ein Woche, eine Beispiel April 16th. examples in algebraic topology

April 16th. examp.

Examples:
Past
closed surface din 2
Hopf surface din 4
K3 surface

CP" CP"

Moore space
Eilenberg - Maclane space
...

- · compute $H_n(X, \mathbb{Z})$, $H^*(X, \mathbb{Z})$, $\pi_n(X, \mathbb{Z})$
- · compute characteristic class and applies the results.
- optional question is X * oriented? * a mfld? of dim n * a cplx mfld? * a Lie group? complex

Today:
$$S^{\infty}$$
; IRP^{n} , $IRIP^{\infty}$; CIP^{n} , CIP^{∞} ; ...

 $S^{\infty} = US^{n}$ $S^{n} \rightarrow S^{m}$ by $(x_{0}, ..., x_{n}) \mapsto (x_{0}, ..., x_{n}, 0, ..., 0)$

1. relations: fiber bundle

 $Z/_{12Z} \rightarrow S^{n}$ $S' \rightarrow S^{2n+1}$ $Z/_{kZ} \rightarrow S^{2n+1}$
 $IRIP^{n}$ CIP^{n} $S^{2n+1}/_{Z/_{kZ}}$ $k \in \mathbb{N}^{+}$, $k > 1$
 $Z/_{12Z} \rightarrow S^{\infty}$ $S' \rightarrow S^{\infty}$ $Z/_{kZ} \rightarrow S^{\infty}$
 $IRIP^{\infty}$ CIP^{m} $S^{\infty}/_{Z/_{kZ}}$

2. (canonical) CW structure.

e.q.								
J. J.	#m-cell	0	1	2	3	4	5	m >5
	2 _r	2	2	2	2	2	2	0
	IRIPS	1	1	1	1	1	1	ο
	CIP'	1	o	1	ა	1	υ	o

$$\Rightarrow \begin{cases} \chi(S^n) = \begin{cases} 0 & n \text{ odd} \\ 1 & n \text{ even} \\ 1 & n \text{ even} \end{cases}$$

$$\chi(|R|p^n) = \begin{cases} 0 & n \text{ odd} \\ 1 & n \text{ even} \end{cases}$$

3. Homology & Cohomology homo<u>log</u>

no <u>l</u>	ogy							
	H: (X,Z)	0	1	2	3	4	5	i >5
	$\mathcal{Z}_{\mathfrak{r}}$	Z	0	0	0	ა	Z	0
	IRIP"	Z	2/22/	O	2/27/	0	Z	0
	CIP'	Z	0	Z	0	Z	0	0
	IRIP4	Z	Z/ _{2]/}	0	7427	o	o	0

Cor. IRIP" is nonoriented; IRIP", 5", CIP" are oriented.

5' 0→Ze' + Ze' +

Rock. The definition of cellular homology uses the homology of
$$S$$
, so seriously]

we can't compute $H_i(S^n, Z)$ by cellular homology.

$$|R||^{5} \quad 0 \quad \longrightarrow \mathbb{Z}e^{5} \longrightarrow \mathbb{Z}e^{4} \longrightarrow \mathbb{Z}e^{3} \longrightarrow \mathbb{Z}e^{2} \longrightarrow$$

Similarly, Hn (500, Z) = fZ n=0 otherwise

$$H_n(IRIP^{\bullet \circ}, \mathbb{Z}) = \begin{cases} \mathbb{Z} & n=0 \\ \mathbb{Z}/_{2\mathbb{Z}} & n \text{ odd} \\ 0 & \text{otherwise} \end{cases}$$

Hn(IRIP^{\infty}, \mathbb{Z}/_{2\mathbb{Z}}) = \mathbb{Z}/_{2\mathbb{Z}}

$$H_n(\mathbb{CP}^{\infty}, \mathbb{Z}) = \sum_{i=1}^{\infty} n_i \text{ even}$$

co homology

···							
H ¹ (X,Z)	0	1	2	3	4	5	i >5
2 _t	7/	٥	0	0	ು	Z	o
IRIP*	Z	O	74274	o	72/274	Z	0
CIP '	Z	0	Z	0	Z	٥	0
IR IP4	7	0	7/27/	0	742	o	0

$$\Rightarrow \begin{cases} H^*(|R|P^{2n}) = \mathbb{Z}[x]/_{(2\times,x^{n+1})} \\ H^*(|R|P^{2n+1}) = \mathbb{Z}[x]/_{(2\times,x^{n+1})} \oplus \mathbb{Z}y \\ H^*(\mathbb{C}|P^n) = \mathbb{Z}[x]/_{(x^{n+1})} \end{cases}$$

prod structure. Use Poincaré duality & cellular cohomology, see [May, P153]. Hy(CPn) ~ Hy(CPn-1) for 9 < n

