$\mathcal{O}_J(\Theta)$

$C \longrightarrow J$

$x_1 + \cdots x_{g-1}$

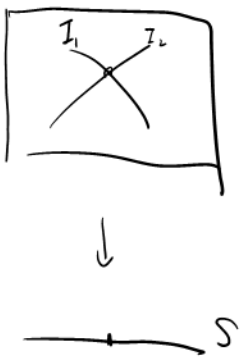
$\shortparallel$

$k - y_1 \cdots y_{g-1}$

$\Uparrow$

$x \in C \cap \Theta \iff x + y_1 + \cdots y_{g-1} = k - (g-2)a.$

$\mathcal{L}$



X

S

$I_1 \in CH^1(X) \quad I_2 \in CH^1(X)$

$I_1 \cdot I_2 \in CH^2(X)$

$\downarrow$

$CH^1(S)$

$$\begin{matrix} \omega_Y & & \omega_X \\ Y & \overset{i}{\hookrightarrow} & X \end{matrix}$$

$$\omega_Y \cong i^*\big(\omega_X \otimes \mathcal{O}_X(Y)\big)$$
$$\cong i^*\,(\omega_X(Y)).$$

$C \xrightarrow{\Delta} C \times C$

$$\omega_C \cong \Delta^*(\omega_C \boxtimes \omega_C \otimes \mathcal{O}_{C\times C}(\Delta))$$
$$\cong \omega_C^2 \otimes \mathcal{O}_{C\times C}(\Delta)$$

$$\Big(\varinjlim_{\mathcal{C}} \mathrm{Pic}(\mathcal{C})_{\mathbb{Q}} \times_{\mathrm{Pic}(C)_{\mathbb{Q}}} \mathrm{Pic}(C)\Big)^{\wedge} = \{\text{metrized line bundles on } C^{an}\}$$

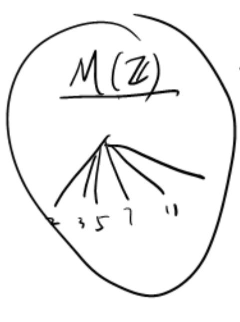
$\searrow \quad \swarrow$

$\mathrm{Pic}(C)$

$C \longrightarrow \mathcal{C}$

$\mathbb{Z}_p.$

$\mathbb{Z}_p$ $\quad \|\cdot\|_p^t$

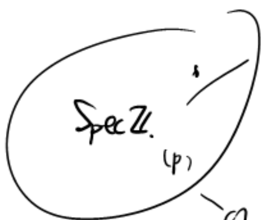


$\mathbb{A}^1_{\mathbb{Z}_p}$

$\mathbb{Z}_p[x].$

$\mathbb{C}[[x]].$

$\mathbb{R} \quad \mathbb{F}_1$



$\mathbb{R}$

$\uparrow$

$\swarrow$ metric.

$\mathbb{Q}_p. - \mathbb{Z}_p \quad \mathbb{F}_p.$

§ 1. Berkovich space

fix $\mathcal{A}$ comm ring $\underline{\|\cdot\|_{\mathcal{A}}}$ norm

Def. (Berk space, affine version).

$$\mathcal{M}(\mathcal{A}, \|\cdot\|_{\mathcal{A}}) = \left\{ \|\cdot\| \;\middle|\; \begin{array}{l} \underline{\text{bounded}}\ \|\cdot\| \le C \|\cdot\|_{\mathcal{A}} \ \forall C \\ \underline{\text{multi}} \\ \text{seminorm} \end{array} \right\}$$

with the weak topo.

$(\forall f \in \mathcal{A})$ $\quad \phi_f : \mathcal{M}(\mathcal{A}, \|\cdot\|_{\mathcal{A}}) \longrightarrow \mathbb{R}$. $\qquad \|\cdot\| \longmapsto \|f\|$ is cont.

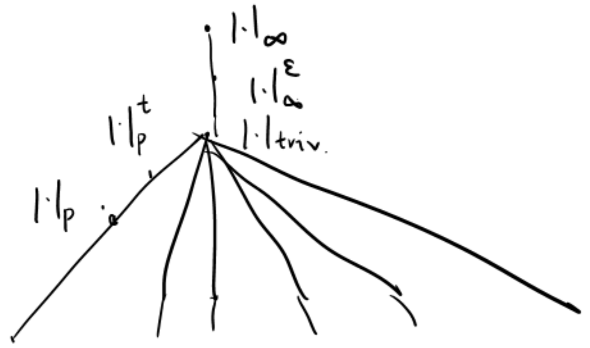
Eg. (Gelfand-Mazur)

fix X. cpt. topo space.

