

Eine Woche, ein Beispiel  
6.30 starting functions.

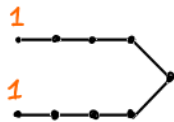
This is a follow-up of [2021.08.15], [2022.02.20] and [2021.05.07]. The combinatorics of starting functions is more intricate than I thought. Therefore, I collect these findings here, and wish somebody can give a rigorous proof for these phenomenonons (e.p. the numbers of 1's).

1. folding & starting fcts

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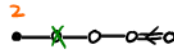
•: minuscule rep  
 x: quasi-minuscule rep  
 x: adjoint rep

E.g.  $C_5$

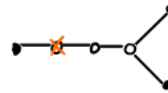


$A_9$

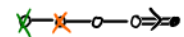
(unfold)



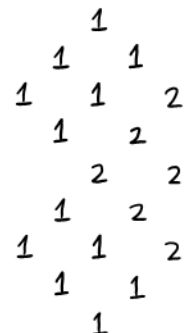
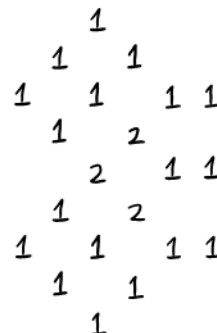
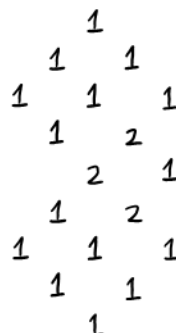
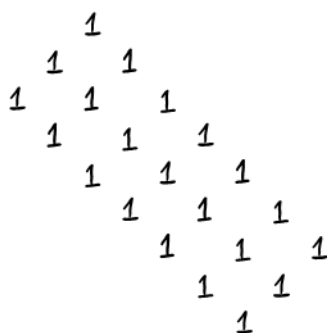
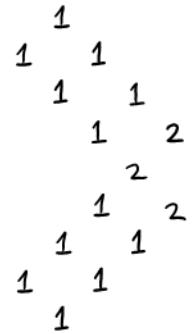
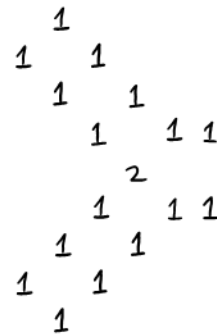
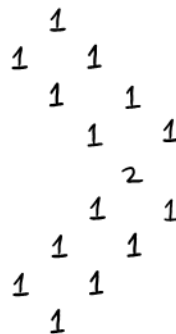
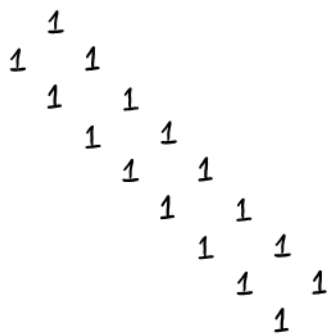
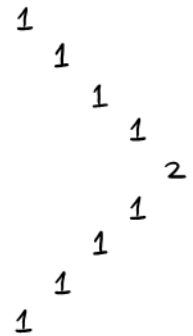
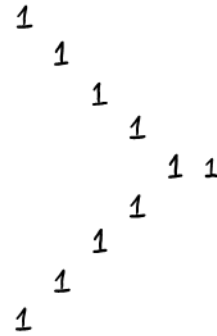
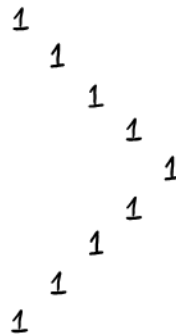
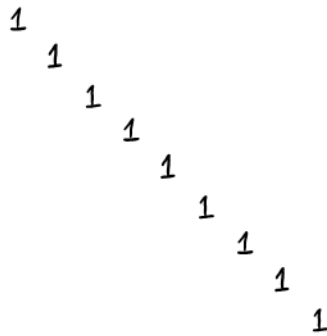
$C_5$

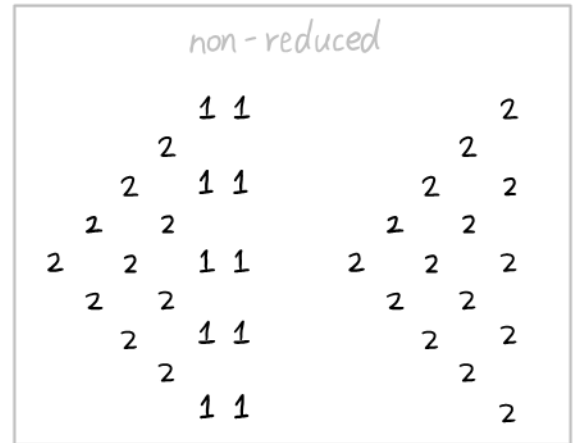
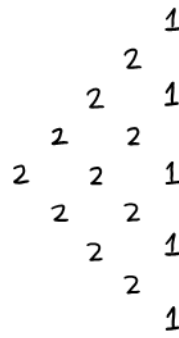
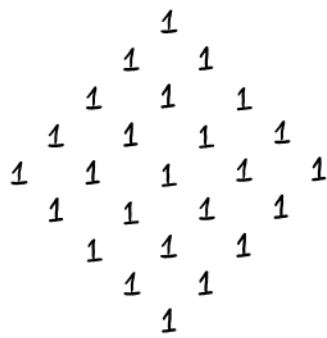
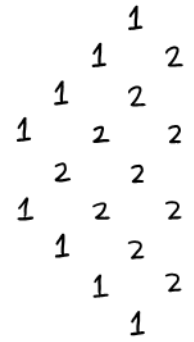
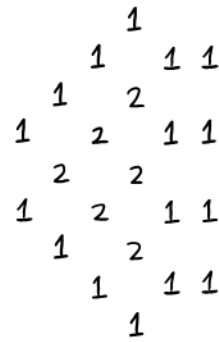
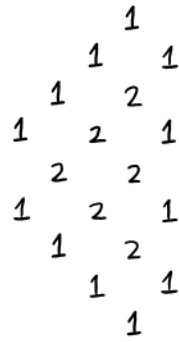
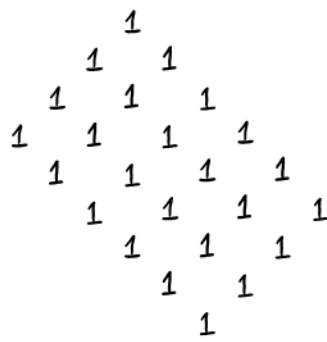


