

# Eine Woche, ein Beispiel

## 10.2 equivariant $K$ -theory of Steinberg variety : notation

This document is written to reorganize the notations in Tomasz Przezdziecki's master thesis:  
[http://www.math.uni-bonn.de/ag/stroppel/Master%27s%20Thesis\\_Tomasz%20Przezdziecki.pdf](http://www.math.uni-bonn.de/ag/stroppel/Master%27s%20Thesis_Tomasz%20Przezdziecki.pdf)

We changed some notation for the convenience of writing.

Task.

1. dimension vector
2. Weyl gp
3. alg group & Lie algebra
4. typical variety
5. equivariant stratifications
6. tangent space, Euler class
7. basis of Hecke alg

We may use two examples for the convenience of presentation.  
Readers can easily distinguish them by the dim vectors.

## 1. dimension vector

$$|d| = 5$$

$$d = (3, 2)$$

$$\underline{d} = \begin{pmatrix} 3, 2 \\ 2, 2 \\ 1, 1 \\ 0, 0 \\ 0, 0 \end{pmatrix} = \begin{array}{c} \diagup \quad \diagdown \\ \diagup \quad \diagdown \\ \diagup \quad \diagdown \end{array} = \begin{array}{c} \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \end{array} \begin{array}{c} \uparrow \quad \rightarrow \quad \uparrow \\ \uparrow \quad \rightarrow \quad \uparrow \\ \uparrow \quad \rightarrow \quad \uparrow \end{array} = \begin{array}{c} \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \end{array} \begin{array}{c} \downarrow \quad \rightarrow \quad \downarrow \\ \downarrow \quad \rightarrow \quad \downarrow \\ \downarrow \quad \rightarrow \quad \downarrow \end{array} \in W_d \backslash W_d \text{ or } \mathcal{M}_{in}(W_{Id}, W_d)$$

Young Tableau  $r_{\infty} = \pi_d^{-1}(F_{\infty})$

## 2. Weyl group

Set

$$W_{Id} = S_5$$

$$W_d = S_3 \times S_2$$

$$W_d \backslash W_{Id} = S_3 \times S_2 / S_5$$

$$\mathcal{M}_{in}(W_{Id}, W_d) = \{ \begin{array}{c} \diagup \quad \diagdown \\ \diagup \quad \diagdown \\ \diagup \quad \diagdown \end{array}, \dots \}$$

element

$$\varpi$$

$$w$$

$$w, \underline{d}$$

$$u$$

special element

$$\varpi_{\max} = \begin{array}{c} \diagup \quad \diagdown \\ \diagup \quad \diagdown \\ \diagup \quad \diagdown \end{array}$$

$$w_{\max} = \begin{array}{c} \diagup \quad \diagdown \\ \diagup \quad \diagdown \\ \diagup \quad \diagdown \end{array}$$

$$\begin{array}{c} \diagup \quad \diagdown \\ \diagup \quad \diagdown \\ \diagup \quad \diagdown \end{array}$$

$$\begin{array}{c} \diagup \quad \diagdown \\ \diagup \quad \diagdown \\ \diagup \quad \diagdown \end{array}$$

others

$$\Pi = \{s_1, s_2, s_3, s_4\}$$

$$\Pi_d = \{s_1, s_2, s_4\}$$

(Comp<sub>d</sub>)

(Shuffled)

$$d = (1, 2) \quad \begin{array}{l} a \rightarrow b \\ \langle v_1 \rangle \rightarrow \langle v_1, v_3 \rangle \end{array}$$

∇ The action on Flag is not the same as in

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	$tu = wu$		$w$	$d = u$	order of basis	$l(w)$	$l(u)$	$B_w$	$B_u$	$wB_u^{-1}$
Id	Id $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$	$\begin{array}{ c c c } \hline \square & \square & \square \\ \hline \end{array}$	$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$	$\begin{array}{ c c c } \hline \square & \square & \square \\ \hline \end{array}$	$\{v_1, v_2, v_3\}$	0	0	$\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$	$\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$	$\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$
t	$(23) \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$	$\begin{array}{ c c c } \hline \square & \times & \square \\ \hline \end{array}$	$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$	$\begin{array}{ c c c } \hline \times & \square & \square \\ \hline \end{array}$	$\{v_1, v_3, v_2\}$	1	1	$\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$	$\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$	$\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$
s	$(12) \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$	$\begin{array}{ c c c } \hline \times & \square & \square \\ \hline \end{array}$	$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$	$\begin{array}{ c c c } \hline \square & \times & \square \\ \hline \end{array}$	$\{v_2, v_1, v_3\}$	1	0	$\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$	$\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$	$\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$
ts	$(132) \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$	$\begin{array}{ c c c } \hline \times & \times & \square \\ \hline \end{array}$	$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$	$\begin{array}{ c c c } \hline \times & \times & \square \\ \hline \end{array}$	$\{v_3, v_1, v_2\}$	2	1	$\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$	$\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$	$\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$
st	$(123) \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$	$\begin{array}{ c c c } \hline \times & \times & \times \\ \hline \end{array}$	$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$	$\begin{array}{ c c c } \hline \square & \times & \times \\ \hline \end{array}$	$\{v_2, v_3, v_1\}$	2	0	$\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$	$\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$	$\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$
sts	$(13) \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$	$\begin{array}{ c c c } \hline \times & \times & \times \\ \hline \end{array}$	$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$	$\begin{array}{ c c c } \hline \times & \times & \times \\ \hline \end{array}$	$\{v_3, v_2, v_1\}$	3	1	$\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$	$\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$	$\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$