Eine Woche, ein Beispiel

11.19. Basic sheaf calculation

Goal: Motivate $f_*, f^*, f_!, f^!$ by connecting them with (co)homology theory

After story: $\rightsquigarrow$ calculation of $\mathrm{Perv}_\Delta(\mathbb{C}\mathbb{P}^1)$
$\rightsquigarrow$ generalize Morse theory
$\rightsquigarrow$ Characteristic classes / cycles
$\rightsquigarrow$ index theorem

Minor advantages from my talk:

- offers examples for derived category.
  (more geometrical compared with examples about quiver reps)
- the first step toward 6-fctor formalism.
  - formal nonsense: adjointness, open-closed, SES(triangles)
  - application: Riemann-Roch, Serre duality, index theorem (guess)
    $\rightsquigarrow$ understand cpt RS, Weil conj, ...
  - glue: open-closed, cellular fibration, Morse theory, ...
    covering: (étale) descent, ramification, ...
    Three types: closed immersion, submersion, covering.

Usual setting: $X \in \mathrm{Top}$

$\mathrm{Obj}(\mathrm{Sh}(X)) = \{\text{sheaves of abelian gps}\}$

e.g. $\mathrm{Sh}(\{*\}) = \mathrm{Abel}$ $\qquad \underline{\mathbb{Q}}_{\{*\}} \longleftrightarrow \underline{\mathbb{Q}}$

0. sheaf
1. f_*, skyscraper sheaf & global sections
2. f^*, constant sheaf & stalks
3. Rf_* & cohomology
4. $f_!$ & global sections with cpt supp
5. $Rf_!$ & cohomology with cpt supp
6. $f^!$ & homology
   Poincaré duality.