THEORETICAL APPROACH TO ESTIMATE THE PROBABILITY OF STRATA CONVERGENCE

A THESIS SUBMITTED IN PARTIAL
FULFILLMENT OF THE REQUIREMENTS FOR
THE DEGREE OF

Bachelor of Engineering
In
Mining Engineering
By

RAKTIM GHOSH

ID: 111212008

Under the guidance of

DR. PRATIK DUTTA

Depart of Mining Engineering

Indian Institute of Engineering Science and Technology, Shibpur

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CERTIFICATE

This is to certify that the thesis entitled, "***Theoretical approach to estimate the probability of strata convergence***" submitted by Mr. Raktim Ghosh in partial fulfilment of the requirement for the award of Bachelor of Technology Degree in Mining Engineering at the Indian Institute of Engineering Science and Technology, Shibpur (Deemed University) is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any University/Institute for the award of any Degree or Diploma.

Prof Pratik Dutta
HOD Dept. of Mining Engg
IIEST,Shibpur

Date-

ACKNOWLEDGEMENT

The most pleasant point of presenting a thesis is the opportunity to thank those who have contributed to it. Unfortunately, the list of expressions of thank no matter how extensive is always incomplete and inadequate. Indeed this page of acknowledgment shall never be able to touch the horizon of generosity of those who tendered their help to me.

First and foremost, I would like to express my gratitude and indebtedness to Dr. Pratik Dutta for his kindness in allowing me to do work in the present topic and for his inspiring guidance, constructive criticism and valuable suggestion throughout this project work. I am sincerely thankful to him for his able guidance and pain taking effort in improving my understanding of this project.

An assemblage of this nature could never have been attempted without reference to and inspiration from the works of others whose details are mentioned in reference section. I acknowledge my indebtedness to all of them.

My sincere thanks to all my friends who have patiently extended all sorts of help for accomplishing this undertaking.

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CONTENTS

ABSTARCT

Estimation of strata **convergence** and the distribution of stresses around the excavated wall as well as the probability of displacement are of paramount importance. **Convergence** is defined as the gradual decrease in the interval between the two specified rock units or geological horizons as a result of thinning of intervening strata. The distribution of stresses with time resulting into simultaneous strata displacement is very difficult to predict. Convergence resulting from enlarging an excavation lowers the overburden and thereby decreases the gravitational potential energy. The mining condition changes with time after the initial exposure of roof and sides.

Various types of factor and their relative contribution and specific correlation are investigated to predict the convergence. It is hypothesized that entire rock continuum or rock mass ,however, as an isotropic , continuous and even elastic medium and geological disturbances (faults, folds, dykes etc) are eliminated and energy transmission of surrounding rock units are consistent and uniform. To predict convergence ,the rock mass has been divided into many small cubes of equal sizes and when one of the cube flows out ,a void is created and is filled by one of the other nine cubes with a definite probability in random patterns.

Two models are developed to estimate the probability of strata convergence. Two sections are taken from the entire rock continuum being hypothesized as geometrically similar. Also it has been hypothesized that entire rock continuum or rock mass ,however, as an isotropic , continuous and even elastic medium and geological disturbances (faults, folds, dykes etc) are eliminated and energy transmission of surrounding rock units are consistent and uniform. In the first model, a vertical section is taken and in the second model, a horizontal section is taken according to a particular crucial sequence.

Theoretical functional equations are predicted so as to construct a probability distribution function and it is established that how the predicted probability will vary according to the area of excavation. In the latter stage, numerical modelling is done to precisely simulate the strata convergence so as to recognize a prolific pattern behind the entire convergence mechanism. As it is a theoretical approach, geometric similarity is taken into account in order to minimize the initial complexity of the problem.

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

Convergence is defined as the gradual decrease in the interval between the two specified rock units or geological horizons as a result of thinning of intervening strata. Roof fall due to convergence is the major contributor towards fatal and serious accidents in underground mines. The mining conditions changes with time after the initial exposure of roofs and sides. Several parameters are thought to be correlated to predict the strata behaviour vis-a-vis convergence. Some examples include ,stress redistribution around the excavation free surface(Kontogianni And Sitros,2005),bodily displacement of the cavity, elastic unloading of the rock and the displacement of the mined rock in the gravity field(Walsh,1977).Convergence resulting from enlarging an excavation lowers the overburden and thereby decrease the gravitational potential energy.

In the present paper, various types of factor and their relative contribution and specific correlation is investigated to predict the convergence. It is hypothesized that entire rock continuum or rock mass ,however, as an isotropic , continuous and even elastic medium and geological disturbances (faults, folds, dykes etc) are eliminated and energy transmission of surrounding rock units are consistent and uniform. To predict convergence ,the rock mass has been divided into many small cubes of equal sizes and when one of the cube flows out ,a void is created and is filled by one of the other nine cubes with a definite probability in random patterns. For generating a random probability distribution function, several factors of convergence has been considered like as elasticity of rock mass , shape factor of every block , density of rock, position of each block with respect to other surrounding blocks and stresses . The key objective is to correlate those factors to get a prolific distribution function and has been hoped to extract an algorithm to define the movement of each and every surrounding blocks with respect to that void. The crucial observation is to give a separate ratings of each block after correlating several factors of convergence. Geometric similarity is explored with a view to minimizing the initial complexity of the problem and as it is a theoretical approach ,a definite ideal situation is considered over the entire analysis and a similarity characteristics value is to be determined .The relations among all those convergence factors and assessing a definite probability is the primary objective.

Analytical Models:

Analytical models are the mathematical models that have a closed form solution, i.e. solution to the equation used to describe changes in a system can be expressed as a mathematical analytic function.

In the present paper, rock continuum or rock mass has been hypothesized as an isotropic, continuous, and even elastic medium. Also the several geological disturbances (fault, folds, dykes etc)have been eliminated and energy transmissions are being takes as a consistent for analysing convergence.

- **Rock Continuum:**

A continuum is an idealised material whose physical properties – strength, density, stiffness, etc- vary continuously and smoothly from point to point. A rock continuum has been shown in this figure below.

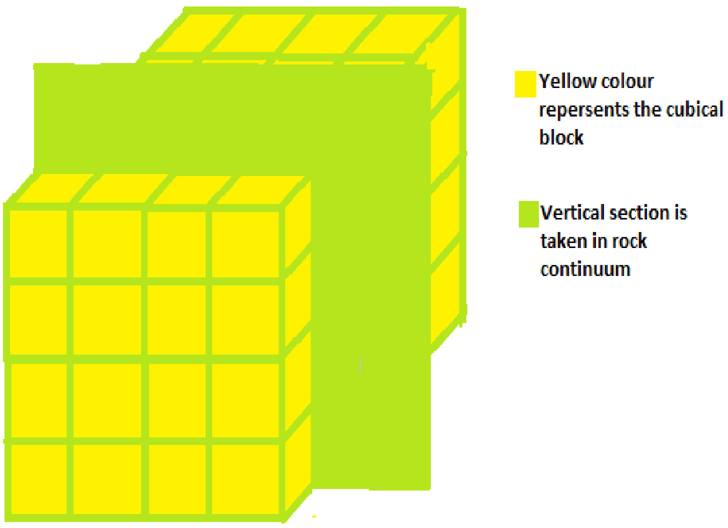

Fig 1: Rock continuum model

In this present figure, a rock continuum has been divided into smaller cubes of equal size and similar properties and characteristics. A vertical section is taken in order to analyse the prediction model of convergence. A probability distribution function is to be predicted and also the block distributional value has been given along with the similarity characteristic value.

- **1.1 Model 1:**

In fact in vertical section, similarity characteristic value will be changed slightly and it is happened due to the location of each block with respect to other surrounding blocks. The upper three rectangles

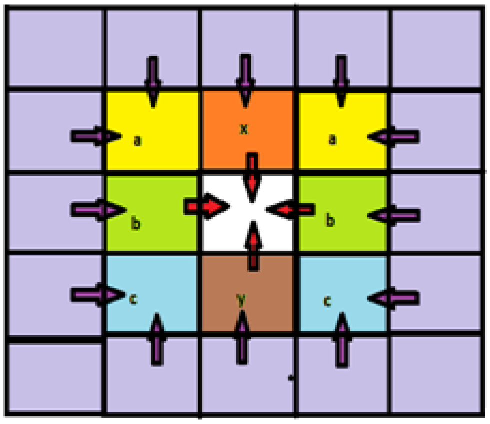

Fig 2 Probability of filling the void

(two yellow and one red) are in favour of gravity . Due to geometric similarity, yellow rectangles are having a definite value of **a** and the red one is having a value of **x**. Also it has been shown in the figure that two green and two sky coloured rectangles are having similar characteristics with respect to their location and geometry and the value is **b** and **c**. The brown rectangle is having a value of y .As the middle rectangle has been extracted out , probability would be zero . So the entire probability of filling the block is one .So the equation becomes as shown,-

$$\sum_{i=1}^{9} P = 2a + 2b + 2c + x + y = 1 \quad \text{..........................(1)}$$

a=probability of two yellow rectangles
b=probability of two green rectangles
c=probability of two sky rectangles
x=probability of one orange rectangle
y=probability of one brown rectangle

From the general consideration, highest value of rating would be given to the red rectangle as it is having a profound gravitational support .So as per the requirement of the project ,the general comparison of the theoretical value which is to be evaluated after generating a probability distribution function and the value estimated from software analysis by a comprehensive modelling shall be compared to get a desired result.

- **1.2 Model 2:**

In the second model, a horizontal section is taken from the entire rock continuum. The figure is shown below.

