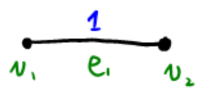
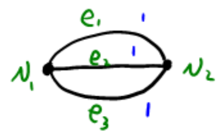
Eine Woche, ein Beispiel

12.26 Average Resistance $\tau(\Gamma)$

Goal: compute parameters in https://arxiv.org/pdf/0901.3945.pdf ← [Cin],
and think of their physical meaning (I need your help!)
If possible, find a way to explain the Cinkir's bound [Cin, Thm 5.21].

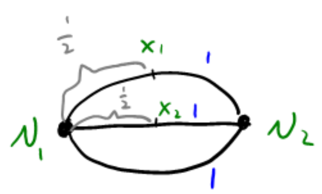
We begin with an undirected weighted connected graph Γ.
(weight is always positive, and can be thought as the length ; Γ have at lease 1 edge)

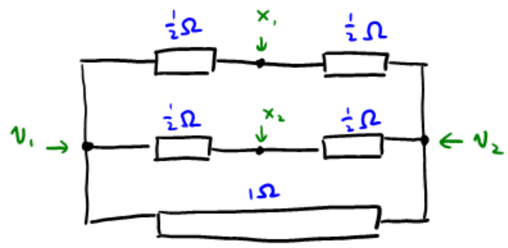
E.g.

			
Vertices $V = V(\Gamma)$	$\{v_1, v_2\}$	$\{v\}$	$\{v_1, v_2\}$
Edges $E = E(\Gamma)$	$\{e_1\}$	$\{e\}$	$\{e_1, e_2, e_3\}$
total length $l = l(\Gamma)$	1	l	3
genus $g = g(\Gamma)$	0	1	2

You can think a graph Γ as some electrical wires with given length and constant resistivity $1\Omega/m$. Then we can compute the resistance between two points $p, q \in \Gamma$, and denote it by $r(p,q)$.
↖ can be points on edges

E.g. In Fig 1, $r(v_1, v_2) = \frac{1}{3}\Omega$, $r(x_1, x_2) = \frac{1}{2}\Omega$





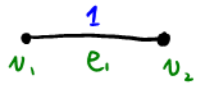
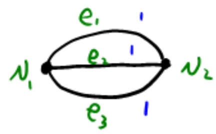
Thm. There exists a unique real signal Borel measure μ_{can} on Γ, satisfying:
(i) $\mu_{can}(\Gamma) = 1$, $|\mu_{can}|(\Gamma) < \infty$
(ii) The expression $(x, y \in \Gamma)$

$$\frac{1}{2}\int_\Gamma r(x,y)\, d\mu_{can}(y)$$

is independent of the variant x.

We denote $\tau = \tau(\Gamma) = \frac{1}{2}\int_\Gamma r(x,y)\, d\mu_{can}(y)$, and call it the average resistance.

E.g.

			
μ_{can}	$\frac{1}{2}\delta_{v_1}+\frac{1}{2}\delta_{v_2}$	$\frac{1}{l}dx$	$-\frac{1}{2}\delta_{v_1}-\frac{1}{2}\delta_{v_2}+\frac{2}{3}dx$
$\tau=\tau(\Gamma)$	$\frac{1}{4}\Omega$	$\frac{1}{12}l\,\Omega$	$\frac{7}{36}\Omega$

Ex. Verify the value of $\tau(\Gamma)$ in the tables. (assuming that μ_{can} is already known)

Q: Do we have any physical explanation for $\tau(\Gamma)$?

Actually we can write down μ_{can} explicitly. For doing so we have to introduce some new concepts.