Eine Woche, ein Beispiel

8.15 indecomposable representation of Dynkin quiver.

AR-quiver is a powerful tool considering about the indecomposable modules and relations among them. Using the AR-quiver, one can find(not totally serious):
- all the indecomposable modules;
- all the morphisms between these indecomposable modules;
- all the irreducible morphisms and AR-sequences;

However, it's not easy to see the coker and ker of some morphisms given by the AR-quiver. See [23.06.01] for partial discussions.

The following AR-quiver pictures are now useless, since everyone can get better pictures at https://www.math.uni-bielefeld.de/~jgeuenich/string-applet/ or https://www.math.uni-bielefeld.de/~wcrawley/#knitting.

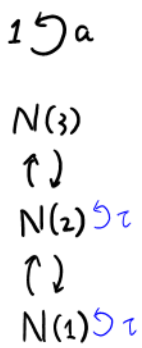
not for Dynkin quiver

Unfortunately, the knitting process can not draw some AR-quivers even in the case where "there are finite iso class of indec modules of quiver"

e.g.

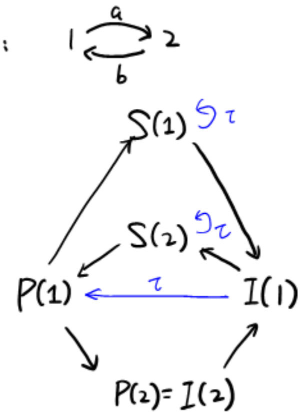
$A = K[T]/(T^3) \cong KQ/(a^3)$

Q:



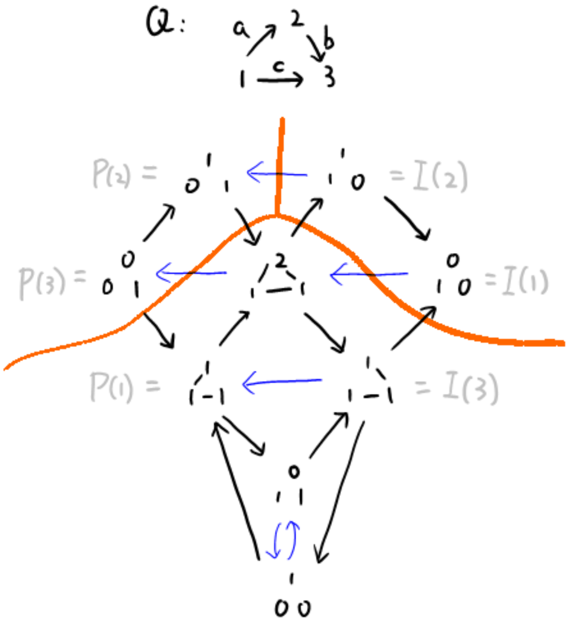
$A = KQ/(ab)$

Q:



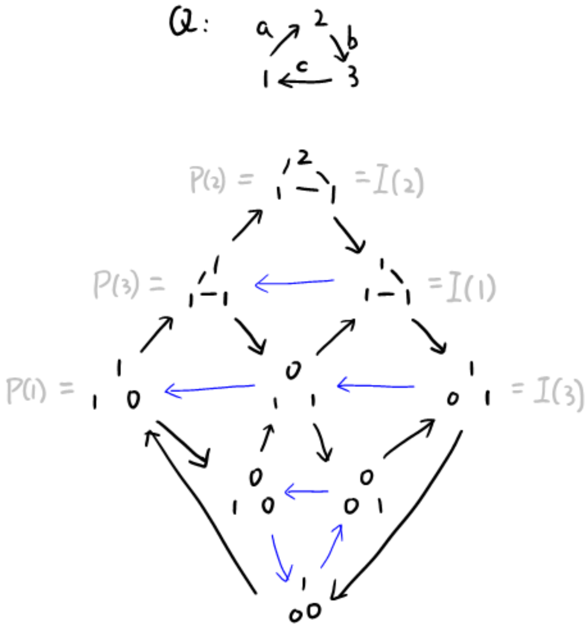
$A = KQ/(ab)$

Q:



$A = KQ/(ab)$

Q:



from different components of the AR-quiver of KQ.

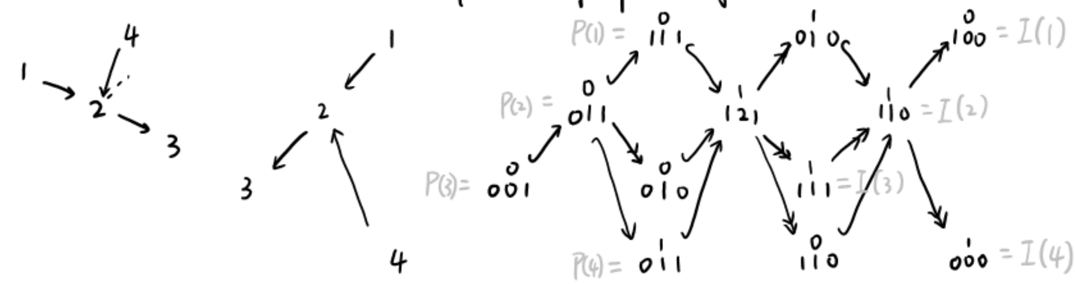
For the description of AR quiver of type A and D by a triangulated (puctured) polygon, see [Quiver Representations by Ralf Schiffler,3.1.3+3.3.3].

Ex. Use the applet to compute $KQ/(a^3, b^2, ab, ba)$

$a \circlearrowleft 1 \circlearrowright b$

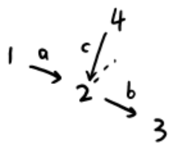
Even for bounded quiver algebra with Dynkin quiver, it is not very clear how the AR-quiver is related with the AR-quiver of path algebra.

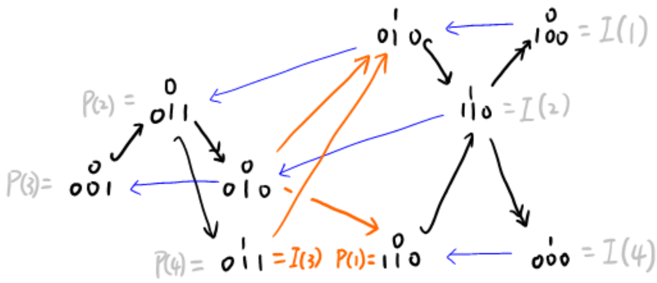
D_4

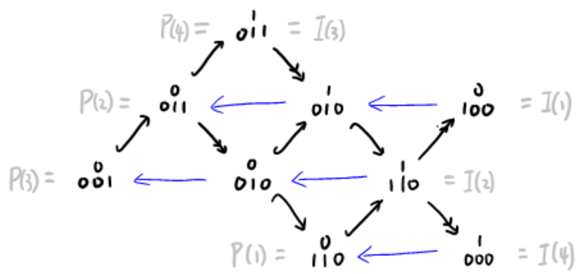


$A = KQ/(ab)$

Q:







Q. Are the nontrivial bounded quiver algebra of affine quiver representation finite?

Expected: yes.

