Eine Woche, ein Beispiel 11,3 normalization of a plane curve

In this document, we try to solve the exercise in [Ar85, I. Ex A-6]. After the discussion with Vincent, I want to record the proof here.

 $[Ar85]: Arbarello, E., M. \ Cornalba, P.\ A.\ Griffiths\ and\ J.\ Harris.\ Geometry\ of\ Algebraic\ Curves\ Volume\ L.\ Grundlehren\ Math.\ Wiss.\ Springer,\ Cham,\ 1985.$