# Eine Woche, ein Beispiel

## 10.2 equivariant K-theory of Steinberg variety

Ref:
[Ginz] Ginzburg's book "Representation Theory and Complex Geometry"
[LCBE] Langlands correspondence and Bezrukavnikov's equivalence
[LW-BWB] The notes by Liao Wang: The Borel-Weil-Bott theorem in examples (can not be found on the internet)
https://people.math.harvard.edu/~gross/preprints/sat.pdf

Task. Complete the following tables.

G here is SL_n but not GL_n (to make sure the correctness of K(St))

| $K^-(-)$ | pt | $\mathcal{B}$ $\quad T^*\mathcal{B}$ | $\mathcal{B}\times\mathcal{B}$ $\quad T^*(\mathcal{B}\times\mathcal{B})$ | St |
|---|---|---|---|---|
| $G$ | $\mathbb{Z}[X^*(T)]^W$ | $\mathbb{Z}[X^*(T)]$ | $\mathbb{Z}[X^*(T)]\otimes_{\mathbb{Z}[X^*(T)]^W}\mathbb{Z}[X^*(T)]$ | $\mathbb{Z}[W_{ext}]$ |
| $B$ | $\mathbb{Z}[X^*(T)]$ | $\mathbb{Z}[X^*(T)]\otimes_{\mathbb{Z}[X^*(T)]^W}\mathbb{Z}[X^*(T)]$ | $\mathbb{Z}[X^*(T)]\otimes_{\mathbb{Z}[X^*(T)]^W}\mathbb{Z}[X^*(T)]\otimes_{\mathbb{Z}[X^*(T)]^W}\mathbb{Z}[X^*(T)]$ | |
| $Id$ | $\mathbb{Z}$ | | | $\mathbb{Z}[X^*(T)]/_{I_T}\rtimes\mathbb{Z}[W_f]$ |
| $G\times\mathbb{C}^\times$ | $\mathbb{Z}[X^*(T)]^W[t^{\pm1}]$ | | | $\mathcal{H}_{ext}$ |
| $B\times\mathbb{C}^\times$ | $\mathbb{Z}[X^*(T)][t^{\pm1}]$ | | | |
| $\mathbb{C}^\times$ | $\mathbb{Z}[t^{\pm1}]$ | | | |

We use the shorthand.

| $K^-(-)$ | pt | $\mathcal{B}$ $\quad T^*\mathcal{B}$ | $\mathcal{B}\times\mathcal{B}$ $\quad T^*(\mathcal{B}\times\mathcal{B})$ | St |
|---|---|---|---|---|
| $G$ | $R(T)^W$ | $R(T)$ | $R(T)\otimes_{R(G)}R(T)$ | $\mathbb{Z}[W_{ext}]$ |
| $B$ | $R(T)$ | $R(T)\otimes_{R(G)}R(T)$ | $R(T)\otimes_{R(G)}R(T)\otimes_{R(G)}R(T)$ | |
| $Id$ | $\mathbb{Z}$ | | | $R(T)/_{I_T}\rtimes\mathbb{Z}[W_f]$ |
| $G\times\mathbb{C}^\times$ | $R(G)[t^{\pm1}]$ | | | $\mathcal{H}_{ext}$ |
| $B\times\mathbb{C}^\times$ | $R(T)[t^{\pm1}]$ | | | |
| $\mathbb{C}^\times$ | $\mathbb{Z}[t^{\pm1}]$ | | | |

$$R(B) = \mathbb{Z}[X^*(T)] = \mathcal{H}(\hat{T}(F), \hat{T}(\mathcal{O}_F))$$
$$R(G) = \mathbb{Z}[X^*(T)]^W \neq \mathcal{H}(\hat{G}(F), \hat{G}(\mathcal{O}_F))$$
$$R(G)[q^{\pm\frac12}] = \mathbb{Z}[X^*(T)]^W[q^{\pm\frac12}] = \mathcal{H}_{sph}[q^{\pm\frac12}]$$
$$R(G\times\mathbb{C}^\times) = \mathbb{Z}[X^*(T)]^W[t^{\pm1}]$$
$$K^{G\times\mathbb{C}^\times}(St) = \mathcal{H}_{ext} \overset{?}{=} \mathcal{H}(\hat{G}(F), I)$$