Q Let R be a f.d. k-algebra with is indecomposable and of rep finite.

Is there only one component of AR-quiver (for R)?

Thm. [Thm 13.27,http://www.math.uni-bonn.de/~schroer/fd-atlas-files/FD_Atlas.pdf]

Let R be a f.d. k-algebra with is indecomposable.

If one component of AR-quiver (for R) have only finite vertexes,

then R is of rep finite.

Conj [Conj 13.28,13.29, http://www.math.uni-bonn.de/~schroer/fd-atlas-files/FD_Atlas.pdf]

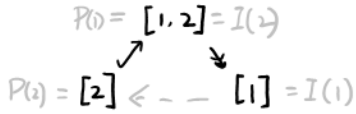
Let R be a f.d. k-algebra.

If the AR-quiver (for R) have only finite component,

then R is of rep finite.

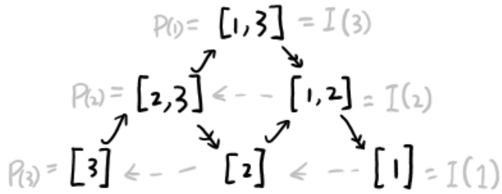
A_2 $\quad 1 \to 2$

(1)



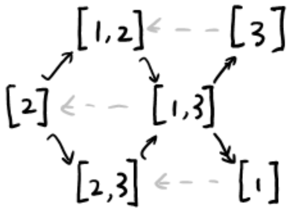
A_3 $\quad 1 \to 2 \to 3$

(2)



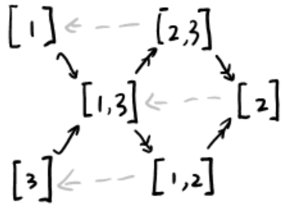
$1 \to 2 \leftarrow 3$

(1,1)



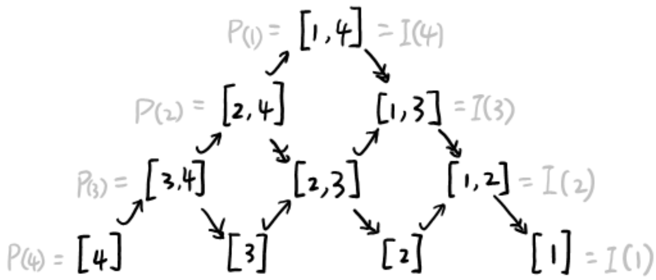
$1 \leftarrow 2 \rightarrow 3$

(1,1)*



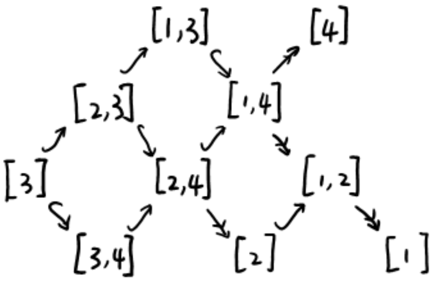
A_4 $\quad 1 \to 2 \to 3 \to 4$

(3)



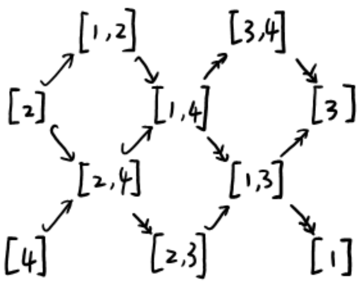
$1 \to 2 \to 3 \leftarrow 4$

(2,1)

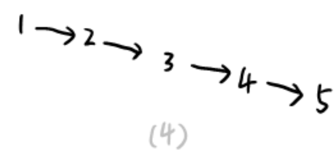


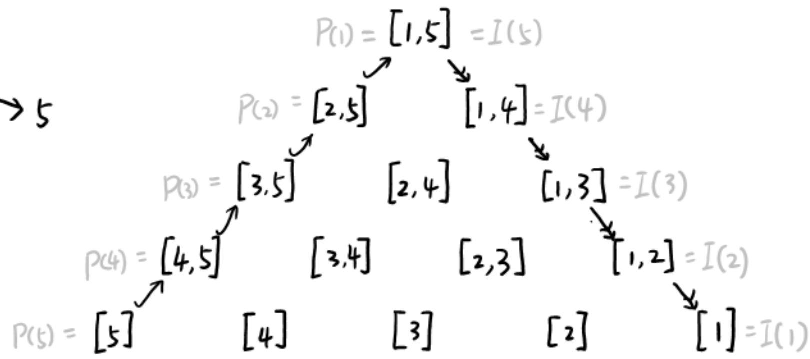
$1 \to 2 \leftarrow 3 \to 4$

(1,1,1)



A_5


1 2 3 4 5
(4)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P(2) = [2,5]
[1,4] = I(4)
P(3) = [3,5]
[2,4]
[1,3] = I(3)
P(4) = [4,5]
[3,4]
[2,3]
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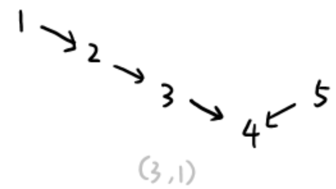
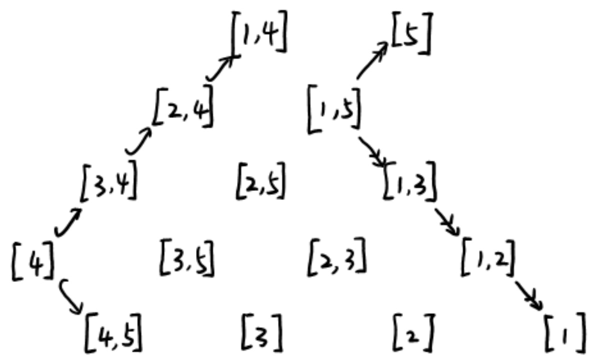
P(5) = [5]
[4]
[3]
[2]
[1] = I(1)


1 2 3 4 5
(3,1)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[1,5]
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[4]
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[2,3]
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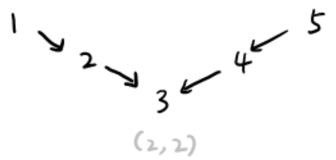
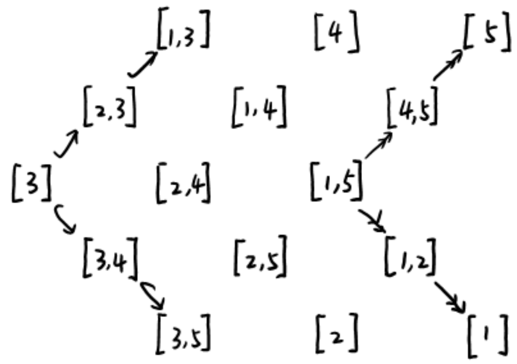
[4,5]
[3]
[2]
[1]


