Eine Woche, ein Beispiel

8.15 indecomposable representation of Dynkin quiver.

AR-quiver is a powerful tool considering about the indecomposable modules and relations among them. Using the AR-quiver, one can find(not totally serious):
- all the indecomposable modules;
- all the morphisms between these indecomposable modules;
- all the irreducible morphisms and AR-sequences;

However, it's not easy to see the coker and ker of some morphisms given by the AR-quiver.

The following AR-quiver pictures are now useless, since everyone can get better pictures at https://www.math.uni-bielefeld.de/~wcrawley/#knitting.

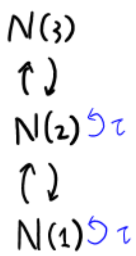
not for Dynkin quiver

Unfortunately, the knitting process can not draw some AR-quivers even in the case where "there are finite iso class of indec modules of quiver"

e.g.

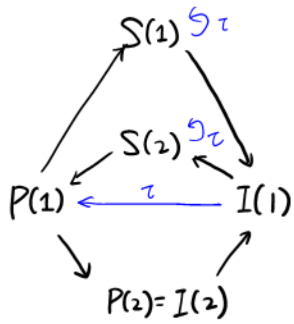
$A = K[T]/(T^3) \cong KQ/(a^3)$

$Q: \quad 1 \circlearrowleft a$



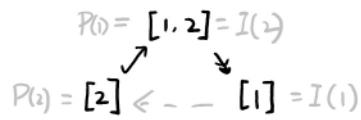
$A = KQ/(ab)$ $\quad \rightleftarrows = 0$

$Q: \quad 1 \underset{b}{\overset{a}{\rightleftarrows}} 2$



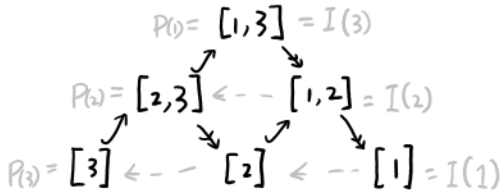
A_2 $1 \to 2$

(1)



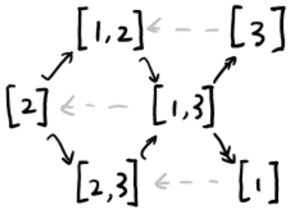
A_3 $1 \to 2 \to 3$

(2)



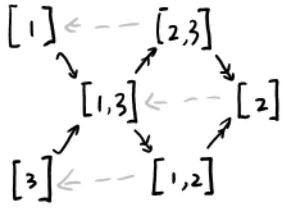
$1 \to 2 \leftarrow 3$

(1,1)



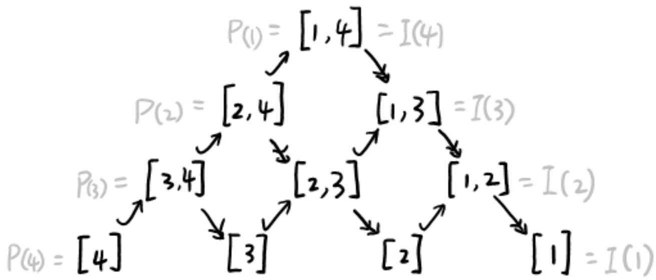
$1 \leftarrow 2 \to 3$

(1,1)*



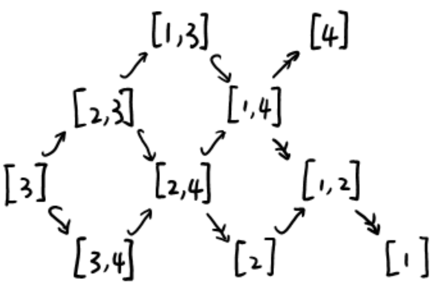
A_4 $1 \to 2 \to 3 \to 4$

(3)



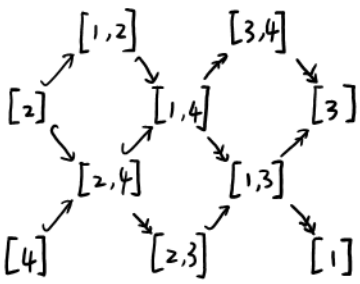
$1 \to 2 \to 3 \leftarrow 4$

(2,1)



$1 \to 2 \leftarrow 3 \to 4$

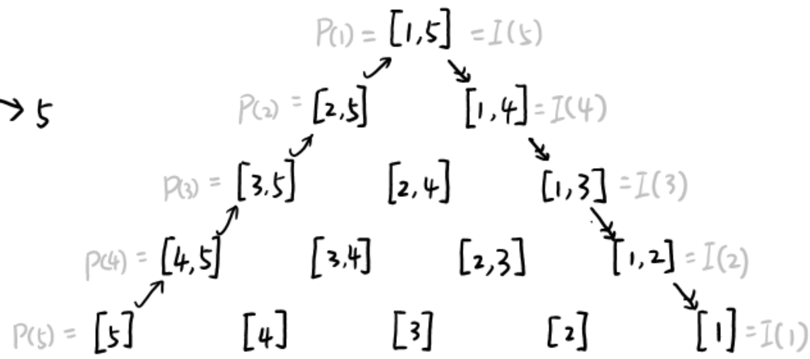
(1,1,1)



A_5

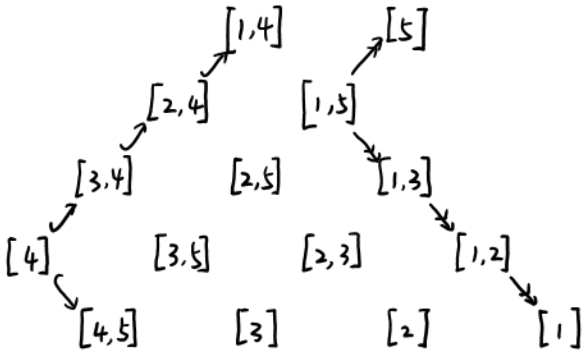
$1 \to 2 \to 3 \to 4 \to 5$

(4)



