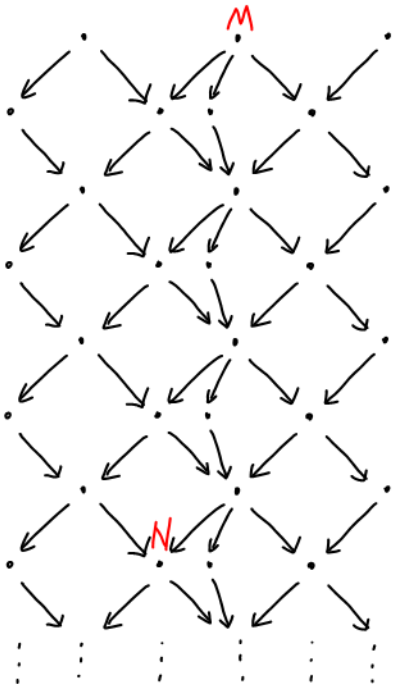
Eine Woche, ein Beispiel

6.11 more combinatorics in AR-quiver

1. dimension of Hom /path
2. database for sectional map

1. dimension of Hom /path

Suppose that you have some arrows : (type E_7)

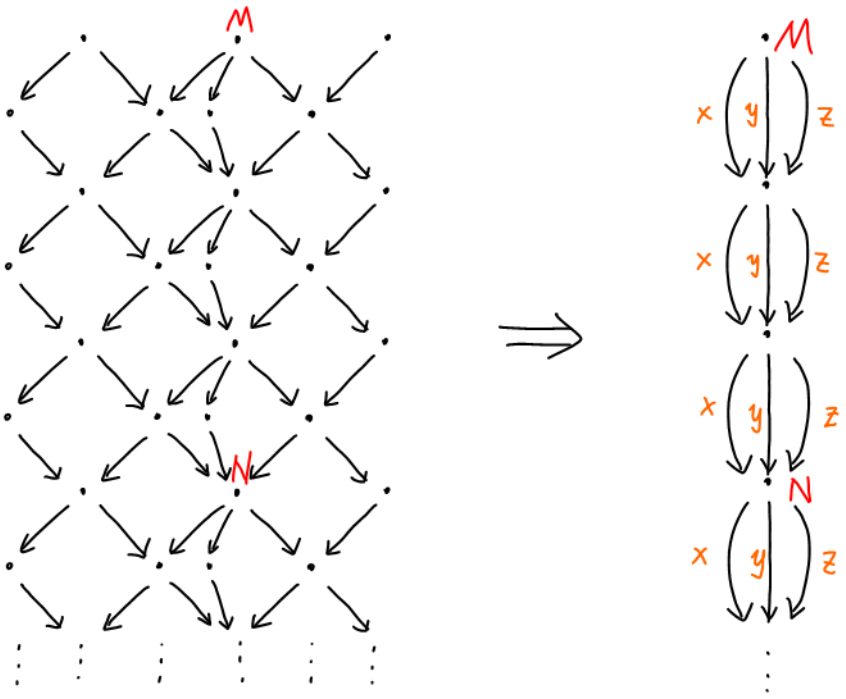


Q: 1. How many paths do we have from M to N?
2. After identifying paths by AR-sequence, how many paths do we have from M to N?

A: (partial) By induction process.
Ex: find a basis for $\mathrm{Hom}(M, N)$.

Mesh categories

Special cases:



$$\dim_{\mathbb{C}} \mathrm{Hom}(M,N) = \dim\left(\mathbb{C}\{x,y,z\}/(x^4,y^2,z^3,x+y+z)\right)_{\deg 4}$$

"Cor" The $\mathbb{Z}$-graded alg

$$A_{a,b,c} := \mathbb{C}\{x,y,z\}/(x^a,y^b,z^c,x+y+z) \qquad a,b,c \in \mathbb{N}_{\geq 1}$$

is f.d. iff $\frac{1}{a}+\frac{1}{b}+\frac{1}{c} > 1$.

We can find a (degree k) monomial basis of $A_{a,b,c}$ by playing the "Tic-tac-toe" game on $\{x,y,z\}^{\oplus k}$.

E.g. We try to find a basis of $A_{4,2,3}$.

