

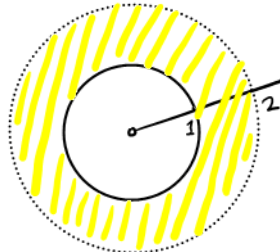
4.1. the complex torus of form $\mathbb{C}^x / \mathbb{Z}\gamma$

$\mathcal{C}_1 = \mathbb{C}^* / \mathbb{Z} \tau \stackrel{\text{topo}}{=} \mathbb{P}^1$ is a cpt Riemannian surface of genus 1.

$$\gamma \in \text{Aut}(\mathbb{C}^*) \quad \gamma(z) = az \quad a \in \mathbb{C}^* \quad |a| > 1$$

Today: $a=2$

1. fundamental set:

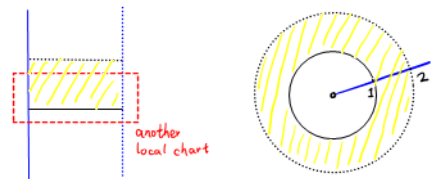


\Rightarrow only need 2 local chart


$$2. \quad 0 \rightarrow \mathbb{Z} \hookrightarrow \mathbb{C} \xrightarrow{f: z \mapsto e^{2\pi iz}} \mathbb{C}^\times \rightarrow 1$$


$$\downarrow +\frac{1}{2\pi i} \ln 2 \quad \downarrow +\frac{1}{2\pi i} \ln 2 \quad \downarrow \times 2$$

$$0 \rightarrow \mathbb{Z} + \frac{1}{2\pi i} \ln 2 \rightarrow \mathbb{C} \longrightarrow \mathbb{C}^\times \rightarrow 1$$



$$\mathbb{C}^* = \mathbb{C}/\mathbb{Z} \Rightarrow \mathbb{C}^*/\mathbb{Z}_Y = \mathbb{C}/(\mathbb{Z} \oplus \frac{1}{2\pi i} \ln 2\mathbb{Z}) \xrightarrow{\text{blue arrow}}$$

better: $a = e^{2\pi} \approx 535.49$ 

$a = e^{-2\pi i} \approx -230.765$ 

3. line bundle on \mathcal{C}

$b \in \mathbb{C}^*$

$$\mathcal{L}_b := \mathbb{C}^* \times \mathbb{C} / (z, \zeta) \sim (z\zeta, b\zeta)$$

$$\downarrow$$

$$\mathcal{C} = \mathbb{C}^* / z \sim z\zeta$$

\Rightarrow ① $\mathcal{L}_b \in \text{Pic}_0(\mathbb{C})$; ($\mathcal{L}_b \sim \mathcal{L}_1 \cong \mathcal{O}_{\mathbb{C}}$)
 (cont. deformation)
 ② $\text{Pic}_0(\mathbb{C}) \cong \mathbb{C} = \mathbb{C}^* / \mathbb{Z}\gamma$
 (naive, base pt $1 \in \mathbb{C}^* / \mathbb{Z}\gamma$)
 $\mathcal{L}_b \xrightarrow{\quad} b$

Reduced to: find a section s on \mathcal{L}_b st $\text{div } s = [b] - [1]$

Reduced to: find a meromorphic functions g on \mathbb{C}^\times s.t

① $g(2z) = b g(z)$ $b \in \mathbb{C}, b \neq 2^k$; e.g. $b=3$

② g has simple poles on 2^n , and simple zeros on $2^n b$ $n \in \mathbb{Z}$

$$b = e^{2\pi i c}, c \in \mathbb{C}$$

$$\tau = \frac{1}{2\pi i} \ln 2$$

$$w(z) = \frac{1}{2\pi i} \ln z$$

$$g(z) = \frac{\theta [1, -z]}{\theta \begin{bmatrix} 1 \\ 1 \end{bmatrix}} (w(z), \tau)$$
 is the required one.

Blue — example

Orange — more than this example

Red — important results

Purple — I don't know the answer/proof

Green — sketch of proof: in a minimal way

Grey — some supplementary explanation. Unimportant assumptions.