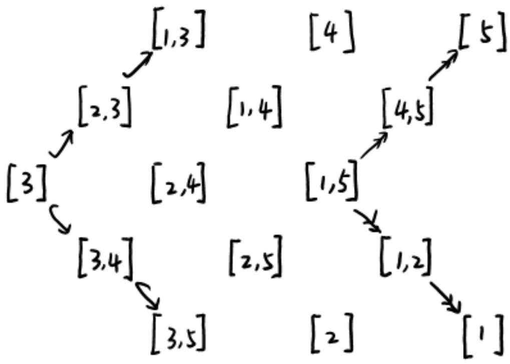
$1 \to 2 \to 3 \to 4 \leftarrow 5$

(3,1)



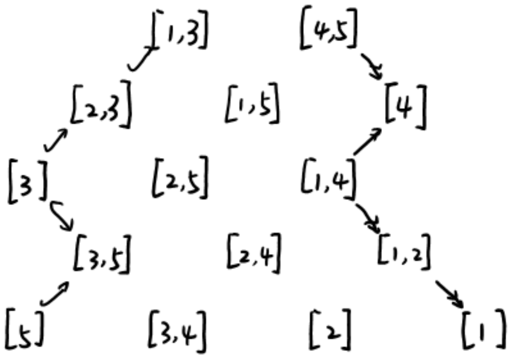
$1 \to 2 \to 3 \leftarrow 4 \leftarrow 5$

(2,2)



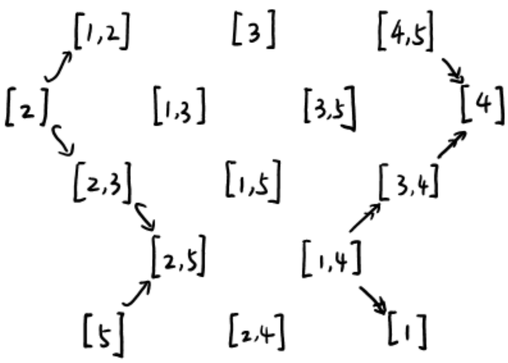
$1 \to 2 \to 3 \leftarrow 4 \to 5$

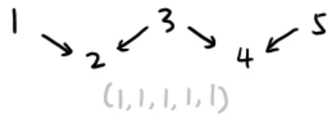
(2,1,1)

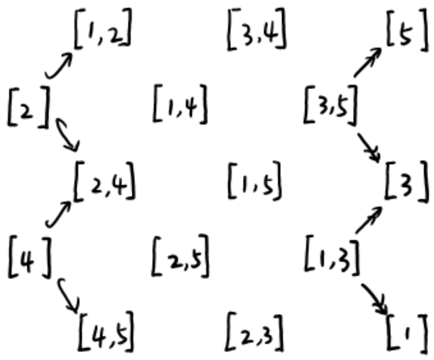


$1 \to 2 \leftarrow 3 \leftarrow 4 \to 5$

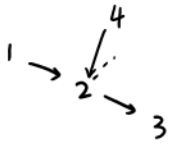
(1,2,1)

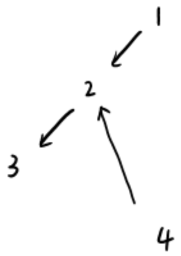


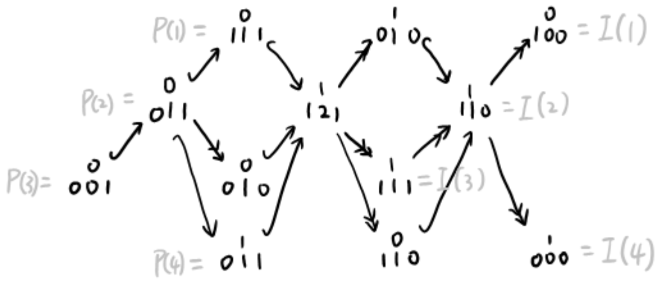

1
2
3
4
5
(1, 1, 1, 1, 1)

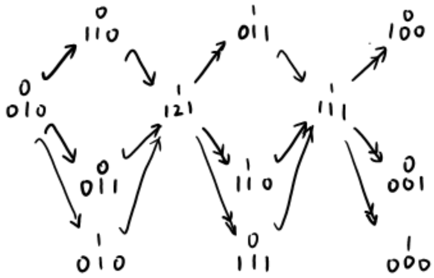

[1, 2]
[3, 4]
[5]
[2]
[1, 4]
[3, 5]
[2, 4]
[1, 5]
[3]
[4]
[2, 5]
[1, 3]
[4, 5]
[2, 3]
[1]

