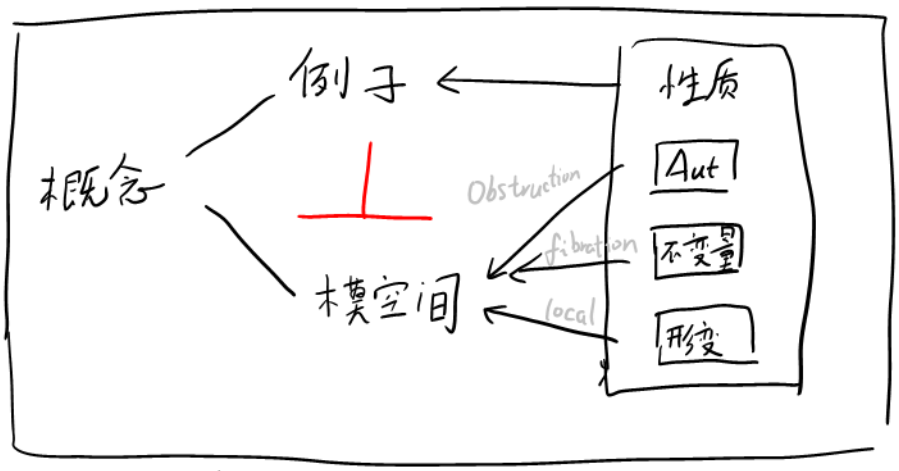
系统性　　严格性

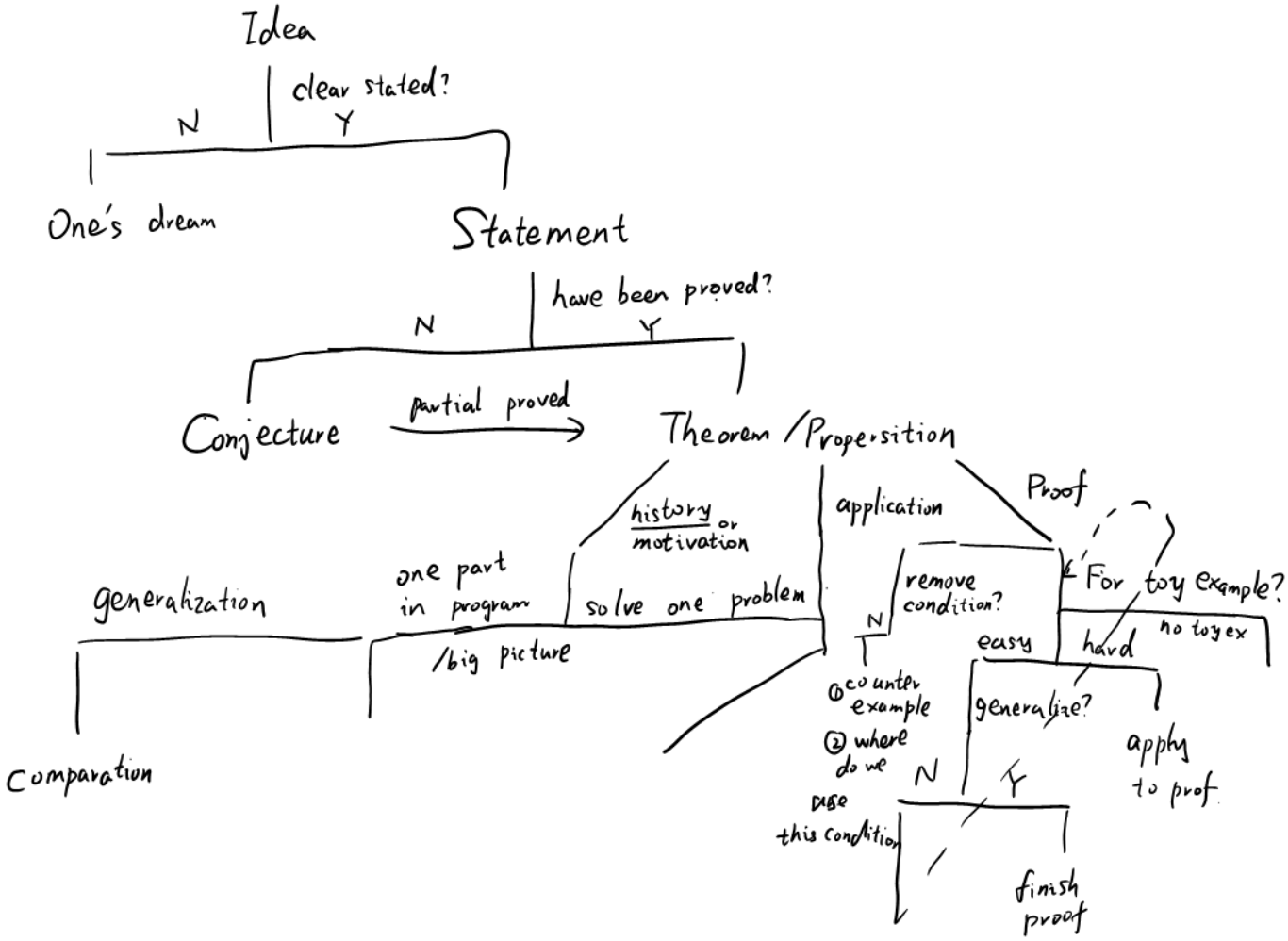
生动性（趣味性）

可行性　　几何（化）性



分类 ≈ 对概念的理解.

词典的重要性,词典的完备性
例子：
把几何对象翻译成代数对象(交换代数申屠老师的词典)
把新的对象翻译成旧对象(Grothendieck topology...)


