Un exemple par jour

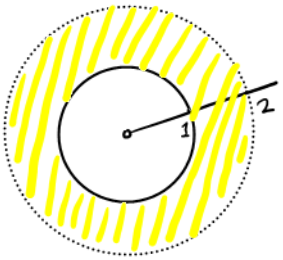
4.1. the complex torus of form $\mathbb{C}^\times/\mathbb{Z}\gamma$

$C := \mathbb{C}^\times/\mathbb{Z}\gamma \overset{\text{topo}}{=} \mathbb{T}^2$ is a cpt Riemannian surface of genus 1.

$\gamma \in Aut(\mathbb{C}^\times)$ $\gamma(z)=az$ $a\in\mathbb{C}^\times$ $|a|>1$

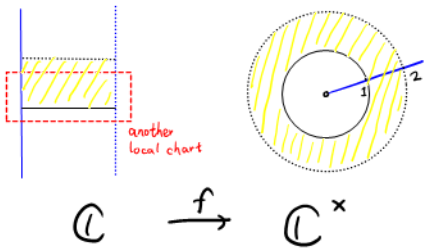
Today: $a=2$

1. fundamental set:



$\Rightarrow$ only need 2 local chart

2.
$$\begin{array}{ccccccc} 0 & \to & \mathbb{Z} & \hookrightarrow & \mathbb{C} & \xrightarrow{f:z\mapsto e^{2\pi i z}} & \mathbb{C}^\times \to 1 \\ & & \downarrow{\scriptstyle +\frac{1}{2\pi i}\ln 2} & & \downarrow{\scriptstyle +\frac{1}{2\pi i}\ln 2} & & \downarrow{\scriptstyle \times 2} \\ 0 & \to & \mathbb{Z}+\frac{1}{2\pi i}\ln 2 & \to & \mathbb{C} & \longrightarrow & \mathbb{C}^\times \to 1 \end{array}$$



$\mathbb{C} \xrightarrow{f} \mathbb{C}^\times$

$\mathbb{C}^\times = \mathbb{C}/\mathbb{Z} \Rightarrow \mathbb{C}^\times/\mathbb{Z}\gamma = \mathbb{C}/(\mathbb{Z}\oplus\frac{1}{2\pi i}\ln 2\,\mathbb{Z})$

better: $a = e^{2\pi} \approx 535.49$

$a = e^{-2\pi i w} \approx -230.765$

3. line bundle on C

$b\in\mathbb{C}^\times$ $\mathcal{L}_b := \mathbb{C}^\times\times\mathbb{C}/(z,\xi)\sim(2z,b\xi)$ $\Rightarrow$ ① $\mathcal{L}_b \in Pic_0(C)$; ($\mathcal{L}_b \overset{\text{cont. deformation}}{\sim} \mathcal{L}_1 \cong \mathcal{O}_C$)

$\downarrow$

$C = \mathbb{C}^\times/z\sim 2z$

② $Pic_0(C) \overset{\text{(naive, base pt } 1\in\mathbb{C}^\times/\mathbb{Z}\gamma)}{\cong} C = \mathbb{C}^\times/\mathbb{Z}\gamma$

$\mathcal{L}_b \longrightarrow b$

Reduced to: find a section s on $\mathcal{L}_b$ st $div\, s = [b]-[1]$

Reduced to: find a meromorphic functions g on $\mathbb{C}^\times$ s.t

① $g(2z) = bg(z)$ $b\in\mathbb{C}^\times$, $b\neq 2^k$; e.g. $b=3$

② g has simple poles on 2^n, and simple zeros on $2^n b$ $n\in\mathbb{Z}$

$b = e^{2\pi i c}$, $c\in\mathbb{C}$

$\tau := \frac{1}{2\pi i}\ln 2$

$w(z) = \frac{1}{2\pi i}\ln z$

$$g(z) := \frac{\theta\left[\begin{smallmatrix}1\\1-2c\end{smallmatrix}\right](w(z),\tau)}{\theta\left[\begin{smallmatrix}1\\1\end{smallmatrix}\right](w(z),\tau)}$$

is the required one.

Blue — example

Orange — more than this example

Red — important results

Purple — I don't know the answer/proof

Green — sketsch of proof : in a minimal way

Grey — some supplementary explanation . Unimportant assumptions.

Hell grey — explanation on well-known notations.

Brown — small title in subsections.

