

# Eine Woche, ein Beispiel

## 8.15 indecomposable representation of Dynkin quiver.

AR-quiver is a powerful tool considering about the indecomposable modules and relations among them. Using the AR-quiver, one can find(not totally serious):

- all the indecomposable modules;
- all the morphisms between these indecomposable modules;
- all the irreducible morphisms and AR-sequences;

However, it's not easy to see the coker and ker of some morphisms given by the AR-quiver.

The following AR-quiver pictures are now useless, since everyone can get better pictures at

<https://www.math.uni-bielefeld.de/~jgeuenich/string-applet/> or <https://www.math.uni-bielefeld.de/~wcrawley/#knitting>.

Unfortunately, the knitting process can not draw some AR-quivers even in the case where "there are finite iso class of indec modules of quiver"

e.g.

$$A = K[T]/(T^3) \cong KQ/(a^3)$$

$$Q: 1 \xrightarrow{a} 2$$

$$\begin{array}{c} N(3) \\ \uparrow \downarrow \\ N(2) \xrightarrow{\tau} \\ \uparrow \downarrow \\ N(1) \xrightarrow{\tau} \end{array}$$

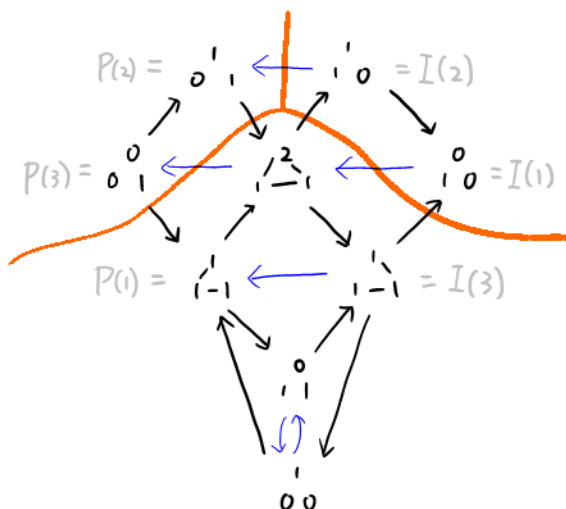
$$A = KQ/(ab)$$

$$Q: 1 \xrightleftharpoons[b]{a} 2$$

$$\begin{array}{ccc} & S(1) \xrightarrow{\tau} & \\ & \swarrow \quad \searrow & \\ P(1) & \xleftarrow{\tau} & I(1) \\ & \searrow \quad \swarrow & \\ & P(2) = I(2) & \end{array}$$

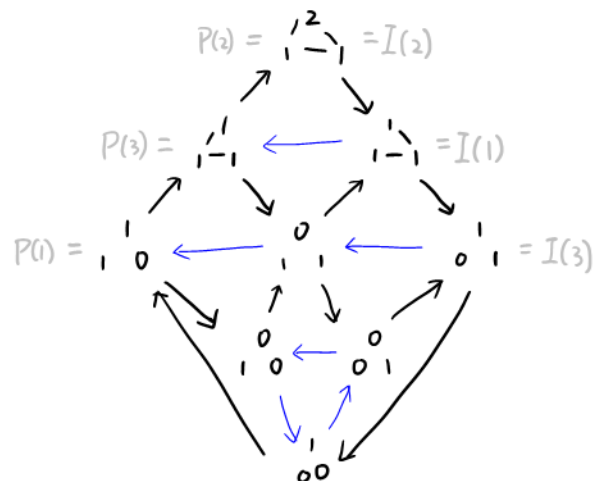
$$A = KQ/(ab)$$

$$Q: \begin{array}{ccc} & 2 & \\ a \nearrow & & \searrow b \\ 1 & \xrightarrow{c} & 3 \end{array}$$



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$$Q: \begin{array}{ccc} & 2 & \\ a \nearrow & & \searrow b \\ 1 & \xleftarrow{c} & 3 \end{array}$$

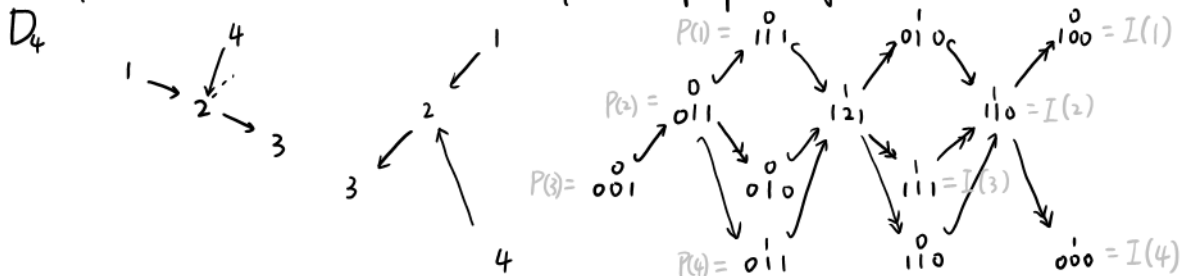


from different components of the AR-quiver of KQ.

For the description of AR quiver of type A and D by a triangulated (puctured) polygon, see [Quiver Representations by Ralf Schiffler, 3.1.3+3.3.3].

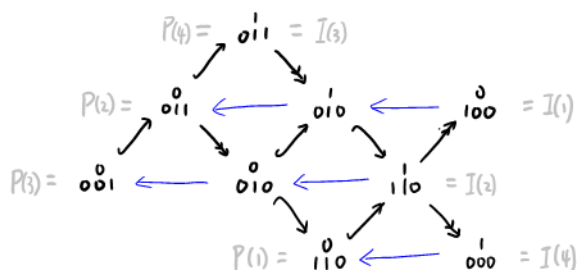
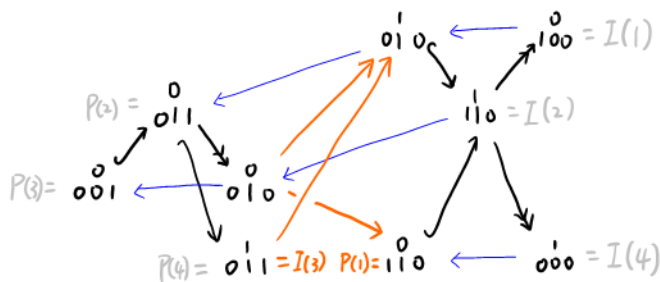
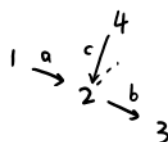
Ex. Use the applet to compute  $KQ/(a^3, b^3, ab, ba)$   
 $aG12b$

Even for bounded quiver algebra with Dynkin quiver, it is not very clear how the AR-quiver is related with the AR-quiver of path algebra.



$A = KQ/(ab)$

$Q:$



Q. Are the nontrivial bounded quiver algebra of affine quiver representation finite?

Expected: yes.

