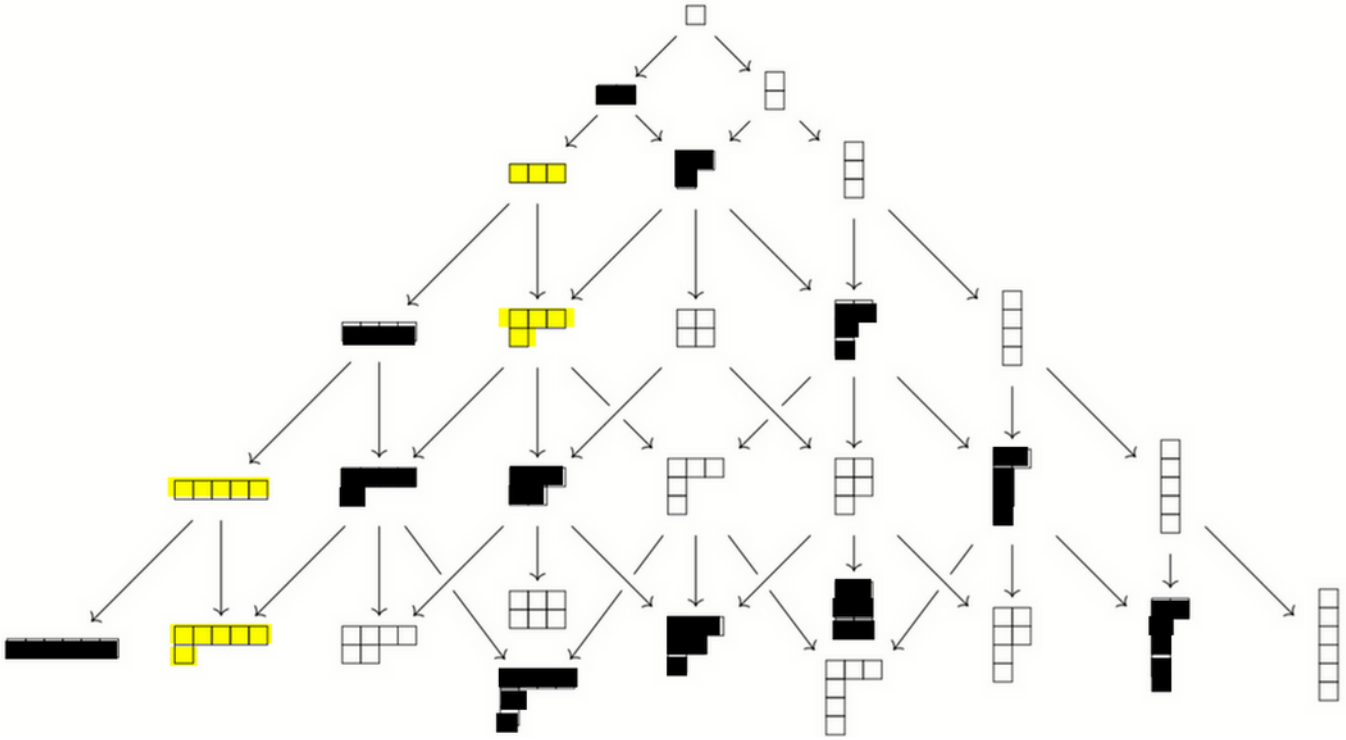
Eine Woche, ein Beispiel

7.25 irreducible representation of S_n / A_n



A000702

S_2 / A_2

	1^2	2^1
	1	(12)
	1	-1
	1	1

S_3 / A_3

	1^3	3^1	$2 \cdot 1$
	Id	(123) (132)	(12)
	1	1	-1
	1	1	1
	1	1 1	
	2	-1	0
	1	ρ ρ^2	
	1	ρ^2 ρ	

S_4/A_4

	1^4	2^2	$3\cdot 1$	$2\cdot 1^2$	4^1
	Id	(12)(34)	(123) (132)	(12)	(1234)
	1	1	1	-1	-1
	1	1	1	1	1
	3	-1	0	-1	1
	3	-1	0	1	-1
	2	2	-1	0	0

S_5/A_5

	1^5	$2^2\cdot 1$	$3\cdot 1^2$	5^1	$2\cdot 1^3$	$3\cdot 2$	$4\cdot 1$
	Id	(12)(34)	(123)	(12345) (12345)	(12)	(123)(45)	(1234)
	1	1	1	1	-1	-1	-1
	1	1	1	1	1	1	1
	4	0	1	-1	-2	1	0
	4	0	1	-1	2	-1	0
	5	1	-1	0	-1	-1	1
	5	1	-1	0	1	1	-1
	6	-2	0	1	0	0	0
	3	-1	0	$\frac{1+\sqrt{5}}{2}$ $\frac{1-\sqrt{5}}{2}$			
	3	-1	0	$\frac{1-\sqrt{5}}{2}$ $\frac{1+\sqrt{5}}{2}$			

S_6 / A_6

	1^6	$2^2 \cdot 1^2$	$3 \cdot 1^3$	3^2	$4 \cdot 2$	$5 \cdot 1$		$2 \cdot 1^4$	2^3	$3 \cdot 2 \cdot 1$	$4 \cdot 1^2$	6
	Id	$(12)(34)$	(123)	$(123)(456)$	$(1234)(56)$	(12345)	(12354)	(12)	$(12)(34)(56)$	$(123)(45)$	(1234)	(123456)
	1	1	1	1	1	1		-1	-1	-1	-1	-1
	1	1	1	1	1	1		1	1	1	1	1
	1	1	1	1	1	1	1					
	5	1	2	-1	-1	0		-3	1	0	-1	1
	5	1	2	-1	-1	0		3	-1	0	1	-1
	5	1	2	-1	-1	0	0					
	9	1	0	0	1	-1		-3	-3	0	1	0
	9	1	0	0	1	-1		3	3	0	-1	0
	9	1	0	0	1	-1	-1					
	10	-2	1	1	0	0		-2	2	1	0	-1
	10	-2	1	1	0	0		2	-2	-1	0	1
	10	-2	1	1	0	0	0					
	5	1	-1	2	-1	0		-1	3	-1	1	0
	5	1	-1	2	-1	0		1	-3	1	-1	0
	5	1	-1	2	-1	0	0					
	16	0	-2	-2	0	1		0	0	0	0	0
	8	0	-1	-1	0	$\frac{1+\sqrt{5}}{2}$	$\frac{1-\sqrt{5}}{2}$					
	8	0	-1	-1	0	$\frac{1-\sqrt{5}}{2}$	$\frac{1+\sqrt{5}}{2}$					

We use Frobenius character formula to compute characters of S_n. For a clear statement of Frobenius character formula, see here: https://mathoverflow.net/questions/323949/frobenius-formula

After this, we can compute characters of A_n by using the theories in Fulton's book [Rep, Lec 5.1].