Eine Woche, ein Beispiel

8.15 indecomposable representation of Dynkin quiver.

AR-quiver is a powerful tool considering about the indecomposable modules and relations among them. Using the AR-quiver, one can find(not totally serious):
- all the indecomposable modules;
- all the morphisms between these indecomposable modules;
- all the irreducible morphisms and AR-sequences;

However, it's not easy to see the coker and ker of some morphisms given by the AR-quiver.

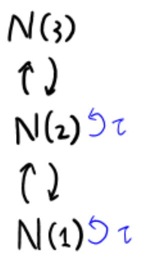
The following AR-quiver pictures are now useless, since everyone can get better pictures at https://www.math.uni-bielefeld.de/~jgeuenich/string-applet/ or https://www.math.uni-bielefeld.de/~wcrawley/#knitting.

Unfortunately, the knitting process can not draw some AR-quivers even in the case (not for Dynkin quiver) where "there are finite iso class of indec modules of quiver"

e.g.

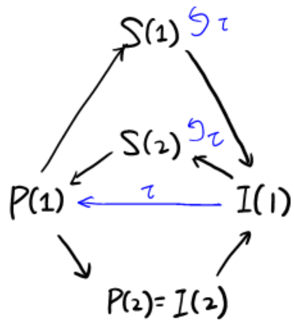
$A = K[T]/(T^3) \cong KQ/(a^3)$

Q: $1 \circlearrowleft a$



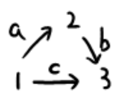
$A = KQ/(ab)$ $= 0$

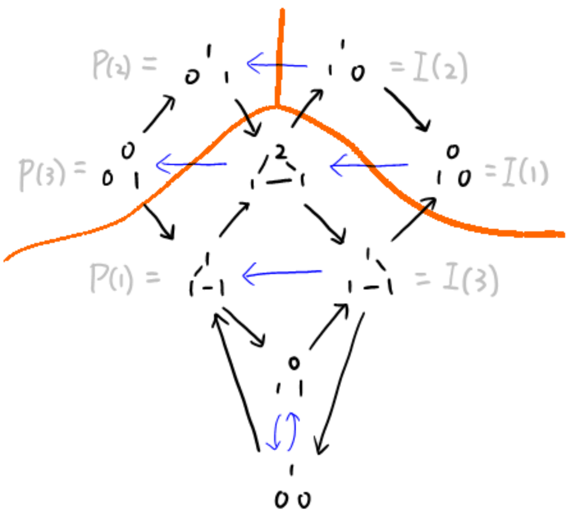
Q: $1 \underset{b}{\overset{a}{\rightleftarrows}} 2$



$A = KQ/(ab)$

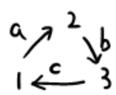
Q:

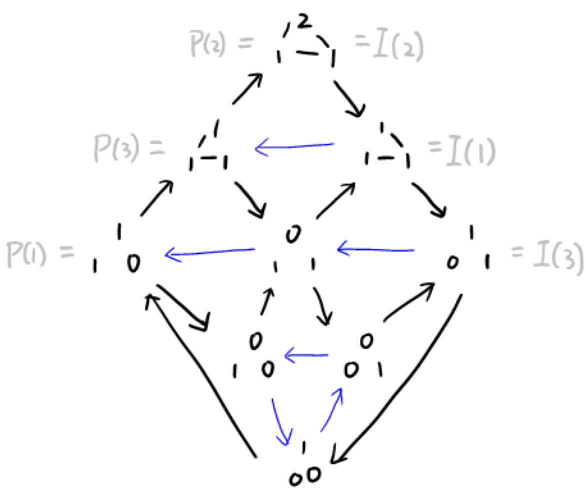




$A = KQ/(ab)$

Q:



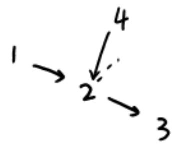


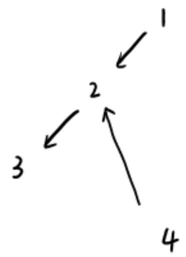
from different component of the AR-quiver of KQ.

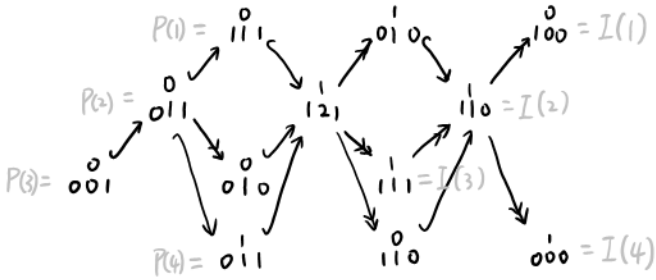
For the description of AR quiver of type A and D by a triangulated (puctured) polygon, see [Quiver Representations by Ralf Schiffler,3.1.3+3.3.3].

Even for bounded quiver algebra with Dynkin quiver, it is not very clear how the AR-quiver is related with the AR-quiver of path algebra.

D_4

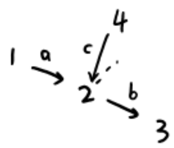


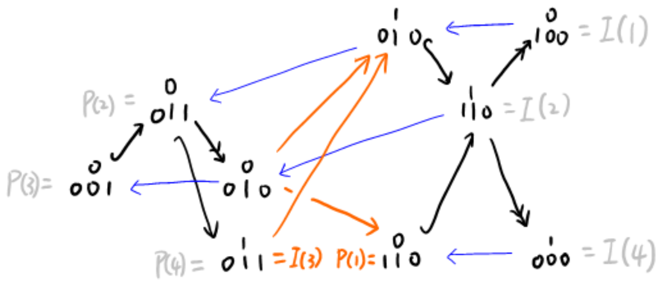


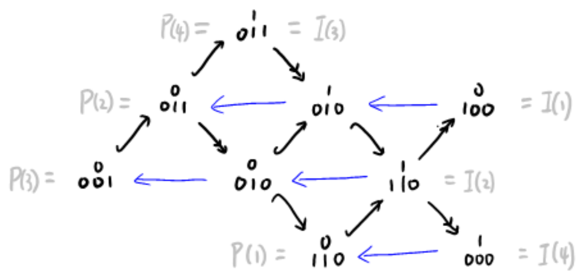


$A = KQ/(ab)$

Q:

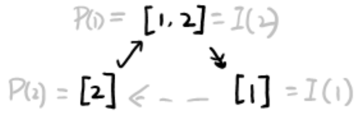






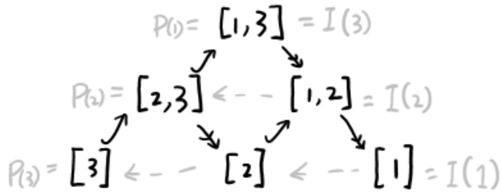
A_2 $\quad 1 \to 2$

(1)



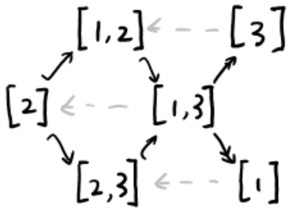
A_3 $\quad 1 \to 2 \to 3$

(2)



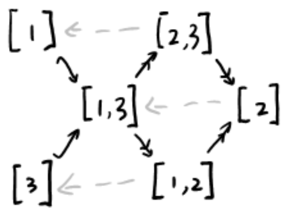
$1 \to 2 \leftarrow 3$

(1,1)