Q Let  $R$  be a f.d.  $k$ -algebra with is indecomposable and of rep finite.

Is there only one component of AR-quiver (for  $R$ )?

Thm. [Thm 13.27, [http://www.math.uni-bonn.de/~schroer/fd-atlas-files/FD\\_Atlas.pdf](http://www.math.uni-bonn.de/~schroer/fd-atlas-files/FD_Atlas.pdf)]

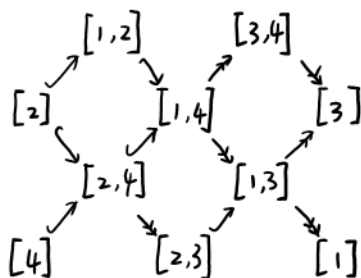
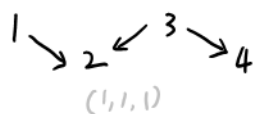
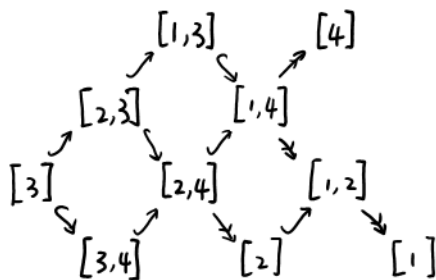
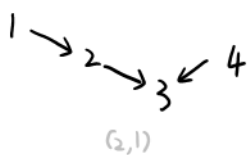
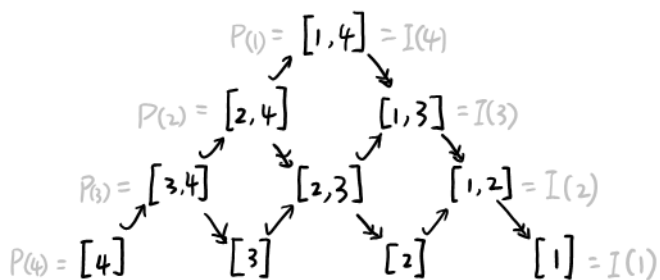
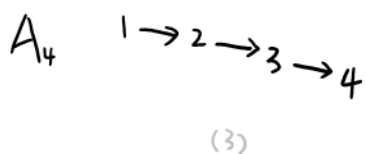
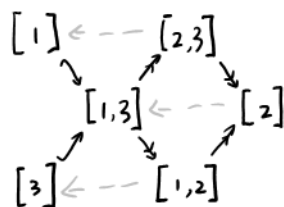
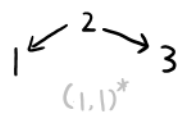
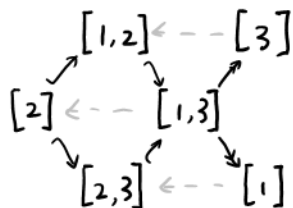
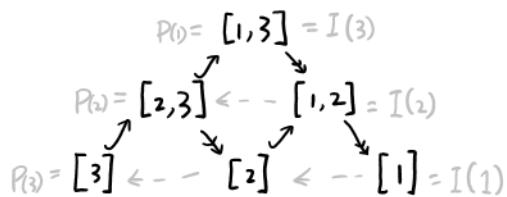
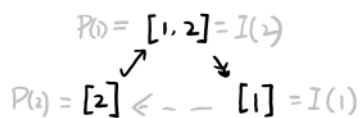
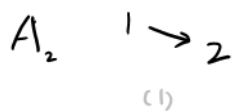
Let  $R$  be a f.d.  $k$ -algebra with is indecomposable.

If one component of AR-quiver (for  $R$ ) have only finite vertexes, then  $R$  is of rep finite.

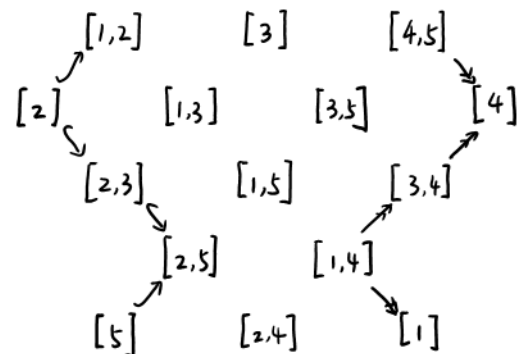
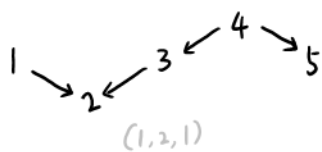
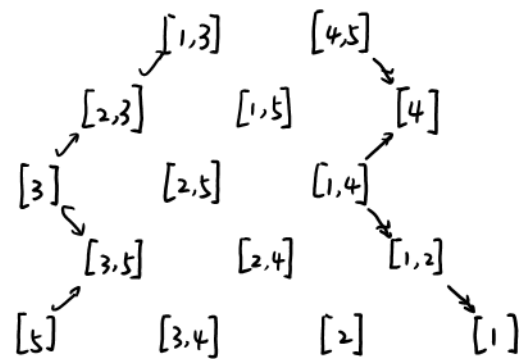
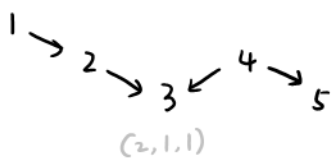
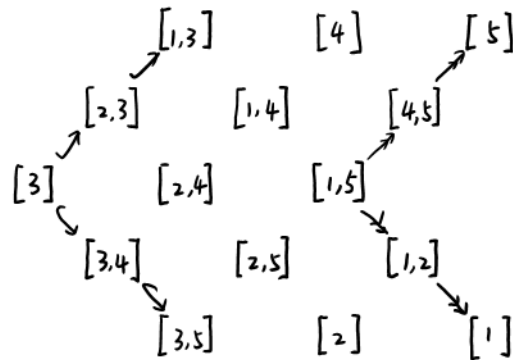
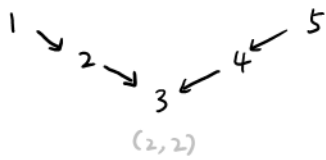
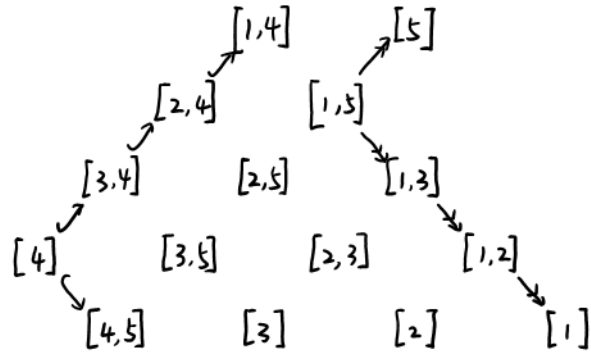
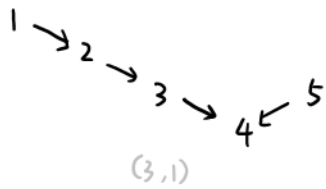
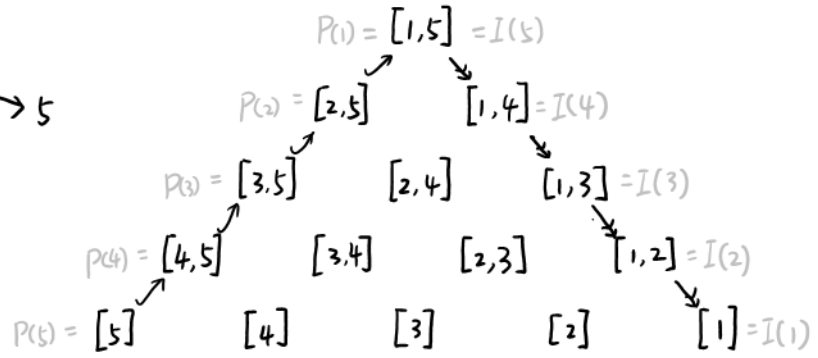
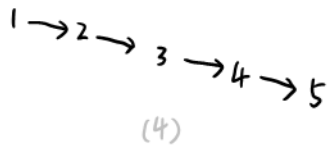
Conj [Conj 13.28, 13.29, [http://www.math.uni-bonn.de/~schroer/fd-atlas-files/FD\\_Atlas.pdf](http://www.math.uni-bonn.de/~schroer/fd-atlas-files/FD_Atlas.pdf)]

