Un exemple par jour

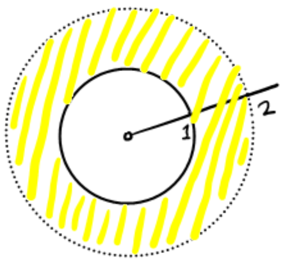
4.1. the complex torus of form $\mathbb{C}^\times/\mathbb{Z}\gamma$

$$\mathcal{C} := \mathbb{C}^\times/\mathbb{Z}\gamma \overset{\text{topo}}{=\!=} \mathbb{T}^2 \text{ is a cpt Riemannian surface of genus } 1.$$

$\gamma \in \mathrm{Aut}(\mathbb{C}^\times)$ $\gamma(z) = az$ $a \in \mathbb{C}^\times$ $|a| > 1$

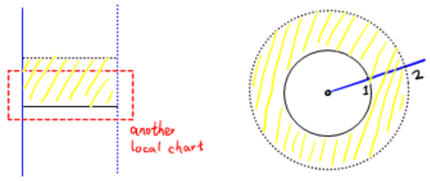
Today: $a = 2$

1. fundamental set:



$\Rightarrow$ only need 2 local chart

2.
$$\begin{array}{ccccccccc} 0 \to & \mathbb{Z} & \hookrightarrow & \mathbb{C} & \xrightarrow{f: z \mapsto e^{2\pi i z}} & \mathbb{C}^\times & \to 1 \\ & \downarrow {\scriptstyle +\frac{1}{2\pi i}\ln 2} & & \downarrow {\scriptstyle +\frac{1}{2\pi i}\ln 2} & & \downarrow {\scriptstyle \times 2} & \\ 0 \to & \mathbb{Z} + \frac{1}{2\pi i}\ln 2 & \to & \mathbb{C} & \longrightarrow & \mathbb{C}^\times & \to 1 \end{array}$$



$$\mathbb{C} \xrightarrow{f} \mathbb{C}^\times$$

$$\mathbb{C}^\times = \mathbb{C}/\mathbb{Z} \Rightarrow \mathbb{C}^\times/\mathbb{Z}\gamma = \mathbb{C}/(\mathbb{Z} \oplus \tfrac{1}{2\pi i}\ln 2\,\mathbb{Z})$$

better: $a = e^{2\pi} \approx 535.49$

$a = e^{-2\pi i w} \approx -230.765$

3. line bundle on $\mathcal{C}$

$b \in \mathbb{C}^\times$
$$\begin{array}{c} \mathcal{L}_b := \mathbb{C}^\times \times \mathbb{C} / (z,\xi) \sim (2z, b\xi) \\ \downarrow \\ \mathcal{C} = \mathbb{C}^\times / z \sim 2z \end{array}$$

$\Rightarrow$ ① $\mathcal{L}_b \in \mathrm{Pic}_0(\mathcal{C})$; ($\mathcal{L}_b \overset{\text{cont. deformation}}{\sim} \mathcal{L}_1 \cong \mathcal{O}_{\mathcal{C}}$)

(naive, base pt $1 \in \mathbb{C}^\times/\mathbb{Z}\gamma$)

② $\mathrm{Pic}_0(\mathcal{C}) \cong \mathcal{C} = \mathbb{C}^\times/\mathbb{Z}\gamma$

$\mathcal{L}_b \longmapsto b$

Reduced to: find a section s on $\mathcal{L}_b$ st $\operatorname{div} s = [b] - [1]$

Reduced to: find a meromorphic functions g on $\mathbb{C}^\times$ s.t

① $g(2z) = b\,g(z)$ $b \in \mathbb{C}^\times$, $b \neq 2^k$; e.g. $b = 3$

② g has simple poles on 2^n, and simple zeros on $2^n b$ $n \in \mathbb{Z}$

$b = e^{2\pi i c}$, $c \in \mathbb{C}$

$\tau := \frac{1}{2\pi i}\ln 2$

$w(z) = \frac{1}{2\pi i}\ln z$

$$g(z) := \frac{\theta\left[\begin{smallmatrix}1 \\ 1-2c\end{smallmatrix}\right](w(z),\tau)}{\theta\left[\begin{smallmatrix}1 \\ 1\end{smallmatrix}\right](w(z),\tau)} \text{ is the required one.}$$

Blue — example
Orange — more than this example
Red — important results
Purple — I don't know the answer/proof
Green — sketsch of proof : in a minimal way
Grey — some supplementary explanation. Unimportant assumptions.
Hell grey — explanation on well-known notations.
Brown — small title in subsections.

