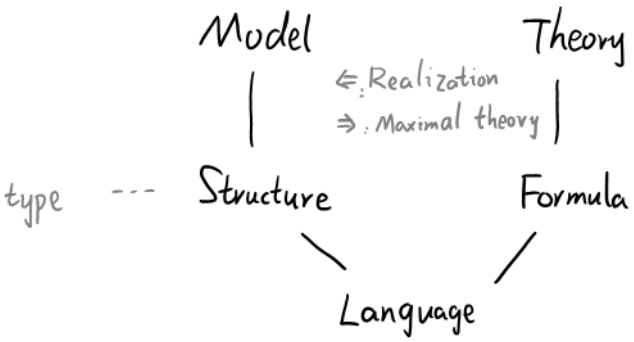
Eine Woche, ein Beispiel

3.27 model theory

Ref: https://philippschlicht.github.io/teaching/files/Lecture.pdf
I heard something from Yilong Zhang, and want to jot down some key points so that I won't be confused next time.



Modern Algebra (H)

Preliminaries

- Logic. set & map, operations on Set. ($\cap \cup -^c$, sub/quotient, two ways of disjoint union, Cartesian product)
  - Axiomatic set theory (ZFC)
  - Russell's paradox
  - type of proof : constructive, algorithm, ...

  Ex. graph

  Classify topologies of $\{1, \dots, n\}$

  First adjunction: $\mathrm{Map}(A \times B, C) \cong \mathrm{Map}(A, \mathrm{Map}(B, C))$
- From $\mathbb{N}$ to $\mathbb{C}$ (the basic of examples, though logically it's not here)
  - Peano axioms. Axiom of induction
  - alg structure, order and topology
  - Completeness axiom.
- Cardinal: the only property of set.
  - naive definition

    https://math.stackexchange.com/questions/1712964/attempt-at-proving-the-class-of-all-cardinals-is-a-proper-class

  - operations on cardinal
  - examples
  - The continuum hypothesis
  - large cardinal axiom

	alg	total order	topo
$\mathbb{N}$	$(+, \times)$	✓	discrete
$\mathbb{Z}$	$(+, -, \times)$	✓	discrete
$\mathbb{Q}$	$(+, -, \times, \div)$	✓	dense but not complete
$\mathbb{R}$	$(+, -, \times, \div)$	✓	complete
$\mathbb{C}$	$(+, -, \times, \div)$	✗	complete

- Order structure
  - def, operations and properties (partial/total/well order)
  - ordinal, relationship with cardinal.

  Ex. "well-order" on class of cardinals.

  Classify subpartial ordered set of {subsets of $\{1, \dots, n\}$}, up to iso. $n \le 5$

A bird eye's view of gp theory

- Group, field and v.s
  - Group: *def
    * initial example: Aut of set. ordered set. graph; $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$.
  - Field: def + example: $\mathbb{Q}$, $\mathbb{Z}/p\mathbb{Z}$, $\mathbb{R}, \mathbb{C}$
  - v.s. def + example: K^n, fct space

    recall: generators, basis, dim; extra alg structures on v.s.
- "Group is symmetry"
  - more examples. Galois gp & matrix gp
  - Cayley's thm
  - group action: on sets, on graph, on v.s., ...

  Ex. concepts of group action on sets

  finite group of $SO(3, \mathbb{R})$

  reminder: $GL_2(\mathbb{R}) \supset H$. foundamental gp, homotopy gp. $E(\mathbb{Q})$, braid group...
- Universal property with group
  - sub/quotient three iso.
  - Ker / Im direct sum & product
  - free group
  - presentation of group

  Ex. coset decomposition

  Ex. $Z(G), [G, G]$, centralizer and normalizer.
- Decomposition of group
  - simple gp, ind gp
  - filstration
  - Zassenhaus lemma, Schreier refinement theorem
  - Split, semi-product gp
  - Results of simple group.

  Ex. cyclic gp case

  abelian gp case. $\rightsquigarrow$ "abelian category"

  Levi decomposition

Down-to-earth analysis.

- f.g. abelian gp + apps.

  Ex. lattice & Crystallographic point gp
- G act on G
- Sylow thm
- app: classifications of gp of small order. research on specific gp.

Ring & module

R(x) R((x)) R{{x}} R<<x>>

- Basic def. e.g. R[x], R[[x]], R{x}, R<x>
- Category, k-algs
- Basic def of modules, abelian category

  E.x. R-algs.

  tensor product, change of basis

  three rep theory
- AG translation, examples.
- Concepts under AG translation
- ED ⇒ PID ⇒ UFD ⇒ domain
- classification of f.g. module over PID.

https://math.stackexchange.com/questions/765787/ring-of-convergent-power-series-in-r-and-c-is-a-local-ring

Field and Galois theory (See [GTM167])

can focus more on $\mathbb{F}_p$, $\mathbb{Q}_p$, $\mathbb{F}_p((t))$, and geometrical point of view.

(Reminder) other structures: norm, metric & topo, measure; sheaf.