Eine Woche, ein Beispiel

11.26 calculation of double point

Final goal: Fill in the tables in the next page.

(for first time, remove the $i^!$ column)

Ref:

[Willians]: Langlands correspondence and Bezrukavnikov's equivalence

calculations from Lukas Bonfert's note (don't forward this to anyone else).

$X = \mathbb{C}\cup_{\{0\}}\mathbb{C} = \{(z_1,z_2)\in\mathbb{C}^2 \mid z_1 z_2 = 0\}$, $Z=\{0\}$, $\mathcal{U} = \mathbb{C}^\times \sqcup \mathbb{C}^\times$

$i_*\mathbb{Q}_Z$

$(0,1,1,1)$

	n	-2	-1	0	1
$\mathcal{U}$	j^*	0	0	0	0
$\{0\}$	i^*	0	0	$\mathbb{Q}$	0
	$i^!$	0	0	$\mathbb{Q}$	0
	$R^n\Gamma$	0	0	$\mathbb{Q}$	0

$\mathbb{Q}_X[1]$

$(-1,-1,-1,-1)$

	n	-2	-1	0	1
$\mathcal{U}$	j^*	0	$\underline{\mathbb{Q}}$	0	0
$\{0\}$	i^*	0	$\mathbb{Q}$	0	0
	$i^!$	0	0	$\mathbb{Q}$	$\mathbb{Q}^2$
	$R^n\Gamma$	0	$\mathbb{Q}$	0	0

perverse sheaf

IC sheaf

? by dim argument

	n	-2	-1	0	1
$\mathcal{U}$	j^*	×		×	×
$\{0\}$	i^*	—		×	×
	$i^!$	×	×	×	
	$R^n\Gamma$	—			

$Rj_*\mathbb{Q}_{\mathcal{U}}[1]$

$(-1,0,0,0)$

	n	-2	-1	0	1
$\mathcal{U}$	j^*	0	$\underline{\mathbb{Q}}$	0	0
$\{0\}$	i^*	0	$\mathbb{Q}^2$	$\mathbb{Q}^2$	0
	$i^!$	0	0	0	0
	$R^n\Gamma$	0	$\mathbb{Q}^2$	$\mathbb{Q}^2$	0
	Γ	0	$\mathbb{Q}^2$	$\mathbb{Q}$	0

$\Gamma^{(n)} = \Gamma(R^n j_*\mathbb{Q}_{\mathcal{U}}[1])$

$j_!\mathbb{Q}_{\mathcal{U}}[1]$

$(-1,0,0,0)$

	n	-2	-1	0	1
$\mathcal{U}$	j^*	0	$\underline{\mathbb{Q}}$	0	0
$\{0\}$	i^*	0	0	0	0
	$i^!$	0	0	$\mathbb{Q}^2$	$\mathbb{Q}^2$
	$R^n\Gamma$	0	0	0	0

$\pi^!\mathbb{Q}[-1]$

$(-1,-1,-1,-1)$

	n	-2	-1	0	1
$\mathcal{U}$	j^*	0	$\underline{\mathbb{Q}}$	0	0
$\{0\}$	i^*	0	$\mathbb{Q}^2$	$\mathbb{Q}$	0
	$i^!$	0	0	0	$\mathbb{Q}$
	$R^n\Gamma$	0	$\mathbb{Q}$	0	0

$X = \mathbb{C}^2$, $Z = \{0\}$, $\mathcal{U} = \mathbb{C}_3 - \{0\} \cong S^3 \times \mathbb{R}_{>0}$

$\mathcal{H}^0(\mathcal{F}) \in Perv_\Lambda(X)$

$i_* \underline{\mathbb{Z}}_Z$
$(0,1,1,1)$

	n	-2	-1	0	1	2
$\mathcal{U}$	j^*	0	0	0	0	0
$\{0\}$	i^*	0	0	$\mathbb{Z}$	0	0
	$i^!$	0	0	$\mathbb{Z}$	0	0
	$R^n\Gamma$	0	0	$\mathbb{Z}$	0	0

