Un exemple par jour
4.1. the complex torus of form $\mathbb{C}^\times/\mathbb{Z}\gamma$

$C := \mathbb{C}^\times/\mathbb{Z}\gamma \overset{topo}{=\!=\!=} \mathbb{T}^2$ is a cpt Riemannian surface of genus 1.
  $\gamma \in \mathrm{Aut}(\mathbb{C}^\times)$ $\gamma(z) = az$ $a \in \mathbb{C}^\times$ $|a| > 1$

Today: $a = 2$
  1. fundamental set:
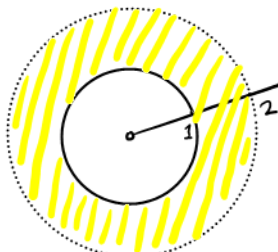
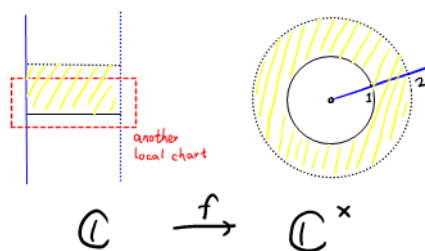$\Rightarrow$ only need 2 local chart

2.
$$0 \to \mathbb{Z} \hookrightarrow \mathbb{C} \xrightarrow{f: z \mapsto e^{2\pi i z}} \mathbb{C}^\times \to 1$$

$\Big\downarrow {+\frac{1}{2\pi i}\ln 2}$ $\Big\downarrow {+\frac{1}{2\pi i}\ln 2}$ $\Big\downarrow {\times 2}$

$$0 \to \mathbb{Z} + \tfrac{1}{2\pi i}\ln 2 \to \mathbb{C} \longrightarrow \mathbb{C}^\times \to 1$$

another local chart

$\mathbb{C} \xrightarrow{f} \mathbb{C}^\times$

$\mathbb{C}^\times = \mathbb{C}/\mathbb{Z} \Rightarrow \mathbb{C}^\times/\mathbb{Z}\gamma = \mathbb{C}/(\mathbb{Z} \oplus \frac{1}{2\pi i}\ln 2\,\mathbb{Z})$

better: $a = e^{2\pi} \approx 535.49$

$a = e^{-2\pi i \omega} \approx -230.765$

3. line bundle on C

$b \in \mathbb{C}^\times$ $\mathcal{L}_b := \mathbb{C}^\times \times \mathbb{C} /_{(z,\xi) \sim (2z, b\xi)}$

$\Big\downarrow$

$C = \mathbb{C}^\times/_{z \sim 2z}$

$\Rightarrow$ ① $\mathcal{L}_b \in \mathrm{Pic}_0(C)$; $(\mathcal{L}_b \sim \mathcal{L}_1 \cong \mathcal{O}_C)$ cont. deformation

② $\mathrm{Pic}_0(C) \overset{\sim}{\cong} C = \mathbb{C}^\times/\mathbb{Z}\gamma$ (naive, base pt $1 \in \mathbb{C}^\times/\mathbb{Z}\gamma$)
$\mathcal{L}_b \longmapsto b$

Reduced to: find a section s on $\mathcal{L}_b$ st div $s = [b] - [1]$
Reduced to: find a meromorphic functions g on $\mathbb{C}^\times$ s.t
  ① $g(2z) = b\,g(z)$ $b \in \mathbb{C}^\times, b \neq 2^k$; e.g. $b = 3$
  ② g has simple poles on 2^n, and simple zeros on $2^n b$ $n \in \mathbb{Z}$
$b = e^{2\pi i c}, c \in \mathbb{C}$
$\tau := \frac{1}{2\pi i}\ln 2$ $g(z) = \dfrac{\theta\!\begin{bmatrix}\frac{1}{1-2c}\end{bmatrix}(w(z), \tau)}{\theta\begin{bmatrix}\frac{1}{1}\end{bmatrix}(w(z), \tau)}$ is the required one.
$w(z) = \frac{1}{2\pi i}\ln z$

Blue — example

Orange — more than this example, slogan

Red — important results

Purple — I don't know the answer/proof

Green — sketsch of proof : in a minimal way

Grey — some supplementary explanation. Unimportant assumptions.

Hell grey— explanation on well-known notations.

Brown — small title in subsections.

My symbol collection set

Letter		Mathbb	Mathsf/Mathcal	Greek

A — abelian variety | α
B |
C | c e | ℂ cplx number | 𝒜 apartment
D | d d | | ℬ building / chamber / category
E — elliptic curve | e — constant / ramification index | | 𝒞
F — field, fiber, formal gp law | f | 𝔽 finite field | 𝒟 Poincaré disk
G — group | g | 𝔾 gp scheme | ℰ
H | h — constant | | ℱ sheaf
I — ideal | i | | 𝒢 g: Lie alg / upper half plane
J | j | | ℋ Hecke alg
K — cos/base field | k ← k | | ℐ ideal of sheaf
L | l | | 𝒥
M — module | m | | 𝒦
N | n | ℕ natural number | ℒ
O | o | 𝕆 | ℳ moduli space
P | p | ℙ proj space | 𝒩
Q | q | ℚ rational number | 𝒪 structure sheaf
R — ring | r | ℝ real number | 𝒫 ℘: Weierstrass ell fct
S — base scheme, test scheme | s | 𝕊 | 𝒬
T — tangent space, translation | t | 𝕋 torus | ℛ
U ← U | u | 𝕌 | 𝒮
V — v.s. | v | 𝕍 | 𝒯
W — witt vector | w | 𝕎 | 𝒰
X | x | 𝕏 | 𝒱 ϑ
Y = Y | y | 𝕐 | 𝒲
Z — center | z | ℤ integer | 𝒳 / 𝒴 / 𝒵