1 2 3 4 5
(2,2)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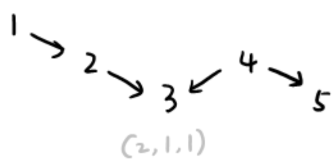
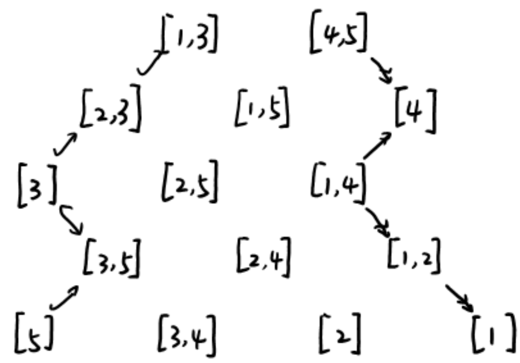
[1,2]
[3,5]
[2]
[1]


1 2 3 4 5
(2,1,1)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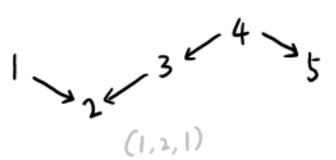
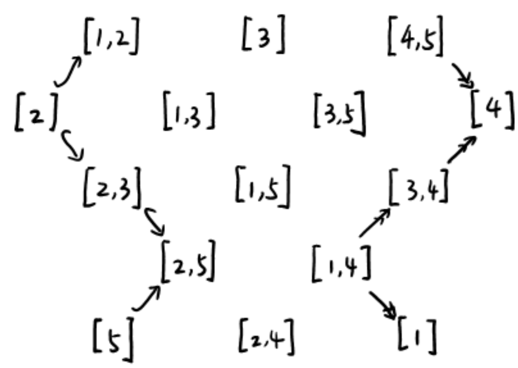

1 2 3 4 5
(1,2,1)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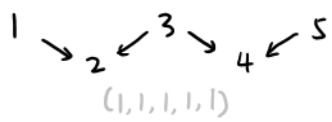
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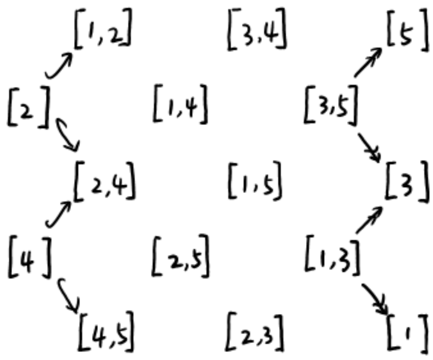
(1,1,1,1,1)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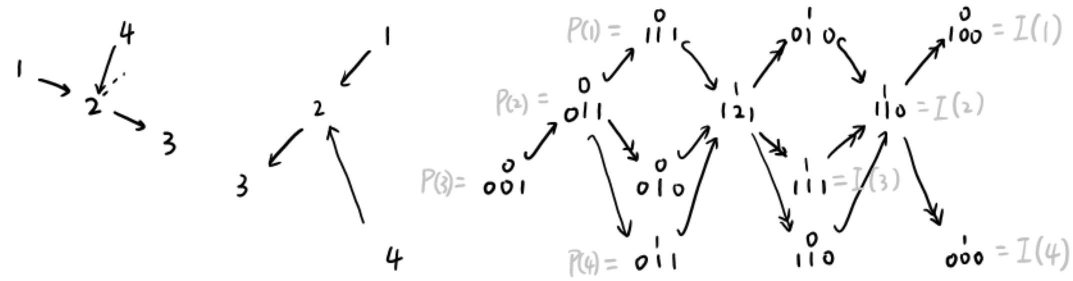
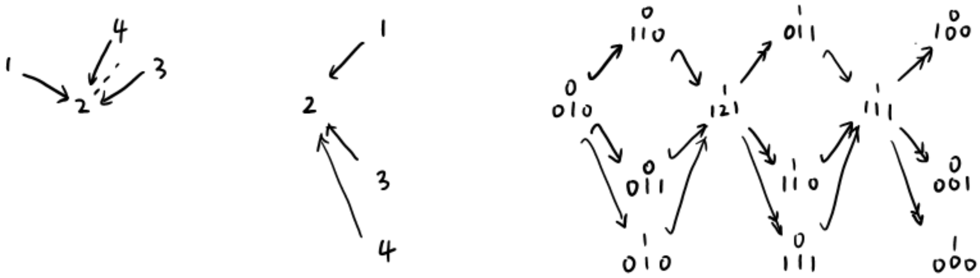
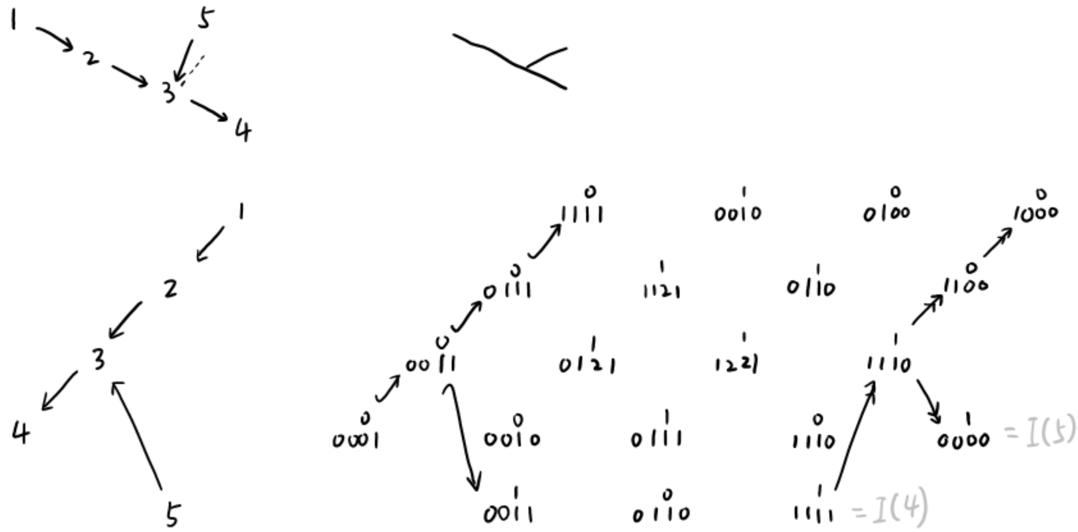
[1,3]
[4,5]
[2,3]
[1]

