Eine Woche, ein Beispiel

9.18 reps of p-adic groups

This is also an unfinished task. I'm afraid that I forget those materials I organized.
main ref: The Local Langlands Conjecture for GL(2)

Process

1. new notations
2. preliminaries
   - group
   - chain order
3. statement of classification (without proof)

realization

4. fin dim
   https://mathoverflow.net/questions/34374/any-finite-dimensional-admissiblesmooth-irreducible-representation-of-gl2-q-p
5. other principal series
   - construction
   - proof
6. Cuspidal reps
7. Applications

1. new notations

I don't want to bother you or make you confused, so I collect my notations here. Often it's not rigorous defined. You can view this section as a dictionary of notations.

https://math.stackexchange.com/questions/299626/the-center-of-operatornamegln-k

from 2022.04.24

F: non-archi local field.

$A = M_{2\times 2}(F)$ $\quad G = GL_2(F)$

$B = \begin{pmatrix} * & * \\ 0 & * \end{pmatrix}$ $\quad T = \begin{pmatrix} * & 0 \\ 0 & * \end{pmatrix}$ $\quad N = \begin{pmatrix} 1 & * \\ 0 & 1 \end{pmatrix}$ $\quad Z = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} = Z(G)$ $\quad S = \begin{pmatrix} * & 0 \\ 0 & 1 \end{pmatrix}$

$w = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\quad T^0 = \begin{pmatrix} \mathcal{O}^\times & 0 \\ 0 & \mathcal{O}^\times \end{pmatrix}$ $\quad N_j = \begin{pmatrix} 1 & \mathfrak{p}^j \\ 0 & 1 \end{pmatrix}$ $\quad N_j' = \begin{pmatrix} 1 & 0 \\ \mathfrak{p}^j & 1 \end{pmatrix}$

page	name	symbol	case $e_{\mathfrak{A}} = 1$	case $e_{\mathfrak{A}} = 2$
86	$\mathcal{O}$-lattice chain	$\mathcal{L}$	$\mathfrak{p}^{-1}\oplus\mathfrak{p}^{-1} \subseteq \mathcal{O}\oplus\mathcal{O} \subseteq \mathfrak{p}\oplus\mathfrak{p} \subseteq \cdots$	$\mathfrak{p}^{-1}\oplus\mathcal{O} \subseteq \mathcal{O}\oplus\mathcal{O} \subseteq \mathcal{O}\oplus\mathfrak{p} \subseteq \cdots$
87	$\mathcal{O}$ - orders chain order	$\mathfrak{A} = \mathfrak{A}_{\mathcal{L}}$	$\mathfrak{M} = \begin{pmatrix} \mathcal{O} & \mathcal{O} \\ \mathcal{O} & \mathcal{O} \end{pmatrix}$	$\mathfrak{J} = \begin{pmatrix} \mathcal{O} & \mathcal{O} \\ \mathfrak{p} & \mathcal{O} \end{pmatrix}$
88	prime element	Π	$\begin{pmatrix} \pi & \\ & \pi \end{pmatrix}$	$\begin{pmatrix} & 1 \\ \pi & \end{pmatrix}$
88	Jacobson radical	$Jac(\mathfrak{A})$	$Jac(\mathfrak{M}) = \begin{pmatrix} \mathfrak{p} & \mathfrak{p} \\ \mathfrak{p} & \mathfrak{p} \end{pmatrix}$	$Jac(\mathfrak{J}) = \begin{pmatrix} \mathfrak{p} & \mathcal{O} \\ \mathfrak{p} & \mathfrak{p} \end{pmatrix}$
88		$U_{\mathfrak{A}} = U_{\mathfrak{A}}^0 = \mathfrak{A}^\times$	$K_0 = \begin{pmatrix} \mathcal{O} & \mathcal{O} \\ \mathcal{O} & \mathcal{O} \end{pmatrix}^\times$	$I_0 = \begin{pmatrix} \mathcal{O} & \mathcal{O} \\ \mathfrak{p} & \mathcal{O} \end{pmatrix}^\times = \begin{pmatrix} \mathcal{O}^\times & \mathcal{O} \\ \mathfrak{p} & \mathcal{O}^\times \end{pmatrix}$
88		$U_{\mathfrak{A}}^n = 1 + Jac(\mathfrak{A})^n$ $n \geq 1$	$K_n = 1 + \begin{pmatrix} \mathfrak{p}^n & \mathfrak{p}^n \\ \mathfrak{p}^n & \mathfrak{p}^n \end{pmatrix}$	$I_{2k-1} = 1 + \begin{pmatrix} \mathfrak{p}^k & \mathfrak{p}^{k-1} \\ \mathfrak{p}^k & \mathfrak{p}^k \end{pmatrix}$ $I_{2k} = 1 + \begin{pmatrix} \mathfrak{p}^k & \mathfrak{p}^k \\ \mathfrak{p}^{k+1} & \mathfrak{p}^k \end{pmatrix}$
89		$K_{\mathfrak{A}}$	$K_0 \rtimes \langle \begin{pmatrix} \pi & \\ & \pi \end{pmatrix} \rangle$	$I_0 \rtimes \begin{pmatrix} \mathfrak{p} & \mathcal{O} \\ \mathfrak{p} & \mathfrak{p} \end{pmatrix}$