Here is an initial example.

| $K^-(-)$ | pt | $\mathcal{B}$ $\quad T^*\mathcal{B}$ | $\mathcal{B}\times\mathcal{B}$ $\quad T^*(\mathcal{B}\times\mathcal{B})$ | St |
|---|---|---|---|---|
| $SL_2$ | $\mathbb{Z}[x]$ | $\mathbb{Z}[z^{\pm1}]$ | $\mathbb{Z}[z_1^{\pm1}, z_2^{\pm1}]/((z_1-z_2)(z_1-z_2^{-1}))$ | $\mathbb{Z}[W_{ext}] = \bigoplus_{w\in W}\mathbb{Z}[z_w^{\pm1}]$ |
| $B$ | $\mathbb{Z}[y^{\pm1}]$ | $\mathbb{Z}[y^{\pm1}, z]/(z-y)(z-y^{-1})$ | $\mathbb{Z}[y^{\pm1}, z_1, z_2]/((z_1-y)(z_1-y^{-1}), (z_2-y)(z_2-y^{-1}))$ | |
| $Id$ | $\mathbb{Z}$ | $\mathbb{Z}[z]/(z-1)^2$ | $\mathbb{Z}[z_1, z_2]/((z_1-1)^2, (z_2-1)^2)$ | $R(T)/_{I_T}\rtimes\mathbb{Z}[W_f] = \bigoplus_{w\in W}\mathbb{Z}[z_w^{\pm1}]/(z_w-1)^2$ |
| $SL_2\times\mathbb{C}^\times$ | $\mathbb{Z}[x, t^{\pm1}]$ | | | $\mathcal{H}_{ext} = \bigoplus_{w\in W}\mathbb{Z}[z_w^{\pm1}, t^{\pm1}]$ |
| $B\times\mathbb{C}^\times$ | $\mathbb{Z}[y^{\pm1}, t^{\pm1}]$ | | | |
| $\mathbb{C}^\times$ | $\mathbb{Z}[t^{\pm1}]$ | | | |

This is our final task. Most of the notations are still not fixed.
G_d here is of type GL.

