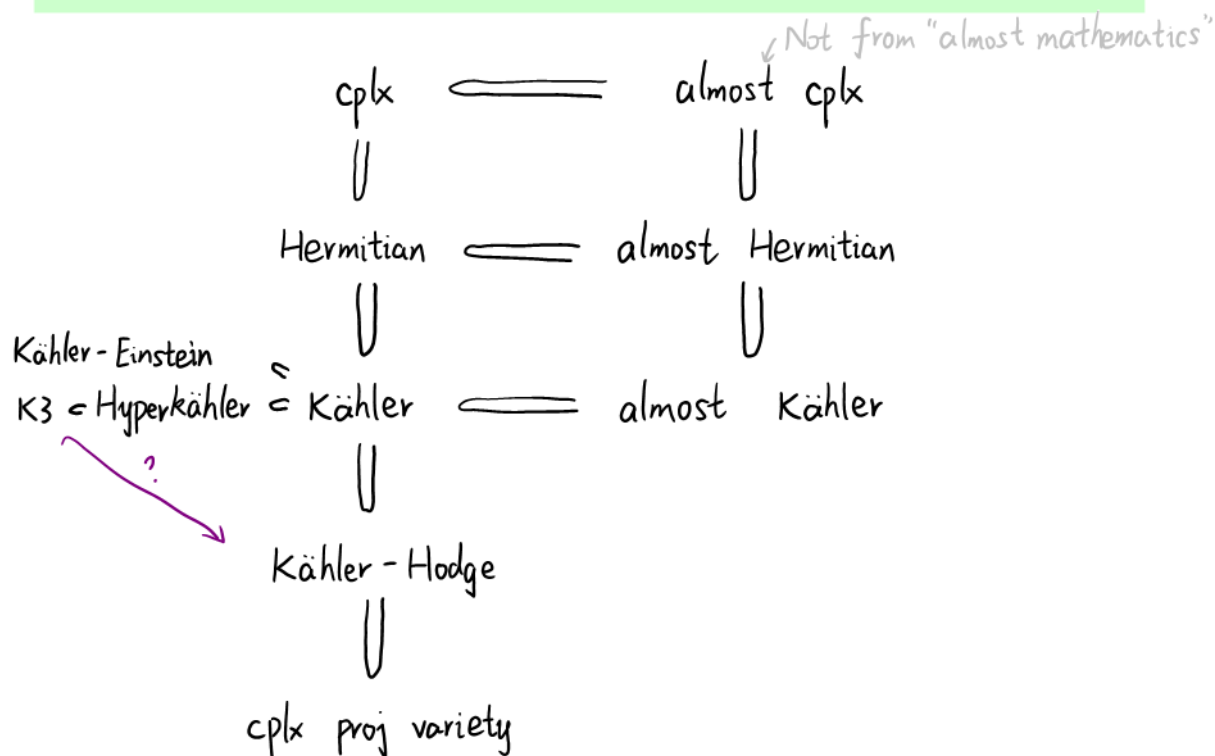


# Eine Woche, ein Beispiel

## 3.17 special complex manifolds

We also take the reference from "Introduction to complex geometry", written by Yalong Shi:  
[http://maths.nju.edu.cn/~yshi/BICMR\\_ComplexGeometry.pdf](http://maths.nju.edu.cn/~yshi/BICMR_ComplexGeometry.pdf)

[V002] Voisin, Claire. Hodge Theory and Complex Algebraic Geometry. I. Translated from the French by Leila Schneps. 卷 76. Camb. Stud. Adv. Math. Cambridge: Cambridge University Press, 2002.



When can the Kähler form be understood as special characteristic class? In the following discussion there are some partial answers:  
<https://mathoverflow.net/questions/197808/line-bundles-over-k%C3%A4hler-hodge-manifolds>  
 Question: are these line bundles defined functorial?

cplx proj variety is Kähler Hodge:

<https://mathworld.wolfram.com/KaehlerForm.html>

"In the special case of a projective algebraic variety, the Kähler form represents an integral cohomology class."

The following equivalent definitions of Kähler metric come from [V002, Theorem 3.13]:

**Theorem 3.13** The following properties are equivalent:

- (i) The metric  $h$  is Kähler. *Hermitian mfl +  $dw=0$*
- (ii) The complex structure endomorphism  $I$  is flat for the Levi-Civita connection. This means that it satisfies

$$\nabla(I\chi) = I\nabla\chi, \quad \forall \chi \in A^0(T_{X,\mathbb{R}}).$$

- (iii) The Chern connection and the Levi-Civita connection coincide on  $T_X$ , identified with  $T_{X,\mathbb{R}}$  via the map  $\Re$ .

Some information from Prof. Xu:

integrable system  $\supset$   $\begin{cases} \text{Hitchin integrable system} \\ \text{ALE, ALF, ALC, ALH, ...} \\ \text{elliptic K3 surface} \end{cases}$