Eine Woche, ein Beispiel

7.2. compactifications of $\mathbb{N}$

⚠ We assume the Zorn's lemma, so that $\beta\mathbb{N}$ exists.

Rmk. As topo gp, we have

$$\operatorname{Spec}\mathbb{Z} \cong \text{cofinite topo with one generic pt}$$

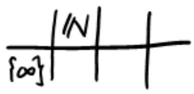
$$\not\cong \quad \{\tfrac{1}{n} \mid n\in\mathbb{Z}_{>0}\}\cup\{0\} \cong \text{one pt compactification of } \mathbb{N}$$

Fort Space on a Countably Infinite Set

$$\not\cong \quad \beta\mathbb{N}$$

https://en.wikipedia.org/wiki/Stone%E2%80%93%C4%8Cech_compactification

Jan van Mill has described βN as a "three headed monster"—the three heads being a smiling and friendly head (the behaviour under the assumption of the continuum hypothesis), the ugly head of independence which constantly tries to confuse you (determining what behaviour is possible in different models of set theory), and the third head is the smallest of all (what you can prove about it in ZFC).

X	$\operatorname{Spec}\mathbb{Z}$	$\{\frac{1}{n}\}\cup\{0\}$	$\beta\mathbb{N}$
$\mathbb{N}\subset X$			?
open / closed	× / ×	✓ / ×	✓ / ?
restricted topo	cofinite	discrete	discrete
cpt / seq cpt	✓ / ✓	✓ / ✓	✓ / ×
Hausdorff	×	✓	✓
second countable	✓	✓	×
connectness	(path) connected	totally disconnected	totally disconnected
$\pi_1(X,*)$	$\{Id\}$		
$H_0(X;\mathbb{Z})$			
$H_n(X;\mathbb{Z})$			
$\pi_n(X,*)$			

You can check these information from this database: https://topology.pi-base.org/

Wait to do: read
https://en.wikipedia.org/wiki/Stone%E2%80%93%C4%8Cech_compactification
try to understand

- What is ultrafilter
- Why do we have

$$\ell^\infty(\mathbb{N}) \cong C(\beta\mathbb{N})$$

$$(\ell^\infty(\mathbb{N}))' \cong \{\text{Borel measures on } \beta\mathbb{N}\}$$

- How is the monoid structure on $\beta\mathbb{N}$ defined.