Eine Woche, ein Beispiel

1.14. reminder on Morse theory

1. Calculations of index

E.g. $f: \mathbb{R}^2 \longrightarrow \mathbb{R}^3 \xrightarrow{g = \|\cdot\|^2} \mathbb{R}$

$(u_1, u_2) \longmapsto (u_1, u_2, 1)$

$x = (x_1, x_2, x_3) \longmapsto \langle x, x \rangle = x_1^2 + x_2^2 + x_3^2$

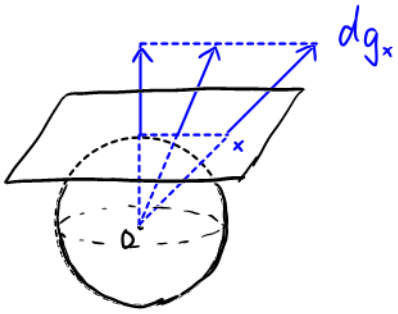
$x = (u_1, u_2, 1)$

$\frac{\partial x}{\partial u_1} = (1, 0, 0)$

$\frac{\partial x}{\partial u_2} = (0, 1, 0)$

$\frac{\partial^2 x}{\partial u_i \partial u_j} = (0, 0, 0) \quad \forall i, j$

$g_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$



$g(x) = \langle x, x \rangle$

$dg_x = 2x\,dx = \sum_i 2x_i\,dx_i$

$dg_x(\vec{v}) = 2\langle x, \vec{v} \rangle \qquad \vec{v} \in T_x\mathbb{R}^3$

$f(u) = \langle u, u \rangle + 1$

$df_u = 2u\,du = \sum_i 2u_i\,du_i$

$df_u(\vec{v}) = 2\langle u, \vec{v} \rangle \qquad \vec{v} \in T_u\mathbb{R}^2$

$f(u_1, u_2) = u_1^2 + u_2^2 + 1$

$\frac{\partial f}{\partial u_1} = 2u_1$

$\frac{\partial f}{\partial u_2} = 2u_2$

$\left(\frac{\partial^2 f}{\partial u_i \partial u_j}\right)_{i,j} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

$\frac{\partial f}{\partial u_i} = df_u(\vec{e}_i) = 2\langle u, \vec{e}_i \rangle = 2u_i$

$\frac{\partial f}{\partial u_i} = \frac{\partial g}{\partial x} \cdot \frac{\partial x}{\partial u_i} = \sum_j \frac{\partial g}{\partial x_j} \cdot \frac{\partial x_j}{\partial u_i}$

$= \sum_j 2x_j \cdot \delta_{ij}$

$= 2u_i$

have one critical pt $(0, 0, 1) \in \mathbb{R}^3$. with Morse index 0.

(attach one 0-cell.)