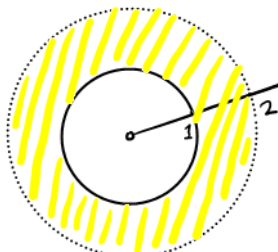


4.1. the complex torus of form $\mathbb{C}^x / \mathbb{Z}\gamma$

$$\gamma \in \text{Aut}(\mathbb{C}^x) \quad \gamma(z) = az \quad a \in \mathbb{C}^x \quad |a| > 1$$

1. fundamental set:

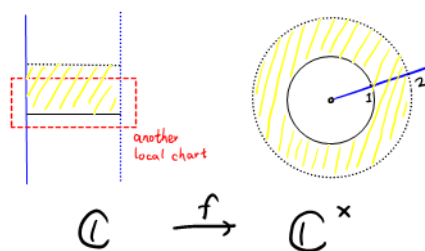


\Rightarrow only need 2 local chart


$$2. \quad 0 \rightarrow \mathbb{Z} \hookrightarrow \mathbb{C} \xrightarrow{f: z \mapsto e^{2\pi i z}} \mathbb{C}^\times \rightarrow 1$$


$$\downarrow + \frac{1}{2\pi i} \ln 2 \quad \downarrow + \frac{1}{2\pi i} \ln 2 \quad \downarrow \times 2$$

$$0 \rightarrow \mathbb{Z} + \frac{1}{2\pi i} \ln 2 \rightarrow \mathbb{C} \longrightarrow \mathbb{C}^\times \rightarrow 1$$



$$\mathbb{C}^* = \mathbb{C}/\mathbb{Z} \Rightarrow \mathbb{C}^*/\mathbb{Z}_Y = \mathbb{C}/(\mathbb{Z} \oplus \frac{1}{2\pi i} \ln 2 \mathbb{Z}) \xrightarrow{\text{blue arrow}}$$

better: $a = e^{2\pi} \approx 535.49$ 

$a = e^{-2\pi i} \approx -230.765$ 

3. line bundle on \mathcal{C}

$b \in \mathbb{C}^*$
 $\mathcal{L}_b := \mathbb{C}^* \times \mathbb{C} / (z, \zeta) \sim (bz, b\zeta)$
 \Rightarrow ① $\mathcal{L}_b \in \text{Pic}_0(C)$; ($\mathcal{L}_b \sim \mathcal{L}_1 \cong \mathcal{O}_C$) ^{cont. deformation}
 \downarrow
 $\mathcal{C} = \mathbb{C}^* / z \sim bz$
 $\text{② } \text{Pic}_0(C) \cong \mathcal{C} = \mathbb{C}^* / z \sim bz$ ^(naive, base pt $1 \in \mathbb{C}^* / z \sim bz$)
 $\mathcal{L}_b \xrightarrow{\quad} b$

Reduced to: find a section s on \mathcal{L}_b st $\text{div } s = [b] - [1]$

Reduced to: find a meromorphic functions g on \mathbb{C}^\times s.t

① $g(2z) = b g(z)$ $b \in \mathbb{C}, b \neq 2^k$; e.g. $b=3$

② g has simple poles on 2^n , and simple zeros on $2^n b$ $n \in \mathbb{Z}$

$$b = e^{2\pi i c}, c \in \mathbb{C}$$

$$\tau = \frac{1}{2\pi i} \ln z$$

$$w(z) = \frac{1}{2\pi i} \ln z$$

$$g(z) = \frac{\theta \begin{bmatrix} 1 & -z_c \end{bmatrix} (w(z), \tau)}{\theta \begin{bmatrix} 1 \\ 1 \end{bmatrix} (w(z), \tau)}$$
 is the required one.

Blue — example

Orange — more than this example, slogan

Red — important results

Purple — I don't know the answer/proof

Green — sketch of proof: in a minimal way

Grey — some supplementary explanation. Unimportant assumptions.

Hell grey — explanation on well-known notations.

Brown — small title in subsections.

My symbol collection set

		Mathbb	Mathrsf/Mathcal	Greek	
A abelian variety	a	A adèles	A apartment		α
B	b	B	B building		β
C	c	C cplx number	C category	Γ gp graph	γ
D	d	D	D Poincare disk	Δ diag embedding	δ
E elliptic curve	e	E	E		ε
F field fiber	f	F finite field	F sheaf		ζ
G formal gp law	g	G gp scheme	G Lie alg		η
H group	h	H	H upper half plane	⊕	θ
I ideal	i	I	I Hecke alg		ι injection
J	j	J	J ideal of sheaf		κ
K cos/base field	k ← k	K	K	Λ lattice	λ
L	l	L	L		μ
M module	m	M	M moduli space		ν
N	n	N natural number	N		ξ root of unity (ξ/ω)
O	o	O	O structure sheaf	Π multi	ζ constant
P	p	P proj space	P Weierstrass		π uniformizer
Q	q	Q rational number	Q ell fct	Σ sum	ρ ← ρ
R ring	r	R real number	R		σ
S base scheme	s	S	S	Φ	τ
T test scheme	t	T torus	T		φ
U tangent space	u	U	-	Ψ	χ character
V translation	v	V	∂	Ω	ψ
W v.s.	w	W	∂		ω ω ≈ ωω
X witt vector	x	X	X		
Y	y	Y	Y	hebrew	Russian
Z center	z	Z integer	Z	ℵ cardinal	Ш sha gp

