

Eine Woche, ein Beispiel

8.15 indecomposable representation of Dynkin quiver.

AR-quiver is a powerful tool considering about the indecomposable modules and relations among them. Using the AR-quiver, one can find(not totally serious):

- all the indecomposable modules;
- all the morphisms between these indecomposable modules;
- all the irreducible morphisms and AR-sequences;

However, it's not easy to see the coker and ker of some morphisms given by the AR-quiver.

The following AR-quiver pictures are now useless, since everyone can get better pictures at <https://www.math.uni-bielefeld.de/~wcrawley/#knitting>.

Unfortunately, the knitting process can not draw some AR-quivers even in the case where "there are finite iso class of indec modules of quiver"

e.g.

$$A = K[T]/(T^3) \cong KQ/(a^3)$$

$$Q: 1 \xrightarrow{a} 2$$

$$N(3)$$

$$\uparrow \downarrow$$

$$N(2) \xrightarrow{\tau}$$

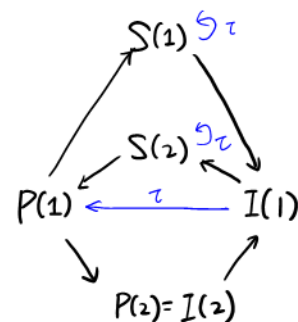
$$\uparrow \downarrow$$

$$N(1) \xrightarrow{\tau}$$

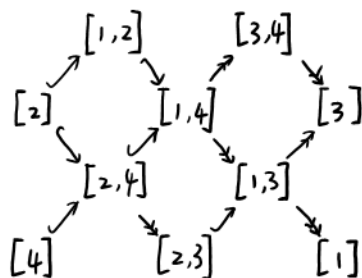
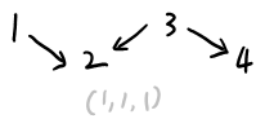
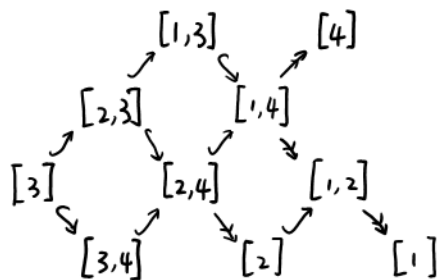
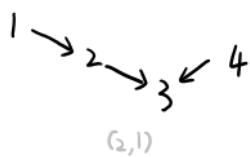
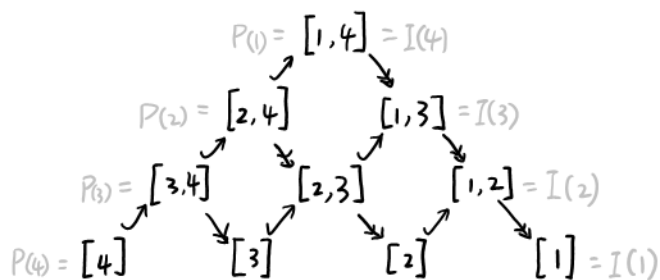
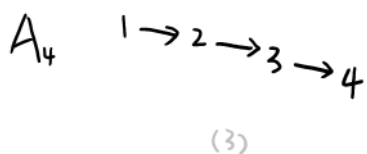
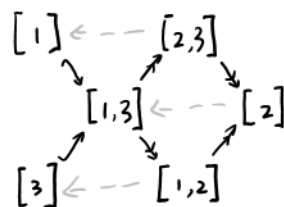
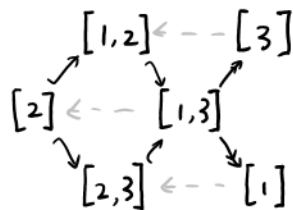
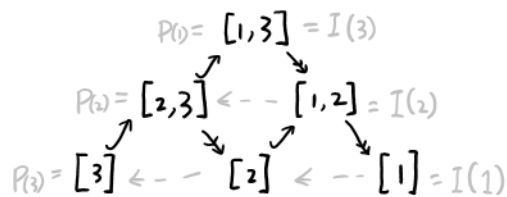
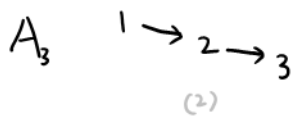
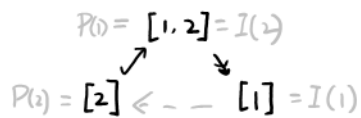
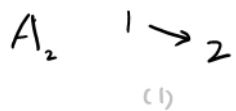
$$A = KQ/(ab)$$

