§4.2. Modular form

https://github.com/ramified/personal_handwritten_collection/tree/main/modular_form
https://github.com/ramified/personal_tex_collection/blob/main/KleinAG_2023Sep_Talk2/KleinAG_talk2_LC_XiaoxiangZhou.pdf

I only add some left materials here. If needed, the content here will move to other documents.

Shimura

$$Sh_G(\mathcal{U}_{fin})(\mathbb{C}) = G(\mathbb{Q}) \backslash G(\mathbb{A}_{\mathbb{Q},fin}) / \mathcal{U}_{fin} \mathcal{U}_\infty$$

$$\mathcal{A}_{cusp}(GL_2, w) = \text{space of cusp auto forms on } GL_2(\mathbb{A}_{\mathbb{Q}}) \text{ with central char } w.$$

Rmk. When w is unitary, i.e., $w : \mathbb{Q}^\times \backslash \mathbb{A}_{\mathbb{Q}}^\times \longrightarrow S^1$,

$$\mathcal{A}_{cusp}(GL_2, w) \subseteq L^2_{cusp}(GL_2(\mathbb{Q}) \backslash GL_2(\mathbb{A}_{\mathbb{Q}}); w)$$

has dense degree, where

$$\langle \phi, \phi' \rangle_{L^2} = \int_{GL_2(\mathbb{Q}) \backslash GL_2(\mathbb{A}_{\mathbb{Q}})} \phi(g) \overline{\phi'(g)} \, dg$$

Hierarchy:

$$\text{Siegel} \Rightarrow \text{PEL} \Rightarrow \text{Hodge} \Rightarrow \text{abelian}$$