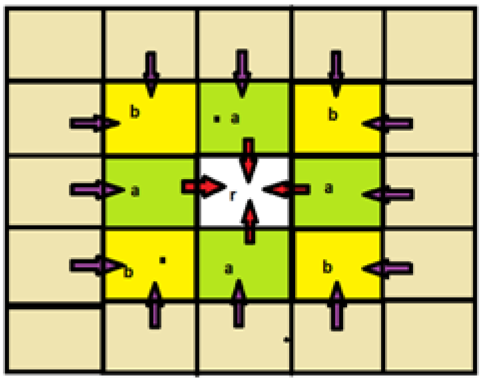

Fig 3 Probability of rectangles to fill the void

In fact , the similarity characteristic value is the probability of the four side rectangles , a. Other probabilities are probability of four rectangles , b, and that of the middle rectangle ,r. The summation of probabilities of the nine rectangles is equal to one,-

$$\sum_{i=1}^{9} P = 4a+4b+r=1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

a= probability of four green rectangles

b=probability of four yellow rectangles

r=probability of the middle rectangle

P=summation of probabilities of all block

When the inner rectangle is extracted out ,probability of r would be zero because it will not be having any influence over the entire convergence mechanism. So the equation is reduced to,-

$$4a+4b=1 \text{ as } r=0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

So the value of **a** and **b** is only the variable. The equation is **a + b = 1/4**. This figure is considered as the horizontal section with respect to the ground surface. As it is generally expected that the four green coloured rectangles are having the major contribution over the convergence .When the ratio of a and b is known and if it is possible to find out the value of a, then the value of b is obtained .The height and width of the rectangles or the shape factor is having a key component for stability in mining .There is an extreme possibility of having a stress distribution correlation among the yellow and green rectangles being influenced by the surrounding rectangles .As it has been decided that the energy transmission is uniform between the surrounding blocks ,the ration of a and b won't be effected and as other section of the entire rock continuum ,the probability distribution has been shown in the figure below.

1.3 Objective

Objective of the present work is to correlate the probable factors of strata convergence and to give separate ratings of each block after correlating those factors with a view to estimating the probability of convergence of each rectangle as specified in predefined models and to recognize a pattern in probability of convergence from numerical modelling by using software.

1.4 Methodology

The above objectives could only be reached if acted upon with a planned approach. The first step towards a goal always starts with knowing everything about it. Thus we began with literature review. As it is a theoretical approach, various functional equations and random functions have to be taken into account and the ratings of each block can be determined by a systematic approach and keeping one variable constant, it will be possible to predict a differential equation and with respect to that, a separate ratings can be given .The entire algorithm could be extracted out if specific objectives are achieved.

After determining the probable ratings of each block, the similar value could be imported to **phase 2** software to get a result for comparison.

In **Phase-2** software, after creating an appropriate excavation boundary and defining the material properties, it will be possible to analyse and interpret the result thereof. Finite element meshing is a simple two-step process. First you must discretize the boundaries, and then the Mesh can be generated. The deformed shape of the excavation boundaries can be graphically illustrated by the use of these options. The deformation is magnified by a scale factor, which can be user-defined in the Display Options dialog.

2.1 Probable Factors of Convergence

To predict convergence, several factors having a definite contribution on the mechanism have been taken into consideration. To minimize the initial complexity of the problem, contribution of energy transformation between surrounding rocks have been eliminated. Also the geological disturbances have not been taken as it is hypothesized as an ideal situation:

- Weight of individual block
- Density and porosity of rock
- In situ stresses
- Depth of overburden
- Shape factor
- Position of each block with respect to others
- Elasticity

2.1.1 Weight of Individual Block

From the model I&II, the weight of each block is having a major contribution over the convergence. With the increase in the weight, the greater rating shall be provided to each and every block for getting a prolific distribution function. Proper investigation of a predictable underlying functions vesting in the mechanism of convergence is required as well as the correlation of weight with the function is to be done.

2.1.2 Density and Porosity of Rock

Density is defined as the mass per volume of a substance and also the porosity is the non solid or pore-volume fraction of a substance. In general, density increases and porosity decrease with depth. There is ample support for the claim because differential pressures increase with depth. As pressure increases, grains will shift and rotate to more dense packing. More force will be imposed on grain contacts. As a result, density and porosity is having a proper contribution on the mechanism of convergence.

2.1.3 In Situ Stress

One of the most important boundary conditions for the analysis of boundary excavations is in-situ stress. In a stressed rock mass, the sequencing and advancement of a face results in the disturbance and redistribution of the primary in situ stress field which will have an adverse effect on convergence. Also the redistribution around the proximity of the boundary is having a functional effect on convergence. The stress state at a given point in a rock mass is generally presented in terms of magnitude and orientation of the principal stresses.

2.1.4 Depth of Overburden

With the increase of the depth of the overburden, the vertical stress around the excavation boundary will increase which will further increase the probability of the convergence. So the depth is having an adverse effect. With the increase in overburden depth , the vertical stress will increase and which will further increase the rate of convergence. For that reason, an encapsulating equation following a specific probability distribution function has to be considered.

2.1.5 Shape Factor

The shape of the excavation boundary may be circular or rectangular or various different forms. Each shape is having a unique contribution over convergence because in cylindrical opening, a unique closed form solution can be found out as well as for elliptical opening. So the shape factor can be defined as the ratio of a area of a particular predesigned excavation boundary with respect to a circular boundary.

2.1.6 Position of Each Block with Respect to Others

In model I&II, position of blocks or the position of a rectangular portion with respect to other rectangular portions is of paramount importance to decide whether the higher rating is being achieved by a particular rectangle or not. The reason behind achieving a higher rating is to be the positional constraint. The figure shown below,-

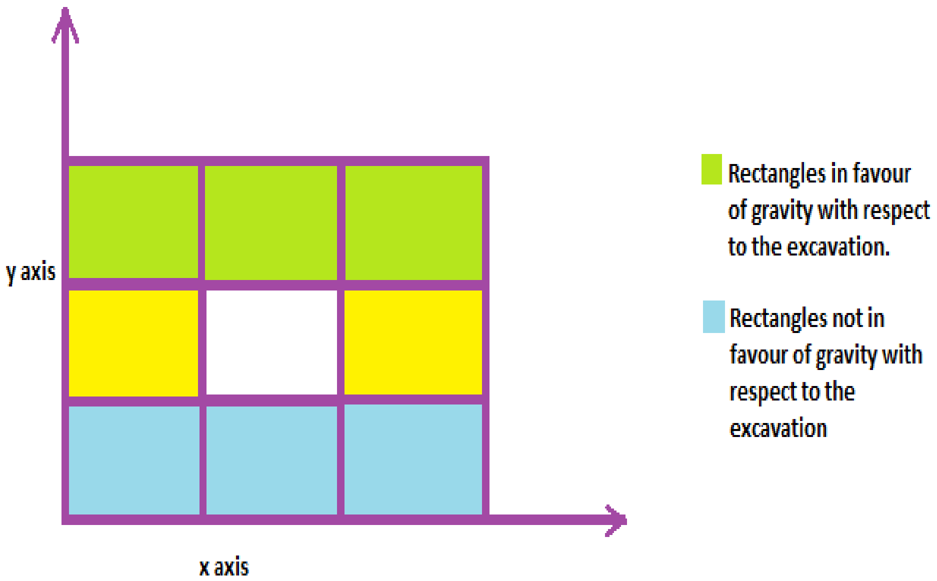

Fig4: Position of rectangles with respect to the excavation

In case of green rectangles having in favour of gravity with respect to the excavated rectangle with higher ratings in respect of other rectangles. Also the sky rectangles are not in favour of gravity with respect to the excavated rectangle, i.e., it gains a lower ratings.

2.1.7 Elasticity

The term convergence is used to describe the elastic deformation, which can be estimated theoretically or by numerical coding. Generally the elastic deformation is instantaneous, while the elastic displacements have a strong time dependent component being initiated by the change in the rock mass by means of advancement. As it refers to the rock's ability to deform in a non-permanent way, meaning that when stress load is removed from the rock it will recover its original form.

CHAPTER 2

LITERATURE REVIEW

Estimation of convergence is a very important issue for engineering construction. A systematic convergence measurement is usually implemented to adjust the design during the whole construction, and consequently deadly hazards can be prevented .However if the trends of the convergence can be known or predicted at early stages, the design can gain both accuracy and time. It is noticed that an accurate prediction of deformation in the design stage is very difficult even though computation methods are properly used. This is due to, for instance, complex geological conditions, unpredictability of rock material behaviours and absence of knowledge about real ground conditions resulting obviously in a kind of uncertain prediction (Lee and Akutagawa, 2009).

In the literature, several simulation models with associated software are nowadays available as a tool for investigating the stability of underground excavations (tunnelling). With the ability to take all relevant factors into account, such as ground heterogeneity, non-linear behaviour of soils, soil structure interaction and construction methods, numerical analyses proved to be the most accurate and realistic approach for the safety control of real projects.

3.1 Finite Element Analysis:

Finite element analysis (FEA) involves solution of engineering problem using computers. Engineering structures that have complex geometry and loads, are either very difficult to analyse or have no theoretical solution. However, in FEA, a structure of this type can be analysed. Commercial FEA programs, written so that a user can solve complex engineering problems without knowing the governing equations or the mathematics: the user is required only to know about the geometry of the structure and its boundary conditions.

In order to become a skilful FEA user, thorough understanding of techniques for modelling a structure, the boundary conditions and, the limitations of the procedure, are very crucial. Complex structures in engineering are very difficult to analyse by classical theory. In this technique the structure is divided into very small but finite elements. Individual behaviour of this element is known and based on this knowledge; behaviour of the entire structure is determined.

FEA solution of engineering problems such as finding stresses and deflections in a structure requires three steps:

- Pre-process or modelling the structure
- Analysis
- Post-processing

Brief description of each these steps are as follows:

Step1:Pre-process or modelling the structure:

Using a CAD that either comes with the FEA software or provided by other source, the structure is modelled. The final FEA model consists of several elements that collectively represent the entire structure. The segments not only represent the entire structure but also simulate its mechanical properties and behaviours. The complex geometrical regions in a structure require significant number of elements in order to gain a prolific accuracy. The selection of proper elements requires prior experience with FEA, knowledge of structure's behaviour, available elements in the software and their characteristics. The elements are joined at the nodes and common points.

In pre-processor phase, along with the geometry of the structure, the constraints, loads and mechanical properties of the structure are defined. The structure represented by the nodes and elements is called "mesh"

Step2: Analysis:

In this step, the geometry, constraints, mechanical properties and loads are applied to generate matrix equations for each element, which are then encapsulated in a matrix equation of a structure. The form of the matrix equations are as follows:

$$\{\boldsymbol{F}\} = [\boldsymbol{K}]\{\boldsymbol{U}\}$$

Where

{F} = External force matrix.