> https://math.stackexchange.com/questions/1128712 /integral-cohomology-ring-of-real-projective-space

By spectral sequence: GTM 82 Example 14.22, 14.32, Ex 18.4, 18.10

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Interlude: LES of homotopy groups x & EBCACX, X top space
   Def relative homotopy group
                            \pi_{h}(X,A,x_{0}) = \left[f: (I^{\gamma},\partial I^{\gamma},J^{\gamma-1}) \longrightarrow (X,A,x_{0})\right] \qquad n\geqslant 1 \quad J^{\gamma-1} = \partial I^{\gamma} - I^{\gamma-1}
          Relations. f \sim g \iff \exists F_t \cdot (I^n, \partial I^n, J^{n-1}) \rightarrow (x, A, x_0) s.t
           (denote by [f] = [g]) Fo = f Fi = g
   Lemma: Suppose f: (I^n, \partial I^n, J^{h-1}) \rightarrow (X, A, x_0),
                f(I^n) \subseteq A, then [f] = [o]
    Thm. we have LES <= ker = Im

\Im \pi_2(A,B,x_0) \longrightarrow \pi_2(X,B,x_0) \longrightarrow \pi_2(X,A,x_0)

          \hookrightarrow \pi_{\iota}(A,B,x_{\bullet}) \longrightarrow \pi_{\iota}(\chi,B,x_{\bullet}) \rightarrow \pi_{\iota}(\chi,A,x_{\bullet})
    Cor we have LES
                             \pi_2(A,x_0) \longrightarrow \pi_2(X,x_0) \longrightarrow \pi_2(X,A,x_0)

\widehat{S\pi}_{i}(A, x_{o}) \longrightarrow \pi_{i}(X, x_{o}) \longrightarrow \pi_{i}(X, A, x_{o})

                             S_{\pi_o(A, X_o)} \longrightarrow \pi_o(X, X_o)
                                                                                       scalled Serve fibration
    Thm. when p: E \rightarrow B has the homotopy lifting property w.r.t. I^k (\forall k \ge 0)
                 then (denote bo & B. x & F . = p - (B))
                          p_*: \pi_n(E, F, x_o) \longrightarrow \pi_n(B, b_o) n \ge 1 I^k \times [o, 1] \longrightarrow E \ni x_o f isomorphism.
                  is an isomorphism.
   Rmk. 1 The proof mainly uses the HLP: homotopy lifting property.
               2. Any fiber bundle p. Ē→B is a Serre fibration.
  Cor Suppose p: E \rightarrow B is the fiber bundle map, then we have LES
           (denote boeB. xoeF .= p (B))
                          \pi_{2}(F, x_{0}) \longrightarrow \pi_{2}(E, x_{0}) \longrightarrow \pi_{2}(B, b_{0})_{>}

\widehat{\Rightarrow} \pi_{\iota}(F, \times_{\circ}) \longrightarrow \pi_{\iota}(E, \times_{\circ}) \longrightarrow \pi_{\iota}(B, b_{\circ})

                      \hookrightarrow \pi_{\circ}(F, x_{\circ}) \longrightarrow \pi_{\circ}(E, x_{\circ})
4. Humotopy: by LES of fibration, we obtain
                                                                           \pi_{m}(\mathbb{C}|\mathbb{P}^{n}) = \begin{cases} 0 & m=1 \\ \mathbb{Z} & m=2 \\ \pi_{n}(\mathbb{C}^{2n+1}) & m > 2 \end{cases}
                \pi_m(|R|p^n) = \begin{cases} 2\ell/2\chi & m=1\\ \pi_m(S^n) & m>1 \end{cases}
      Rmk. 500 is contractable by the argument in https://mathoverflow.net/questions/198
      Cor IRIP is of type K(2/12, 1)
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CIP is of type K(Z, 2)

