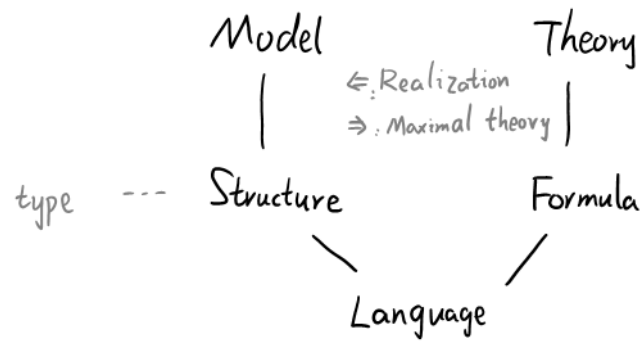


Eine Woche, ein Beispiel

3.27 model theory

Ref: <https://philippschlicht.github.io/teaching/files/Lecture.pdf>

I heard something from Yilong Zhang, and want to jot down some key points so that I won't be confused next time.



Modern Algebra (H)

Preliminaries

- Logic, set & map, operations on Set. $\left(\cap \cup -^c, \text{sub/quotient}, \right.$
 $\left. \text{two ways of disjoint union} \right)$
 - Axiomatic set theory (ZFC)
 - Russell's paradox
 - type of proof: constructive, algorithm, ...

Ex. graph

- From \mathbb{N} to \mathbb{C} (the basic of examples, though logically it's not here)

- Peano axioms. Axiom of induction
- alg structure, order and topology
- Completeness axiom.

- Cardinal: the only property of set.
 - naive definition

<https://math.stackexchange.com/questions/1712964/attempt-at-proving-the-class-of-all-cardinals-is-a-proper-class>

	alg	total order	topo
\mathbb{N}	$(+, \times)$	✓	discrete
\mathbb{Z}	$(+, -, \times)$	✓	discrete
\mathbb{Q}	$(+, -, \times, \div)$	✓	dense but not complete
\mathbb{R}	$(+, -, \times, \div)$	✓	complete
\mathbb{C}	$(+, -, \times, \div)$	X	complete

- operations on cardinal
- examples
- The continuum hypothesis
- large cardinal axiom

- Order structure
 - def, operations and properties (partial/total/well order)
 - ordinal, relationship with cardinal.

Ex. "well-order" on class of cardinals.

Classify subpartial set of $\{\text{subsets of } \{1, \dots, n\}\}$, up to iso. $n \leq 5$

A bird eye's view of gp theory

- Group, field and v.s.
 - Group: *def
 - * initial example: Aut of set, ordered set, graph; $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$.
 - Field: def + example: $\mathbb{Q}, \mathbb{Z}/p\mathbb{Z}, \mathbb{R}, \mathbb{C}$
 - v.s. def + example: K^n , fct space
 - recall: generators, basis, dim; extra alg structures on v.s.
- "Group is symmetry"
 - more examples. Galois gp & matrix gp
 - Cayley's thm
 - group action: on sets, on graph, on v.s., ...

Ex. concepts of group action on sets

reminder: $GL_2(\mathbb{R})$, GH , fundamental gp, homotopy gp, $E(\mathbb{Q})$, braid group, ...

- Universal property with group
 - sub/quotient three iso.
 - Ker / Im direct sum & product
 - free group
 - presentation of group

Ex. coset decomposition

Ex. $Z(G), [G, G]$, centralizer and normalizer

- Decomposition of group
 - simple gp, ind gp
 - filtration
 - Zassenhaus lemma, Schreier refinement theorem
 - Split, semi-product gp
 - Results of simple group.

Ex. cyclic gp case

abelian gp case. \leadsto "abelian category"

Down-to-earth analysis.

- f.g. abelian gp + apps.
 - Ex. lattice & Crystallographic point gp
- G act on G
- Sylow thm
- app: classifications of gp of small order. research on specific gp.

Ring & module

- Basic def, e.g. $R[x]$, $R[[x]]$, ...
- Category, k -algs
- Basic def of modules, abelian category

E.x. R -algs.

three rep theory

- AG translation, examples.
- Concepts under AG translation
- $ED \Rightarrow PID \Rightarrow UFD \Rightarrow \text{domain}$
- classification of f.g. module over PID.

Field and Galois theory (See [GTM167])

can focus more on \mathbb{F}_p , \mathbb{Q}_p , $\mathbb{F}_p((t))$, and geometrical point of view.
(Reminder) other structures: norm, metric & topo; measure; sheaf.