$$\geq 0$$

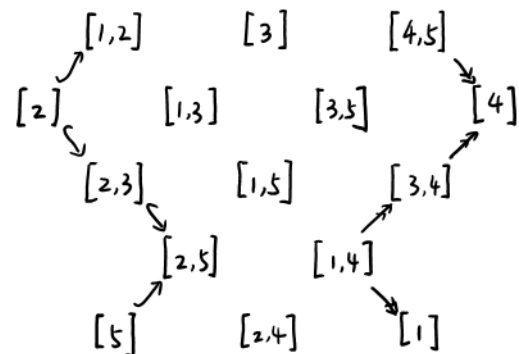
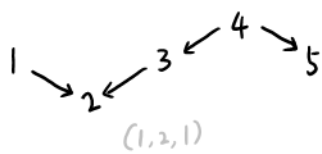
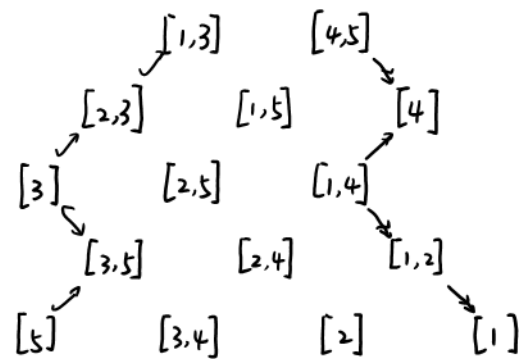
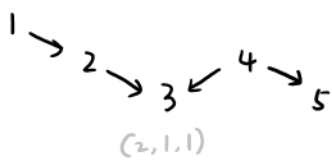
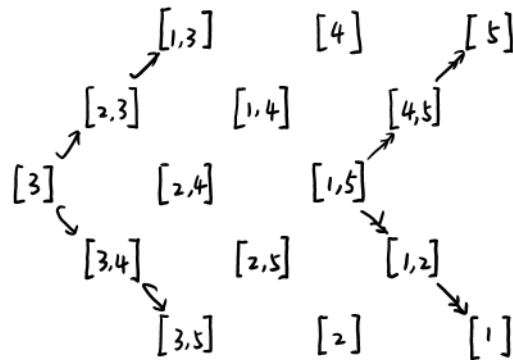
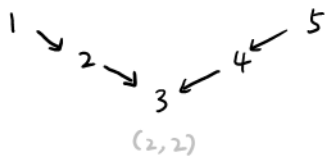
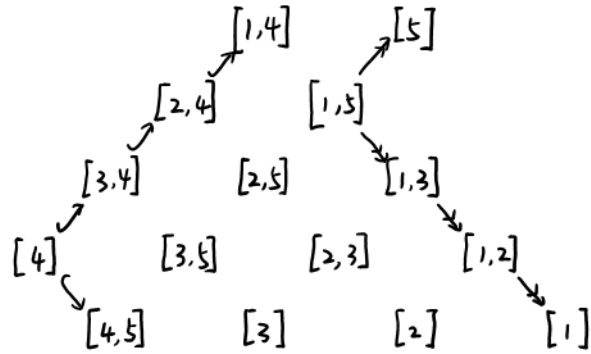
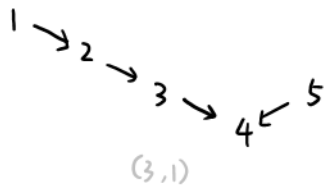
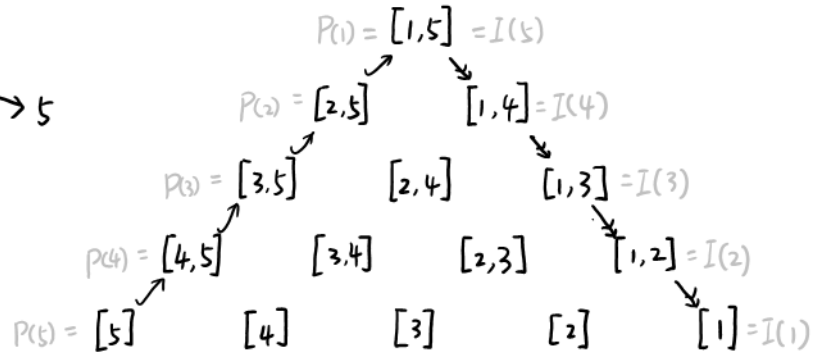
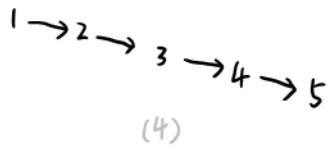
$$Q: 1 \xrightleftharpoons[b]{a} 2$$

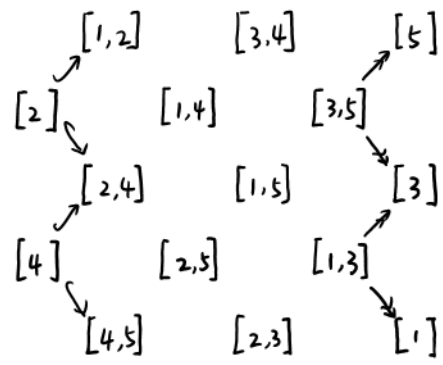
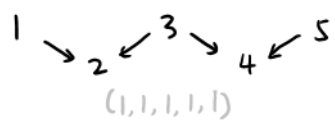


For the description of AR quiver of type A and D by a triangulated (puctured) polygon, see [Quiver Representations by Ralf Schiffler, 3.1.3+3.3.3].