D_4

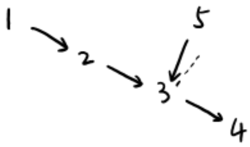


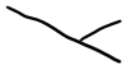


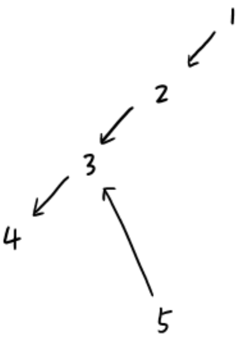


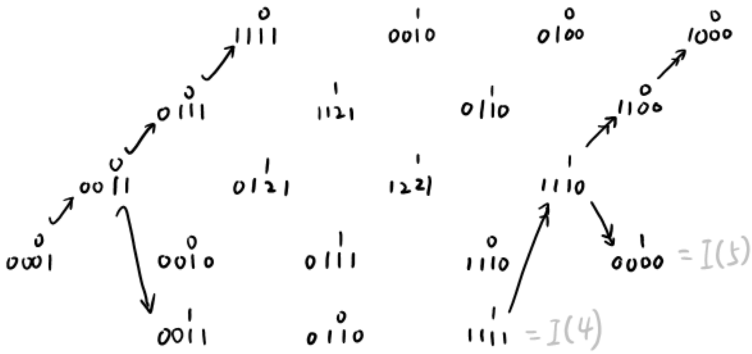


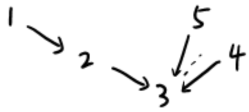
D_5

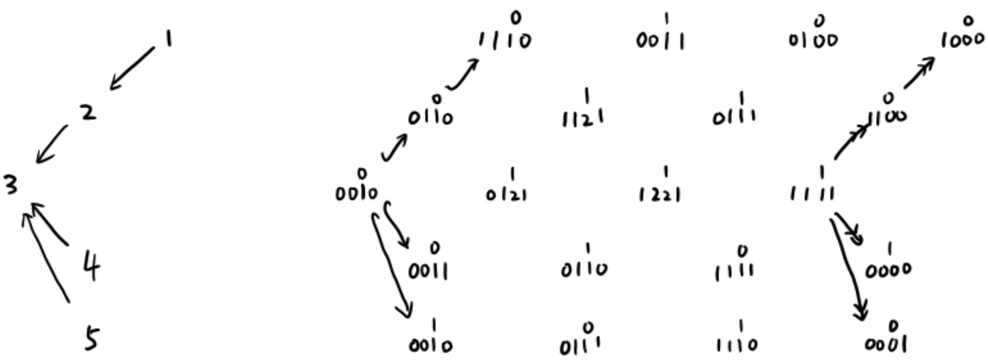


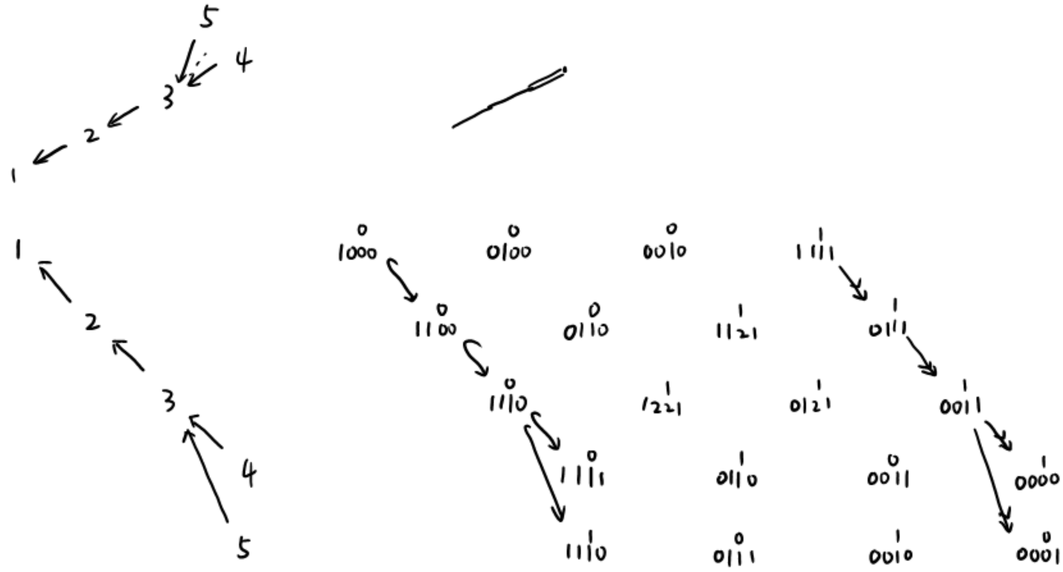


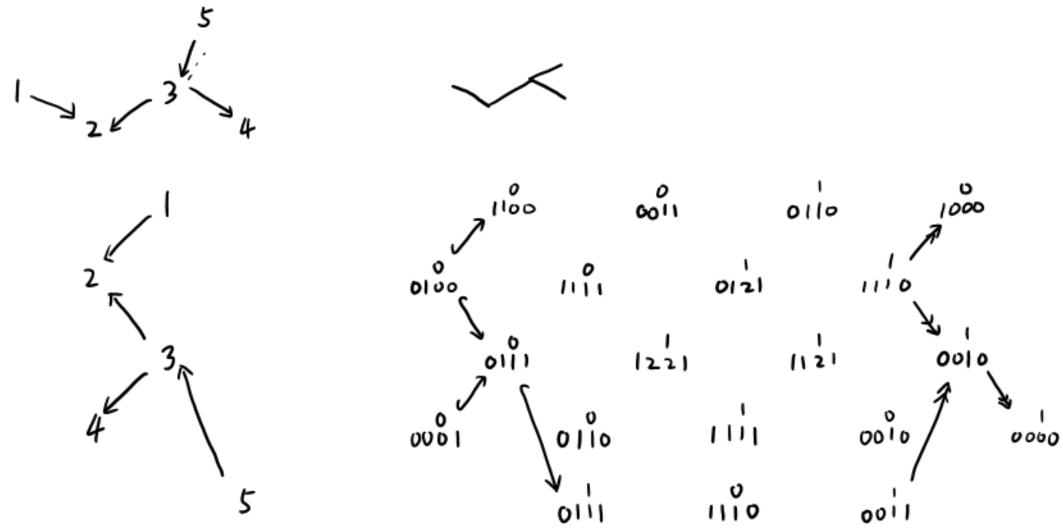