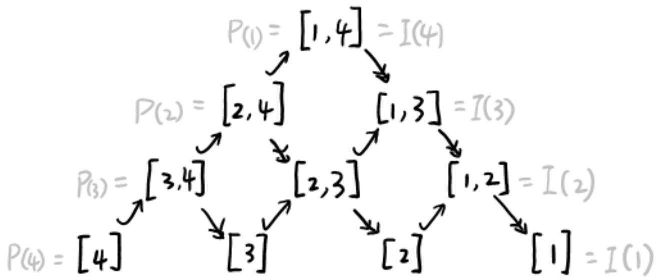
$1 \leftarrow 2 \to 3$

(1,1)*



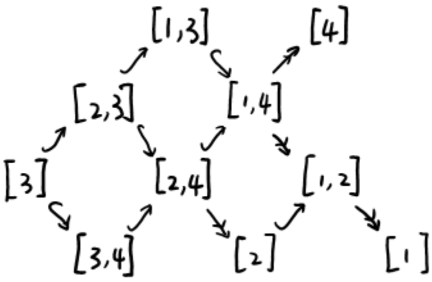
A_4 $\quad 1 \to 2 \to 3 \to 4$

(3)



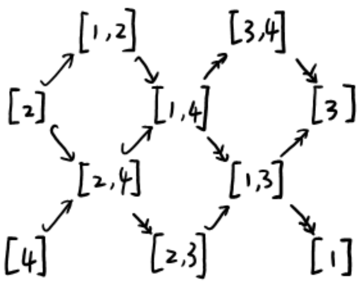
$1 \to 2 \to 3 \leftarrow 4$

(2,1)

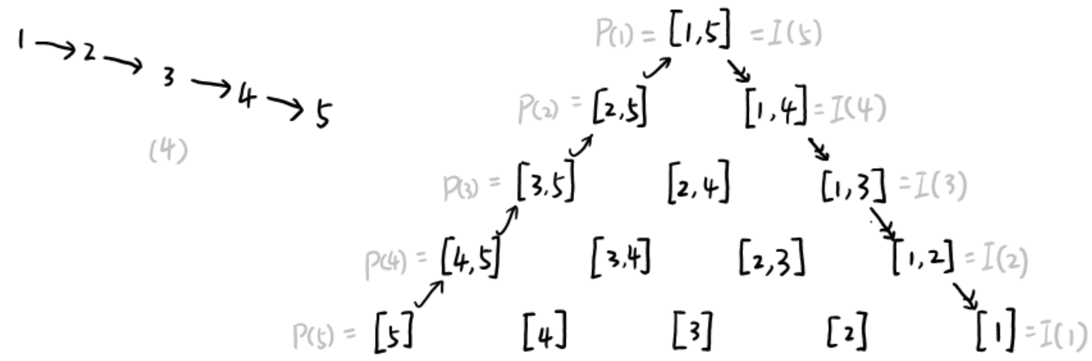


$1 \to 2 \leftarrow 3 \to 4$

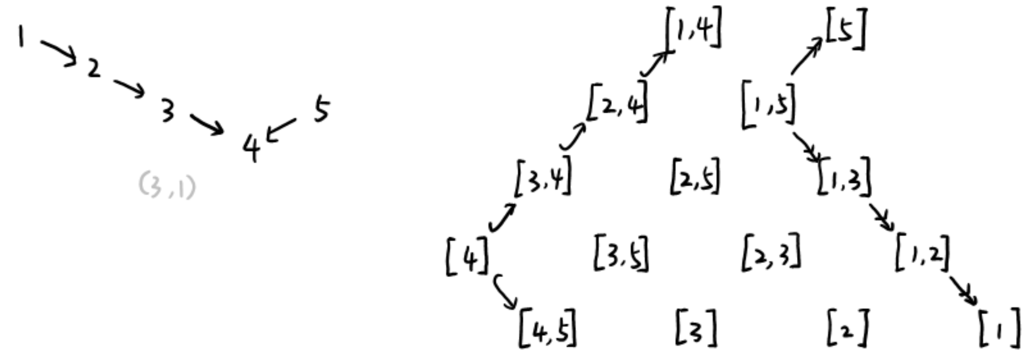
(1,1,1)



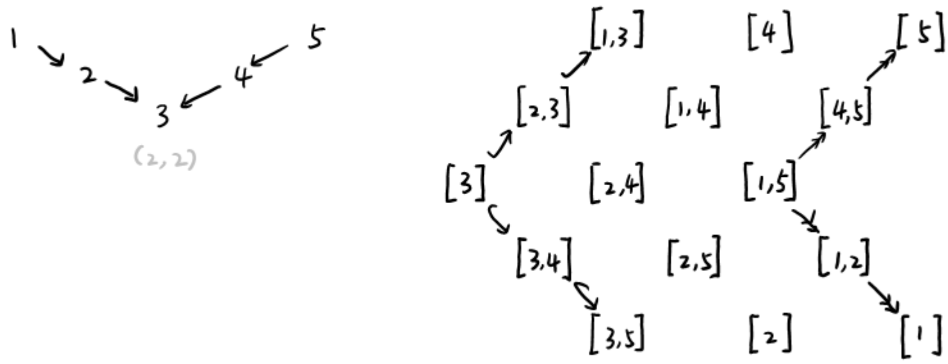
A_5



1 2 3 4 5
(4)
P(1) = [1,5] = I(5)
P(2) = [2,5]
[1,4] = I(4)
P(3) = [3,5]
[2,4]
[1,3] = I(3)
P(4) = [4,5]
[3,4]
[2,3]
[1,2] = I(2)
P(5) = [5]
[4]
[3]
[2]
[1] = I(1)



1 2 3 4 5
(3,1)
[1,4]
[5]
[2,4]
[1,5]
[3,4]
[2,5]
[1,3]
[4]
[3,5]
[2,3]
[1,2]
[4,5]
[3]
[2]
[1]



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(2,2)
[1,3]
[4]
[5]
[2,3]
[1,4]
[4,5]
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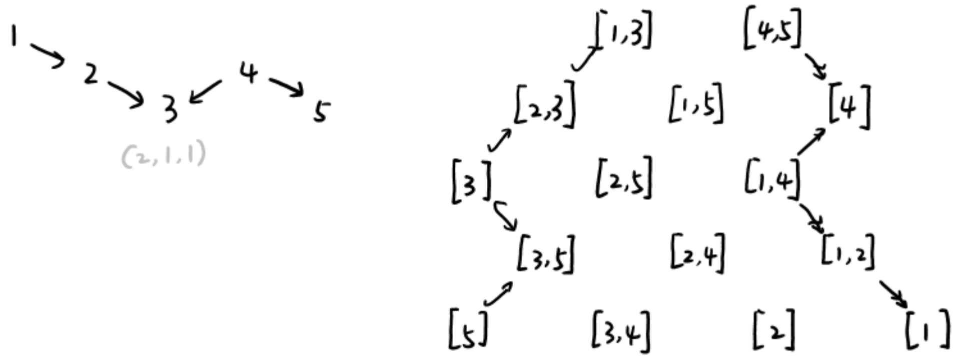
[2,4]
[1,5]
[3,4]
[2,5]
[1,2]
[3,5]
[2]
[1]



