Un exemple par jour

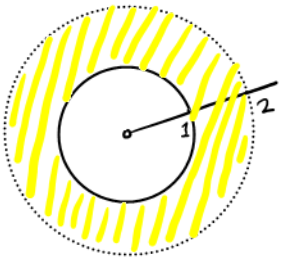
4.1. the complex torus of form $\mathbb{C}^\times/\mathbb{Z}\gamma$

$C := \mathbb{C}^\times/\mathbb{Z}\gamma \overset{\text{topo}}{=} T^2$ is a cpt Riemannian surface of genus 1.

$\gamma \in \mathrm{Aut}(\mathbb{C}^\times)$ $\gamma(z) = az$ $a \in \mathbb{C}^\times$ $|a| > 1$

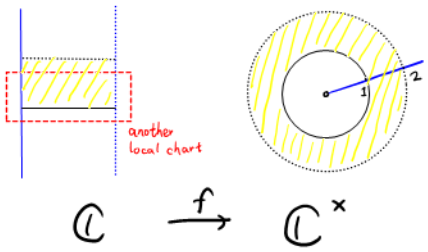
Today: $a = 2$

1. fundamental set:



$\Rightarrow$ only need 2 local chart

2.
$$\begin{array}{ccccccc} 0 & \to & \mathbb{Z} & \hookrightarrow & \mathbb{C} & \xrightarrow{f: z \mapsto e^{2\pi i z}} & \mathbb{C}^\times \to 1 \\ & & \downarrow{\scriptstyle +\frac{1}{2\pi i}\ln 2} & & \downarrow{\scriptstyle +\frac{1}{2\pi i}\ln 2} & & \downarrow{\scriptstyle \times 2} \\ 0 & \to & \mathbb{Z}+\frac{1}{2\pi i}\ln 2 & \to & \mathbb{C} & \longrightarrow & \mathbb{C}^\times \to 1 \end{array}$$



$\mathbb{C} \xrightarrow{f} \mathbb{C}^\times$

$\mathbb{C}^\times = \mathbb{C}/\mathbb{Z} \Rightarrow \mathbb{C}^\times/\mathbb{Z}\gamma = \mathbb{C}/(\mathbb{Z} \oplus \frac{1}{2\pi i}\ln 2\,\mathbb{Z})$

better: $a = e^{2\pi} \approx 535.49$

$a = e^{-2\pi i w} \approx -230.765$

3. line bundle on C

$b \in \mathbb{C}^\times$ $\mathcal{L}_b := \mathbb{C}^\times \times \mathbb{C}/(z,\xi) \sim (2z, b\xi)$ $\Rightarrow$ ① $\mathcal{L}_b \in \mathrm{Pic}_0(C)$; ($\mathcal{L}_b \overset{\text{cont. deformation}}{\sim} \mathcal{L}_1 \cong \mathcal{O}_C$)

$\downarrow$

$C = \mathbb{C}^\times/z \sim 2z$

② $\mathrm{Pic}_0(C) \overset{\text{(naive, base pt } 1 \in \mathbb{C}^\times/\mathbb{Z}\gamma)}{\cong} C = \mathbb{C}^\times/\mathbb{Z}\gamma$

$\mathcal{L}_b \longrightarrow b$

Reduced to: find a section s on $\mathcal{L}_b$ s.t $\mathrm{div}\, s = [b]-[1]$

Reduced to: find a meromorphic functions g on $\mathbb{C}^\times$ s.t

① $g(2z) = bg(z)$ $b \in \mathbb{C}^\times$, $b \neq 2^k$; e.g. $b = 3$

② g has simple poles on 2^n, and simple zeros on $2^n b$ $n \in \mathbb{Z}$

$b = e^{2\pi i c}$, $c \in \mathbb{C}$

$\tau := \frac{1}{2\pi i}\ln 2$

$w(z) = \frac{1}{2\pi i}\ln z$

$$g(z) := \frac{\theta\left[\begin{smallmatrix}1\\1-2c\end{smallmatrix}\right](w(z),\tau)}{\theta\left[\begin{smallmatrix}1\\1\end{smallmatrix}\right](w(z),\tau)}$$

is the required one.

Blue — example

Orange — more than this example

Red — important results

Purple — I don't know the answer/proof

Green — sketsch of proof : in a minimal way

Grey — some supplementary explanation . Unimportant assumptions.

Hell grey — explanation on well-known notations.

Brown — small title in subsections.

