Eine Woche, ein Beispiel

5.8. exponential and logarithm

I hear about this point of view from Qirui Li.
polynomial is fundamental in Elementary function; (Before I thought exponential to be fundamental. They're both "basis" in Fourier theory)
Taylor expansion is binomial expansion for exp and log.

$$e^x = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{xn} = \lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n \overset{\text{viewed as}}{=\!=} (1+\varepsilon x)^{\infty}$$

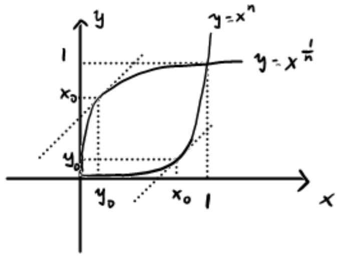
$\infty \sim n$
$\varepsilon \sim \frac{1}{n}$

$$= \lim_{n\to\infty} A(n)\left(\left(\frac{x}{A(n)}+x_0(n)\right)^n - x_0(n)^n\right)+1$$

$$\ln y = \lim_{n\to\infty} n\left(y^{\frac{1}{n}}-1\right) \overset{\text{viewed as}}{=\!=} \frac{y^{\varepsilon}-1}{\varepsilon}$$

$$= \lim_{n\to\infty} A(n)\left(\frac{1}{A(n)}(y-1)+y_0(n)\right)^{\frac{1}{n}} - A(n)\,y_0(n)^{\frac{1}{n}}$$

$$\text{where} \begin{cases} A(n) = n^{\frac{n}{n-1}} \\ x_0(n) = n^{-\frac{1}{n-1}} \\ y_0(n) = n^{-\frac{n}{n-1}} \end{cases}$$



$$\text{So} \quad e^x = (1+\varepsilon x)^{\infty}$$
$$= 1+\binom{\infty}{1}\varepsilon x + \binom{\infty}{2}(\varepsilon x)^2 + \binom{\infty}{3}(\varepsilon x)^3 + \cdots$$
$$= 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots$$

$$\ln(1+x) = \frac{(1+x)^{\varepsilon}-1}{\varepsilon}$$
$$= \frac{1}{\varepsilon}\left(1+\varepsilon x + \binom{\varepsilon}{2}x^2 + \binom{\varepsilon}{3}x^3 + \cdots - 1\right)$$
$$= x + \frac{\varepsilon-1}{2!}x^2 + \frac{(\varepsilon-1)(\varepsilon-2)}{3!} + \cdots$$
$$= x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \cdots$$