$C^0(X, \mathbb{C})$ cplx cont map on X.

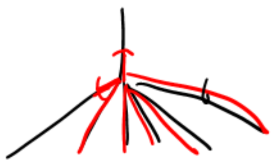
then $\mathcal{M}(C^0(X,\mathbb{C}), |\cdot|_\infty) \overset{\text{homeo}}{\cong} X$

E.g. $\underline{\mathcal{M}(\mathbb{Z}, |\cdot|_\infty)}$ $\qquad \mathbb{Z} \hookrightarrow \mathbb{R}$.



$|\cdot|_p^\infty : \mathbb{Z} \xrightarrow{\pi} \mathbb{F}_p \xrightarrow{|\cdot|} \mathbb{R}$.

· Topo.



Prop. $\mathcal{M}(\mathbb{Z})$ is conn. cpt. Hausdorff space.

E.g. $\mathcal{M}(\mathbb{Q}, \underline{|\cdot|_\infty}) \cong \{|\cdot|_\infty\}$

Rmk. $\mathcal{M}(\underline{\mathcal{A}}, \underline{\|\cdot\|})$ is non-empty and $\underline{\text{cpt}}$.

Def. (Berk Space, relative version)

fix k. comm Banach ring with 1 $\qquad (k = \underline{(\mathbb{Z}, |\cdot|_\infty)}, \underline{(\mathbb{Q}_p, |\cdot|_p)})$

X/k scheme.

a) $X = \operatorname{Spec} A$, then

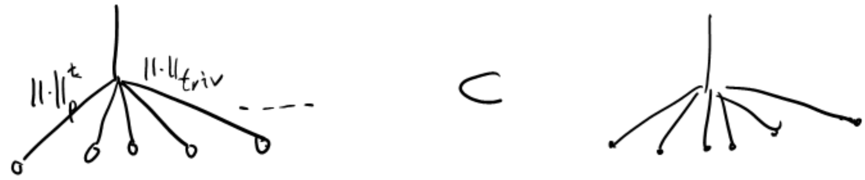
$$X^{an} := \mathcal{M}(A/k) := \left\{ \|\cdot\| \;\middle|\; \begin{array}{l} \|\cdot\| \text{ multi seminorm} \\ \|\cdot\|_k \text{ is bounded.} \end{array} \right\}$$

with the weak topology.

b). Globally.

E.g. $k = (\mathbb{Z}, |\cdot|_\infty)$. $X = \operatorname{Spec} \mathbb{Q}$. over k.

$X^{an} =$



$t \in [0, +\infty)$

E.g. $k = (\mathbb{C}, |\cdot|_\infty)$ X/k finite type.

then

$$X^{an} \overset{homeo}{\cong} X(\mathbb{C})$$

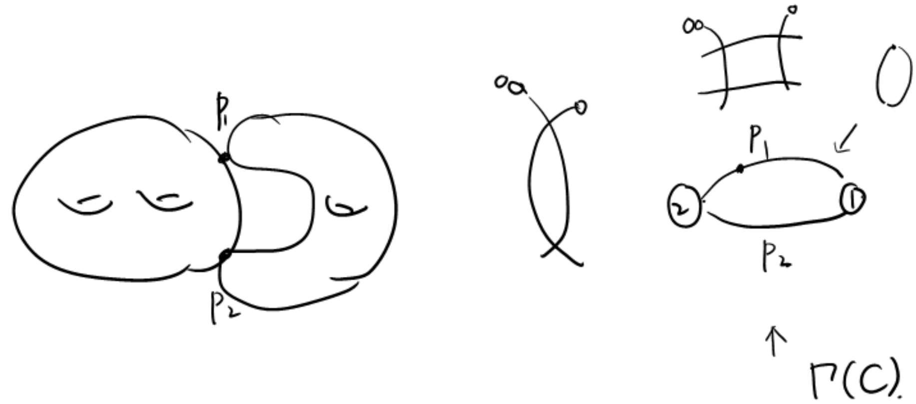
E.g. k: non-archi field.

C/k proj sm curve of genus $g > 1$.

