Eine Woche, ein Beispiel

8.17 tropical hypersurface

Ref:

https://arxiv.org/abs/1311.2360

How to draw these tropical curves:
https://mathoverflow.net/questions/328342/how-to-draw-tropical-curves
https://ntiggemann.github.io/coding.html#Plotting_tropical_curves

K: valued field $v: K \longrightarrow \mathbb{R} \cup \{+\infty\}$ most time: $\mathbb{Z} \cup \{+\infty\}$
$X \subseteq \mathbb{A}^n_K$ variety

$$x \in X(K) \quad \Rightarrow \quad -v(x) \in \mathrm{Trop}(X)$$
$$Y \subseteq X \quad \Rightarrow \quad \mathrm{Trop}(Y) \subset \mathrm{Trop}(X)$$

⚠ If we want compatability of $v(x)$ with $\oplus$, then we should define $u \oplus v = \min(u, v)$.
Usually tropical people don't do this, they want

"addition of positive number should go up".

We respect the conventions.

$$v(x+y) \geqslant \min(v(x), v(y))$$
$$v(xy) = v(x) + v(y)$$
$$v(0) = +\infty$$

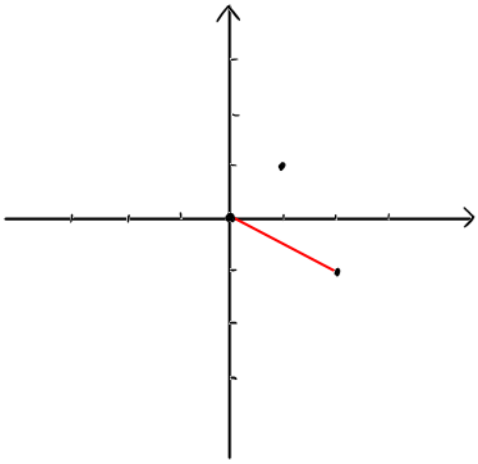
$u \oplus' v = \min(u, v)$
$u \otimes' v = u + v$
$+\infty$ $\mathbb{T} = \mathbb{R} \cup \{+\infty\}$
read from bottom

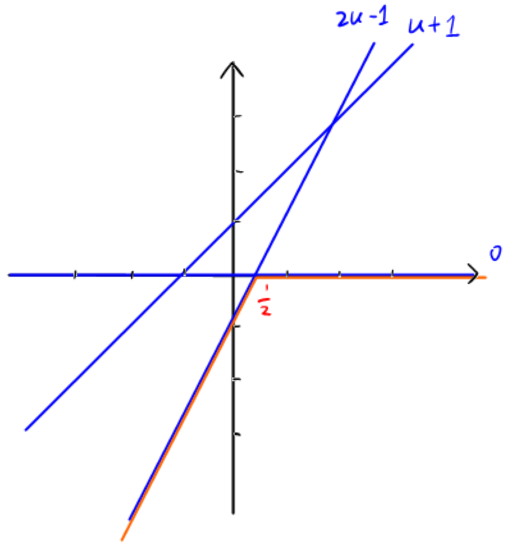
$u \oplus v = \max(u, v)$
$u \otimes' v = u + v$
$-\infty$
read from above

in calculation

Relation with Newton polygon

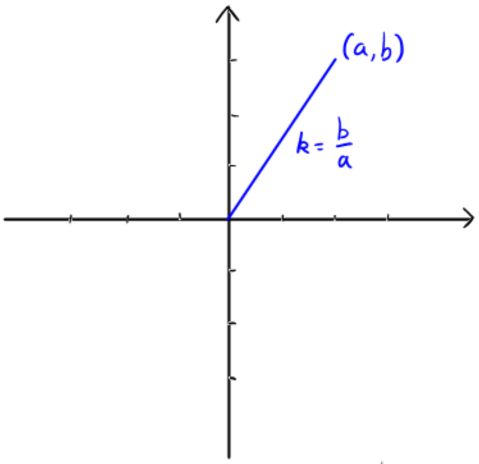
E.g.

