


# Eine Woche, ein Beispiel

## 6.4. basics of fields

This document is aimed for people who have enough mathematical maturity, but miss the chance and time to study Galois theory. For a (relative) complete study of Galois theory which takes time, please see [GTM167].

1. classical motivation
2. common confusion
3. field extension

### 1. classical motivation

	ruler-and-compass construction 尺规作图	solving higher degree equations 求根公式
possible	 17-gon	$\cos \frac{2\pi}{5}$ $\frac{1}{5}$ $\cos \frac{2\pi}{17}$ $\frac{1}{17}$ $\deg F \leq 4$
impossible	Squaring the circle Doubling the cube Angle trisection	$\pi$ 化圆为方 $\sqrt[3]{2}$ 倍立方 $x: 4x^3 - 3x - a = 0$ 三等分角 $\deg F \geq 5$

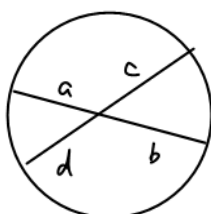
Ex. Denote

$$F_R := \{z \in \mathbb{C} \mid z \text{ can be drawn by ruler-and-compass, given } 0, 1\}$$

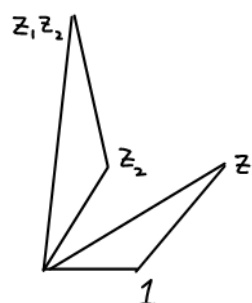
$$F_{\text{根}} := \{z \in \mathbb{C} \mid z \text{ can be expressed by } +, -, \times, \div, \text{ radicals}\}$$

Verify that  $F_R, F_{\text{根}}$  are fields.

Hint. Verify that  $\mathbb{Q} \subseteq F_R$  to get some intuition.



$$ab = cd$$



Ex. Given  $1, a \in \mathbb{R}^+$ , try to draw  $\sqrt{a}$  by ruler-and-compass.