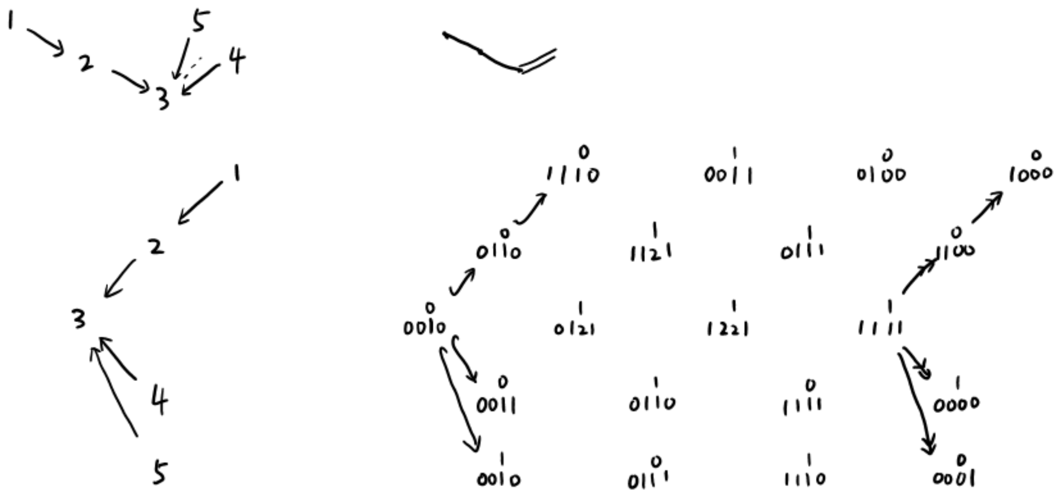
D_4

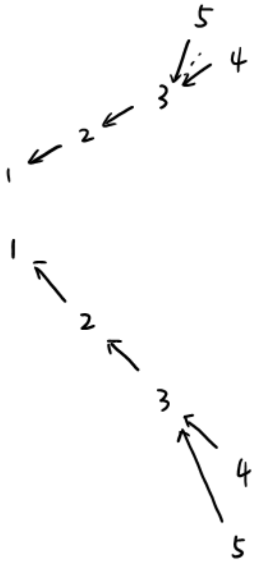


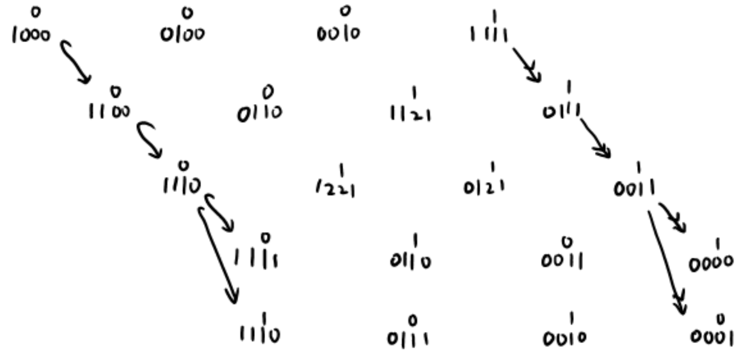


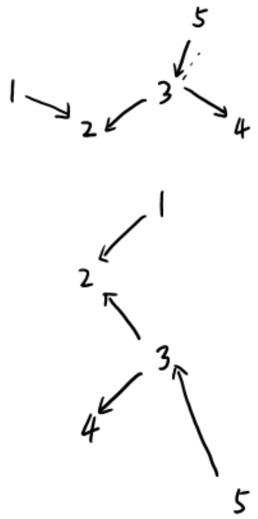
D_5

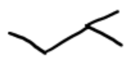


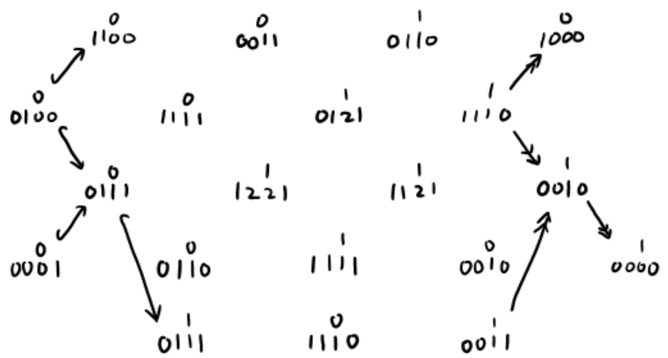


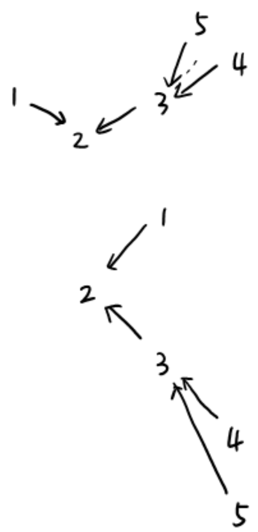


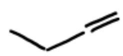


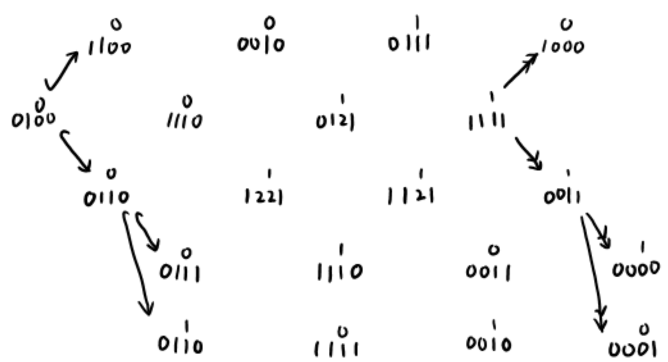


