Eine Woche, ein Beispiel

7.17 formalism

Here I collect some formalisms which enhance well-known theories.
Somehow it also explains the polularity of the abstraction in mathematics. We want to have a better understand, so we invent new languages and theories, which in turn causes more troubles for beginners. Hug those formalisms!

母题: classification, symmetry, glue and ramified covering

Axiomatic set Theory ⇝ Grothendieck Universe
Both let us be away from paradoxes. The latter is more convenient.

Category Theory ⇝ Infinite Category Theory
[adjoint fctor ⇔ preserve (co)limits] is some cases

(·/Braided/Symmetric) Monoidal Categories ⇝ Categorification
Extract combinatorical informations from categories ⇕

Examples ⇝ Classification
Specify and generalization. ⇑

See https://github.com/ramified/personal_tex_collection/blob/main/%E8%9B%99%E9%B8%A39-7/latest%20version.pdf

Parameter space ⇝ Moduli space
Structures on parameter spaces are important.

See https://github.com/ramified/moduli_in_algebraic_geometry

Topology ⇝ Grothendieck Topology
Get étale topology.

Topological space ⇝ Condensed Set
Get abelian category.

Scheme ⇝ Functor
Get more objects. e.g. Ind-Sch, stack, ...

Snake lemma ⇝ homological algebra, spectral sequence
Diagram chasing all the time!

(co)homdogy ⇝ derived category, six-fctor formalism.
Reduce important properties to categorical non-sense.

https://www.math.uni-bonn.de/people/schwede/EnhancedSeminar-WS2223.pdf

Finite field ⇝ ???
Should contain "field with one element".

Moreover, AR theory tells us the structure of indecomposable reps,
Bruhat-Tits theory tells us the structure of p-adic groups,
Artin-Schreier theory tells us the structure of deg p extensions. $\mathrm{char}\, F = p$

invariant, structure $\rightsquigarrow$ forgetful functors

e.g. $\mathrm{Perv} \begin{cases} \nearrow \text{characteristic cycle} \\ \rightsquigarrow \text{Hodge structure} \end{cases} \Rightarrow \mathrm{Rep}(G) \begin{cases} \nearrow \mathrm{Rep}(T_u \rtimes \mathrm{Gal}(\gamma)) \\ \searrow \mathrm{Rep}(\mathbb{G}_m^2) \end{cases}$

subcat of