Eine Woche, ein Beispiel

1.12 monodromy of the Gauss map

$A \cong \mathbb{C}^g / \Lambda$

$Z \subset A$ sm of dim r e.g. $\ell \subset \mathrm{Jac}(\ell)$

$\rightsquigarrow$ Gauss map

tautological bd

$$\phi: \underset{TZ}{Z} \longrightarrow \underset{\mathscr{S}}{Gr(r,g)} \qquad z \longmapsto [T_z Z]$$

$$\cup \qquad\qquad \cup$$

$$\{p_1, \ldots, p_d\} \longrightarrow Gr(r, g-1)$$

$$d = \deg \Lambda_Z = \phi^* \sigma_{(1)^r} = (-1)^r \phi^* c_r(\mathscr{S}) = (-1)^r c_r(TZ)$$

$$\begin{array}{c} I_{mon} \subset Z \times Gr(g-1, g) \xrightarrow{\quad \subset A \times Gr(g-1,g) \quad} Gr(r,g) \times Gr(g-1,g) \\ \downarrow \\ Gr(g-1, g) \end{array}$$

$\rightsquigarrow$ monodromy gp

$$Gal(\gamma) := \left(\text{mon gp of } \quad I_{mon} \longrightarrow Gr(g-1, g) \right)$$

Q: How can we compute $Gal(\gamma)$?

E.g. 1. $Z = \mathcal{C}$ non-hyperelliptic, $Gr(1,g) = \mathbb{P}^{g-1}$,

$$\phi = |\omega_c| : \mathcal{C} \longrightarrow \mathbb{P}^{g-1}$$
$$\cup \qquad\qquad \cup$$
$$\{p_1, \ldots, p_{2g-2}\} \longmapsto H$$

$$Gal(\gamma) = S_{2g-2} = W(A_{2g-3})$$

E.g. 2. $Z = \mathcal{C}$ hyperelliptic, $Gr(1,g) = \mathbb{P}^{g-1}$,

$$\phi = |\omega_c| : \mathcal{C} \longrightarrow \mathbb{P}^1 \longrightarrow \mathbb{P}^{g-1}$$
$$\cup \qquad\qquad \cup \qquad\qquad \cup$$
$$\{p_1, \ldots, p_{2g-2}\} \to \{z_1, \ldots, z_{g-1}\} \longrightarrow H$$

$$Gal(\gamma) = S_{g-1} \rtimes (\mathbb{Z}/2\mathbb{Z})^{g-1} = W(C_{g-1})$$

E.g. 3. X : cubic threefold $\subseteq \mathbb{P}^4$
$F = F(X)$: Fano surface $\subseteq Gr(2,5)$
$F \subset Alb(F) \leftarrow dim = 5$

$$\rightsquigarrow \phi : F \longrightarrow Gr(2,5)$$
$$\cup \qquad\qquad \cup$$
$$F(X \cap H) \longrightarrow Gr(2,4)$$
$$\|$$
$$\{p_1, \ldots, p_{27}\}$$

$$Gal(\gamma) = Aut(\text{Schläfli graph}) = W(E_6)$$

Q: Can we find $Z \subset A$ sm s.t. $Gal(\gamma) = W(E_7)$?