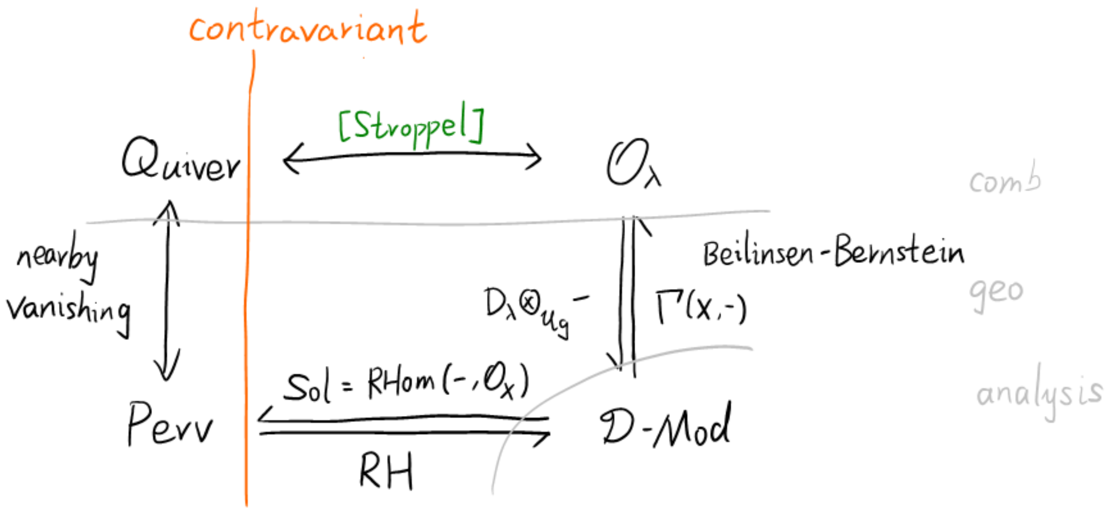
Eine Woche, ein Beispiel

11.10 5 indecomposible representations

This document is the continuation of [2013.11.26]. After the discussion with Renzhi Liang and Aaron, the last piece of the puzzle has been put together.

extra ref:
[Stroppel]: Category O: Quivers and endomorphism rings of projectives
https://www.math.uni-bonn.de/ag/stroppel/Quivers.pdf



$$\begin{tikzcd} \overset{0}{\psi} \arrow[r, bend left, "\mathrm{can}"] & \overset{-2\rho}{\phi} \arrow[l, bend left, "\mathrm{var}"] \end{tikzcd}$$

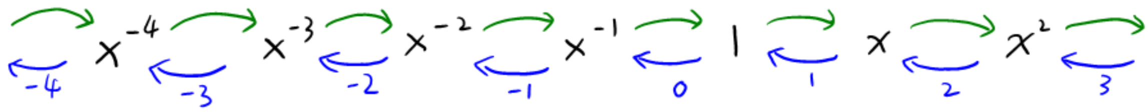
L : irreducible rep
M : Verma module
P : proj rep
I : inj rep

$\mathrm{var} \circ \mathrm{can} = 0$

Quiver	$0 \underset{0}{\overset{0}{\rightleftarrows}} \mathbb{Q}$	$\mathbb{Q} \underset{0}{\overset{0}{\rightleftarrows}} 0$	$\mathbb{Q} \underset{1}{\overset{0}{\rightleftarrows}} \mathbb{Q}$	$\mathbb{Q} \underset{0}{\overset{1}{\rightleftarrows}} \mathbb{Q}$	$\mathbb{Q} \underset{(1,0)}{\overset{\binom{0}{1}}{\rightleftarrows}} \mathbb{Q}^2$
filtration					
Perv	$i_*\underline{\mathbb{Q}}_{\{\infty\}}$	$\underline{\mathbb{Q}}_{\mathbb{CP}^1}[1]$	$Rj_*\underline{\mathbb{Q}}_{\mathbb{C}}[1]$	$j_!\underline{\mathbb{Q}}_{\mathbb{C}}[1]$	
alias	IC_0	IC_∞	$I(\psi)$	$P(\psi)$	$P(\phi) = I(\phi)$
$\mathcal{D}$-mod	A_1/A_1x	$A_1/A_1\partial$	$A_1/A_1x\partial$	$A_1/A_1\partial x$	$A_1/A_1x\partial x$
	$k[\partial]$	$k[x]$	$k[\partial, \partial^{-1}]$	$k[x, x^{-1}]$	
$\mathcal{O}_\lambda$	$L(-2\rho)$ $M(-2\rho)$ $M^*(-2\rho)$	$L(0)$	$M(0)$ $P(0)$	$M(0)^*$ $I(0)$ dual	$P(-2\rho) = I(-2\rho)$