| $K^-(-)$ | pt | $\mathcal{F}_{\underline{d}}$ $\quad\widetilde{Rep}_{\underline{d}}(Q)$ | $\mathcal{F}_{\underline{d}} \times \mathcal{F}_{\underline{d}'}$ | $\mathcal{Z}_{\underline{d},\underline{d}'}$ |
|---|---|---|---|---|
| $G_d$ | $R(T_d)^{W_d}$ | $R(T_d)$ | $R(T_d)\otimes_{R(G_d)}R(T_d)$ | |
| $B_d$ | $R(T_d)$ <br> $\bigoplus_{w\in W_d} R(G_d)$ | $R(T_d)\otimes_{R(G_d)}R(T_d)$ <br> $\bigoplus_{w\in W_d} R(T_d)[\overline{\Omega_w}]^{T_d}$ | $R(T_d)\otimes_{R(G_d)}R(T_d)\otimes_{R(G_d)}R(T_d)$ <br> $\bigoplus_{w,w'\in W_d} R(T_d)[\overline{\Omega_{w,w'}}]^{T_d}$ | $\bigoplus_{w,w'\in W_d} R(T_d)[\overline{\tilde{\Omega}_{w,w'}}]^{T_d}$ |
| $Id$ | $\mathbb{Z}$ | $\bigoplus_{w\in W_d} \mathbb{Z}[\overline{\Omega_w}]$ | $\bigoplus_{w,w'\in W_d} \mathbb{Z}[\overline{\Omega_{w,w'}}]$ | $\bigoplus_{w,w'\in W_d} \mathbb{Z}[\overline{\tilde{\Omega}_{w,w'}}]$ |
| $G_d\times\mathbb{C}^*$ | $R(G_d)[t^{\pm1}]$ | | | |
| $B_d\times\mathbb{C}^*$ | $R(T_d)[t^{\pm1}]$ <br> $\bigoplus_{w\in W_d} R(G_d\times\mathbb{C}^*)$ | $\bigoplus_{w\in W_d} R(T_d\times\mathbb{C}^*)[\overline{\Omega_w}]^{T_d\times\mathbb{C}^*}$ | $\bigoplus_{w,w'\in W_d} R(T_d\times\mathbb{C}^*)[\overline{\Omega_{w,w'}}]^{T_d\times\mathbb{C}^*}$ | $\bigoplus_{w,w'\in W_d} R(T_d\times\mathbb{C}^*)[\overline{\tilde{\Omega}_{w,w'}}]^{T_d\times\mathbb{C}^*}$ |
| $\mathbb{C}^*$ | $\mathbb{Z}[t^{\pm1}]$ | $\bigoplus_{w\in W_d} R(\mathbb{C}^*)[\overline{\Omega_w}]^{\mathbb{C}^*}$ | $\bigoplus_{w,w'\in W_d} R(\mathbb{C}^*)[\overline{\Omega_{w,w'}}]^{\mathbb{C}^*}$ | $\bigoplus_{w,w'\in W_d} R(\mathbb{C}^*)[\overline{\tilde{\Omega}_{w,w'}}]^{\mathbb{C}^*}$ |

| $K^-(-)$ | pt | $\mathcal{F}_d$ $\quad\widetilde{Rep}_{d}(Q)$ | $\mathcal{F}_d \times \mathcal{F}_d$ | $\mathcal{Z}_d = \coprod_{\underline{d},\underline{d}'} \mathcal{Z}_{\underline{d},\underline{d}'}$ |
|---|---|---|---|---|