[K] = Global stiffness matrix

{U} = Displacement matrix

The equation is then solved for deflection. Using the values of strain, stress etc, and several reactions are calculated. All the results are then stored. The stored results can be used to create graphic plots and charts in the post analysis.

Step3: Post processing:

This is the last step in a finite element analysis. Results obtained in step2 are usually in form of raw data and difficult to interpret. In post analysis, a CAD program is utilized to manipulate the data for generating deflected shape of the structure, creating stress plots, animation etc. A graphical representation of the results is very useful in understanding behaviour of the structure.

3.1.1 How FEA works – within software

The following steps can summarize FEA procedure that works inside software:

- Using users input, the predefined structure is graphically divided into small elements (segments or regions) so that each and every element's mechanical behaviour can be defined by a set of differential equations.
- The differential equations are then converted into algebraic equation and then into matrix equations, suitable for a computer-aided solution.
- The element equations are combined and a global structural equation is obtained.
- Appropriate load and boundary conditions, supplied by user, and are incorporated into the structural matrix.
- The structural matrix is solved and deflections are calculated at each node.
- A node can be shared by several elements and the deflections at the shared node represent deflection of the sharing elements at the location of the node.
- Deflection at any other point in the element is calculated by interpolation of all the node points in the element.
- An element can have a linear or higher order interpolation function.

The individual element matrix equations are assembled into a combined structure equation of the form {F} = [k]{u}.

For static finite element analysis, the equation characterising equilibrium can be written in the following matrix form:

$$\mathbf{K}\Delta\mathbf{U} = \mathbf{P} - \mathbf{F}$$

where **P** represents the vector of applied loads, **F** the vector of internal forces, and Δ**U** the vector of current nodal displacements. In non-linear analysis the load **P** is applied in a series of load steps **P(1) , P(2) , P(3)** , …etc.

3.2 Phase 2:

Phase 2 is an extremely versatile 2D finite element stress analysis program for designing underground and surface excavations and their support systems.

In Phase-2 software, after creating an appropriate excavation boundary and defining the material properties, it would be possible to analyse and interpret the result thereof. Finite element meshing is a simple two-step process. First you must discretize the boundaries, and then the Mesh can be generated. The deformed shape of the excavation boundaries can be graphically illustrated by the use of these options. The deformation is magnified by a scale factor, which can be user-defined in the Display Options dialog.

Also a proper **modelling** can be done by using this versatile software. In **modelling**, the crucial features include:-

- Interactive geometry entry
- Intuitive workflow tabs
- Boundaries-external, material, excavation, stage, joint ,piezo, structural interface
- Grid/vertex/object snapping
- Sequential staging of excavation and support(up to 300 stage)
- Plain strain or axisymmetric analysis
- One-click material assignment
- Import/export in DXF format
- Unlimited undo/redo
- Right click editing shortcut
- Tunnel Wizard

Finite element meshing is a crucial step which needs to be done to analyse the model. In elements & meshing, the key features include the triangular and quadrilateral finite elements with 3or 6 node triangles and 4 or 8 node quadrilaterals. Mesh can be generated by clicking once at a time. Mapped meshing and custom meshing is very handy for users. Quality of the mesh can be checked and controlled as per requirement. Boundary conditions can be easily applied.

Prior to analysing the model, properties of the material shall be defined which includes elastic or non-linear with the strength criteria of Mohr-Coulomb, Generalized Hoek-Brown, Cam-Clay, Modified Cam-Clay etc.

After analysing the results of a particular model with definite material properties as well as specific boundary conditions, interpretation of the result is very important because improper interpretation will fetch negative results. The data interpretation in Phase 2 includes several key features like-

- View stress, displacement, strength factor contours
- Effective stress, pore pressure contours
- Contour user-defined data
- Stress/failure trajectories, deformation vectors
- Display deformations to user defined scale
- Query and graph material, support, joint data
- Export to Excel
- Show values directly on model
- Highlight yielded material, support, joint elements
- And iso-contours

3.3 Discreet Element Modelling:

Another dominant approach to predict convergence is discreet element modelling **(DEM).**The pivotal concept of DEM is that domain of interest is treated as an assemblage of deformable blocks or bodies or particles and contacts among them need to be identified and updated during the entire deformation process and represented by proper constitutive model. The constitutive model is very important to be considered.

During the time of applying distinct element code 3DEC in numerical modelling based on explicit time marching finite difference method, several boundary conditions are to be taken into account. After several interpretations, it would be possible to analyse the convergence. The time requirement can be reduced with hybrid continuum-discontinuum modelling.

3.3.1 The developed software for DEM:

The software was developed using C++ and DirectX technology for three-dimensional rendering. The environment is 3D and the computations are performed in real time. The software development was focused on pre and post-processors. The solver used was developed by AGEIA, based on NovoDEX, a physics engine developed NovoDEX AG, spinoff of ETH Zurich. The solver uses the graphics processing unit (GPU) to accelerate the computations required in contacts of DEM.

3.3.2 Computation

The computations involved in a DEM approach is quite trivial, alternating two numerical steps (Malone and Xu, 2008)- the second law of Newton and a force displacement law in the contacts. The second law of Newton handles the particle movements under applied forces, while the force displacement law is used to compute the contact forces out of virtual displacements.

3.4 ANFIS:

Various types of prediction models have been developed for the assessment of excavation damaged zone (**EDZ**).The adaptive network-based fuzzy inference system (**ANFIS)** with modulus of deformation as input was used to build a prediction model for the assessment of **EDZ**. An adaptive neural network is a network structure consisting of several nodes connected through directional links. In every nodes, a distinct node function is defined by assigning every adjustable parameters .After initializing fuzzy inference system, neural network algorithms can be used to determine the unknown parameters. Due to the optimization procedure the system is called adaptive.

3.4.1 Analysis with FIS:

For the analysis of a fuzzy system whose inputs and outputs are described by linguistic variables, the following steps have to be carried out:

- **Fuzzification:**

The linguistic variables of the fuzzy rules are expressed in the form of fuzzy sets where these Variables are defined in terms of their degree of associated membership functions. These methods of calculation the degree of belongingness of the crisp input in the fuzzy set is called the fuzzification. The membership functions may be triangular, trapezoidal, and Gaussian or bell shaped. As the information about the degree of the membership is used for further processing, considerable amount of information may be lost during the course of fuzzification. This is because the procedure can be seen as a nonlinear transformation of the inputs. For example in the case of triangular or trapezoidal membership functions information is lost in the regions of membership functions where the slope is zero, as at these points the membership functions are not differentiable. Therefore fuzzy systems having triangular or trapezoidal membership function can encounter problems of learning from data. Smoother membership functions like Gaussian or bell function may be used to overcome this difficulty.

- **Aggregation:**

After the degree of each linguistic statement is evaluated, they are combined by logical operators such as AND and OR. The conjunction of these linguistic statements is carried out by logical t-norm and the t-conorm operator to a large number of linguistic statements. Max and Min operators are used for classification task. For the purpose of approximation and identification the product and algebraic product operators are better suited due to their smoothness and differentiability. Similarly the bounded sum and difference operators offer several advantages to some neuro-fuzzy learning schemes.

- **Activation:**

Here the degree of rule fulfilment is used to calculate the output activations of the rules.

- **Accumulation:**

In this step the output activations of all the rules are joined together to give rise to the fuzzy output of the system.

- **Defuzzification:**

If a crisp value of the system is required, the final fuzzy output has to be defuzzified. This can be done by different methods like centre of gravity, bisector of area, mean of maximum (mom), smallest (absolute) of maximum (som) and largest (absolute) of maximum (lom).

3.5 Artificial Neural Network:

There has been an upsurge in interest in Artificial Neural Networks over the last five years. An artificial neural network is a computational paradigm that differs substantially from those based on the standard von Neumann architecture. ANN's generally learn from experience rather than being explicitly programmed with rules like in conventional artificial intelligence. It is considered as a form of artificial intelligence attempting to imitate the proper functions of brain with the purpose of recognizing the relationship among the data presented to them. They consist of densely interconnected simple processing referred as neurons that can perform large parallel computations. In each neuron n input data are processed and a single output is obtained as below:

$$\mathbf{Y=f}\ (\textstyle\sum_{i=1}^{n} \boldsymbol{w\,x + b})$$

Here x is n*1 input vector; b and y 1*1 bias and output vectors, respectively; w is 1*n weight matrix and f is 1*1 vectors representing the activation function. ANN models can allow accurate estimation of convergence because the main influential parameters can be taken into considerations by learning from available field data.

3.5.1 Advantages of ANN:

There are several advantages of ANN and also successful applications can be found in certain well-constrained environments. These include-

- Massive parallelism
- Distributed representation and computation
- Learning ability
- Generalization ability
- Adaptivity
- Inherent contextual information processing
- Fault tolerance and
- Low energy consumption

3.6 Back Analysis:

Back analysis is able to forecast controlling parameters of system through analysing its output behaviour. Back analysis problems may be solved in two different ways: inverse and direct. In inverse method, mathematical formulation is just opposite the typical analysis; however, the direct method is based on optimization in which trial values of unknowns are corrected in a way that the difference between values measured and calculated is minimised. This method can be used for non-linear relations as well. However, the mentioned method needs a lot of time to carry out repetitive calculations.

3.7 Multivariate Adaptive regression Splines:

MARS was introduced by Friedman (1991). It is a nonlinear and nonparametric statistical method that simulates the non-linear responses between the inputs and the output of a system by the means of piecewise linear segments called splines with differing gradients. The segments are delimited by knots which mark subdivision between two data regions in such a way piecewise curves are obtained. These piecewise curves are referred to as basis functions. In addition, the functional relationship between input variables and the output is not specifically required. This allows for greater flexibility, bends, thresholds, and several kinds of basis functions in modelling.