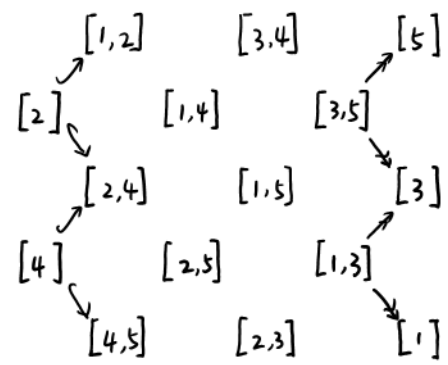
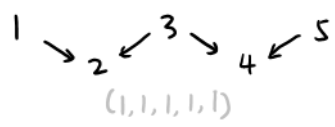
Let  $R$  be a f.d.  $k$ -algebra.

If the AR-quiver (for  $R$ ) have only finite component, then  $R$  is of rep finite.

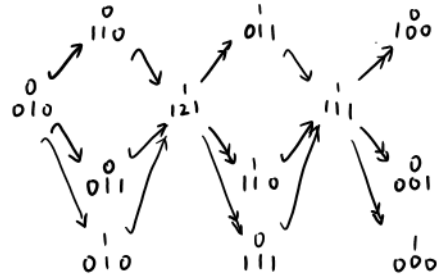
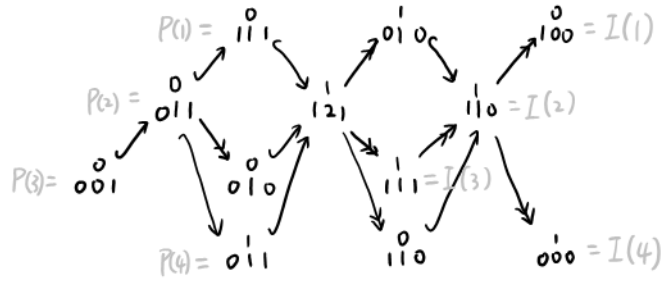
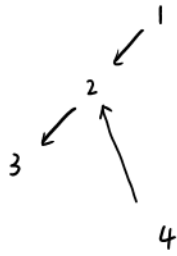
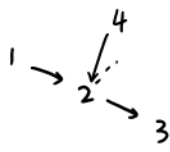


$A_5$

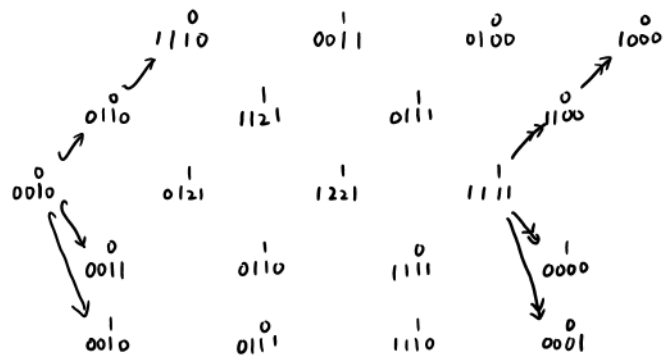
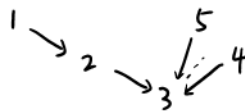
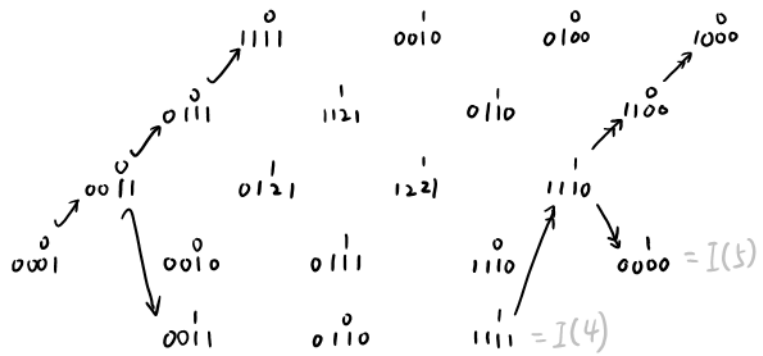
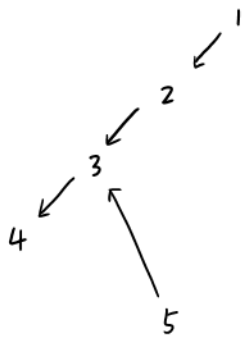
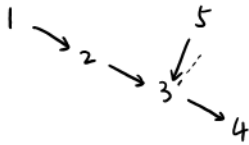