| $G_d$ | $R(T_d)^{W_d}$ | $\bigoplus_{\underline{d}} R(T_d)$ | $\bigoplus_{\underline{d}} R(T_d)\otimes_{R(G_d)}R(T_d)$ | |
| $B_d$ | $R(T_d)$ <br> $\bigoplus_{w\in W_d} R(G_d)$ | $\bigoplus_{\underline{d}} R(T_d)\otimes_{R(G_d)}R(T_d)$ <br> $\bigoplus_{\varpi\in W_{\lvert d\rvert}} R(T_d)[\overline{\mathcal{O}_\varpi}]^{T_d}$ | $\bigoplus_{\underline{d}} R(T_d)\otimes_{R(G_d)}R(T_d)\otimes_{R(G_d)}R(T_d)$ <br> $\bigoplus_{\varpi,\varpi'\in W_{\lvert d\rvert}} R(T_d)[\overline{\mathcal{O}_{\varpi,\varpi'}}]^{T_d}$ | $\bigoplus_{\varpi,\varpi'\in W_{\lvert d\rvert}} R(T_d)[\overline{\tilde{\mathcal{O}}_{\varpi,\varpi'}}]^{T_d}$ |
| $Id$ | $\mathbb{Z}$ | $\bigoplus_{\varpi\in W_{\lvert d\rvert}} \mathbb{Z}[\overline{\mathcal{O}_\varpi}]$ | $\bigoplus_{\varpi,\varpi'\in W_{\lvert d\rvert}} \mathbb{Z}[\overline{\mathcal{O}_{\varpi,\varpi'}}]$ | $\bigoplus_{\varpi,\varpi'\in W_{\lvert d\rvert}} \mathbb{Z}[\overline{\tilde{\mathcal{O}}_{\varpi,\varpi'}}]$ |
| $G_d\times\mathbb{C}^*$ | $R(G_d)[t^{\pm1}]$ | | | |
| $B_d\times\mathbb{C}^*$ | $R(T_d)[t^{\pm1}]$ <br> $\bigoplus_{w\in W_d} R(G_d\times\mathbb{C}^*)$ | $\bigoplus_{\varpi\in W_{\lvert d\rvert}} R(T_d\times\mathbb{C}^*)[\overline{\mathcal{O}_\varpi}]^{T_d\times\mathbb{C}^*}$ | $\bigoplus_{\varpi,\varpi'\in W_{\lvert d\rvert}} R(T_d\times\mathbb{C}^*)[\overline{\mathcal{O}_{\varpi,\varpi'}}]^{T_d\times\mathbb{C}^*}$ | $\bigoplus_{\varpi,\varpi'\in W_{\lvert d\rvert}} R(T_d\times\mathbb{C}^*)[\overline{\tilde{\mathcal{O}}_{\varpi,\varpi'}}]^{T_d\times\mathbb{C}^*}$ |
| $\mathbb{C}^*$ | $\mathbb{Z}[t^{\pm1}]$ | $\bigoplus_{\varpi\in W_{\lvert d\rvert}} R(\mathbb{C}^*)[\overline{\mathcal{O}_\varpi}]^{\mathbb{C}^*}$ | $\bigoplus_{\varpi,\varpi'\in W_{\lvert d\rvert}} R(\mathbb{C}^*)[\overline{\mathcal{O}_{\varpi,\varpi'}}]^{\mathbb{C}^*}$ | $\bigoplus_{\varpi,\varpi'\in W_{\lvert d\rvert}} R(\mathbb{C}^*)[\overline{\tilde{\mathcal{O}}_{\varpi,\varpi'}}]^{\mathbb{C}^*}$ |

