Eine Woche, ein Beispiel

4.21 cohomology calculation sheet

All the cohomology in this sheet are for constant sheaves.

Appetizer

Try to compute

$$H_c^\bullet([0,1)) \quad \& \quad H_\bullet^{BM}([0,1))$$

$$H_c^\bullet(\text{Möbius strip})$$

For the following spaces, compute $H^\bullet$, $H_\bullet$, $H_c^\bullet$ & $H_\bullet^{BM}$.

Lasy guys can just compute $H^\bullet$

means: I don't know the answer

Easy mode

$$\mathbb{C}^n - \{0\}$$

$$\mathbb{C} \cup_{\{0\}} \mathbb{C}$$

for $K \subseteq S^3$ knot $\qquad S^3 - K \qquad$ 801770

Hyperplane mode:

compute $H^\bullet(\mathbb{C}^n - Z)$, $H^\bullet(Z - \{0\})$ & $H_c^\bullet(Z)$, where $Z \subseteq \mathbb{C}^n$

$$Z = \{z_1 z_2 = 0\}$$

$$\{z_1 z_2 z_3 = 0\}$$

$$\{z_1 z_2 z_3 (z_1 + z_2 + z_3) = 0\}$$

$$\{z_1^2 + z_2^2 + z_3^2 = 0\}$$

Hint for the last case: consider the Morse fct

$$\|\cdot\| : Z \longrightarrow \mathbb{R} \qquad z \longmapsto \|z\|$$

Infinite mode

$\mathbb{Z}$

$\{\frac{1}{n}\} \cup \{0\}$

$\beta\mathbb{N}$

$\operatorname{Spec}\mathbb{Z}$ (non-Haus, only computes $H^\cdot$)

$\mathbb{C} - \mathbb{Z}$

$\mathbb{R}^\infty$

$\mathbb{C}\mathbb{P}^\infty$

$\mathbb{R}\mathbb{P}^\infty$

Q: Why can't one compute $H_c^\cdot(\mathbb{R}^\infty)$?
Can one argue by

$$H_c^\cdot(\mathbb{R}^\infty) = H^\cdot(S^\infty, \{\infty\}) \ ?$$

A: No.