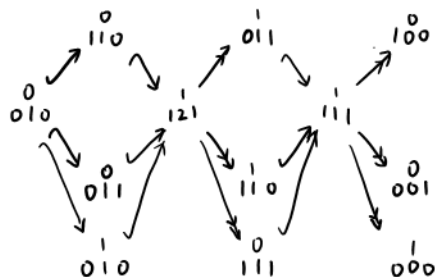
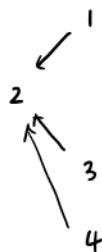
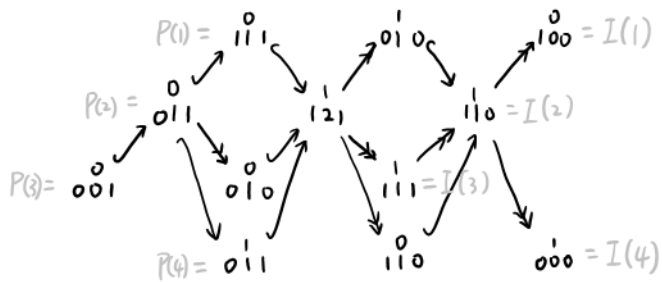
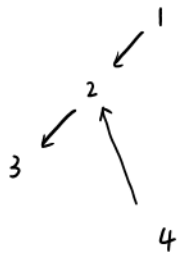
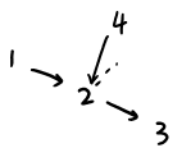


A_5

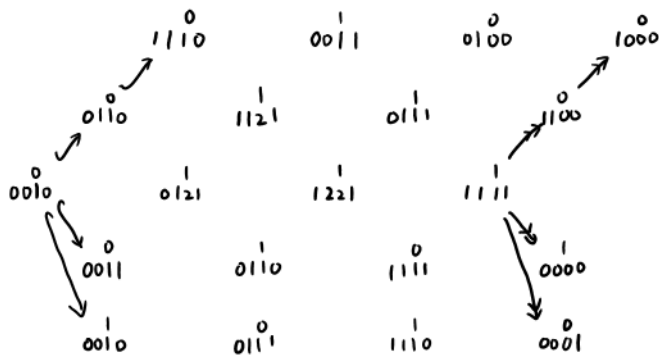
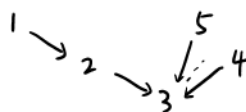
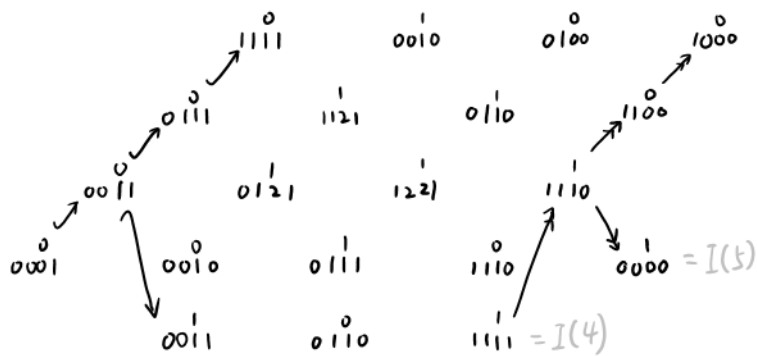
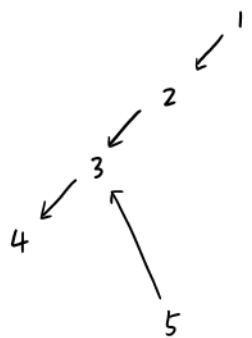
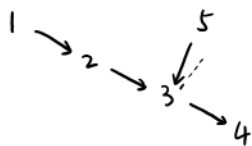


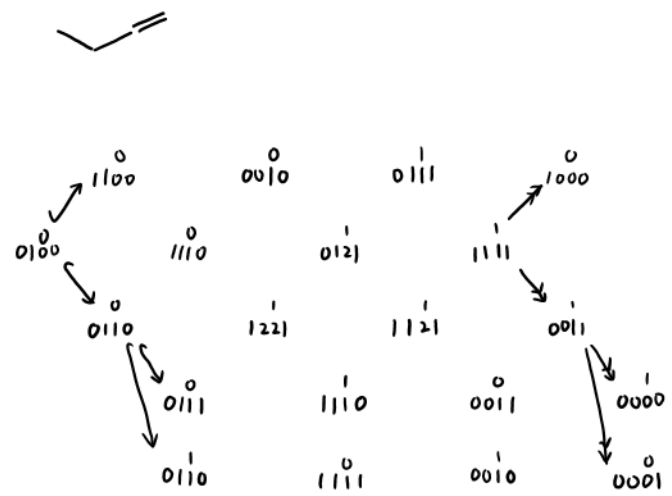
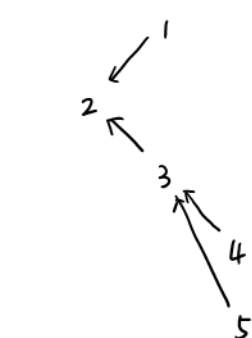
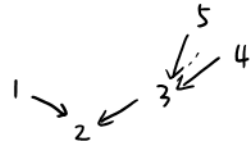
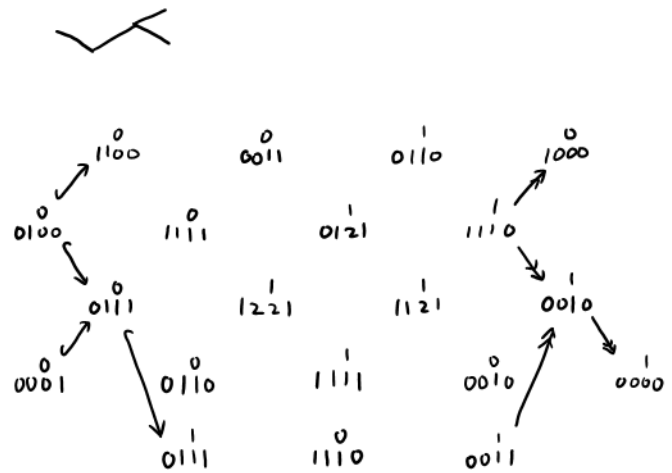
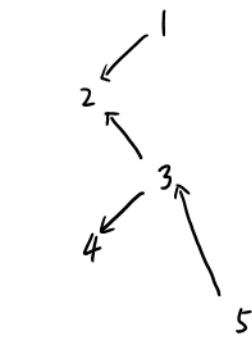
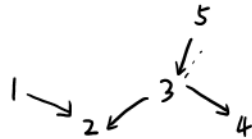
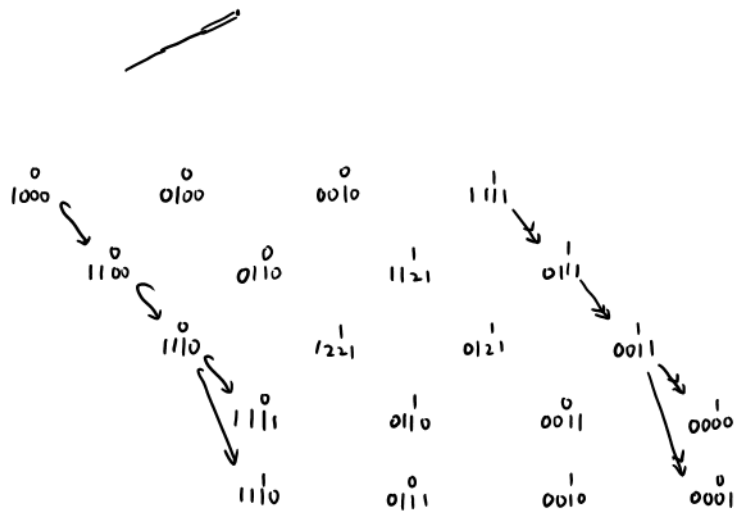
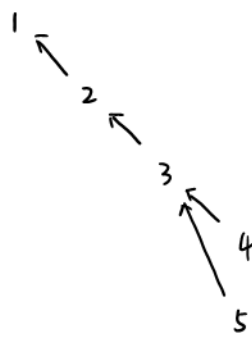
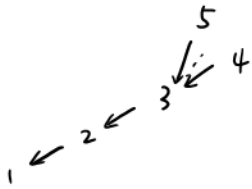