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[1,3]
[4,5]
[2,3]
[1,5]
[4]
[3]
[2,5]
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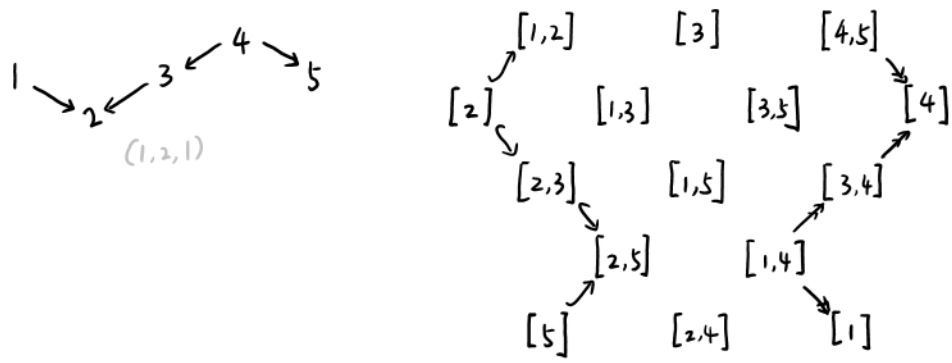
[3,4]
[2]
[1]



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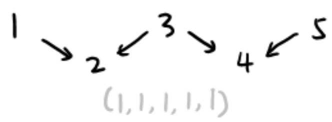
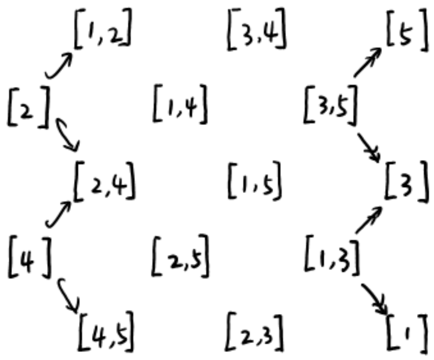
(1,2,1)
[1,2]
[3]
[4,5]
[2]
[1,3]
[3,5]
[4]
[2,3]
[1,5]
[3,4]
[2,5]
[1,4]
[5]
[2,4]
[1]


1
2
3
4
5
(1,1,1,1,1)


[1,2]
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[1,5]
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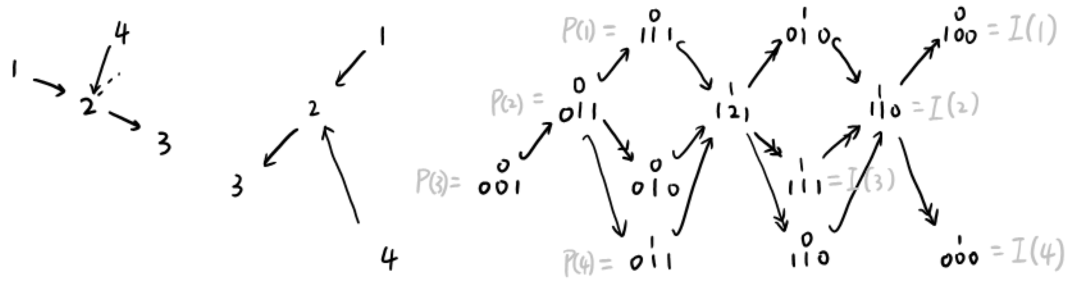
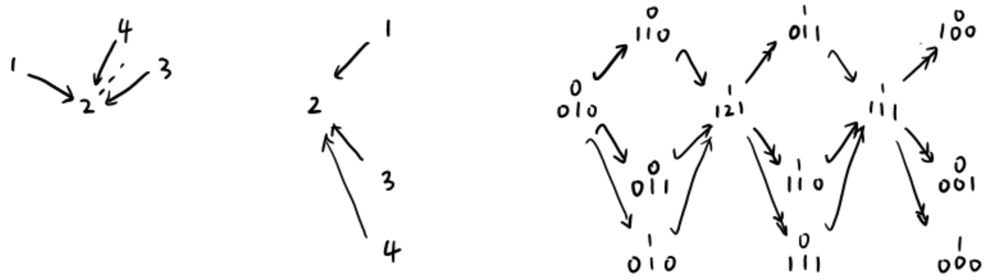
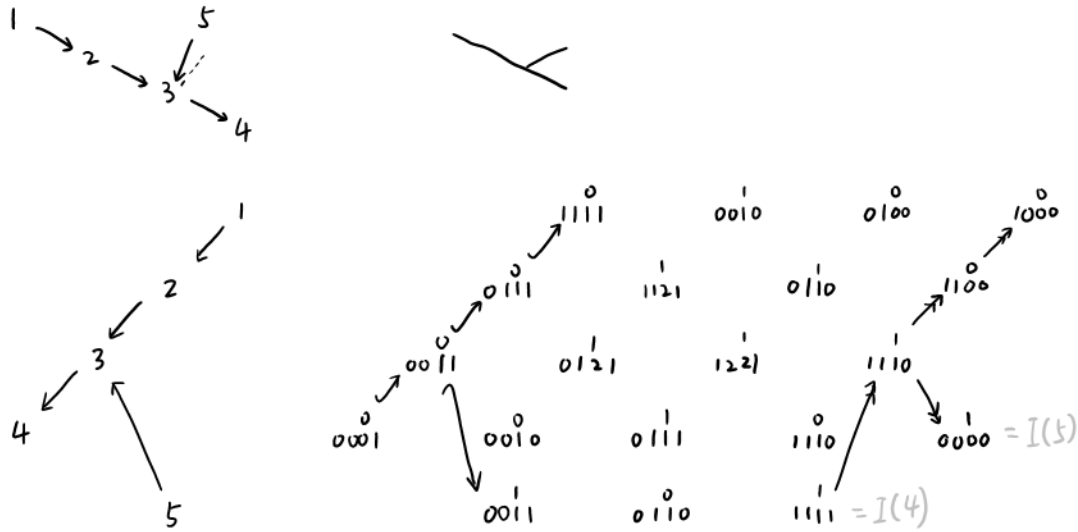
[2,5]
[1,3]
[4,5]
[2,3]
[1]

