

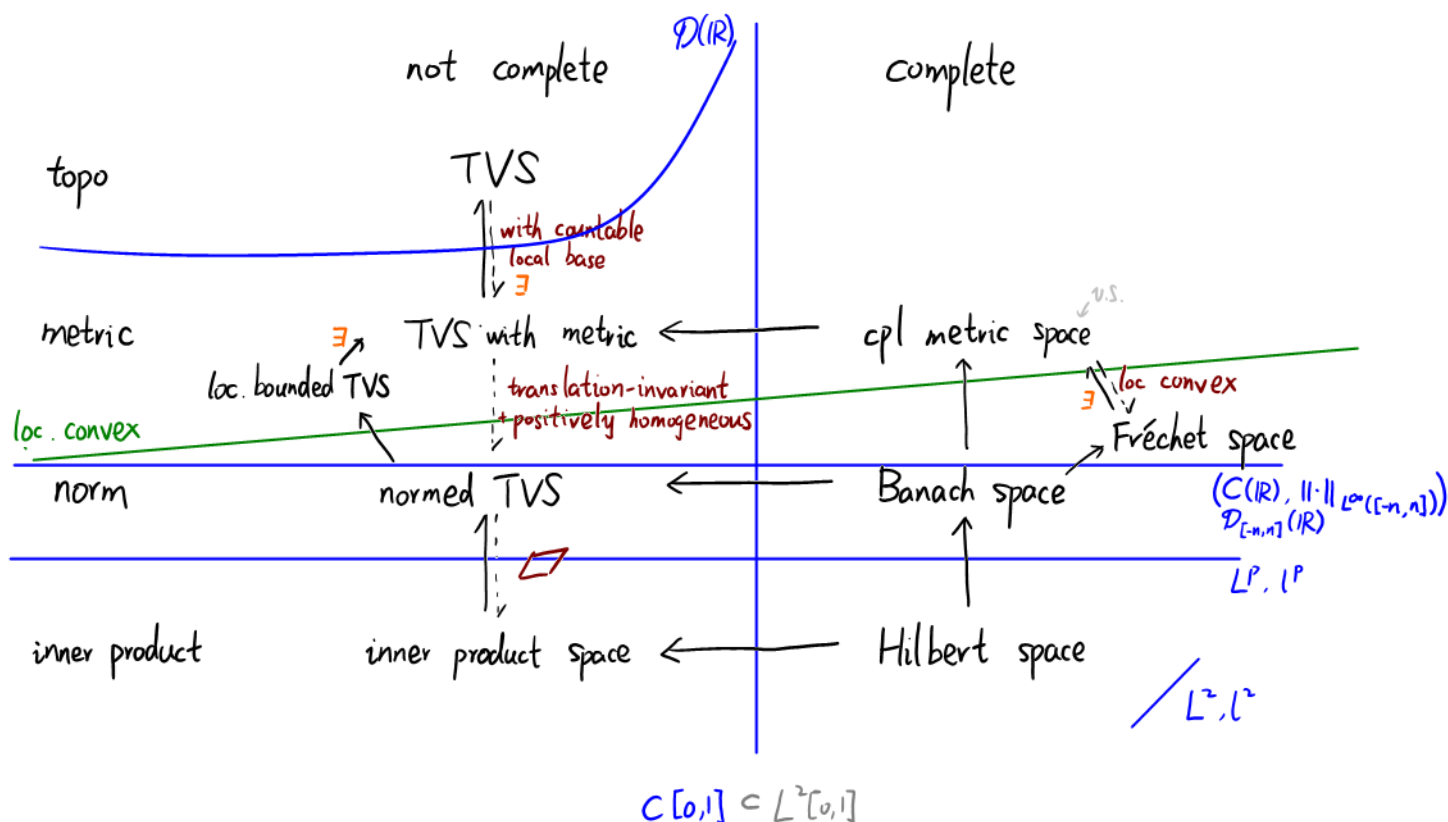
Eine Woche, ein Beispiel
4.30 TVS = topological vector space

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Lec 1-7: <http://staff.ustc.edu.cn/~wangzuoq/Courses/15F-FA/index.html>

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In this document, we don't worry about extra structure here, and we assume Hausdorff.



\exists : exists a metric

metrizable TVS = TVS with countable local base

seminormable TVS = TVS with **convex** local base

normable TVS = TVS with a convex bounded nbhd

F-space = TVS + \exists cpl tran-inv metric

<https://math.stackexchange.com/questions/3356421/example-of-a-locally-convex-topological-vector-space-which-is-not-metrizable>

Fréchet space = countable sep seminorms $\{p_n\}$ + \exists cpl tran-inv metric
 = loc convex or: the induced metric is cpl.
 + \exists cpl tran-inv metric

Rmk. There are two definitions of boundedness in metrizable TVS X , and they coincide if the metric is translation-invariant.

Def. (boundedness for TVS)

$E \subset X$ is bounded if $\forall U \in \mathcal{U}$ open, $\exists s > 0$ s.t.
 $\forall t > s, \quad E \subset tU$

In this case,

$$E \text{ is bounded} \Leftrightarrow \left[\begin{array}{l} \forall \{x_n\} \subset E, \{ \alpha_n \} \subseteq \mathbb{R} \text{ or } \mathbb{C}, \\ \alpha_n \rightarrow 0 \Rightarrow \alpha_n x_n \rightarrow 0 \end{array} \right]$$

Def. (boundedness for metric space)

Fix $x_0 \in X$. boundedness does not depend on x_0 .

$E \subset X$ is bounded if $\exists r > 0, \quad E \subset B_{x_0}(r)$.

Rmk. 1. For loc convex TVS with metric, all open balls are convex.
 2. cpl metric space $\Rightarrow 2^{\text{nd}}$ category.

1st category set

<https://math.stackexchange.com/questions/1237159/understanding-the-definition-of-nowhere-dense-sets-in-abbotts-understanding-ana>

Def. A closed subset $A \subset X$ is nowhere dense if A^c is dense in X , i.e., $A^\circ = \emptyset$



A closed

Def. $A \subset X$ is of 1st category, if

$$A \subset \bigcup_{i \in \mathbb{Z}_{>0}} A_i \quad \text{for some } A_i = \overline{A_i} \text{ nowhere dense.}$$