$D_6$



$B_5$





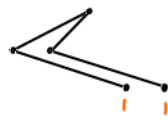
E.g.  $B_3$



$D_4$



$B_3$



$A_5$

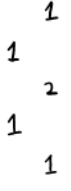
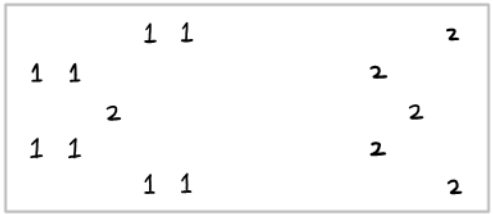
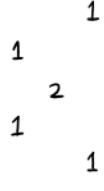
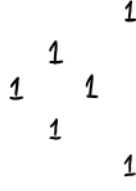
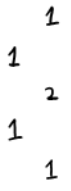
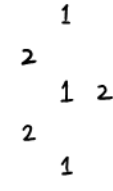
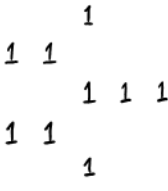
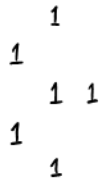
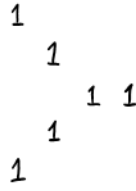
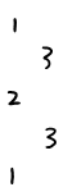
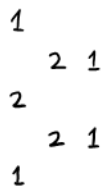
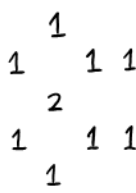


$C_3$



$G_2$

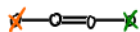
(unfold)



E.g.  $F_4$



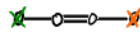
$E_6$



$F_4$



$E_6$



$F_4$

(unfold)



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1  
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1 1 1 1  
2  
1 1 1  
1 1 1  
1  
1

1  
1  
2  
1 2  
1 2  
1 2  
1 2  
1  
1

1  
1 1 1  
1 2 1  
2 1 2  
1 3 1  
2 2 2  
1 3 1  
2 1 2  
1 2 1  
1 1 1  
1

1  
1 2  
2 2  
1 4  
3 2  
2 4  
3 2  
1 4  
2 2  
1 2  
1

1  
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1 2 1  
2 1 1  
1 2  
1 1 1  
1 1  
1

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1 1  
1 2  
2 1  
1 3  
2 1  
1 2  
1 1  
1

non-reduced

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1 1  
1 1

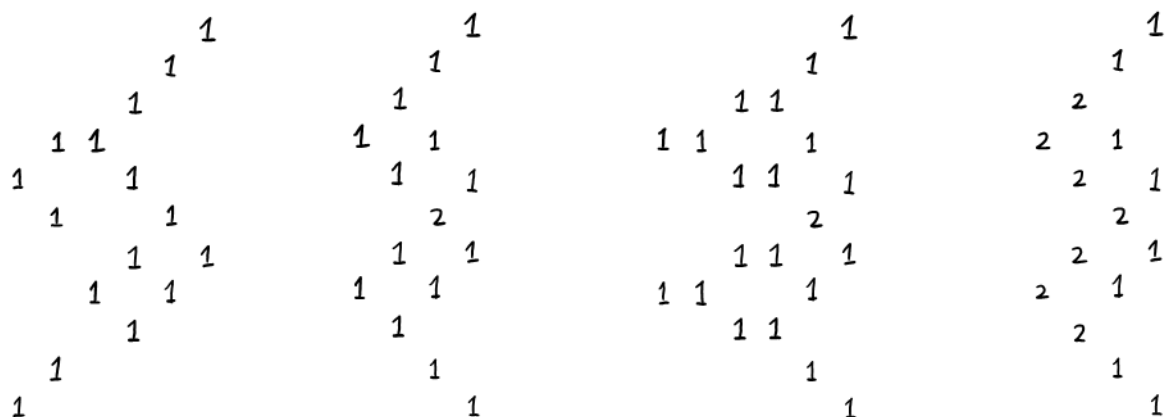
2  
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2

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1 1 4  
3 3 2  
2 2 4  
3 3 2  
1 1 4  
2 2 2  
1 1 2  
1 1

2  
2 2  
4 2  
2 4  
6 2  
4 4  
6 2  
2 4  
4 2  
2 2  
2

1  
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1 1 2  
2 2 1  
1 1 3  
2 2 1  
1 1 2  
1 1 1  
1

1  
2 1  
2 2  
4 1  
2 3  
4 1  
2 2  
2 1  
1



Rmk. Adjoint rep is compatible with normal folding, while  
 quasi-minuscule rep } are compatible with reversed folding  
 minuscule rep }

E.p. simply-laced  $\Leftrightarrow$  (quasi-minuscule = adjoint)

For minuscule rep,  $(\circ) \Rightarrow (\circ)$   $\Rightarrow$   $(\circ)$   
 keep

<https://mathoverflow.net/questions/111469/dual-versions-of-folding-symmetric-ade-dynkin-diagrams>