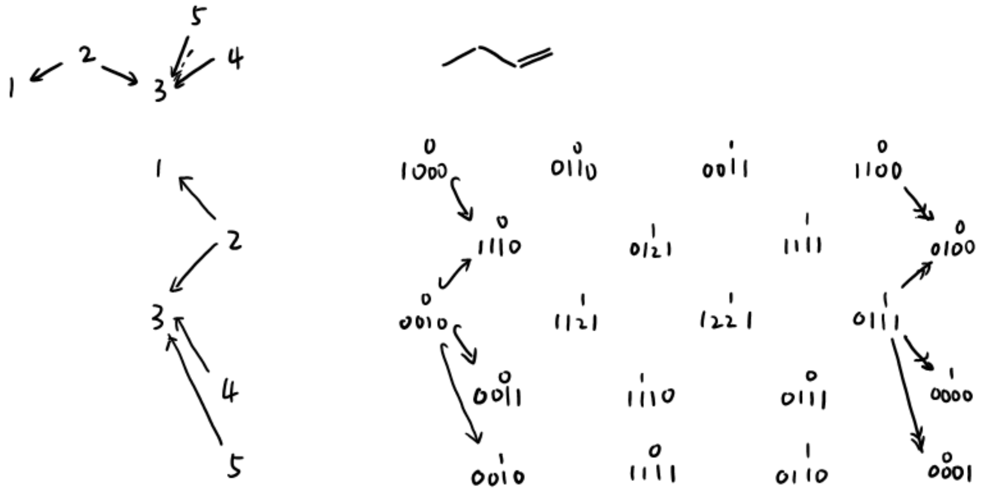




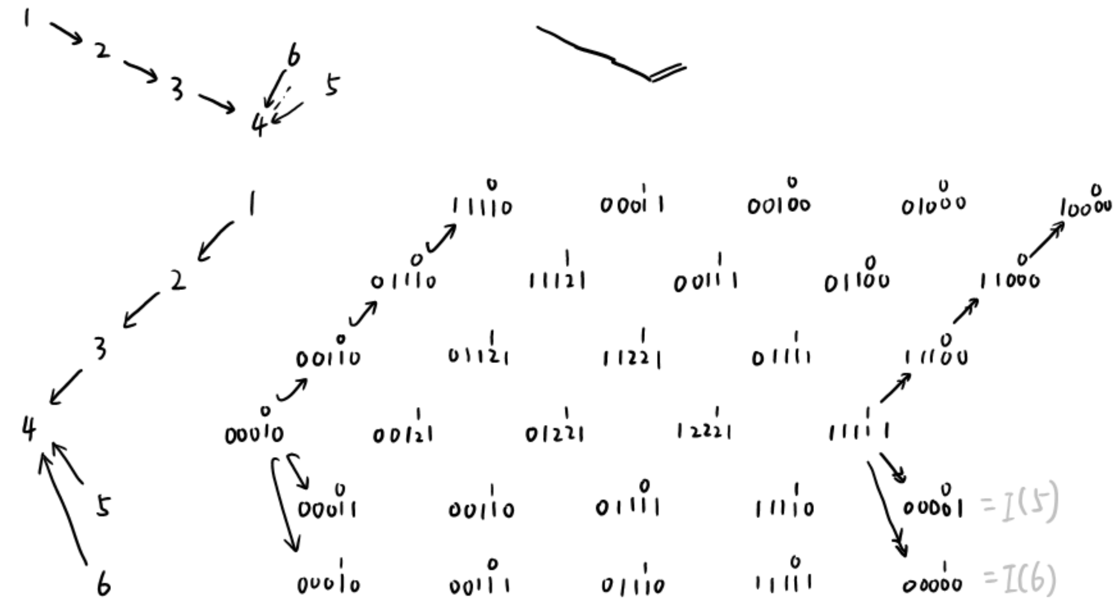




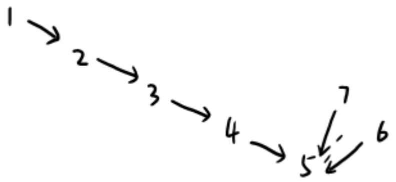




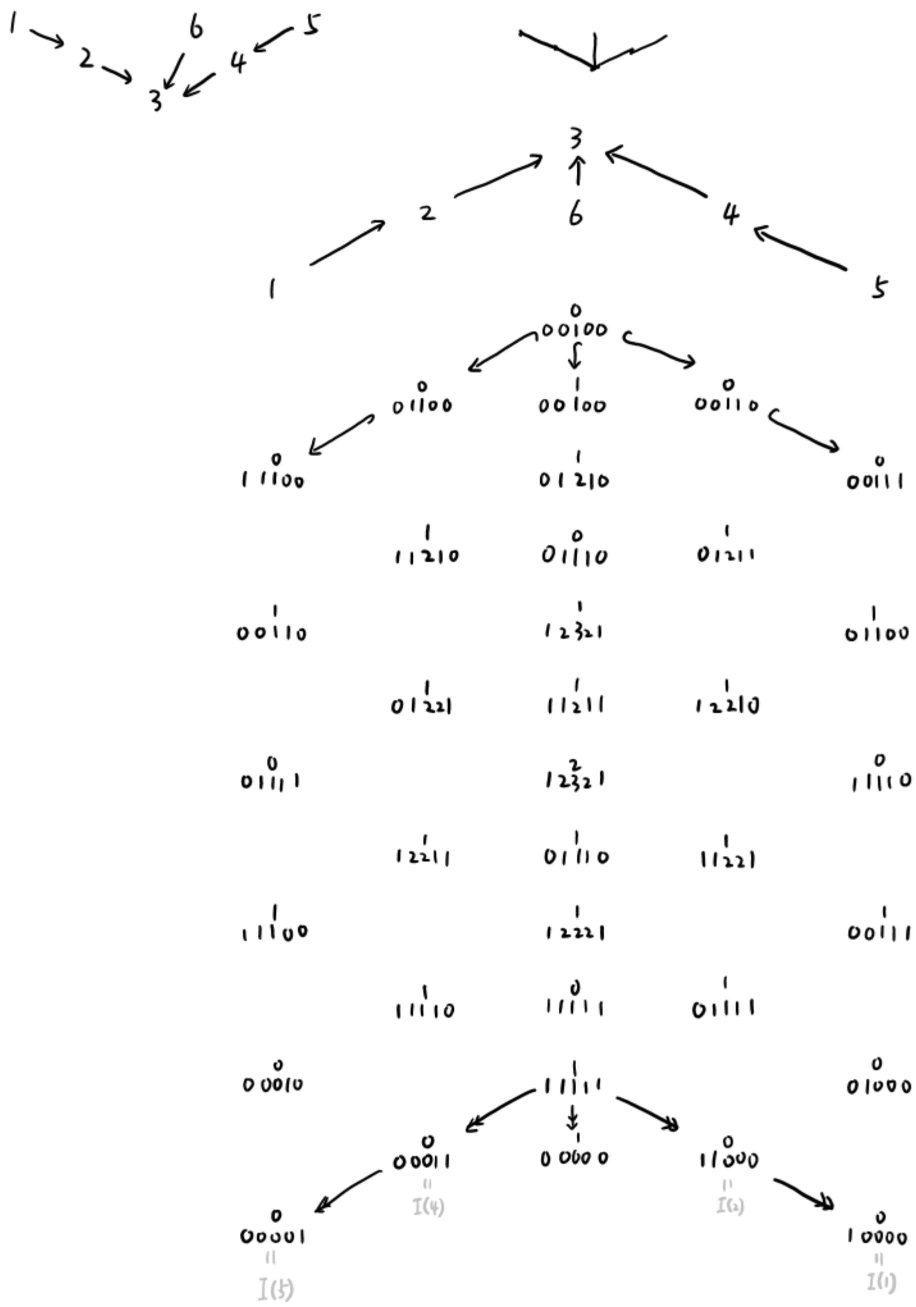
D_6



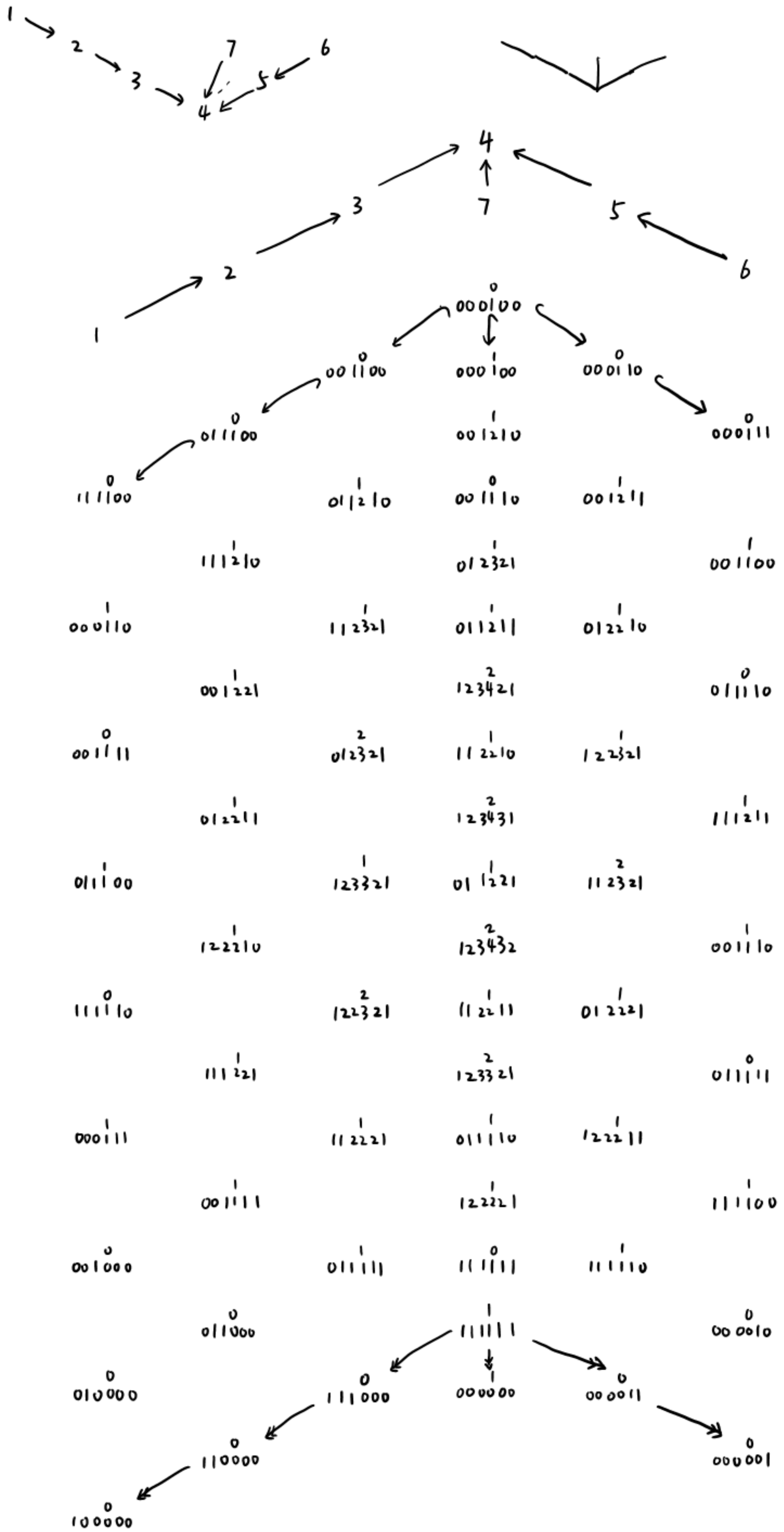
D_7



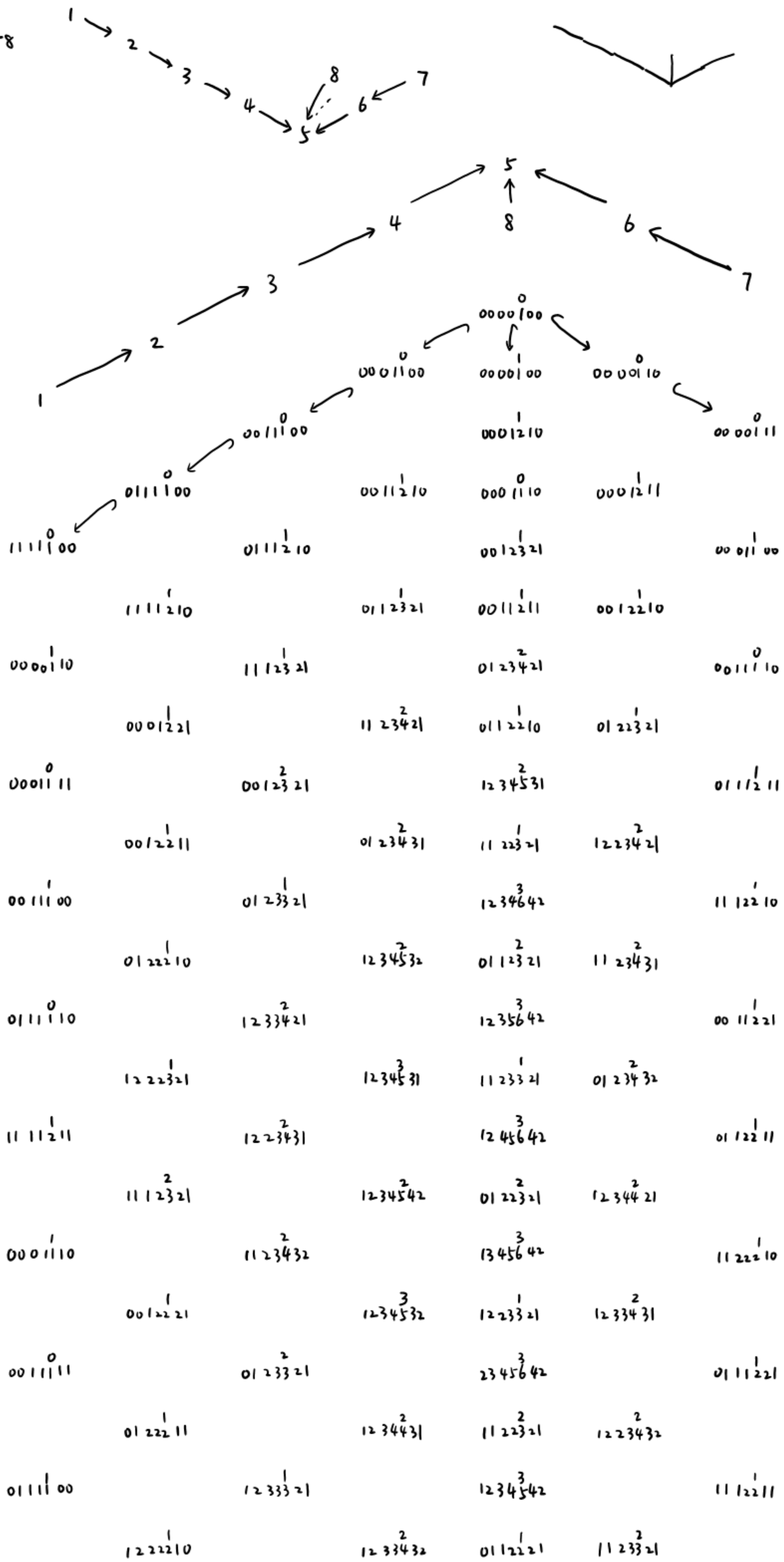
E_6

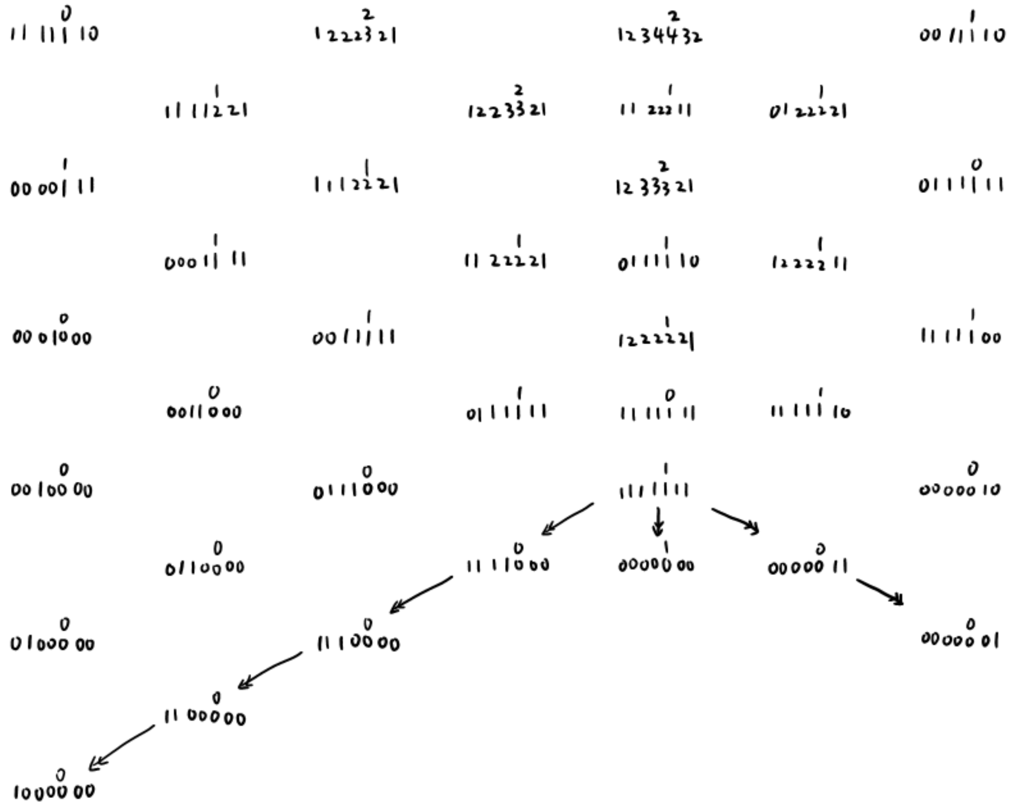


E_7



E_8





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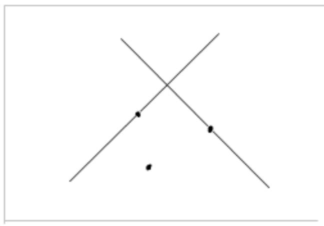
