$$\begin{array}{l} M \\ \mid ① \\ L \\ \mid ② \\ K \end{array}\Bigg\}③ \qquad\qquad \begin{tikzcd}[column sep=small] & LM \arrow[dl, dash] \arrow[dr, dash] \arrow[ddr, dash, "⑥"'] & \\ L \arrow[dr, dash, "④"'] & & M \arrow[dl, dash, "⑤"] \\ & K & \end{tikzcd}$$

$$① \xrightarrow{③\ \text{pure ins}} ③$$

normal:	③ ⇒ ①	③ ⇏ ②	① + ② ⇏ ③	⑥ ⇏ ④	④ + ⑤ ⇒ ⑥
seperable:		① + ② = ③			④ + ⑤ = ⑥
Galois:	③ ⇒ ①	③ ⇏ ②	① + ② ⇏ ③	⑥ ⇏ ④	④ + ⑤ ⇒ ⑥
purely inseparable		① + ② = ③			④ + ⑤ = ⑥

only 1 root for minimal poly

[GTM 167, Thm 4.13] char $F = p$. then
F perfect $\Leftrightarrow F^p = F$

$\overline{K}$	$\overline{\mathbb{F}}_p$	$\overline{\mathbb{F}}_p$	$\overline{\mathbb{F}}_p$	
$\mid$ closed subgroup	$\mid\ \mathbb{Z}_l$	$\mid\ \prod_{p\neq l}\mathbb{Z}_p$	$\mid\ d\hat{\mathbb{Z}}$	$\hat{\mathbb{Z}} = \prod_p \mathbb{Z}_p$
L	$\bigcup_{l\nmid a} \mathbb{F}_{p^a}$	$\bigcup_n \mathbb{F}_{p^{l^n}}$	$\mathbb{F}_q$	$(q = p^d)$
(finite) $\mid$ quotient group.	$\mid\ \prod_{p\neq l}\mathbb{Z}_p$	$\mid\ \mathbb{Z}_l$	$\mid\ \mathbb{Z}/d\mathbb{Z}$	
K	$\mathbb{F}_p$	$\mathbb{F}_p$	$\mathbb{F}_p$	

open subgroup $\overset{\{1\}\subseteq\mathbb{Z}_p}{\subsetneq}$ closed subgroup $= \{Gal(\overline{K}/L) \mid L/k \text{ ext}\} \overset{\mathbb{Z}\subseteq\mathbb{Z}_p}{\subsetneq}$ subgroup

Lem. A subgroup of a profinite group is open iff it's closed and has finite index.

Ref: https://ctnt-summer.math.uconn.edu/wp-content/uploads/sites/1632/2020/06/CTNT-InfGaloisTheory.pdf

Some wonderful exercises for Galois correspondence:

Let E/F be field ext of deg n, $m \mid n$. prove: $\exists$ subfield ext of deg m.
(Sylow thm & $Z(G)\neq\{1\}$ for a p-gp & classification of f.g. abelian gp)

Cor. For p prime, F field, one can define ${}^p\overline{F} := \bigcup_{[E:F]=p^k} E$, and

$$\overline{F} = \prod_{p \text{ prime}} {}^p\overline{F}$$