Eine Woche, ein Beispiel

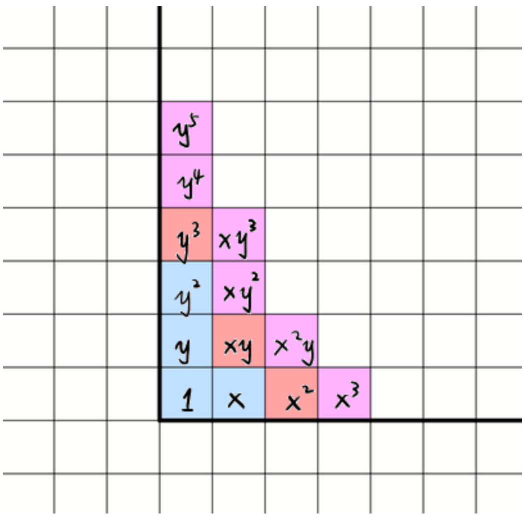
6.22 tangent space of $(\mathbb{A}^n)^{[k]}$

The tangent space of the Hilbert scheme has an element combinatorial description, while there are still some open problems (about the upper bound of the dimension of tangent space)

Ref:

Dori Bejleri, David Stapleton, The tangent space of the punctual Hilbert scheme
Miller Ezra, Bernd Sturmfels. Combinatorial Commutative Algebra. Graduate Texts in Mathemat

E.g.



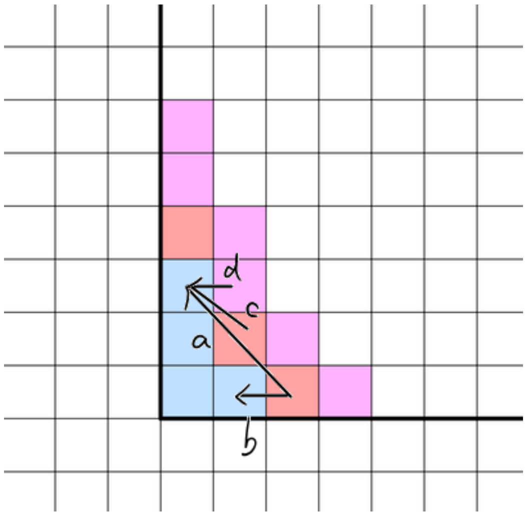
$R = \mathbb{C}[x,y]$

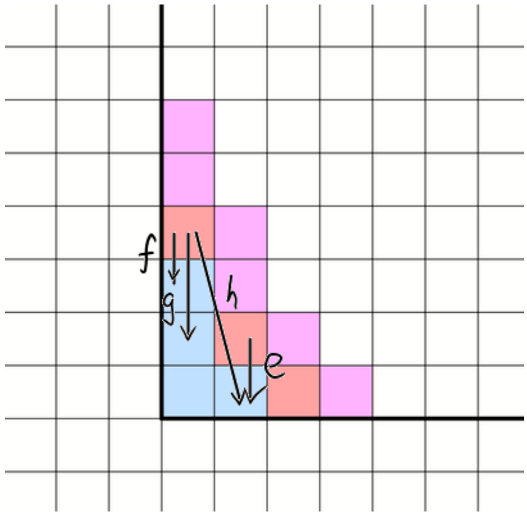
$I = \langle x^2, xy, y^3 \rangle$

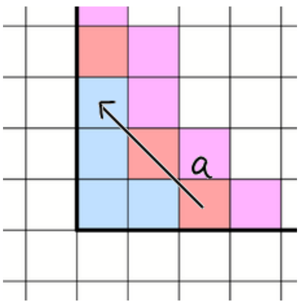
$R/I = \langle 1, x, y, y^2 \rangle_{v.s.} = \mathbb{C}^4$

$I/I^2 = \langle x^2, x^3, xy, \dots, y^5 \rangle_{v.s.} = \mathbb{C}^9$

$\mathrm{Hom}_R(I, R/I) = \langle a, b, c, \dots, h \rangle = \mathbb{C}^8$, where



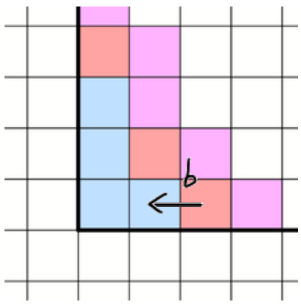




$a: I/I^2 \longrightarrow R/I$

$x^2 \longmapsto y^2$

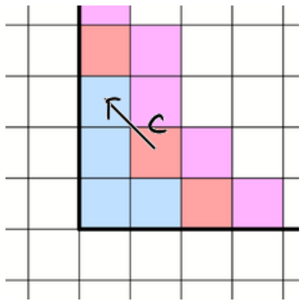
other basis $\longmapsto 0$



$b: I/I^2 \longrightarrow R/I$

$x^2 \longmapsto x$

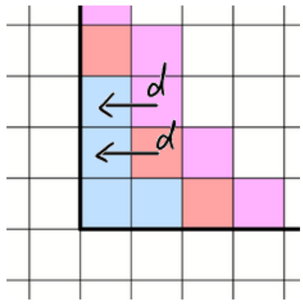
other basis $\longmapsto 0$



$c: I/I^2 \longrightarrow R/I$

$xy \longmapsto y^2$

other basis $\longmapsto 0$



$d: I/I^2 \longrightarrow R/I$

$xy^2 \longmapsto y^2$

$xy \longmapsto y$

other $\longmapsto 0$