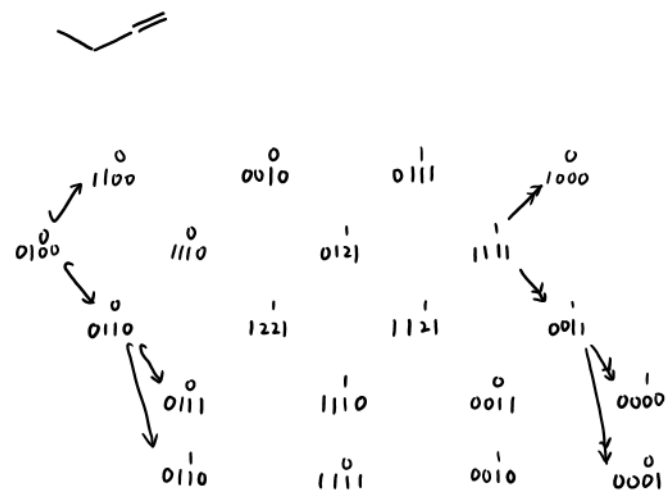
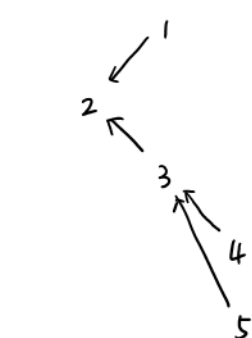
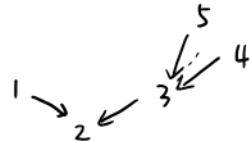
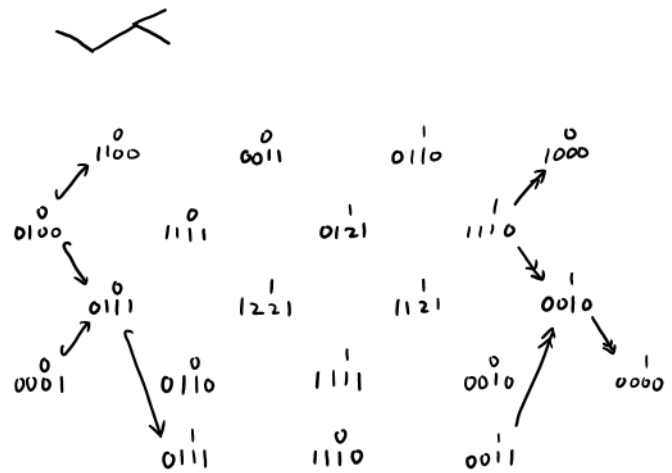
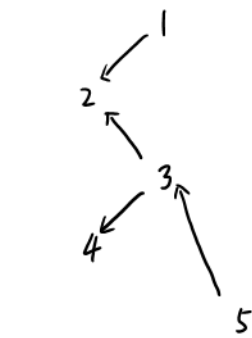
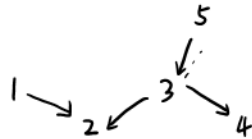
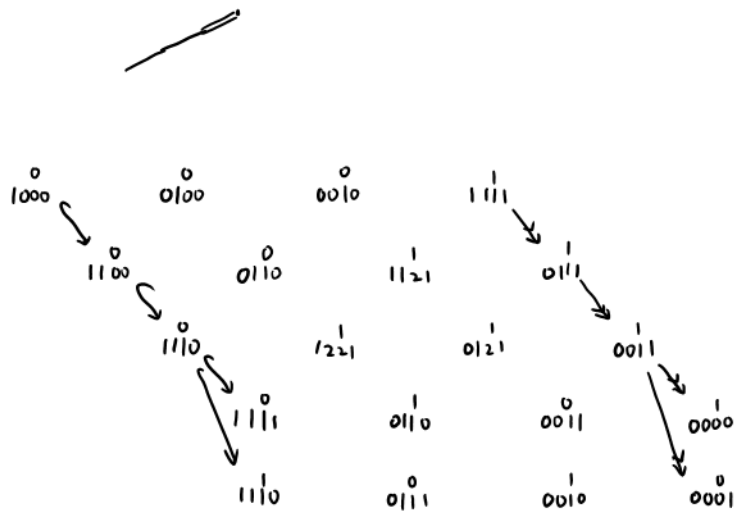
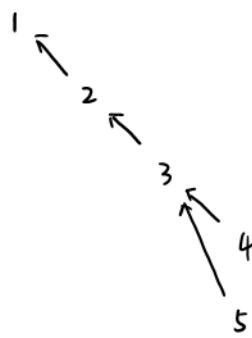
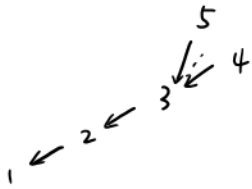


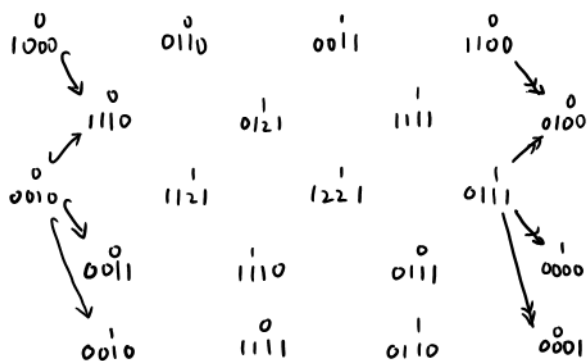
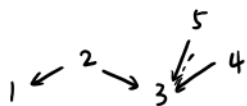
$D_4$



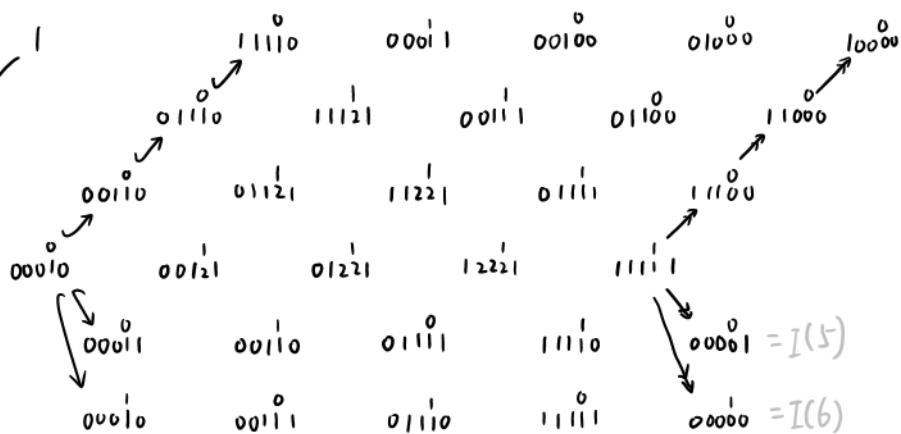
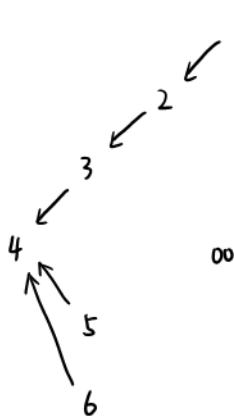
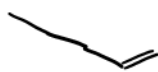
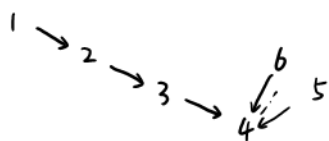
$D_5$