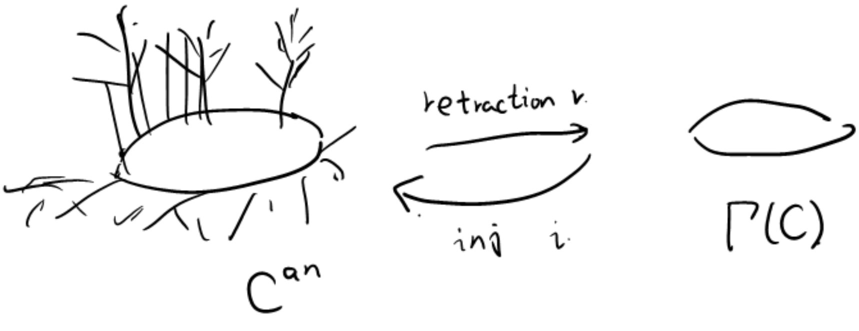
We assume • $C(k) \neq \phi$. & C has split semistable red $\mathcal{C}$ over $\mathcal{O}_k$

$$\begin{tikzcd} & \mathcal{C}_s \arrow[dr] \arrow[d] & \\ C \arrow[rr] \arrow[d] & \operatorname{Spec} k \arrow[dr] & \mathcal{C} \arrow[d] \\ \operatorname{Spec} k \arrow[rr] & & \operatorname{Spec} \mathcal{O}_k \end{tikzcd}$$

$\mathcal{C}_s$



C^{an}.



§2. structures on X^{an}

§ metrized line bundle over X.

Rmk. $\left(\varinjlim_{\mathcal{C}} \left(\operatorname{Pic} C_{\mathbb{Q}} \times_{\operatorname{Pic} \mathcal{C}_{\mathbb{Q}}} \operatorname{Pic} \mathcal{C} \right) \right)^{\vee} = \{ \text{metrize line bundle over } C \}$

$$\begin{array}{ccc} C & \to & \mathcal{C} \\ \downarrow & & \downarrow \\ \operatorname{Spec} K & \to & \operatorname{Spec} \mathcal{O}_K \end{array}$$

$(\mathcal{L} \times \mathcal{M}) \longmapsto \left(\|s\|_x : \longrightarrow \inf \right)$

$\mathcal{L}_x \cong h^* \mathcal{M}_x$

$$\begin{array}{ccc} \operatorname{Spec}(\mathcal{H}_x, \|\cdot\|) & \xrightarrow{h} & \operatorname{Spec} \mathcal{O}_x \\ \downarrow & & \downarrow \\ C & \hookrightarrow & \mathcal{C} \end{array}$$

$\mathcal{L}$, $\mathcal{M}_x$, $\mathcal{M}$

Def (residue field of $\|\cdot\|_x \in (\operatorname{Spec} A)^{an}$)

$$(\mathcal{H}_x, \|\cdot\|_x) = \left(A / \ker \|\cdot\|_x , \|\cdot\| \right)^{\wedge}$$

$\mathbb{Q}_p \longleftarrow \mathbb{Z}_p$

$\|1\| = 1$

$\|p\| = \frac{1}{p}$

X^{an} $\quad \mathcal{L} \in \operatorname{Pic}(X)$

Def. (fiber of $\mathcal{L}$ over $x \in X^{an}$).

$$\mathcal{L}^{an}(x) := \mathcal{H}_x \otimes_{k(\bar{x})} \mathcal{L}(\bar{x})$$

$$\begin{array}{ccc} \mathcal{L}^{an}(x) & & \mathcal{L}(\bar{x}) \\ | & & | \\ x & \longmapsto & \bar{x} \\ \in & & \in \\ X^{an} & \xrightarrow{k} & X \end{array}$$

$\mathcal{L}$

Def. (metric at $x \in X^{an}$).

$$\|\cdot\|(x) : \mathcal{L}^{an}(x) \longrightarrow \mathbb{R}_{\geq 0}$$

① $\|\cdot\|(x)$ is a norm

② $\|f l\|(x) = |f|(x) \, \|l\|(x)$ for $\forall f \in \mathcal{H}_x$, $l \in \mathcal{L}^{an}(x)$

Def. (metric)

$$\|\cdot\| = \{ \|\cdot\|(x) \mid x \in X^{an} \} \quad \text{collection.}$$

$(\mathcal{L}, \|\cdot\|)$ is called metrized line bundle.

E.g. (metric over $\mathcal{O}_X$) can be viewed as an positive real fct on $X(\bar{k})$

Prop.