CHAPTER 3

THEORETICAL PREDICTION OF FUNCTIONS

The investigation of underlying function of convergence is of paramount importance. Hence a theoretical approach of developing a functional equation from model I&II has been done. So an investigation of a function to interpret the theoretical study area is the prominent part of the entire area of research. In the figure shown below;-

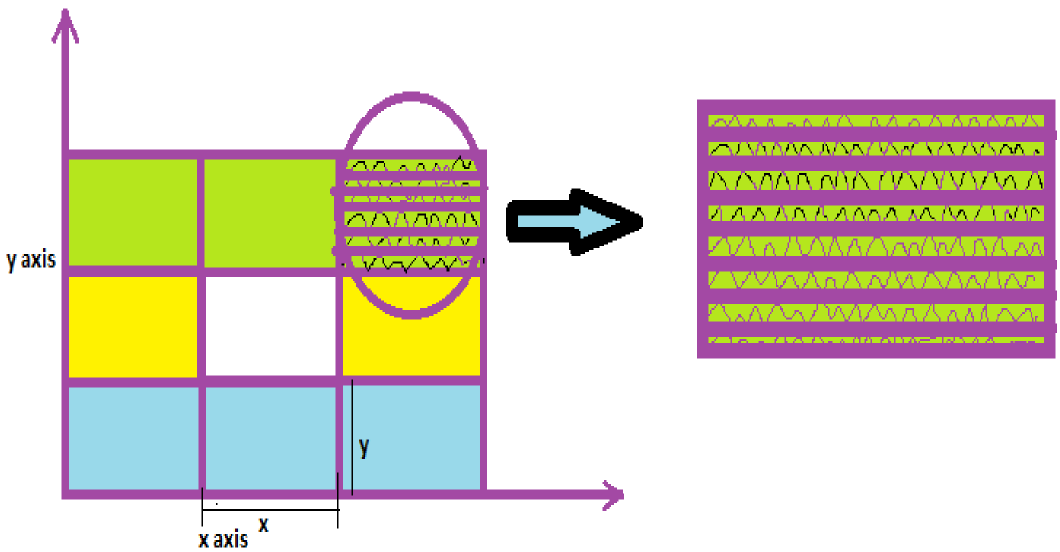

Fig 5 Prediction of probable function by dividing every rectangle into triangles of similar height

In this figure, a rectangle has been selected for the analysis. As it is aforementioned that every rectangle is geometrically similar, the length is considered here is **x** and the breath of every rectangle is **y.** So naturally the area of the rectangle will be **xy.** Now let us assume that **y** is a function of **x** and if we replace y as **bx** , the area of each rectangle will be $\mathbf{bx^2}$**.**

Length of rectangle = **x**

Breath of rectangle = **y**

Area of rectangle = **xy**

Assuming **y** as a function of x and **y = bx**

Area of rectangle = $\mathbf{bx^2}$

Now weight of each rectangle will be $\alpha b x^2$ if α is the density of the rectangular block. Now there is no doubt about the fact that the αb is a constant function. So it is concluded that weight of each block is dependent on the square of the value of **x.**

Now in figure 5, a rectangular block is selected and it is divided into smaller rectangle keeping the height constant along the breath. Also each rectangle has been divided into triangle of finite area with a constant height as shown in the figure.

Now the area of a triangle is $\frac{1}{2}\, \delta x_i \times \delta y_i$

Now in this particular rectangle, the smaller triangles are having equal height of **m** and the base of the triangle is having a series of values $x_1, x_2, x_3 \ldots\ldots x_n$. Here **n** is very large.

Now all the values are being chosen in such a way that after plotting it into a graph, the values are following a function which is strictly decreasing as shown below;-

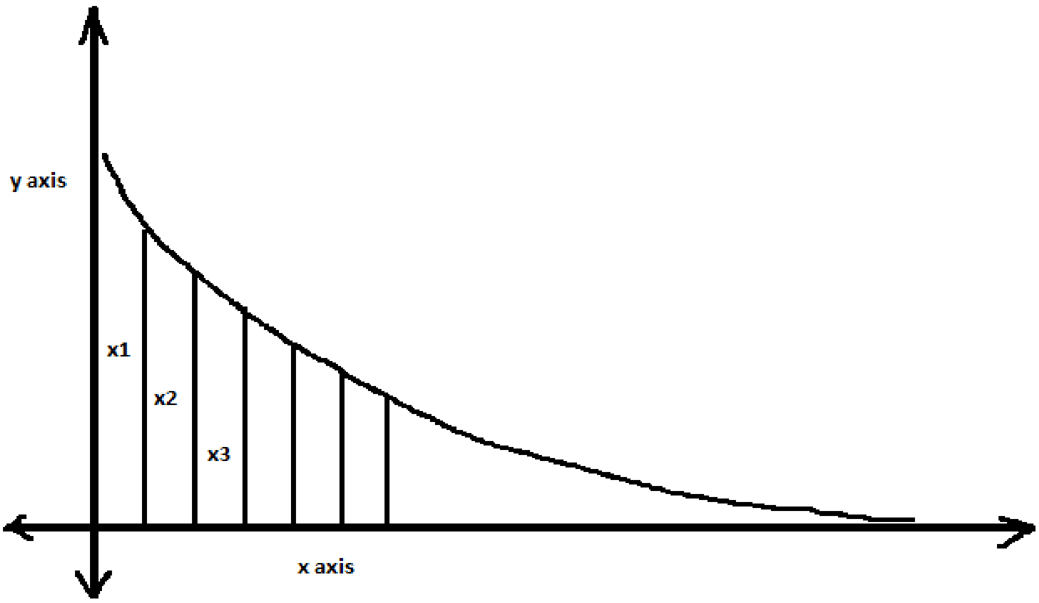

Fig6: Base of triangles from increasing to decreasing order.

Now in the above figure, the base of triangles have been plotted in a very crucial fashion as to predict the underlying function. So the graph shown in figure is a strictly decreasing function. As it is shown on the graph, it is a negative exponential function, it can be taken as a function of $f(x) = e^{-\Psi x}g(x)$. Where;-

- **f(x)=**exponential function in figure 6
- **Ψ** is the constant
- **g(x)** is an arbitrary function chosen to fit into the graph

If the value of the functions are taken from increasing to decreasing order, all other functions will be looking like as follows;-

$f_1(x) = e^{-\Psi 1x}g_1(x)$.......................1

$f_2(x) = e^{-\Psi 2x}g_2(x)$..................................2

$f_3(x) = e^{-\Psi 3x}g_3(x)$..3

....

....

....

$f_n(x) = e^{-\Psi nx}g_n(x)$..n

Here **n** is a very large number and after combining them all, an encapsulated equation has been developed.

$$\sum_{i=1}^{n} fi(x) = \sum_{i=1}^{n} exp(-\Psi ixgi(x))$$

This distribution function is very important to explore the internal mechanism of convergence. With the proper justification of the function $g_i(x)$, several valuable function equations related to the rectangular block can be derived and an exponential distribution function can be predicted.

4.1 Investigation of Functional Equation:

A functional equation is an equation of unknown function. In this paper, a prediction has been made over model I&II with a view to extracting a functional equation. The solution of the predicted equation has been made classically. Prior to solution, several assumptions have been made specifically for minimizing the initial complexity of the problem. Let us define the following dependence with the help of a diagram as shown below;-

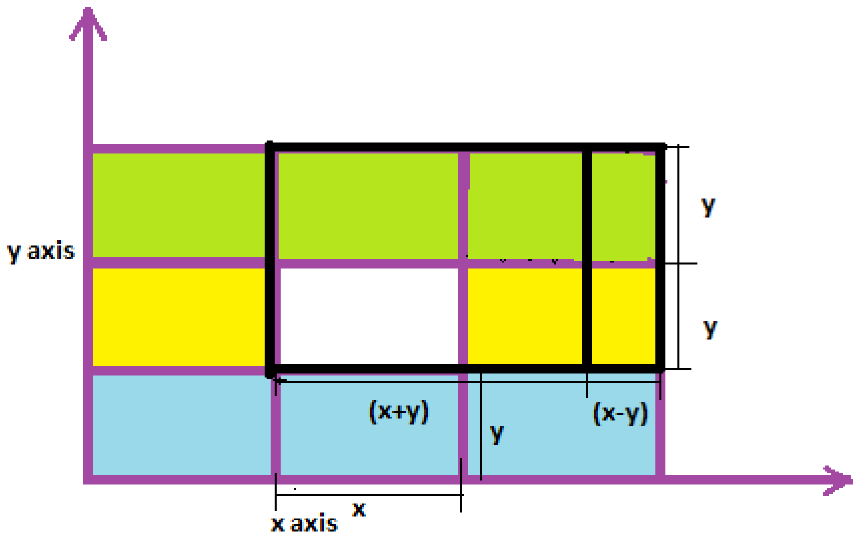

Fig7: Prediction of a functional equation considering four rectangles

In this figure, four rectangles have been chosen for predicting the functional equation. The four rectangles have been bordered by black colour as shown in figure. A vertical black line has been chosen inside the four rectangles in such a way as to make a distance of $x + y$ and $x - y$ as shown in the figure.

- Suppose a function of $f(x + y)$ on the distance of $(x + y)$ and another function of $g(x - y)$ on the distance of $(x - y)$ is following the convergence mechanism over the entire distance of 2y.
- In another way if we consider that a function of $h(x)$ is following the convergence mechanism over the distance of x and another function of $h(y)$ is following the same over the distance of y.