My symbol collection set

		Mathbb	Mathrsf/Mathcal	Greek	
A abelian variety	a	$\mathbb{A}$ adèles	$\mathscr{A}$ apartment		α
B	b	$\mathbb{B}$	$\mathscr{B}$ building		β
C	c $\mathbb{c}$ e	$\mathbb{C}$ cplx number	$\mathscr{C}$ chamber category	Γ gp graph $\mathbb{\Gamma}$	γ
D	d $\mathbb{d}$	$\mathbb{D}$	$\mathscr{D}$ Poincare disk	Δ diag embedding	δ
E elliptic curve	e constant ramification index	$\mathbb{E}$	$\mathscr{E}$		ε
F field	f	$\mathbb{F}$ finite field	$\mathscr{F}$ sheaf		ζ
G group	g	$\mathbb{G}$ gp scheme	$\mathscr{G}$ $\mathfrak{g}$: Lie alg		η
H	h constant	$\mathbb{H}$	$\mathscr{H}$ upper half plane Hecke alg	Θ	θ
I ideal	i	$\mathbb{I}$	$\mathscr{I}$ ideal of sheaf		ι injection
J	j	$\mathbb{J}$	$\mathscr{J}$		κ
K cos/base field	k ← k	$\mathbb{K}$	$\mathscr{K}$	Λ lattice $\mathbb{\Lambda}$	λ
L	l	$\mathbb{L}$	$\mathscr{L}$		μ
M module	m	$\mathbb{M}$	$\mathscr{M}$ moduli space		ν
N	n	$\mathbb{N}$ natural number	$\mathscr{N}$		ξ root of unity (ζ_n)
O	o	$\mathbb{O}$	$\mathscr{O}$ structure sheaf $\mathcal{O}$	Π multi	π constant uniformizer Projection
P	p	$\mathbb{P}$ proj space	$\mathscr{P}$ Weierstrass $\wp$: ell fct		ρ ← ϱ
Q	q	$\mathbb{Q}$ rational number	$\mathscr{Q}$	Σ sum	σ
R ring	r	$\mathbb{R}$ real number	$\mathscr{R}$		τ
S base scheme	s	$\mathbb{S}$	$\mathscr{S}$	Φ	φ
T test scheme tangent space translation	t	$\mathbb{T}$ torus	$\mathscr{T}$		χ character
U ← U	u	$\mathbb{U}$	-	Ψ	ψ
V v.s.	v	$\mathbb{V}$	$\mathscr{V}$ $\mathbb{\mathscr{V}}$	Ω $\mathbb{\Omega}$	ω $\mathbb{\omega}$ $\approx$ $\varpi\omega$
W witt vector	w	$\mathbb{W}$	-		
X	x	$\mathbb{X}$	$\mathscr{X}$		
Y = Y	y	$\mathbb{Y}$	$\mathscr{Y}$	hebrew	Russian
Z center	z	$\mathbb{Z}$ integer	$\mathscr{Z}$	$\aleph$ cardinal	Ш sha gp

Green: number / basic stuffs in senior high school

Orange: scheme - related

Darkyellow: advanced algebra

Don't mix: w/ω, ζ/ξ, $k/\kappa/\mathcal{K}/K$ ↙ Don't use them simultaneously! (usually)

$1/\mathrm{l}/\iota$, $\times/x/\chi/\mathrm{X}$,

φ/ψ, ϵ/e, γ/ν

$\varpi\omega$ `$\varpi\boldsymbol{\omega}$` (need amsbsy package)

https://en.wikipedia.org/wiki/Greek_letters_used_in_mathematics,_science,_and_engineering
https://mathoverflow.net/questions/8295/origins-of-mathematical-symbols-names

Japanese mathematician and their Chiese translations.

	Matsumoto	松本	Hideya Matsumoto	松本英野	
1860	Sawayama	沢山	Yuzaburo Sawayama	沢山勇三郎	
1875.4			Teiji Takagi	高木貞治	高木 貞治
1901.4	Oka	冈	Kiyoshi Oka	冈洁	岡 潔
1902.8	Akizuki	秋月	Yasuo Akizuki	秋月康夫	秋月 康夫
1908.12	Tannakian	淡中的	Tadao Tannaka	淡中忠郎	
1912.7	Nakayama	中山	Tadashi Nakayama	中山正	
1915.3	Kodaira	小平	Kunihiko Kodaira	小平邦彦	
1917.11	Iwasawa	岩泽	Kenkichi Iwasawa	岩泽健吉	岩澤 健吉
1924.2	Tomita	富田	Minoru Tomita	富田稔	冨田 稔
1925.11	Tamagawa	玉河	Tsuneo Tamagawa	玉河恒夫	
1926	Iwahori	岩堀	Nagayoshi Iwahori	岩堀长庆	岩堀 長慶
1927.11	Taniyama	谷山	Yutaka Taniyama	谷山丰	谷山 豊
1927.12	Satake	佐武	Ichirō Satake	佐武一郎	
1928			Hiroshi Toda	户田宏	戸田 宏
1928.4	Sato	佐藤	Mikio Sato	佐藤干夫	佐藤 幹夫
1930.2	Shimura	志村	Gorō Shimura	志村五郎	
1930.3	Yoneda	米田	Nobuo Yoneda	米田信夫	
1930	Matsumura	松村	Hideyuki Matsumura	松村英之	
1931.4	Hironaka	广中	Heisuke Hironaka	广中平祐	広中 平祐
1933.7	Takesaki	竹崎	Masamichi Takesaki	竹崎正道	竹崎 正道
1944.3			Toshitsune Miyake	三宅敏恒	
1947.1			Masaki Kashiwara	柏原正树	柏原 正樹
1951.2	Mori	森	Shigefumi Mori	森重文	
1952.1			Kazuya Kato	加藤和也	
1953.12.8	Mukai	向井	Shigeru Mukai	向井茂	
1959.3	Fukaya	深谷	Kenji Fukaya	深谷贤治	
1961.9	Saito	斋藤	Takeshi Saito	斋藤毅	斎藤 毅
1962.11	Nakajima	中岛	Hiraku Nakajima	中岛启	中島 啓
1969.3			Shinichi Mochizuki	望月新一	

Confusion list:

1. Ring has unit. Don't consider 0-Ring.
2. Read the diagram from top to bottom.
3. countable = finite + inf countable (at most countable)
4. g fix set A: $\forall a \in A,\ ga = a$ (use "stabilized" instead)
5. $\subset$: only mean a subset, or an injective map
   (uncompatible structures are allowed, e.g. $L^\infty([0,1]) \subset L^1([0,1])$)
6. definition of norm/seminorms
7. $HK \neq H \times K$ $\quad HK = \{g \in G \mid g = hk \text{ for some } h \in H, k \in K\}$