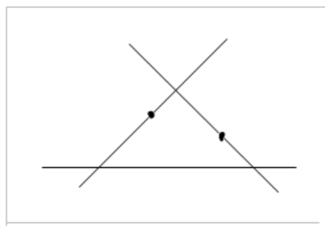
Bonus: subspace case (projective space version)

$1\overset{1}{2}1$

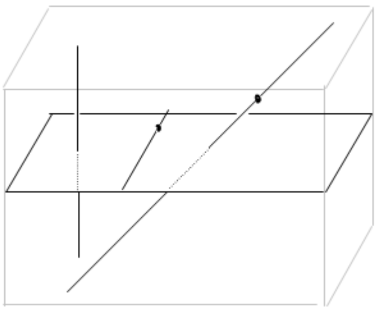
$12\overset{1}{3}21$



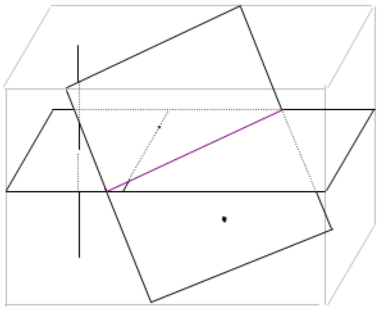
$12\overset{2}{3}21$



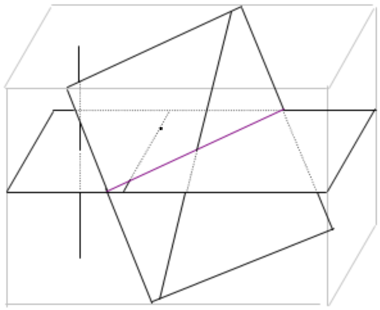
$123\overset{2}{4}21$



$123\overset{2}{4}31$

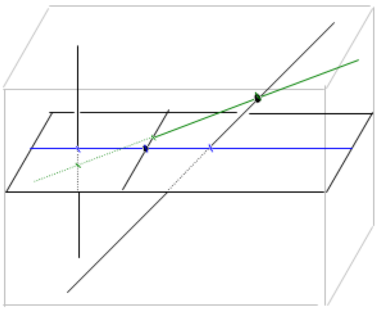


$123\overset{2}{4}32$