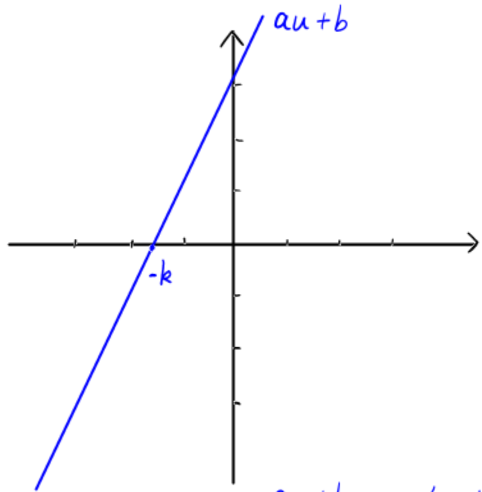


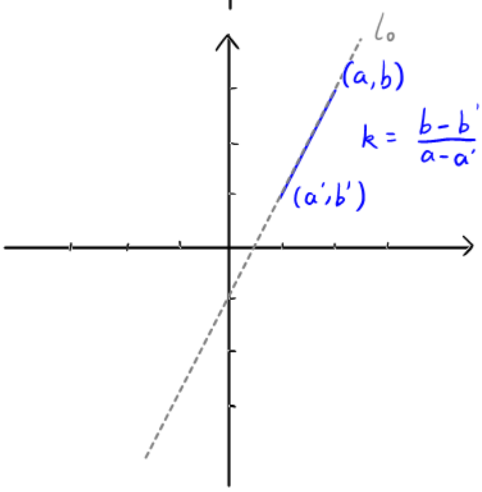


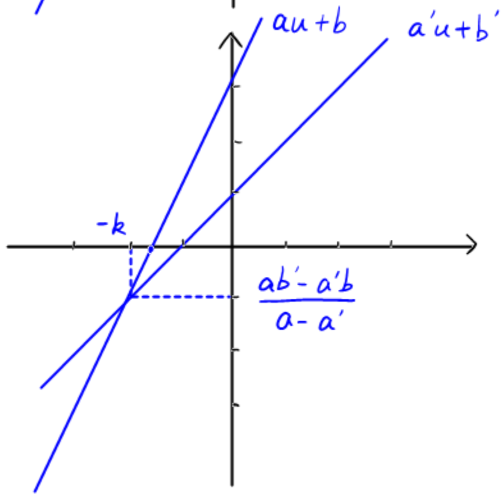
$1 + 5z + \frac{1}{5} z$

$0 \oplus' (u+1) \oplus' (2u-1)$



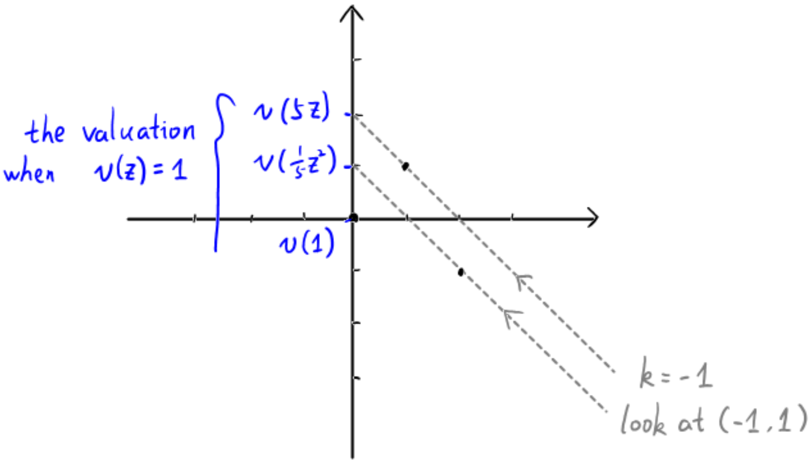






Rmk. $p = (x, y) \in l_0 \iff l_p = $ "$xu+y$" passes through $\left(-k, \frac{ab'-a'b}{a-a'}\right)$

better point of view

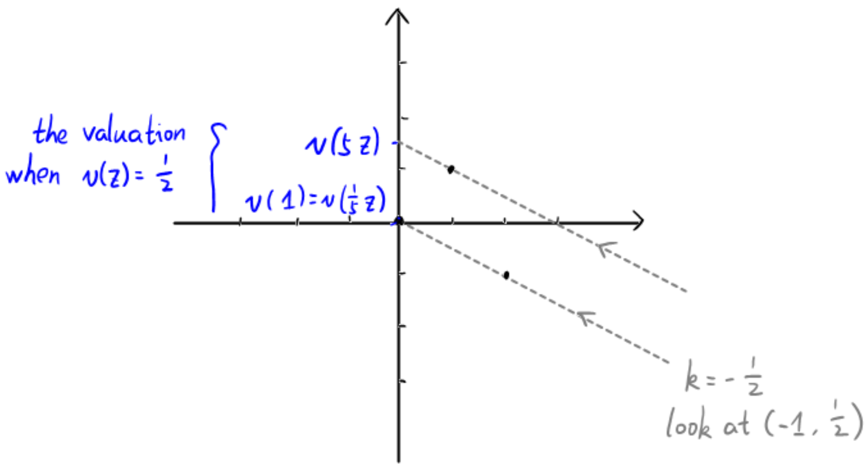


$1+5z+\frac{1}{5}z$

A special valuation of z may be seen as a kind of projection.

You can then read the value as though from the markings of a graduated cylinder.

It is curious that mathematicians read numbers from unexpected angles, rather than from the usual horizontal view.



$1+5z+\frac{1}{5}z$

When the two values meet and rest at the very bottom among all values, we have the possibility that v(f)=+\infty.

This happens when the gaze brings the two points into perfect alignment; the negative of the slope of this sightline is v(z).

That line is exactly the lower convex edge of the Newton polygon.

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