

Eine Woche, ein Beispiel
6.30 starting functions.

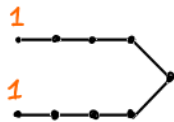
This is a follow-up of [2021.08.15], [2022.02.20] and [2021.05.07]. The combinatorics of starting functions is more intricate than I thought. Therefore, I collect these findings here, and wish somebody can give a rigorous proof for these phenomenonons (e.p. the numbers of 1's).

1. folding & starting fcts

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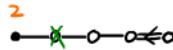
•: minuscule rep
 x: quasi-minuscule rep
 x: adjoint rep

E.g. C_5

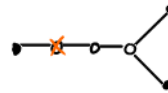


A_9

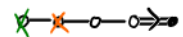
(unfold)



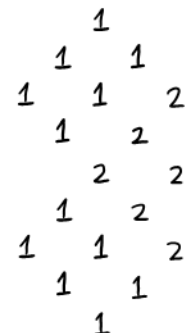
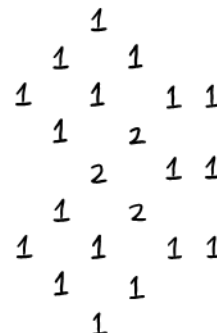
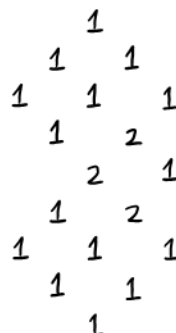
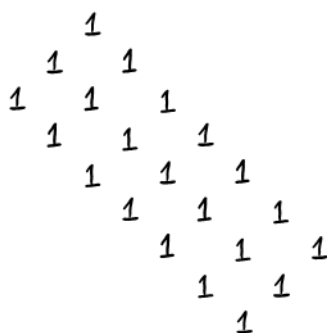
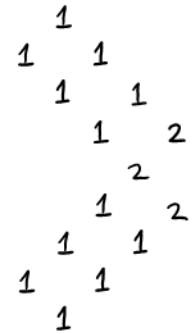
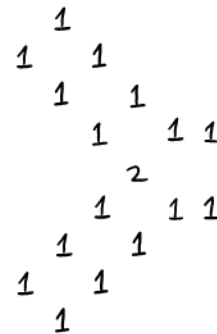
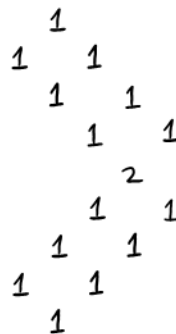
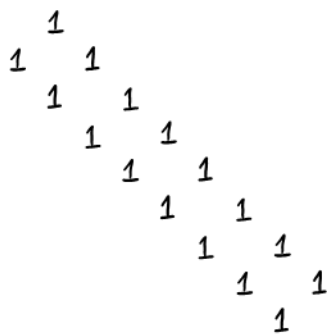
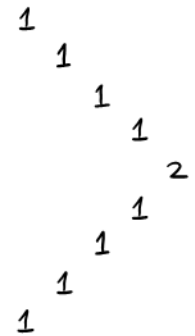
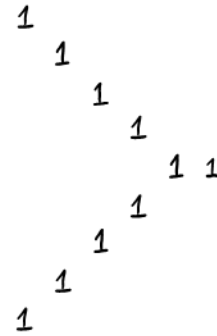
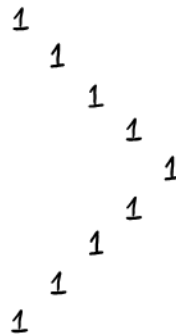
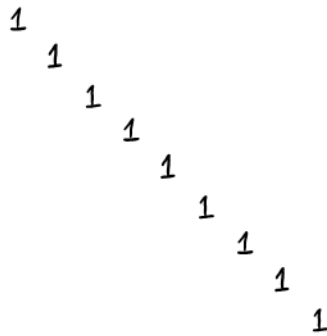
C_5

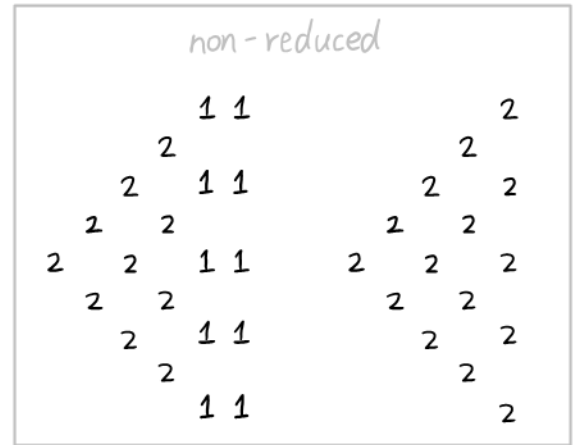
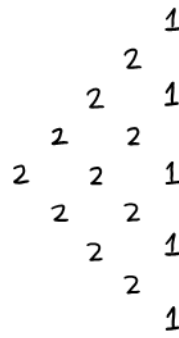
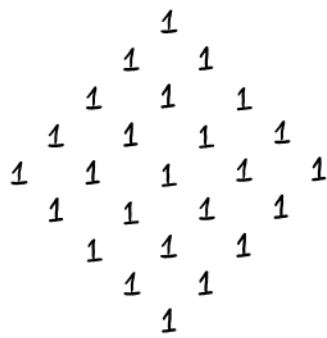
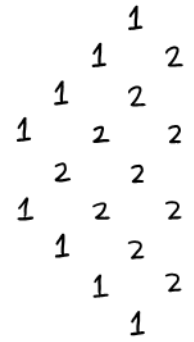
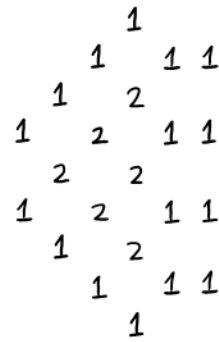
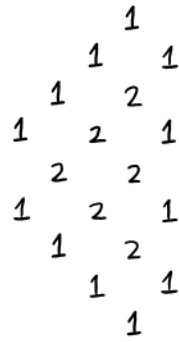
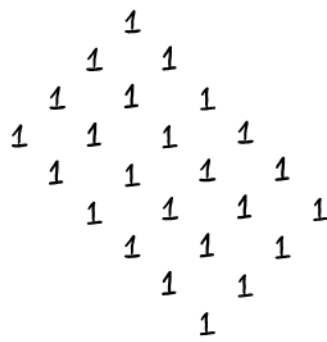