If we combine logically the above statements, the following functions ***f(x + y), g(x – y), h(x)*** and ***h(y)*** shall be connected to form a functional equation. The functional equation will be as follows;-

$$f(x + y) + g(x - y) = 2h(x) + 2\,h(y) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$$

4.2 Solution of investigated functional equation:

The functional equation as above mentioned can be solved logically if we are able to express **f** and **g** in terms of **h**. The function equation is written as follows;-

$$f(x + y) + g(x - y) = 2h(x) + 2\,h(y) \qquad\qquad f, g, h : \mathbf{R} \rightarrow \mathbf{R}$$

First if we take **y = x** and **g(0) = a ,** we get the following equation $f(2x) = 4h(x) - a$ and furthermore putting $y = 0$ leads to an equation of $f(x) + g(x) = 2h(x) + 2h(0)$ and after rearranging the initial equation we get $g(x) = 2h(x) + 2b - 4h(\frac{x}{2}) + a$, where $h(0) = b$. As we know that if we replace $2x$ by x in the equation $f(2x) = 4h(x) - a$, it will take the form as $f(x) = 4h(x/2) - a$ and after replacing x by $(x + y)$ in the previous equation**,** it will be rewritten as $f(x + y) = 4h(\frac{x+y}{2}) - a$, after replacing x by $x - y$ in the function of $g(x)$, it will take the form of $g(x - y) = 2h(x - y) + 2b - 4h(\frac{x-y}{2}) + a$. Now after putting the value of $f(x + y)$ and $g(x - y)$ in **equation 1,** it will take the form as follows**;-**

$$2\left[h\left(\frac{x+y}{2}\right) + h\left(\frac{x-y}{2}\right)\right] + h(x - y) + b = h(x) + h(y) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$

Let $H(x) = h(x) - b$. These longer linear expressions can be easily handled if we express functions in form of the sum of an even and odd function, i.e. $H(x) = H_e(x) + H_o(x)$. Substituting this into **(2)** and writing the same expression for $(-x, y)$ and $(x, -y)$ we can add them together and get:

$$2\left[He\left(\frac{x-y}{2}\right) + He\left(\frac{x+y}{2}\right)\right] + H_e(x - y) = H_e(x) + H_e(y) \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3$$

If we set –**y** in this expression and add to **3** we get using $H_e(y) = H_e(-y)$ **,**the expression can be rewritten as;-

$$H_e(x + y) - H_e(x - y) = 2H_e(x) + 2H_e(y) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4$$

4.2.1 Solution of Equation 4:

In order to solve the equation 4, initially we have to put $x = y = 0$ in equation 4 and it can be rewritten as;-

$H_e(0) - H_e(0) = 2H_e(0) + 2H_e(0)$ and so it will lead to $H_e(0) = 0$ and our goal is to restructure the equation 4 in one variable functional equation as it will minimize the initial complexity of the problem. Putting $y = x$ and it will lead to the equation **4** in other form like as $H_e(2x) - H_e(0) = 2H_e(x) + 2H_e(x)$ and it will further lead to a very interesting equation of like $H_e(2x) = 4H_e(x)$. Now as it is aforementioned that this function is a real valued function mapped from real to real. So it is differentiable. Now we are differentiating the equation and it will be $2H_e'(2x) = 4H_e'(x)$ and which will be further differentiated to get the form of as in $4H_e''(2x) = 4H_e''(x)$. The final expression can be reduced to as follows**;-**

$$H_e''(2x) = H_e''(x) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 5$$

Substituting $2x$ by x in the previous equation lead to

$$H_e''(x) = H_e''\left(\frac{x}{2}\right)$$

If we further substitute x by x/2 it will lead to the following equation

$$H_e''\left(\frac{x}{2}\right) = H_e''\left(\frac{x}{4}\right)$$

Combining the following equations and if we continue this process it will give an expression of like;-

$$H_e''(x) = H_e''\left(\frac{x}{2}\right) = H_e''\left(\frac{x}{2^2}\right) = \ldots\ldots\ldots\ldots = H_e''\left(\frac{x}{2^n}\right)$$

Now we know that $\lim_{n\to\infty}\left(\frac{x}{2^n}\right) = 0$ so the function $H_e''\left(\lim_{n\to\infty}\frac{x}{2^n}\right) = H_e''(0) = C$ which is a constant function.

So it can be rewritten as $H_e''(x) = H_e''(0) = C$

Now integrating the above differential equation;-

$$\int He''(x)\,dx = \int C\,dx$$

So now further integrating

$\int \boldsymbol{He}''(\boldsymbol{x})\, \boldsymbol{dx} = \int \boldsymbol{Cdx}$

Now the next step will be;-

$H_e'(x) = Cx + C_1$

So after that

$\int \boldsymbol{He}'(\boldsymbol{x})\, \boldsymbol{dx} = \int \boldsymbol{Cx}\, \boldsymbol{dx} + \int \boldsymbol{C1}\, \boldsymbol{dx}$

Now it will be

$H_e(x) = Cx^2/2 + C_1x + C_2$

Now if we take **x = 0** ,it will be

$H_e(0) = 0 + 0 + C_2$ and as the value of $H_e(0) = 0$,the value of C_2 will also be zero.

If we put **x = 0** in the equation of $H_e'(x) = Cx + C_1$, it will be

$H_e'(0) = 0 + C_1$ and as it has been previous proved that $H_e'(0) = 0$**,** the value of C_1 will be and the final expression will be;-

$$H_e(x) = Cx^2/2 \qquad \ldots\ldots 6$$

Similar method gives the simple relation for H_o

$$H_o(x + y) + H_o(x - y) = 2\, H_o(x) \qquad \ldots\ldots 7$$

4.2.2 Solution of Equation 7:

In order to solve the equation **7,** initially we have to replace and rearrange the equation in a very crucial way. So it is very important to note that there is a very interesting transformation in the equation of $H_o(x + y) + H_o(x - y) = 2\, H_o(x)$

Now if we rearrange the equation as follows;-

$$H_o(x + y) + H_o(x - y) = 2\, H_o\left[\frac{(x+y) + (x-y)}{2}\right] \qquad \ldots\ldots 8$$

If we replace ***(x + y)*** by ***x*** and ***(x – y)*** by ***y*** the **equation 8** will be

$H_o(x) + H_o(y) = 2H_o\left(\frac{x + y}{2}\right)$

Now after if we do it in a very different way

$$H_o\left(\frac{x+y}{2}\right) = \frac{Ho(x) + Ho(y)}{2} \text{...} 9$$

4.2.3 Graphical Solution of Equation 9:

In order to solve equation 9, initially we have to take the help of a graph. It can also be solved analytically. Now we have the equation 9 as below;-

$$H_o\left(\frac{x+y}{2}\right) = \frac{Ho(x) + Ho(y)}{2}$$

Replacing ***$H_o(x)$*** by ***f(x)*** and ***$H_o(x)$*** by ***f(y)*** and we will proceed further with the help of a graph as shown below;-

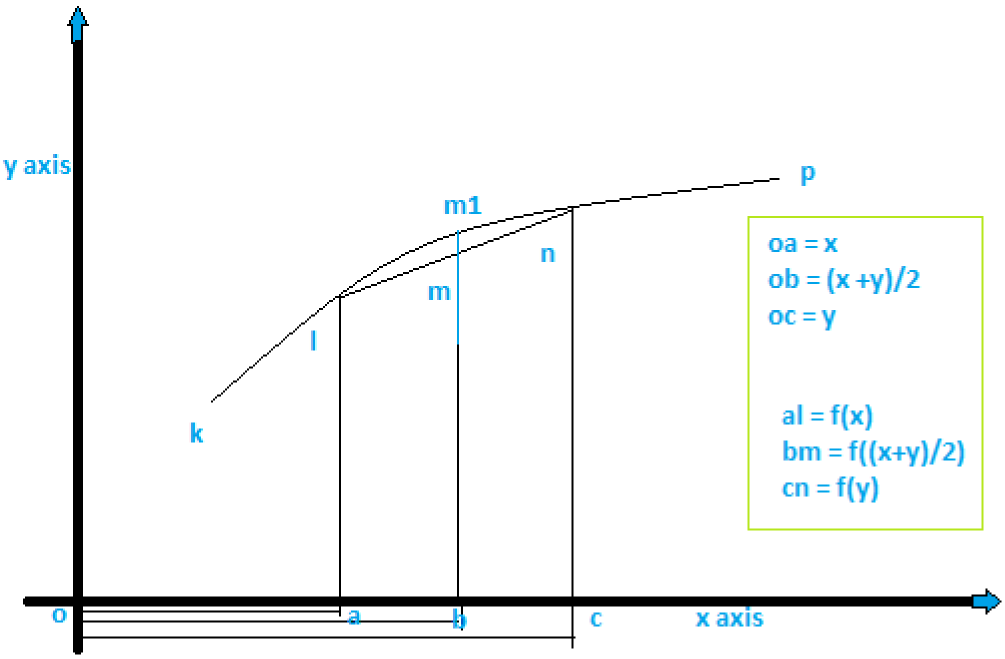

Fig 8 : Graphical solution of equation 9

Let ***l (x, f(x))*** and ***n (y, f(y))*** be any two points on the curve ***y = f(x).***

Let **m** be the midpoint of **ln**

Therefore, co-ordinates of **m** are $\left(\frac{x+y}{2}, \frac{f(x)+f(y)}{2}\right)$

Co-ordinates of **m1** are $\left(\frac{x+y}{2}, f\left(\frac{x+y}{2}\right)\right)$

and **m1b > mb**

Therefore $\quad f\left(\frac{x+y}{2}\right) > \frac{f(x)+f(y)}{2}$

But we know that $\quad f\left(\frac{x+y}{2}\right) = \frac{f(x)+f(y)}{2}$

Hence $y = f(x)$ must be a linear function

So it can be written as $f(x) = £x + €$…………………………………………………………………10

As we replaced previously $H_o(x)$ by $f(x)$, conversely it can be written as $H_o(x) = £x + €$ from equation **10**. As we know from previous analysis, $H(x) = H_e(x) + H_o(x)$ and after putting the value of $H_e(x)$ and $H_o(x)$ we will get a prolific result i.e. the value of $H(x)$ which is inextricably linked with the value of $f(x)$.

