Eine Woche, ein Beispiel
7.3.　le programme de Fontain

Adjoint
$$W_{\mathbb{Z}_p} \dashv (-)^b$$

$W_{\mathbb{Z}_p}$: p-complete　p-torsion free　$\mathbb{Z}_p$-Alg　$\longrightarrow$　$\mathbb{F}_p$-Alg

$(-)^b$: tilting

$E/\mathbb{Q}_p$　$F \in \mathbb{F}_p$-Alg　$W_{\mathcal{O}_E}(F) := W(F) \otimes_{\mathbb{Z}_p} \mathcal{O}_E$　$\color{red}{C = \mathbb{C}_p}$

$A_{inf} := W_{\mathbb{Z}_p}(\mathcal{O}_C^b)$　Guess: $\mathcal{O}_C^b = \mathcal{O}_{C^b}$

$$A_{inf} \overset{\theta}{\twoheadrightarrow} \mathcal{O}_C$$
$$\downarrow \qquad \downarrow$$
$$0 \longrightarrow (t) \longrightarrow A_{inf}[\tfrac{1}{p}] \twoheadrightarrow C \longrightarrow 0$$

where　$t = [\omega] - p$, and
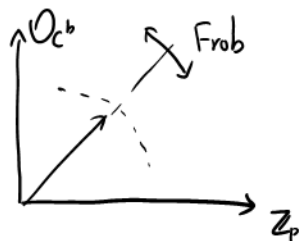
$$\mathcal{O}_{C^b} \longrightarrow W(\mathcal{O}_{C^b})$$
$$\omega \longmapsto [\omega]$$

A_{crys} = pd hull of A_{inf}
$\qquad = A_{inf}[\tfrac{t^n}{n!}]\hat{\ }$　completion at (t)

$B_{dR}^+ = A_{inf}[\tfrac{1}{p}]\hat{\ }$　completion at (t)

<u>Cohen structure thm</u>　$C[[t]]$

$B_{dR} = B_{dR}^+[\tfrac{1}{t}]$
$B_{crys}^+ = A_{crys}[\tfrac{1}{p}]$
$B_{crys} = B_{crys}^+[\tfrac{1}{t}]$

Only　B_{dR} and B_{crys} are fields. All rings are completed.
Even falser, but easier to remember:

$A_{inf} = \mathcal{O}_C[[t]]$
$A_{crys} = \mathcal{O}_C[[t]][\tfrac{t^n}{n!}]$　$\leadsto$ Crys coh
$B_{dR}^+ = C[[t]]$
$B_{dR} = C((t))$　$\leadsto$ de Rham coh
$B_{crys}^+ = C[[t]][\tfrac{t^n}{n!}]$
$B_{crys} = C((t))[\tfrac{t^n}{n!}]$