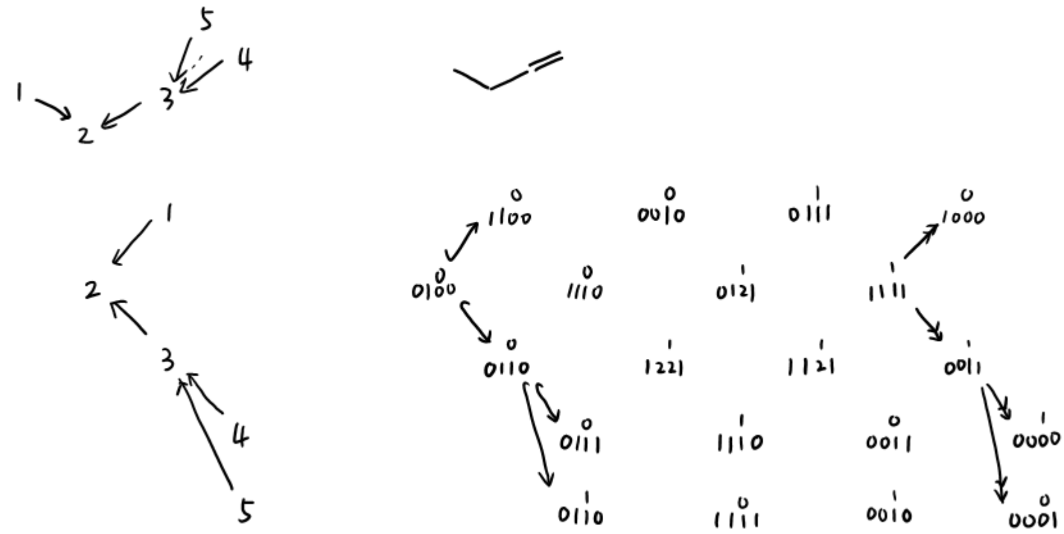


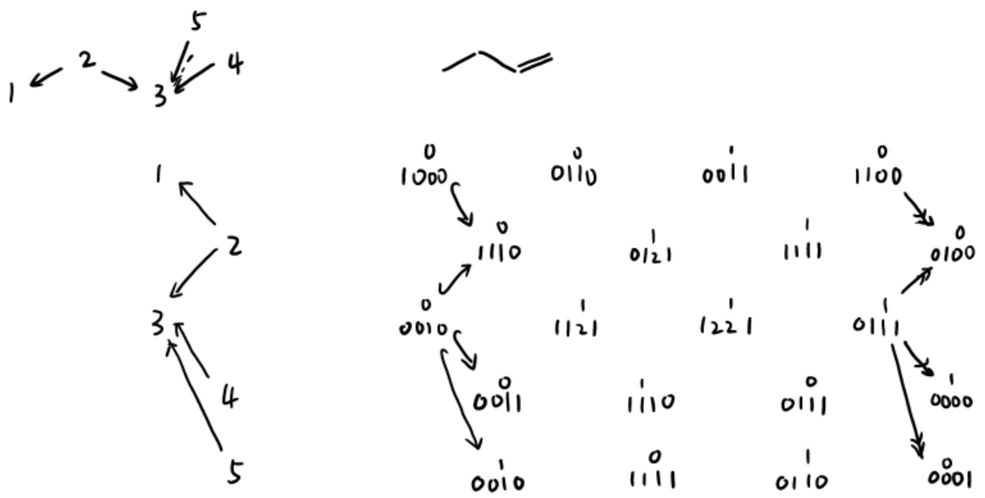




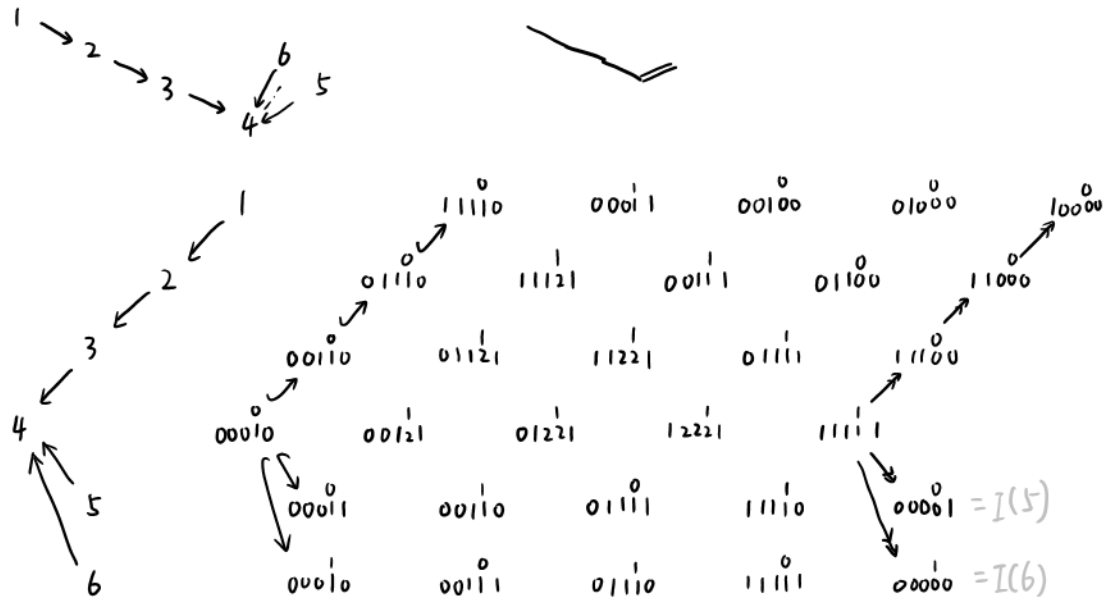






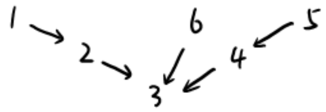


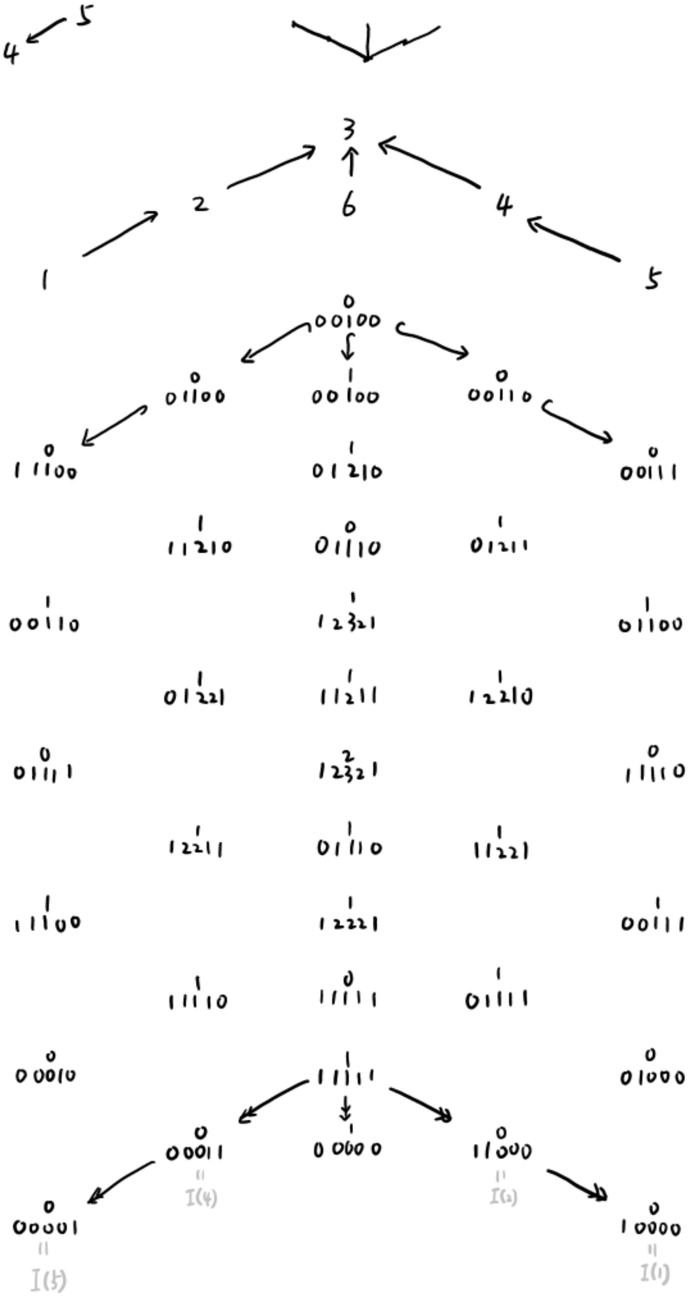
D_6



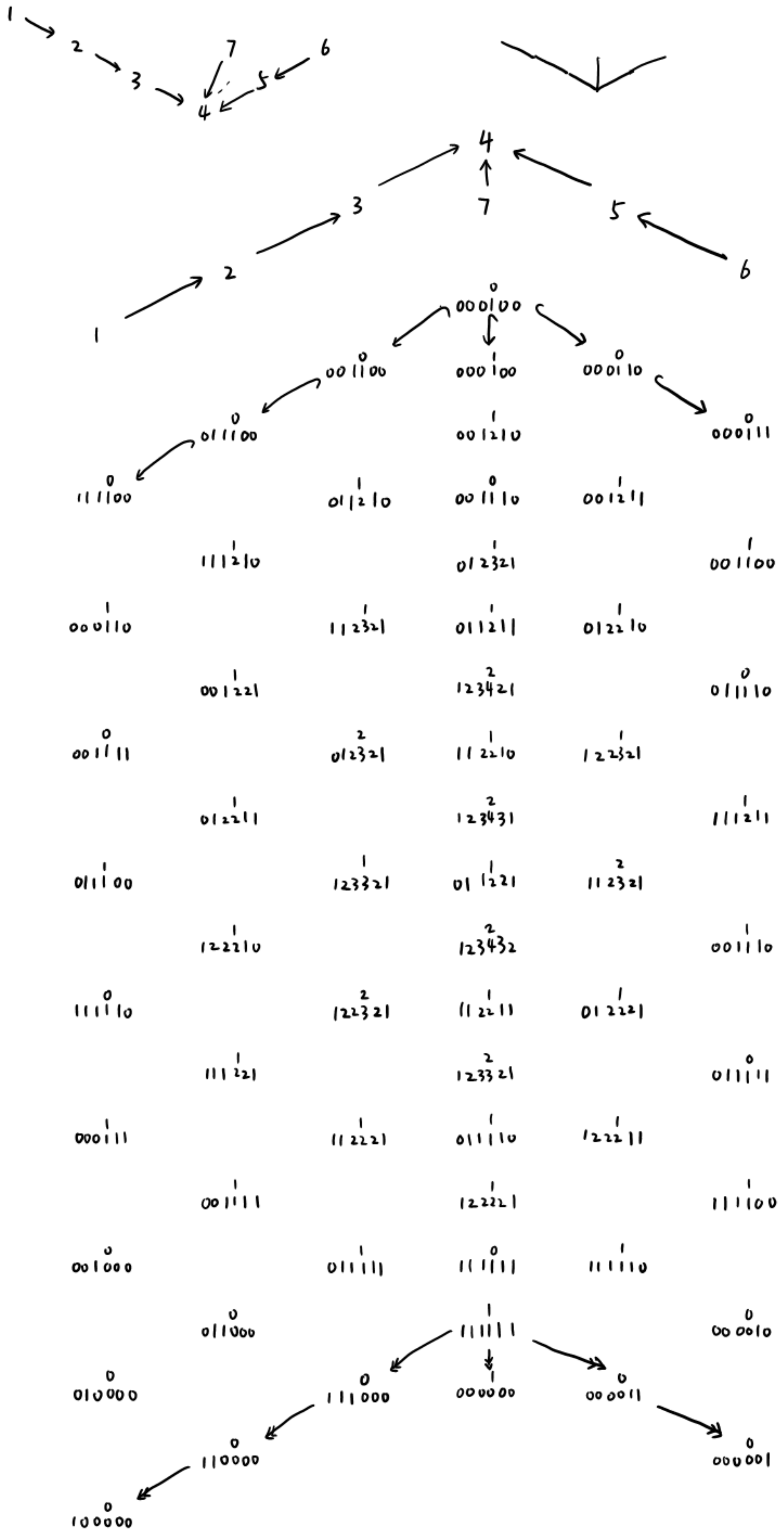
D_7

E_6

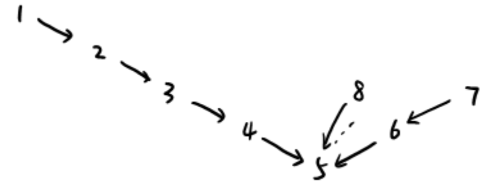


