Eine Woche, ein Beispiel
5.19. Weierstrass point

references:

https://en.wikipedia.org/wiki/Weierstrass_point

https://en.wikipedia.org/wiki/Inflection_point

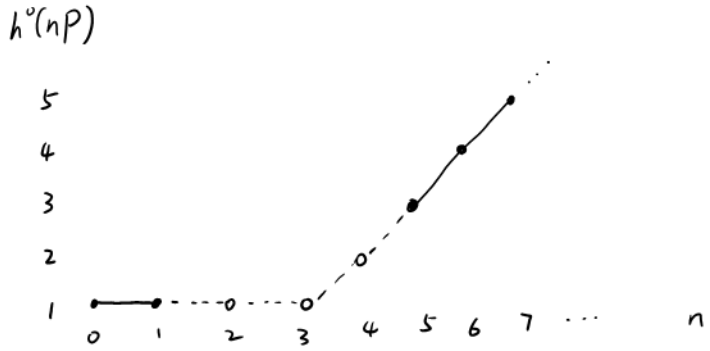
Klein quartic has 24 inflection points:
https://www.uio.no/studier/emner/matnat/math/MAT2000/v24/projects/2023_the_klein_quartic_and_its_n_weierstrass_points.pdf

curve of genus >0 don't have single simple pole:
https://math.stackexchange.com/questions/2841459/finding-a-meromorphic-function-on-a-compact-riemann-surface-with-prescribed-zero

Setting: C: proj sm curve $/k$ $\quad \bar{k} = k$, $\operatorname{char} k = 0$

	$h^0(\mathcal{O}(nP))$ \ n / $g(C)$	0	1	2	3	4	5	6	7	8	$g(g^2-1)$
	0	1	2	3	4	5	6	7	8	9	0
	1	1	1	2	3	4	5	6	7	8	0
	2	1	1	?	2	3	4	5	6	7	6
	3	1	1	?	?	?	3	4	5	6	24
	4	1	1	?	?	?	?	?	4	5	60
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$g=3$:	non-Weierstrass	1	1	1	1	2	3	4	5	6	$\emptyset$
	W: general quartic / e.g. Klein quartic	1	1	1	2	2	3	4	5	6	1×24
	W: Fermat quartic	1	1	1	2	3	3	4	5	6	2×12
	W: hyperelliptic case	1	1	2	2	3	3	4	5	6	3×8

$P \in C$ non-Weierstrass:



$g = 3$ case

$\Leftrightarrow h^0(\mathcal{O}(gP)) = 1$

$P \in C$ Weierstrass $\Leftrightarrow h^0(\mathcal{O}(gP)) \geq 2$

$\Leftrightarrow \exists\ f \in K(C)$. f has a single pole at P, with $\mathrm{ord}_P(f) \geq -g$

e.g.

$g = 2$: $P \in C$ Weierstrass $\Leftrightarrow h^0(\mathcal{O}(2P)) = 2$

$\Leftrightarrow \exists\ f \in K(C)$. f has a single double pole at P

$g = 3$: $P \in C$ Weierstrass $\Leftrightarrow h^0(\mathcal{O}(3P)) \geq 2$

$\Leftarrow \exists\ f \in K(C)$. f has a single triple pole at P