$\begin{array}{cccccccccccccccccccccccccccccccccccc$						in	GTM	82	Who	at I	Can	prove now
\$\frac{1}{2^2} \cdot 0 0		π ₁	π2	π ₃	π ₄		H					
5. Characteristic class. We have both toutological vector bundle and tangent bundle for S^n, IRIP^n, CIP^n. Elph' by bittps://en.wikipedia.org/wiki/Chem.class , $c(\mathbb{CP}^n)^{\text{def}}(T\mathbb{CP}^n) = c(\mathcal{O}_{\mathbb{CP}^n}(1))^{n+1} = (1+a)^{n+1}$. where a is the canonical generator of the cohomology group $H^2(\mathbb{CP}^n, \mathbb{Z})$: $tautological$ bundle $\mathcal{O}_{\mathbb{CP}^n}(-1)$ are not spin; \mathbb{CIP}^n is not a boundary. IRIP^n. Similarly, $W(Y_n) = 1+t$ $W(IRIP^n) = W(Y_n)^{n+1} = (1+t)^{n+1}$. Cor. Yn' is not orientable only when $n = 1 \mod 2$: $TIRIP^n$ is orientable only when $n = 3 \mod 4$ or $n = 1$. S^n : Lemma π^n : $H^n(IRIP^n, \mathbb{Z}/2\mathbb{Z}) \longrightarrow H^n(S^n, \mathbb{Z}/2\mathbb{Z})$ is zero. Proof by computation. $\mathbb{C}(S^n, \mathbb{Z}/2\mathbb{Z})$ $0 \longrightarrow e^{\frac{t}{2}} = e^{\frac{t}{2}} \longrightarrow e^{\frac{t}{2}} = e^{\frac{t}{2}} \longrightarrow e^{\frac{t}{2}} = e^{\frac{t}{2}} \longrightarrow e^{\frac{t}{2}} = e$	S ⁰	0	0	0	0	0	0	0	0	0	0	
5. Characteristic class We have both teurological vector bundle and tangent bundle for S^n , $IRIP^n$. Where a is the canonical generator of the cohomology group $H^2(IRIP^n, IRIP^n) = I - a$. Cor. $IRIP^n$, IR	S ¹	Z	0	0	0	0	0	0	0	0	0	
5. Characteristic class We have both teurological vector bundle and tangent bundle for S^n , $IRIP^n$. Where a is the canonical generator of the cohomology group $H^2(IRIP^n, IRIP^n) = I - a$. Cor. $IRIP^n$, IR	S ²	0	Z	Z	\mathbb{Z}_2	\mathbb{Z}_2	Z ₁₂	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}_3	Z ₁₅	and the first to
5. Characteristic closs We have both toutological vector bundle and tangent bundle for S^n , $[R]P^n$, $C[P^n]$. by https://en.wikipedia.org/wiki/Chem.class, $(CP^n)^{\frac{1}{2}}=(TCP^n)^{\frac{1}{2}}=(CCP^n)^{\frac{1}{2}}=(1+a)^{n+1}$. where a is the canonical generator of the cohomology group $H^2(CP^n, \mathbb{Z})$: toutological bundle $O_{QP^n}(-1): c(O_{Q} P^n(-1))=1-a$ $Cor. TCP^n$, $O_{QP^n}(-1)$ are not spin; $C[P^n]$ is not a boundary. IRIP^n: Similarly, $W(Y_n)^2=1+t$ $W(IRIP^n)=W(Y_n')^{n+1}=(1+t)^{n+1}$ $Cor. Y_n'$ is not orientable; $TIRP^n$ is orientable only when $n \equiv 1 \mod 2$: $TIRP^n$ is orientable only when $n \equiv 3 \mod 4$ or $n = 1$. $Cor. Y_n' = (1+t)^{n+1}$ $Cor. Y_n' = (1+t)^$	S ³	0	0	Z	\mathbb{Z}_2	\mathbb{Z}_2	Z ₁₂	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}_3	Z ₁₅	by Hopf fibration
