# Eine Woche, ein Beispiel

## 4.9 group cohomology

Etingof–Gelaki–Nikshych–Ostrik: Tensor Categories

A much better document is this:
https://users.math.msu.edu/users/ruiterj2/math/Documents/Spring%202019/Comprehensive%20exam/Group%20cohomology,%20Brauer%20groups,%20and%20algebraic%20K-theory.pdf

A: Abelian

$$H^0(G,A) = A^G$$

$$H^1(G,A) = \{f: G \to A \rtimes G\}/A\text{-conj} \underset{G \curvearrowright A \text{ triv}}{=\!=} \mathrm{Hom}_G(G,A)/A\text{-conj}$$

$$H^2(G,A) = \{0 \to A \to X \to G \to 0 \mid \text{extension of } G \text{ by } A\}$$

($G \curvearrowright A$ coincide)

$$\underset{G \curvearrowright A \text{ triv}}{=\!=} \{0 \to A \to X \to G \to 0 \mid X \text{ central ext of } G\}$$

$A \rightsquigarrow M$ for comparison with Hochchild cohomology

moved to [21.05.21]

Rmk.

$$H^n(G,M) = Ext^n_{\mathbb{Z}[G]}(\mathbb{Z},M) \qquad R\mathrm{Hom}_{\mathbb{Z}[G]}(\mathbb{Z},M)$$
$$H_n(G,M) = Tor_n^{\mathbb{Z}[G]}(\mathbb{Z},M) \qquad \mathbb{Z} \overset{L}{\otimes}_{\mathbb{Z}[G]} M$$

$$HH^n(A,M) = Ext^n_{A^e}(A,M) \qquad R\mathrm{Hom}_{A^e}(A,M)$$
$$HH_n(A,M) = Tor_n^{A^e}(A,M) \qquad A \overset{L}{\otimes}_{A^e} M$$