

Eine Woche, ein Beispiel

7.17 formalism

Here I collect some formalisms which enhance well-known theories.

Somehow it also explains the popularity of the abstraction in mathematics. We want to have a better understand, so we invent new languages and theories, which in turn causes more troubles for beginners. Hug those formalisms!

Axiomatic set Theory

→ Grothendieck Universe

Both let us be away from paradoxes. The latter is more convenient.

Category Theory

→ Infinite Category Theory

[adjoint fctor \Leftrightarrow preserve (co)limits] is some cases

(/Braided/Symmetric) Monoidal Categories → Categorification

Extract combinatorial informations from categories

Examples

→ Classification

Specify and generalization.

See https://github.com/ramified/personal_tex_collection/blob/main/%E8%9B%99%E9%B8%A3-7/latest%20version.pdf

Parameter space

→ Moduli space

Structures on parameter spaces are important.

See https://github.com/ramified/moduli_in_algebraic_geometry

Topology

→ Grothendieck Topology

Get étale topology.

Topological space

→ Condensed Set

Get abelian category.

Scheme

→ Functor

Get more objects. e.g. Ind-Sch, stack, ...

Snake lemma

→ homological algebra, spectral sequence

Diagram chasing all the time!

(co)homology

→ derived category, six-fctor formalism.

Reduce important properties to categorical non-sense.

Finite field

→ ???

Should contain "field with one element".

Moreover, AR theory tells us the structure of indecomposable reps,

Bruhat-Tits theory tells us the structure of p -adic groups,

Artin-Schreier theory tells us the structure of $\deg p$ extensions. $\text{char } F = p$