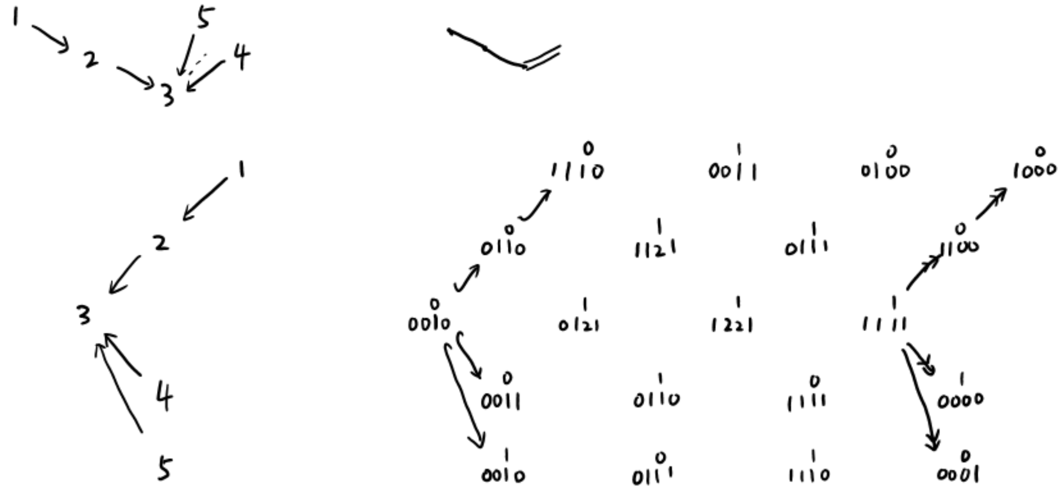
D_4

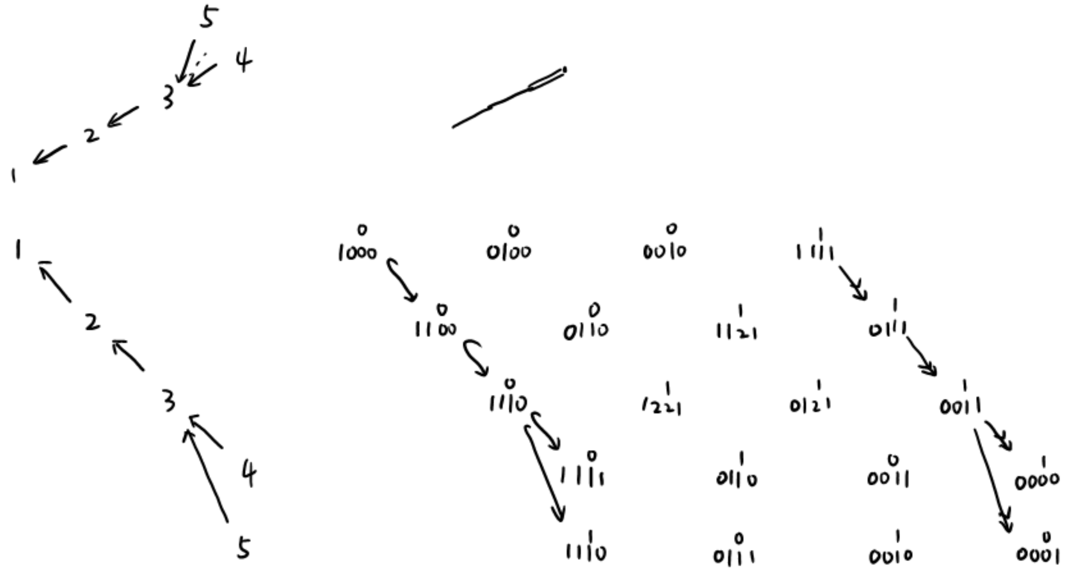




D_5






5
4
3
2
1
1
2
3
4
5
1000
0100
0010
1111
1100
0110
1121
0111
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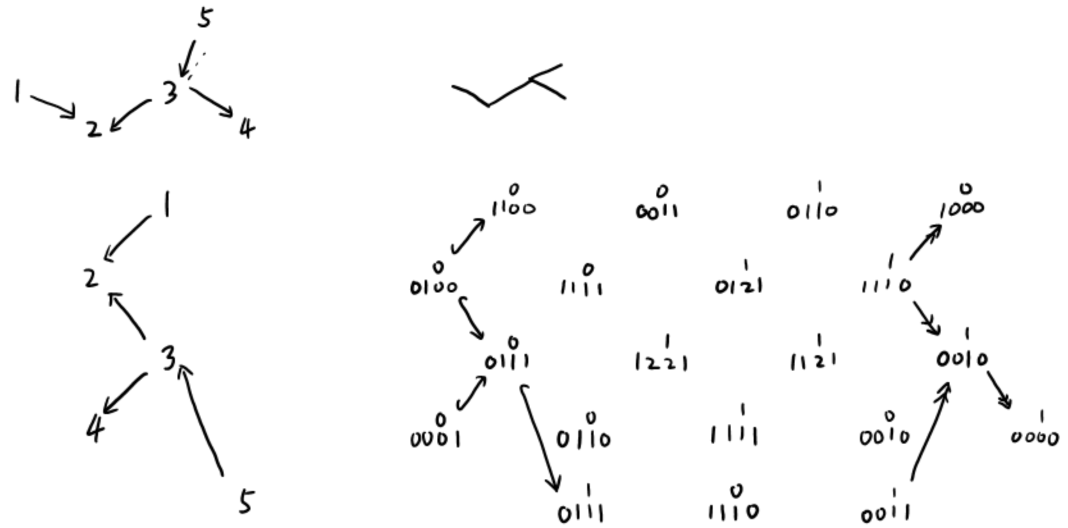
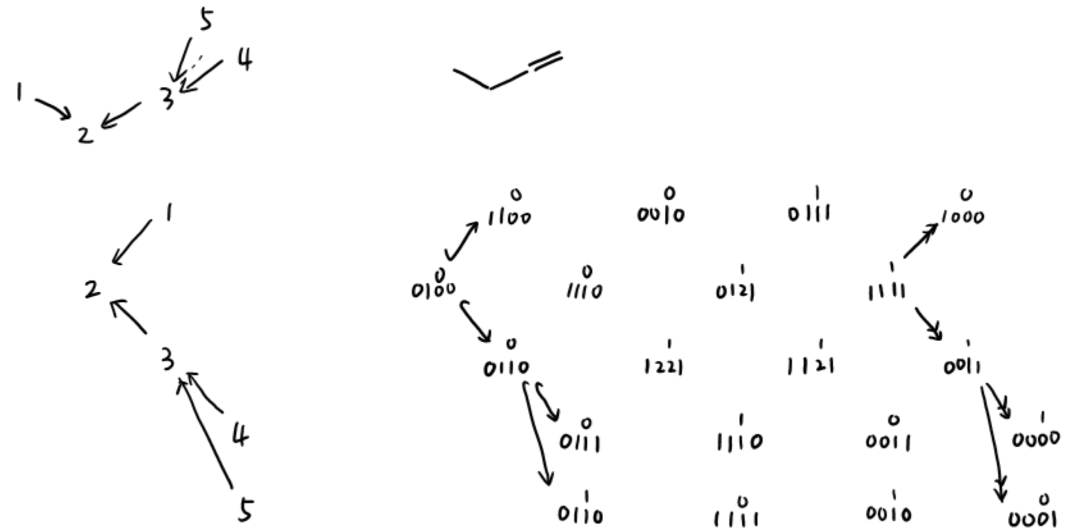
0000
1110
0111
0010
0001


5
1
3
2
4
1
2
3
4
5
1100
0011
0110
1000
0100
1111
0121
1110
0111
1221
1121
0010
0001
0110
1111
0010
0000
0111
1110
0011


