapdoor function is a fn that is easy to compute in one direction, yet difficult to compute in the opposite direction (finding its inverse) without special information, called the trapdoor.



easy if given "t" \rightarrow trapdoor value.

$A \rightarrow B$.

Refer wikipedia for details.

Let $E_p(a,b)$ be the elliptic curve.
 consider the equation $| Q = kP |$

where $Q, P \rightarrow$ points on curve and $k < n$.

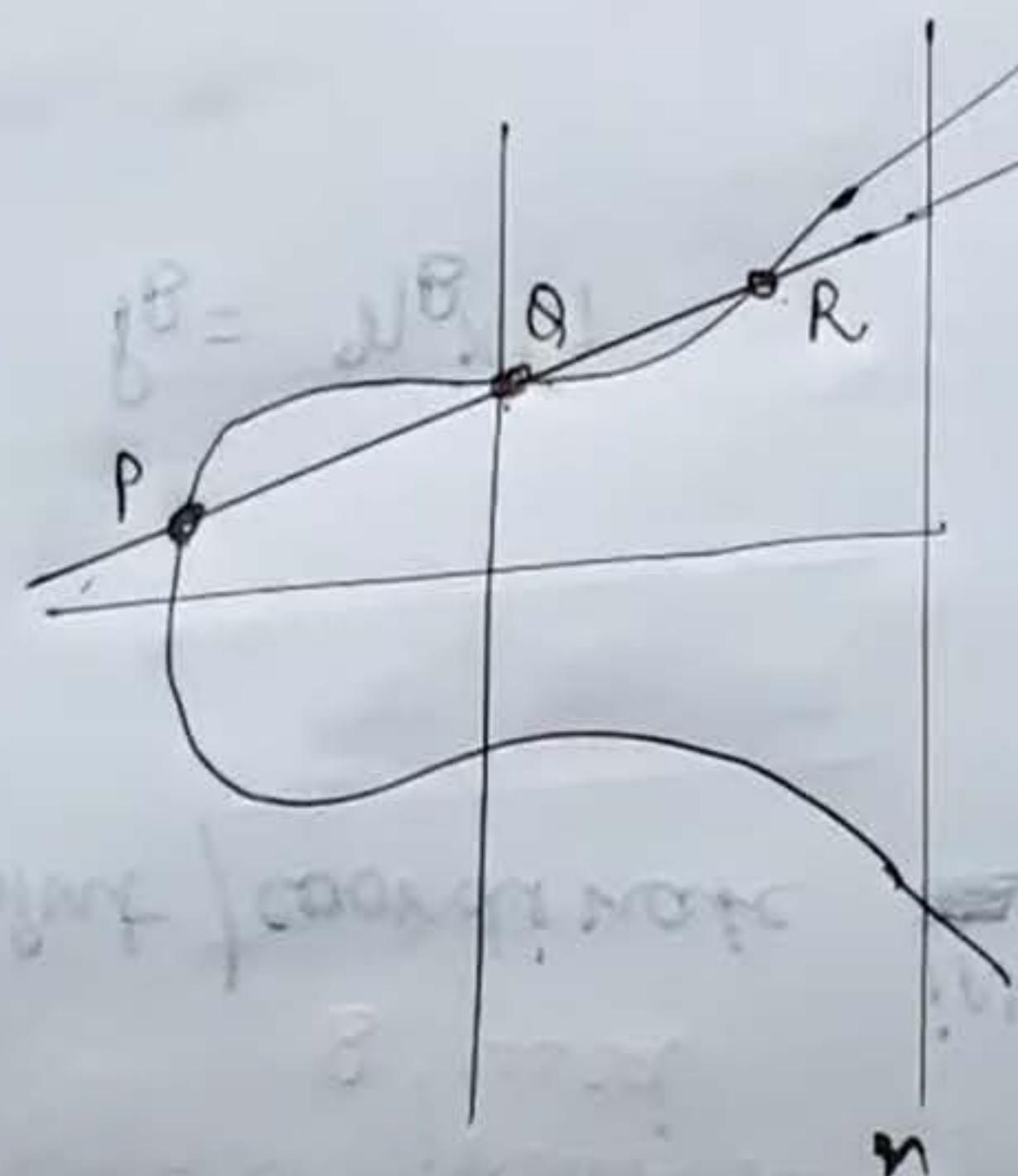
(If k and $P \rightarrow$ given, it should be easy to find Q)

but if we know Q and P , it should be extremely
difficult to find k .)

This is called the
discrete logarithm problem
 for elliptic curves

ie $f_n \rightarrow$ Trap door fn.

ie $A \rightarrow B$ is easy
 but $B \rightarrow A$
 is difficult.