So the final expression will be;-

$$H(x) = Cx^2/2 + £x + € \quad \ldots\ldots 11$$

As we know $H(x) = h(x) – b$

So $h(x) = Cx^2/2 + £x + € + b$

Also $f(2x) = 4h(x) – a$

$f(2x) = 4(Cx^2/2 + £x + € + b) – a$

Simplifying we get

$f(2x) = 2Cx^2 + 4£x + 4(€ + b) – a$

Replacing **2x** and **x,** we get

$f(x) = Cx^2/2 + 2£x + 4(€ + b) – a$

Finally it will be;-

$$f(x) = \alpha x^2 + \beta x + \gamma \quad \ldots\ldots 12$$

4.6 Probability Distribution Function of Model II:

A probability distribution is a table or an equation that links each outcome of a statistical experiment with its probability of occurrence. In the figure below

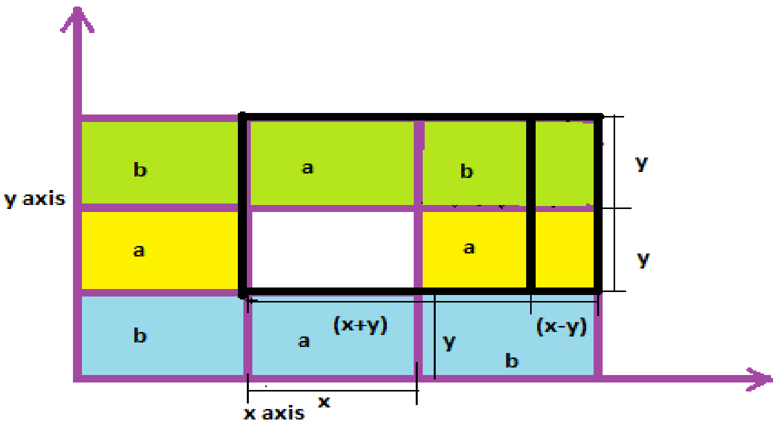

Fig9: Probability of all rectangles in model 2

Probability of all rectangles in ***model 2*** as shown in ***Fig9*** is inextricably linked with the solution of the functional equation ***1*** and the value of ***f(x)*** which has been derived previously as $\boldsymbol{\alpha x^2 + \beta x + \gamma}$. Now as in ***model 2,*** summation of probability of all eight rectangles will be ***4a + 4b = 1.*** Now the functional equation ***f(x + y) + g(x – y) = 2h(x) + 2 h(y)*** as predicted previously was restricted to four rectangles as shown in ***Fig9***. So the corresponding probability will be ***2a + b*** as in ***Fig9.*** So the value of ***b*** will be obtained from equation ***4a + 4b = 1*** after some rearrangement will be as shown;-

4a + 4b = 1

It can be written as

2(2a + b) + 2b = 1

So it will be

$\boldsymbol{b = 1/2\{1 - 2(2a + b)\}}$

The value of the probability (***2a + b)*** as shown above is having an intricate relationship with the aforementioned function ***{2h(x) + 2 h(y)}.*** The value of ***h(x)*** has been derived previously and it was ***h(x) =*** $\boldsymbol{Cx^2/2}$ ***+ £x + € + b .***

After replacing ***x*** by ***y***

h(y) = $\boldsymbol{Cy^2/2}$ ***+ £y + € + b***

So the value of ***{2h(x) + 2 h(y)} =*** $\boldsymbol{C/2(x^2 + y^2)}$ ***+ £(x + y) + 2(€ + b)***

This is a function of ***x*** and ***y*** and it can be written as ***m(x , y).***

So the value of probability ***b*** is a function of ***m(x , y).***

$\boldsymbol{b = 1/2\{1 - 2(m(x, y))\}}$…………………………………………………………………***13***

An important observation from ***equation 13*** will be

Range of the probability ***b*** will be $\mathbf{0} < b \le 1$

So $\boldsymbol{1/2\{1 - 2(m(x, y))\}} > 0$

As from the previous line

$\boldsymbol{\{1 - 2(m(x, y))\}} > 0$

It will be $\boldsymbol{m(x, y)} < \mathbf{1/2}$

That is the function

$\boldsymbol{C/2(x^2 + y^2) + £(x + y) + 2(€ + b)} < \mathbf{1/2}$

5.1 Numerical Modelling Study:

The gallery has been modelled using PHASE 2 in order to analyse strata convergence.

The modelling procedure is

a) OPEN PHASE 2
b) Select: Analysis → Project Settings
c) Select: Boundaries → Add Excavation
d) Select: Boundaries → Add External
e) Select: Mesh → Mesh Setup
f) Select: Mesh → Discretize
g) Select: Mesh → Mesh
h) Select the Loads & Restraints workflow tab, then select Field Stress from the toolbar or the Loading menu.
i) Select: Loading → Field Stress
j) Select: Properties → Define Materials
k) Select: Properties → Assign Properties

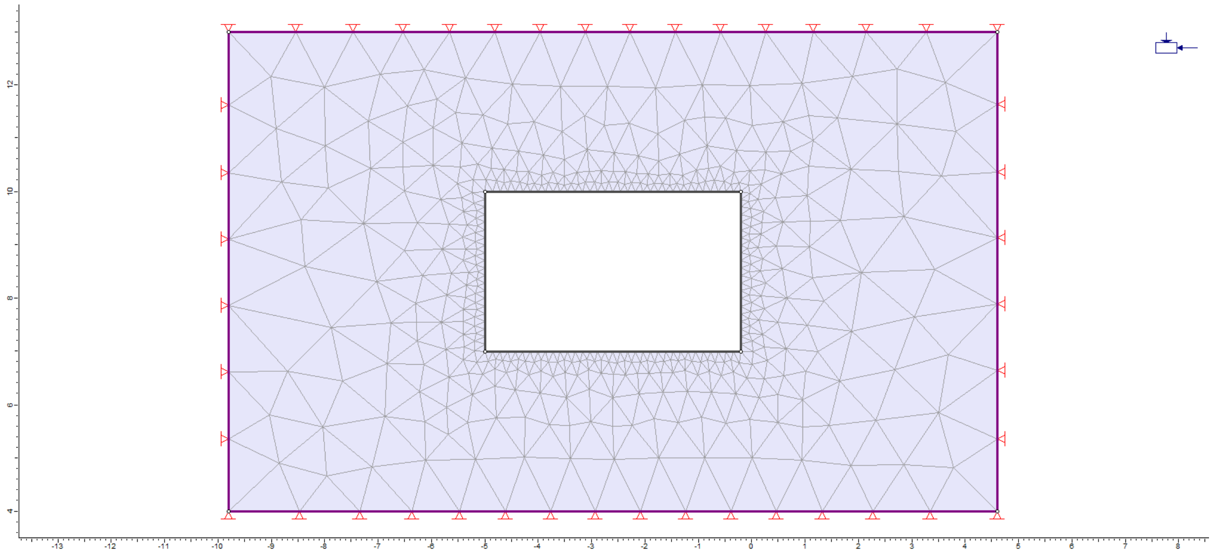

Fig10 Discretization of numerical model showing gallery (3×4.8 m^2)

5.2 Computation & Interpretation of Finished model:

Prior to analyzing the finished model, it shall be saved as a file with **.fez** filename extension. After saving the file, now it is ready for computation and interpretation with following steps.

- Select: Analysis → Compute
- Select: Analysis → Interpret

5.2.1 Principal Stress

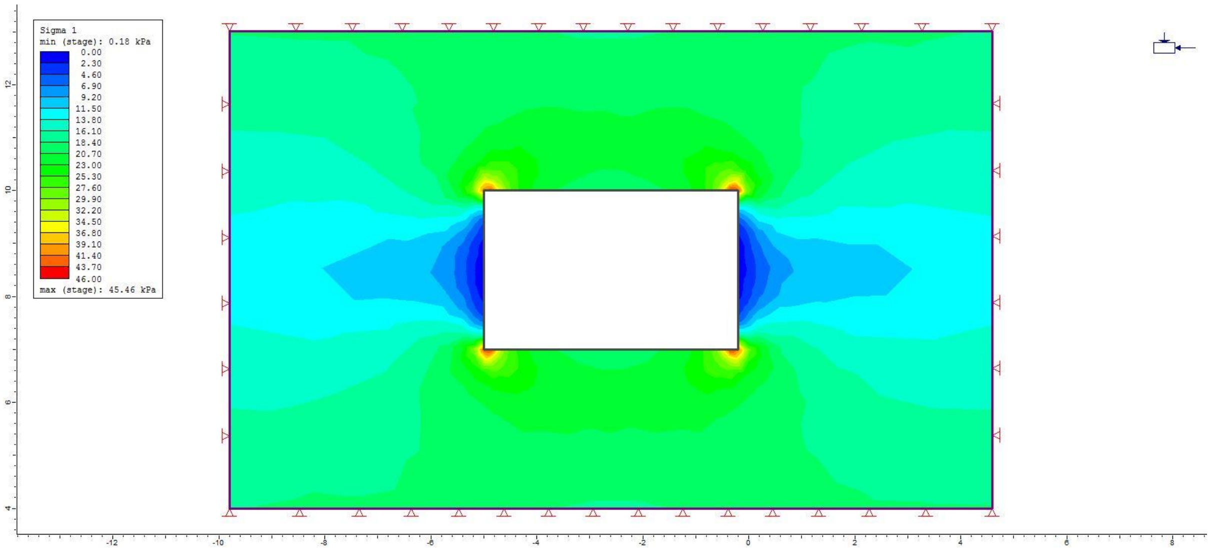

Fig11: Contours of Major Principal Stress

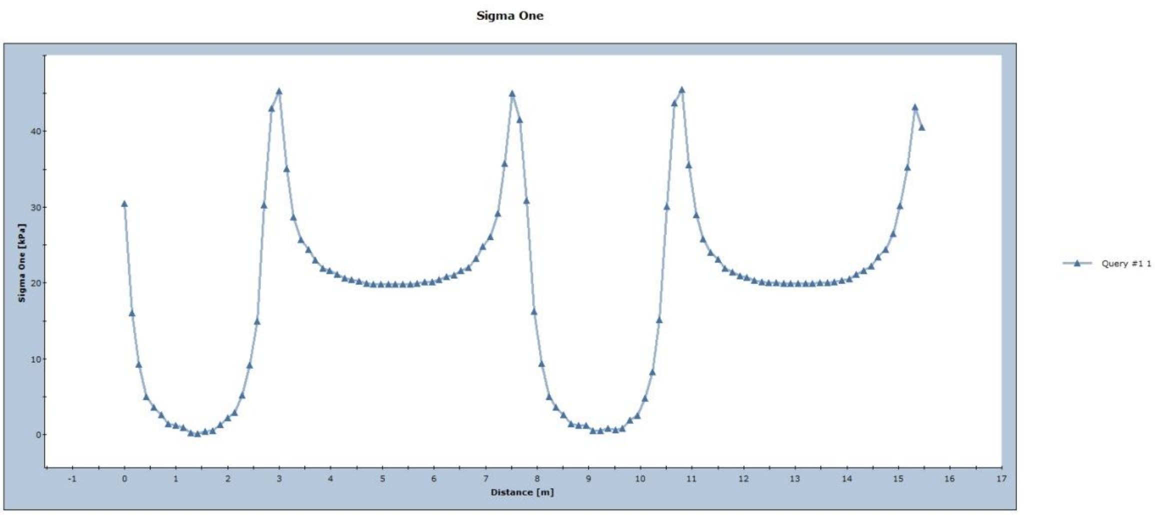

Fig12: Principal stress on excavation boundary

5.2.2 Displacements

The objective in solving a rock mechanics problem is to calculate the displacement vector **u** at every point in the rock mass or rock specimen based on the knowledge of the applied surface tractions and body forces, and the boundary conditions.

