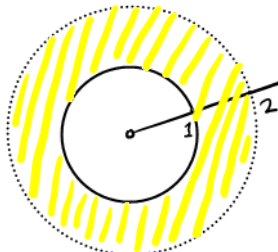


4.1. the complex torus of form $\mathbb{C}^x / \mathbb{Z}\gamma$

$$\gamma \in \text{Aut}(\mathbb{C}^x) \quad \gamma(z) = az \quad a \in \mathbb{C}^x \quad |a| > 1$$

1. fundamental set:

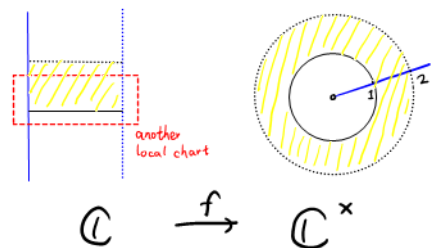


\Rightarrow only need 2 local chart


$$2. \quad 0 \rightarrow \mathbb{Z} \hookrightarrow \mathbb{C} \xrightarrow{f: z \mapsto e^{2\pi i z}} \mathbb{C}^\times \rightarrow 1$$


$$\downarrow +\frac{1}{2\pi i} \ln 2 \quad \downarrow +\frac{1}{2\pi i} \ln 2 \quad \downarrow \times 2$$

$$0 \rightarrow \mathbb{Z} + \frac{1}{2\pi i} \ln 2 \rightarrow \mathbb{C} \longrightarrow \mathbb{C}^\times \rightarrow 1$$



$$\mathbb{C}^* = \mathbb{C}/\mathbb{Z} \Rightarrow \mathbb{C}^*/\mathbb{Z}_Y = \mathbb{C}/(\mathbb{Z} \oplus \frac{1}{2\pi i} \ln 2\mathbb{Z}) \xrightarrow{\text{blue arrow}}$$

better: $a = e^{2\pi} \approx 535.49$ 

$a = e^{-2\pi i} \approx -230.765$ 

3. line bundle on \mathcal{C}

$$b \in \mathbb{C}^* \quad \mathcal{L}_b := \mathbb{C}^* \times \mathbb{C} / (z, \zeta) \sim (z\zeta, b\zeta) \quad \Rightarrow \quad \textcircled{1} \quad \mathcal{L}_b \in \text{Pic}_0(\mathbb{C}); \quad (\mathcal{L}_b \sim \mathcal{L}_1 \cong \mathcal{O}_{\mathbb{C}})$$

$$\downarrow$$

$$\mathcal{C} = \mathbb{C}^* / z \sim z\zeta$$

$$\textcircled{2} \quad \text{Pic}_0(\mathbb{C}) \cong \mathbb{C} = \mathbb{C}^* / \mathbb{Z} \quad \begin{matrix} \text{(naive, base pt } 1 \in \mathbb{C}^* / \mathbb{Z}) \\ \mathcal{L}_b \longmapsto b \end{matrix}$$

Reduced to: find a section s on \mathcal{L}_b st $\text{div } s = [b] - [1]$

Reduced to: find a meromorphic functions g on \mathbb{C}^\times s.t

① $g(2z) = b g(z)$ $b \in \mathbb{C}, b \neq 2^k$; e.g. $b=3$

② g has simple poles on 2^n , and simple zeros on $2^n b$ $n \in \mathbb{Z}$

$$b = e^{2\pi i c}, c \in \mathbb{C}$$

$$\tau = \frac{1}{2\pi i} \ln 2$$

$$w(z) = \frac{1}{2\pi i} \ln z$$

$$g(z) = \frac{\theta [1, z] (w(z), \tau)}{\theta \begin{bmatrix} 1 \\ 1 \end{bmatrix} (w(z), \tau)}$$
 is the required one.

Blue — example

Orange — more than this example

Red — important results

Purple — I don't know the answer/proof

Green — sketch of proof: in a minimal way

Grey — some supplementary explanation. Unimportant assumptions.

Hell grey — explanation on well-known notations.

Brown — small title in subsections.

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