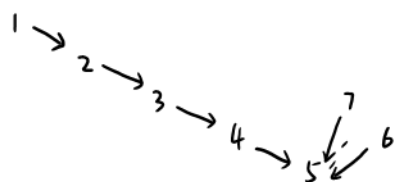




$D_6$



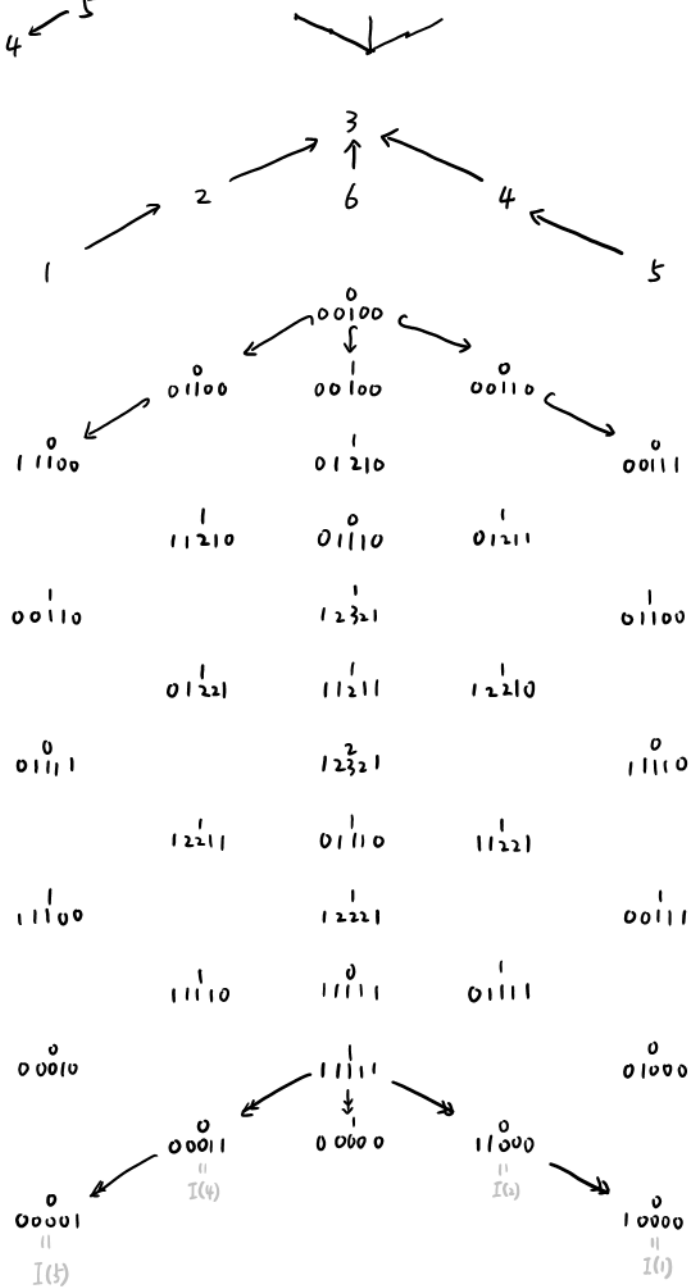
$D_7$



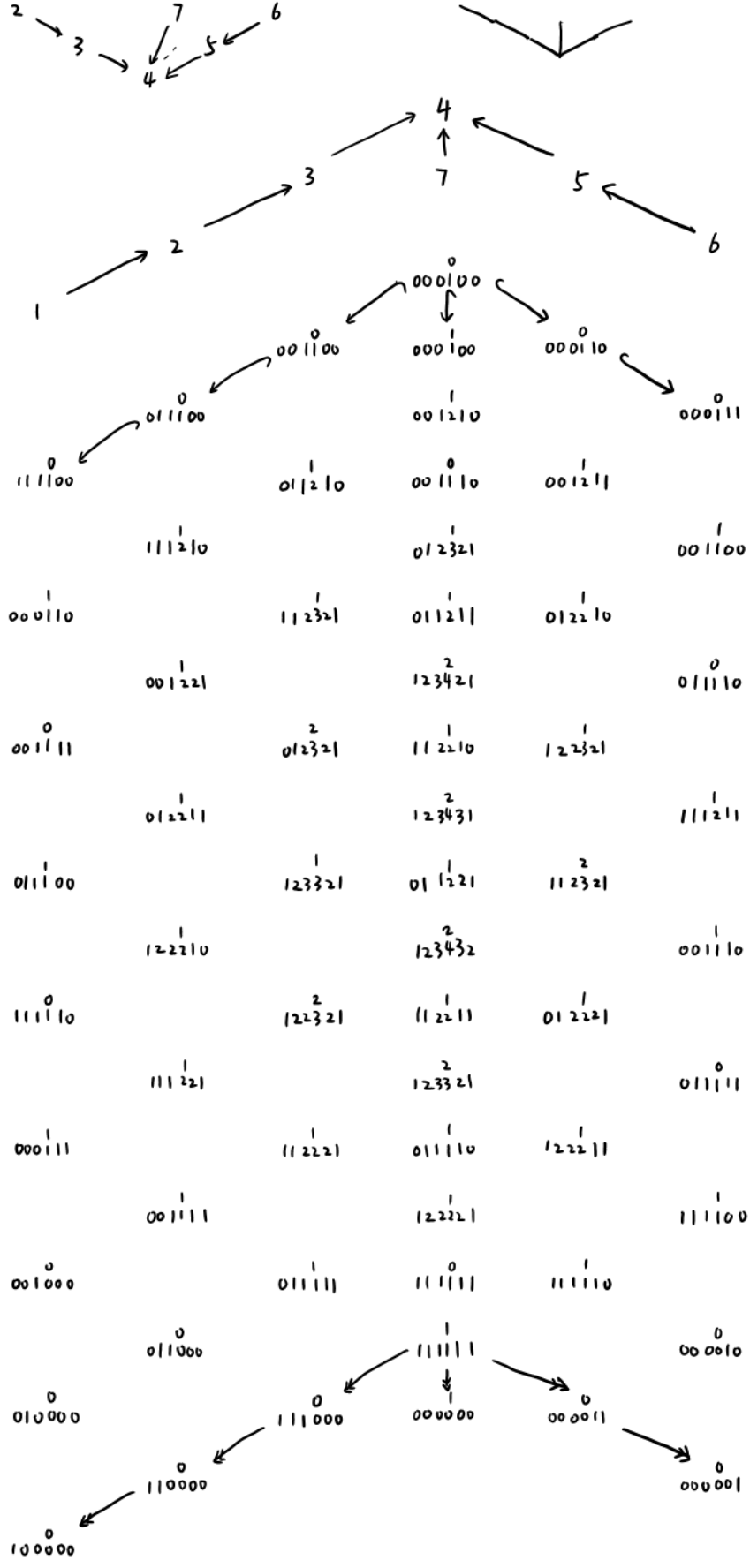
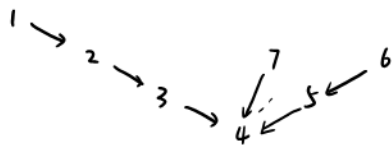
$$P(7) = 000010 \leftarrow \dots \leftarrow 000001 = P(6)$$

