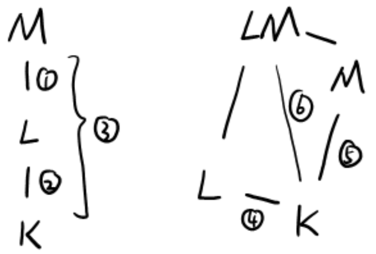

$$① \xrightarrow{② \text{ pure ins}} ③$$

normal: $③ \Rightarrow ①$ $\quad ③ \not\Rightarrow ②$ $\quad ① + ② \not\Rightarrow ③$ $\quad ⑥ \not\Rightarrow ④$ $\quad ④ + ⑤ \Rightarrow ⑥$

+

seperable: $① + ② = ③$ $\quad ④ + ⑤ = ⑥$

=

Galois: $③ \Rightarrow ①$ $\quad ③ \not\Rightarrow ②$ $\quad ① + ② \not\Rightarrow ③$ $\quad ⑥ \not\Rightarrow ④$ $\quad ④ + ⑤ \Rightarrow ⑥$

purely inseparable $① + ② = ③$ $\quad ④ + ⑤ = ⑥$

only 1 root for minimal poly

[GTM 167, Thm 4.13] char $F = p$. then

F perfect $\Leftrightarrow F^p = F$

$$\begin{array}{c} \overline{K} \\ | \\ L \\ | \\ K \end{array} \quad \begin{array}{l} \\ \text{closed subgroup} \\ \\ \text{(finite) quotient group} \\ \\ \end{array} \qquad \left.\begin{array}{ccc} \overline{\mathbb{F}_p} & \overline{\mathbb{F}_p} & \overline{\mathbb{F}_p} \\ |\ \mathbb{Z}_l & |\ \prod_{p \nmid l} \mathbb{Z}_p & |\ d\hat{\mathbb{Z}} \\ \bigcup_{l \nmid a} \mathbb{F}_{p^a} & \bigcup_n \mathbb{F}_{p^{l^n}} & \mathbb{F}_q \\ |\ \prod_{p \nmid l} \mathbb{Z}_p & |\ \mathbb{Z}_l & |\ \mathbb{Z}/d\mathbb{Z} \\ \mathbb{F}_p & \mathbb{F}_p & \mathbb{F}_p \end{array}\right] \quad \hat{\mathbb{Z}} = \prod_p \mathbb{Z}_p \quad (q = p^d)$$

open subgroup $\overset{\{1\} \subseteq \mathbb{Z}_p}{\subsetneq}$ closed subgroup $= \{\mathrm{Gal}(\overline{K}/L) \mid L/K \text{ ext}\}$ $\overset{\mathbb{Z} \subseteq \mathbb{Z}_p}{\subsetneq}$ subgroup

Lem. A subgroup of a profinite group is open iff it's closed and has finite index.

Ref: https://ctnt-summer.math.uconn.edu/wp-content/uploads/sites/1632/2020/06/CTNT-InfGaloisTheory.pdf