

Eine Woche, ein Beispiel

6.27 adèles and idèles

I would recommend this paper: <https://people.math.umass.edu/~weston/oldpapers/idele.pdf>.

After reading it, you may learn:

- The definition of two topology space adèles and idèles
- Basic properties of them (subspaces, canonical map...)
- Use this to prove the finiteness of the ideal class group and the generalized Dirichlet unit theorem

discrete subtopology
compact quotient

Slogan: A good ambient space can make researched objects into "full lattice"!

objects

$$\begin{aligned} \mathcal{O}_K \\ \mathcal{O}_K^\times / U_K \\ K \\ K^\times \end{aligned}$$

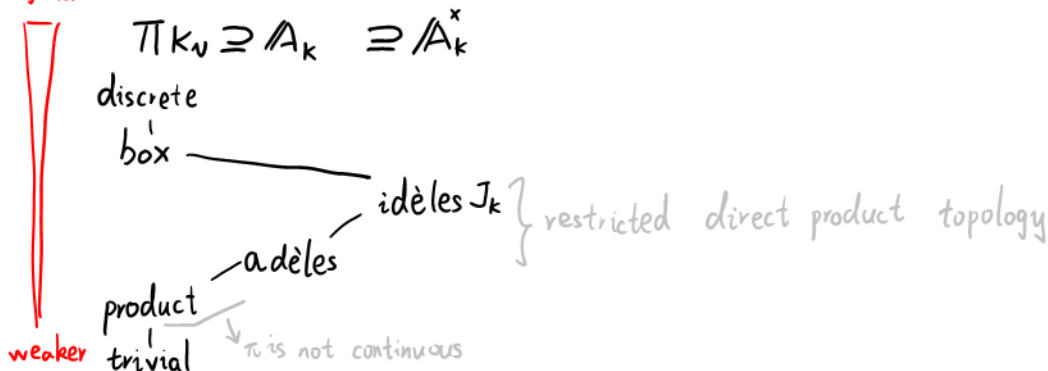
ambient space

$$\begin{aligned} \mathbb{R}^{r_1} \times \mathbb{C}^{r_2} \\ H \subseteq \mathbb{R}^{r_1+r_2} \\ \mathbb{A}_K \\ J_K^\circ \subseteq J_K \end{aligned}$$

K : number field

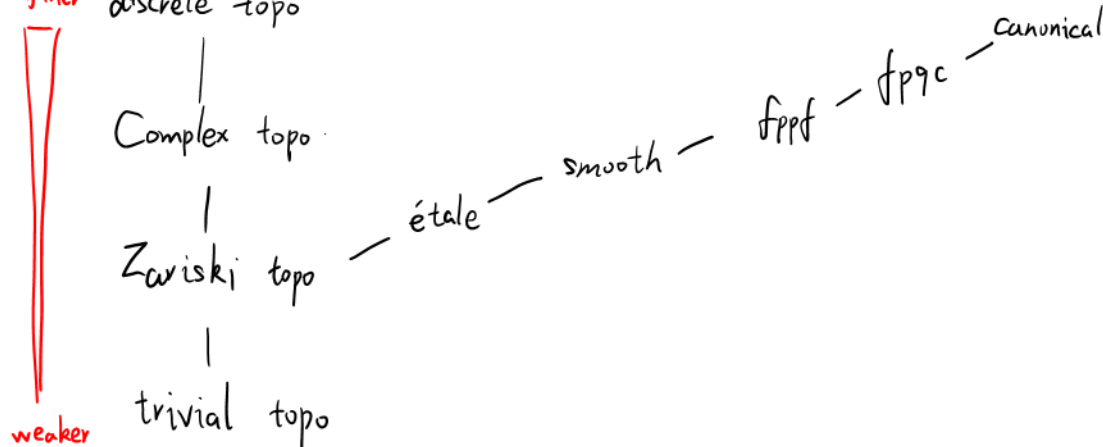
\mathbb{A}_K, J_K° are not linear space

stronger
finer



weaker

stronger
finer



weaker

From [<https://math.stackexchange.com/questions/2869928/definition-of-the-weil-group-question-about-exact-sequence-with-inertia-group-a>]
: A caveat is that the topology on the Weil group is not the subspace topology, but finer than the subspace topology. We require that (the image of) the inertia group be open.