My symbol collection set

		Mathbb	Mathrsf/Mathcal	Greek	
A abelian variety	a	$\mathbb{A}$ Adèles	$\mathcal{A}$		α
B	b	$\mathbb{B}$	$\mathcal{B}$ building		β
C	c	$\mathbb{C}$ cplx number	$\mathcal{C}$ category	Γ gp graph	γ
D	d	$\mathbb{D}$	$\mathcal{D}$ Poincare disk	Δ diag embedding	δ
E elliptic curve	e constant ramification index	$\mathbb{E}$	$\mathcal{E}$		ε
F field	f	$\mathbb{F}$ finite field	$\mathcal{F}$ sheaf		ζ
G group	g	$\mathbb{G}$ gp scheme	$\mathcal{G}$ $\mathfrak{g}$: Lie alg upper half plane		η
H	h constant	$\mathbb{H}$	$\mathcal{H}$ Hecke alg	Θ	θ
I ideal	i	$\mathbb{I}$	$\mathcal{I}$ ideal of sheaf		ι injection
J	j	$\mathbb{J}$	$\mathcal{J}$		κ
K cos/base field	$k \leftarrow k$	$\mathbb{K}$	$\mathcal{K}$	Λ lattice	λ
L	l	$\mathbb{L}$	$\mathcal{L}$		μ
M module	m	$\mathbb{M}$	$\mathcal{M}$ moduli space		ν root of unity (ζ/ω)
N	n	$\mathbb{N}$ natural number	$\mathcal{N}$		ξ
O	o	$\mathbb{O}$	$\mathcal{O}$ structure sheaf	Π multi	π constant uniformizer projection
P	p	$\mathbb{P}$ proj space	$\mathcal{P}$ Weierstrass $\wp$: ell fct		$\rho \leftarrow \rho$
Q	q	$\mathbb{Q}$ rational number	$\mathcal{Q}$	Σ sum	σ
R ring	r	$\mathbb{R}$ real number	$\mathcal{R}$		τ
S base scheme	s	$\mathbb{S}$	–	Φ	φ
T test scheme tangent space translation	t	$\mathbb{T}$ torus	$\mathcal{T}$		χ character
$U \leftarrow U$	u	$\mathbb{U}$	–	Ψ	ψ
V v.s.	v	$\mathbb{V}$	–	Ω	ω
W witt vector	w	$\mathbb{W}$	–		
X	x	$\mathbb{X}$	$\mathcal{X}$		
$Y = Y$	y	$\mathbb{Y}$	–	hebrew	Russian
Z center	z	$\mathbb{Z}$ integer	$\mathcal{Z}$	$\aleph$ cardinal	Ш sha gp

Green: number / basic stuffs in senior high school

Orange: scheme - related

Darkyellow: advanced algebra

Don't use them simultaneously!

Don't mix: w/ω, ζ/ξ, $k/\kappa/\mathcal{K}$, $1/l/\iota$, $\times/x/\chi/\mathcal{X}$,

φ/ψ

Japanese mathematician and their Chiese translations.

1860	Sawayama	沢山	Yuzaburo Sawayama	沢山勇三郎	
1908.12	Tannakian	淡中的	Tadao Tannaka	淡中忠郎	
1912.7	Nakayama	中山	Tadashi Nakayama	中山正	
1915.3	Kodaira	小平	Kunihiko Kodaira	小平邦彦	
1917.11	Iwasawa	岩泽	Kenkichi Iwasawa	岩泽健吉	岩澤 健吉
1925.11	Tamagawa	玉河	Tsuneo Tamagawa	玉河恒夫	
1927.11			Yutaka Taniyama	谷山丰	谷山 豊
1927.12	Satake	佐武	Ichirō Satake	佐武一郎	
1928			Hiroshi Toda	户田宏	戸田 宏
1930.2	Shimura	志村	Gorō Shimura	志村五郎	
1930.3	Yoneda	米田	Nobuo Yoneda	米田信夫	
1931.4	Hironaka	广中	Heisuke Hironaka	广中平祐	広中 平祐
1947.1			Masaki Kashiwara	柏原正树	柏原 正樹
1951.2			Shigefumi Mori	森重文	
1952.1			Kazuya Kato	加藤和也	
1959.3	Fukaya	深谷	Kenji Fukaya	深谷贤治	
1969.3			Shinichi Mochizuki	望月新一	