$k=1$

x	y	z
①	②	X

① x
② y

$k=2$

	x	y	z
x	①	②	X
y	③		X
z	X	X	X

① x^2
② xy
③ yx

$k=3$.

x

	x	y	z
x	①	②	X
y	③		X
z	X	X	X

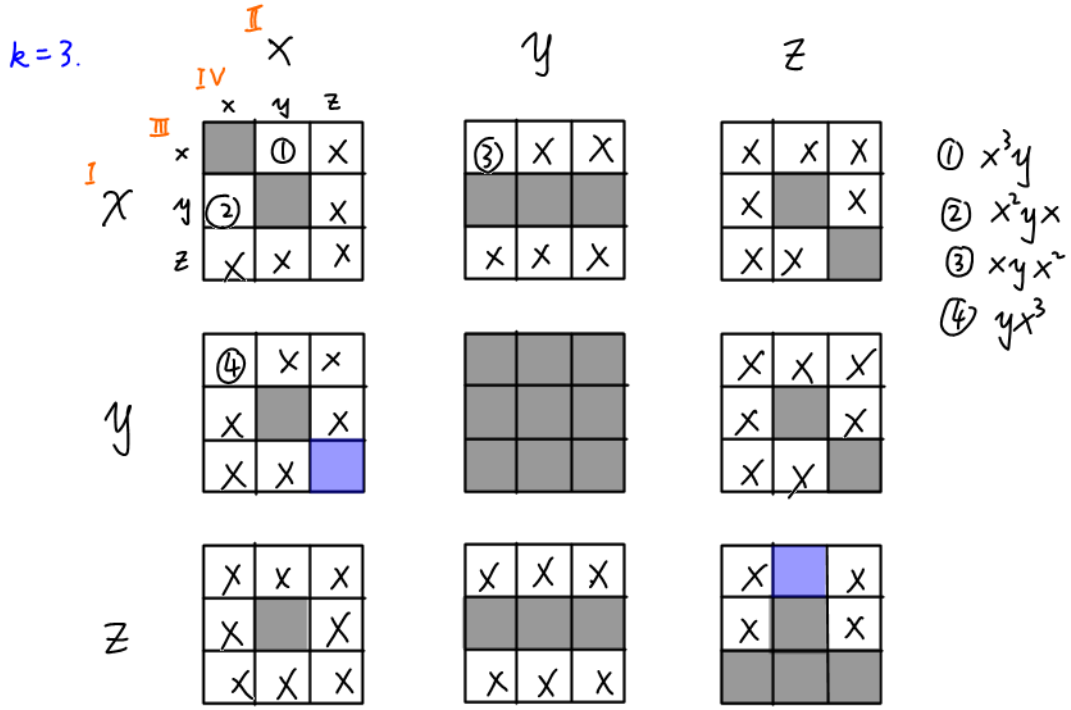
y

X	X	X
X	X	X

z

X	X	X
X		X
X	X	

① x^3
② x^2y
③ xyx
④ yx^2



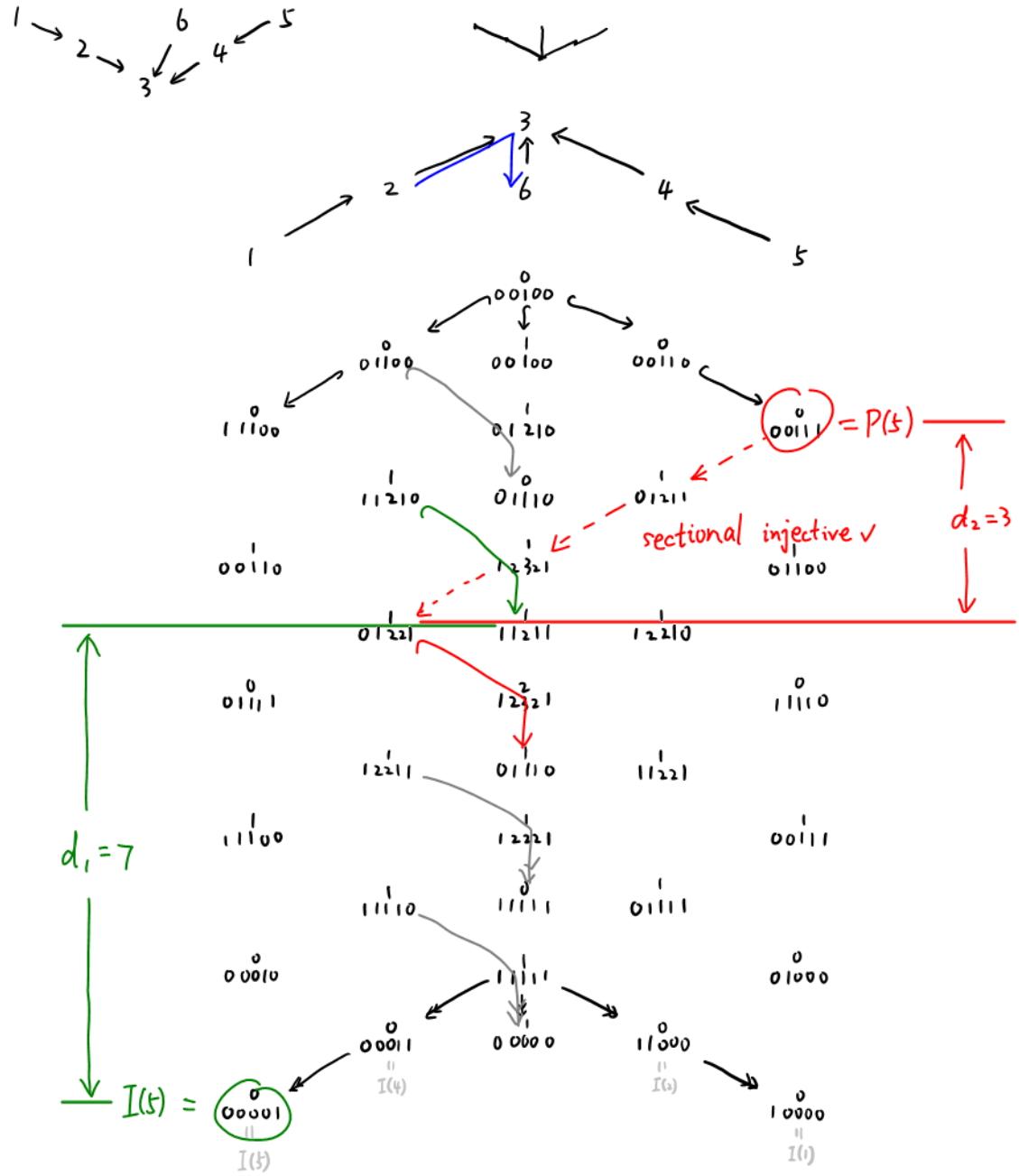
2. database for sectional map

We want to compute kernel & cokernel of some map, which turns out to be rather complicated (even though doable, and some techniques can speed up the computation).

We collect informations into tables, just like how people create character tables.

E.g.

E_6



Consider the sectional map from 2 to 6.

	6
2	5

index of kernel or cokernel ; e.g. P(5)

	6
2	X

sec surj?

	6
2	✓