5
4
1
3
2
1
2
3
4
5
1100
0010
0111
1000
0100
1110
0121
1111
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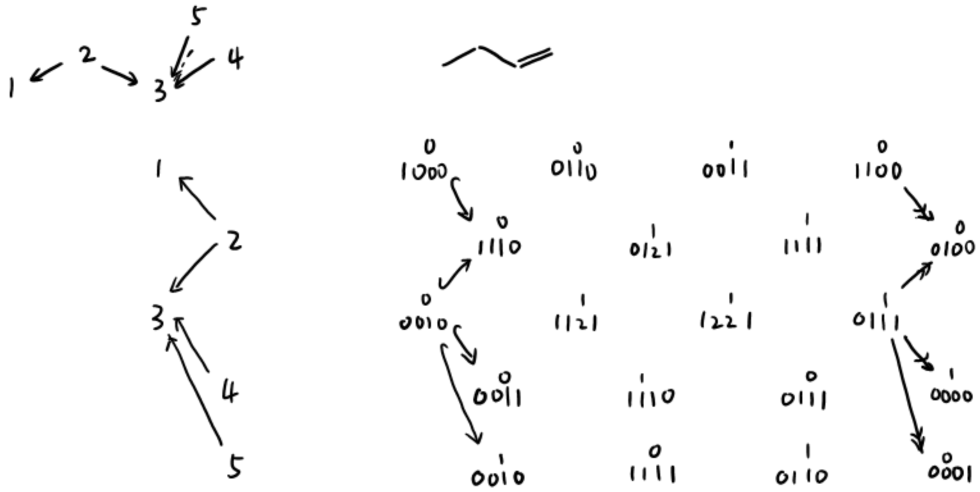
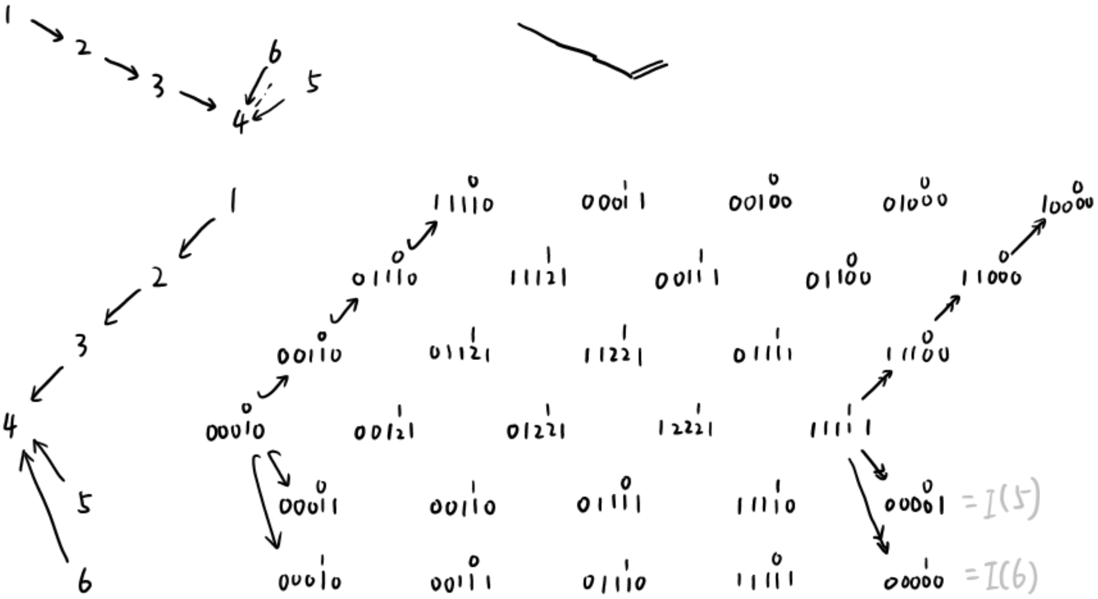
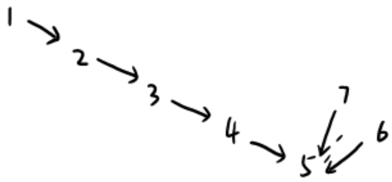
0000
0110
1111
0010
0001