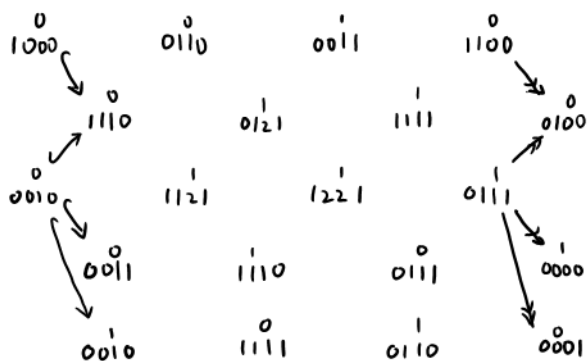
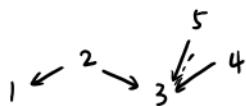
D_4



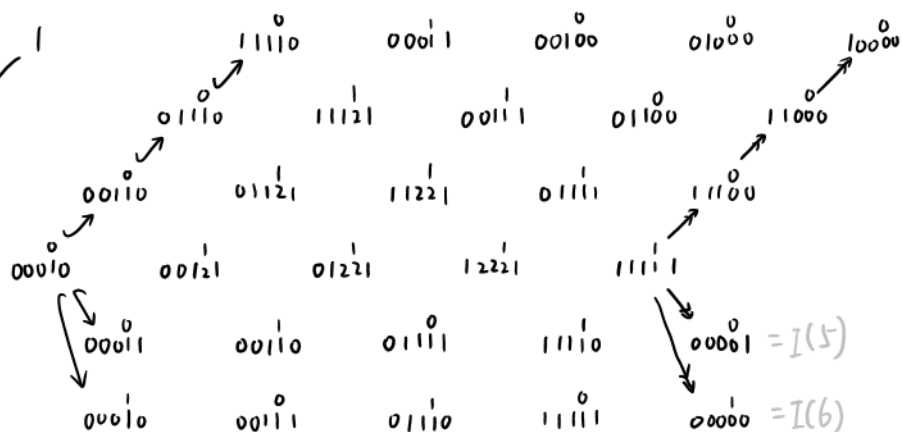
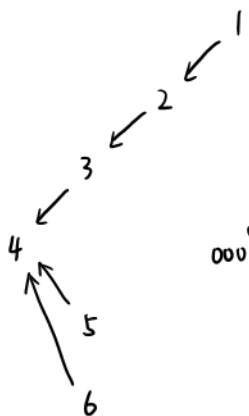
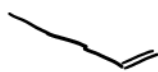
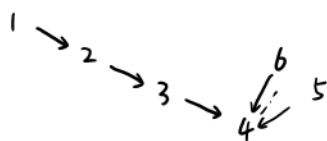
D_5