Algo is somewhat similar to Set'

ECC - ALGORITHM

ECC - Key Exchange

Global Public Elements

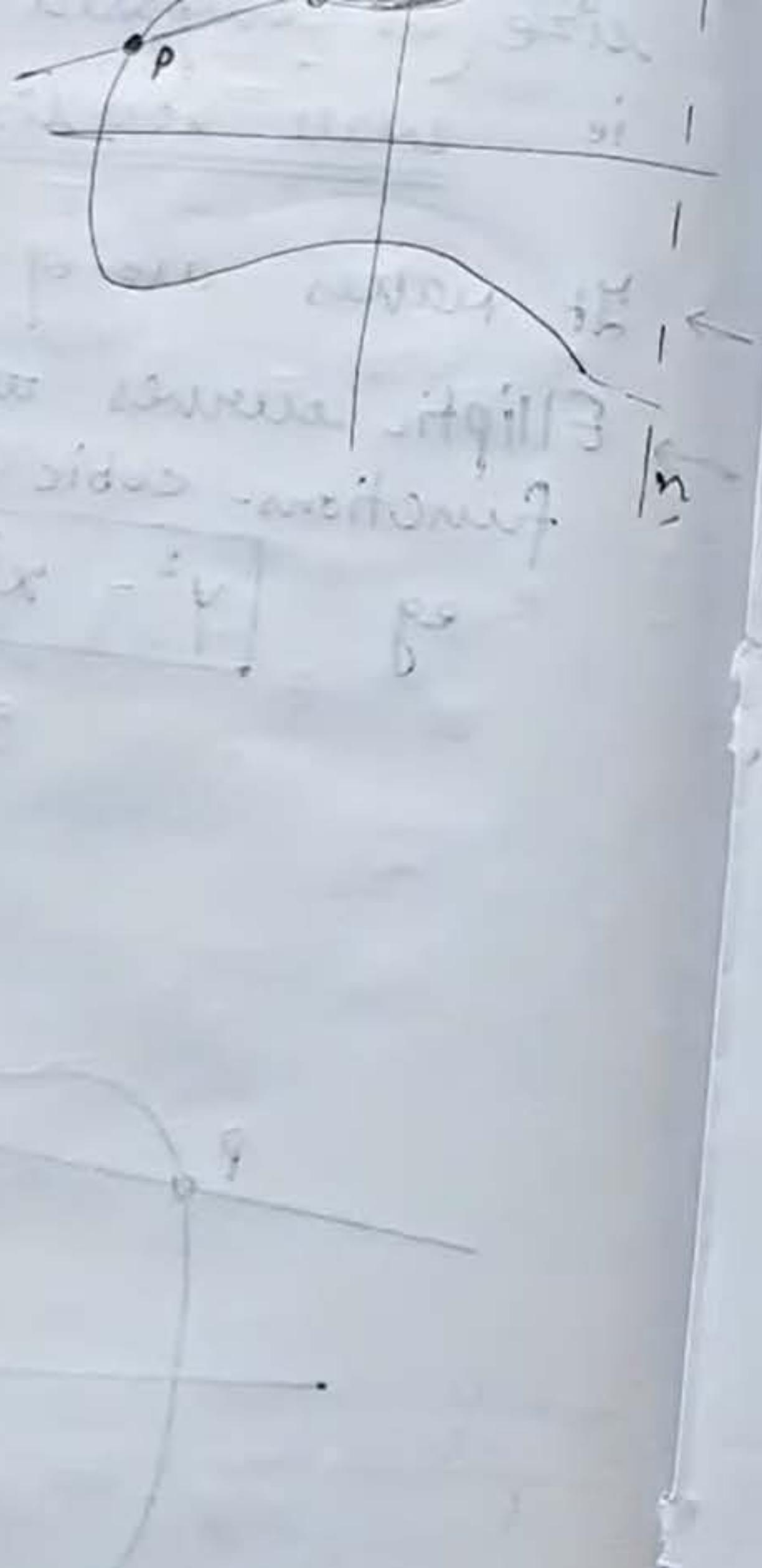
$E_q(a,b)$: elliptic curve with parameters a, b
and \boxed{q}

prime no. or an integer of the
form 2^m .

G_1 Point on the curve/elliptic curve whose
order is large value of n .

User key generation

private key n_A $n_A < n$



Elliptic curve with parameters a, b and q
prime no. or an integer of the form 2^m .

G_1 : Point on the curve/elliptic curve whose order is large value of n .

User A key generation

Select private key n_A
calculate public key P_A

$$n_A < n$$
$$P_A = n_A \times G_1$$

User B key generation

Select private key n_B
calculate public key P_B

$$n_B < n$$
$$P_B = n_B \times G_1$$

Calculation of secret

$$K = r$$

Calculation of

$C_m = \{ kG_1, P_m + kP_B \}$ This point will be sent to the receiver
for encryption public key of B used

DECRYPTION

for decryption, multiply 1st point in the pair with receiver's secret key
ie $kG_1 * n_B$ // for decryption private key of B used

Then subtract it from 2nd point / coordinate in
the pair

$$\text{ie } P_m + kP_B - (kG_1 * n_B)$$

but we know $P_B = n_B \times G_1$

$$= P_m + kP_B - kP_B$$

$$= P_m \quad (\text{original point}).$$

→ So receiver gets same point