Eine Woche, ein Beispiel
7.7 special irreducible representations of simple Lie algebras

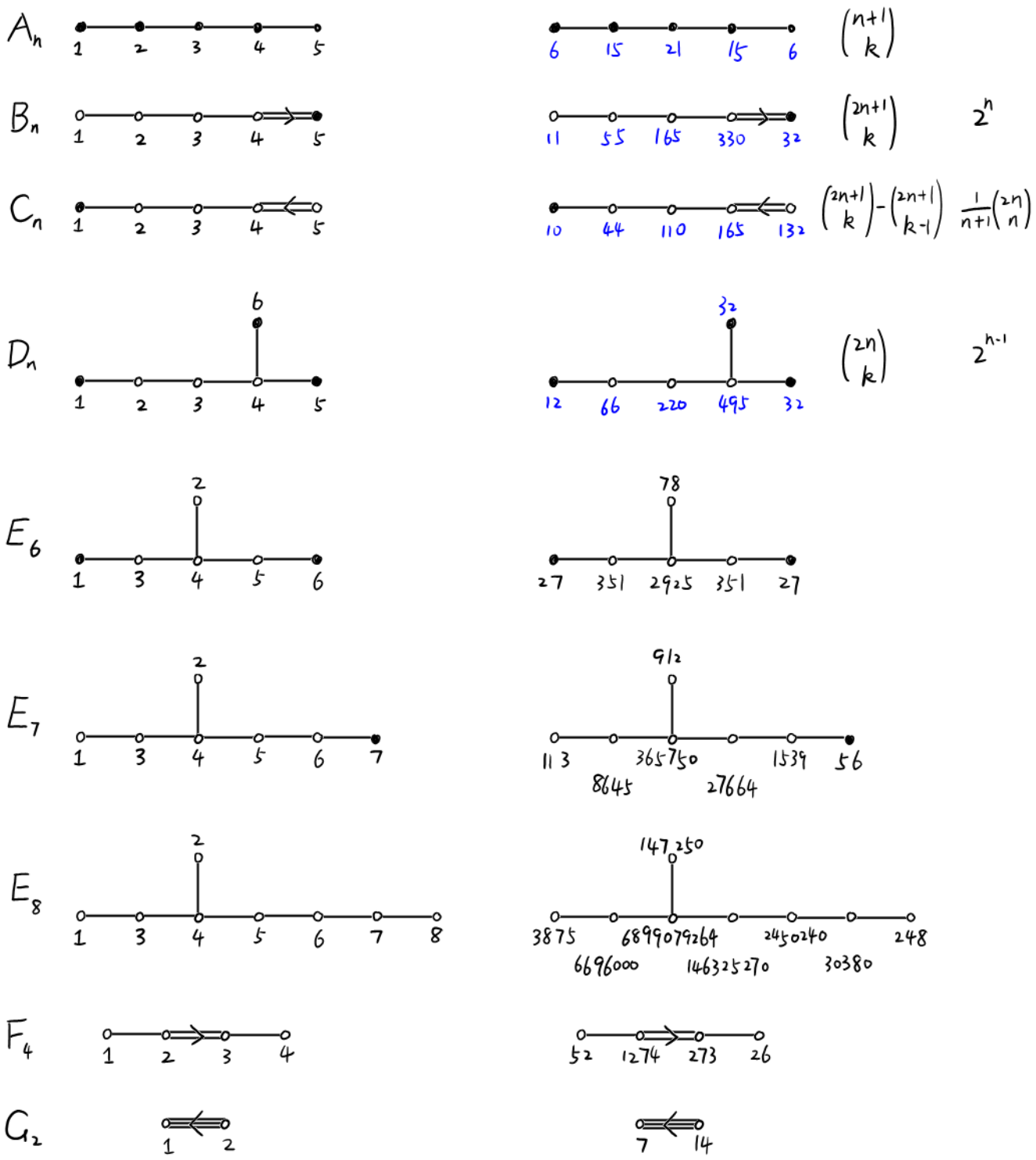
This document is a continuation for [2021.05.07_liegroup], [2021.07.18_irr_rep_of_semi_Lie_alg], [2021.07.25_irr_rep_of_SnAn], [2024.06.30_starting_functions].

