Eine Woche, ein Beispiel

9.3. field extension with RS

Goal: construct an equivalence between two categories.

$$\mathrm{RS}^{cc} \overset{\text{cpt conn}}{=} \left\{ \begin{array}{l} \text{Obj: cpt conn RS} \\ \text{Mor: non-const holo morphisms} \end{array} \right\} \longleftrightarrow \left\{ \begin{array}{l} \text{Obj: } F/\mathbb{C} \text{ field ext s.t.} \\ \qquad \mathrm{trdeg}_{\mathbb{C}} F = 1 \\ \qquad F/\mathbb{C} \text{ f.g as a field} \\ \text{Mor: morphism as fields}/\mathbb{C} \end{array} \right\}^{op} = \mathrm{field}^{op}_{\mathbb{C}(t)/\mathbb{C}}$$

$$\begin{array}{ccc} Y & & \mathcal{M}(Y) \\ \big\downarrow f & \Longrightarrow & \big\uparrow f^* \\ X & & \mathcal{M}(X) \end{array}$$

which obeys the following slogan:

$$\boxed{\text{(ramified) covering} \quad \approx \quad \text{(function) field extension}}$$

Rmk. 1. For requiring $F/\mathbb{C}$ f.g. as a field, we avoid examples like $\overline{\mathbb{C}(t)}$.
Do they corresponds to some non-cpt Riemann surface?
If so, how to enlarge the category RS^{cc}?

2. $\mathrm{field}_{\mathbb{C}(t)/\mathbb{C}}$ means fields over $\mathbb{C}$ which are fin ext of $\mathbb{C}(t)$ abstractly;
morphisms don't need to fix $\mathbb{C}(t)$.
Do you have a better name for RS^{cc} and $\mathrm{field}_{\mathbb{C}(t)/\mathbb{C}}$?

https://math.stackexchange.com/questions/633628/threefold-category-equivalence-algebraic-curves-riemann-surfaces-and-fields-of
https://math.stackexchange.com/questions/1286286/link-between-riemann-surfaces-and-galois-theory

1. field of meromorphic functions
2. Galois covering
3. valuations
4. quadratic extension of $\mathbb{C}(x)$: hyperelliptic curve
5. miscellaneous.