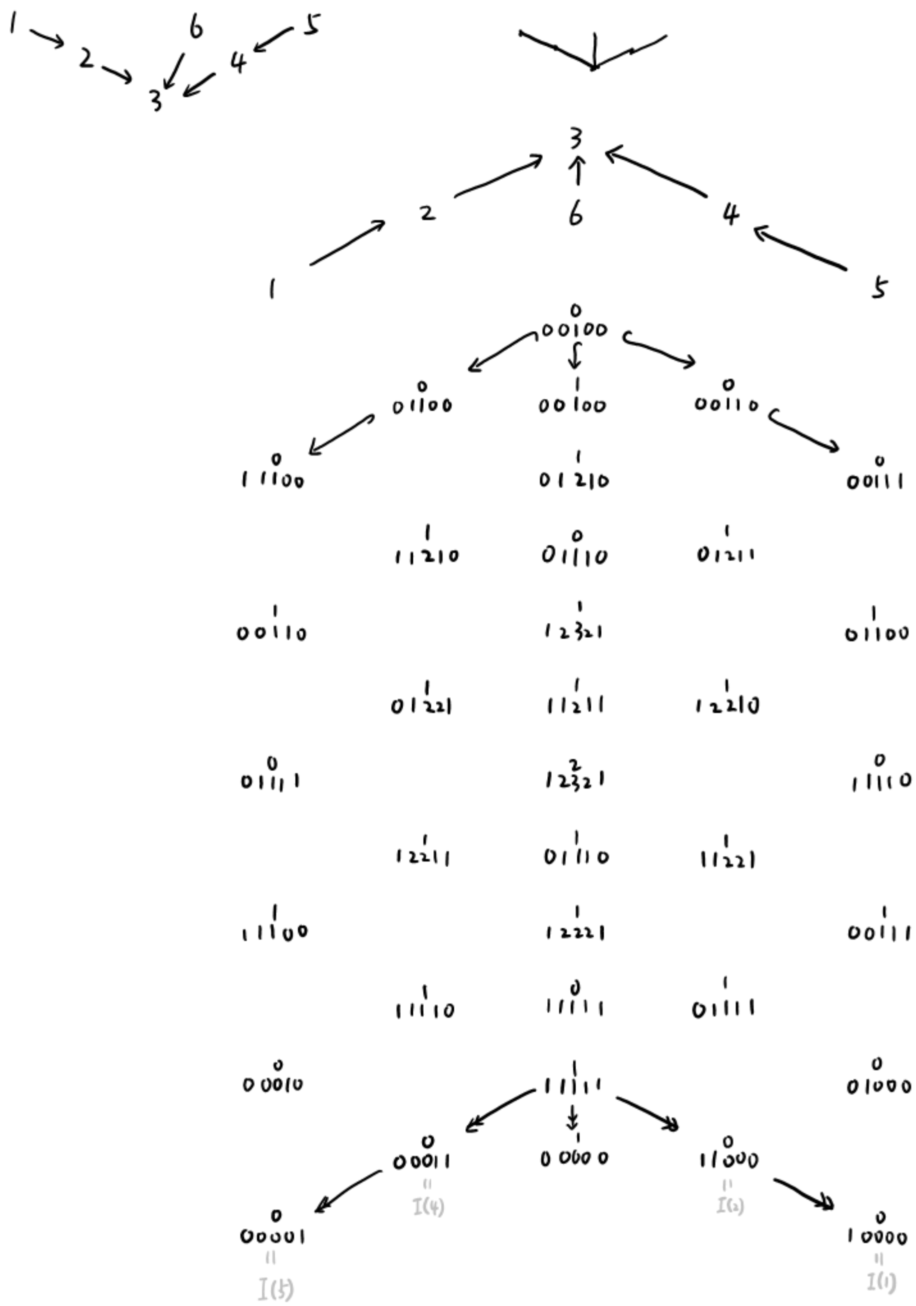
D_6



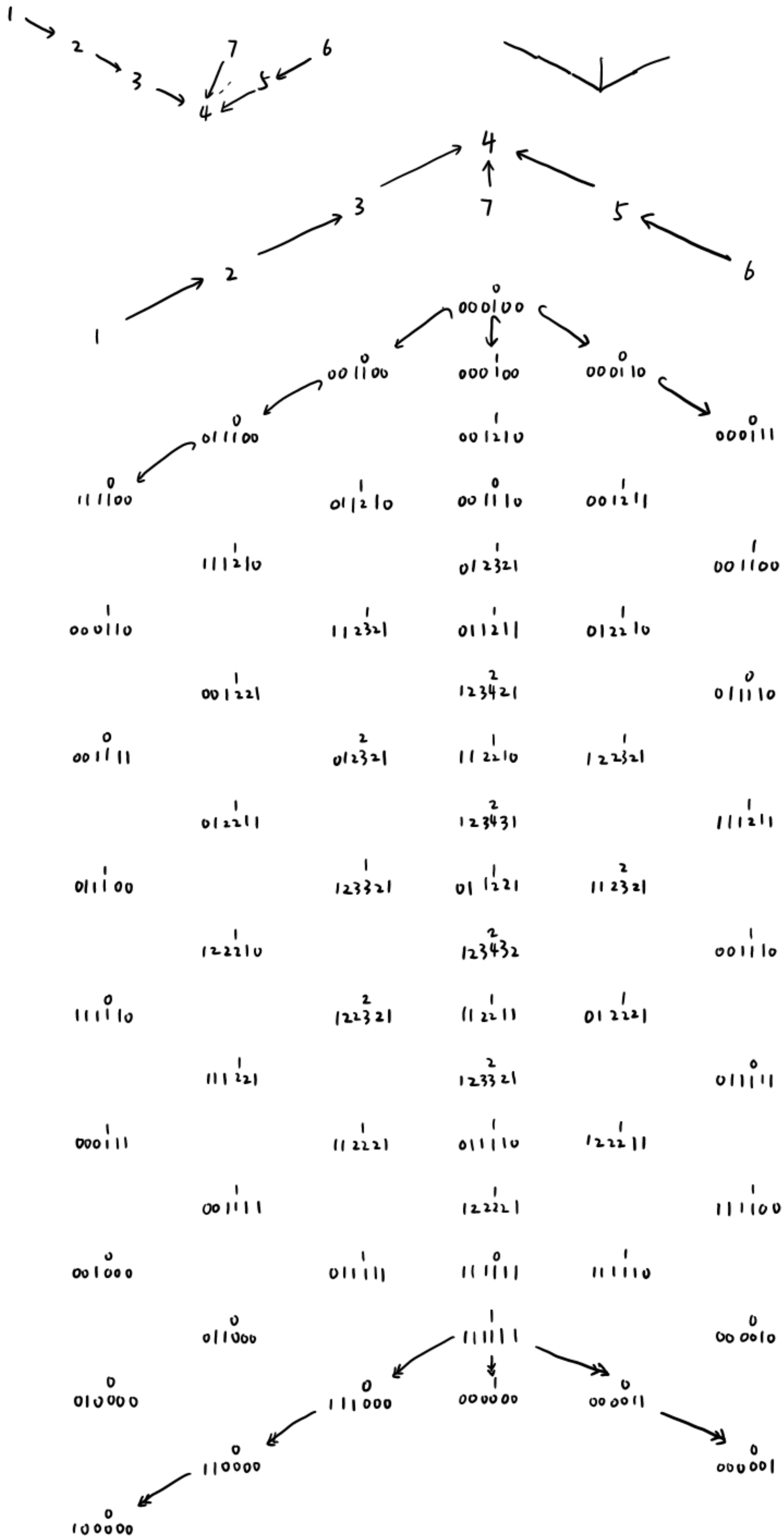
D_7



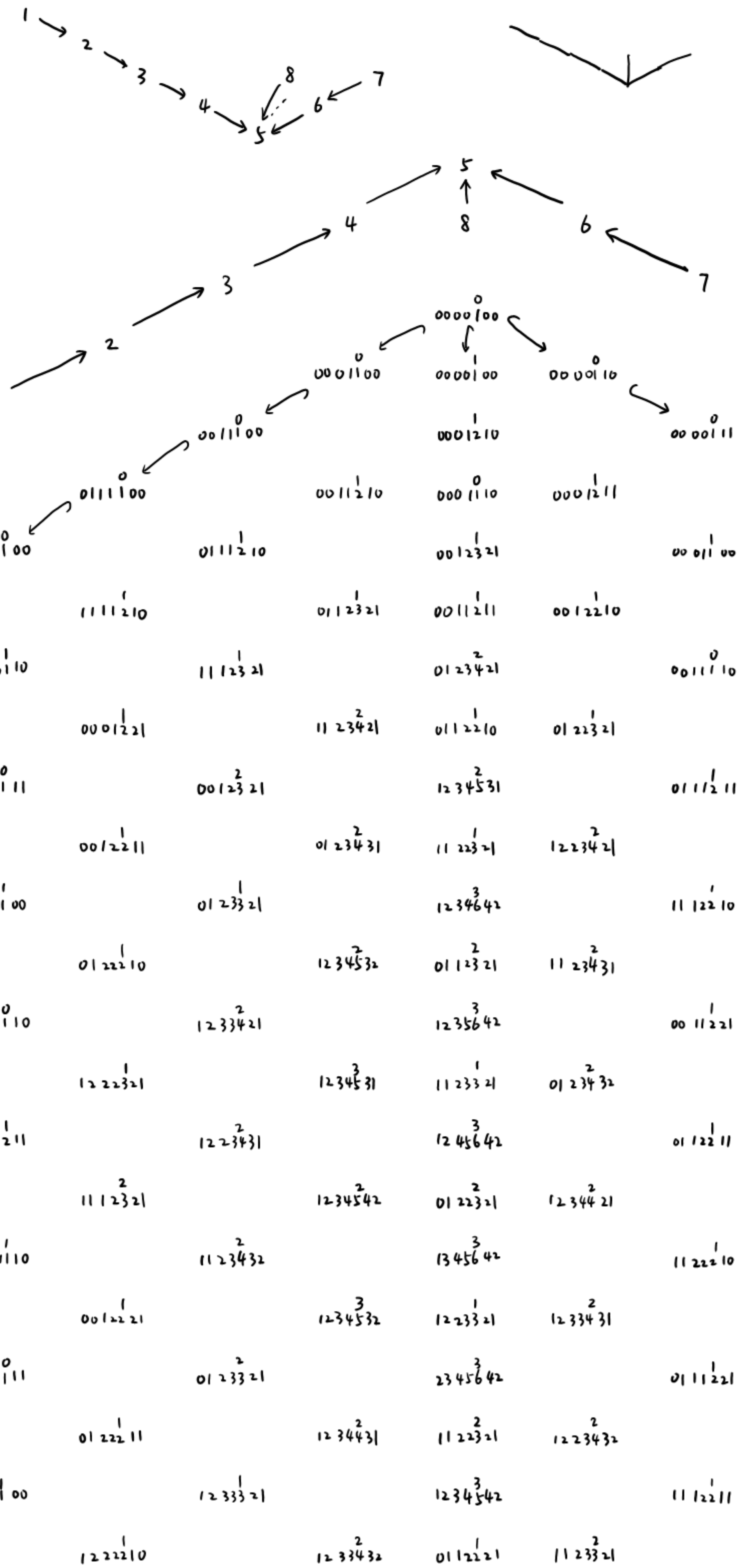
E_6



E_7



E_8



1 2 3 4 5 6 7 8
0 0000100
0 0001100 1 0000100 0 0000010
0 0011100 1 0001210 0 0000011
0 0111100 1 0011210 0 0001110 1 0001211
0 1111100 1 0111210 1 0012321 1 0001100
1 1111210 1 0112321 1 0011211 1 0012210
1 0000110 1 1112321 2 0123421 0 0011110
1 0001221 2 1123421 1 0112210 1 0122321
0 0001111 2 0012321 2 1234531 1 0111211
1 0012211 2 0123431 1 1122321 2 1223421
1 0011100 1 0123321 3 1234642 1 1112210
1 0122210 2 1234532 2 0112321 2 1123431
0 0111110 2 1233421 3 1235642 1 0011221
1 1222321 3 1234531 1 1123321 2 0123432
1 1111211 2 1223431 3 1245642 1 0112211
2 1112321 2 1234542 2 0122321 2 1234421
1 0001110 2 1123432 3 1345642 1 1122210
1 0012221 3 1234532 1 1223321 2 1233431
0 0011111 2 0123321 3 2345642 1 0111221
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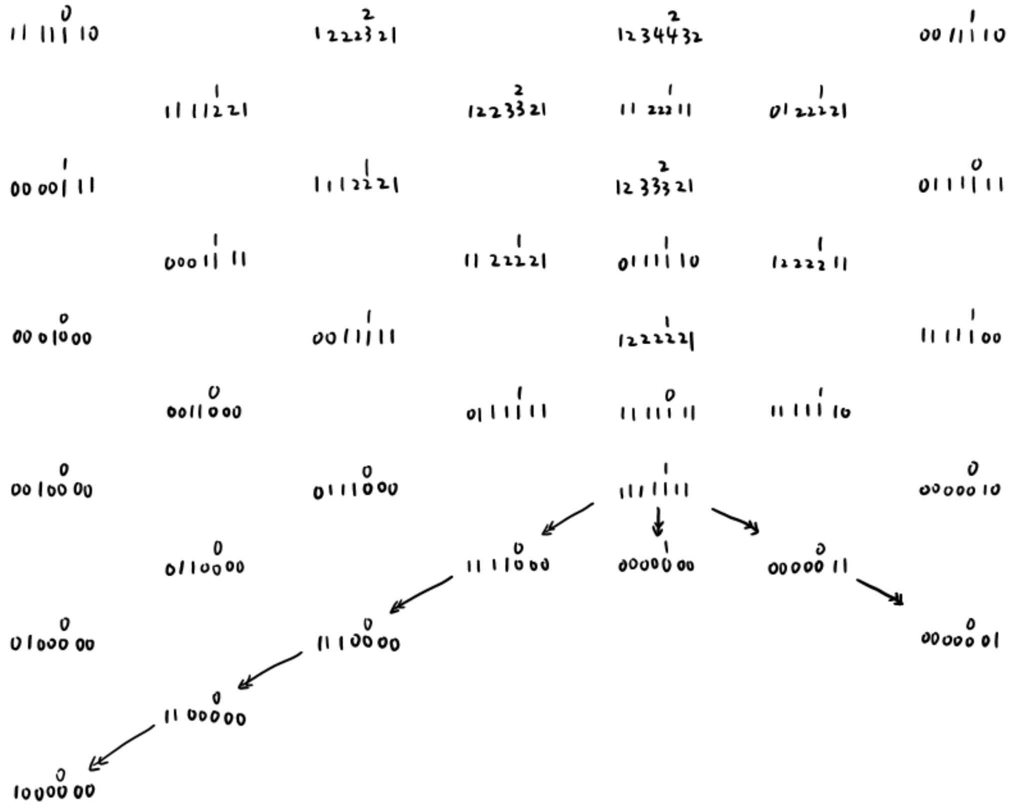
1 0111100 1 1233321 3 1234542 1 1112211
1 1222210 2 1233432 1 0112221 2 1123321



0
1111110
2
1222321
2
1234432
1
0011110
1
1111221
2
1223321
1
1122211
1
0122221
1
0000111
1
1112221
2
1233321
0
0111111
1
0001111
1
1122221
1
0111110
1
1222211
0
