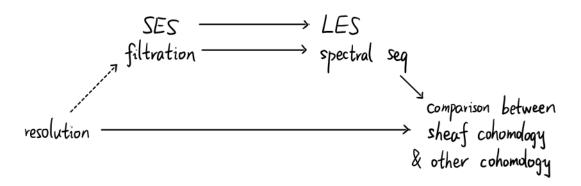
slogan:

SES induces LES, filtration induces spectral sequence.

To expend a little bit,



Even though "filtration \Rightarrow spectral seq" is the most general statement, people start with "SES \Rightarrow LES" and "acyclic resolution \Rightarrow other coh \approx hyper coh". Let us leave spectral seq in other people's notes.

method	spectral seq	LES	Cohomology/resolution
check by stalks	for stratifications	relative coh seq	Simplicial/cellular
	Čech-to-derived fctor	MV	Čech
al	Grothendieck		
filtration by HG)	Leray-Serre	Gysin	<u>Fuler class</u>
filtration by Fi	Hodge-de Rham		Hodge-Tate de Rham, Hodge-de Rham Dolbeault $H^{P}(X, \Omega^{q}) = H^{P, q}(X)$
need resolution to get "another" complex	Frölicher		$H^{p,q}(X) \Rightarrow H^{p+q}(X)$ "composition Singular
	Adams Atiyah-Hivzebruch		for stable homotopy gp for top K-theory
spectral sequences	Bar Bockstein		for group homology
which I don't know	Cartan - Levay Eilenberg - Moore Green		for Koszul cohomology
	:		:

For more spectral sequences, see: https://en.wikipedia.org/wiki/Spectral_sequence https://github.com/CubicBear/SpectralSequences/blob/main/SpecralSequences.pdf

- 1. open-closed formalism

- 2. open Cover
 3. filtration by H'(F')
 4. Hodge related filtration

1. open-closed formalism related: comparison of j! & j* one-point compactification.

Observe the following pictures:

$$Z \xrightarrow{i} X \xleftarrow{j} U$$

$$\mathcal{D}(z) \xrightarrow{i^* = i_!} \mathcal{D}(x) \xrightarrow{j^* = j^!} \mathcal{D}(u)$$

Black box:

- 0. We assume some nice conditions.
 e.g. in the category Haus loc. cpt, and Z C X is loc. contractable.
 Under these conditions.
- 1. $i_* = i!$, $j^* = j!$ 2. j!, i^* , j^* , i_* are exact.

Ex. 1. Shows that
$$i^*i_* = i^!i_* = Id_{\mathcal{D}(z)}$$
 $j^*j_! = j^*Rj_* = Id_{\mathcal{D}(u)}$ $i^*j_! = o$, $i^!Rj_* = o$

- 2. (for category fans)

 i*, j*, j! are fully faithful, and

 i*, i!, j*, Rj* preserve injectives.
- 3. One has SES $0 \longrightarrow j_! j_! \mathcal{F} \longrightarrow \mathcal{F} \longrightarrow i_* i^* \mathcal{F} \longrightarrow 0 \qquad (1)$

Ex for (1). 1. Apply the $R\pi_{X,*}$ to (1), take $F = Q_X$, what do we get?

In general, what do we get when applying $R\pi_{X,*}$ & $R\pi_{X,!}$? Discuss 2 spectural cases $\mathcal{F} = \mathcal{Q}_X$ \mathcal{D}_X $\mathcal{D}_X = \pi_X^{\perp} \mathcal{Q}_{\{*\}} = \mathcal{D}_X(\mathcal{Q}_X)$

- 2. Derive from (1) the SES $0 \longrightarrow j_! F \longrightarrow Rj_* F \longrightarrow i_* i^* Rj_* F \longrightarrow 0$ which measures the difference between $j_! F$ & $j_* F$.
- 3. Shows that $H_c(X) \cong H'(\overline{X}, \mathscr{F}_{o}); \mathbb{Z})$ for one pt compactification $(: X \hookrightarrow \overline{X})$. Try to compute $H_c(\mathbb{R}^n)$ in this way.

It seems that we get only half of the results.

Verdier dual

Def. The Verdier dual/dualizing functor is defined as

$$D_{x} \cdot D^{b}(X;Q) \longrightarrow D^{b}(X;Q)$$
 $D_{x}\mathcal{F}' = \underbrace{Hom}_{\mathcal{D}^{b}(X;Q)} (\mathcal{F}', \pi_{x}' \underline{Q}_{\{k\}})$

We know that

When $F \in D^b(X, Q)$ is constructable, then D'F & F

Therefore, in the constructable setting, $f^* Dx = Dy f!$ $Rf_!DY = D_XRf_*$ For exact statements about IDx, see [MS21, Cor211] [IHPS, Thm 5.3 9]

Ex. Derive from (1) the triangle

$$i_{1}i_{\mathcal{F}} \longrightarrow \mathcal{F} \longrightarrow Rj_{*}j^{*}\mathcal{F} \xrightarrow{+1} \qquad (2)$$

for F ∈Db(X;Q) constructable.

Ex for (2). Do the same arguments in "Ex for (1)".