$$H_*^{G_d}(\mathcal{Z}_{\underline{d},\underline{d}}) \cong \bigotimes_{d_i} NH_{d_i}$$

$$K^{G_d}(\mathcal{Z}_{\underline{d},\underline{d}}) \cong R(T_d)\otimes_{R(G_d)}R(T_d) \cong \bigotimes_{d_i} R(T_{d_i})\otimes_{R(G_{d_i})}R(T_{d_i})$$

Black: know the alg structure under tensor prod (not convolution prod)

Orange: only know the $R(G_{rp})$-module structure, and the alg structure is yet not known

light yellow: $R(G_d)$-module + $W_d$-equiv iso

$d = (1,2)$ $\qquad$ $\begin{array}{c} a \longrightarrow b \\ \langle v_1 \rangle \longrightarrow \langle v_2, v_3 \rangle \end{array}$

⚠ The action on Flag is not the same as in http://www.math.uni-bonn.de/ag/stroppel/Master%27s%20Thesis_Tomasz%20Przezdziecki.pdf

| $\varpi = wu$ | | | | | $w$ | $\underline{d} = u$ | | order of basis | $l(\varpi)$ | $l(w)$ | $\mathbb{B}_\varpi$ | $B_\varpi$ | $\varpi B \varpi^{-1}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Id | Id | $\binom{1\,2\,3}{1\,2\,3}$ | | $\begin{bmatrix}1&&\\&1&\\&&1\end{bmatrix}$ | | abb | | $\{v_1, v_2, v_3\}$ | 0 | 0 | $\begin{bmatrix}*&*&*\\&*&*\\&&*\end{bmatrix}$ | $\begin{bmatrix}*&&\\&*&*\\&&*\end{bmatrix}$ | $\begin{bmatrix}*&&\\&*&*\\&&*\end{bmatrix}$ |
| $t$ | (23) | $\binom{1\,2\,3}{1\,3\,2}$ | | $\begin{bmatrix}1&&\\&&1\\&1&\end{bmatrix}$ | | abb | | $\{v_1, v_3, v_2\}$ | 1 | 1 | $\begin{bmatrix}*&*&*\\&*&\\&*&*\end{bmatrix}$ | $\begin{bmatrix}*&&\\&*&\\&*&*\end{bmatrix}$ | $\begin{bmatrix}*&&\\&*&\\&*&*\end{bmatrix}$ |
| $s$ | (12) | $\binom{1\,2\,3}{2\,1\,3}$ | | $\begin{bmatrix}&1&\\1&&\\&&1\end{bmatrix}$ | | bab | | $\{v_2, v_1, v_3\}$ | 1 | 0 | $\begin{bmatrix}*&&*\\*&*&*\\&&*\end{bmatrix}$ | $\begin{bmatrix}*&&\\&*&*\\&&*\end{bmatrix}$ | $\begin{bmatrix}*&&*\\&*&\\&&*\end{bmatrix}$ |
| $ts$ | (132) | $\binom{1\,2\,3}{3\,1\,2}$ | | $\begin{bmatrix}&1&\\&&1\\1&&\end{bmatrix}$ | | bab | | $\{v_3, v_1, v_2\}$ | 2 | 1 | $\begin{bmatrix}*&*&\\&*&\\*&*&*\end{bmatrix}$ | $\begin{bmatrix}*&&\\&*&\\&*&*\end{bmatrix}$ | $\begin{bmatrix}*&*&\\&*&\\&&*\end{bmatrix}$ |
| $st$ | (123) | $\binom{1\,2\,3}{2\,3\,1}$ | | $\begin{bmatrix}&&1\\1&&\\&1&\end{bmatrix}$ | | bba | | $\{v_2, v_3, v_1\}$ | 2 | 0 | $\begin{bmatrix}*&&\\*&*&*\\*&&*\end{bmatrix}$ | $\begin{bmatrix}*&&\\&*&*\\&&*\end{bmatrix}$ | $\begin{bmatrix}*&&\\&*&\\*&&*\end{bmatrix}$ |
| $sts$ | (13) | $\binom{1\,2\,3}{3\,2\,1}$ | | $\begin{bmatrix}&&1\\&1&\\1&&\end{bmatrix}$ | | bba | | $\{v_3, v_2, v_1\}$ | 3 | 1 | $\begin{bmatrix}*&&\\*&*&\\*&*&*\end{bmatrix}$ | $\begin{bmatrix}*&&\\&*&\\&*&*\end{bmatrix}$ | $\begin{bmatrix}*&&\\*&*&\\&&*\end{bmatrix}$ |

## Conclusions on the summer vacation.

I guess that most part of my tasks coincide with this paper:
http://www.math.uni-bonn.de/ag/stroppel/Master%27s%20Thesis_Tomasz%20Przezdziecki.pdf
Sadly, I only found it in the last week of the vacation.

### Some possible tasks to work on.

1. Work out what $K_0^{Id}(\mathcal{B})$ is.

ref:
In [3264], the author computes the Chow group of G(2,4).
https://pbelmans.ncag.info/blog/2018/08/22/rank-flag-varieties/
http://www.math.tau.ac.il/~bernstei/Publication_list/publication_texts/BGG-SchubCells-Usp.pdf

The module structure is easy, see
[https://math.stackexchange.com/questions/1012699/when-does-a-smooth-projective-variety-x-have-a-free-grothendieck-group]

2. Work out what $\mathcal{H}(G(F), I)$ is, i.e.
   - Bernstein presentation
   - try to understand the center of $\mathcal{H}(G(F), I)$
   - How does $\mathcal{H}(G(F), I)$ reflect informations on the rep theory
   - How can the Hecke algebra be realized as a Hecke algebra?

ref. [Hecke, Sec 10-17], [Williamson 11.4-12.2]

3. Try to understand what the Hall algebra / Quantum group is.

ref: [Lec 1-4, Appendix 4, https://arxiv.org/pdf/math/0611617.pdf]