These shapes should be as general as possible, otherwise it may be not indecomposable:

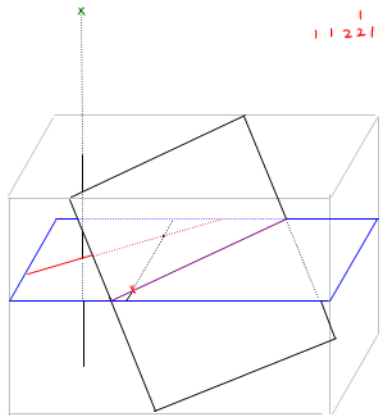
e.g. "$123\overset{2}{4}21$" $= 112\overset{1}{2}10 \oplus 011\overset{1}{2}11$



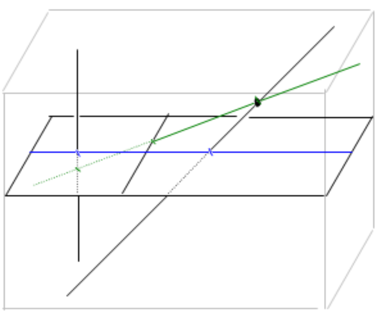
"$123\overset{2}{4}3$" $=$ "$123\overset{1}{3}2$" $\oplus\ 000\overset{1}{1}1$

$\shortparallel$

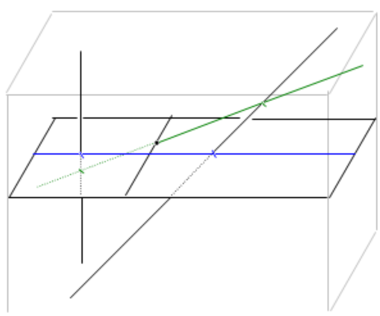
$112\overset{1}{2}1 \oplus 011\overset{0}{1}1$