My symbol collection set

		Mathbb	Mathrsf/Mathcal	Greek	
A abelian variety	a	$\mathbb{A}$ adèles	$\mathscr{A}$ apartment		α
B	b	$\mathbb{B}$	$\mathscr{B}$ building		β
C	c	$\mathbb{C}$ cplx number	$\mathscr{C}$ chamber, category	Γ gp, graph	γ
D	d	$\mathbb{D}$	$\mathscr{D}$ Poincare disk	Δ diag embedding	δ
E elliptic curve	e constant, ramification index	$\mathbb{E}$	$\mathscr{E}$		ε
F field	f	$\mathbb{F}$ finite field	$\mathscr{F}$ sheaf		ζ
G group	g	$\mathbb{G}$ gp scheme	$\mathscr{G}$ $\mathfrak{g}$: Lie alg		η
H	h constant	$\mathbb{H}$	$\mathscr{H}$ upper half plane, Hecke alg	Θ	θ
I ideal	i	$\mathbb{I}$	$\mathscr{I}$ ideal of sheaf		ι injection
J	j	$\mathbb{J}$	$\mathscr{J}$		κ
K cos/base field	$k \leftarrow \Bbbk$	$\mathbb{K}$	$\mathscr{K}$	Λ lattice	λ
L	l	$\mathbb{L}$	$\mathscr{L}$		μ
M module	m	$\mathbb{M}$	$\mathscr{M}$ moduli space		ν
N	n	$\mathbb{N}$ natural number	$\mathscr{N}$		ξ root of unity (ζ_n)
O	o	$\mathbb{O}$	$\mathscr{O}$ structure sheaf	Π multi	π constant, uniformizer, projection
P	p	$\mathbb{P}$ proj space	$\mathscr{P}$ Weierstrass $\wp$: ell fct		$\rho \leftarrow \varrho$
Q	q	$\mathbb{Q}$ rational number	$\mathscr{Q}$	Σ sum	σ
R ring	r	$\mathbb{R}$ real number	$\mathscr{R}$		τ
S base scheme	s	$\mathbb{S}$	-	Φ	φ
T test scheme, tangent space, translation	t	$\mathbb{T}$ torus	$\mathscr{T}$		χ character
$U \leftarrow \mathcal{U}$	u	$\mathbb{U}$	-	Ψ	ψ
V v.s.	v	$\mathbb{V}$	-	Ω	ω
W witt vector	w	$\mathbb{W}$	-		
X	x	$\mathbb{X}$	$\mathscr{X}$		
$Y = \mathcal{Y}$	y	$\mathbb{Y}$	$\mathscr{Y}$	hebrew	Russian
Z center	z	$\mathbb{Z}$ integer	$\mathscr{Z}$	$\aleph$ cardinal	Ш sha gp

Green: number / basic stuffs in senior high school

Orange: scheme-related

Darkyellow: advanced algebra

Don't use them simultaneously!

Don't mix: w/ω, ζ/ξ, $k/\kappa/\mathscr{K}$, $1/l/\iota$, $\times/x/\chi/\mathcal{X}$, φ/ψ

Japanese mathematician and their Chiese translations.

1860	Sawayama	沢山	Yuzaburo	Sawayama	沢山勇三郎	
1875.4			Teiji	Takagi	高木貞治	高木 貞治
1908.12	Tannakian	淡中的	Tadao	Tannaka	淡中忠郎	
1912.7	Nakayama	中山	Tadashi	Nakayama	中山正	
1915.3	Kodaira	小平	Kunihiko	Kodaira	小平邦彦	
1917.11	Iwasawa	岩泽	Kenkichi	Iwasawa	岩泽健吉	岩澤 健吉
1925.11	Tamagawa	玉河	Tsuneo	Tamagawa	玉河恒夫	
1927.11			Yutaka	Taniyama	谷山丰	谷山 豊
1927.12	Satake	佐武	Ichirō	Satake	佐武一郎	
1928			Hiroshi	Toda	户田宏	戸田 宏
1928.4	Sato	佐藤	Mikio	Sato	佐藤干夫	佐藤 幹夫
1930.2	Shimura	志村	Gorō	Shimura	志村五郎	
1930.3	Yoneda	米田	Nobuo	Yoneda	米田信夫	
1930	Matsumura	松村	Hideyuki	Matsumura	松村英之	
1931.4	Hironaka	广中	Heisuke	Hironaka	广中平祐	広中 平祐
1944.3			Toshitsune	Miyake	三宅敏恒	
1947.1			Masaki	Kashiwara	柏原正树	柏原 正樹
1951.2	Mori	森	Shigefumi	Mori	森重文	
1952.1			Kazuya	Kato	加藤和也	
1959.3	Fukaya	深谷	Kenji	Fukaya	深谷贤治	
1962.11	Nakajima	中岛	Hiraku	Nakajima	中岛启	中島 啓
1969.3			Shinichi	Mochizuki	望月新一	