5. Characteristic closs We have both toutological vector bundle and tangent bundle for S^n , $[R]P^n$, $C[P^n]$. We have both toutological vector bundle and tangent bundle for S^n , $[R]P^n$, $C[P^n]$. Other by https://en.wikipedia.org/wiki/Chem.class , $c(\mathbb{CP}^n)^{def} c(T\mathbb{CP}^n) = c(\mathbb{C}_{\mathbb{CP}^n}(1))^{n+1} = (1+a)^{n+1}$. Where a is the canonical generator of the cohomology group $H^2(\mathbb{CP}^n, \mathbb{Z})$; tautological bundle $\mathcal{O}_{Q}P^n(-1)$: $c(\mathcal{O}_{\mathbb{C}}P^n(-1)) = 1-a$. Cor. $TC[P^n]$, $(\mathbb{C}P^n)^{en}(-1)$ are not spin; $C[P^n]$ is not a boundary. IRIP^h. Similarly, $w(Y_n) = 1+t$ $w(IR[P^n] = w(Y_n)^{n+1} = (1+t)^{n+1}$. Cor. Y_n is not orientable; $T[RP^n]$ is spin only when $n \equiv 1 \mod 2$; $T[RP^n]$ is spin only when $n \equiv 3 \mod 4$ or $n = 1$. Solventia. C. $(RP^n, Y_n)^{en}(RP^n, Y_$	S ⁴	0	0	0	Z				\mathbb{Z}_2^2	\mathbb{Z}_2^2	$\mathbb{Z}_{24} \times \mathbb{Z}_3$	
5. Characteristic class. We have both toutological vector bundle and tangent bundle for S^n , $[R]P^n$, $C[P^n]$. We have both toutological vector bundle and tangent bundle for S^n , $[R]P^n$, $C[P^n]$. CIP ⁿ : by https://en.wikipedia.org/wiki/Chern_class , $c(CP^n)^{\frac{1}{2}}e^{-c}(TCP^n) = c(C_{CP^n}(1))^{n+1} = (1+a)^{n+1}$. Where a is the canonical generator of the cohomology group $H^2(CP^n, \mathbb{Z})$; toutological bundle $O_{QP^n}(-1)$: $c(O_{Q P^n}(-1)) = 1-a$ Cor. $TC[P^n]$, $O_{QP^n}(-1)$ are not spin; $C[P^n]$ is not a boundary. IRIP ⁿ : Similarly, $W(Y_n) = 1+t$ $W(IR[P^n] = W(Y_n)^{n+1} = (1+t)^{n+1}$ Cor. Y_n is not orientable; $T[RP^n]$ is spin only when $n \equiv 1 \mod 2$; $T[RP^n]$ is spin only when $n \equiv 3 \mod 4$ or $n = 1$. Solventially $Y_n = Y_n =$	S ⁵	0	0	0	0	Z			Z ₂₄			
We have both toutological vector bundle and tangent bundle for S^n , $ R P^n$, $C P^n$. Of the horizontal parameter of the cohomology group $H^2(\mathbb{CP}^n, \mathbb{Z})$; where a is the canonical generator of the cohomology group $H^2(\mathbb{CP}^n, \mathbb{Z})$; tautological bundle $U_{CP^n}(-1): c(\mathcal{O}_{CP^n}(-1)) = 1-a$ Cor. TCP^n , $(\mathcal{O}_{CP^n}(-1))$ are not spin; (CP^n) is not a boundary. IRIP^n: Similarly, $(Y_n) = 1+t$ $(IR P^n) = W(Y_n)^{n+1} = (1+t)^{n+1}$ Cor. Y_n is not orientable; $T R P^n$ is orientable only when $n \equiv 1 \mod 2$; $T R P^n$ is orientable only when $n \equiv 3 \mod 4$ or $n = 1$. Solvential $Y_n = Y_n = Y$	ţ	$\pi_{\sigma}(S^7)$										
$ \begin{array}{c} \text{OIP}^n: \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \$	0.							مامام	aà co	ا ا	ctor bu	andle and torsent bundle for Sn IDIP (IP)
$c(\mathbb{CP}^n) \stackrel{\mathrm{def}}{=} c(T\mathbb{CP}^n) = c(O_{\mathbb{CP}^n}(1))^{n+1} = (1+a)^{n+1}.$ where a is the canonical generator of the cohomology group $H^2(\mathbb{CP}^n, \mathbb{Z})$; $tauto \log_{\mathbb{C}}(a) bundle \mathcal{O}_{GP^n}(-1) : c(\mathcal{O}_{GP^n}(-1)) = 1-a$ $\text{Cor. } TCP^n, (\mathcal{O}_{\mathbb{C}P^n}(-1)) are not spin; CIP^n is not a boundary.$ $IRIP^n: similarly, w(\gamma_n') = 1+t w(IRIP^n) = w(\gamma_n')^{n+1} = (1+t)^{n+1}.$ $\text{Cor. } \gamma_n' \text{is not orientable}; TIRIP^n: is \text{orientable}; TIRIP^n: is \text{orientable}; not n = 1 mod z: TIRIP^n: is spin only \text{when} n = 3 \text{mod} \text{for } n = 1.$ $\text{S}^n: Lemma. } \pi^*: H^n(IRIP^n, \mathbb{Z}/2\mathbb{Z}) \longrightarrow H^n(\mathbb{S}^n, \mathbb{Z}/2\mathbb{Z}) \text{is zero.}$ $\text{Proof} \text{by computation.} \text{C.}(IRIP^3, \mathbb{Z}/2\mathbb{Z}) \text{o} \longrightarrow e^{\frac{t}{2}} \longrightarrow e^{t$	(L)											note and largest buriate for 3, 1211, 211.