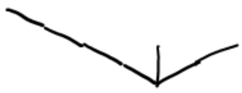


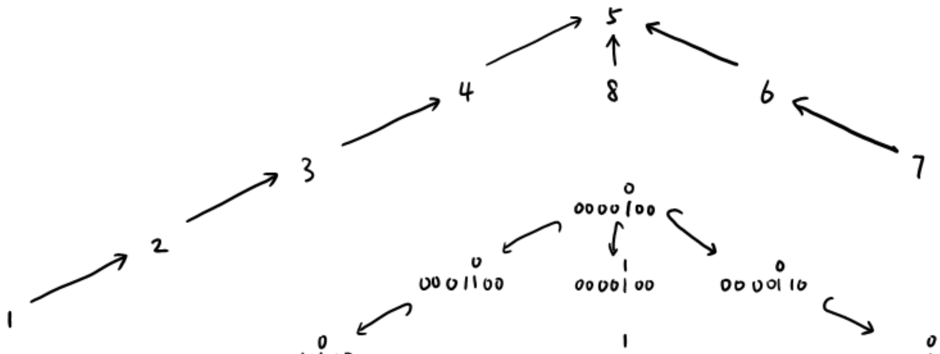
E_7



E_8







$\overset{0}{0000100}$

$\overset{0}{0001100}$ $\overset{1}{0000100}$ $\overset{0}{0000010}$

$\overset{0}{0011100}$ $\overset{1}{0001210}$ $\overset{0}{0000011}$

$\overset{0}{0111100}$ $\overset{1}{0011210}$ $\overset{0}{0001110}$ $\overset{1}{0001211}$