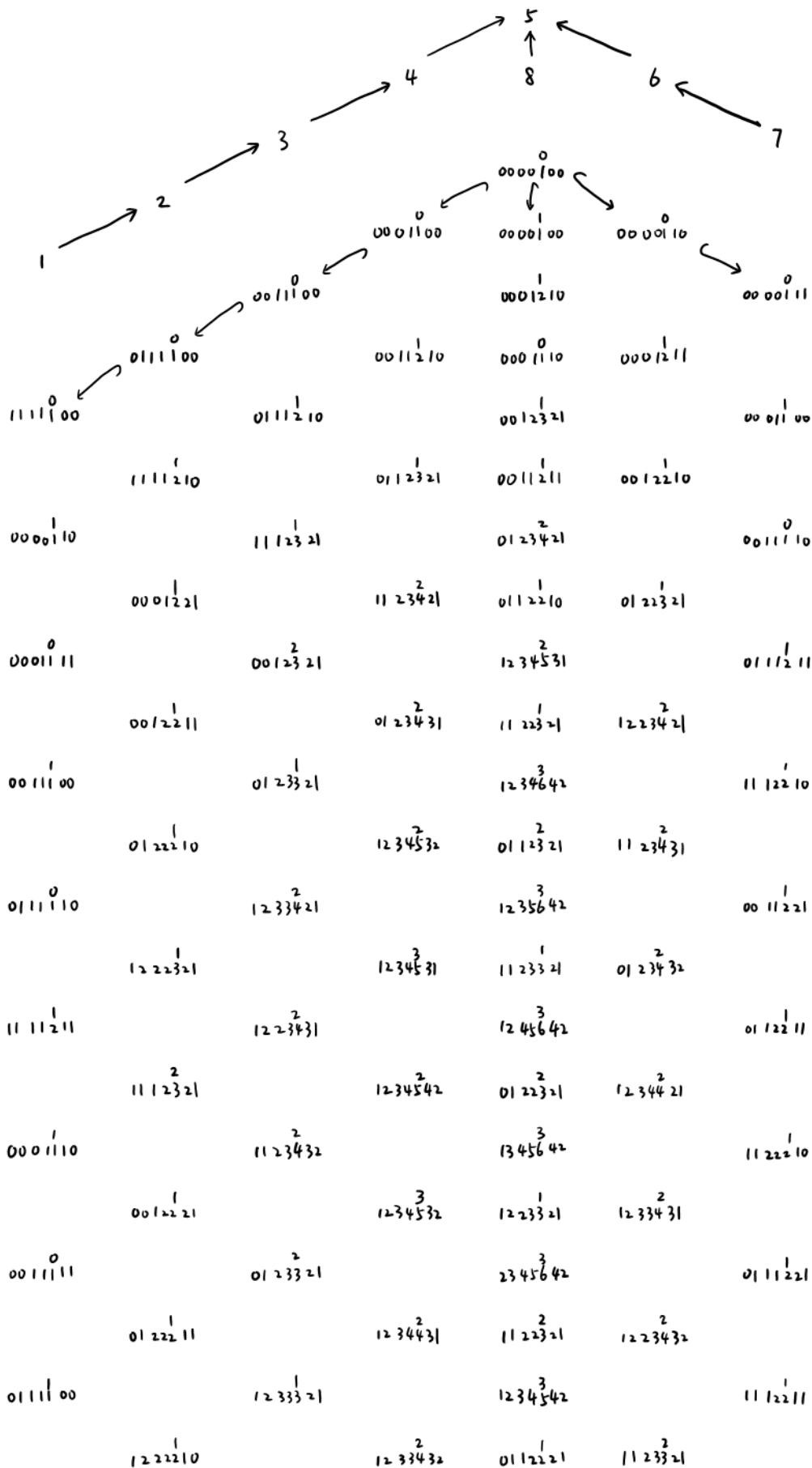
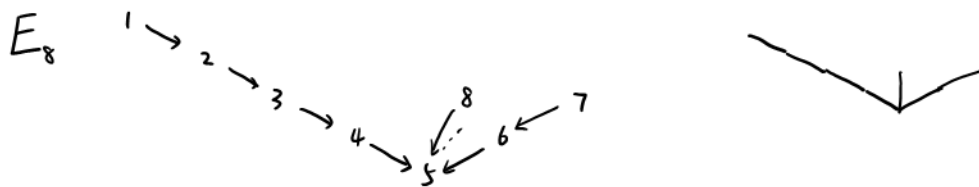


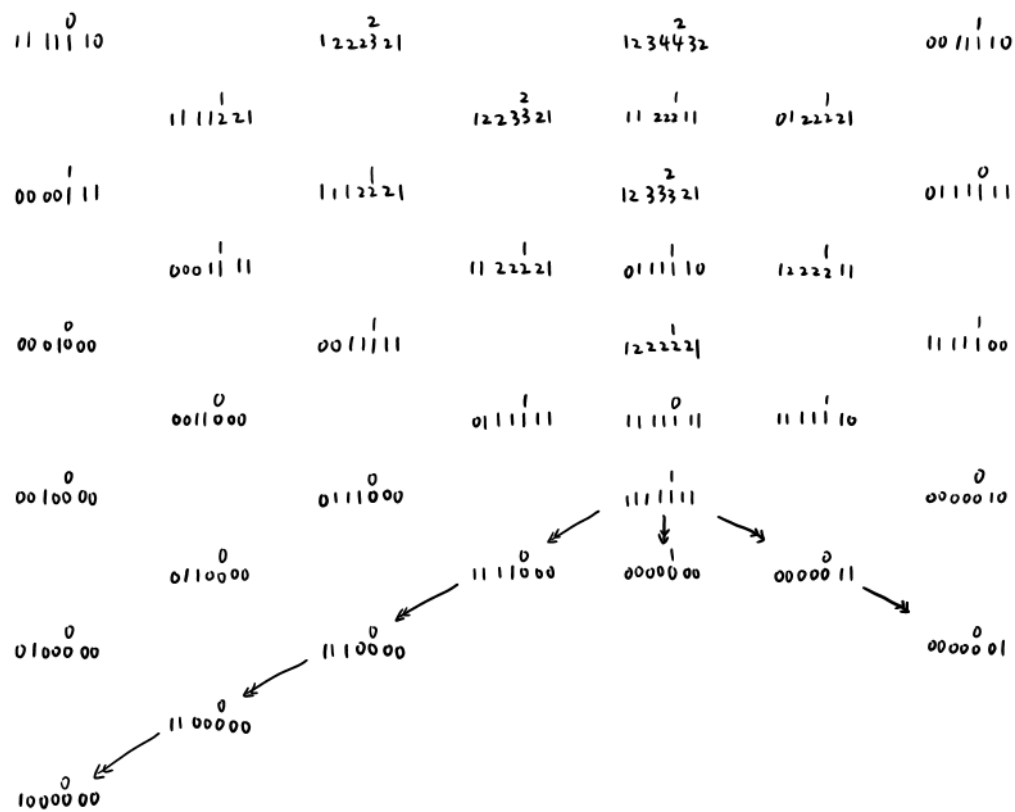
$E_6$   $1 \rightarrow 2 \rightarrow 3 \xleftarrow{6} 4 \leftarrow 5$



