Eine Woche, ein Beispiel

1.28 conormal bundle
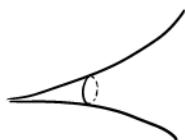
1. conceptions describing the singularity

$$\text{tangent cone} \quad \subseteq \quad \bigcup \text{ limit of tangent spaces} \quad \subseteq \text{ tangent space}$$

in Grassmannian, for affine variety

e.g.

$$\{\omega^2 = z^3\} \qquad \{\omega^2 = 0\} \qquad = \{v \in \mathbb{C}^2 \mid \langle v, d\omega \rangle = 0\} \qquad \subseteq \qquad \mathbb{C}^2$$

$$\{z_1^2 + z_2^3 + z_3^4 = 0\} \quad \{z_1^2 = 0\} \qquad = \left\{ v \in \mathbb{C}^3 \;\middle|\; \begin{array}{l} \langle v, a\,dz_1 + b\,dz_2 \rangle = 0 \\ \text{for some } (a,b) \in \mathbb{C}^2 - 0 \end{array} \right\} \subseteq \qquad \mathbb{C}^3$$

Apart from these conceptions, we have topological cone:

$$\{\omega^2 = z^3\}$$
with link $S^1 \subseteq S^3$

singularity $\quad$ link
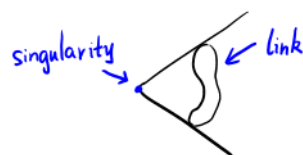
$$\{z_1^2 + z_2^3 + z_3^4 = 0\}$$
with link $\quad S^3/_{\mathbb{Z}/3\mathbb{Z}} \subseteq S^5$