# Eine Woche, ein Beispiel

## 10.2 equivariant K-theory of Steinberg variety : notation

This document is written to reorganize the notations in Tomasz Przezdziecki's master thesis:
http://www.math.uni-bonn.de/ag/stroppel/Master%27s%20Thesis_Tomasz%20Przezdziecki.pdf

We changed some notation for the convenience of writing.

Task.

1. dimension vector
2. Weyl gp
3. alg group & Lie algebra
4. typical variety
5. (equivariant) stratifications
6. tangent space, Euler class
7. basis of Hecke alg

We may use two examples for the convenience of presentation.
Readers can easily distinguish them by the dim vectors.

## 1. dimension vector

$|d| = 5$

$d = (3,2)$

$\underline{d} = \begin{pmatrix} 3,2 \\ 2,2 \\ 2,1 \\ 1,1 \\ 0,0 \\ 0,0 \end{pmatrix} =$  $\in W_d \backslash \mathbb{W}_d$ or $\mathcal{M}in(\mathbb{W}_{|d|}, W_d)$

Young Tableax $\quad r_{\varpi} = \pi_{\underline{d}}^{-1}(F_{\varpi})$

## 2. Weyl group

| Set | element | special element | others |
|---|---|---|---|
| $\mathbb{W}_{|d|} = S_5$ | $\varpi$ | $\varpi_{max} =$ | $\Pi = \{s_1, s_2, s_3, s_4\}$ |
| $W_d = S_3 \times S_2$ | $w$ | $w_{max} =$ | $\Pi_d = \{s_1, s_2, \quad s_4\}$ |
| $W_d \backslash \mathbb{W}_{|d|} = S_3 \times S_2 \backslash S_5$ | $\varpi, \underline{d}$ | | (Comp$_d$) |
| $\mathcal{M}in(\mathbb{W}_{|d|}, W_d) = \{ \ , \cdots \}$ | $u$ | | (Shuffled) |

$$\begin{tikzcd} & & & \mathcal{M}in(\mathbb{W}_{|d|}, W_d) \arrow[dl] \arrow[d, "\cong"] & \\ 0 \arrow[r] & W_d \arrow[r] & \mathbb{W}_{|d|} \arrow[r] & \mathbb{W}_{|d|} \backslash W_d \arrow[r] & 0 \end{tikzcd}$$

$$\begin{array}{c} u \\ \downarrow \\ \varpi = wu \longmapsto \underline{d} \end{array}$$

$\varpi =$

$u =$

$w =$

Another example: $\quad d = (1,2) \qquad \begin{array}{c} a \longrightarrow b \\ \langle v_1 \rangle \longrightarrow \langle v_2, v_3 \rangle \end{array}$

| $\varpi = wu$ | | | $w$ | $\underline{d}$ | order of basis | $l(\varpi)$ | $l(w)$ | $\mathbb{B}_{\varpi}$ | $B_{\varpi}$ | $\varpi B \varpi^{-1}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Id | Id | $\begin{pmatrix} 1&2&3 \\ 1&2&3 \end{pmatrix}$ | | abb | $\{v_1, v_2, v_3\}$ | 0 | 0 | $\begin{bmatrix} *&*&* \\ &*&* \\ &&* \end{bmatrix}$ | $\begin{bmatrix} *&& \\ &*&* \\ &&* \end{bmatrix}$ | $\begin{bmatrix} *&& \\ &*&* \\ &&* \end{bmatrix}$ |
| t | (23) | $\begin{pmatrix} 1&2&3 \\ 1&3&2 \end{pmatrix}$ | | abb | $\{v_1, v_3, v_2\}$ | 1 | 1 | $\begin{bmatrix} *&*&* \\ &*& \\ &*&* \end{bmatrix}$ | $\begin{bmatrix} *&& \\ &*& \\ &*&* \end{bmatrix}$ | $\begin{bmatrix} *&& \\ &*& \\ &*&* \end{bmatrix}$ |
| s | (12) | $\begin{pmatrix} 1&2&3 \\ 2&1&3 \end{pmatrix}$ | | bab | $\{v_2, v_1, v_3\}$ | 1 | 0 | $\begin{bmatrix} *&&* \\ *&*&* \\ &&* \end{bmatrix}$ | $\begin{bmatrix} *&& \\ &*&* \\ &&* \end{bmatrix}$ | $\begin{bmatrix} *&&* \\ &*& \\ &&* \end{bmatrix}$ |
| ts | (132) | $\begin{pmatrix} 1&2&3 \\ 3&1&2 \end{pmatrix}$ | | bab | $\{v_3, v_1, v_2\}$ | 2 | 1 | $\begin{bmatrix} *&*& \\ &*& \\ *&*&* \end{bmatrix}$ | $\begin{bmatrix} *&& \\ &*& \\ &*&* \end{bmatrix}$ | $\begin{bmatrix} *&*& \\ &*& \\ &&* \end{bmatrix}$ |
| st | (123) | $\begin{pmatrix} 1&2&3 \\ 2&3&1 \end{pmatrix}$ | | bba | $\{v_2, v_3, v_1\}$ | 2 | 0 | $\begin{bmatrix} *&& \\ *&*&* \\ *&&* \end{bmatrix}$ | $\begin{bmatrix} *&& \\ &*&* \\ &&* \end{bmatrix}$ | $\begin{bmatrix} *&& \\ &*& \\ *&&* \end{bmatrix}$ |
| sts | (13) | $\begin{pmatrix} 1&2&3 \\ 3&2&1 \end{pmatrix}$ | | bba | $\{v_3, v_2, v_1\}$ | 3 | 1 | $\begin{bmatrix} *&& \\ *&*& \\ *&*&* \end{bmatrix}$ | $\begin{bmatrix} *&& \\ &*& \\ &*&* \end{bmatrix}$ | $\begin{bmatrix} *&& \\ *&*& \\ &&* \end{bmatrix}$ |

⚠ The action on Flag is not the same as in

http://www.math.uni-bonn.de/ag/stroppel/Master%27s%20Thesis_Tomasz%20Przezdziecki.pdf

3. alg group & Lie algebra

$$\mathbb{G}_{|d|},\ \mathbb{B}_{|d|},\ \mathbb{T}_{|d|},\ \mathbb{W}_{|d|} = N_{\mathbb{G}_{|d|}}(\mathbb{T}_{|d|})$$