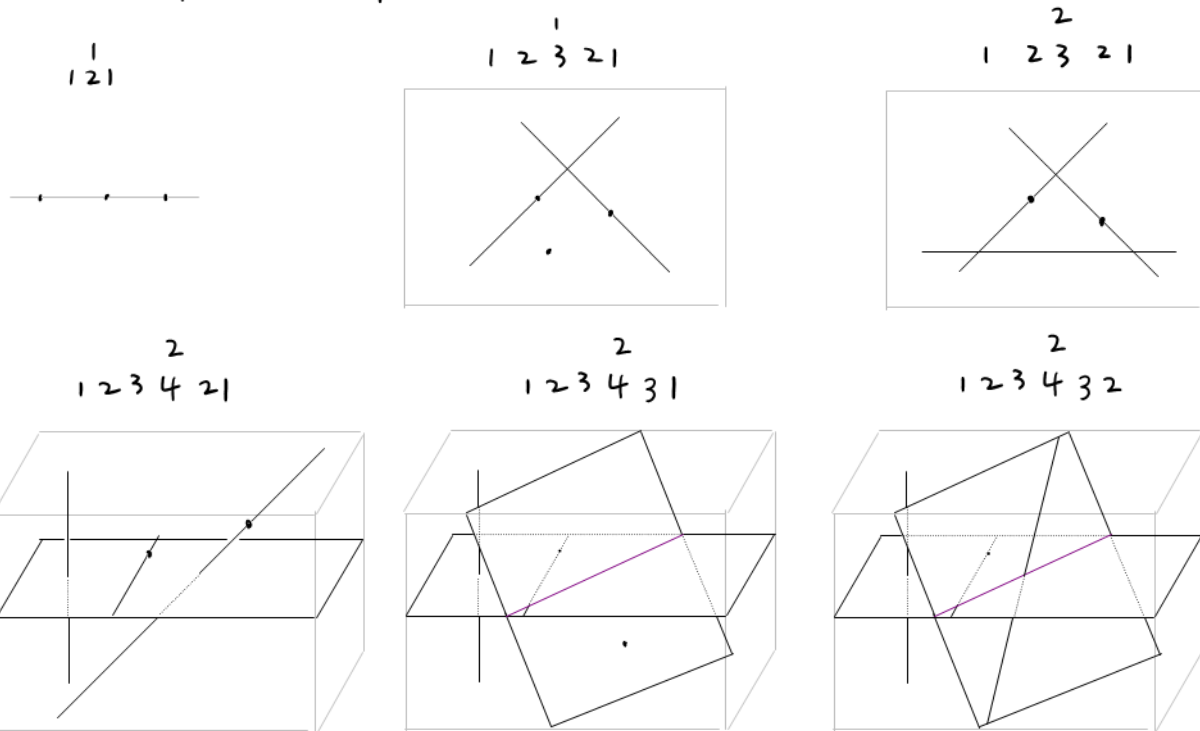
$E_7$





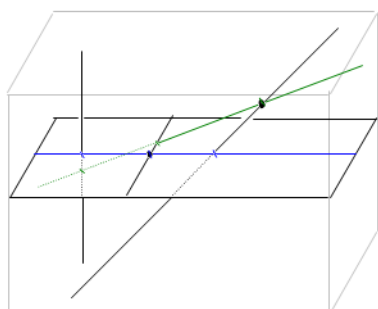


# Bonus: subspace case (projective space version)



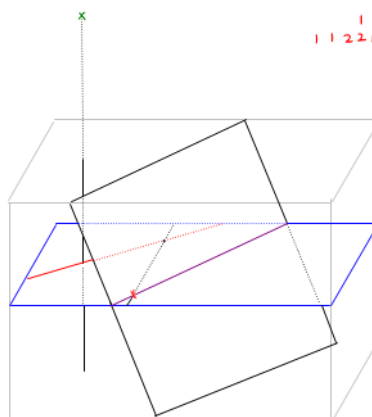
These shapes should be as general as possible, otherwise it may be not indecomposable:

e.g.  $\begin{smallmatrix} 2 \\ 1\ 2\ 3\ 4\ 2\ 1 \end{smallmatrix} = \begin{smallmatrix} 1 \\ 1\ 1\ 2\ 2\ 1\ 0 \end{smallmatrix} \oplus \begin{smallmatrix} 1 \\ 0\ 1\ 1\ 2\ 1\ 1 \end{smallmatrix}$

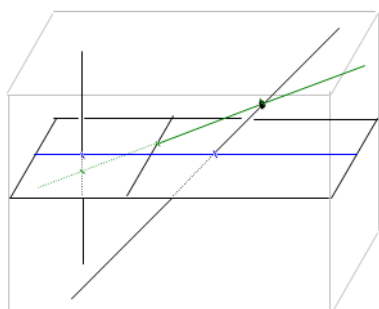


$\begin{smallmatrix} 2 \\ 1\ 2\ 3\ 4\ 3 \end{smallmatrix} = \begin{smallmatrix} 1 \\ 1\ 2\ 3\ 3\ 2 \end{smallmatrix} \oplus \begin{smallmatrix} 1 \\ 0\ 0\ 0\ 1\ 1 \end{smallmatrix}$

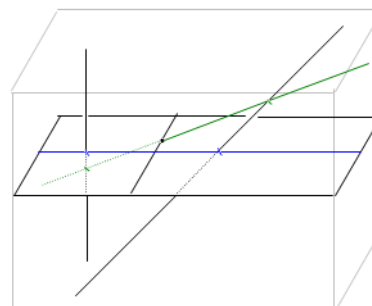
$\begin{smallmatrix} 1 \\ 1\ 1\ 2\ 2\ 1 \end{smallmatrix} \oplus \begin{smallmatrix} 1 \\ 0\ 1\ 1\ 1\ 1 \end{smallmatrix}$