$\overset{0}{1111100}$ $\overset{1}{0111210}$ $\overset{1}{0012321}$ $\overset{1}{0001100}$

$\overset{1}{1111210}$ $\overset{1}{0112321}$ $\overset{1}{0011211}$ $\overset{1}{0012210}$

$\overset{1}{0000110}$ $\overset{1}{1112321}$ $\overset{2}{0123421}$ $\overset{0}{0011110}$

$\overset{1}{0001221}$ $\overset{2}{1123421}$ $\overset{1}{0112210}$ $\overset{1}{0122321}$

$\overset{0}{0001111}$ $\overset{2}{0012321}$ $\overset{2}{1234531}$ $\overset{1}{0111211}$

$\overset{1}{0012211}$ $\overset{2}{0123431}$ $\overset{1}{1122321}$ $\overset{2}{1223421}$

$\overset{1}{0011100}$ $\overset{1}{0123321}$ $\overset{3}{1234642}$ $\overset{1}{1112210}$

$\overset{1}{0122210}$ $\overset{2}{1234532}$ $\overset{2}{0112321}$ $\overset{2}{1123431}$

$\overset{0}{0111110}$ $\overset{2}{1233421}$ $\overset{3}{1235642}$ $\overset{1}{0011221}$

$\overset{1}{1222321}$ $\overset{3}{1234531}$ $\overset{1}{1123321}$ $\overset{2}{0123432}$

$\overset{1}{1111211}$ $\overset{2}{1223431}$ $\overset{3}{1245642}$ $\overset{1}{0112211}$

$\overset{2}{1112321}$ $\overset{2}{1234542}$ $\overset{2}{0122321}$ $\overset{2}{1234421}$

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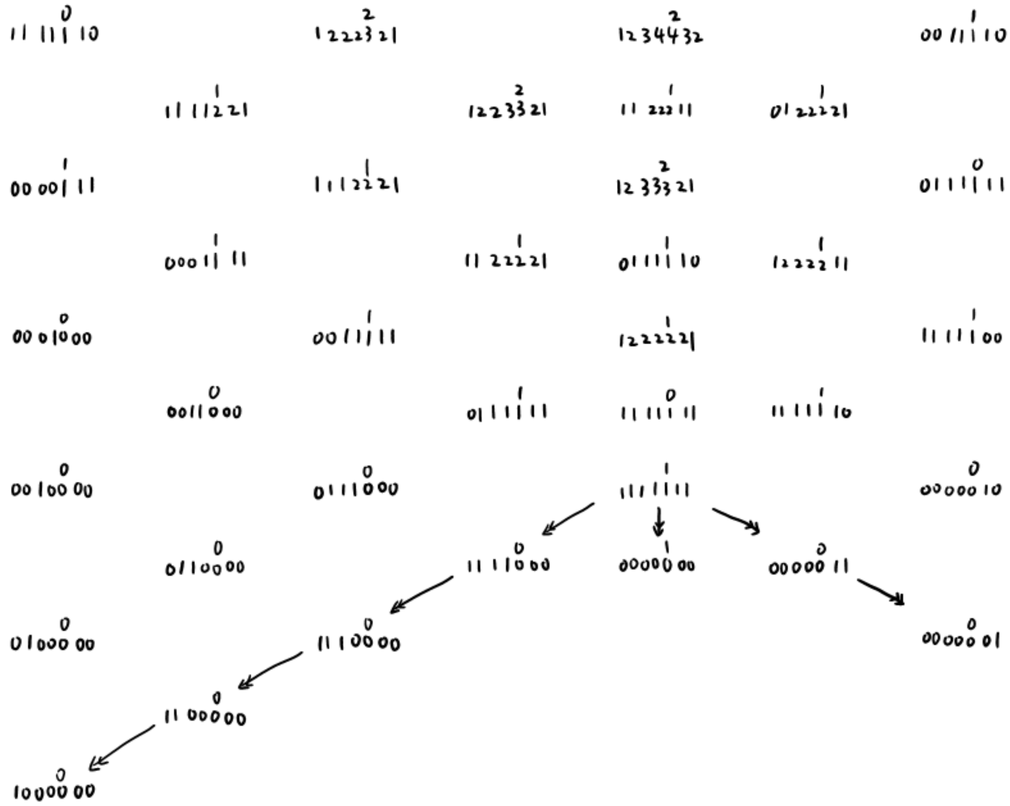
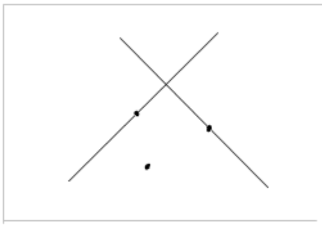
$\overset{1}{1222210}$ $\overset{2}{1233432}$ $\overset{1}{0112221}$ $\overset{2}{1123321}$



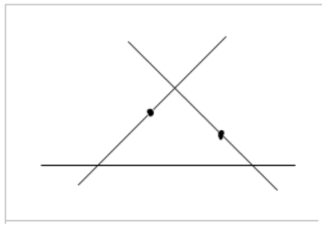
Bonus: subspace case (projective space version)

$\begin{smallmatrix} & 1 & \\ 1 & 2 & 1 \end{smallmatrix}$

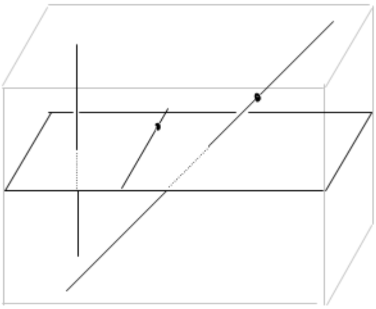
$\begin{smallmatrix} & & 1 & & \\ 1 & 2 & 3 & 2 & 1 \end{smallmatrix}$