where a is the canonical generator of the cohomology group $H^2(\mathbb{CP}^n, \mathbb{Z})$: $tautological$ bundle $U_{C4^n}(-1): c(U_{C P^n}(-1))=1-a$ $Cor. TCP^n, (O_{C P^n}(-1))$ are not spin; CP^n is not a boundary. IRIP^n. Similarly, $W(Y_n')=1+t$ $W(IRIP^n)=W(Y_n')^{n+1}=(1+t)^{n+1}$ $Cor. Y_n'$ is not orientable; $TIRIP^n$ is orientable only when $n \equiv 1 \mod 2$; $TIRIP^n$ is spin only when $n \equiv 3 \mod 4$ or $n = 1$. $S^n: Lemma. \pi^*. H^n(IRIP^n, \mathbb{Z}_{2\mathbb{Z}}) \to H^n(S^n, \mathbb{Z}_{2\mathbb{Z}})$ is zero. $Proof. by computation.$ $C.(IRIP^3, \mathbb{Z}_{1\mathbb{Z}})$ o $Oolongooder = 0$	UII	; 0	J								/1	_\n+1
tautological bundle $\mathcal{O}_{GP}^{n}(-1)$: $c(\mathcal{O}_{GP}^{n}(-1)) = 1-a$ Cor. TCP^{n} , $\mathcal{O}_{CP}^{n}(-1)$ are not spin; CP^{n} is not a boundary. IRIP ⁿ . Similarly, $w(Y_{n}^{i}) = 1+t$ $w(IRIP^{n}) = w(Y_{n}^{i})^{n+1} = (1+t)^{n+1}$ Cor. Y_{n}^{i} is not orientable; $TIRIP^{n}$ is orientable only when $n = 1 \mod 2$; $TIRIP^{n}$ is spin only when $n = 3 \mod 4$ or $n = 1$. S ⁿ . Lemma. π^{*} : $H^{n}(IRIP^{n}, \mathbb{Z}_{2}) \to H^{n}(S^{n}, \mathbb{Z}_{2})$ is zero. Proof. by computation. $C.(IRIP^{3}, \mathbb{Z}_{2})$ o $\longrightarrow e^{\frac{t}{2}} \longrightarrow e^{\frac{t}{2}} \to e^{\frac{t}{2}}$												
Cor. TCP^n , $Och^n(-1)$ are not spin; CP^n is not a boundary. IRP^n: similarly, $w(x_n') = 1+t$ $w(IRP^n) = w(x_n')^{n+1} = (1+t)^{n+1}$ Cor. Y_n' is not orientable; $TIRP^n$ is orientable only when $n \equiv 1 \mod 2$; $TIRP^n$ is spin only when $n \equiv 3 \mod 4$ or $n = 1$. S ⁿ : Lemma. π^n : $H^n(IRP^n, \mathbb{Z}/2\mathbb{Z}) \longrightarrow H^n(S^n, \mathbb{Z}/2\mathbb{Z})$ is zero. Proof by computation. C.(IRP^s, $\mathbb{Z}/2\mathbb{Z})$ o $\longrightarrow e^s$, $e^s \longrightarrow e^a \longrightarrow e^$										_		
IRIP ^h . Similarly, $w(x_n') = 1+t$ $w(IRIP^n) = w(x_n')^{n+1} = (1+t)^{n+1}$ Cor x_n' is not orientable; TIRIP ^h is orientable only when $n = 1 \mod 2$; TIRIP ⁿ is spin only when $n = 3 \mod 4$ or $n = 1$. S ⁿ : Lemma π^* : $H^*(IRIP^n, \frac{7}{2}/2Z) \longrightarrow H^*(S^n, \frac{7}{2}/2Z)$ is zero. Proof by computation. C.(IRIP ^s , $\frac{7}{2}/2Z$) $o \longrightarrow e^{\frac{1}{2}} \longrightarrow e^{\frac{1}{2}$			_		•					- 1		
Cor $\forall n'$ is not orientable; TIRIP* is orientable only when $n \equiv 1 \mod 2$: TIRIP* is spin only when $n \equiv 3 \mod 4$ or $n = 1$. S** Lemma π^* : H*(IIRIP*, $\mathbb{Z}/2\mathbb{Z}$) \longrightarrow H*(S**, $\mathbb{Z}/2\mathbb{Z}$) is zero. Proof by computation. C.(IRIP\$, $\mathbb{Z}/2\mathbb{Z}$) $0 \longrightarrow e^{\frac{1}{2}} \longrightarrow e^{\frac{1}{2}$												