$(\mathcal{L}_1, \|\cdot\|_1)$ $(\mathcal{L}_2, \|\cdot\|_2)$

$(\mathcal{L}_1, \|\cdot\|)^\vee$

$(\mathcal{O}_X, \|\cdot\|_{triv})$.

Prop. pull back

$$\begin{tikzcd} i_*\|\cdot\|_{\mathcal{L}} \arrow[d, dash] & \|\cdot\|_{\mathcal{L}} \arrow[d, dash] \\ i_*\mathcal{L} \arrow[d, dash] & \mathcal{L} \arrow[d, dash] \\ X \arrow[r, "i"] & Y \end{tikzcd}$$

→ $\mathcal{L}$ integral $\Leftrightarrow \mathcal{L} \cong \mathcal{L}_1 \otimes \mathcal{L}_2^\vee$. where $\mathcal{L}_1, \mathcal{L}_2$ strongly nef.

→ $(\mathcal{L}, \|\cdot\|)$ integral $\Leftrightarrow \mathcal{L} \cong \mathcal{L}_1 \otimes \mathcal{L}_2^\vee$ $\mathcal{L}_1, \mathcal{L}_2$ is semipositive. $\deg \mathcal{L}|_C > 0$

harmonic analysis. on C^{an}

§ 2.2. Chambert-Loir measure C^{an}

$$\begin{tikzcd} \|\cdot\| \arrow[d, dash] \\ \mathcal{L} \arrow[d, dash] \\ C \end{tikzcd}$$

$$c_1(\mathcal{L}, \|\cdot\|) := \frac{i}{2\pi} \partial\bar{\partial} \log\|s\|_h^2 \quad \text{over } \mathcal{U}$$

$\left(s \in \mathcal{L}(\mathcal{U}) \text{ non-zero} \right)$

Black box.

(1) $c_1(\mathcal{L}, \|\cdot\|)$ is a measure. over C^{an}

(2) $c_1(\mathcal{L}_1 \otimes \mathcal{L}_2, \|\cdot\|_1 \cdot \|\cdot\|_2) = c_1(\mathcal{L}_1, \|\cdot\|_1) + c_1(\mathcal{L}_2, \|\cdot\|_2)$

$c_1(\mathcal{L}_1^\vee, \|\cdot\|^{-1}) = -c_1(\quad)$

(3) (non-archi Calabi thm)

$\|\cdot\|$: integrable metric on $\mathcal{L}$. then

$c_1(\mathcal{O}_C, \|\cdot\|) = 0 \Leftrightarrow \|\cdot\| \equiv c_0$ on C^{an}

$$\begin{tikzcd} C \arrow[dr, "(\mathrm{Id}\times)"] & & C \\ C \arrow[r, "\Delta"] & C\times C \arrow[ur, "p_1"] \arrow[dr, "p_2"] & \\ C \arrow[ur, "(\times, \mathrm{Id})"'] & & C \end{tikzcd}$$

$$\begin{tikzcd} \mathcal{O}(x) \arrow[dr, dash] & \omega_C \arrow[d, dash] & \omega_C^\vee \arrow[dl, dash] & (\mathcal{O}(\Delta), \|\cdot\|_{\Delta,a}) \arrow[d, dash] \\ & C \arrow[rr, "\Delta"] & & C\times C \end{tikzcd}$$

$\Delta^*(\mathcal{O}(\Delta), \|\cdot\|_{\Delta,a}) := (\omega_C^\vee, \|\cdot\|_a^{-1})$

$(x, \mathrm{Id})^*(\mathcal{O}(\Delta), \|\cdot\|_{\Delta,a}) = (\mathcal{O}(x), \|\cdot\|_x)$.

Green fct on $C\times C$.

$$g_\Delta(x,y) := -\log \|1\|_{\Delta,a}(x,y) : (C^2)^{an} - \Delta^{an} \longrightarrow \mathbb{R} \rightsquigarrow \text{sym}$$

$$\| \quad g_x(y) := -\log \|1\|_x(y) : C^{an} - \{x\} \longrightarrow \mathbb{R}_{\geq 0} \quad = g_\Delta(x,y)$$

§ 3. Main thm. (Zhang metric).

