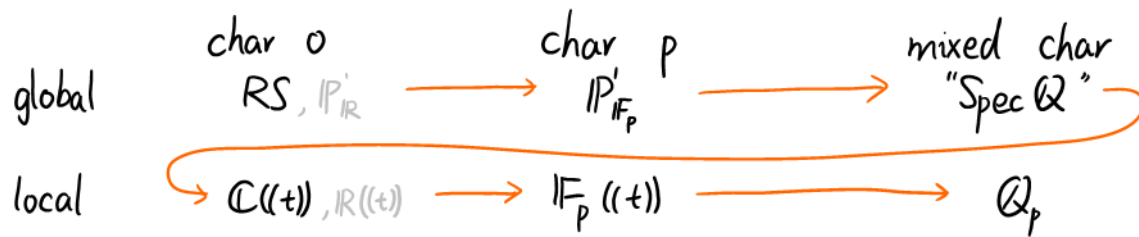


Eine Woche, ein Beispiel
 8.27. ramified covering: RS case



\longrightarrow : route in this series

For having the best geometrical intuition, we design this route. People may prefer working with local objects first (and then global objects), since global objects are glued by local objects. However, you don't have to sharpen your tools before playing the puzzles.

play global ——— tools local ——— play again local-global principle

Today: We work on Riemann Surface (RS), the most intuitive case.
 The relationship with field extension is left to next time.

1. standard ramified covering
2. definition
3. examples
 - morphisms with explicit expressions
 - RS defined by equations
 - infinite pt case
 - morphisms defined by quotients.

1. standard ramified covering

For practice, we only consider ramified covering with finite ramification index.

Observation. Consider

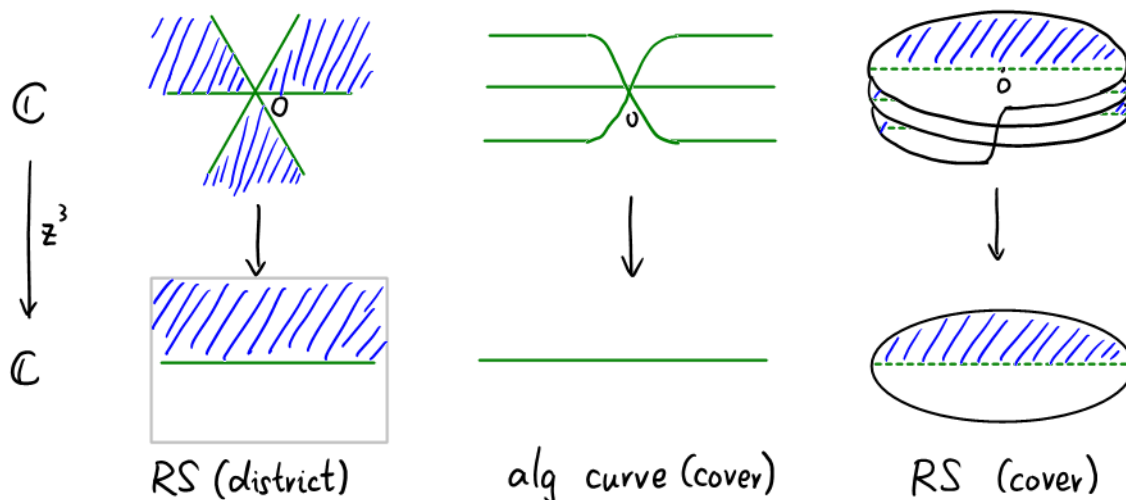
$$f: \mathbb{C} \rightarrow \mathbb{C} \quad z \mapsto z^e$$

how to understand this fct?

- f is holomorphic; $f \in \mathbb{C}[z]$
- $f^*: \mathcal{M}(\mathbb{C}) \rightarrow \mathcal{M}(\mathbb{C})$ field extension of deg e
- f is "roughly a cover":
 - $f^{-1}(z) = \begin{cases} e \text{ pts}, & z \neq 0 \\ \{0\}, & z = 0 \end{cases}$
 - $f|_{\mathbb{C} - \{0\}}: \mathbb{C} - \{0\} \rightarrow \mathbb{C} - \{0\}$ is a cover

Once we divide $\mathbb{C} (= \text{Im } f)$ by several districts, with 0 lying in the boundary, we can divide domain by several districts, and see the movement of pts easily (as long as they don't pass 0).

e.g. $e=3$



We will only draw the first two pictures later on, since the last one is too difficult to draw.

Fact (show in next document)

For a ramified covering $f: X \rightarrow Y$ of deg e ,
 $f^*: \mathcal{M}(Y) \rightarrow \mathcal{M}(X)$ is a field extension of deg e .

Notice that we don't assume X, Y to be cpt.