$\underline{\mathbb{Z}}_X[2]$
$(1,1,1,1)$

	n	-2	-1	0	1	2
$\mathcal{U}$	j^*	$\underline{\mathbb{Z}}$	0	0	0	0
$\{0\}$	i^*	$\mathbb{Z}$	0	0	0	0
	$i^!$	0	0	0	0	$\mathbb{Z}$
	$R^n\Gamma$	$\mathbb{Z}$	0	0	0	0

$Rj_* \underline{\mathbb{Z}}_{\mathcal{U}}[2] \in {}^pD^{\geq 0}(X) - {}^pD^{\leq 0}(X)$
$(1,0,0,0)$

	n	-2	-1	0	1	2
$\mathcal{U}$	j^*	$\underline{\mathbb{Z}}$	0	0	0	0
$\{0\}$	i^*	$\mathbb{Z}$	0	0	$\mathbb{Z}$	0
	$i^!$	0	0	0	0	0
	$R^n\Gamma$	$\mathbb{Z}$	0	0	$\mathbb{Z}$	0

	n	-2	-1	0	1	2
$\mathcal{U}$	j^*	$\underline{\mathbb{Z}}$	0	0	0	0
$\{0\}$	i^*	$\mathbb{Z}$	0	0	0	0
	$i^!$	0	0	0	0	$\mathbb{Z}$
	$R^n\Gamma$	$\mathbb{Z}$	0	0	0	0

$j_! \underline{\mathbb{Z}}_{\mathcal{U}}[2] \in {}^pD^{\leq 0}(X) - {}^pD^{\geq 0}(X)$
$(1,0,0,0)$

	n	-2	-1	0	1	2
$\mathcal{U}$	j^*	$\underline{\mathbb{Z}}$	0	0	0	0
$\{0\}$	i^*	0	0	0	0	0
	$i^!$	0	$\mathbb{Z}$	0	0	$\mathbb{Z}$
	$R^n\Gamma$	0	0	0	0	0

	n	-2	-1	0	1	2
$\mathcal{U}$	j^*	$\underline{\mathbb{Z}}$	0	0	0	0
$\{0\}$	i^*	$\mathbb{Z}$	0	0	0	0
	$i^!$	0	0	0	0	$\mathbb{Z}$
	$R^n\Gamma$	$\mathbb{Z}$	0	0	0	0

$\pi^! \mathbb{Z}[-2]$
$(1,1,1,1)$

	n	-2	-1	0	1	2
$\mathcal{U}$	j^*	$\underline{\mathbb{Z}}$	0	0	0	0
$\{0\}$	i^*	$\mathbb{Z}$	0	0	0	0
	$i^!$	0	0	0	0	$\mathbb{Z}$
	$R^n\Gamma$	$\mathbb{Z}$	0	0	0	0

$X = X_3 = \{(z_1, z_2, z_3) \in \mathbb{C}^3 \mid z_1^2 + z_2^2 + z_3^2 = 0\}$, $Z = \{0\}$, $\mathcal{U} = X_3 - \{0\} = \mathbb{C}^\times$-bd over $\mathbb{CP}^1$ (twisted)

$\mathcal{H}^0(\mathcal{F}) \in Perv_\Lambda(X)$

$i_* \underline{\mathbb{Z}}_Z$

$(0,1,1,1)$

	n	-2	-1	0	1	2
$\mathcal{U}$	j^*	0	0	0	0	0
$\{0\}$	i^*	0	0	$\mathbb{Z}$	0	0
	$i^!$	0	0	$\mathbb{Z}$	0	0
	$R^n\Gamma$	0	0	$\mathbb{Z}$	0	0

$\underline{\mathbb{Z}}_X[2]$

$(1,1,1,1)$

	n	-2	-1	0	1	2
$\mathcal{U}$	j^*	$\underline{\mathbb{Z}}$	0	0	0	0
$\{0\}$	i^*	$\mathbb{Z}$	0	0	0	0
	$i^!$	0	0	0	$\mathbb{Z}/2\mathbb{Z}$	$\mathbb{Z}$
	$R^n\Gamma$	$\mathbb{Z}$	0	0	0	0

