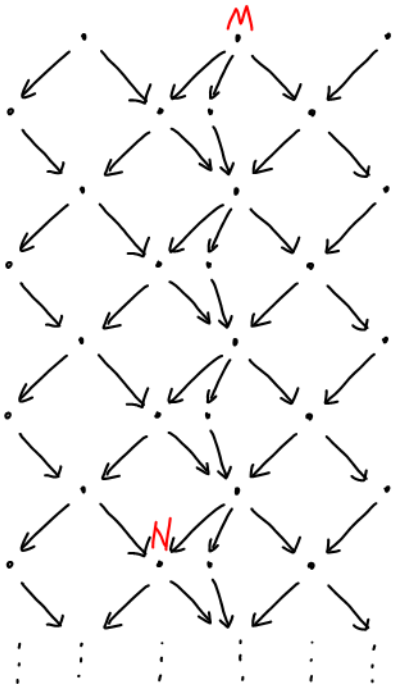
Eine Woche, ein Beispiel

6.11 more combinatorics in AR-quiver

1. dimension of Hom /path
2. database for sectional map

1. dimension of Hom /path

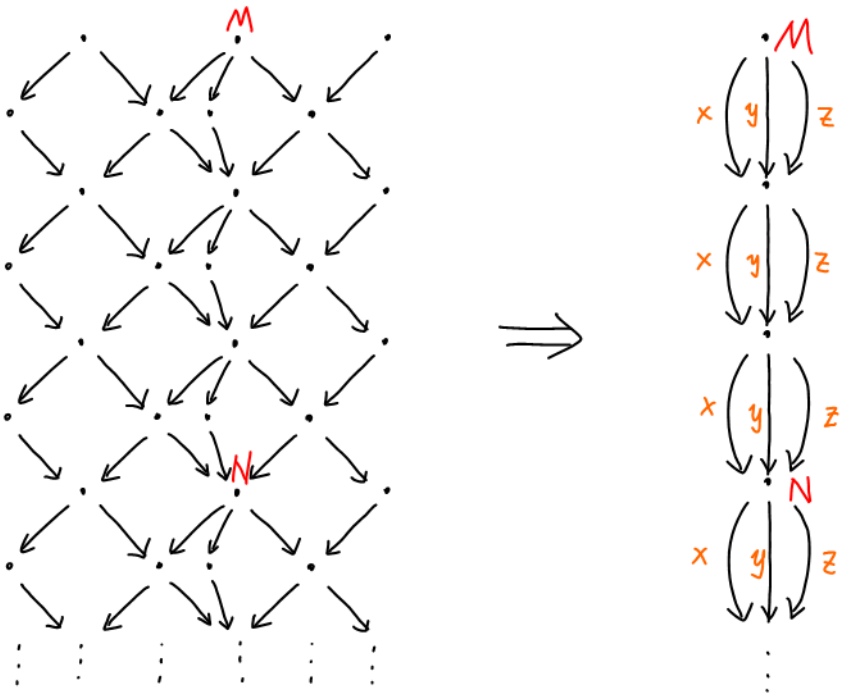
Suppose that you have some arrows: (type E_7)



Q: 1. How many paths do we have from M to N?
2. After identifying paths by AR-sequence, how many paths do we have from M to N?

A: (partial) By induction process.
Ex: find a basis for $\mathrm{Hom}(M, N)$.

Special cases:



$$\dim_{\mathbb{C}} \mathrm{Hom}(M,N) = \dim\left(\mathbb{C}\{x,y,z\}/(x^4, y^2, z^3, x+y+z)\right)_{\deg 4}$$

"Cor" The $\mathbb{Z}$-graded alg

$$A_{a,b,c} := \mathbb{C}\{x,y,z\}/(x^a, y^b, z^c, x+y+z) \qquad a,b,c \in \mathbb{N}_{\geq 1}$$

is f.d. iff $\frac{1}{a}+\frac{1}{b}+\frac{1}{c} > 1$.

We can find a (degree k) monomial basis of $A_{a,b,c}$ by playing the "Tic-tac-toe" game on $\{x,y,z\}^{\otimes k}$.

E.g. We try to find a basis of $A_{4,2,3}$.

$k=1$

x	y	z
①	②	X

① x
② y

$k=2$

	x	y	z
x	①	②	X
y	③		X
z	X	X	X

① x^2
② xy
③ yx

$k=3$.

x

	x	y	z
x	①	②	X
y	③		X
z	X	X	X

y

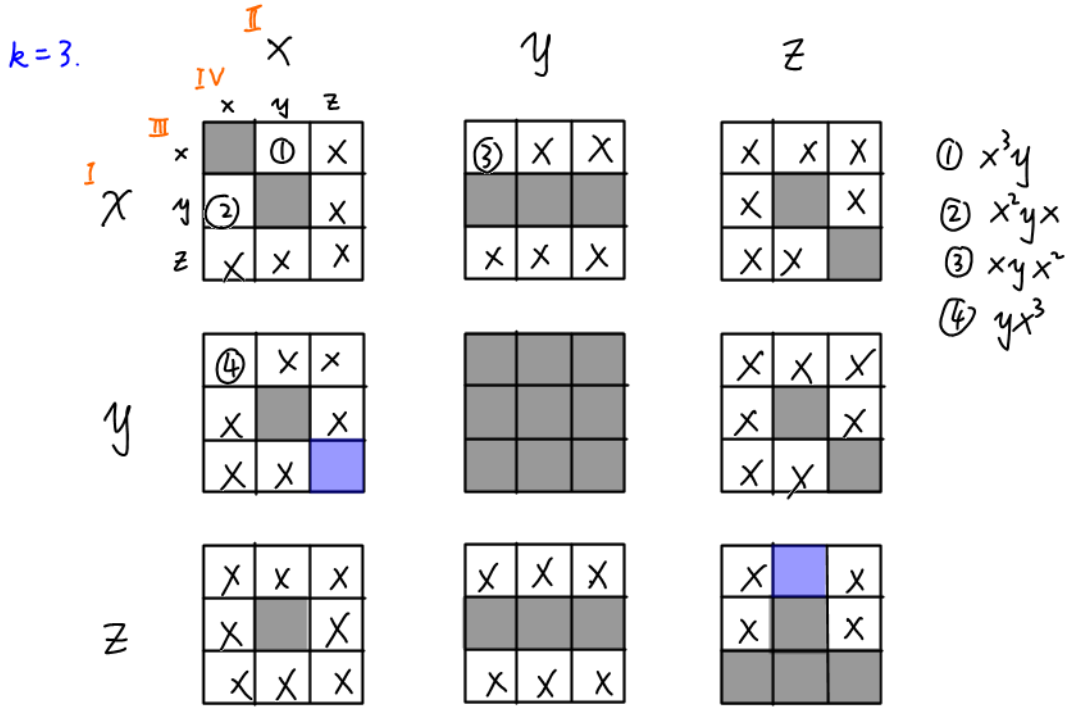
X	X	X
x	x	x

z

X	X	X
X		X
X	X	

① x^3
② x^2y
③ xyx
④ yx^2

$k = 3$.



① x^3y

② x^2yx

③ xyx^2

④ yx^3