Eine Woche, ein Beispiel

12.17 calculation of NMD

Goal: compute normal Morse data (NMD) $\{f\geq 0\} \overset{\tilde{i}}{\hookrightarrow} X \overset{\tilde{j}}{\hookleftarrow} \{f<0\}$

$$\begin{aligned}
NMD(\mathcal{F}^\cdot, S) &= (R\Gamma_{\{f|_{N\cap X} \geq f(x)\}}(\mathcal{F}^\cdot|_{N\cap X}))_x \\
&\overset{\substack{S=\{*\} \\ X \text{ is cone} \\ f(x)=0}}{\underset{\text{compatible}}{=}} (R\Gamma_{\{f\geq 0\}}(\mathcal{F}^\cdot))_x \\
&= \iota_x^* i^! \mathcal{F}^\cdot \\
&= R\Gamma(X, \{f<0\}, \mathcal{F}^\cdot) \\
&= \mathrm{Fiber}\left(R\Gamma(X, \mathcal{F}^\cdot) \longrightarrow R\Gamma(\{f<0\}, \mathcal{F}^\cdot)\right) \\
&= \mathrm{Fiber}\left(\mathcal{F}_x \longrightarrow R\Gamma(\iota_x, \mathcal{F}^\cdot)\right)
\end{aligned}$$

E.g. $X = \mathbb{CP}^1$ $\quad f : \mathbb{CP}^1 \dashrightarrow \mathbb{C} \xrightarrow{\operatorname{Re} z} \mathbb{R}$ $\quad L_x = \{*\}$ $\quad S = \{\infty\}$

$\infty \longmapsto 0$

$\mathcal{F}$	$NMD(\mathcal{F}^\cdot, S)$	$\mathcal{F}_x$	$R\Gamma(L_x, \mathcal{F}^\cdot)$
$i_* \underline{\mathbb{Q}}_{\{\infty\}}$	$\mathbb{Q}$	$\mathbb{Q}$	0
$\underline{\mathbb{Q}}_{\mathbb{CP}^1}[1]$	0	$\mathbb{Q}[1]$	$\mathbb{Q}[1]$
$Rj_* \underline{\mathbb{Q}}_{\mathbb{C}}[1]$	$\mathbb{Q}$	$\mathbb{Q} \oplus \mathbb{Q}[1]$	$\mathbb{Q}[1]$
$j_! \underline{\mathbb{Q}}_{\mathbb{C}}[1]$	$\mathbb{Q}$	0	$\mathbb{Q}[1]$
$P(\phi)$	$\mathbb{Q}^2$	$\mathbb{Q}$	$\mathbb{Q}[1]$

E.g. $X = \{z_2^2 = z_1^3\}$ $\quad f : X \hookrightarrow \mathbb{C}^2 \xrightarrow{z_1} \mathbb{C} \xrightarrow{\operatorname{Re} z} \mathbb{R}$ $\quad L_x = \{a, b\}$ $\quad S = \{0\}$

$\mathcal{F}$	$NMD(\mathcal{F}^\cdot, S)$	$\mathcal{F}_x$	$R\Gamma(L_x, \mathcal{F}^\cdot)$
$i_* \underline{\mathbb{Q}}_Z$	$\mathbb{Q}$	$\mathbb{Q}$	0
$\underline{\mathbb{Q}}_X[1]$	$\mathbb{Q}$	$\mathbb{Q}[1]$	$\mathbb{Q}^2[1]$
$Rj_* \underline{\mathbb{Q}}_U[1]$	$\mathbb{Q}^2$	$\mathbb{Q} \oplus \mathbb{Q}[1]$	$\mathbb{Q}^2[1]$
$j_! \underline{\mathbb{Q}}_U[1]$	$\mathbb{Q}^2$	0	$\mathbb{Q}^2[1]$
$P(\phi)$	$\mathbb{Q}^3$	$\mathbb{Q}$	$\mathbb{Q}^2[1]$

E.g. $X = \mathbb{C} \cup_{\{0\}} \mathbb{C} = \{(z_1, z_2) \in \mathbb{C}^2 \mid z_1 z_2 = 0\}$

$f: X \hookrightarrow \mathbb{C}^2 \xrightarrow{z_1 + z_2} \mathbb{C} \xrightarrow{\mathrm{Re}\, z} \mathbb{R}$ $\quad L_X = \{a, b\}$ $\quad S = \{0\}$

$\mathcal{F}$	$NMD(\mathcal{F}^\cdot, S)$	$\mathcal{F}_x$	$R\Gamma(L_X, \mathcal{F}^\cdot)$
$i_* \mathbb{Q}_Z$	$\mathbb{Q}$	$\mathbb{Q}$	0
$\mathbb{Q}_X[1]$	$\mathbb{Q}$	$\mathbb{Q}[1]$	$\mathbb{Q}^2[1]$
$Rj_* \mathbb{Q}_U[1]$	$\mathbb{Q}^2$	$\mathbb{Q}^2 \oplus \mathbb{Q}^2[1]$	$\mathbb{Q}^2[1]$
$j_! \mathbb{Q}_U[1]$	$\mathbb{Q}^2$	0	$\mathbb{Q}^2[1]$
$\pi^! \mathbb{Q}[-1]$	$\mathbb{Q}$	$\mathbb{Q} \oplus \mathbb{Q}^2[1]$	$\mathbb{Q}^2[1]$
$IC(\mathbb{Q}_U[1])$	0	$\mathbb{Q}^2[1]$	$\mathbb{Q}^2[1]$

E.g. $X = X_3$ $\quad f: X \hookrightarrow \mathbb{C}^3 \xrightarrow{z_3} \mathbb{C} \xrightarrow{\mathrm{Re}\, z} \mathbb{R}$ $\quad L_X = \mathbb{C}^\times$ $\quad S = \{0\}$

$\mathcal{F}$	$NMD(\mathcal{F}^\cdot, S)$	$\mathcal{F}_x$	$R\Gamma(L_X, \mathcal{F}^\cdot)$
$i_* \mathbb{Q}_Z$	$\mathbb{Q}$	$\mathbb{Q}$	0
$\mathbb{Q}_X[2] = \pi^! \mathbb{Q}[-2]$	$\mathbb{Q}$	$\mathbb{Q}[2]$	$\mathbb{Q}[1] \oplus \mathbb{Q}[2]$
$Rj_* \mathbb{Q}_U[2]$	$\mathbb{Q} \oplus \mathbb{Q}[-1]$	$\mathbb{Q}[2] \oplus \mathbb{Q}[-1]$	$\mathbb{Q}[1] \oplus \mathbb{Q}[2]$
$j_! \mathbb{Q}_U[2]$	$\mathbb{Q} \oplus \mathbb{Q}[1]$	0	$\mathbb{Q}[1] \oplus \mathbb{Q}[2]$
$IC(\mathbb{Q}_U[2])$	$\mathbb{Q}$	$\mathbb{Q}[2]$	$\mathbb{Q}[1] \oplus \mathbb{Q}[2]$