

STUDENT SEMINAR: MODULI OF VECTOR BUNDLES

XIAOXIANG ZHOU

Talk 1: Introduction.

Talk 2: Explicit constructions of semistable bundles on elliptic curves. In this talk, we introduce semistability of vector bundle on curves, and provide first non-trivial example of semistable vector bundles.

- Define slope stability [2, Definition 2.3]. State some basic properties, e.g., [2, Exercise 2.4], [1, 14.1].
- Recall the classification of vector bundles on \mathbb{P}^1 , and determine when they are (semi)stable.
- Show the picture of Ford circles. Fix an elliptic curve (E, p_0) . For each rational number $\mu = \frac{d}{r} > 0$, construct a stable vector bundle V_μ of rank r and degree d such that $\det V_\mu \cong \mathcal{O}_E(dp_0)$. [1, 14.3]
- Verify the stability of V_μ by induction. Shows that

$$\dim \operatorname{Hom}(V_{\mu_1}, V_{\mu_2}) = \dim \operatorname{Ext}(V_{\mu_2}, V_{\mu_1}) = \begin{cases} d_2 r_1 - d_1 r_2, & \mu_1 < \mu_2 \\ 1, & \mu_1 = \mu_2 \\ 0, & \mu_1 > \mu_2. \end{cases}$$

Discuss further properties, including [1, Corollary 14.11]. In particular, describe $H^\bullet(V_\mu)$.

- For each $r \geq 1$ and $\mathcal{L} \in \operatorname{Pic}^0(E)$, construct a semistable vector bundle $V_{r,\mathcal{L}}$ of rank r with $\det V_{r,\mathcal{L}} \cong \mathcal{L}$.
- Conclude the talk by stating [2, Example 2.7] as a theorem.

Talk 3: Fourier–Mukai transform on elliptic curve.

Talk 4: Vector bundles on curves of genus ≥ 2 .

Talk 5: Semistable sheaves of degree 0. [2, 2.4]

Talk 6: Stability manifold of \mathbb{P}^1 .

Talk 7: Equivariant vector bundles on Grassmannian.

Talk 8: Chern class.

Talk 9: Exceptional vector bundles on \mathbb{P}^2 .

Talk 10: Stability manifold of surfaces.

Talk 11: vector bundles on K3 surfaces.

Talk 12: vector bundles on threefolds.

REFERENCES

- [1] Joseph Le Potier. *Lectures on vector bundles*, volume 54 of *Camb. Stud. Adv. Math.* Cambridge: Cambridge University Press, 1997.
- [2] Emanuele Macrì and Benjamin Schmidt. Lectures on Bridgeland stability. In *Moduli of curves. CIMAT Guanajuato, Mexico 2016. Lecture notes of a CIMP-ICTP school, Guanajuato, Mexico, February 22 – March 4, 2016*, pages 139–211. Cham: Springer, 2017.

INSTITUT FÜR MATHEMATIK, HUMBOLDT-UNIVERSITÄT ZU BERLIN, BERLIN, 12489, GERMANY,
Email address: `email:xiaoxiang.zhou@hu-berlin.de`