

Eine Woche, ein Beispiel

3.26. double coset decomposition

Double coset decompositions are quite impressive!

This document follows and repeats 2022.09.04_Hecke_algebra_for_matrix_groups. Some new ideas come, so I have to write a new.

Ref:

[JL18]: John M. Lee, Introduction to Riemannian Manifolds

[Gorodski]: Claudio Gorodski, An Introduction to Riemannian Symmetric Spaces

<https://www.ime.usp.br/~gorodski/ps/symmetric-spaces.pdf>

[KWL10]: Kai-Wen Lan: An example-based introduction to Shimura varieties

<https://www-users.cse.umn.edu/~kwlan/articles/intro-sh-ex.pdf>

<https://www.mathi.uni-heidelberg.de/~pozzetti/References/Iozzi.pdf>

<https://www.mathi.uni-heidelberg.de/~lee/seminarSS16.html>

1. G-space
2. double coset decomposition: schedule
3. examples (draw Table)
4. special case: v.b on \mathbb{P}^1 .

In this document, stratification = disjoint union of sets

1. G-space

Recall: Group action $G \curvearrowright X$

discrete \Rightarrow fundamental domain
non discrete \Rightarrow stratification by G/G_x

$$\Lambda \in \mathbb{C} \\ S' \in S^2$$

$$SL_2(\mathbb{Z}) \in \mathcal{H} \\ \mathbb{C}^* \in \mathbb{CP}^1$$

Rmk. Many familiar spaces are homogeneous spaces.

E.g. $\text{Flag}(V) \cong GL(V)/P$ e.g. Grassmannian, \mathbb{P}^n

$$S^n \cong O(n+1)/O(n) \cong SO(n+1)/SO(n)$$

$$O(n) := O(n, \mathbb{R})$$

$$SO(n) := SO(n, \mathbb{R})$$

\leadsto Stiefel mflld [21.11.14]

$$\mathbb{A}^n = \mathbb{A}^n$$

$$\mathcal{H}^n \cong O^*(1, n)/O(n)$$

$$\mathcal{H} \cong GL_2(\mathbb{R})/O(2) \cdot \mathbb{R}_{>0} \cong SL_2(\mathbb{R})/SO_2(\mathbb{R})$$

\leadsto Hermitian symmetric space

where $\mathcal{H}^n := \{v = (v_i)_{i=1}^{n+1} \in \mathbb{R}^{n+1} \mid \langle v, v \rangle = -1, v_{n+1} > 0\}$
 $\langle \cdot, \cdot \rangle: \mathbb{R}^{n+1} \times \mathbb{R}^{n+1} \rightarrow \mathbb{R}$
 $\langle v, w \rangle = v^T \begin{pmatrix} 1 & & \\ & \ddots & \\ & & -1 \end{pmatrix} w$

$$O(n, 1) := \text{Aut}(\mathbb{R}^{n+1}, \langle \cdot, \cdot \rangle) \subseteq GL_{n+1}(\mathbb{R})$$

$$O^*(n, 1) := \{g \in O(n, 1) \mid g\mathcal{H}^n \subset \mathcal{H}^n\}$$

For more informations about \mathcal{H}^n , see [JL18, P62-67].