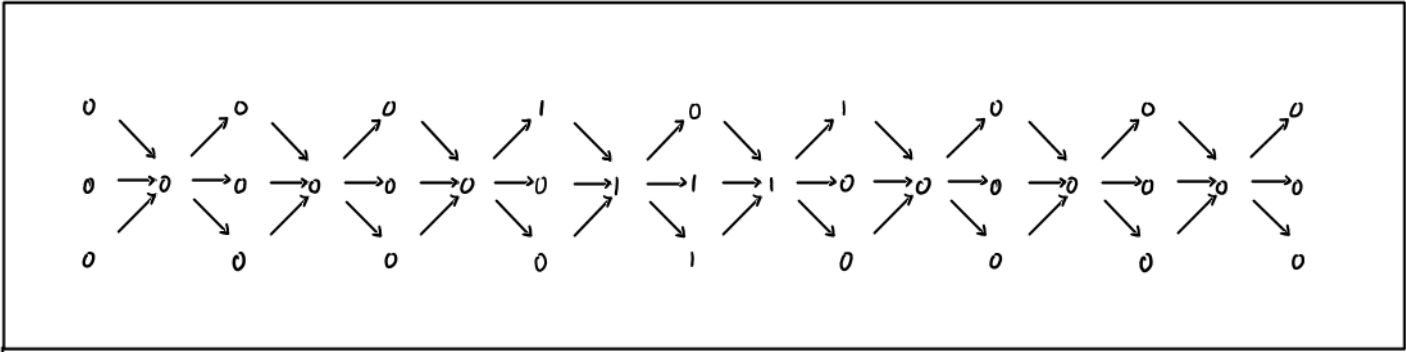
$\begin{smallmatrix} 2 \\ 2\ 3\ 4\ 2\ 1 \end{smallmatrix} = \begin{smallmatrix} 1 \\ 1\ 2\ 2\ 1\ 0 \end{smallmatrix} \oplus \begin{smallmatrix} 1 \\ 1\ 1\ 1\ 2\ 1\ 1 \end{smallmatrix}$



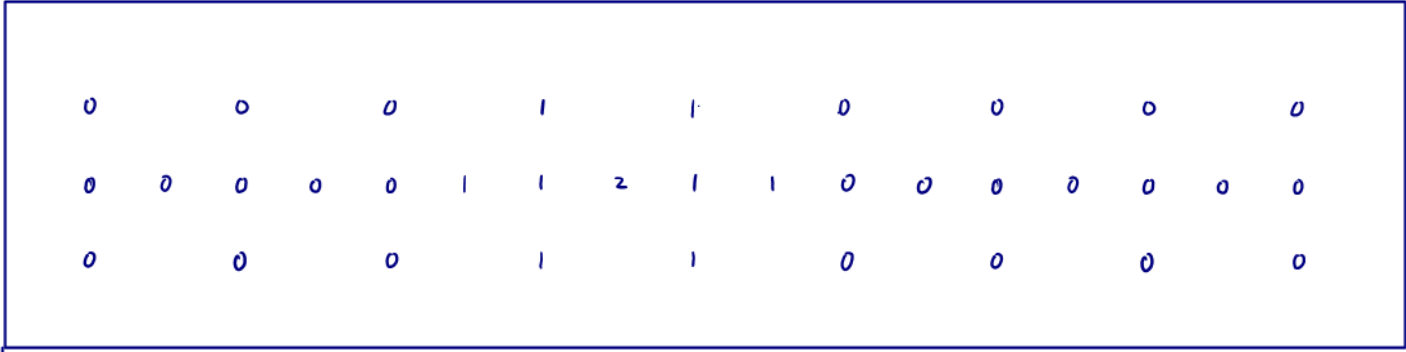
$\begin{smallmatrix} 2 \\ 1\ 2\ 3\ 4\ 2 \end{smallmatrix} = \begin{smallmatrix} 1 \\ 0\ 1\ 2\ 2\ 1 \end{smallmatrix} \oplus \begin{smallmatrix} 1 \\ 1\ 1\ 1\ 2\ 1 \end{smallmatrix}$



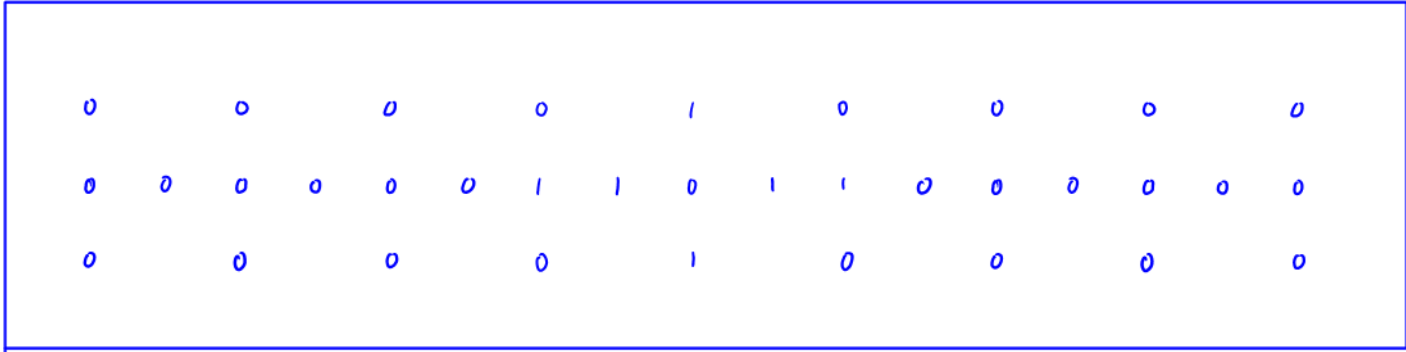
It's not easy to read the informations of them, but AR-quivers can.



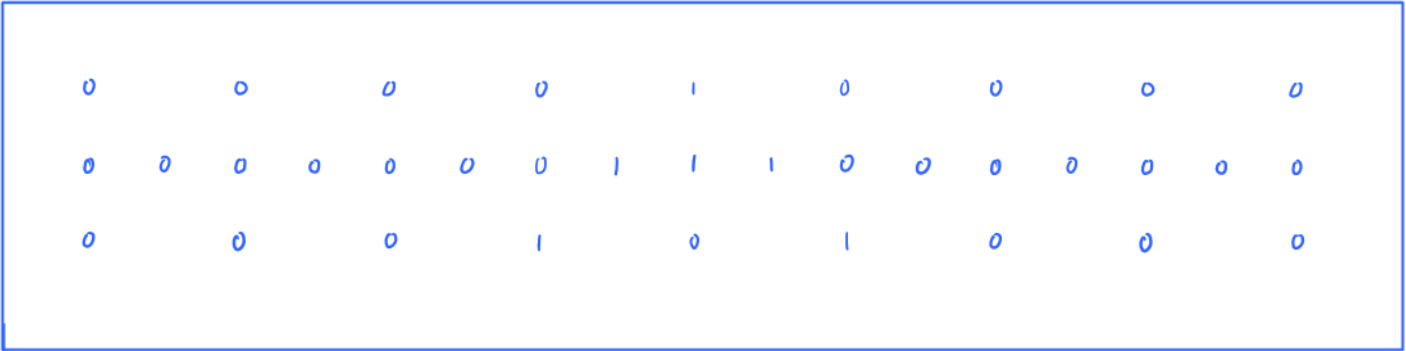
P(1)



P(2)



P(3)



P(4)