# Eine Woche, ein Beispiel

## 11.26 calculation of double point

Final goal: Fill in the tables in the next page.
(for first time, remove the $i^!$ column)

Ref:

[Willians]: Langlands correspondence and Bezrukavnikov's equivalence

calculations from Lukas Bonfert's note (don't forward this to anyone else).

$X = \mathbb{C} \cup_{\{0\}} \mathbb{C} = \{(z_1, z_2) \in \mathbb{C}^2 \mid z_1 z_2 = 0\}$, $Z = \{0\}$, $U = \mathbb{C}^\times \sqcup \mathbb{C}^\times$

$i_* \underline{\mathbb{Q}}_Z$
$(0,1,1,1)$

| | $n$ | $-2$ | $-1$ | $0$ | $1$ |
|---|---|---|---|---|---|
| $U$ | $j^*$ | $0$ | $0$ | $0$ | $0$ |
| $\{0\}$ | $i^*$ | $0$ | $0$ | $\mathbb{Q}$ | $0$ |
| | $i^!$ | $0$ | $0$ | $\mathbb{Q}$ | $0$ |
| | $R^n\Gamma$ | $0$ | $0$ | $\mathbb{Q}$ | $0$ |

$\underline{\mathbb{Q}}_X[1]$
$(-1,-1,-1,-1)$

| | $n$ | $-2$ | $-1$ | $0$ | $1$ |
|---|---|---|---|---|---|
| $U$ | $j^*$ | $0$ | $\underline{\mathbb{Q}}$ | $0$ | $0$ |
| $\{0\}$ | $i^*$ | $0$ | $\mathbb{Q}$ | $0$ | $0$ |
| | $i^!$ | $0$ | $0$ | $\mathbb{Q}$ | $\mathbb{Q}^2$ |
| | $R^n\Gamma$ | $0$ | $\mathbb{Q}$ | $0$ | $0$ |

perverse sheaf IC sheaf — ? by dim argument

| | $n$ | $-2$ | $-1$ | $0$ | $1$ |
|---|---|---|---|---|---|
| $U$ | $j^*$ | $\times$ | | $\times$ | $\times$ |
| $\{0\}$ | $i^*$ | — | | $\times$ | $\times$ |
| | $i^!$ | $\times$ | $\times$ | $\times$ | |
| | $R^n\Gamma$ | — | | | |

$Rj_*\underline{\mathbb{Q}}_U[1]$
$(-1,0,0,0)$

| | $n$ | $-2$ | $-1$ | $0$ | $1$ |
|---|---|---|---|---|---|
| $U$ | $j^*$ | $0$ | $\underline{\mathbb{Q}}$ | $0$ | $0$ |
| $\{0\}$ | $i^*$ | $0$ | $\mathbb{Q}^2$ | $\mathbb{Q}^2$ | $0$ |
| | $i^!$ | $0$ | $0$ | $0$ | $0$ |
| | $R^n\Gamma$ | $0$ | $\mathbb{Q}^2$ | $\mathbb{Q}^2$ | $0$ |
| | $\Gamma$ | $0$ | $\mathbb{Q}^2$ | $\mathbb{Q}$ | $0$ |

$\Gamma^{(n)} = \Gamma(R^n j_* \underline{\mathbb{Q}}_U[1])$

$j_! \underline{\mathbb{Q}}_U[1]$
$(-1,0,0,0)$

| | $n$ | $-2$ | $-1$ | $0$ | $1$ |
|---|---|---|---|---|---|
| $U$ | $j^*$ | $0$ | $\underline{\mathbb{Q}}$ | $0$ | $0$ |
| $\{0\}$ | $i^*$ | $0$ | $0$ | $0$ | $0$ |
| | $i^!$ | $0$ | $0$ | $\mathbb{Q}^2$ | $\mathbb{Q}^2$ |
| | $R^n\Gamma$ | $0$ | $0$ | $0$ | $0$ |

$\pi^! \mathbb{Q}[-1]$
$(-1,-1,-1,-1)$

| | $n$ | $-2$ | $-1$ | $0$ | $1$ |
|---|---|---|---|---|---|
| $U$ | $j^*$ | $0$ | $\underline{\mathbb{Q}}$ | $0$ | $0$ |
| $\{0\}$ | $i^*$ | $0$ | $\mathbb{Q}^2$ | $\mathbb{Q}$ | $0$ |
| | $i^!$ | $0$ | $0$ | $0$ | $\mathbb{Q}$ |
| | $R^n\Gamma$ | $0$ | $\mathbb{Q}$ | $0$ | $0$ |

$X = X_3 = \{(z_1,z_2,z_3) \in \mathbb{C}^3 \mid z_1^2+z_2^2+z_3^2 = 0\}$, $Z = \{0\}$, $U = X_3 - \{0\} = \mathbb{C}^\times$-bd over $\mathbb{CP}^1$ (twisted)

$\mathcal{H}^\circ(\mathcal{F}) \in Perv_\Lambda(X)$

$i_+\underline{\mathbb{Z}}_Z$
$(0,1,1,1)$

| | $n$ | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|
| $U$ | $j^*$ | 0 | 0 | 0 | 0 | 0 |
| $\{0\}$ | $i^*$ | 0 | 0 | $\mathbb{Z}$ | 0 | 0 |
| | $i^!$ | 0 | 0 | $\mathbb{Z}$ | 0 | 0 |
| | $R^n\Gamma$ | 0 | 0 | $\mathbb{Z}$ | 0 | 0 |

