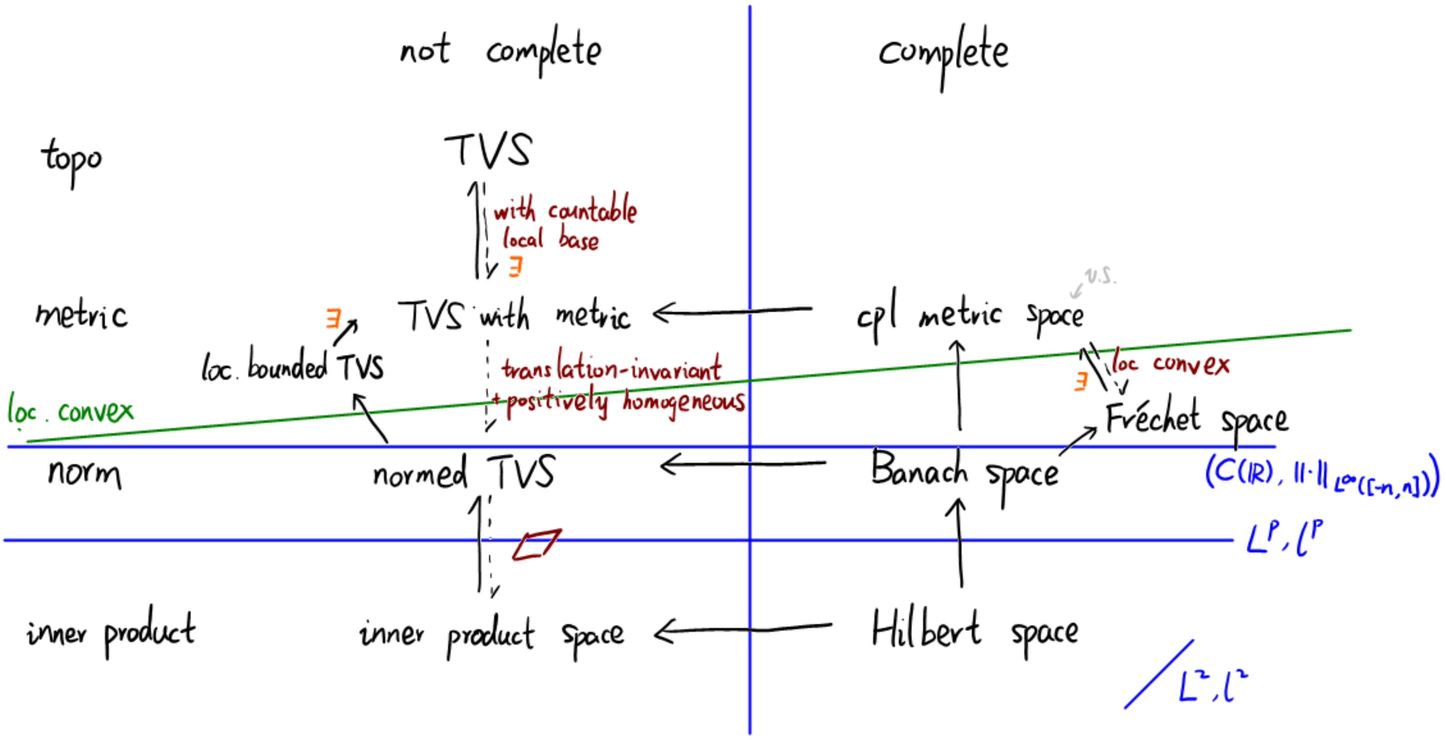
Eine Woche, ein Beispiel

4.30 TVS = topological vector space

Ref:
Lec 1-7: http://staff.ustc.edu.cn/~wangzuoq/Courses/15F-FA/index.html

In this document, we don't worry about extra structure here, and we assume Hausdorff.



$C[0,1] \subset L^2[0,1]$

∃ : exists a metric

metrizable TVS = TVS with countable local base

normable TVS = TVS with a convex bounded nbhd

Fréchet space = countable sep seminorms $\{p_n\}$ + ∃ cpl tran-inv metric

or: the induced metric is cpl.

= loc convex + ∃ cpl tran-inv metric

Rmk. There are two definitions of boundedness in metrizable TVS X, and they coincide if the metric is translation-invariant.

Def. (boundness for TVS)

$E \subset X$ is bounded if $\forall\, 0 \in \mathcal{U}$ open, $\exists s > 0$ s.t.
$\forall t > s$, $E \subset t\mathcal{U}$

In this case,

$$E \text{ is bounded} \iff \left[\begin{array}{c} \forall \{x_n\} \subset E, \quad \{\alpha_n\} \subseteq \mathbb{R} \text{ or } \mathbb{C}, \\ \alpha_n \to 0 \Rightarrow \alpha_n x_n \to 0 \end{array} \right]$$

Def. (boundness for metric space)

Fix $x_0 \in X$. boundness does not depend on x_0.

$E \subset X$ is bounded if $\exists\, r > 0$, $E \subset B_{x_0}(r)$.

Rmk. 1. For loc convex TVS with metric, all open balls are convex.

2. cpl metric space $\Rightarrow$ 2^{nd} category.