0001000
1
0011111
1
1222221
1
1111100
0
0011000
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1111111
1
1111110
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0010000
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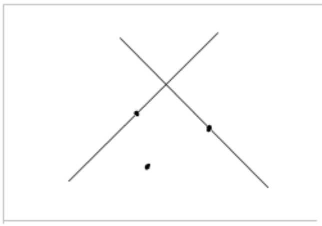
0111000
1
1111111
0
0000010
0
0110000
0
1111000
1
0000000
0
0000011
0
0100000
0
1110000
0
0000001
0
1100000
0
1000000

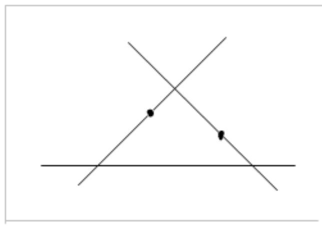
Bonus: subspace case (projective space version)

$1\overset{1}{2}1$

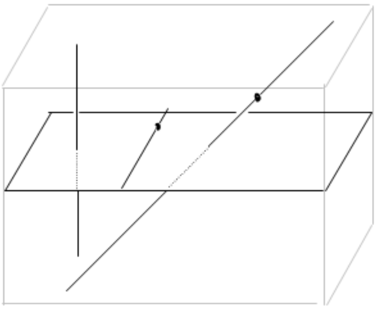
$12\overset{1}{3}21$



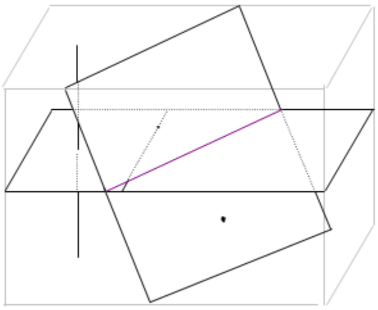
$1\,2\overset{2}{3}\,21$



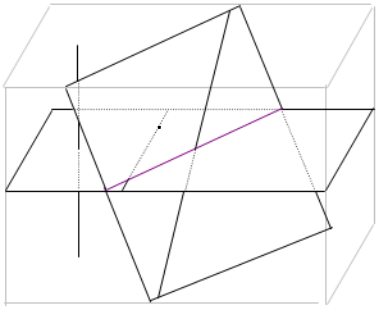
$123\overset{2}{4}21$



$123\overset{2}{4}31$

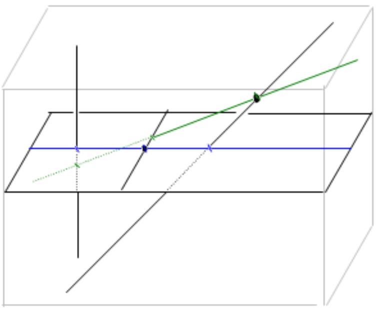


$123\overset{2}{4}32$



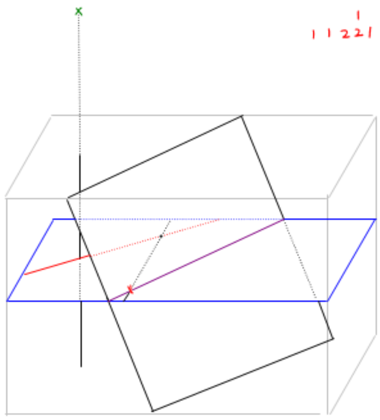
These shapes should be as general as possible, otherwise it may be not indecomposable:

e.g. "$123\overset{2}{4}21$" $= 112\overset{1}{2}10 \oplus 011\overset{1}{2}11$

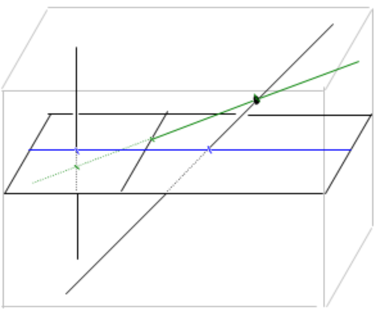


"$123\overset{2}{4}3$" $=$ "$123\overset{1}{3}2$" $\oplus\ 000\overset{1}{1}1$

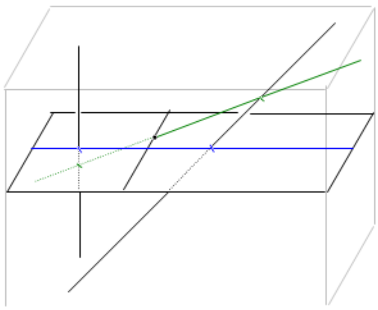
"$=$" $112\overset{1}{2}1 \oplus 011\overset{0}{1}1$