$\underline{\mathbb{Z}}_X[2]$
$(1,1,1,1)$

| | $n$ | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|
| $U$ | $j^*$ | $\underline{\mathbb{Z}}$ | 0 | 0 | 0 | 0 |
| $\{0\}$ | $i^*$ | $\mathbb{Z}$ | 0 | 0 | 0 | 0 |
| | $i^!$ | 0 | 0 | 0 | $\mathbb{Z}/2\mathbb{Z}$ | $\mathbb{Z}$ |
| | $R^n\Gamma$ | $\mathbb{Z}$ | 0 | 0 | 0 | 0 |

$Rj_*\underline{\mathbb{Z}}_U[2] \in {}^pD^{\geq 0}(X) - {}^pD^{\leq 0}(X)$
$(1,0,0,0)$

| | $n$ | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|
| $U$ | $j^*$ | $\underline{\mathbb{Z}}$ | 0 | 0 | 0 | 0 |
| $\{0\}$ | $i^*$ | $\mathbb{Z}$ | 0 | $\mathbb{Z}/2\mathbb{Z}$ | $\mathbb{Z}$ | 0 |
| | $i^!$ | 0 | 0 | 0 | 0 | 0 |
| | $R^n\Gamma$ | $\mathbb{Z}$ | 0 | $\mathbb{Z}/2\mathbb{Z}$ | $\mathbb{Z}$ | 0 |

| | $n$ | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|
| $U$ | $j^*$ | $\underline{\mathbb{Z}}$ | 0 | 0 | 0 | 0 |
| $\{0\}$ | $i^*$ | $\mathbb{Z}$ | 0 | $\mathbb{Z}/2\mathbb{Z}$ | 0 | 0 |
| | $i^!$ | 0 | 0 | 0 | 0 | $\mathbb{Z}$ |
| | $R^n\Gamma$ | $\mathbb{Z}$ | 0 | $\mathbb{Z}/2\mathbb{Z}$ | 0 | 0 |

$j_!\underline{\mathbb{Z}}_U[2] \in {}^pD^{\leq 0}(X) - {}^pD^{\geq 0}(X)$
$(1,0,0,0)$

| | $n$ | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|
| $U$ | $j^*$ | $\underline{\mathbb{Z}}$ | 0 | 0 | 0 | 0 |
| $\{0\}$ | $i^*$ | 0 | 0 | 0 | 0 | 0 |
| | $i^!$ | 0 | $\mathbb{Z}$ | 0 | $\mathbb{Z}/2\mathbb{Z}$ | $\mathbb{Z}$ |
| | $R^n\Gamma$ | 0 | 0 | 0 | 0 | 0 |

| | $n$ | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|
| $U$ | $j^*$ | $\underline{\mathbb{Z}}$ | 0 | 0 | 0 | 0 |
| $\{0\}$ | $i^*$ | $\mathbb{Z}$ | 0 | 0 | 0 | 0 |
| | $i^!$ | 0 | 0 | 0 | $\mathbb{Z}/2\mathbb{Z}$ | $\mathbb{Z}$ |
| | $R^n\Gamma$ | $\mathbb{Z}$ | 0 | 0 | 0 | 0 |

$\pi^!\mathbb{Z}[-2]$
$(1,1,1,1)$

| | $n$ | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|
| $U$ | $j^*$ | $\underline{\mathbb{Z}}$ | 0 | 0 | 0 | 0 |
| $\{0\}$ | $i^*$ | $\mathbb{Z}$ | 0 | $\mathbb{Z}/2\mathbb{Z}$ | 0 | 0 |
| | $i^!$ | 0 | 0 | 0 | 0 | $\mathbb{Z}$ |
| | $R^n\Gamma$ | $\mathbb{Z}$ | 0 | $\mathbb{Z}/2\mathbb{Z}$ | 0 | 0 |

perverse sheaf
IC sheaf

| | n | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|
| $\mathcal{U}$ | $j^*$ | | × | × | × | × |
| $\{0\}$ | $i^*$ | | | × | × | × |
| | $i^!$ | × | × | × | | |
| | $R^n\Gamma$ | | | | | |

Conclusion. $\subseteq$ IC $\subseteq$ Perv $\subseteq$ Constructable

$\mathbb{Q}$-coefficient $i_*\underline{\mathbb{Q}}_Z$ $\quad$ $Rj_*\underline{\mathbb{Q}}_U[2]$, $j_!\underline{\mathbb{Q}}_U[2]$

$\underline{\mathbb{Q}}_X[2] \cong \pi^!\mathbb{Q}[-2]$

$\mathbb{Z}$-coefficient $i_*\underline{\mathbb{Z}}_Z$ $\quad$ $Rj_*\underline{\mathbb{Z}}_U[2]$, $j_!\underline{\mathbb{Z}}_U[2]$

$\underline{\mathbb{Z}}_X[2]$ $\quad$ $\pi^!\mathbb{Z}[-2]$