Eine Woche, ein Beispiel

8.15 indecomposable representation of Affine quiver.

Task: give some examples to correct my misunderstanding of AR theory.

1: $\widetilde{A}_2$ $\quad 1 \rightrightarrows 2$

Ind rep: $\quad M_{n+1,n}$: $\quad K^{n+1} \overset{(Id_n|0)}{\underset{(0|Id_n)}{\rightrightarrows}} K^n$

$\mathrm{End}(M_{n+1,n}) = K \qquad [M_{n+1,n}, M_{n+1,n}]' = 0$

They all corresponds to preproj/pre inj.

M_A : $\quad K^n \overset{A}{\underset{Id}{\rightrightarrows}} K^n \qquad A \in M_n(K)$

$M_A \sim M_{A'} \iff A$ conj to A'

They're not all ind reps of dim vector (n,n).

2. $\widetilde{A}_3$

$$\begin{tikzcd} & 2 \arrow[dr] & \\ 1 \arrow[ur] \arrow[rr] & & 3 \end{tikzcd}$$

$C_P = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{pmatrix} \qquad C_I = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

$\Phi_A = -C_I C_P^{-1} = -\begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 & -2 \\ 2 & 0 & -1 \\ 1 & 1 & -1 \end{pmatrix}$

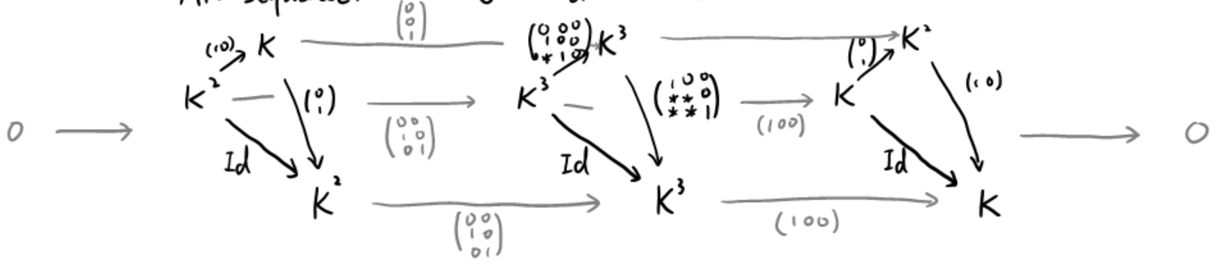
Ind rep: $\quad M$:

$$\begin{tikzcd} & K^2 \arrow[dr, "(1\,0)"] & \\ K \arrow[ur, "\binom{0}{1}"] \arrow[rr, "1"'] & & K \end{tikzcd}$$

$\mathrm{End}(M) = \begin{pmatrix} \alpha & 0 \\ \beta & \alpha \end{pmatrix} \qquad [M,M]' = 2$

$\underline{\dim}\ \tau(M) = \Phi_A \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$

AR sequence: $\quad 0 \to \tau M \to ? \to M \to 0$



non-split sequence which is not AR sequence: ? $\quad 0 \to \tau M \to ? \to M \to 0$