TIRIP ⁿ is orientable only when $n = 1 \mod 2$; TIRIP ⁿ is spin only when $n = 3 \mod 4$ or $n = 1$. S ⁿ . Lemma. π^* : H ⁿ (IIR IP ⁿ , $\mathbb{Z}/2\mathbb{Z}$) \longrightarrow H ⁿ (S ⁿ , $\mathbb{Z}/2\mathbb{Z}$) is zero. Proof by computation. C.(IRIP ^s , $\mathbb{Z}/2\mathbb{Z}$) \longrightarrow $e^{\frac{1}{2}}$ \longrightarrow $e^{$	IRI	P ":	S ì	mila	r ly	,	w (۲¦) =	1+	t	W	$(R p^n) = W(\chi_1^n)^{n+1} = (1+t)^{n+1}$
TIRIP is spin only when $n \equiv 3 \mod 4$ or $n = 1$. Solvent Lemma π^* : $H^{\circ}(IRIP^{\circ}, \mathbb{Z}/2Z) \longrightarrow H^{\circ}(S^{\circ}, \mathbb{Z}/2Z)$ is zero. Proof by computation. C.(IRIPs, $\mathbb{Z}/2Z$) $O \longrightarrow e^{\frac{1}{5}} \longrightarrow e^{$			Con	, .	8n'	is	not	ori	ental	de :	,	
Lemma π^* : $H^*(IRIP^*, \mathbb{Z}/2Z) \longrightarrow H^*(S^*, \mathbb{Z}/2Z)$ is zero. Proof by computation. C.(IRIP^*, $\mathbb{Z}/2Z$) $0 \longrightarrow e^{\frac{1}{5}} \longrightarrow e^{\frac{1}{5}}$				-	TIRIP	,^ is	. 0	rient	able		only	when $n = 1 \mod 2$;
Lemma π^* : $H^*(IRIP^*, \mathbb{Z}/2Z) \longrightarrow H^*(S^*, \mathbb{Z}/2Z)$ is zero. Proof by computation. C.(IRIP^*, $\mathbb{Z}/2Z$) $0 \longrightarrow e^{\frac{1}{5}} \longrightarrow e^{\frac{1}{5}}$				-	TIRIF	'n i	s 5	pin			only	when $n \equiv 3 \mod 4$ or $n = 1$.
Proof by computation. C.(IRIP's, \mathbb{Z}_{12}) $0 \longrightarrow e^{\frac{1}{5}} \longrightarrow e^{e^{+}} \longrightarrow e^{3} \longrightarrow e^{1} \longrightarrow e^{1} \longrightarrow e^{0} \longrightarrow 0$ $C.(S^{3}, \mathbb{Z}_{12}) 0 \longrightarrow e^{3}, e^{3}_{1} \longrightarrow e^{1}_{1}, e^{3}_{1} \longrightarrow e^{3}_{1}, e^{3}_{2} \longrightarrow e^{3}_{1}, e^{3}_{1} \longrightarrow e^{3}_{1}, e^{3}_{2} \longrightarrow e^{3}_{2} \longrightarrow e^{3}_{1}, e^{3}_{2} \longrightarrow e^{3}_{1}$	5	4	Le	mm	a	π*	. <i>F</i>	1^(1R	P".	7427	() -	$\rightarrow H^{1}(S^{1}, \mathbb{Z}/_{2})$ is zero.
$C.(R P^{5}, \mathbb{Z}/_{2}\mathbb{Z}) \qquad 0 \longrightarrow e^{\frac{c}{5}} \longrightarrow e^{4} \longrightarrow e^{3} \longrightarrow e^{2} \longrightarrow e^{1} \longrightarrow e^{0} \longrightarrow 0$ $C.(S^{5}, \mathbb{Z}/_{2}\mathbb{Z}) \qquad 0 \longrightarrow e^{5}, e^{5}_{2} \longrightarrow e^{4}, e^{4}_{2} \longrightarrow e^{3}, e^{3}_{2} \longrightarrow e^{7}_{1}, e^{7}_{2} \longrightarrow e^{7}_{1} \longrightarrow e^{7}_{1}, e^{7}_{2} \longrightarrow e^{7}_{1}, e^{7}_{2} \longrightarrow e^{7}_{2} \longrightarrow e^{7}_{1}, e^{7}_{2} \longrightarrow e^{7}_{1}, e^{7}_{2} \longrightarrow e^{7}_{1} \longrightarrow e^{7}_{1}, e^{7}_{2} \longrightarrow e^{7}_{1}, e^{7}_{2} \longrightarrow e^{7}_{1}, e^{7}_{2} \longrightarrow e^{7}_{1}, e^{7}_{2} \longrightarrow e^{7}_{1} \longrightarrow e^{7}_$		Ì		_						,	•	
$C.(S^{s}, \mathbb{Z}/_{2}) \qquad 0 \longrightarrow e^{s}, e^{s} \longrightarrow e^{u}, e^{u} \longrightarrow e^{3}, e^{3} \longrightarrow e^{1}, e^{1} \longrightarrow e^{1}, $				-) ·	ל לכן	7/ ~`	. yui	0			ځم	$\longrightarrow \rho^{4} \longrightarrow \rho^{3} \longrightarrow \rho^{2} \longrightarrow \rho^{0} \longrightarrow \rho^{0} \longrightarrow \rho^{0}$