Ex. the A_1-module structure of $k[x,x^{-1}]$

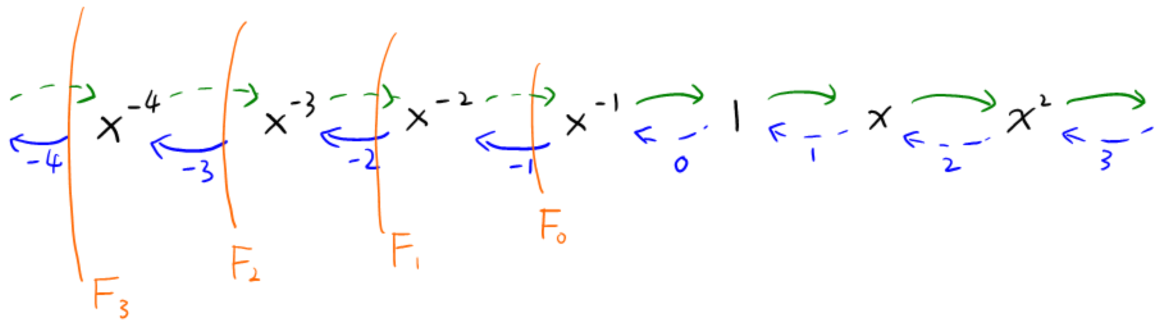
Basis with action:



$\rightarrow$: x-action

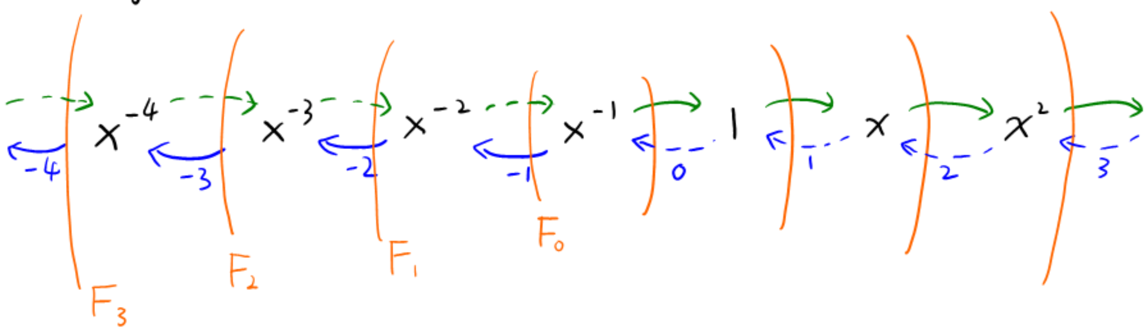
$\rightarrow$: ∂-action

Order filtration:



$$\Rightarrow \operatorname{gr}_{F_\cdot^{ord}} k[x,x^{-1}] = k[x,\partial]/_{x\partial}$$

Bernstein filtration:



$$\Rightarrow \operatorname{gr}_{F_\cdot^{B}} k[x,x^{-1}] = k[x,\partial]/_{x\partial}$$