Let K : non-archi field

C/K curve $g > 0$

Then there is a unique symmetric integrable metric $\underline{\|\cdot\|_{\Delta,a}}$ over $\mathcal{O}_{C^2}(\Delta)$

s.t. ($\forall$ any K'/K $x, y \in C(K')$)

① (compatibility)

$$c_1(\mathcal{O}(x), \|\cdot\|_x) = c_1(\mathcal{O}(y), \|\cdot\|_y)$$

$$\rightarrow \quad (2g-2)\, c_1(\mathcal{O}(x), \|\cdot\|_x) = c_1(\omega_C, \|\cdot\|_a) \quad \leftarrow \text{"admissible"}$$

$\rightarrow$ ② (normalization).

$$\|\cdot\|'_{\Delta,a} := \|\cdot\|_{\Delta.a} \cdot \underline{c}$$

$$\int_{(C_{K'})^{an}} g_{\underline{x}}(-)\, c_1(\mathcal{O}(x) \cdot \|\cdot\|_x) = 0.$$

Proof. (uniqueness).

$$\|\cdot\|_{\Delta,a} \rightsquigarrow \|\cdot\|_a \quad \|\cdot\|_x \qquad ① ②$$

$$\|\cdot\|'_{\Delta,a} \rightsquigarrow \|\cdot\|'_a \quad \|\cdot\|'_x \qquad ① ②$$

$$c_1\left(\mathcal{O}_C, \frac{\|\cdot\|'_a}{\|\cdot\|_a}\right) = c_1(\omega_C, \|\cdot\|'_a) - c_1(\omega_C, \|\cdot\|_a)$$

$$\| ①$$

$$\underline{(2g-2)}\, c_1\left(\underline{\mathcal{O}_C}, \frac{\|\cdot\|'_{x_0}}{\|\cdot\|_{x_0}}\right) = (2g-2)\left(c_1(\mathcal{O}(x_0), \|\cdot\|'_{x_0}) - c_1(\mathcal{O}(x_0), \|\cdot\|_{x_0})\right)$$

$$\| \quad \frac{\|1\|'_{x_0}(y)}{\|1\|_{x_0}(y)} : C \longrightarrow \mathbb{R}_{\geq 0} \quad = \underline{e^{\varphi(y)}}$$

$$c_1(\mathcal{O}_C, e^{(2g-2)\varphi(y)})$$

$$c_1\left(\mathcal{O}_C, \frac{\|\cdot\|'_a}{\|\cdot\|_a} e^{-\varphi(y)}\right) = 0 \iff \frac{\|\cdot\|'_a(y)}{\|\cdot\|_a(y)} = c_0 \cdot e^{(2g-2)\varphi(y)}$$

$$\frac{\|\cdot\|'_a}{\|\cdot\|_a} = c_0 \cdot e^{(2g-2)\varphi}$$

similarly. $$\frac{\|\cdot\|'_x(y)}{\|\cdot\|_x(y)} = e^{\psi(x)} e^{\varphi(y)} = e^{\varphi(x) + \varphi(y) + c_1}$$

$$g'_\Delta(x,y) - g_\Delta(x,y) = g'_x(y) - g_x(y)$$

$$= -\log \frac{\|1\|'_x(y)}{\|1\|_x(y)} = -\psi(x) - \varphi(y)$$

$$-\psi(x) - \varphi(y) = -\psi(y) - \varphi(x)$$

$$\psi(x) - \varphi(x) = \psi(y) - \varphi(y)$$

$$\psi - \varphi \equiv c_1$$

$$\begin{tikzcd} & (\omega_C^\vee, \|\cdot\|_a^{-1}) \arrow[d, dash] & (\mathcal{O}_{C^2}(\Delta), \|\cdot\|_{\Delta,a}) \arrow[d, dash] \\ \|\cdot\|_a^{-1}(x) \arrow[d, dash] & C \arrow[r, "\Delta"] & C\times C \\ x \arrow[ur, "x"] \arrow[r, "x"] \arrow[d, dash] & C \arrow[ur, "{(x,id)}"'] \arrow[r, dash] & (\mathcal{O}_C(x), \|\cdot\|_x) \\ \|\cdot\|_x(x) & & \end{tikzcd}$$

$$\|1\|_a^{-1}(x) = \|1\|_x(x)$$

$$\Rightarrow \|1\|_a(x) \cdot \|1\|_x(x) = 1$$

$$\Rightarrow \frac{\|\cdot\|_a'(x)}{\|\cdot\|_a(x)} \cdot \frac{\|\cdot\|_x'(x)}{\|\cdot\|_x(x)} = 1$$