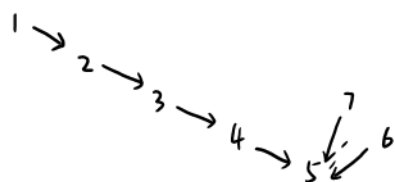




D_6

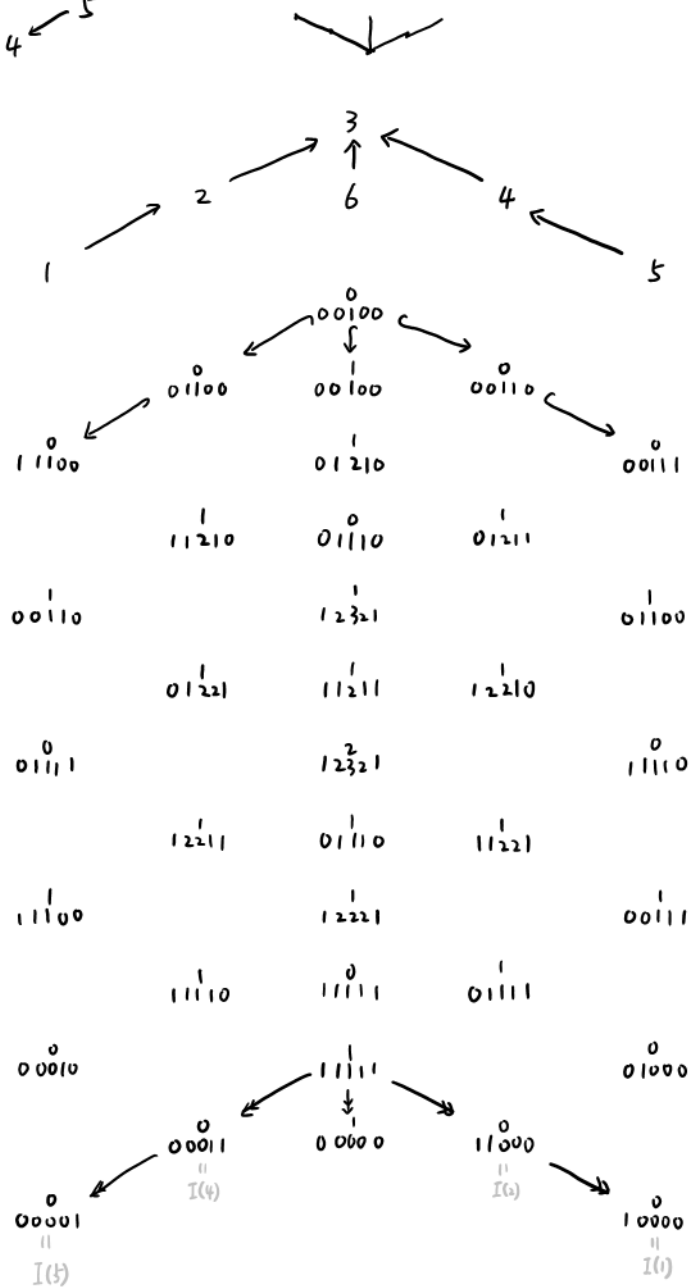


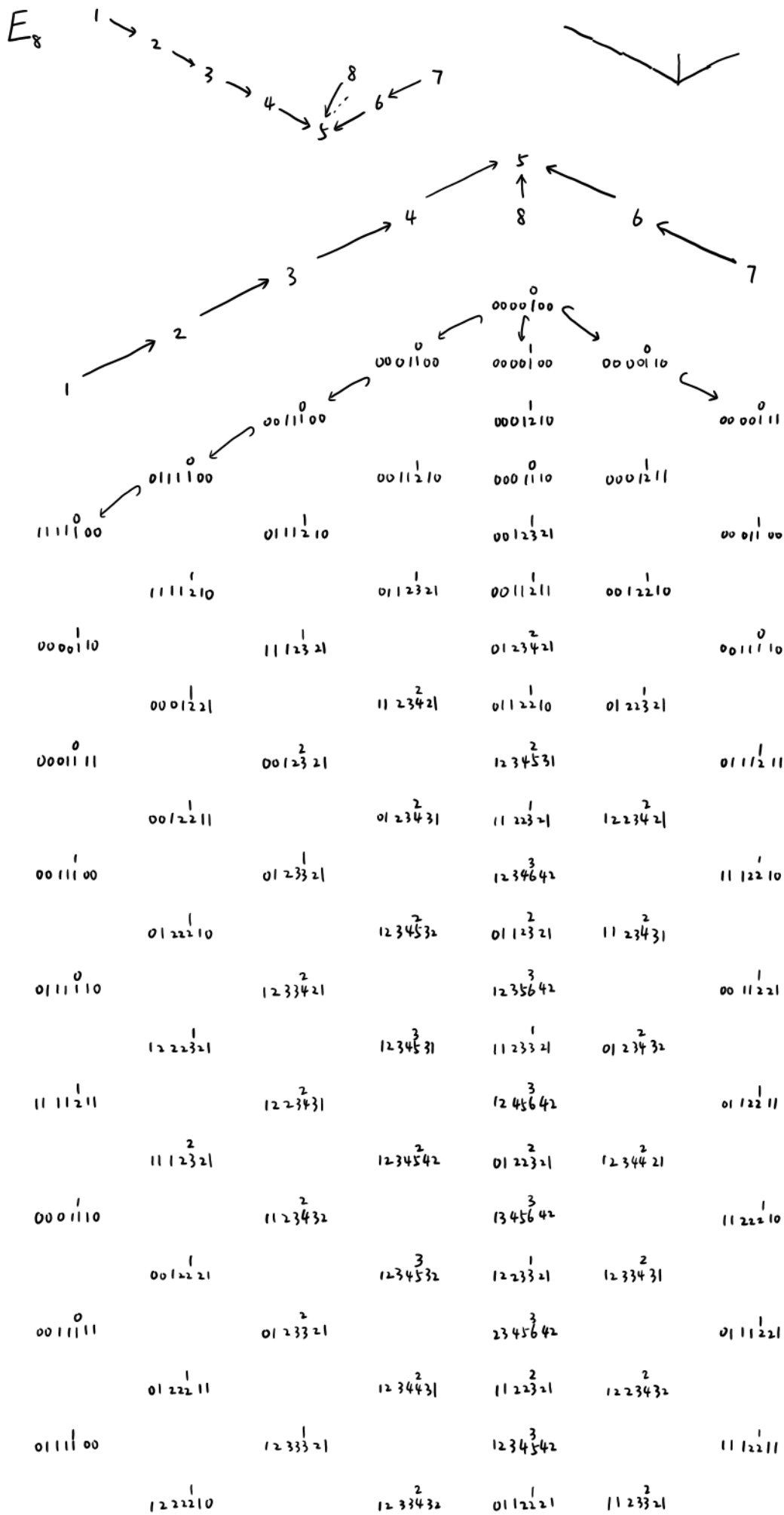
D_7

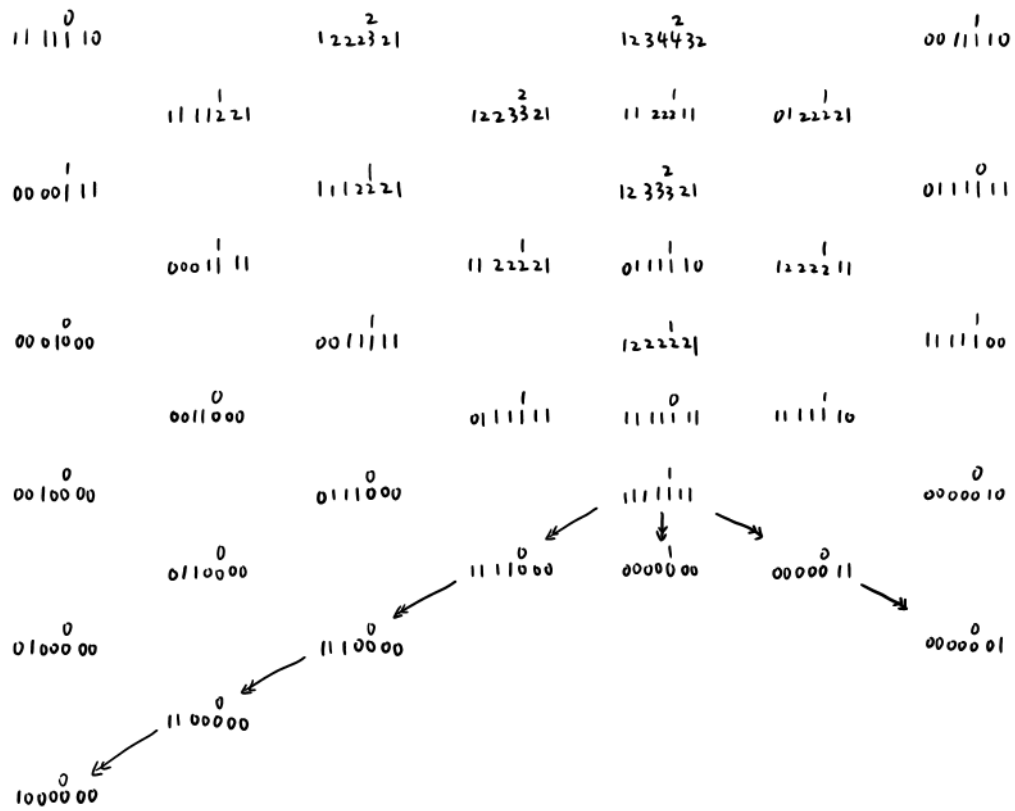


$$P(7) = 000010 \leftarrow \dots \leftarrow 000001 = P(6)$$

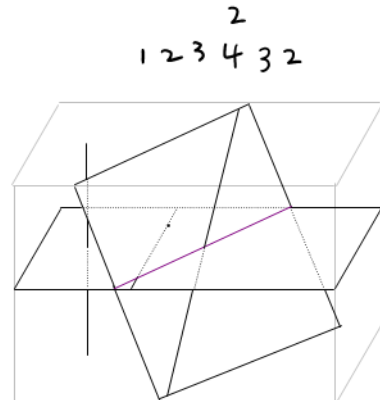
