Eine Woche, ein Beispiel

11.26 calculation of double point

Final goal: Fill in the tables in the next page.

(for presentation, remove the i' column)

Ref:

[Willians]: Langlands correspondence and Bezrukavnikov's equivalence

calculations from Lukas Bonfert's note (don't forward this to anyone else).

$X = \mathbb{C} \cup_{\{0\}} \mathbb{C} = \{(z_1, z_2) \in \mathbb{C}^2 \mid z_1 z_2 = 0\}$, $Z = \{0\}$, $\mathcal{U} = \mathbb{C}^\times \sqcup \mathbb{C}^\times$

$i_* \mathbb{Q}_Z$
$(0, 1, 1, 1)$

	n	-2	-1	0	1
$\mathcal{U}$	j^*	0	0	0	0
$\{0\}$	i^*	0	0	$\mathbb{Q}$	0
	$i^!$	0	0	$\mathbb{Q}$	0
	$R^n\Gamma$	0	0	$\mathbb{Q}$	0

$\mathbb{Q}_X[1]$
$(-1, -1, -1, -1)$

	n	-2	-1	0	1
$\mathcal{U}$	j^*	0	$\mathbb{Q}$	0	0
$\{0\}$	i^*	0	$\mathbb{Q}$	0	0
	$i^!$	0	0	$\mathbb{Q}$	$\mathbb{Q}^2$
	$R^n\Gamma$	0	$\mathbb{Q}$	0	0

perverse sheaf
IC sheaf
? by dim argument

	n	-2	-1	0	1
$\mathcal{U}$	j^*	×		×	×
$\{0\}$	i^*	—		×	×
	$i^!$	×	×	×	
	$R^n\Gamma$	—			

$Rj_* \mathbb{Q}_\mathcal{U}[1]$
$(-1, 0, 0, 0)$

	n	-2	-1	0	1
$\mathcal{U}$	j^*	0	$\mathbb{Q}$	0	0
$\{0\}$	i^*	0	$\mathbb{Q}^2$	$\mathbb{Q}^2$	0
	$i^!$	0	0	0	0
	$R^n\Gamma$	0	$\mathbb{Q}^2$	$\mathbb{Q}^2$	0
	Γ	0	$\mathbb{Q}^2$	$\mathbb{Q}$	0

$\Gamma^{(n)} = \Gamma(R^n j_* \mathbb{Q}_\mathcal{U}[1])$

$j_! \mathbb{Q}_\mathcal{U}[1]$
$(-1, 0, 0, 0)$

	n	-2	-1	0	1
$\mathcal{U}$	j^*	0	$\mathbb{Q}$	0	0
$\{0\}$	i^*	0	0	0	0
	$i^!$	0	0	$\mathbb{Q}^2$	$\mathbb{Q}^2$
	$R^n\Gamma$	0	0	0	0

$\pi^! \mathbb{Q}[-1]$
$(-1, -1, -1, -1)$

	n	-2	-1	0	1
$\mathcal{U}$	j^*	0	$\mathbb{Q}$	0	0
$\{0\}$	i^*	0	$\mathbb{Q}^2$	$\mathbb{Q}$	0
	$i^!$	0	0	0	$\mathbb{Q}$
	$R^n\Gamma$	0	$\mathbb{Q}$	0	0

$X = X_3 = \{(z_1, z_2, z_3) \in \mathbb{C}^3 \mid z_1^2 + z_2^2 + z_3^2 = 0\}$, $Z = \{0\}$, $\mathcal{U} = X_3 - \{0\}$ = $\mathbb{C}^\times$-bd over $\mathbb{CP}^1$ twisted

$i_* \underline{\mathbb{Q}}_Z$
$(0,1,1,1)$

	n	-2	-1	0	1	2
$\mathcal{U}$	j^*	0	0	0	0	0
$\{0\}$	i^*	0	0	$\mathbb{Q}$	0	0
	$i^!$	0	0	$\mathbb{Q}$	0	0
	$R^n\Gamma$	0	0	$\mathbb{Q}$	0	0

