Un exemple par jour

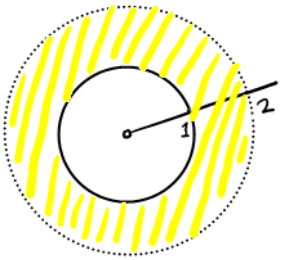
4.1. the complex torus of form $\mathbb{C}^\times/\mathbb{Z}\gamma$

$C := \mathbb{C}^\times/\mathbb{Z}\gamma \overset{\text{topo}}{=} \mathbb{T}^2$ is a cpt Riemannian surface of genus 1.

$\gamma \in Aut(\mathbb{C}^\times)$ $\gamma(z)=az$ $a\in\mathbb{C}^\times$ $|a|>1$

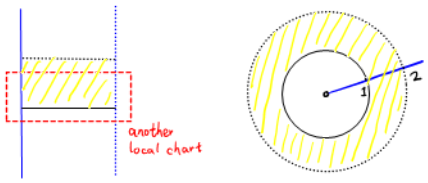
Today: $a=2$

1. fundamental set:



$\Rightarrow$ only need 2 local chart

2.
$$\begin{array}{ccccccc} 0 & \to & \mathbb{Z} & \hookrightarrow & \mathbb{C} & \xrightarrow{f:z\mapsto e^{2\pi i z}} & \mathbb{C}^\times \to 1 \\ & & \downarrow{\scriptstyle +\frac{1}{2\pi i}\ln 2} & & \downarrow{\scriptstyle +\frac{1}{2\pi i}\ln 2} & & \downarrow{\scriptstyle \times 2} \\ 0 & \to & \mathbb{Z}+\frac{1}{2\pi i}\ln 2 & \to & \mathbb{C} & \longrightarrow & \mathbb{C}^\times \to 1 \end{array}$$



$\mathbb{C} \xrightarrow{f} \mathbb{C}^\times$

$\mathbb{C}^\times = \mathbb{C}/\mathbb{Z} \Rightarrow \mathbb{C}^\times/\mathbb{Z}\gamma = \mathbb{C}/(\mathbb{Z}\oplus\frac{1}{2\pi i}\ln 2\,\mathbb{Z})$

better: $a = e^{2\pi} \approx 535.49$

$a = e^{-2\pi i w} \approx -230.765$

3. line bundle on C

$b\in\mathbb{C}^\times$ $\mathcal{L}_b := \mathbb{C}^\times\times\mathbb{C}/(z,\xi)\sim(2z,b\xi)$ $\Rightarrow$ ① $\mathcal{L}_b \in Pic_0(C)$; ($\mathcal{L}_b \overset{\text{cont. deformation}}{\sim} \mathcal{L}_1 \cong \mathcal{O}_C$)

$\downarrow$

$C = \mathbb{C}^\times/z\sim 2z$

② $Pic_0(C) \overset{\text{(naive, base pt } 1\in\mathbb{C}^\times/\mathbb{Z}\gamma)}{\cong} C = \mathbb{C}^\times/\mathbb{Z}\gamma$

$\mathcal{L}_b \longrightarrow b$

Reduced to: find a section s on $\mathcal{L}_b$ st $div\, s = [b]-[1]$

Reduced to: find a meromorphic functions g on $\mathbb{C}^\times$ s.t

① $g(2z) = bg(z)$ $b\in\mathbb{C}^\times$, $b\neq 2^k$; e.g. $b=3$

② g has simple poles on 2^n, and simple zeros on $2^n b$ $n\in\mathbb{Z}$

$b = e^{2\pi i c}$, $c\in\mathbb{C}$

$\tau := \frac{1}{2\pi i}\ln 2$

$w(z) = \frac{1}{2\pi i}\ln z$

$$g(z) := \frac{\theta\left[\begin{smallmatrix}1\\1-2c\end{smallmatrix}\right](w(z),\tau)}{\theta\left[\begin{smallmatrix}1\\1\end{smallmatrix}\right](w(z),\tau)}$$ is the required one.

Blue — example

Orange — more than this example

Red — important results

Purple — I don't know the answer/proof

Green — sketsch of proof : in a minimal way

Grey — some supplementary explanation . Unimportant assumptions.

Hell grey — explanation on well-known notations.

Brown — small title in subsections.

My symbol collection set

		Mathbb	Mathrsf/Mathcal	Greek	
A	a	$\mathbb{A}$ adèles	$\mathscr{A}$		α
B	b	$\mathbb{B}$	$\mathscr{B}$ building		β
C	c	$\mathbb{C}$ cplx number	$\mathscr{C}$ category	Γ gp / graph	γ
D	d constant	$\mathbb{D}$	$\mathscr{D}$ Poincare disk	Δ diag embedding	δ
E	e	$\mathbb{E}$	$\mathscr{E}$		ε
F	f	$\mathbb{F}$ finite field	$\mathscr{F}$ sheaf		ζ
G group	g	$\mathbb{G}$ gp scheme	$\mathscr{G}$ $\mathfrak{g}$: Lie alg		η
H	h constant	$\mathbb{H}$	$\mathscr{H}$ upper half plane / Hecke alg	Θ	θ
I ideal	i	$\mathbb{I}$	$\mathscr{I}$ ideal of sheaf		ι injection
J	j	$\mathbb{J}$	$\mathscr{J}$		κ
K cos/base field	$k \leftarrow \Bbbk$	$\mathbb{K}$	$\mathscr{K}$	Λ lattice	λ
L	l	$\mathbb{L}$	$\mathscr{L}$		μ
M module	m	$\mathbb{M}$	$\mathscr{M}$		ν
N	n	$\mathbb{N}$ natural number	$\mathscr{N}$		ξ root of unity (ξ/ω)
O	o	$\mathbb{O}$	$\mathscr{O}$ structure sheaf	Π multi	π constant / uniformizer / Projection
P	p	$\mathbb{P}$ proj space	$\mathscr{P}$ Weierstrass $\wp$: ell fct		$\rho \leftarrow \varrho$
Q	q	$\mathbb{Q}$ rational number	$\mathscr{Q}$	Σ sum	σ
R ring	r	$\mathbb{R}$ real number	$\mathscr{R}$		τ
S	s	$\mathbb{S}$	-	Φ	φ
T	t	$\mathbb{T}$	$\mathscr{T}$		χ character
$U \leftarrow \mathcal{U}$	u	$\mathbb{U}$	-	Ψ	ψ
V v.s.	v	$\mathbb{V}$	-	Ω	ω
W witt vector	w	$\mathbb{W}$	-		
X	x	$\mathbb{X}$	$\mathscr{X}$		
$Y \leftarrow \mathcal{Y}$	y	$\mathbb{Y}$	-	hebrew	Russian
Z	z	$\mathbb{Z}$ integer	$\mathscr{Z}$	$\aleph$ cardinal	Ш sha gp

Green: number / basic stuffs in senior high school

Orange: scheme-related

Darkyellow: advanced algebra

Don't use them simultaneously!

Don't mix: w/ω, ζ/ξ, $k/\kappa/\mathscr{K}$, $1/\mathbb{1}/\iota$, $\times/x/\chi/\mathcal{X}$, φ/ψ