

Eine Woche, ein Beispiel

4.24 irreducible representation of the Mirabolic group

Main reference: The Local Langlands Conjecture for $GL(2)$ by Colin J. Bushnell and Guy Henniart.
[https://link.springer.com/book/10.1007/3-540-31511-X]

Process

1. Notations
2. Constructions
3. Classification
4. Applications
 - Computation of $V(N), V_N, V(\psi), V_\psi$.
 - Dual, Sym^m, \wedge^m, \dots
 - Decompose $Res_B^G Rep_B^G Ind_B^G \chi$ (not today, need knowledge of $G \& B$)
 - Trace formula
5. Irr rep of B ?

1. Notations. F : non-arch local field.

<https://math.stackexchange.com/questions/299626/the-center-of-operatornamegl-n-k>

$$A = M_{2 \times 2}(F) \quad G = GL_2(F)$$

$$B = \begin{pmatrix} * & * \\ 0 & * \end{pmatrix} \quad T = \begin{pmatrix} * & 0 \\ 0 & * \end{pmatrix} \quad N = \begin{pmatrix} 1 & * \\ 0 & 1 \end{pmatrix} \quad Z = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \stackrel{\downarrow}{=} Z(G) \quad S = \begin{pmatrix} * & 0 \\ 0 & 1 \end{pmatrix}$$

$$\omega = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad T^0 = \begin{pmatrix} 0^x & 0^x \\ 0 & 0^x \end{pmatrix} \quad N_j = \begin{pmatrix} 1 & p^j \\ 0 & 1 \end{pmatrix} \quad N_j' = \begin{pmatrix} 1 & 0 \\ p^j & 1 \end{pmatrix}$$

Temporarily, $P = \left\{ \begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix} \in GL_2(F) \right\} = F \rtimes F^\times = N \rtimes S$ $0 \rightarrow (F, +) \xrightarrow{N} P \xrightarrow{S} F^\times \rightarrow 0$

\uparrow
parabolic subgp