$\underline{\mathbb{Q}}_X[2]$
$(1,1,1,1)$

	n	-2	-1	0	1	2
$\mathcal{U}$	j^*	$\underline{\mathbb{Q}}$	0	0	0	0
$\{0\}$	i^*	$\underline{\mathbb{Q}}$	0	0	0	0
	$i^!$	0	$\mathbb{Z} \xrightarrow{\times 2}$ 0	$\mathbb{Z}$ $\mathbb{Z}/2\mathbb{Z}$ $\mathbb{Z}$	$\xrightarrow{\times 2} \mathbb{Z}$ $\mathbb{Z}/2\mathbb{Z}$	$\mathbb{Q}$
	$R^n\Gamma$	$\mathbb{Q}$	0	0	0	0

$Rj_* \underline{\mathbb{Q}}_{\mathcal{U}}[2] \in {}^pD^{\geq 0}(X) - {}^pD^{\leq 0}(X)$
$(1,0,0,0)$

	n	-2	-1	0	1	2
$\mathcal{U}$	j^*	$\underline{\mathbb{Q}}$	0	0	0	0
$\{0\}$	i^*	$\mathbb{Q}$	$\mathbb{Q}$	$\mathbb{Q}$	$\mathbb{Q}$	0
	$i^!$	0	0	0	0	0
	$R^n\Gamma$	$\mathbb{Q}$	$\mathbb{Q}$	$\mathbb{Q}$	$\mathbb{Q}$	0
	Γ	$\mathbb{Q}$	$\mathbb{Q}$	$\mathbb{Q}$	$\mathbb{Q}$	

$j_! \underline{\mathbb{Q}}_{\mathcal{U}}[2] \in {}^pD^{\leq 0}(X) - {}^pD^{\geq 0}(X)$
$(1,0,0,0)$

	n	-2	-1	0	1	2
$\mathcal{U}$	j^*	$\underline{\mathbb{Q}}$	0	0	0	0
$\{0\}$	i^*	0	0	0	0	0
	$i^!$	0	$\mathbb{Q}$	$\mathbb{Q}$	$\mathbb{Q}$	$\mathbb{Q}$
	$R^n\Gamma$	0	0	0	0	0

$\pi^! \mathbb{Q}[-2]$
$(1,1,1,1)$

	n	-2	-1	0	1	2
$\mathcal{U}$	j^*	$\underline{\mathbb{Q}}$	0	0	0	0
$\{0\}$	i^*	$\mathbb{Q}$	0 $\mathbb{Z} \xrightarrow{\times 2}$	$\mathbb{Z}$ $\mathbb{Z}$ $\mathbb{Z}/2\mathbb{Z}$	$\xrightarrow{\times 2} \mathbb{Z}$ $\mathbb{Z}/2\mathbb{Z}$	0
	$i^!$	0	0	0	0	$\mathbb{Q}$
	$R^n\Gamma$	$\mathbb{Q}$	0 $\mathbb{Z} \xrightarrow{\times 2}$	$\mathbb{Z}$ $\mathbb{Z}$ $\mathbb{Z}/2\mathbb{Z}$	$\xrightarrow{\times 2} \mathbb{Z}$ $\mathbb{Z}/2\mathbb{Z}$	0

perverse sheaf

IC sheaf

n		-2	-1	0	1	2
$\mathcal{U}$	j^*		×	×	×	×
$\{0\}$	i^*			×	×	×
	$i^!$	×	×	×		
	$R^n\Gamma$					

Conclusion. $\subsetneq$ IC $\subsetneq$ Perv $\subsetneq$ Constructable

$\mathbb{Q}$-coefficient $i_*\underline{\mathbb{Q}}_Z$ $\quad Rj_*\underline{\mathbb{Q}}_{\mathcal{U}}[2],\ j_!\underline{\mathbb{Q}}_{\mathcal{U}}[2]$

$\underline{\mathbb{Q}}_X[2] \cong \pi^!\mathbb{Q}[-2]$

$\mathbb{Z}$-coefficient $i_*\underline{\mathbb{Z}}_Z$ $\quad \underline{\mathbb{Z}}_X[2]$ $\quad Rj_*\underline{\mathbb{Z}}_{\mathcal{U}}[2],\ j_!\underline{\mathbb{Z}}_{\mathcal{U}}[2]$

$\pi^!\mathbb{Z}[-2]$