$C'(R P^{5}, Z/22) \qquad 0 \longrightarrow e^{5*} \longleftarrow e^{4*} \longleftarrow e^{3*} \longleftarrow e^{2*} \longleftarrow e^{1*} \longleftarrow e^{0*} \longleftarrow 0$ $C'(R P^{5}, Z/22) \qquad 0 \longleftarrow e^{5*} \longleftarrow e^{4*} \longleftarrow e^{4*} \longleftarrow e^{3*} \longleftarrow e^{2*} \longleftarrow e^{1*} \longleftarrow e^{0*} \longleftarrow 0$ $C'(R P^{5}, Z/22) \qquad 0 \longleftarrow e^{5*} \bigoplus_{i=1}^{5*} \bigoplus_{i=1}^{5*} \bigoplus_{i=1}^{4*} \bigoplus_{i=1}^{4*} \bigoplus_{i=1}^{4*} e^{2*} \longleftarrow e^{1*} \bigoplus_{i=1}^{4*} e^{1*} \bigoplus_{i=1}^{4*} \bigoplus_{i=1}^{4*} e^{1*} \bigoplus_{i=1}^{4*} \bigoplus_{i=1}^{4*} e^{1*} \bigoplus_{i=1}^{4*} \bigoplus_{i=1$			ٔ ا	CINI	, ,	4261		· ·			1 ei	$^{5}\mapsto$ 6 5 4 4 4 4
$C'(R P^{5}, Z/22) \qquad 0 \longrightarrow e^{5*} \longleftarrow e^{4*} \longleftarrow e^{3*} \longleftarrow e^{2*} \longleftarrow e^{1*} \longleftarrow e^{0*} \longleftarrow 0$ $C'(R P^{5}, Z/22) \qquad 0 \longleftarrow e^{5*} \longleftarrow e^{4*} \longleftarrow e^{4*} \longleftarrow e^{3*} \longleftarrow e^{2*} \longleftarrow e^{1*} \longleftarrow e^{0*} \longleftarrow 0$ $C'(R P^{5}, Z/22) \qquad 0 \longleftarrow e^{5*} \bigoplus_{i=1}^{5*} \bigoplus_{i=1}^{5*} \bigoplus_{i=1}^{4*} \bigoplus_{i=1}^{4*} \bigoplus_{i=1}^{4*} e^{2*} \longleftarrow e^{1*} \bigoplus_{i=1}^{4*} e^{1*} \bigoplus_{i=1}^{4*} \bigoplus_{i=1}^{4*} e^{1*} \bigoplus_{i=1}^{4*} \bigoplus_{i=1}^{4*} e^{1*} \bigoplus_{i=1}^{4*} \bigoplus_{i=1$			_	(5	٠ - ك	7/ -	`	_			و ا	
$C'(IRIP^{5}, Z/_{2}) \qquad 0 \iff e^{5*} \iff e^{4*} \iff e^{3*} \iff e^{2*} \iff e^{4*} \iff e^{0*} \iff e$												
C'($\mathbb{R}\mathbb{P}^{5}$, \mathbb{Z}_{22}) $0 \leftarrow e^{\frac{5\pi}{4}} e^{\frac{5\pi}{4}} \leftarrow e^{\frac{5\pi}{4}} e^{\frac{5\pi}{4}} \leftarrow e^{\frac{5\pi}{4}} e^{\frac{3\pi}{4}} \leftarrow e^{\frac{3\pi}{4}} e^{\frac{3\pi}{4}} \leftarrow e^{\frac{3\pi}{4}} e^{\frac{3\pi}{4}} \leftarrow e^{\frac{5\pi}{4}} e^{\frac{5\pi}{4}} \leftarrow e^{\frac{5\pi}{4$											- 1	
C'($\mathbb{R}\mathbb{P}^{5}$, \mathbb{Z}_{22}) $0 \leftarrow e^{\frac{5\pi}{4}} e^{\frac{5\pi}{4}} \leftarrow e^{\frac{5\pi}{4}} e^{\frac{5\pi}{4}} \leftarrow e^{\frac{5\pi}{4}} e^{\frac{3\pi}{4}} \leftarrow e^{\frac{3\pi}{4}} e^{\frac{3\pi}{4}} \leftarrow e^{\frac{3\pi}{4}} e^{\frac{3\pi}{4}} \leftarrow e^{\frac{5\pi}{4}} e^{\frac{5\pi}{4}} \leftarrow e^{\frac{5\pi}{4$												u* 34 2* 1* 0*
es*-es* - e14*			C) (II	ЦР°,	1/27	<u>(</u>)					
es*-es* - e14*											les	
			C	(IR	ribz,	, 24/2	<u>v</u>)	(0 ←	_	es*, e	$e_{i}^{*} \leftarrow e_{i}^{*}, e_{i}^{*} \leftarrow e_{i}^{*$
· · · · · · · · · · · · · · · · · · ·												
$-e^{x^{*}} + e^{x^{*}} \leftarrow e^{x^{*}}$										- (25#+ 1	e'* - e'*
btw. when n is odd, $H^{n}(IRIP^{n}, \mathbb{Z}) \longrightarrow H^{n}(S^{n}, \mathbb{Z})$			6	łw.	wh	en i	h ¿	s od	d.	۲	1"(IR	$P^{n}, \mathbb{Z}) \longrightarrow H^{n}(S^{n}, \mathbb{Z})$
$\frac{115}{7} \xrightarrow{\times 2} \frac{165}{7}$				•				•	,			
												Z/ Z/