- understand $\mathcal{H}_{\mathrm{Rep}_k^{nil}(Q)}$ where $Q = \cdot \quad \cdot\!\to\!\cdot \quad \cdot\!\circlearrowleft$ [Lec 2-3]
- understand [Lec 4]

$$\begin{array}{ccc} \mathcal{H}_{\mathbb{P}^1} & \cong & \mathcal{U}_v(\widehat{\mathfrak{sl}_2}) \\ \cup & & \cup \\ \mathcal{H}_{\mathrm{Tor}(\mathbb{P}^1)} & \cong & \bigotimes_{x \in \mathbb{P}^1} \mathcal{H}_{\mathrm{Tor}\, x} \end{array}$$

- define (Quantum) Kac-Moody / loop algs [Appendix 4]
- Why is that

graded

$$K_0(\mathrm{Rep}^{\mathbb{Z}}(R)) = \mathcal{U}_q(\mathfrak{n}(Q))$$

where

$$R = \bigoplus_{\underline{d}} H_{\bullet}^{G \times \mathbb{C}^\times, BM}(Z_{\underline{d}})$$

and what is

$$K_0\left(\mathrm{Rep}^{\mathbb{Z}}\left(\bigoplus_{\underline{d}} K_0^{G \times \mathbb{C}^\times}(Z_{\underline{d}})\right)\right) ?$$

## 4. Work out the big picture

$$
\begin{tikzcd}
 & & \text{Hall alg} \arrow[r, "\text{realize}"] & \text{(half) quantum gp} & \text{Verma module} \\
 & K_0(R\text{-mod}) \arrow[ur, "\sim"] & & & \\
 & & \text{Hecke alg} \arrow[ul, "\text{get}"] & & \text{modular form} \\
\text{Chow group} \arrow[r, phantom, "\sim"] & K_0^{Grp} \arrow[ur] \arrow[dr, "\sim"] & & & \\
 & & \text{convolution alg} & & K_0^{Grp}(\text{Springer fiber})
\end{tikzcd}
$$

Universal enveloping alg

KLR alg = quiver Hecke alg

Nil Hecke ring

## 5. A closer check of Satake iso

| $K_0$ | combinations | Hecke alg |
|---|---|---|
| $R(B) =$ | $\mathbb{Z}[X^*(T)]$ | $= \mathcal{H}(\hat{T}(F), \hat{T}(\mathcal{O}_F))$ |
| $R(G) =$ | $\mathbb{Z}[X^*(T)]^W$ | $\neq \mathcal{H}(\hat{G}(F), \hat{G}(\mathcal{O}_F))$ |
| $R(G)[q^{\pm\frac{1}{2}}] =$ | $\mathbb{Z}[X^*(T)]^W[q^{\pm\frac{1}{2}}]$ | $= \mathcal{H}_{sph}[q^{\pm\frac{1}{2}}]$ |
| $R(G\times\mathbb{C}^\times) =$ | $\mathbb{Z}[X^*(T)]^W[t^{\pm 1}]$ | |
| $R(T)\otimes_{R(G)}R(T) =$ | $NH_n$ $c\mathrm{End}_{\mathbb{Z}}(\mathbb{Z}[X^*(T)])$ | |
| $K^{G\times\mathbb{C}^\times}(St) =$ | $\mathcal{H}_{ext}$ | $\overset{?}{\neq} \mathcal{H}(\hat{G}(F), I)$ |

$G = GL_n$

It's claimed by my schoolmate that

$$K_0(\mathrm{Perv}_B(G/B)) \cong \mathcal{H}(G, B)$$

sym monoidal structure induced from the convolution

then, what is

$$K_0^B(\mathcal{B}) \cong ?$$

$$K_0^{Id}(\mathcal{B}) \cong ?$$

$$? \cong \mathcal{H}(S_{m+n}, S_m\times S_n)$$

Now, about Steinberg varieties.