D_6



B_5





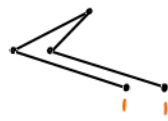
E.g. B_3



D_4



B_3



A_5

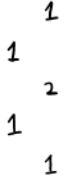
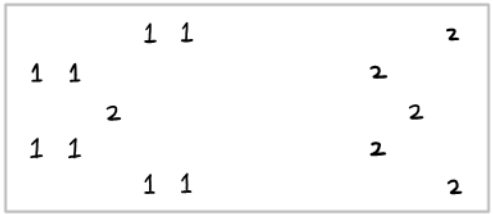
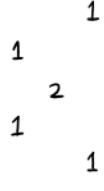
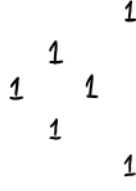
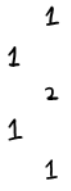
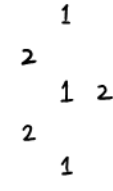
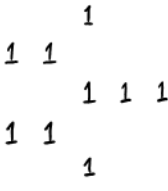
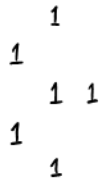
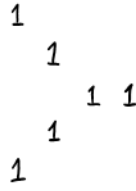
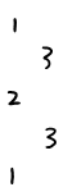
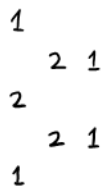
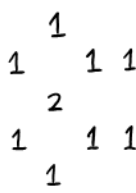


C_3



G_2

(unfold)



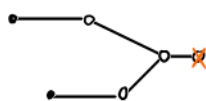
E.g. F_4



E_6



F_4



E_6



F_4

(unfold)



1
1
1 1 1
1 1 1
2
1 1 1
1 1 1
1
1

1
1
2
1 2
1 2
1 2
1 2
1
1

1
1 1 1
1 2 1
1 3 1
2 2 2
1 3 1
2 1 2
1 2 1
1 1 1
1

1
1 2
2 2
1 4
3 2
2 4
3 2
1 4
2 2
1 2
1

1
1 1 1
1 2 1
2 1 1
1 2
1 1 1
1 1
1

1
1 1
1 2
2 1
1 3
2 1
1 2
1 1
1

non-reduced

1 1
1 1
2
1 1 2
1 1 2
2 2
1 1 2
1 1 2
1 1 2
1 1
1 1

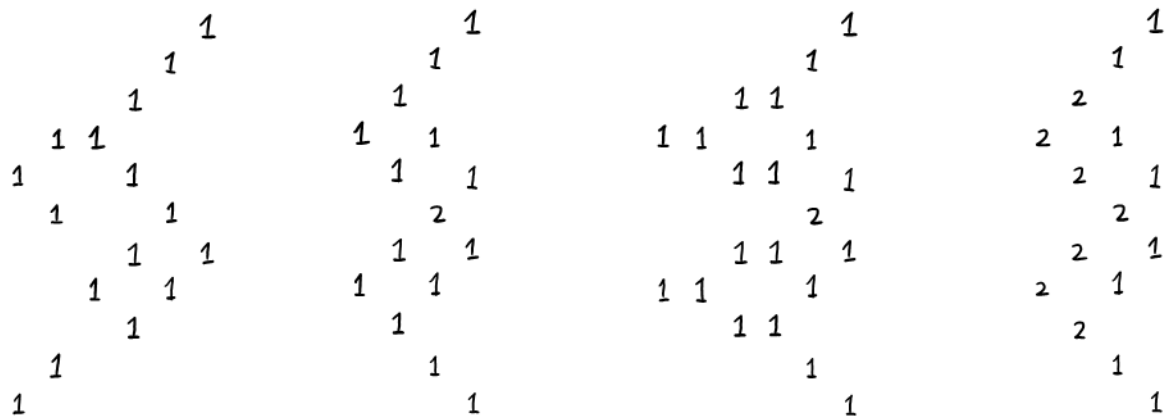
2
2
2
2 2
2 2
2
2 2
2 2
2
2
2

1 1
1 1 2
2 2 2
1 1 4
3 3 2
2 2 4
3 3 2
1 1 4
2 2 2
1 1 2
1 1

2
2 2
4 2
2 4
6 2
4 4
6 2
2 4
4 2
2 2
2

1
1 1 1
1 1 2
2 2 1
1 1 3
2 2 1
1 1 2
1 1 1
1

1
2 1
2 2
4 1
2 3
4 1
2 2
2 1
1



Rmk. Adjoint rep is compatible with normal folding, while
 quasi-minuscule rep } are compatible with reversed folding
 minuscule rep }

E.p. simply-laced \Leftrightarrow (quasi-minuscule = adjoint)

For minuscule rep, $(\Rightarrow) \Rightarrow (\text{keep})$ $(\Leftarrow) \Rightarrow (\text{keep})$

long roots \rightarrow short roots
 short weights \rightarrow long weights

<https://mathoverflow.net/questions/111469/dual-versions-of-folding-symmetric-ade-dynkin-diagrams>