Eine Woche, ein Beispiel

10.29. hermitian metric

Goal. Give examples of

- hermitian metric of $\mathcal{O}(n)$ on $\mathbb{P}_{\mathbb{C}}^{1}$ } Fubini-study metric
- hermitian metric of $\Omega_{\mathbb{P}_{\mathbb{C}}^{n}/\mathbb{C}}$ on $\mathbb{P}_{\mathbb{C}}^{n}$
- hermitian metric of curves of genus ≥ 1 Arakelov Kähler form
- hermitian metric of proj space,

  eg. Klein quartic curve, cubic surface, etc.

E.g. on $\mathbb{P}_{\mathbb{C}}^{2}$ $U = [1 : z_1 : z_2]$

$$[g_{i\bar{j}}] = \frac{1}{(1+|z_1|^2+|z_2|^2)^2} \begin{bmatrix} 1+|z_2|^2 & -\bar{z}_1 z_2 \\ -\bar{z}_2 z_1 & 1+|z_1|^2 \end{bmatrix}$$

$$\partial_i \bar{\partial}_j \log(|z_0|^2 + |z_1|^2 + |z_2|^2)$$

$$= \partial_i \frac{\bar{z}_j}{|z_0|^2+|z_1|^2+|z_2|^2} d\bar{z}_j$$

$$= \frac{\delta_{ij}(|z_0|^2+|z_1|^2+|z_2|^2) - \bar{z}_i z_j}{(|z_0|^2+|z_1|^2+|z_2|^2)^2} dz_i d\bar{z}_j$$