$$e^{(2g-2)\varphi(x)} \cdot e^{2\varphi(x)+c_1} \equiv 1 \qquad \Rightarrow \varphi(x) \equiv c_2$$

$$e^{2\varphi(x)+c_1} \equiv 1$$

$$2\varphi$$

$$\int_{(C_{K'})^{an}} g_x(y)\, c_1(\mathcal{O}(x), \|\cdot\|_x) = \int -\log \|1\|_x(y)\, c_1(\mathcal{O}(x, \|\cdot\|_x))$$

$$\|$$

$$\int -\log \|1\|_x'(y)\, c_1(\mathcal{O}$$

Existence

$$\begin{tikzcd} & \mathcal{C}_s \arrow[dr] & \\ C \arrow[rr] \arrow[d] & & \mathcal{C} \arrow[d] \\ \operatorname{Spec} K \arrow[rr] & \operatorname{Spec} k \arrow[r] & \operatorname{Spec} \mathcal{O}_K \end{tikzcd}$$

$$(\mathcal{C}, \omega_{\mathcal{C}}) \rightsquigarrow (\omega_C, \|\cdot\|_{Ar})$$

$\mathcal{C}_s$

$\Gamma(C)$

$$C^{an} \times C^{an} \to \Gamma(C) \times \Gamma(C)$$

$$\cancel{g_\mu(\ ,\ )\ (C)^{an}}$$

$$\|\cdot\|_a(x) = \|\cdot\|_{Ar}(x) \cdot e^{g_\mu(x,x)}$$

$g_{\Delta,a}(x,y) := i(x,y)\log e_K + g_\mu(x,y)\log e_K.$ $\qquad \mathcal{O}(\Delta).$

$K \quad \mathcal{O}_K \quad \varpi \in \pi_K \qquad e_K := |\varpi|^{-1}$

e.g $K = \mathbb{Q}_p \quad \mathcal{O}_K = \mathbb{Z}_p \quad \varpi = p \in p\mathbb{Z}_p \quad e_K = p \in \mathbb{R}.$

$$\begin{array}{ccc} x,y & \longmapsto & \bar{x},\bar{y} \\ \Cap & & \Cap \\ C & \longrightarrow & \mathcal{C} \end{array}$$

$i(x,y) =$

§4. admissible line bundle.

Def (admissible — over A).
$\mathcal{L} \in \mathrm{Pic}(A)$.

(1). $\mathcal{L} \cong [-1]^*\mathcal{L}. \Rightarrow [2]^*\mathcal{L} \cong \mathcal{L}^{\otimes 4}.$

$(\mathcal{L}, \|\cdot\|_{\mathcal{L}})$ is adm $\Leftrightarrow [2]^*(\mathcal{L}, \|\cdot\|_{\mathcal{L}}) \cong (\mathcal{L}, \|\cdot\|_{\mathcal{L}})^{\otimes 4}.$

(2) $\mathcal{L} \cong ([-1]^*\mathcal{L})^\vee \Rightarrow [2]^*\mathcal{L} \cong \mathcal{L}^{\otimes 2}$

is adm.

(3). $\mathcal{L} \quad \Leftrightarrow \quad \mathcal{L} \otimes [-1]^*\mathcal{L} \qquad \mathcal{L} \otimes ([-1]^*\mathcal{L})^\vee$

Prop. ($\exists !$ admiss. metric, Abelian vari. case)

$(\mathcal{L}, \|\cdot\|_0). \qquad [2]: A \longrightarrow A^{\mathcal{L}}$

$$(\mathcal{L}, \|\cdot\|_i) := \left([2]^*(\mathcal{L}, \|\cdot\|_{i-1})\right)^{\frac{1}{4}}$$

Def. (adm line bundle over C)
Suppose $\underline{\mathrm{Pic}^1(C) \neq \phi}$.
fix $\alpha \in \mathrm{Pic}^1(C) \rightsquigarrow i_\alpha : C \longrightarrow J$

$(\mathcal{L}, \|\cdot\|_{\mathcal{L}})$ over C^{an} is called <u>admissible</u> if
$\exists\ (\mathcal{M}, \|\cdot\|_{\mathcal{M}})$ over J