- Select **Solid Displacement > Total Displacement** from the data list in the tool bar.
- Select the **Deformed Boundaries** and **Deformed Vectors** buttons in the toolbar.

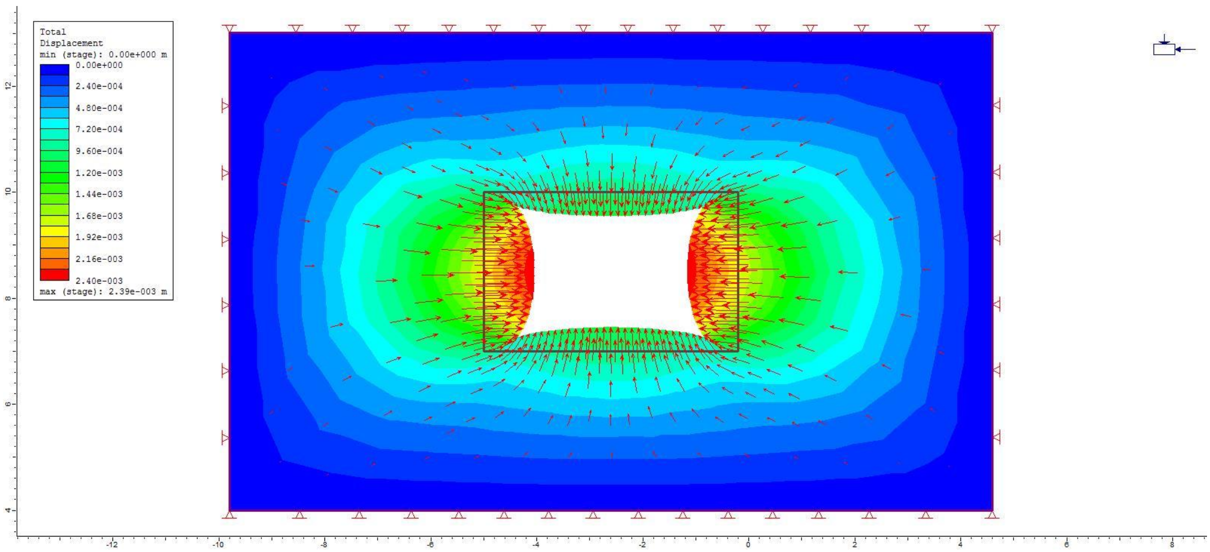

Fig13: Total displacement contours, with deformation vectors and deformed boundaries displayed

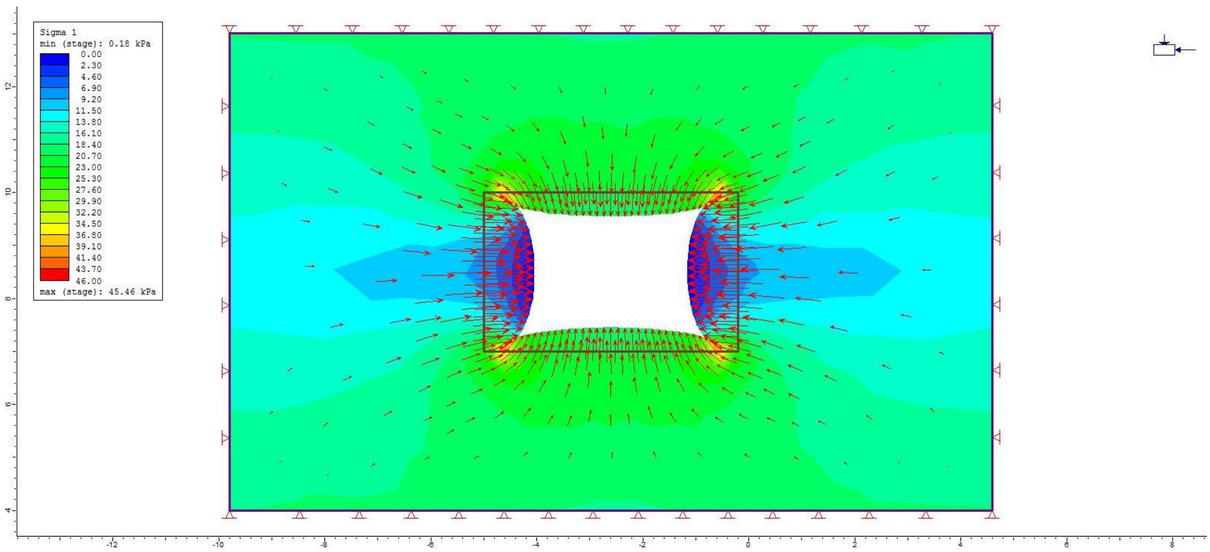

Fig14: Displacement contours of major principal stress with deformed boundaries

5.2.3 Strength Factor

Contours of Strength Factor can be viewed by selecting **Strength Factor** from the toolbar list. The Strength Factor represents the ration of material strength to induced stress at a given point.

- Material strength is based on the strength properties of defined material
- Induced stresses are determined by the elastic stress distribution computed from the boundary element analysis.

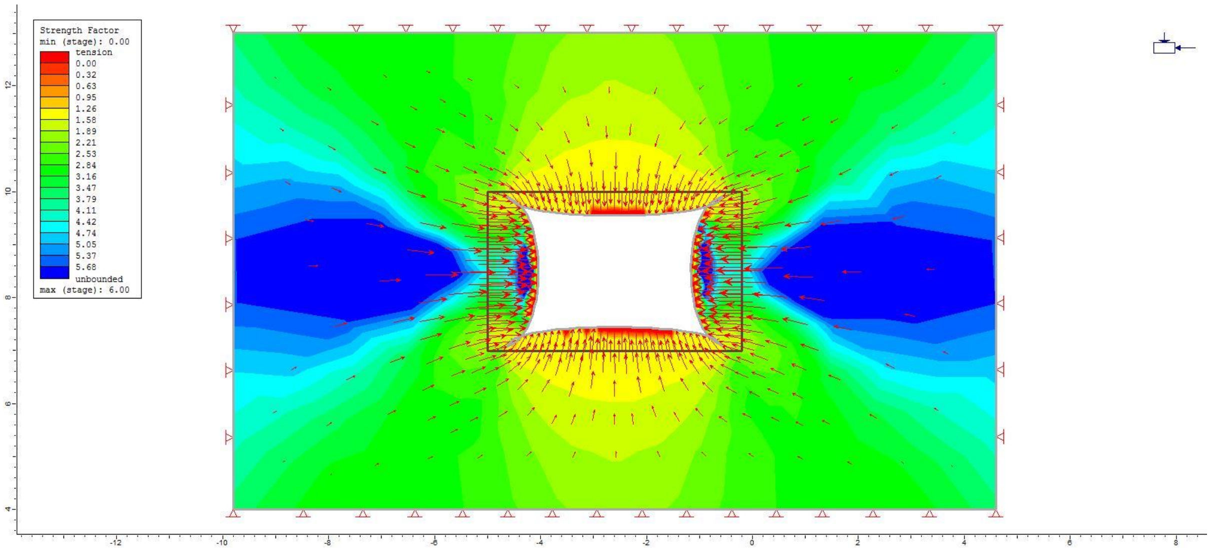

Fig15: Contours and trajectories of strength factor

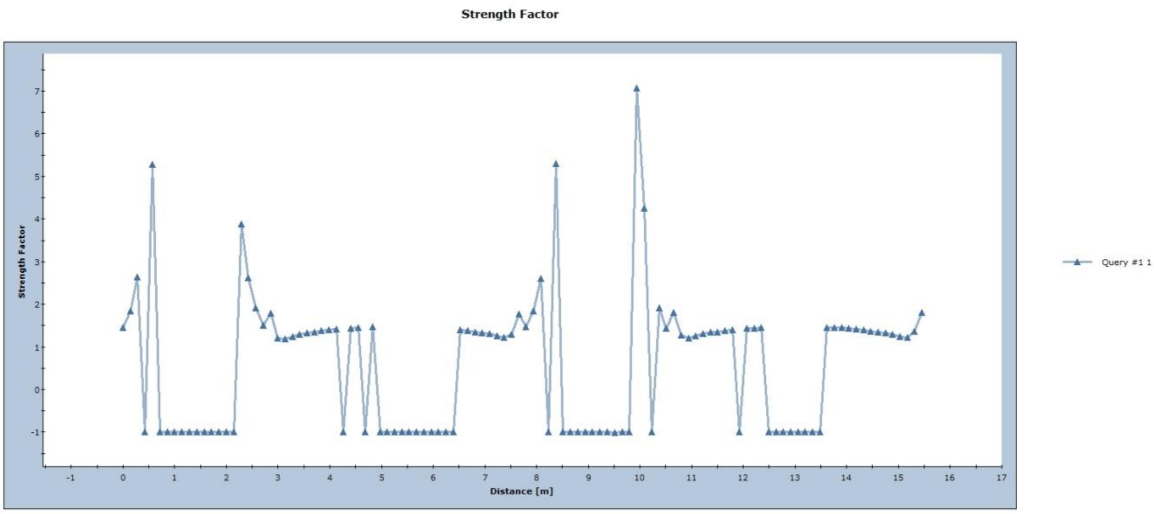

Fig16: Graphical representation of strength factor with the distance

CHAPTER 4

ANALYSIS AND RESULTS

6.1 Theoretical Approach

In theoretical approach, several models were investigated according to the principle of geometric similarity as well as few equations of probability distributions were also established. Based on the similarity characteristic value in **model I & II,** two equations were constructed.