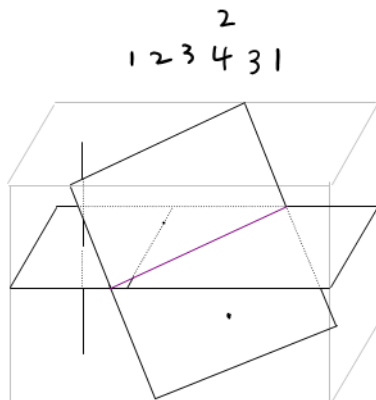
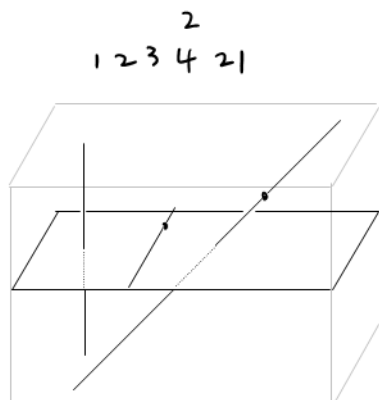
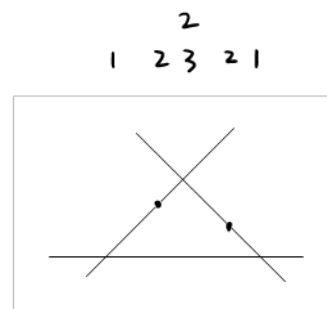
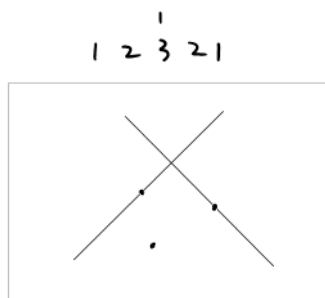
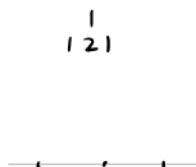
E_6 $1 \rightarrow 2 \rightarrow 3 \xleftarrow{6} 4 \leftarrow 5$