The goal is to collect enough information on the representation sides, and then verify it on the perverse sheaf side.

Setting: We consider simple Lie algebras over $\mathbb{C}$.

1. labeling & basic rep dim
2. quasi-minuscule & adjoint reps

1. labeling & basic rep dim



long roots -> short roots
short weights -> long weights

1. labeling & basic rep dim

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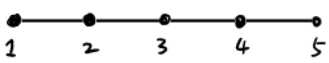
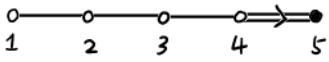
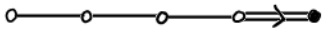
A_n

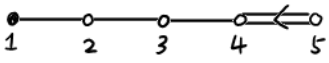


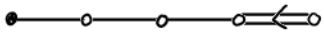
B_n



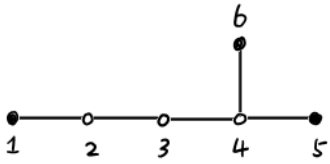


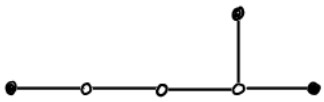
C_n



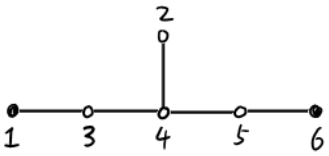


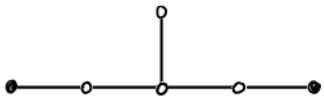
D_n



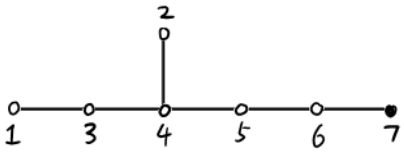


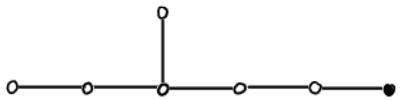
E_6



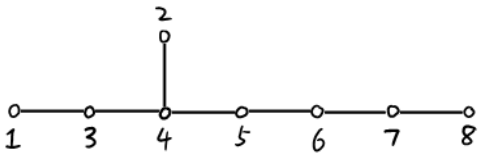


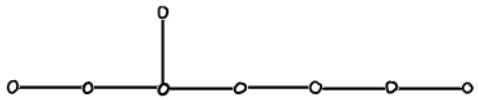
E_7



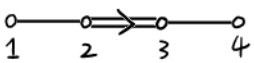


E_8

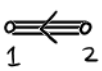




F_4

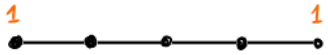
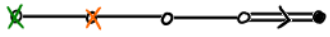
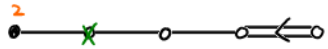
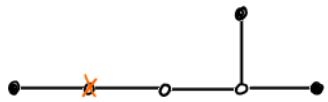
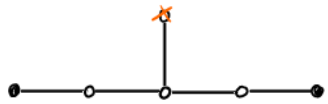
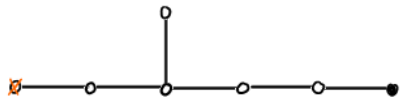
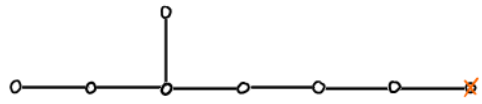
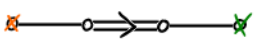
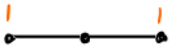


G_2



2. quasi-minuscule & adjoint reps

•: minuscule rep
×: quasi-minuscule rep
×: adjoint rep

		dim/dim	rk/rk
A_n		n^2+2n	n
B_n		$2n^2+n$ / $2n+1$	n / 1
C_n		$2n^2+n$ / $2n^2-n-1$	n / $n-1$
D_n		$2n^2-n$	n
E_6		78	6
E_7		133	7
E_8		248	8
F_4		52/26	4/2
G_2		14/7	2/1
A_1		3	1
B_2		10/5	2/1
D_3		15	3

quasi-minuscule and adjoint reps are exquisite, in the sense that their weights have only 2 or 3 orbits, and one of the orbit is the trivial weight. When the diagram is not simply-laced, the quasi-minuscule rep has orbits consisting of short roots and origin, while the adjoint rep has orbits consisting of long roots, short roots and origin.