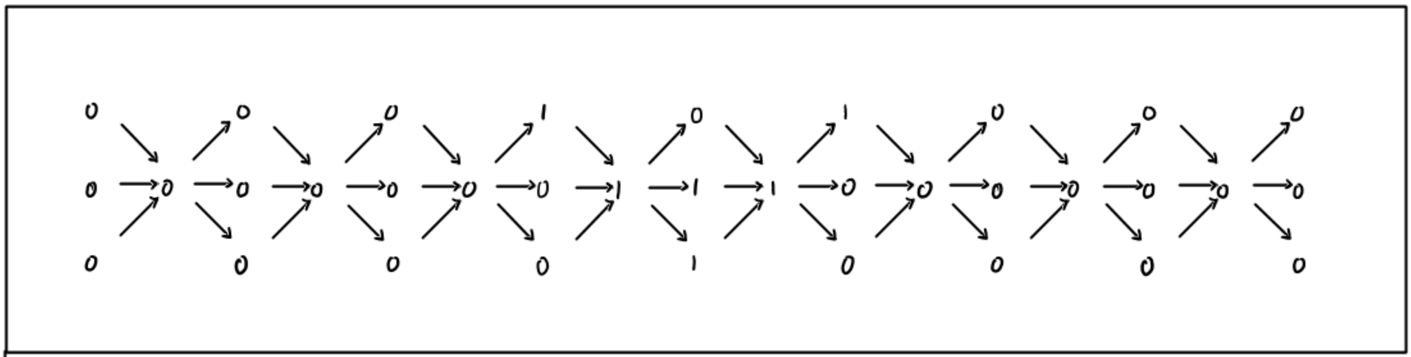
"$23\overset{2}{4}21$" $= 12\overset{1}{2}10 \oplus 11\overset{1}{2}11$



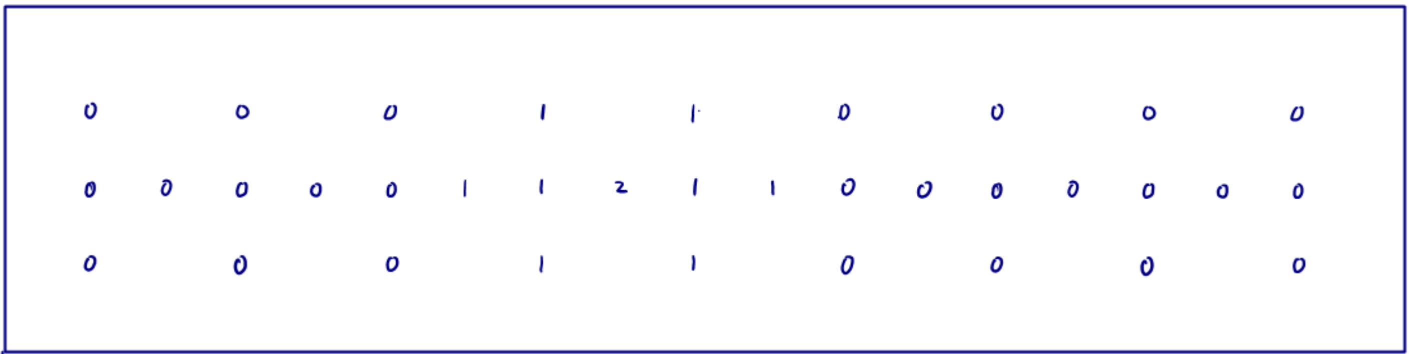
"$123\overset{2}{4}2$" $= 012\overset{1}{2}1 \oplus 111\overset{1}{2}1$



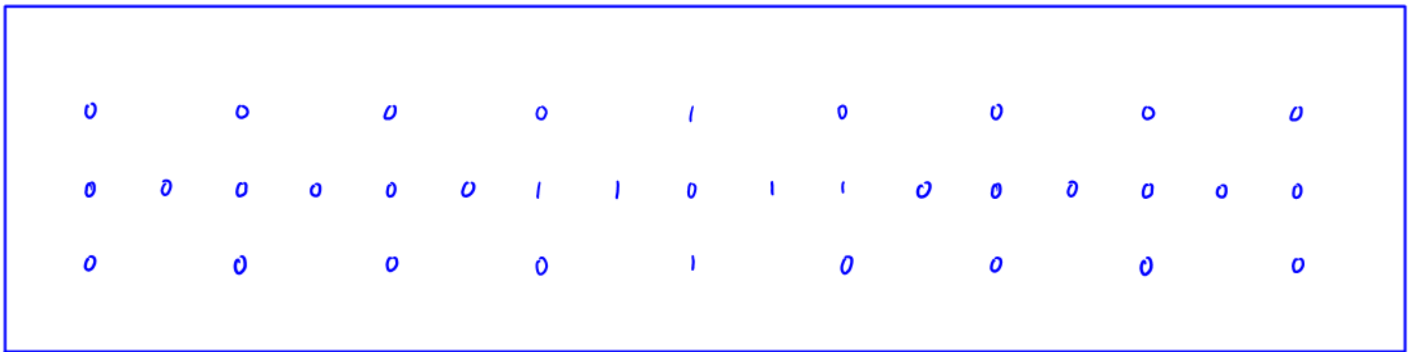
It's not easy to read the informations of them, but AR-quivers can.



P(1)



P(2)



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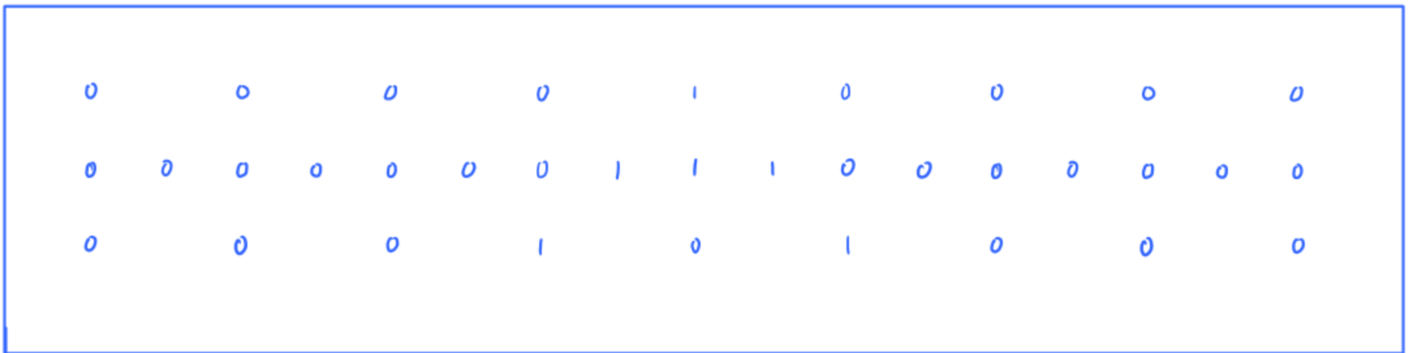

P(4)

AR-quiver cards : made in China