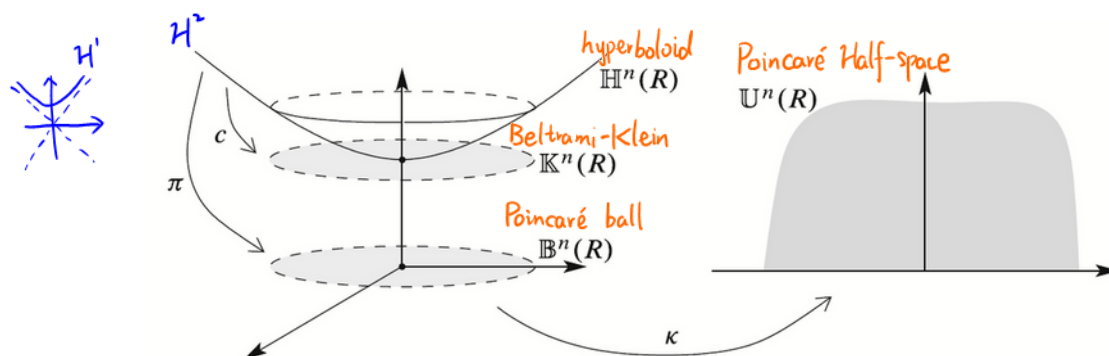
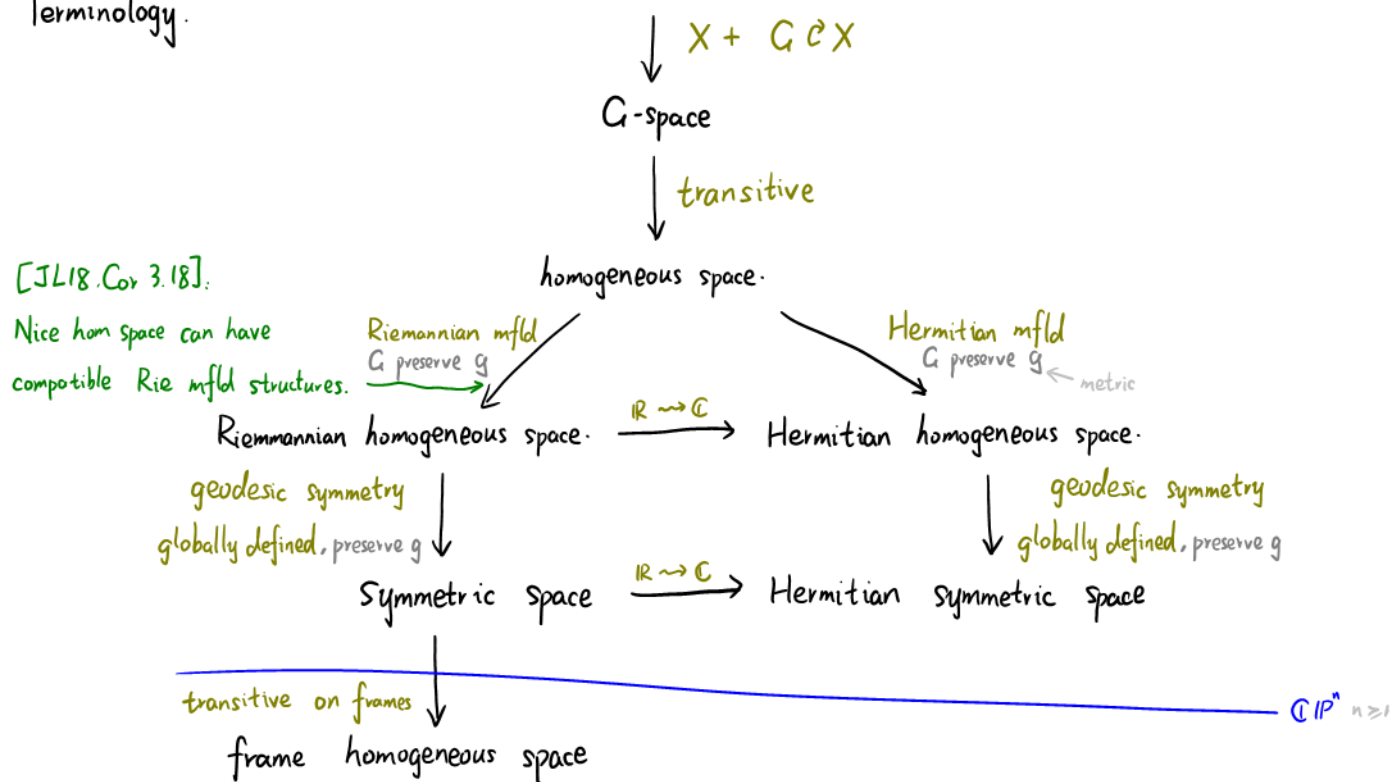


Fig. 3.3: Isometries among the hyperbolic models [JL18, P63]

Terminology.



Rmk. Sym spaces & Hermitian sym spaces are fully classified.
See [Gorodski, Thm 2.38] and [KWL10, §3] for the result.

Q: Can we define and classify sym spaces in p-adic world?