

# Eine Woche, ein Beispiel

5.15  $\mathbb{F}_1$

Ref: <https://core.ac.uk/download/pdf/33527661.pdf>

It's a pretty fancy topic. I discussed with Qirui Li, and finally we don't figure out the essential.

[https://en.wikipedia.org/wiki/Field\\_with\\_one\\_element](https://en.wikipedia.org/wiki/Field_with_one_element)

There are a lot of pretty tables here: <https://ncatlab.org/nlab/show/field+with+one+element>

$F$	$\mathbb{F}_1$	$\mathbb{F}_{1^n}$	$\mathbb{F}_q$
$\#F$	$1 + \epsilon$	$1 + k\epsilon$	$q$
$\#F^x$	$\epsilon$	$k\epsilon$	$q-1$
v.s. of $\dim d$	set of elements	free $\mu_k$ -set	$V$
$\#v.s.$	$1 + d\epsilon$	$1 + kd\epsilon$	$q^d$
$GL_n(F)$	$B \rtimes S_n$		$GL_n(\mathbb{F}_q)$
$\#GL_n(F)$	$\epsilon^n n!$		$(q-1)^n [n]_q! \cdot q^{\binom{n}{2}}$
simple alg gp	Weyl gp		$PGL_n(\mathbb{F}_q), \dots$
$\bar{F}$			
Frob			
Galois gp			
$W(F)$			
$\text{Spec } F$			
$\pi_{\text{ét}}(\text{Spec } F)$			
Riemann conj			

Task: Understand  $\mathbb{F}_{1^n} \otimes_{\mathbb{F}_1} \mathbb{Z} = \mathbb{Z}[T]/(T^n - 1)$

<https://mathoverflow.net/questions/272498/analogies-supporting-heuristic-weyl-groups-algebraic-groups-over-field-with-o>