# Eine Woche, ein Beispiel
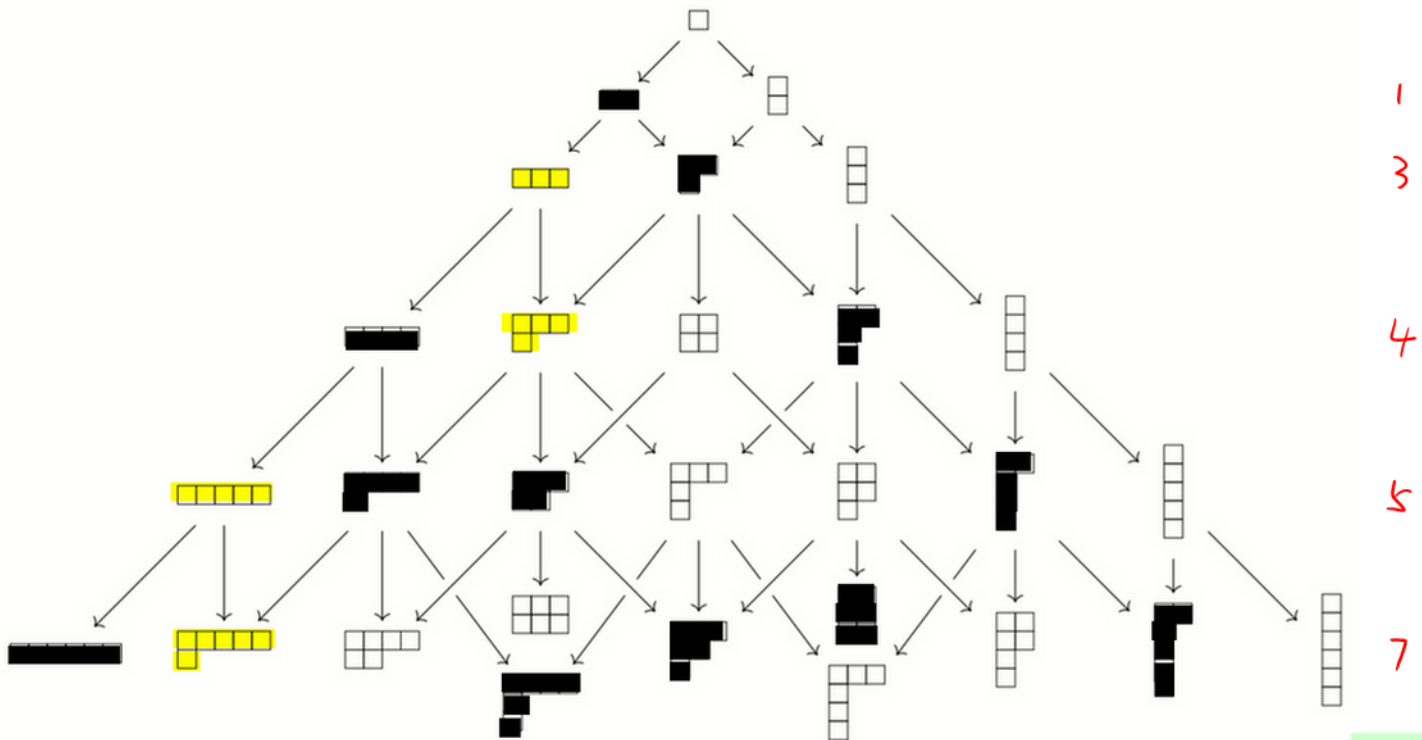**7.25 irreducible representation of** $S_n / A_n$

1

3

4

5

7
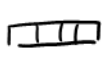
## $S_2 / A_2$
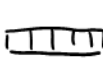
| | $\boxed{\square\atop\square}$  $1^2$  $1$ | $\boxed{\square\square}$  $2'$  $(12)$ |
|---|---|---|
| $\boxed{\square\atop\square}$ | $1$  $1$  $1$ | $-1$  $1$ |

## $S_3 / A_3$

| | $\boxed{\square\atop\square\atop\square}$  $1^3$  $Id$ | $\boxed{\square\square\square}$  $3'$  $(123)$   $(132)$ | $\boxed{\square\atop\square}'$  $2\cdot1$  $(12)$ |
|---|---|---|---|
| $\boxed{\square\atop\square}$ | $1$  $1$  $1$ | $1$  $1$  $1$   $1$  $1$ | $-1$  $1$ |
| $\boxed{\square\square\atop\square}$ | $2$  $1$  $1$ | $-1$  $\rho$  $\rho^2$   $\rho^2$  $\rho$ | $0$ |

## $S_4/A_4$

| | $1^4$ Id | $2^2$ (12)(34) | $3\cdot1$ (123) (132) | $2\cdot1^2$ (12) | $4^1$ (1234) |
|---|---|---|---|---|---|
| | 1 | 1 | 1 | $-1$ | $-1$ |
| | 1 | 1 | 1 | 1 | 1 |
| | <span style="color:red">1</span> | <span style="color:red">1</span> | <span style="color:red">1</span>   <span style="color:red">1</span> | | |
| | 3 | $-1$ | 0 | $-1$ | 1 |
| | 3 | $-1$ | 0 | 1 | $-1$ |
| | <span style="color:red">3</span> | <span style="color:red">$-1$</span> | <span style="color:red">0</span>   <span style="color:red">0</span> | | |
| | 2 | 2 | $-1$ | 0 | 0 |
| | <span style="color:red">1</span> | <span style="color:red">1</span> | <span style="color:red">$\rho^2$</span>   <span style="color:red">$\rho$</span> | | |

