

# Eine Woche, ein Beispiel

## 7.3. le programme de Fontain

This document is full of mistakes. Don't believe me! I just record something from my schoolmate and the "p-adic Hodge theory" course, and of course it's totally uncompleted.

The motivation of p-adic Hodge theory (as well as these period rings) can be seen here (in Chinese):  
<https://www.zhihu.com/question/30816753/answer/1442810702>  
<https://www.zhihu.com/question/51273075/answer/1505155046>

Adjoint

$$W_{\mathbb{Z}_p} \dashv (-)^b$$

$$W_{\mathbb{Z}_p} : \begin{matrix} p\text{-complete} \\ (-)^b : \text{tilting} \end{matrix} \quad p\text{-torsion free } \mathbb{Z}_p\text{-Alg} \longrightarrow \mathbb{F}_p\text{-Alg}$$

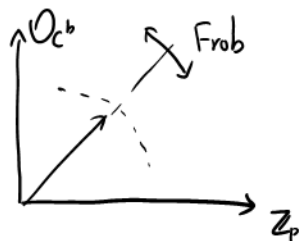
$$E/\mathbb{Q}_p \quad F \in \mathbb{F}_p\text{-Alg} \quad W_{\mathbb{O}_E}(F) := W(F) \otimes_{\mathbb{Z}_p} \mathbb{O}_E \quad C = \mathbb{C}_p$$

$$A_{\text{inf}} := W_{\mathbb{Z}_p}(\mathbb{O}_C^b) \quad \text{Guess: } \mathbb{O}_C^b = \mathbb{O}_{C^b}$$

$$A_{\text{inf}} \xrightarrow{\vartheta} \mathbb{O}_C$$

$$0 \longrightarrow (t) \longrightarrow A_{\text{inf}}\left[\frac{1}{p}\right] \longrightarrow C \longrightarrow 0$$

where  $t = [w] - p$ , and



$$\begin{matrix} \mathbb{O}_C^b & \longrightarrow & W(\mathbb{O}_C^b) \\ \varpi & \longmapsto & [w] \end{matrix}$$

$$A_{\text{crys}} = \text{pd hull of } A_{\text{inf}} = A_{\text{inf}}\left[\frac{t^n}{n!}\right]^\wedge$$

$$B_{\text{dR}}^+ = A_{\text{inf}}\left[\frac{1}{p}\right]^\wedge \quad \text{Cohen structure thm} \quad C[[t]]$$

completion at  $(t)$

completion at  $(t)$

$$B_{\text{dR}} = B_{\text{dR}}^+\left[\frac{1}{t}\right]$$

$$B_{\text{crys}}^+ = A_{\text{crys}}\left[\frac{1}{p}\right]$$

$$B_{\text{crys}} = B_{\text{crys}}^+\left[\frac{1}{t}\right]$$

Only  $B_{\text{dR}}$  and  $B_{\text{crys}}$  are fields. All rings are completed.  
 Even false, but easier to remember:

$$A_{\text{inf}} = \mathbb{O}_C[[t]]$$

$$A_{\text{crys}} = \mathbb{O}_C[[t]]\left[\frac{t^n}{n!}\right]$$

$\leadsto$  Crys coh

$$B_{\text{dR}}^+ = C[[t]]$$

$$B_{\text{dR}} = C((t))$$

$\leadsto$  de Rham coh

$$B_{\text{crys}}^+ = C[[t]]\left[\frac{t^n}{n!}\right]$$

$$B_{\text{crys}} = C((t))\left[\frac{t^n}{n!}\right]$$