All the equations constructed for both models are discussed below:

- For analysing model 1, the total probability distribution after taking a vertical section from the entire rock continuum under a similar geometric scenario was derives as $\sum_{i=1}^{9} \mathbf{P} = \mathbf{2a + 2b + 2c + x + y = 1}$**,** where the different parameters as **a, b, c, x, y** represent the probability of yellow, green, sky, orange and brown rectangles respectively.
- For analysing model 2, the total probability distribution after taking a horizontal section from the entire rock continuum under a similar geometric scenario was derived as $\sum_{i=1}^{9} \mathbf{P} = \mathbf{4a+4b+r=1}$ where the different parameters as **a, b, r** represent the probability of green, yellow and middle rectangles respectively as it was known from the previous analysis that the value of **r** is zero so the new equation was **4a+4b=1**.

Theoretical prediction of several functions had been done previously by analysing a rectangle in **model 1&2.** An exponential function was constructed by dividing a rectangle into several smaller rectangles so as to constitute an encapsulated equation following the convergence mechanism. The equation was derived as;-

$$\sum_{i=1}^{n} \boldsymbol{fi}(\boldsymbol{x}) = \sum_{i=1}^{n} \boldsymbol{exp}(-\Psi \boldsymbol{ixgi}(\boldsymbol{x}))$$

A functional equation was investigate to establish a probability distribution function in order to express the value of the probability of **model 1&2** in terms of **x** and **y**. The initial functional equation was $\boldsymbol{f(x+y) + g(x-y) = 2h(x) + 2\,h(y)}$. The solution of this functional equation was $\boldsymbol{h(x) = Cx^2/2 + £x + € + b}$ and also the value of $\boldsymbol{f(x)}$ was $\boldsymbol{Cx^2/2 + 2£x + 4(€ + b) - a}$. After finding the value of ***h(x)*** and ***f(x)*** , it had been possible to merge the value with previously defined probability equation of ***4a+4b=1.***

6.2 Numerical Modelling by PHASE 2:

In this particular model, a gallery has been chosen with a dimension of **4.8m × 3m**. The stress distribution around the excavated wall has been interpreted after giving certain value of principal stresses as **σ_1 = 20** and **σ_2 = 10.** The failure criterion has been chosen as **Mohr-Coulomb** criteria. In this particular problem, the tensile strength is **0** and the friction angle is **35**° and the material type is **elastic**. The value of the **Young's Modulus** is **20000 kPa** and the **Poisson's Ratio** has been chosen as **0.2.** The elastic type is isotropic. After giving the value of multiple variables, the strength factor, displacements and stress contour have been derived as well as simulated by the application. Also the inclination has been chosen as **0.**

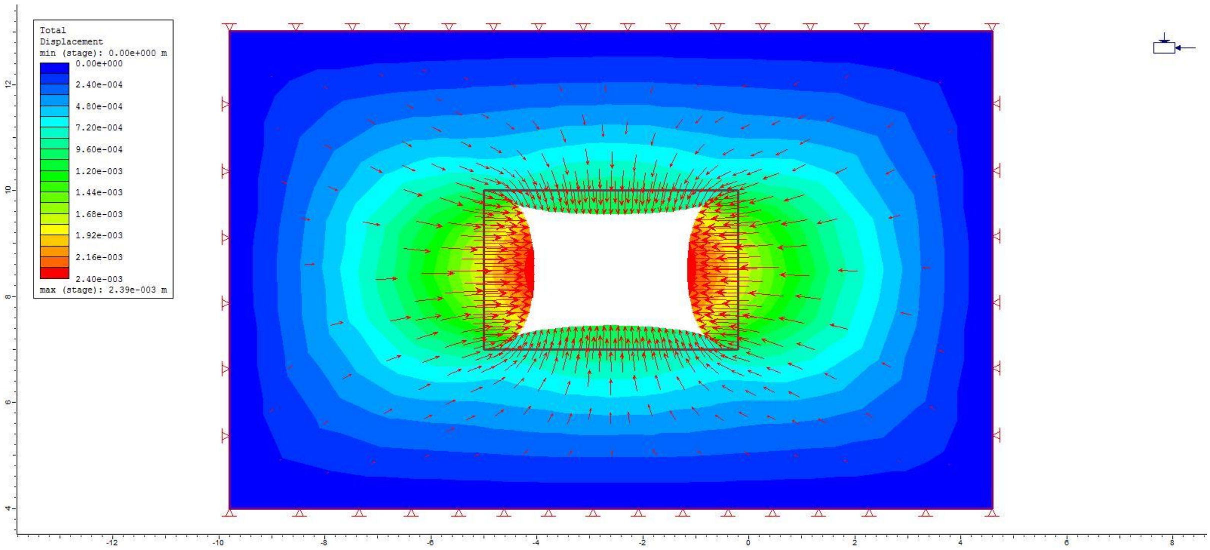

Fig17: Total displacement contours, with deformation vectors and deformed boundaries displayed

The maximum displacement has been occurred in the lateral direction which is around **0.8m**. So it is the value of the maximum convergence according to the interpreted result. Also the maximum strength factor has been in between 7 and 8.The stress trajectories have been shown in the figure 17.

CHAPTER 5

DISCUSSIONS

Initial objective

Initial objective of the present work was to correlate the probable factors of strata convergence and to give separate ratings of each block after correlating those factors with a view to estimating the probability of convergence of each rectangle as specified in predefined models and to recognize a pattern in probability of convergence from numerical modelling by using software. Finally the planned objective has been deviated due to several reasons.

Reason for deviation of initial objective

During the time of correlation of the probable factors of convergence, the weight of the individual block as well as the density of each block was being correlated by a crucial method and the specific equation was

$$\sum_{i=1}^{n} \boldsymbol{fi}(\boldsymbol{x}) = \sum_{i=1}^{n} \boldsymbol{exp}(-\Psi \boldsymbol{ixgi}(\,\boldsymbol{x}))$$

- In this function, the geometrical significance of the function $g_i(x)$ was not known properly and the value of the constant function Ψ was not being determined. Also the number of probable factors of convergence was unknown. In this paper only 7 factors of convergence were taken into account. There might be other factors. Also the correlation between the positions of every rectangle with elasticity was too difficult.
- Also a proper failure criterion with the respective correlation of various factors of convergence was required for imposing ratings to the each block. As a matter of fact, a specific rating was impossible to be given to each rectangle.

Alternative approach to estimate the probability

As the initial approach was not fulfilled, a functional equation was derived in order to estimate the probability of each rectangle in **model 1 & 2.** The equation which was constructed for establishing the relationship of x and y with the probability was $b = 1/2\{1 - 2(m(\,x\,,y\,))\}$ where the value of $m(x, y)$ is $C/2(x^2 + y^2) + £(x + y) + 2(€ + b)$. The value of x and y will be known to us and also the constant function will be known to us.

CHAPTER 6

CONCLUSION AND RECOMMENDATIONS

Based on crucial observation and theoretical approach as well as numerical analysis, the following conclusions and recommendations can be done:

CONCLUSION:

1. for theoretical approach

1.1 The developed models are based on some ideal conditions which were being hypothesized previously in order to minimize the initial complexity of the problem. However, the real geologic or site specific conditions may not be ideal which has been chosen here. As a matter of fact, there is wider variation in real world engineering problems with the theoretical approach. So the value of the probability which was being chosen earlier would have been different.

1.2 In considering the functional equation as a tool for estimating the probability, several constant functions have been considered in each step of solutions. However, the geometric significance of the constant function is not known and it may seem like this constant may be depend upon various trigonometric or exponential functions in real plane rather it could be a polynomial function. Parameter on which the constant function depends as well as the form of proportionality has been neglected.

2. for numerical analysis

2.1 On the basis of numerical analysis, it has been possible to simulate the convergence phenomenon as well as the displacement contour. Also from the graph, it will be possible to detect a pattern as well as the various types of statistical estimation can be carried out. Also if we graph those numerical values of displacements and strength factor at any system, patterns will emerge.

RECOMMENDATIONS:

1. This type of theoretical approach can be used to estimate the probability of strata convergence for ideal conditions. However, it can also be used to design various complex probability equations which will further be used to detect the displacements of strata as well as the frequency of occurrence.

REFERENCES

(1) Cundall P.A., Strack O.D.L.,(1979) " A discrete numerical model for granular assemblies" , *Geotechnique*, vol 29, April 1979 , pp.- 47-65.

(2) Pastarus J.R., Nikitin O.,(2001) " Mining block stability estimation in Estonian oil shale mines" , *International Symposium on Geotechnical Issue of Underground Space Use for Environmentally Protected World*, vol 26, June 2001, pp.- 121-125.

(3) Adoko A.C., Wu L.,(2011) " Fuzzy inference system-based approaches in geotechnical engineering" , *Electronic Journal of Geotechnical Engineering*, vol 16, July 2011, pp.- 1275-1295.

(4) Lee J., Akutagawa S.,(2009) " Quick prediction of tunnel displacements using artificial neural network and field measurement results", *International Journal of the JCRM*, vol 5, February 2009, pp.- 53-62.

(5) Kontogianni V., Sitros S.,(2005) " Induced deformation during tunnel excavation: evidence from geodetic monitoring*" Engineering Geology*, vol 79, June 2005, pp.- 115-126.

(6) Adoko A.C., Wu L.,(2012) " Estimation of convergence of a high-speed railway tunnel in weak rocks using an adaptive neuro-fuzzy inference system (ANFIS)", *Journal of Rock Mechanics and Geotechnical Engineering*, vol 4, January 2012, pp.-11-18.

(7) Rinne M., Shen B., Backers T.,(2013) " Modelling fracture propagation and failure in a rock pillar under mechanical and thermal loadings", *Journal of Rock Mechanics and Geotechnical Engineering*, vol 5, April 2013, pp.- 73-83.

(8) Malone K., Xu B.,(2008) " Determination of contact parameters for discrete element method simulations of granular systems", *Particuology*, vol 6, December 2008, pp.-521-528.