$Rj_* \underline{\mathbb{Z}}_\mathcal{U}[2] \in {}^pD^{\geq 0}(X) - {}^pD^{\leq 0}(X)$

$(1,0,0,0)$

	n	-2	-1	0	1	2
$\mathcal{U}$	j^*	$\underline{\mathbb{Z}}$	0	0	0	0
$\{0\}$	i^*	$\mathbb{Z}$	0	$\mathbb{Z}/2\mathbb{Z}$	$\mathbb{Z}$	0
	$i^!$	0	0	0	0	0
	$R^n\Gamma$	$\mathbb{Z}$	0	$\mathbb{Z}/2\mathbb{Z}$	$\mathbb{Z}$	0

	n	-2	-1	0	1	2
$\mathcal{U}$	j^*	$\underline{\mathbb{Z}}$	0	0	0	0
$\{0\}$	i^*	$\mathbb{Z}$	0	$\mathbb{Z}/2\mathbb{Z}$	0	0
	$i^!$	0	0	0	0	$\mathbb{Z}$
	$R^n\Gamma$	$\mathbb{Z}$	0	$\mathbb{Z}/2\mathbb{Z}$	0	0

$j_! \underline{\mathbb{Z}}_\mathcal{U}[2] \in {}^pD^{\leq 0}(X) - {}^pD^{\geq 0}(X)$

$(1,0,0,0)$

	n	-2	-1	0	1	2
$\mathcal{U}$	j^*	$\underline{\mathbb{Z}}$	0	0	0	0
$\{0\}$	i^*	0	0	0	0	0
	$i^!$	0	$\mathbb{Z}$	0	$\mathbb{Z}/2\mathbb{Z}$	$\mathbb{Z}$
	$R^n\Gamma$	0	0	0	0	0

	n	-2	-1	0	1	2
$\mathcal{U}$	j^*	$\underline{\mathbb{Z}}$	0	0	0	0
$\{0\}$	i^*	$\mathbb{Z}$	0	0	0	0
	$i^!$	0	0	0	$\mathbb{Z}/2\mathbb{Z}$	$\mathbb{Z}$
	$R^n\Gamma$	$\mathbb{Z}$	0	0	0	0

$\pi^! \mathbb{Z}[-2]$

$(1,1,1,1)$

	n	-2	-1	0	1	2
$\mathcal{U}$	j^*	$\underline{\mathbb{Z}}$	0	0	0	0
$\{0\}$	i^*	$\mathbb{Z}$	0	$\mathbb{Z}/2\mathbb{Z}$	0	0
	$i^!$	0	0	0	0	$\mathbb{Z}$
	$R^n\Gamma$	$\mathbb{Z}$	0	$\mathbb{Z}/2\mathbb{Z}$	0	0

perverse sheaf

IC sheaf

n	-2	-1	0	1	2
$\mathcal{U}$ $\quad j^*$		×	×	×	×
$\{0\}$ $\quad i^*$			×	×	×
$i^!$	×	×	×		
$R^n\Gamma$					

Conclusion. $\quad\subsetneq\quad IC \quad\subsetneq\quad \mathrm{Perv} \quad\subsetneq\quad$ Constructable

$\mathbb{Q}$-coefficient $\quad i_*\underline{\mathbb{Q}}_Z \qquad\qquad Rj_*\underline{\mathbb{Q}}_{\mathcal{U}}[2],\ j_!\underline{\mathbb{Q}}_{\mathcal{U}}[2]$

$\underline{\mathbb{Q}}_X[2] \cong \pi^!\mathbb{Q}[-2]$

$\mathbb{Z}$-coefficient $\quad i_*\underline{\mathbb{Z}}_Z \qquad\qquad Rj_*\underline{\mathbb{Z}}_{\mathcal{U}}[2],\ j_!\underline{\mathbb{Z}}_{\mathcal{U}}[2]$

$\underline{\mathbb{Z}}_X[2] \qquad\qquad \pi^!\mathbb{Z}[-2]$