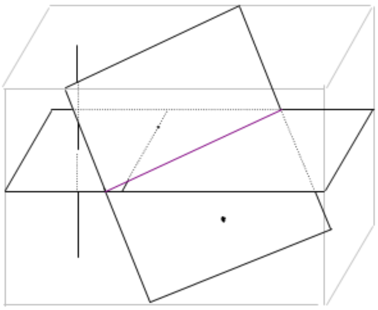
$\begin{smallmatrix} & & 2 & & \\ 1 & 2 & 3 & 2 & 1 \end{smallmatrix}$



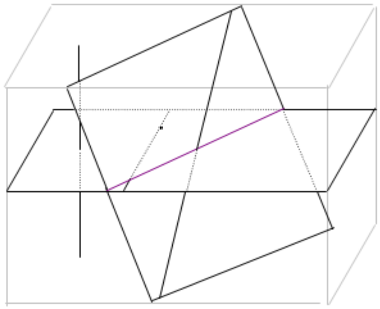
$\begin{smallmatrix} & & & 2 & & \\ 1 & 2 & 3 & 4 & 2 & 1 \end{smallmatrix}$



$\begin{smallmatrix} & & & 2 & & \\ 1 & 2 & 3 & 4 & 3 & 1 \end{smallmatrix}$

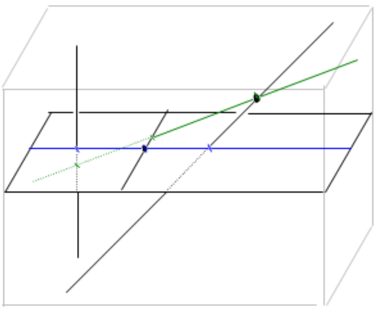


$\begin{smallmatrix} & & & 2 & & \\ 1 & 2 & 3 & 4 & 3 & 2 \end{smallmatrix}$



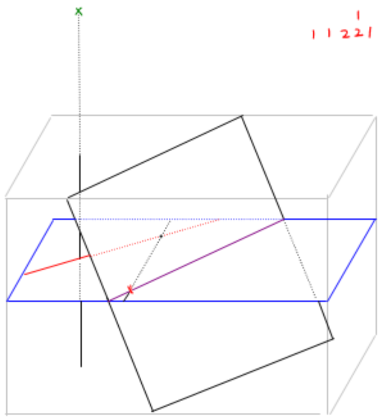
These shapes should be as general as possible, otherwise it may be not indecomposable:

e.g. "$\begin{smallmatrix} & & & 2 & & \\ 1 & 2 & 3 & 4 & 2 & 1 \end{smallmatrix}$" $=$ $\begin{smallmatrix} & & & 1 & & \\ 1 & 1 & 2 & 2 & 1 & 0 \end{smallmatrix}$ $\oplus$ $\begin{smallmatrix} & & & 1 & & \\ 0 & 1 & 1 & 2 & 1 & 1 \end{smallmatrix}$

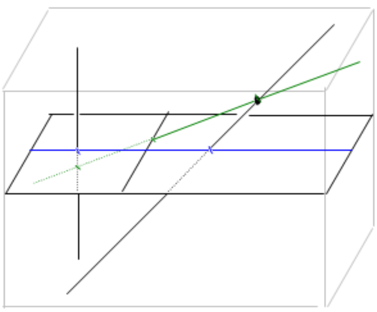


"$\begin{smallmatrix} & & & 2 & \\ 1 & 2 & 3 & 4 & 3 \end{smallmatrix}$" $=$ "$\begin{smallmatrix} & & & 1 & \\ 1 & 2 & 3 & 3 & 2 \end{smallmatrix}$" $\oplus$ $\begin{smallmatrix} & & & 1 & \\ 0 & 0 & 0 & 1 & 1 \end{smallmatrix}$

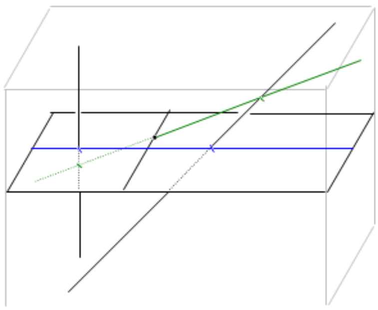
$\begin{smallmatrix} & & & 1 & \\ 1 & 1 & 2 & 2 & 1 \end{smallmatrix}$ " $\oplus$ $\begin{smallmatrix} & & & 0 & \\ 0 & 1 & 1 & 1 & 1 \end{smallmatrix}$



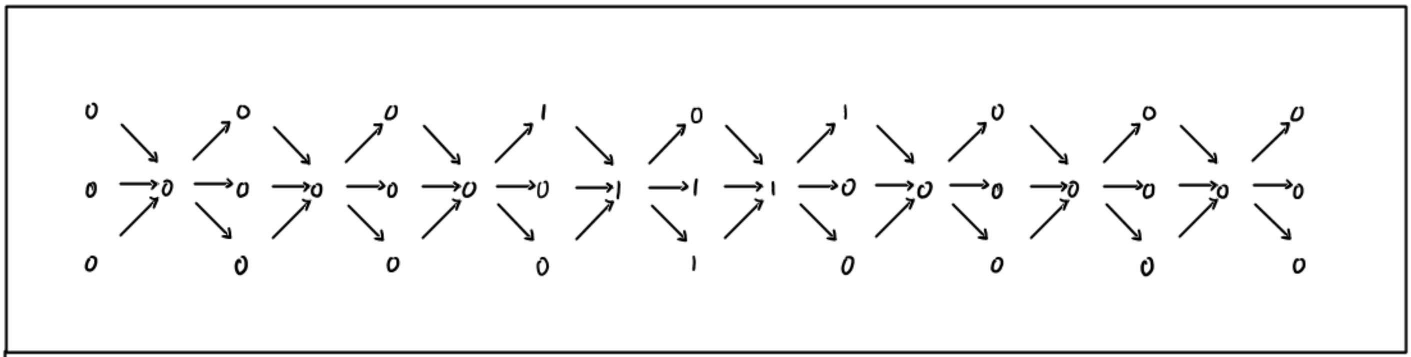
"$\begin{smallmatrix} & & 2 & & \\ 2 & 3 & 4 & 2 & 1 \end{smallmatrix}$" $=$ $\begin{smallmatrix} & & 1 & & \\ 1 & 2 & 2 & 1 & 0 \end{smallmatrix}$ $\oplus$ $\begin{smallmatrix} & & 1 & & \\ 1 & 1 & 2 & 1 & 1 \end{smallmatrix}$