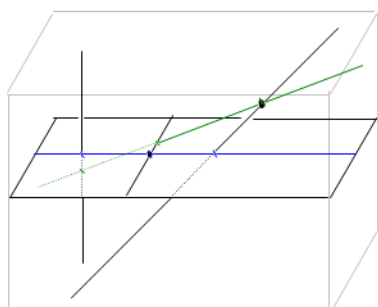


Bonus: subspace case (projective space version)

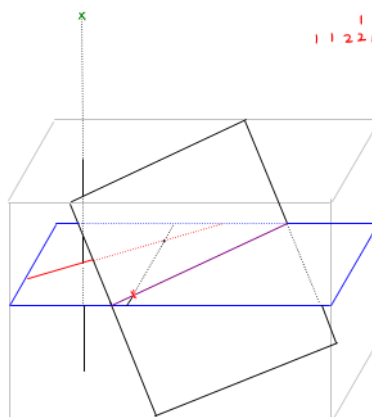


These shapes should be as general as possible, otherwise it may be not indecomposable:

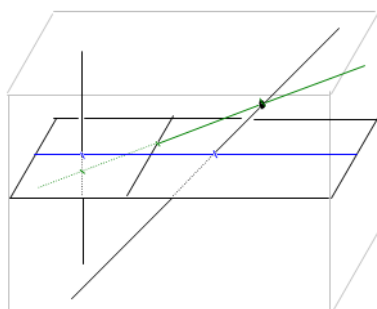
e.g. $\begin{matrix} 2 \\ 123421 \end{matrix} = \begin{matrix} 1 \\ 112210 \end{matrix} \oplus \begin{matrix} 1 \\ 011211 \end{matrix}$



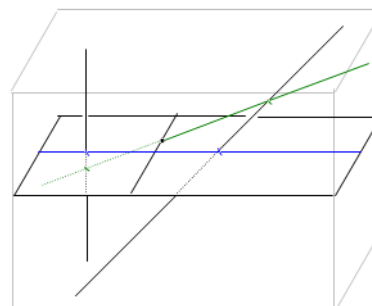
$\begin{matrix} 2 \\ 12343 \end{matrix} = \begin{matrix} 1 \\ 12332 \end{matrix} \oplus \begin{matrix} 1 \\ 00011 \end{matrix}$
 $\begin{matrix} 1 \\ 11221 \end{matrix} \oplus \begin{matrix} 1 \\ 01111 \end{matrix}$



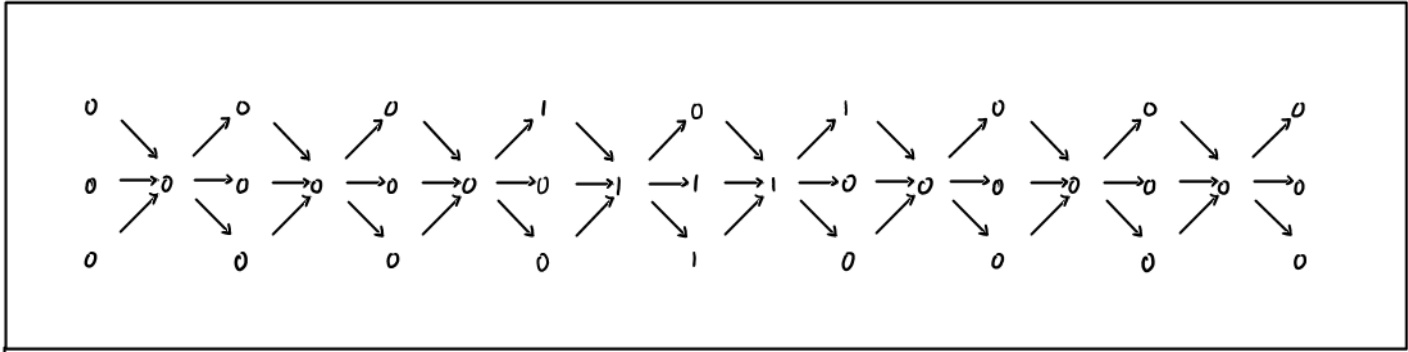
$\begin{matrix} 2 \\ 23421 \end{matrix} = \begin{matrix} 1 \\ 12210 \end{matrix} \oplus \begin{matrix} 1 \\ 11211 \end{matrix}$



