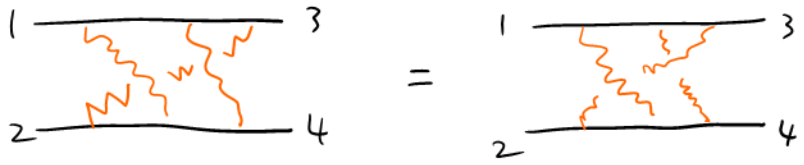
Eine Woche, ein Beispiel

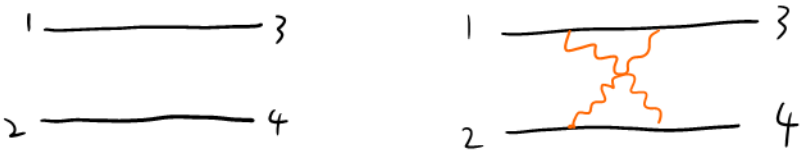
5.22. Feynman diagram

This is a document calculating one enumeration problem about the Feynman diagram.
basic information: https://www.youtube.com/watch?v=X-FEU4mQWtE

1. Requirement

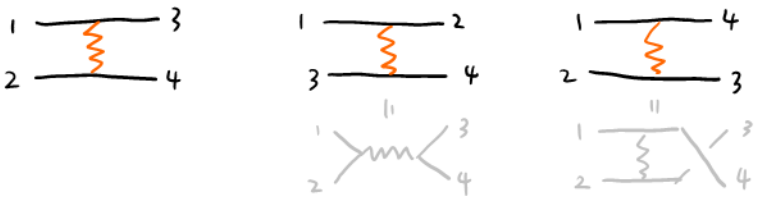


① two homeomorphic diagrams are viewed as the same;



② require to be connected ③ no interaction between photons.

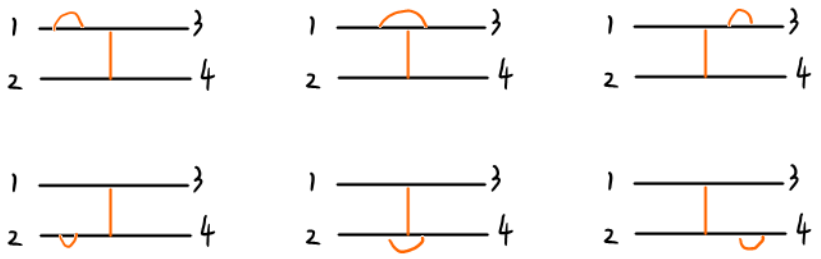
Ex. e^1



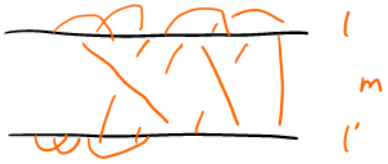
For convenience, later we omit the permutation of {1,2,3,4} (always ask 1 connected with 3) and draw photoes by lines.

e^2: two lines:

one lines:



$$3e^2 + 24e^4 + 207e^6 + 2088e^8 + 24615e^{10} + 334800e^{12} + 5181435e^{14} + \cdots$$



$$\sum_{n=1}^{+\infty} \left(\sum_{\substack{m \geq 1 \\ m+l+l'=n}} m! \cdot \binom{2l+m}{m} \binom{2l'+m}{m} t_l t_{l'} \right) e^{2n} \qquad m, l, l' \in \mathbb{N}_{\geq 0}$$

where $t_l = \#\{\text{ways of pairs of } \{1, 2, \cdots, 2l\}\}$

$$= \frac{\binom{2l}{2}\binom{2l-2}{2}\binom{2l-4}{2}\cdots\binom{2}{2}}{l!}$$

$$= \frac{(2l)!}{2^l l!}$$