Cor.
$$w(x_n,s^n) = \pi^* w(x_n', |R|p^n) = 1$$

 $w(TS^n) = \pi^* w(T|R|P^n) = 1$
 x_n',s^n , TS^n are spin, $S^n = \partial D^n$.

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6. Cplx mfld
                   CIPh is undoubtedly projectix mfld.
                   IRIP<sup>2n-1</sup>, S<sup>2n-1</sup> are not oply milds since they're of odd dim.

IRIP<sup>2n</sup> is not solv all
                    IR IP is not cplx mfld since it's not orientable. S^{n}(n>6), S^{4} are not cplx mflds, see https://mathoverflow.net/questions/11664/complex-structure-on-sn
                    Whether S<sup>6</sup> is a cplx mfld is still an open problem, see https://mathoverflow.net/questions/1973/is-there-a-complex-structure-on-the-6-sphere
                     related problems is the cplx structure of CIP unique? Still open, see
                                https://mathoverflow.net/questions/382442/is-the-complex-structure-of-mathbb-cpn-unique
7 Lie group. S', S3, IRIP', IRIP', we have IRIP' = S' and
                                                                  S' = SU2 = {q∈ H | q1 = 1}
                                                                  \int_{\mathbb{R}}
                                                                  |R|^3 \approx 50_3 https://math.stackexchange.com/questions/4065801/collecting-proofs-so3-cong-bbb-r-p3
                                                                                                  But a better way to see it is here: https://www.youtube.com/watch?v=ACZC_XEyg9U
           for 51: https://math.stackexchange.com/questions/12453/is-there-an-easy-way-to-show-which-spheres-can-be-lie-groups
         for IRIP": lemma. a Lie/topological group structure lifts to a covering space
                                        \textit{Proof:} \quad \textit{see} \quad \text{https://math.stackexchange.com/questions/5391/covering-of-a-topological-group-is-a-topological-group} \\
                                       Cor. IRIP" (n>3) is not a Lie group
          for Oph lemma for the connected Lie group G, \pi_3(G) = 0 \pi_3(G) has no torsion!
                                        broof 566 https://mathoverflow.net/questions/8957/homotopy-groups-of-lie-groups
                                       Cor. CIP" is not a Lie group.
                                        different proof of this Cor: https://math.stackexchange.com/questions/3043483/lie-group-structure-o
          Interesting results during the ways of searching
                                      Lemma: a opt Lie group is either abelian => torus
                                                                                                                                               nonabelian & have nonzero H3
                                                           https://math.stack exchange.com/questions/3421788/topological-lie-group-structure-on-projective-spaces and the projective of the project
                                        See
                                        Lemma
                                                           every compact Lie group has zero Euler characteristic since it is parallelizable
                                                            https://math.stackexchange.com/questions/829928/can-s2-be-turned-into-a-topological-group/
                                         Spo
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