## $S_5/A_5$

| | $1^5$ Id | $2^2\cdot1$ (12)(34) | $3\cdot1^2$ (123) | $5^1$ (12345) (12345) | $2\cdot1^3$ (12) | $3\cdot2$ (123)(45) | $4\cdot1$ (1234) |
|---|---|---|---|---|---|---|---|
| | 1 | 1 | 1 | 1 | $-1$ | $-1$ | $-1$ |
| | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | <span style="color:red">1</span> | <span style="color:red">1</span> | <span style="color:red">1</span> | <span style="color:red">1</span>  <span style="color:red">1</span> | | | |
| | 4 | 0 | 1 | $-1$ | $-2$ | 1 | 0 |
| | 4 | 0 | 1 | $-1$ | 2 | $-1$ | 0 |
| | <span style="color:red">4</span> | <span style="color:red">0</span> | <span style="color:red">1</span> | <span style="color:red">$-1$</span>  <span style="color:red">$-1$</span> | | | |
| | 5 | 1 | $-1$ | 0 | $-1$ | $-1$ | 1 |
| | 5 | 1 | $-1$ | 0 | 1 | 1 | $-1$ |
| | <span style="color:red">5</span> | <span style="color:red">1</span> | <span style="color:red">$-1$</span> | <span style="color:red">0</span>  <span style="color:red">0</span> | | | |
| | 6 | $-2$ | 0 | 1 | 0 | 0 | 0 |
| | <span style="color:red">3</span> | <span style="color:red">$-1$</span> | <span style="color:red">0</span> | <span style="color:red">$\frac{1+\sqrt5}{2}$</span>  <span style="color:red">$\frac{1-\sqrt5}{2}$</span> | | | |
| | <span style="color:red">3</span> | <span style="color:red">$-1$</span> | <span style="color:red">0</span> | <span style="color:red">$\frac{1-\sqrt5}{2}$</span>  <span style="color:red">$\frac{1+\sqrt5}{2}$</span> | | | |

# $S_6/A_6$ — G: outer automorphism action on representation.

| | $1^6$ Id | $2^2\cdot1^2$ (12)(34) | $3\cdot1^3$ (123) | $3^2$ (123)(456) | $4\cdot2$ (1234)(56) | $5\cdot1$ (12345) / (12354) | $2\cdot1^4$ (12) | $2^3$ (12)(34)(56) | $3\cdot2\cdot1$ (123)(45) | $4\cdot1^2$ (1234) | $6$ (123456) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| G ⌐ (sign) | 1 | 1 | 1 | 1 | 1 | 1 | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ |
| G ⌐ (triv) | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $A_6$ | 1 | 1 | 1 | 1 | 1 | 1 | | | | | |
| | 5 | 1 | 2 | $-1$ | $-1$ | 0 | $-3$ | 1 | 0 | $-1$ | 1 |
| | 5 | 1 | 2 | $-1$ | $-1$ | 0 | 3 | $-1$ | 0 | 1 | $-1$ |
| $A_6$ | 5 | 1 | 2 | $-1$ | $-1$ | 0 / 0 | | | | | |
| G | 9 | 1 | 0 | 0 | 1 | $-1$ | $-3$ | $-3$ | 0 | 1 | 0 |
| G | 9 | 1 | 0 | 0 | 1 | $-1$ | 3 | 3 | 0 | $-1$ | 0 |
| $A_6$ | 9 | 1 | 0 | 0 | 1 | $-1$ / $-1$ | | | | | |
| | 10 | $-2$ | 1 | 1 | 0 | 0 | $-2$ | 2 | 1 | 0 | $-1$ |
| | 10 | $-2$ | 1 | 1 | 0 | 0 | 2 | $-2$ | $-1$ | 0 | 1 |
| $A_6$ | 10 | $-2$ | 1 | 1 | 0 | 0 / 0 | | | | | |
| | 5 | 1 | $-1$ | 2 | $-1$ | 0 | $-1$ | 3 | $-1$ | 1 | 0 |
| | 5 | 1 | $-1$ | 2 | $-1$ | 0 | 1 | $-3$ | 1 | $-1$ | 0 |
| $A_6$ | 5 | 1 | $-1$ | 2 | $-1$ | 0 / 0 | | | | | |
| G | 16 | 0 | $-2$ | $-2$ | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| $A_6$ | 8 | 0 | $-1$ | $-1$ | 0 | $\frac{1+\sqrt5}{2}$ / $\frac{1-\sqrt5}{2}$ | | | | | |
| $A_6$ | 8 | 0 | $-1$ | $-1$ | 0 | $\frac{1-\sqrt5}{2}$ / $\frac{1+\sqrt5}{2}$ | | | | | |

We use Frobenius character formula to compute characters of S_n. For a clear statement of Frobenius character formula, see here:
https://mathoverflow.net/questions/323949/frobenius-formula

After this, we can compute characters of A_n by using the theories in Fulton's book [Rep, Lec 5.1].

For a deeper research on the symmetry of the representation table, see here: http://www.ma.rhul.ac.uk/~uvah099/Maths/labels.pdf

To compute character tables of S_n in computer, see
https://mathoverflow.net/questions/162478/character-table-of-s-7/162482

For the relationship of representations of subgroup of index 2, see
https://math.stackexchange.com/questions/1965967/decomposition-of-induced-representation-from-index-2-subgroup
[Theorem 23.3,Clifford's Theorem][https://www.mathematik.uni-kl.de/~lassueur/en/teaching/DTWS1920/DT1920/SkriptMD1920.pdf]