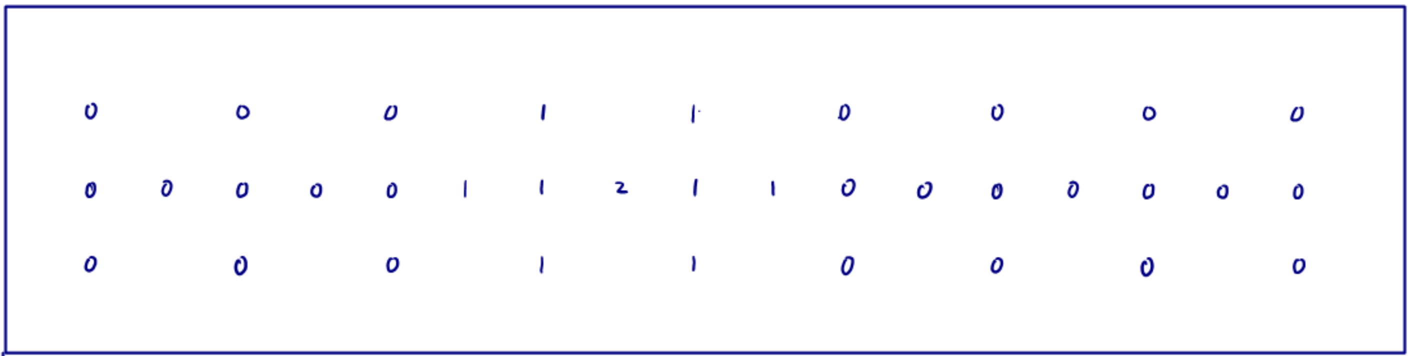
"$\begin{smallmatrix} & & & 2 & \\ 1 & 2 & 3 & 4 & 2 \end{smallmatrix}$" $=$ $\begin{smallmatrix} & & & 1 & \\ 0 & 1 & 2 & 2 & 1 \end{smallmatrix}$ $\oplus$ $\begin{smallmatrix} & & & 1 & \\ 1 & 1 & 1 & 2 & 1 \end{smallmatrix}$



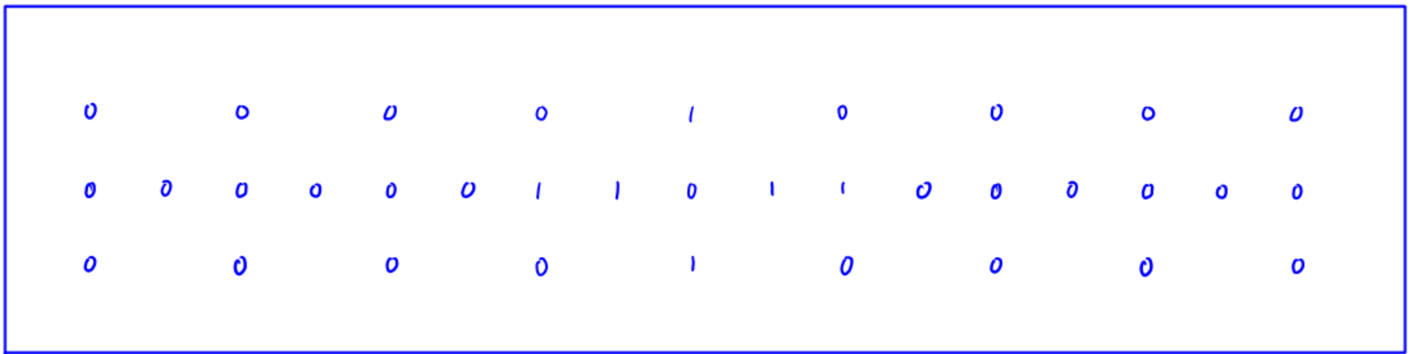
It's not easy to read the informations of them, but AR-quivers can.



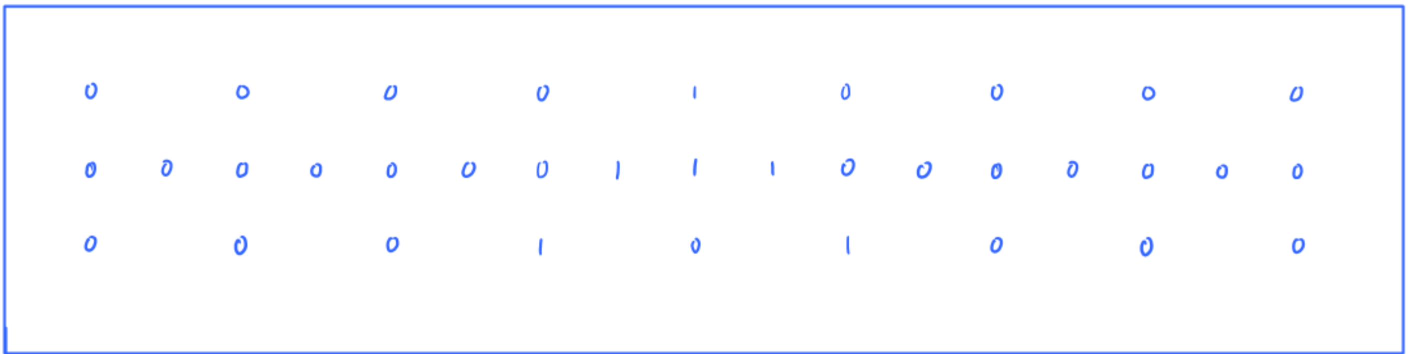
P(1)



P(2)



P(3)



P(4)

AR-quiver cards : made in China