Hell grey — explanation on well-known notations.

Brown — small title in subsections.

My symbol collection set

		Mathbb	Mathrsf/Mathcal	Greek	
A abelian variety	a	A adèles	A apartment		α
B	b	B	B building		β
C	c	C cplx number	C category	Γ gp graph	γ
D	d	D	D Poincare disk	Δ diag embedding	δ
E elliptic curve	e	E	E		ε
F field	f	F finite field	F sheaf		ζ
G group	g	G gp scheme	G Lie alg upper half plane		η
H	h	H	H Hecke alg	⊕	θ
I ideal	i	I	I ideal of sheaf		ι injection
J	j	J	J		κ
K cos/base field	k ← k	K	K	Λ lattice	λ
L	l	L	L		μ
M module	m	M	M moduli space		ν
N	n	N natural number	N		ξ root of unity (ξ/ω)
O	o	O	O structure sheaf	Π multi	ζ constant
P	p	P proj space	P Weierstrass ell fct		π uniformizer
Q	q	Q rational number	Q	Σ sum	ρ ← ρ
R ring	r	R real number	R		σ
S base scheme	s	S	S	Φ	τ
T tangent space	t	T torus	T		φ
U ← U	u	U	-	Ψ	χ character
V v.s.	v	V	∇	Ω	ψ
W witt vector	w	W	-		ω ω ≈ ωω
X	x	X	X		
Y = Y	y	Y	Y	hebrew	Russian
Z center	z	Z integer	Z	N cardinal	III sha gp

Green: number / basic stuffs in senior high school

Orange: scheme - related

Darkyellow: advanced algebra

↙ Don't use them simultaneously!

Don't mix: ω/ω, ξ/ξ, k/k/K, ,

1/1/ι, * / x / X / X ,

φ / ψ , e / c

ωω `\varpi\boldsymbol{\omega}` (need amsbsy package)

Japanese mathematician and their Chinese translations.

	Matsumoto	松本	Hideya Matsumoto	松本英野	
1860	Sawayama	沢山	Yuzaburo Sawayama	沢山勇三郎	
1875.4			Teiji Takagi	高木贞治	高木 貞治
1908.12	Tannakian	淡中の	Tadao Tannaka	淡中忠郎	
1912.7	Nakayama	中山	Tadashi Nakayama	中山正	
1915.3	Kodaira	小平	Kunihiko Kodaira	小平邦彦	
1917.11	Iwasawa	岩泽	Kenkichi Iwasawa	岩泽健吉	岩澤 健吉
1925.11	Tamagawa	玉河	Tsuneo Tamagawa	玉河恒夫	
1926	Iwahori	岩堀	Nagayoshi Iwahori	岩堀长庆	岩堀 長慶
1927.11	Taniyama	谷山	Yutaka Taniyama	谷山丰	谷山 豊
1927.12	Satake	佐武	Ichirō Satake	佐武一郎	
1928			Hiroshi Toda	戸田宏	戸田 宏
1928.4	Sato	佐藤	Mikio Sato	佐藤干夫	佐藤 幹夫
1930.2	Shimura	志村	Gorō Shimura	志村五郎	
1930.3	Yoneda	米田	Nobuo Yoneda	米田信夫	
1930	Matsumura	松村	Hideyuki Matsumura	松村英之	
1931.4	Hironaka	广中	Heisuke Hironaka	广中平祐	広中 平祐
1944.3			Toshitsune Miyake	三宅敏恒	
1947.1			Masaki Kashiwara	柏原正树	柏原 正樹
1951.2	Mori	森	Shigefumi Mori	森重文	
1952.1			Kazuya Kato	加藤和也	
1953.12.8	Mukai	向井	Shigeru Mukai	向井茂	
1959.3	Fukaya	深谷	Kenji Fukaya	深谷贤治	
1961.9	Saito	斎藤	Takeshi Saito	斎藤毅	斎藤 毅
1962.11	Nakajima	中岛	Hiraku Nakajima	中岛启	中島 啓
1969.3			Shinichi Mochizuki	望月新一	