Eine Woche, ein Beispiel

7.7 special irreducible representations of simple Lie algebras

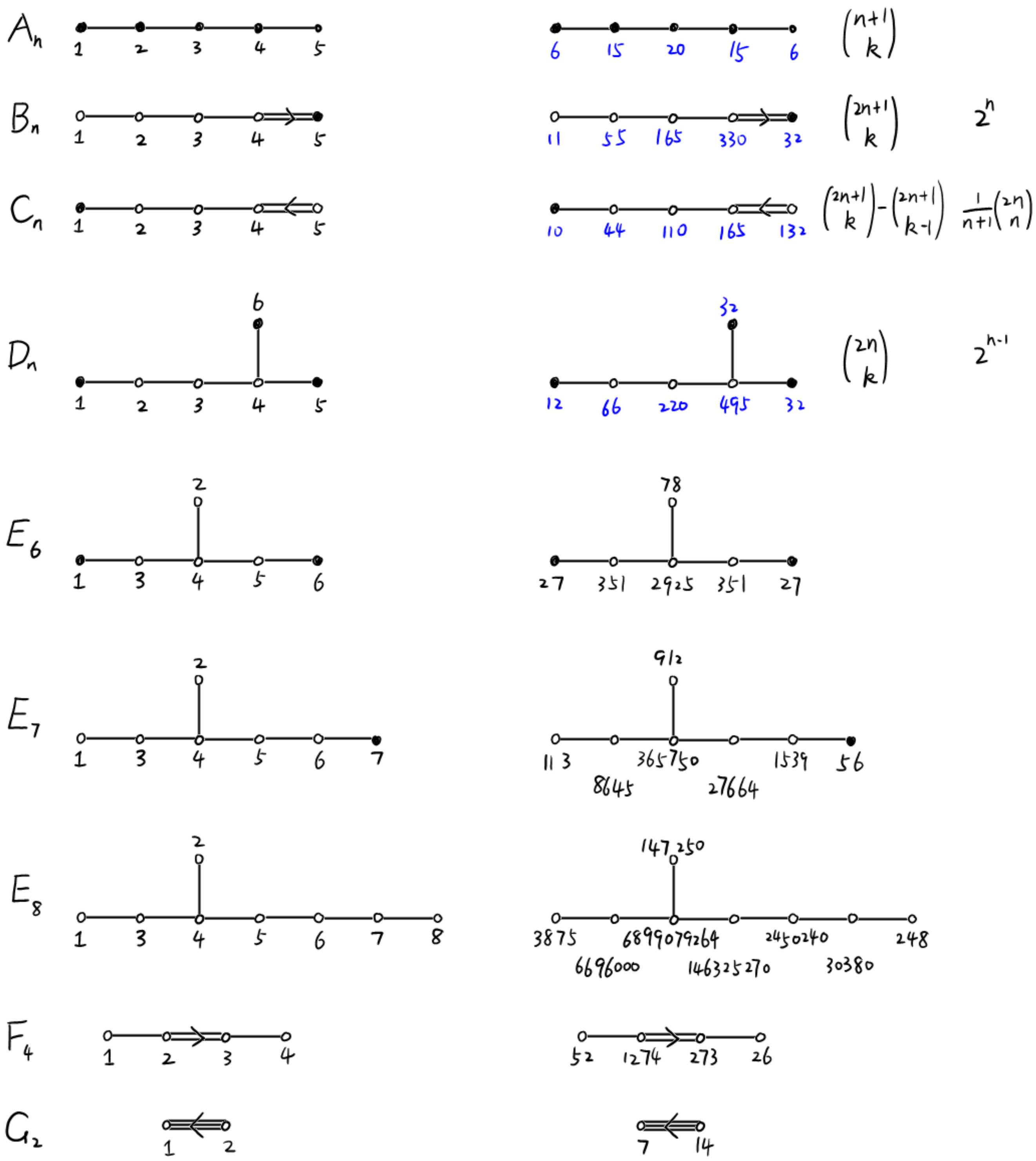
This document is a continuation for [2021.05.07_liegroup], [2021.07.18_irr_rep_of_semi_Lie_alg], [2021.07.25_irr_rep_of_SnAn], [2024.06.30_starting_functions].

The goal is to collect enough information on the representation sides, and then verify it on the perverse sheaf side.

Setting: We consider simple Lie algebras over $\mathbb{C}$.

1. labeling & basic rep dim
2. quasi-minuscule & adjoint reps

1. labeling & basic rep dim



long roots -> short roots
long weights -> short weights
short coroots -> long coroots
short coweights -> long coweights

1. labeling & basic rep dim

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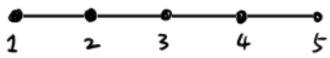
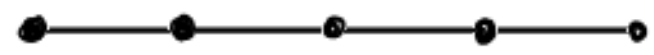
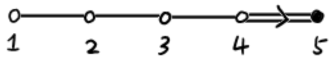
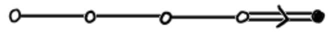
A_n