Green: number / basic stuffs in senior high school

Orange: scheme - related

Darkyellow: advanced algebra

Don't use them simultaneously! (usually)

Don't mix: $w/\omega, \xi/\zeta, k/\kappa/\mathcal{K}/K$

$1/l/\iota, x/\chi/\mathcal{X}$,

$\varphi/\psi, e/\epsilon, \gamma/\nu$

$\omega\omega$ `\varpi\boldsymbol{\omega}` (need amsbsy package)

Japanese mathematicians and their Chinese translations.

	Matsumoto	松本	Hideya Matsumoto	松本英野		
1860	Sawayama	沢山	Yuzaburo Sawayama	沢山勇三郎		
1875.4			Teiji Takagi	高木贞治	高木	貞治
1901.4	Oka	岡	Kiyoshi Oka	岡 洁	岡	潔
1902.7	Yamanouchi	山内	Takahiko Yamanouchi	山内恭彦		
1902.8	Akizuki	秋月	Yasuo Akizuki	秋月康夫	秋月	康夫
1904.01			Hideyaka Teresaka	寺阪英孝	寺阪	英孝
1908.12	Tannakian	淡中の	Tadao Tannaka	淡中忠郎		
1912.7	Nakayama	中山	Tadashi Nakayama	中山正		
1915.3	Kodaira	小平	Kunihiko Kodaira	小平邦彦		
1917.11	Iwasawa	岩泽	Kenkichi Iwasawa	岩泽健吉	岩泽	健吉
1924.1	Igusa	井草	Jun-Ichi Igusa	井草准一	井草	准一
1924.2	Tomita	富田	Minoru Tomita	富田稔	富田	稔
1925.1			Toichiro Kinoshita	木下东一郎	木下	東一郎
1925.11	Tamagawa	玉河	Tsuneo Tamagawa	玉河恒夫		
1926	Iwahori	岩堀	Nagayoshi Iwahori	岩堀长庆	岩堀	長慶
1927.11	Taniyama	谷山	Yutaka Taniyama	谷山丰	谷山	豊
1927.12	Satake	佐武	Ichirō Satake	佐武一郎		
1928.12	Fujita	藤田	Hiroshi Fujita	藤田宏	藤田	宏
1928	Toda	户田	Hiroshi Toda	户田宏	户田	宏
1928.4	Sato	佐藤	Mikio Sato	佐藤干夫	佐藤	幹夫
1930.2	Shimura	志村	Gorō Shimura	志村五郎		
1930.3	Yoneda	米田	Nobuo Yoneda	米田信夫		
1930	Matsumura	松村	Hideyuki Matsumura	松村英之		
1931.4	Hironaka	广中	Heisuke Hironaka	广中平祐	广中	平祐
1932.1			Shoshichi Kobayashi	小林昭七	小林	昭七
1933.7	Takesaki	竹崎	Masamichi Takesaki	竹崎正道	竹崎	正道
1944.3			Toshitsune Miyake	三宅敏恒		
1947.1	Kashiwara	正树	Masaki Kashiwara	柏原正树	柏原	正樹

This is an issue of non-standard language usage. Strictly speaking, "沢山" should be written as "泽山", but since everyone uses "沢山", I also write "沢山" instead of "泽山".

1951.2	Mori	森	Shigefumi Mori	森重文	
1952.1	Kato	加藤	Kazuya Kato	加藤和也	
1952.8	Hida	肥田	Haruzo Hida	肥田晴三	肥田 晴三
1953.12.8	Mukai	向井	Shigeru Mukai	向井茂	
1959.3	Fukaya	深谷	Kenji Fukaya	深谷贤治	
1961.9	Saito	斎藤	Takeshi Saito	斎藤毅	斎藤 毅
1962.11	Nakajima	中島	Hiraku Nakajima	中島启	中島 啓
1969.3			Shinichi Mochizuki	望月新一	
1972.8			Takuro Mochizuki	望月拓郎	

Confusion list:

1. Ring has unit. Don't consider 0-Ring.
2. Read the diagram from top to bottom.
3. countable = finite + inf countable (at most countable)
4. g fix set A : $\forall a \in A, ga = a$ (use "stabilized" instead)
5. \subset only mean a subset, or an injective map
(incompatible structures are allowed, e.g. $L^\infty([0,1]) \subset L'([0,1])$)
6. definition of norm/seminorms
7. $HK \neq H \times K$ $HK = \{g \in G \mid g = hk \text{ for some } h \in H, k \in K\}$