6. Draw a picture, indicating the shape/generalization of the following spaces.
(e.g. in the case of $\cdot$, $\cdot\circlearrowleft$, $\cdot \to \cdot$)

$G, B, T$

$\mathcal{B}, T^*\mathcal{B}, St$

$\mathfrak{g}, \tilde{\mathfrak{g}}, \mathfrak{g}^{sr}, \tilde{\mathfrak{g}}^{sr}, \mathcal{N}, \tilde{\mathcal{N}}, \mathfrak{h}, \mathfrak{n}$

$\tilde{\mathfrak{g}}^h, \mathcal{O}_h, \Lambda_w^h$

7. Try to understand what Kazhdan-Lusztig polynomials are [KL2], and
- Compute the transformation matrix between
$\{[T_w^*], w \in W_f\}$ and $\{[\Lambda_w^h], w \in W_f\}$ ? [KaSai]?
- understand what standard/crystal basis is
- understand the relationship between KL poly and crystal basis
- see if it is related to two basis in Rep(G) (irr reps & multiplicative basis)

Answer from my schoolmate:

standard basis $\xleftrightarrow{\text{KL-poly}}$ canonical basis $\overset{\text{fix } q}{\rightsquigarrow}$ crystal basis

$\{[T_w^*], w \in W_f\}$ $\{[\Lambda_w^h], w \in W_f\}$

irr reps multiplicative basis

8. Try to understand the module part, i.e.,
- numbers of components of the Springer fiber
- how does $K_0^{Grp}(St)$ act on $K_0^{Grp}$(Springer fiber) also act on $K_0^{Grp}(Rep_{\vec{d}}(Q))$
- does that occupy "all rep" of $K_0^{Grp}(St)$

9. Ways of finding multiplication structure

1 By direct computation (with techniques)

Hecke algebra — double coset calculus

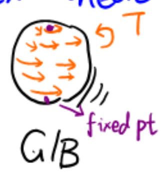
2. By formulas as alg-isos

$K_0^G(\mathcal{B})$ — induction formula

3. By geometrical computation

cohomology — cup product ? de Rham calculus

index theorem




Chow group — intersection theory

4. By deformation (indirect)

$H^{BM}_{top}(St)$

$K_0^{G\times\mathbb{C}^*}(St)$

How to get a clear intersection formula of Grothendieck? It's claimed in the introduction of [https://www.uni-due.de/~adc301m/staff.uni-duisburg-essen.de/Publications_files/excessgw.pdf], but I can not find any other references about this excess intersection formula.
How is it related to the Riemann-Roch theorem?

10. Different views on the double coset

$$B\backslash G/B = (*/B)\times_{*/G}(*/B)$$

- as a set
- as flag variety quotient B-action
- as a stack
- groupoid structure

Some excuses for not working a lot on the project:

| | |
|---|---|
| Preparation for summer school | 2 weeks |
| Summer school of the modular form | 1 week |
| Tourism in Paris | 1 week |
| Conference in Antwerp | 1 week |
| Reading [Ginz, Chap 5] | 2 weeks |
| Computing $\mathcal{H}(G,B)$, $\mathcal{H}_{sph}$, ($\mathcal{H}_{aff}$) | 1 week |
| Applying for tutorials, extend the residence permit, preparation for TOEFL exam, Klein AG, ... | 2 weeks |
| Summer school on Langlands & ICM watch (part) | 1 week |
| In total | 11 weeks |

tough new semester:

- 3 Seminars (+ Master Thesis Seminar)
- Tutorial
- TOEFL exam on 15th Oct.
- The seminar handout and other materials are not completed.
  - L-parameters
  - moduli in AG
  - some following developments of the modular form (different type of gps, Hecke operators, ...)
  - reps of $GL_2(\mathbb{Q}_p)$
- applying for the PhD program.