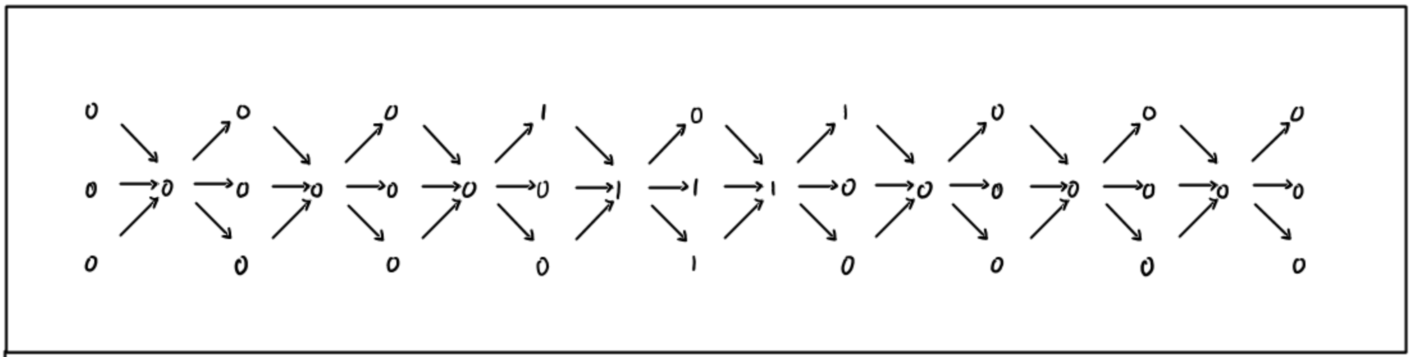
"$23\overset{2}{4}21$" $= 12\overset{1}{2}10 \oplus 11\overset{1}{2}11$



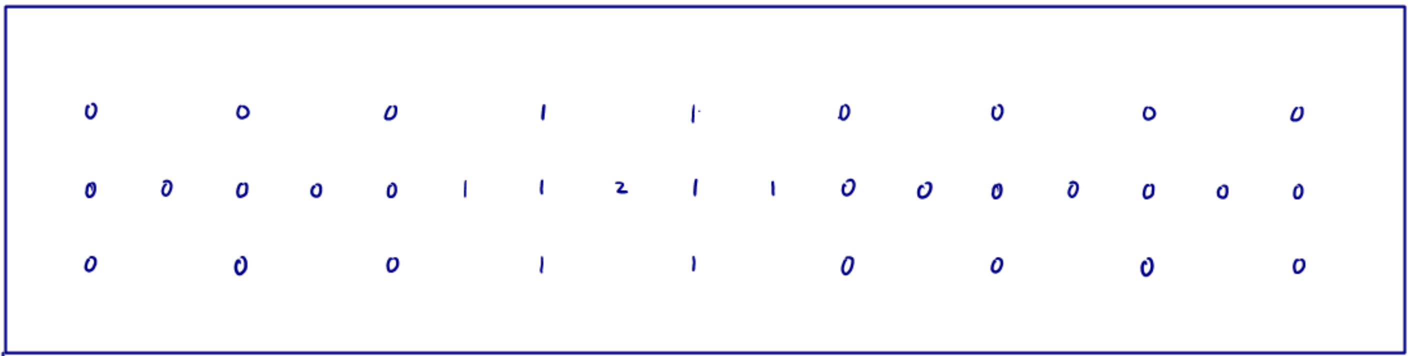
"$123\overset{2}{4}2$" $= 012\overset{1}{2}1 \oplus 111\overset{1}{2}1$



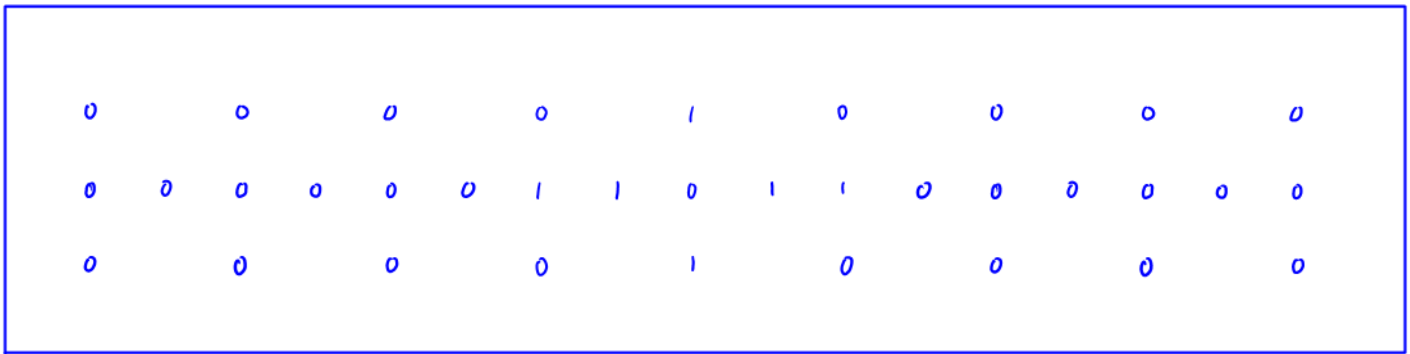
It's not easy to read the informations of them, but AR-quivers can.



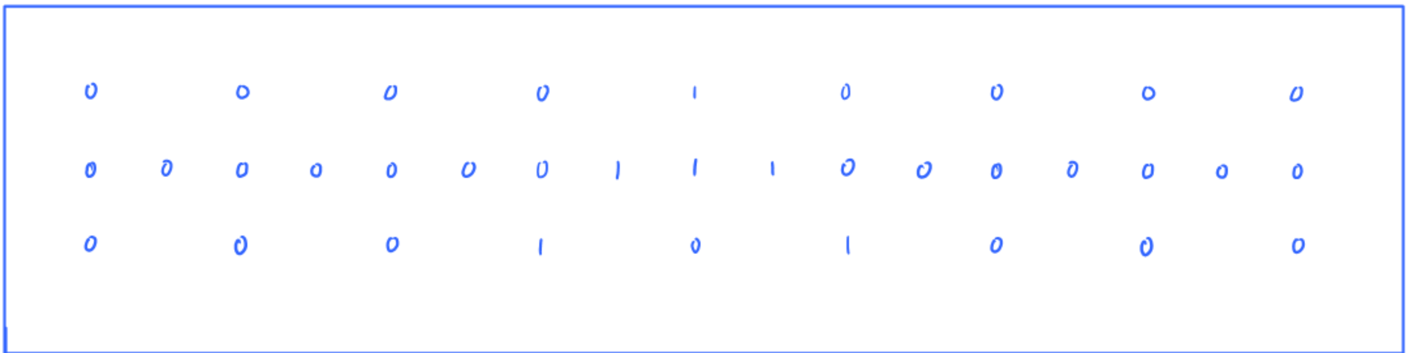
P(1)



P(2)



P(3)



P(4)

AR-quiver cards : made in China