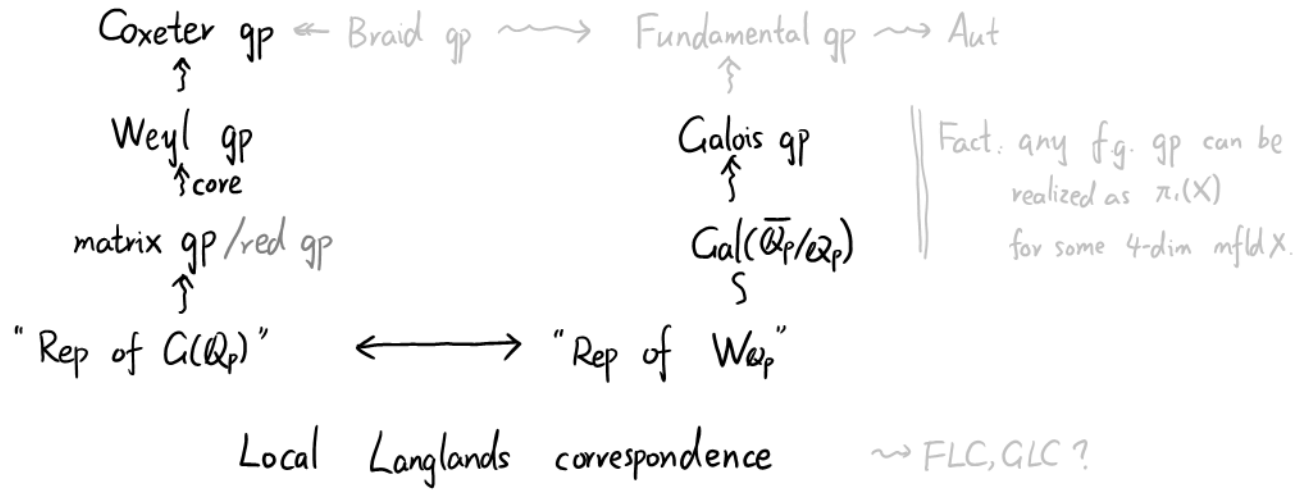
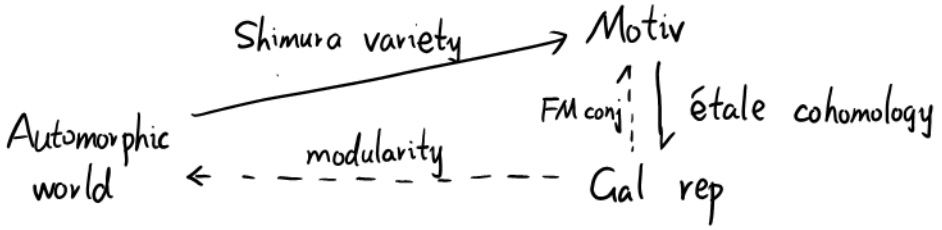


Roadmap

1. Group ⇝ Representation ⇝ Geometrical object.

structure	character	construction
· set	classification	geo rep
· topology	L-fct	cohomology, intersection, ζ-fct
· Tits system, BT-theory.		

2.



3.

	finite field		local field			global field		Adèle	
			Archi	NA					
base field	$\mathbb{F}_1$	$\mathbb{F}_p$	$\mathbb{R}$ or $\mathbb{C}$	$\mathbb{Q}_p$	$\mathbb{F}_p((t))$	$\mathbb{Q}$	$\mathbb{F}_p(t)$	$\mathbb{A}_K$	only analog
integral ring	—	—	—	$\mathbb{Z}_p$	$\mathbb{F}_p[[t]]$	$\mathbb{Z}$	$\mathbb{F}_p[t]$	K	

4. ($G(F)$-case)

G	$\mathbb{A}^1$	$\mathbb{G}_m$	GL_n	red gp (C, B, Unipotent, ...)
$G(F)$	F	$F^\times$	$GL_n(F)$	$G(F)$
$G(\mathbb{A}_K)$	$\mathbb{A}_K$	I_K	$GL_n(\mathbb{A}_K)$	$G(\mathbb{A}_K)$

(both $G(F)$ & Galois)

coefficient ring Λ: $\mathbb{C}$, $\overline{\mathbb{Q}}_p$, $\overline{\mathbb{F}}_p$, $\overline{\mathbb{Z}}_p$, ...

Roughly, need to solve $3 \times 2 \times 8 \times 4 = 192$ cases
+ much more connections

The arrow roughly means:
2 | 1
1
1
2 | 1
1
1

Usual route: fix 3 & 4, [2 G or A] → 1

Anna's route: fix 4 (GL_1), [2 G, A, M] → 3 → 1

Our route: 1 → [2 G, A] → 3 + 4

connections in 2 are delayed.

Program

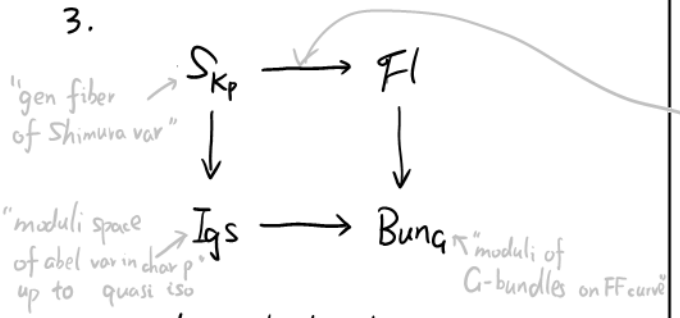
§1. No rep
- ① 1. Structure of finite/local/global field — Galois gp, Frob, Weil gp
- ② 2. Structure of reductive gp (GL_n) — Tits system. BT-theory

§2. 1-dim rep
- 1. Character of Galois gp — local class field theory
- 2. Character of red gp — $\widehat{F^*}$ & $\widehat{F^*}$, $\widehat{\mathcal{O}_F^*}$ & $\widehat{\mathcal{O}_F^*}$, Hecke character

§3. Rep
- 1. Galois rep — DW-rep; $l \neq p$: l-adic monodromy thm; $l = p$: Hierarchy of p-adic Galois rep; global: Chebychev density thm; fin / NA / IR / $\mathbb{A}_K$
- 2. Rep theory of red gp — preliminary; Hecke alg; classification (Hierarchy) { principal series, cuspidal, ...

§4. Geometrical rep
- ④ 1. EC — étale cohomology. Fontaine-Mazur conj
- ③ 2. MF (Moduli space, Shimura variety, Modular curve) — Shimura data; equiv def of MF
- 3. Flag variety — Rep II

§5. Connections
- 1. MF $\xrightarrow{ES\ iso}$ Gal rep — ES iso, ES relation; Deligne-Serre thm
- ⑤ 2. MF $\xleftarrow{modularity}$ Gal rep — Modularity
- 3.



Mingjia's work: HT period map; Torelli theorem

§6. Non-classical Langlands
1. Geometrical Langlands — - Fargues-Scholze; - Chenji's work
2. Categorical geometrical Langlands

https://mathoverflow.net/questions/56571/a-precise-statement-of-the-categorical-version-of-geometric-langlands-conjecture

Also, in each part:

- Describe L-fction
- Describe connections in section / among sections / with last part.