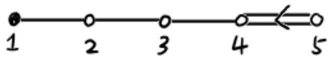


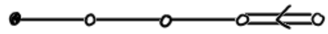
B_n



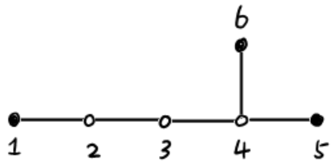


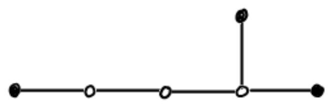
C_n



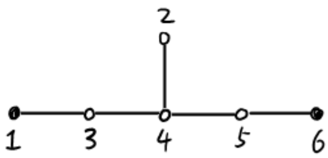


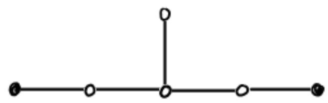
D_n



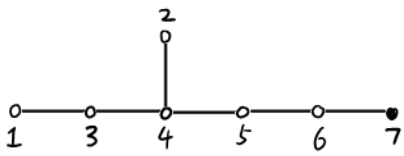


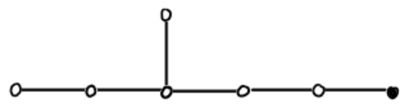
E_6



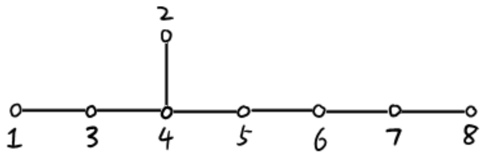


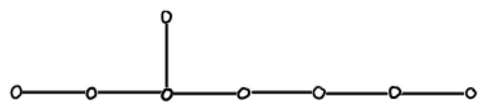
E_7



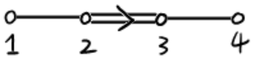


E_8

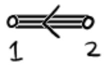




F_4

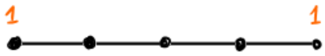
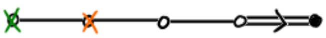
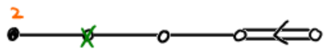
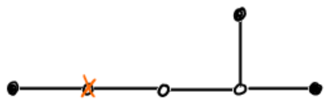
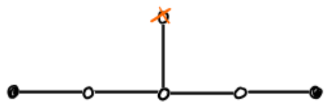
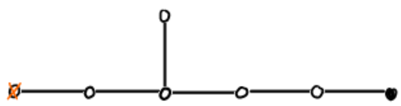
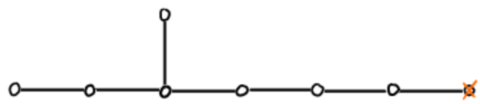
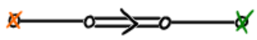
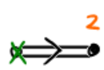
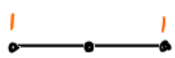


G_2



2. quasi-minuscule & adjoint reps

- • : minuscule rep
- × : quasi-minuscule rep
- × : adjoint rep

		dim/dim	rk/rk
A_n		n^2+2n	n
B_n		$2n^2+n/2n+1$	$n/1$
C_n		$2n^2+n/2n^2-n-1$	$n/n-1$
D_n		$2n^2-n$	n
E_6		78	6
E_7		133	7
E_8		248	8
F_4		$52/26$	$4/2$
G_2		$14/7$	$2/1$
A_1		3	1
B_2		$10/5$	$2/1$
D_3		15	3

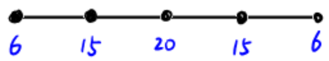
quasi-minuscule and adjoint reps are exquisite, in the sense that their weights have only 2 or 3 orbits, and one of the orbit is the trivial weight. When the diagram is not simply-laced, the quasi-minuscule rep has orbits consisting of short roots and origin, while the adjoint rep has orbits consisting of long roots, short roots and origin.

3. orbit of weights

this can be easily computed from the order of Weyl groups. But I still want to see it very quickly, since they corresponds to the degree of some varieties.

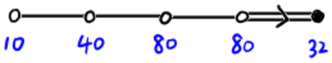
$|W|$

A_n



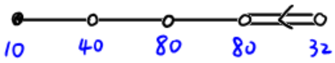
$(n+1)! = n! \times (n+1)$

B_n



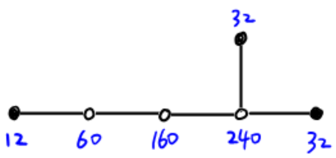
$n! \times 2^n$

C_n



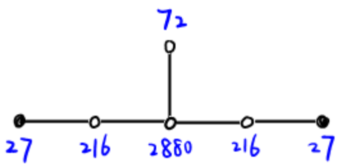
$n! \times 2^n$

D_n



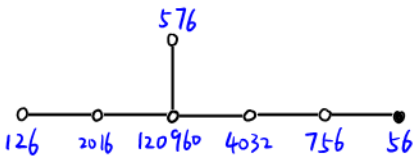
$n! \times 2^{n-1}$

E_6



$51840 = 6! \times 72$

E_7

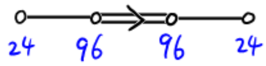


$2903040 = 7! \times 576$

E_8

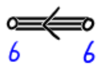
$696729600 = 8! \times 17280$

F_4



$1152 = 4! \times 48$

G_2



$12 = 2! \times 6$