

Roadmap

- Group structure** \rightsquigarrow **Representation** \rightsquigarrow **Geometrical object.**

 - Group structure:** set, topology, root system, Tits system, BT-theory.
 - Representation:** character classification, L-fct.
 - Geometrical object:** geo rep, cohomology, intersection, \int -fct.

- Automorphic world** $\xrightarrow{\text{Shimura variety}} \text{Motive}$

$\text{Automorphic world} \xleftarrow{\text{modularity}} \text{Gal rep}$

$\text{Motive} \xrightarrow{\text{FM conj}} \text{étale cohomology}$

- | | finite field | | local field | field | | global field | | Adèle |
|---------------|----------------|----------------|------------------------------|----------------|---------------------|--------------|-------------------|----------------|
| | | | Archi | NA | | | | |
| base field | \mathbb{F}_l | \mathbb{F}_p | \mathbb{R} or \mathbb{C} | \mathbb{Q}_p | $\mathbb{F}_p((t))$ | \mathbb{Q} | $\mathbb{F}_p(t)$ | \mathbb{A}_K |
| integral ring | — | — | — | \mathbb{Z}_p | $\mathbb{F}_p[[t]]$ | \mathbb{Z} | $\mathbb{F}_p[t]$ | K |

only analog

- $(G(F)$ -case)**

G	A'	G_m	GL_n	red gp	$(C, B, \text{Unipotent}, \dots)$
$G(F)$	F	F^\times	$GL_n(F)$	$G(F)$	
$G(\mathbb{A}_K)$	\mathbb{A}_K	\mathbb{I}_K	$GL_n(\mathbb{A}_K)$	$G(\mathbb{A}_K)$	

Pseudo-reductive Groups

(both $G(F)$ & Galois)

coefficient ring Δ : $\mathbb{C}, \bar{\mathbb{Q}}_p, \bar{\mathbb{F}}_p, \bar{\mathbb{Z}}_p, \dots$

Roughly, need to solve $3 \times 2 \times 8 \times 4 = 192$ cases
 + much more connections

The arrow roughly means.

$$\begin{array}{c} 2 \mid \\ \frac{1}{1} \\ 2 \mid \\ \frac{1}{1} \end{array}$$

Usual route: fix 3 & 4, $\boxed{\begin{array}{c} 2 \\ G \text{ or } A \end{array}} \rightarrow 1$

Anna's route: fix 4 (GL), $\boxed{\begin{array}{c} 2 \\ G, A, M \end{array}} \rightarrow 3 \rightarrow 1$

Our route: $1 \rightarrow \boxed{\begin{array}{c} 2 \\ G, A \end{array}} \rightarrow 3 + 4$

connections in 2 are delayed.

Program

§ 1. No rep

- ① [1. Structure of finite/local/global field
- ② [2. Structure of reductive gp (GL_n)

§ 2. 1-dim rep

- [1. Character of Galois gp ↗ corresponds to rep of F^\times
- [2. Character of red gp

§ 3. Rep

- [1. Galois rep
- [2. Rep theory of red gp

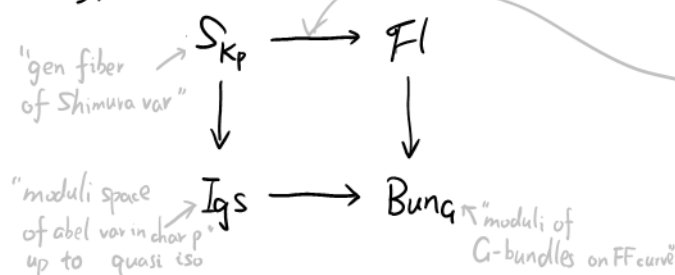
§ 4. Geometrical rep

- ④ [1. EC
- ③ [2. MF $\left(\begin{array}{l} \text{Moduli space} \\ \text{Shimura variety} \\ \text{Modular curve} \end{array} \right)$

3. Flag variety

§ 5. Connections

- [1. MF $\xrightarrow{\text{ES iso}}$ Gal rep
- ⑤ [2. MF $\xleftarrow{\text{modularity}}$ Gal rep
- 3.



§ 6. Non-classical Langlands

1. Geometrical Langlands
2. Categorical geometrical Langlands

Galois gp, Frob, Weil gp
Tits system. BT-theory

local class field theory
 \hat{F}^\times & $\hat{F}^{\times*}$, $\hat{\mathcal{O}}_F^\times$ & $\hat{\mathcal{O}}_F^{\times*}$, Hecke character

WD-rep

$l \neq p$: l -adic monodromy thm
 $l = p$: Hierarchy of p -adic Galois rep
global: Chebotarev density thm
 $f_{in} / NA / IR / A_K$

preliminary

Hecke alg
classification (Hierarchy) $\left\{ \begin{array}{l} \text{principal series} \\ \text{cuspidal} \\ \dots \end{array} \right.$

étale cohomology, Fontaine-Mazur conj
Shimura data
equiv def of MF

Rep II

ES iso, ES relation
Deligne-Serre thm
Modularity

Mingjia's work:
HT period map
Torelli theorem

- Farques-Schulze
- Chenji's work

<https://mathoverflow.net/questions/56571/a-precise-statement-of-the-categorical-version-of-geometric-langlands-conjecture>

Also, in each part:

- Describe L-fcton
- Describe connections in section / among sections / with last part.