Eine Woche, ein Beispiel
3.27 model theory

$$\begin{array}{ccc}
\text{Model} & & \text{Theory} \\
| & \Leftarrow : \text{Realization} & | \\
& \Rightarrow : \text{Maximal theory} & \\
\text{type} \;---\; \text{Structure} & & \text{Formula} \\
& \searrow \qquad \swarrow & \\
& \text{Language} &
\end{array}$$

Modern Algebra (H)

Preliminaries

- Logic. set & map, operations on Set. $\left(\begin{array}{l} \cap \cup -^c, \text{ sub/quotient,} \\ \text{two ways of disjoint union} \\ \text{Cartesian product} \end{array} \right)$
  - Axiomatic set theory (ZFC)
  - Russell's paradox
  - type of proof : constructive, algorithm,...

  Ex. graph

  Classify topologies of $\{1,\dots,n\}$

  First adjunction: $\mathrm{Map}(A \times B, C) \cong \mathrm{Map}(A, \mathrm{Map}(B,C))$

- From $\mathbb{N}$ to $\mathbb{C}$ (the basic of examples, though logically it's not here)
  - Peano axioms. Axiom of induction
  - alg structure, order and topology
  - Completeness axiom.

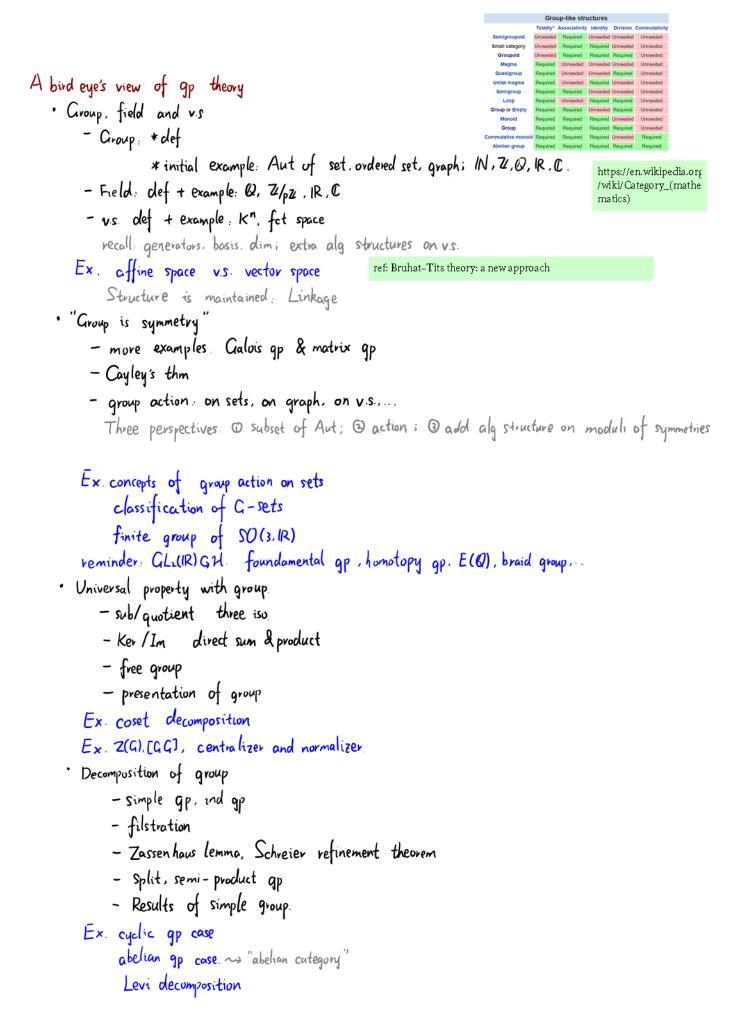- Cardinal: the only property of set.
  - naive definition

	alg	total order	topo
$\mathbb{N}$	$(+,\times)$	✓	discrete
$\mathbb{Z}$	$(+,-,\times)$	✓	discrete
$\mathbb{Q}$	$(+,-,\times,\div)$	✓	dense but not complete
$\mathbb{R}$	$(+,-,\times,\div)$	✓	complete
$\mathbb{C}$	$(+,-,\times,\div)$	✗	complete

  - operations on cardinal
  - examples
  - The continuum hypothesis
  - large cardinal axiom

- Order structure
  - def, operations and properties (partial/total/well order)
  - ordinal. relationship with cardinal.

  Ex. "well-order" on class of cardinals.

  Classify subpartial ordered set of $\{$subsets of $\{1,..,n\}\}$, up to iso. $n \leq 5$

A bird eye's view of gp theory

- Group, field and v.s
  - Group: * def
    * initial example: Aut of set, ordered set, graph; $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$.
  - Field: def + example: $\mathbb{Q}, \mathbb{Z}/p\mathbb{Z}, \mathbb{R}, \mathbb{C}$
  - v.s. def + example: K^n, fct space
    recall: generators, basis, dim; extra alg structures on v.s.

  Ex. affine space v.s. vector space

  Structure is maintained: Linkage

- "Group is symmetry"
  - more examples. Galois gp & matrix gp
  - Cayley's thm
  - group action: on sets, on graph, on v.s., ...
    Three perspectives: ① subset of Aut; ② action; ③ add alg structure on moduli of symmetries

  Ex. concepts of group action on sets
    classification of G-sets
    finite group of $SO(3, \mathbb{R})$
  reminder: $GL_2(\mathbb{R}) \curvearrowright \mathcal{H}$. foundamental gp, homotopy gp. $E(\mathbb{Q})$, braid group, ...

- Universal property with group.
  - sub/quotient three iso.
  - Ker/Im direct sum & product
  - free group
  - presentation of group

  Ex. coset decomposition

  Ex. $Z(G), [G, G]$, centralizer and normalizer

- Decomposition of group
  - simple gp, ind gp
  - filstration
  - Zassenhaus lemma, Schreier refinement theorem
  - Split, semi-product gp
  - Results of simple group.

  Ex. cyclic gp case
    abelian gp case. ⤳ "abelian category"
      Levi decomposition

Down-to-earth analysis.

- f.g. abelian gp + apps.
  Ex. lattice & Crystallographic point gp
      with/without origin
      sublattice count
- G act on G
- Sylow thm
- app: classifications of gp of small order. research on specific gp.

Ring & module

$R(x)$ $R((x))$ $R\{\{x\}\}$ $R\langle\langle x\rangle\rangle$

- Basic def. e.g. $R[x]$, $R[[x]]$, $R\{x\}$, $R\langle x\rangle$
- Category, k-algs
  - Ex. groupoid
- Basic def of modules, abelian category
  - Ex. R-algs.
    - tensor product, change of basis
    - three rep theory
- AG translation, examples.
- Concepts under AG translation
- ED $\Rightarrow$ PID $\Rightarrow$ UFD $\Rightarrow$ domain
- classification of f.g. module over PID.

Field and Galois theory (See [GTM 167])

can focus more on $\mathbb{F}_p$, $\mathbb{Q}_p$, $\mathbb{F}_p((t))$, and geometrical point of view.

(Reminder) other structures: norm, metric & topo; measure; sheaf.