sec inj?

d_1	6
2	7

d_2	6
2	3

E_6 case

	1	2	3	4	5	6
1	–	5	4	6	5	1
2	1	–	5	1	6	5
3	2	1	–	5	4	6
4	6	5	1	–	5	1
5	1	6	2	1	–	5
6	5	1	6	5	1	–

	1	2	3	4	5	6
1	–	✓	✓	✓	✓	✓
2	×	–	✓	×	×	×
3	×	×	–	×	×	×
4	×	×	✓	–	×	×
5	✓	✓	✓	✓	–	✓
6	×	✓	✓	✓	×	–

sec surj?

	1	2	3	4	5	6
1	–	×	×	×	✓	×
2	✓	–	×	×	✓	✓
3	✓	✓	–	✓	✓	✓
4	✓	×	×	–	✓	✓
5	✓	×	×	×	–	×
6	✓	×	×	×	✓	–

sec mono?

d_1	1	2	3	4	5	6
1	–	1	1	2	4	3
2	10	–	2	5	7	7
3	9	9	–	9	9	10
4	7	5	2	–	10	7
5	4	2	1	1	–	3
6	6	3	1	3	6	–

d_2	1	2	3	4	5	6
1	–	10	9	7	4	6
2	1	–	9	5	2	3
3	1	2	–	2	1	1
4	2	5	9	–	1	3
5	4	7	9	10	–	6
6	3	7	10	7	3	–

$d_1 + d_2 + \text{length} = 12$

Rmk. The information inside  is well-regulated, while it is a mess outside  .