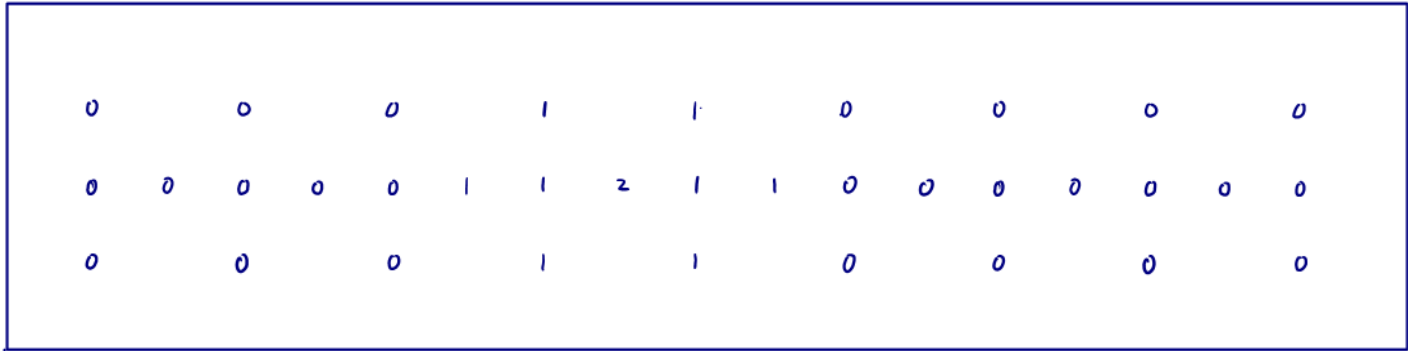
$\begin{matrix} 2 \\ 12342 \end{matrix} = \begin{matrix} 1 \\ 01221 \end{matrix} \oplus \begin{matrix} 1 \\ 11121 \end{matrix}$



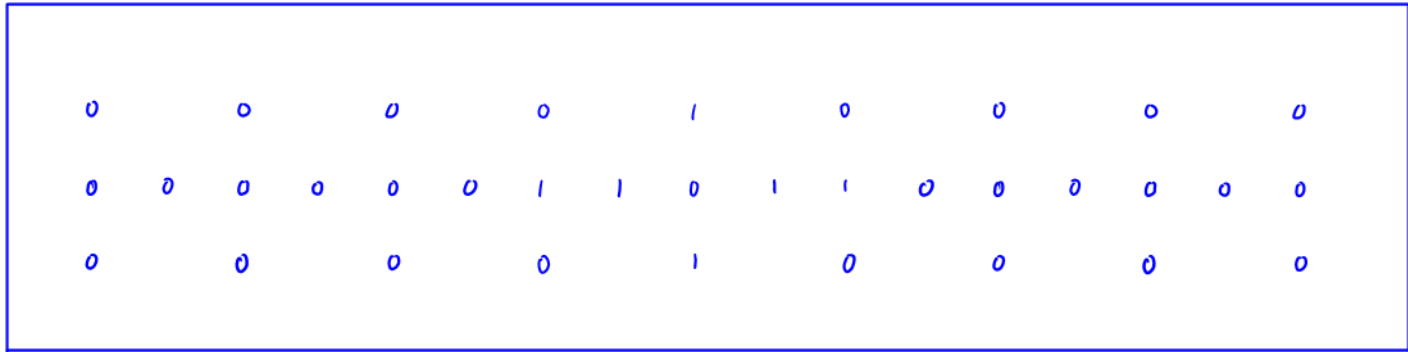
It's not easy to read the informations of them, but AR-quivers can.



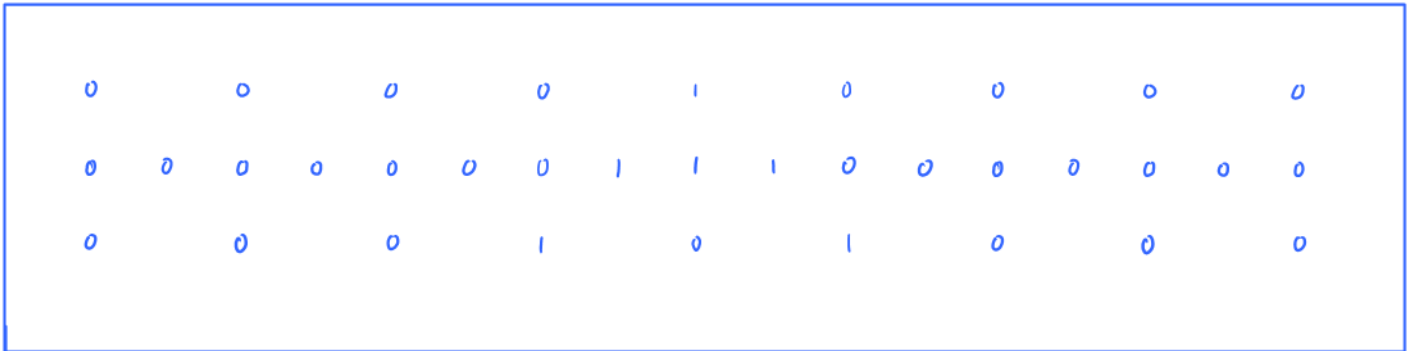
$P(1)$



$P(2)$



$P(3)$



$P(4)$