Idea
clear stated?
N
Y
One's dream
Statement
have been proved?
N
Y
Conjecture
partial proved
Theorem / Propersition
history or motivation
application
Proof
generalization
one part in program
solve one problem
/big picture
remove condition?
N
For toy example?
no toy ex
easy
hard
① counter example
② where do we use this condition
generalize?
apply to prof
N
Y
finish proof
Comparation

从Grothendieck拓扑个是拓扑谈起
数学概念的命名往往带有无意的误导性。
同名不同义：平坦(flat), seperated scheme/map and seperated presheaf
同源不同义：reduced & reducible 约化的不可约概形
同名多类(支持变量多个类型)：拓扑空间的基本群，根系的基本群，概形的étale基本群
affine scheme, affine map, affine scheme over S: 是affine scheme + over S还是scheme over S+ affine map?
exact functor(of abelian category/triangulated category)

历史遗留问题：
presheaf, seperated presheaf, sheaf(我们按照现代观点，或者说Vakil的note)

概念的含混性：Borel-Moore同调是同调吗？紧支上同调是上同调吗？
推广而非限制：Grothendieck拓扑和étale拓扑都不是拓扑
含混义：descent

我大致知道的同调上同调：
Borel-Moore homology
compact supported cohomology

de Rham cohomology
cellular (co)homology
simplicial (co)homology
singular (co)homology

我大致不知道的同调上同调：
Cêch cohomology
sheaf cohomology
Étale cohomology
Galois cohomology

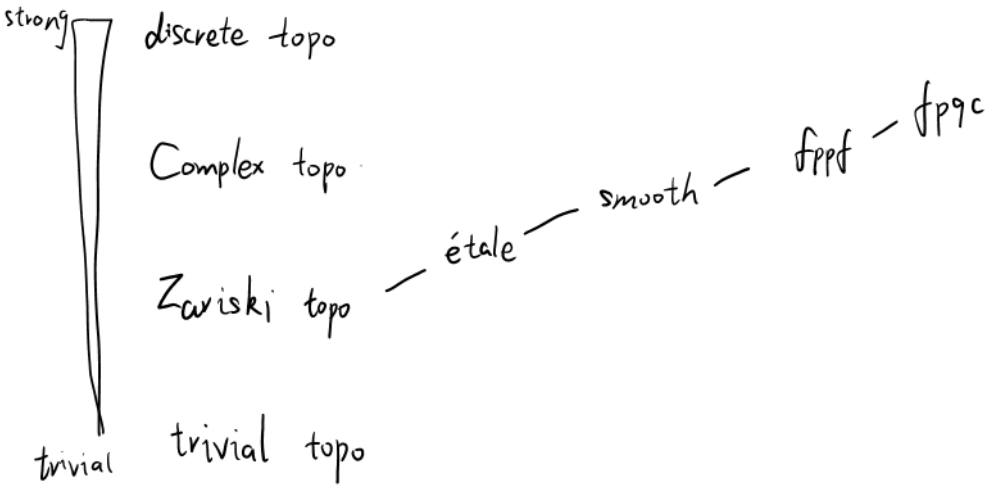
Group (co)homology
Hochchild (co)homology

l-adic cohomology
intersection (co)homology
crystalline cohomology
elliptic cohomology
flat cohomology
infinitesimal cohomology

代数拓扑需要学啥？
Poincaré duality
推出和拉回
Lefschetz trace formula
Lefschetz hyperplane theorem

https://mathoverflow.net/questions/257928/difference-between-the-small-and-big-%c3%a9tale-flat-site

		Object "Open set"	Mor "Covering of open set"
			Grothendieck Top
$X = \mathrm{Spec}\, k$, $\{Id\}$	small Zariski site	X_{Zar} open subset of $X \subseteq Sch/X$ open immersion over X	*
$\{Y \to \mathrm{Spec}\, k\}$	big Zariski site	$(Sch/X)_{Zar}$ Sch/X	* open immersion
$\{\coprod_i \mathrm{Spec}\, l \to \mathrm{Spec}\, k\}$	small étale site	$X_{ét}$ étale + l.f.p over X	* $\{U_i \xrightarrow{f_i} U\}_{i\in I}$, $\bigcup_{i\in I} \mathrm{Im} f_i = U$
	big étale site	$(Sch/X)_{ét}$ Sch/X	* f_i ét + l.f.p
	big smooth site	$(Sch/X)_{sm}$ Sch/X	* smooth + l.f.p. (same topos)
	big fppf site	$(Sch/X)_{fppf}$ Sch/X	f.flat + l.f.p
two different def *	big fpqc site	$(Sch/X)_{fpqc}$ Sch/X	f.flat + q.c的 原像有开覆盖 ^ q.c的


strong
discrete topo
Complex topo
Zariski topo
étale
smooth
fppf
fpqc
trivial
trivial topo

Ex. of Sheaves on $X_{ét}$

Let G: finite abelian group

① $\underline{G}_{pre}: \; - \longmapsto G$ is a presheaf over $X_{ét}$

② $\underline{G} : \mathrm{Hom}_{Sch/X}(-, \underline{G} \otimes_{Spec\,\mathbb{Z}} X)$ is a sheaf over $X_{ét}$

③ $\underline{G} = \underline{G}_{pre}^{sh}$

③ reduced to $\underline{G}_p = G$ for $\forall p \in X$ i.e.

$$\varinjlim_{\substack{p \in U \\ U \to X \text{ ét + lf } p}} \mathrm{Hom}_{Sch/X}\left(U, \prod_{g \in G} X\right) = G$$