E_7 case

	1	2	3	4	5	6	7
1	–	1	2	3	7	1	6
2	1	–	1	2	6	6	1
3	2	1	–	1	1	7	6
4	3	2	1	–	6	5	7
5	7	6	1	6	–	6	1
6	1	6	7	5	6	–	1
7	6	1	6	7	1	1	–

	1	2	3	4	5	6	7
1	–	✓	✓	✓	✓	✓	✓
2	×	–	✓	✓	×	×	×
3	×	×	–	✓	×	×	×
4	×	×	×	–	×	×	×
5	×	×	×	✓	–	×	×
6	×	✓	✓	✓	✓	–	✓
7	×	×	✓	✓	✓	×	–

sec surj?

	1	2	3	4	5	6	7
1	–	×	×	×	×	×	×
2	✓	–	×	×	×	✓	×
3	✓	✓	–	×	×	✓	✓
4	✓	✓	✓	–	✓	✓	✓
5	✓	×	×	×	–	✓	✓
6	✓	×	×	×	×	–	×
7	✓	×	×	×	×	✓	–

sec inj?

d_1	1	2	3	4	5	6	7
1	–	1	1	1	2	5	3
2	16	–	2	2	5	10	8
3	15	15	–	3	10	13	13
4	14	14	14	–	15	15	16
5	12	10	6	2	–	16	12
6	8	4	2	1	1	–	4
7	11	7	3	1	4	11	–

d_2	1	2	3	4	5	6	7
1	–	16	15	14	12	8	11
2	1	–	15	14	10	4	7
3	1	2	–	14	6	2	3
4	1	2	3	–	2	1	1
5	2	5	10	15	–	1	4
6	5	10	13	15	16	–	11
7	3	8	13	16	12	4	–

$d_1 + d_2 + \text{length} = 18$

E_8 case

	1	2	3	4	5	6	7	8
1	–	1	2	3	4	8	1	7
2	1	–	1	2	3	7	1	1
3	2	1	–	1	2	1	7	1
4	3	2	1	–	1	1	8	7
5	4	3	2	1	–	7	6	8
6	8	7	1	1	7	–	7	1
7	1	1	7	8	6	7	–	1
8	7	1	1	7	8	1	1	–

	1	2	3	4	5	6	7	8
1	–	✓	✓	✓	✓	✓	✓	✓
2	×	–	✓	✓	✓	×	×	×
3	×	×	–	✓	✓	×	×	×
4	×	×	×	–	✓	×	×	×
5	×	×	×	×	–	×	×	×
6	×	×	×	×	✓	–	×	×
7	×	×	✓	✓	✓	✓	–	✓
8	×	×	×	✓	✓	✓	×	–

sec surj?

	1	2	3	4	5	6	7	8
1	–	×	×	×	×	×	×	×
2	✓	–	×	×	×	×	×	×
3	✓	✓	–	×	×	×	✓	×
4	✓	✓	✓	–	×	×	✓	✓
5	✓	✓	✓	✓	–	✓	✓	✓
6	✓	×	×	×	×	–	✓	✓
7	✓	×	×	×	×	×	–	×
8	✓	×	×	×	×	×	✓	–

sec inj?

d_1	1	2	3	4	5	6	7	8
1	–	1	1	1	1	2	6	3
2	28	–	2	2	2	5	16	9
3	27	27	–	3	3	11	22	19
4	26	26	26	–	4	21	25	25
5	25	25	25	25	–	27	27	28
6	23	21	16	7	2	–	28	23
7	18	9	4	2	1	1	–	5
8	22	17	8	3	1	5	22	–

d_2	1	2	3	4	5	6	7	8
1	–	28	27	26	25	23	18	22
2	1	–	27	26	25	21	9	17
3	1	2	–	26	25	16	4	8
4	1	2	3	–	25	7	2	3
5	1	2	3	4	–	2	1	1
6	2	5	11	21	27	–	1	5
7	6	16	22	25	27	28	–	22
8	3	9	19	25	28	23	5	–

$d_1 + d_2 + \text{length} = 30$