Since $\quad \partial x \cdot \frac{1}{x} = (x\partial + 1)\cdot \frac{1}{x} = 0$, we get

$$k[x, x^{-1}] \cong A_1/A_1\partial x \cong A_1/A_1(x\partial+1)$$

We have a SES:

$$\begin{tikzcd}
0 \arrow[r] & k[x] \arrow[r] \arrow[d, phantom, "\cong" rotate=90] & k[x,x^{-1}] \arrow[r] \arrow[d, phantom, "\cong" rotate=90] & k[x,x^{-1}]/k[x] \arrow[r] \arrow[d, phantom, "\cong" rotate=90] & 0 \\
0 \arrow[r] & A_1/A_1\partial \arrow[r, "\cdot x"] & A_1/A_1\partial x \arrow[r] & A_1/A_1 x \arrow[r] & 0
\end{tikzcd}$$

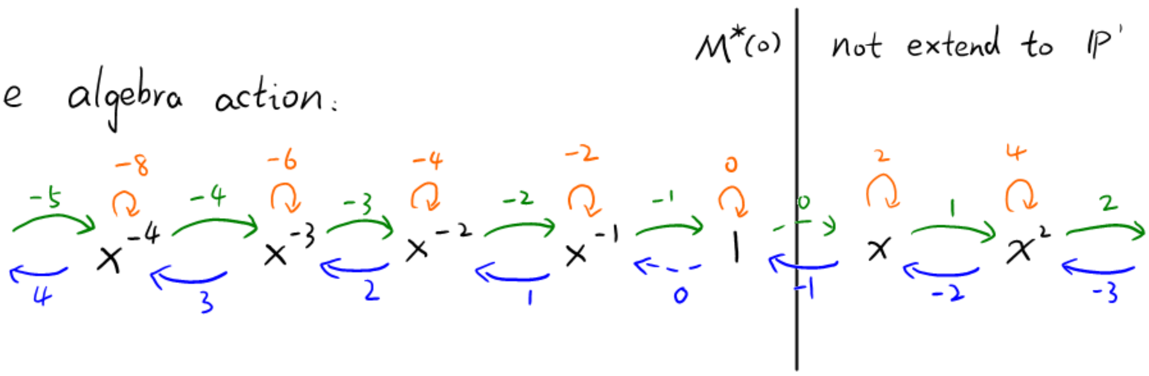
$$0 \longleftarrow \underline{\mathbb{Q}}_{\mathbb{CP}^1}[1] \longleftarrow j_! \underline{\mathbb{Q}}_{\mathbb{C}}[1] \longleftarrow i_* \underline{\mathbb{Q}} \longleftarrow 0$$

It does not split.

Restrict to $D_{\mathbb{C}^\times} = k[x, x^{-1}]\langle \partial \rangle$:

$$D_{\mathbb{C}^\times}/D_{\mathbb{C}^\times}\partial x = D_{\mathbb{C}^\times}/D_{\mathbb{C}^\times}\partial = k[x, x^{-1}]$$

Lie algebra action:



$\longrightarrow$: ∂_e-action
$\longrightarrow$: ∂_f-action
$\longrightarrow$: ∂_h-action

$$\begin{cases} \partial_e = x^2\partial \\ \partial_f = -\partial \\ \partial_h = 2x\partial \end{cases}$$

Perverse sheaf interpolation:

$$R\underline{Hom}_{A_1}(k[x,x^{-1}], \mathcal{O}_{\mathbb{A}^1})$$

$$= R\underline{Hom}_{A_1}(A_1/A_1\partial x, A_1/A_1\partial)$$

$$= R\underline{Hom}_{A_1}([\underset{-1}{A_1} \xrightarrow{\partial x} \underset{0}{A_1}], A_1/A_1\partial)$$

$$= [\underset{0}{A_1/A_1\partial} \xleftarrow{\partial x\cdot} \underset{0}{A_1/A_1\partial}]$$

$$= [k[x] \xleftarrow{\partial x\cdot} k[x]] \qquad ???$$