

# Eine Woche, ein Beispiel

## 7.17 formalism

Here I collect some formalisms which enhance well-known theories.

Somehow it also explains the popularity of the abstraction in mathematics. We want to have a better understand, so we invent new languages and theories, which in turn causes more troubles for beginners. Hug those formalisms!

母题: classification, symmetry, glue and ramified covering

Axiomatic set Theory

→ Grothendieck Universe

Both let us be away from paradoxes. The latter is more convenient.

Category Theory

→ Infinite Category Theory

[adjoint fctor  $\Leftrightarrow$  preserve (co)limits] is some cases

( $\cdot$  / Braided / Symmetric) Monoidal Categories → Categorification

Extract combinatorial informations from categories  $\downarrow$

Examples

→ Classification

Specify and generalization.

See [https://github.com/ramified/personal\\_tex\\_collection/blob/main/%E8%9B%99%E9%B8%A3%97/latest%20version.pdf](https://github.com/ramified/personal_tex_collection/blob/main/%E8%9B%99%E9%B8%A3%97/latest%20version.pdf)

Parameter space

→ Moduli space

Structures on parameter spaces are important.

See [https://github.com/ramified/moduli\\_in\\_algebraic\\_geometry](https://github.com/ramified/moduli_in_algebraic_geometry)

Topology

→ Grothendieck Topology

Get étale topology.

Topological space

→ Condensed Set

Get abelian category.

Scheme

→ Functor

Get more objects. e.g. Ind-Sch, stack, ...

Snake lemma

→ homological algebra, spectral sequence

Diagram chasing all the time!

(co)homology

→ derived category, six-fctor formalism.

Reduce important properties to categorical non-sense.

<https://www.math.uni-bonn.de/people/schwede/EnhancedSeminar-WS2223.pdf>

Finite field

→ ???

Should contain "field with one element".

Moreover, AR theory tells us the structure of indecomposable reps,

Bruhat-Tits theory tells us the structure of  $p$ -adic groups,

Artin-Schreier theory tells us the structure of  $\deg p$  extensions.  $\text{char } F = p$

invariant, structure  $\rightsquigarrow$  forgetful functors

e.g.  $\text{Perv} \begin{matrix} \nearrow \text{characteristic cycle} \\ \searrow \text{Hodge structure} \end{matrix} \Rightarrow \text{Rep}(G) \begin{matrix} \nearrow \text{Rep}(T_U \rtimes \text{Gal}(\bar{\mathbb{Q}})) \\ \searrow \text{Rep}(\mathbb{G}_m^2) \end{matrix}$