- adm
- $\mathcal{M} \overset{alg}{\sim} \mathcal{O}_J(n\Theta_\alpha)$
- $i_\alpha^*(\mathcal{M}, \|\cdot\|_{\mathcal{M}}) \cong (\mathcal{L}, \|\cdot\|_{\mathcal{L}})^{\otimes m}$

$$\begin{array}{cccc} \|\cdot\|_{\mathcal{L}} & \|\cdot\|_{\mathcal{L}}^m & \|\cdot\|_{\mathcal{M}} & \\ \mathcal{L} & \mathcal{L}^{\otimes m} & \mathcal{M} & \mathcal{O}_J(n\Theta_\alpha) \\ & C & \longrightarrow & J \end{array}$$

$$\begin{array}{ccc} \mathcal{L} & & \mathcal{M} \\ C & \longrightarrow & J \end{array}$$

Prop. ($\exists !$ of adm metric)

Prop. (well-defined. + stable under base change. + .)

Prop. ("canonical definition").

$\exists$ measure $d\mu_a$ over $\underline{C^{an}}$ s.t. for $\forall (\mathcal{L}, \|\cdot\|_{\mathcal{L}})$ over C,

$\|\cdot\|_{\mathcal{L}}$ is adm $\Leftrightarrow$ $\|\cdot\|_{\mathcal{L}}$ is integrable

$$\overset{\|\cdot\|_{adm}}{\mathcal{M}} \sim 0 \qquad c_1(\mathcal{L}, \|\cdot\|_{\mathcal{L}}) = \deg(\mathcal{L})\, d\mu_a$$

Proof

① $C \xhookrightarrow{i_\alpha} J$

$$c_1(i_\alpha^* \mathcal{M}, i_\alpha^* \|\cdot\|_{adm}) = 0$$

② $$\int_{C^{an}} c_1(\mathcal{O}(x), \|\cdot\|_{adm}) = 1$$

$$d\mu_a = \frac{1}{\deg i_\alpha^* \mathcal{O}_J(\Theta_\alpha)} c_1(i_\alpha^*(\mathcal{O}_J(\Theta_\alpha), \|\cdot\|_{adm}))$$

$$\begin{array}{ccc} \|\cdot\|_{\mathcal{L}} & & \|\cdot\|_{\mathcal{M}} \\ \mathcal{L} & & \mathcal{M} \sim \underline{\mathcal{O}_J(n\Theta)} \\ C & \longrightarrow & J \end{array}$$

$$\begin{tikzcd} & \mathcal{O}_J(\Theta) \arrow[d, dash] & \mathcal{O}_{C^2}(\Delta) & P \arrow[d, dash] & \\ \mathcal{O}_C(x) \arrow[dr, dash] & J \arrow[rr, "④"] & & J \times J \arrow[r, "m"] \arrow[ur] \arrow[dr] & J \\ & C \arrow[u, "i_\alpha\ ①", "\omega_C"'] \arrow[r, "②"] & C \times C \arrow[ur, "③\ (i_\alpha, i_\alpha)"'] & & \end{tikzcd}$$

$$P := -m^* \mathcal{O}_J(\Theta_\alpha) + p_1^* \mathcal{O}_J(\Theta_\alpha) + p_2^* \mathcal{O}_J(\Theta_\alpha)$$

$\alpha \in \mathrm{Pic}^1(C)$ $\quad \beta \in J$

Lemma.

① $\cancel{i_\alpha^* \mathcal{O}_J(\Theta)} \cong \omega_C + (2-g)\alpha$

$i_\alpha^* \mathcal{O}_J([-1]^*\Theta) \cong g\alpha$. $\quad \rightsquigarrow \alpha$.

② $\Delta^*(\mathcal{O}_{C^2}(\Delta)) \cong -\omega_C$

$(x, id)^*(\mathcal{O}_{C^2}(\Delta)) \cong \mathcal{O}_C(x)$

③ $(i_\alpha, i_\alpha)^*(P) \cong \mathcal{O}_{C^2}(\Delta) - p_1^*\alpha - p_2^*\alpha$. $\quad \rightsquigarrow (\mathcal{O}_{C^2}(\Delta), \|\cdot\|_{\Delta,a})$

④ $\Delta_J^*(P) \cong -\mathcal{O}_J(\Theta_\alpha) - \mathcal{O}_J([-1]^*\Theta_\alpha)$

$(\beta, id)^*(P) \cong -T_\beta^* \mathcal{O}(\Theta_\alpha) + \mathcal{O}(\Theta_\alpha)$.

Claim. $\|\cdot\|_{\Delta,a} \rightsquigarrow \|\cdot\|_a, \|\cdot\|_x$ is admissible.