Eine Woche, ein Beispiel

4.21 cohomology calculation sheet

All the cohomology in this sheet are for constant sheaves.

Appetizer

Try to compute

$H_c^\cdot([0,1))$ & $H^{BM}_\cdot([0,1))$

$H_c^\cdot$ (Möbius strip)

For the following spaces, compute $H^\cdot$, $H_\cdot$, $H_c^\cdot$ & $H^{BM}_\cdot$.

Lasy guys can just compute $H^\cdot$

? means: I don't know the answer

Easy mode

$\mathbb{C}^n - \{0\}$

$\mathbb{C} \cup_{\{0\}} \mathbb{C}$

for $K \subseteq S^3$ knot $\quad S^3 - K$ 801770

$\mathbb{CP}^2$

$\{[x:y:z] \in \mathbb{CP}^2 \mid y^2 z = x(x+z)(x-z)\}$

? (without using Hodge) $\{[z_0:z_1:z_2:z_3] \in \mathbb{CP}^3 \mid z_0^3 + z_1^3 + z_2^3 + z_3^3 = 0\}$ [21.04.08]

Hyperplane mode

compute $H^\cdot(\mathbb{C}^n - Z)$, $H^\cdot(Z - \{0\})$ & $H_c^\cdot(Z)$, where $Z \subseteq \mathbb{C}^n$

$Z =$

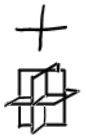
$\{z_1 z_2 = 0\}$

$\{z_1 z_2 z_3 = 0\}$

$\{z_1 z_2 z_3 (z_1 + z_2 + z_3) = 0\}$

$\{z_1^2 + z_2^2 + z_3^3 = 0\}$

$\{z_1^2 + z_2^2 + z_3^3 = 1\}$



Hint for the last case: consider the Morse fct

$\|\cdot\| : Z \longrightarrow \mathbb{R} \qquad z \longmapsto \|z\|$

$\{z_2^2 z_3 = z_1(z_1 + z_3)(z_1 - z_3)\}$

$\{z_2^2 = z_1^3 + z_1^5 z_2^4 + z_2^{10} + z_1^{100}\}$

Infinite mode

$\mathbb{Z}$

$\{\frac{1}{n}\} \cup \{0\}$

$\beta\mathbb{N}$

$\operatorname{Spec}\mathbb{Z}$ (non-Haus, only computes H^i)

$\mathbb{C} - \mathbb{Z}$

$\mathbb{R}^\infty$

$\mathbb{C}P^\infty$

$\mathbb{R}P^\infty$

Q. Why can't one compute $H_c^i(\mathbb{R}^\infty)$?
Can one argue by

$$H_c^i(\mathbb{R}^\infty) = H^i(S^\infty, \{\infty\})\ ?$$

A. No.