Greek column:

ℶ / Γ gp, graph / Γ
β
γ
δ
Δ diag embedding
ε
ζ
η
θ / Θ
ι injection
κ
λ / Λ lattice / ⋀
μ
ν
ξ root of unity (ξ/ω)
π constant, uniformizer, projection / Π multi
ρ ← ρ
σ / Σ sum
τ
υ / Φ
χ character
ψ / Ψ
ω / Ω ϖ ≈ ϖω

hebrew: ℵ cardinal

Russian: Ш sha gp

Green: number / basic stuffs in senior high school

Orange: scheme-related

Darkyellow: advanced algebra

↙ Don't use them simultaneously! (usually)

Don't mix: ω/ω, ζ/ξ, k/κ/𝒦/K   1/ι/υ, ×/x/χ/𝒳,
φ/ψ, e/ι, γ/ν

ϖω $\varpi\boldsymbol{\omega}$ (need amsbsy package)

https://en.wikipedia.org/wiki/Greek_letters_used_in_mathematics,_science,_and_engineering
https://mathoverflow.net/questions/8295/origins-of-mathematical-symbols-names

Japanese mathematicians and their Chiese translations.

	Matsumoto	松本	Hideya Matsumoto	松本英野		
1860	Sawayama	沢山	Yuzaburo Sawayama	沢山勇三郎		
1875.4			Teiji Takagi	高木贞治	高木 貞治	
1901.4	Oka	冈	Kiyoshi Oka	冈洁	岡 潔	
1902.7	Yamanouchi	山内	Takahiko Yamanouchi	山内恭彦		
1902.8	Akizuki	秋月	Yasuo Akizuki	秋月康夫	秋月 康夫	
1904.1			Hidetaka Teresaka	寺阪英孝	寺阪 英孝	
1908.12	Tannakian	淡中的	Tadao Tannaka	淡中忠郎		
1912.7	Nakayama	中山	Tadashi Nakayama	中山正		
1915.3	Kodaira	小平	Kunihiko Kodaira	小平邦彦		
1917.11	Iwasawa	岩泽	Kenkichi Iwasawa	岩泽健吉	岩澤 健吉	
1924.1	Igusa	井草	Jun-Ichi Igusa	井草准一	井草 準一	
1924.2	Tomita	富田	Minoru Tomita	富田稔	冨田 稔	
1924.7	Kuranishi	仓西	Masatake Kuranishi	仓西正武	倉西 正武	
1925.1			Toichiro Kinoshita	木下东一郎	木下 東一郎	
1925.11	Tamagawa	玉河	Tsuneo Tamagawa	玉河恒夫		
1926	Iwahori	岩堀	Nagayoshi Iwahori	岩堀长庆	岩堀 長慶	
1927.11	Taniyama	谷山	Yutaka Taniyama	谷山丰	谷山 豊	
1927.12	Satake	佐武	Ichirō Satake	佐武一郎		
1928.12	Fujita	藤田	Hiroshi Fujita	藤田宏	藤田 宏	
1928	Toda	户田	Hiroshi Toda	户田宏	戸田 宏	
1928.4	Sato	佐藤	Mikio Sato	佐藤干夫	佐藤 幹夫	
1930.2	Shimura	志村	Gorō Shimura	志村五郎		
1930.3	Yoneda	米田	Nobuo Yoneda	米田信夫		
1930	Matsumura	松村	Hideyuki Matsumura	松村英之		
1930	Kubota	久保田	Tomio Kubota	久保田富雄		
1931.4	Hironaka	广中	Heisuke Hironaka	广中平祐	広中 平祐	
1932.1			Shoshichi Kobayashi	小林昭七	小林 昭七	
1933.7	Takesaki	竹崎	Masamichi Takesaki	竹崎正道	竹崎 正道	
1941	Hotta	堀田	Ryoshi Hotta	堀田良之		
1944.3			Toshitsune Miyake	三宅敏恒		
1944			Masaki Maruyama	丸山正树	丸山 正樹	
1947.1	Kashiwara	柏原	Masaki Kashiwara	柏原正树	柏原 正樹	

This is an issue of non-standard language usage. Strictly speaking, "沢山" should be written as "泽山", but since everyone uses "沢山", I also write "沢山" instead of "泽山".

1951.2	Mori	森	Shigefumi Mori	森重文		
1952.1	Kato	加藤	Kazuya Kato	加藤和也		
1952.8	Hida	肥田	Haruzo Hida	肥田晴三	肥田 晴三	
1953.12.8	Mukai	向井	Shigeru Mukai	向井茂		
1955			Toshiyuki Tanisaki	谷崎俊之		
1957.10	Saito	斎藤	Shuji Saito	斎藤秀司	斎藤 秀司	
1959.3	Fukaya	深谷	Kenji Fukaya	深谷賢治		
1961.9	Saito	斎藤	Takeshi Saito	斎藤毅	斎藤 毅	
1962.11	Nakajima	中島	Hiraku Nakajima	中島啓	中島 啓	
1969.3			Shinichi Mochizuki	望月新一		
1972.8			Takurō Mochizuki	望月拓郎		
? 1995 Ph.D.			Kiyoshi Takeuchi	竹内潔		

Confusion list:

1. Ring has unit. Don't consider 0-Ring.
2. Read the diagram from top to bottom.
3. countable $=$ finite $+$ inf countable   (at most countable)
4. g fix set A: $\forall a \in A$, $ga = a$ (use "stabilized" instead)
5. $\subset$: only mean a subset, or an injective map
   (uncompatible structures are allowed. e.g. $L^{\infty}([0,1]) \subset L^{1}([0,1])$)
6. definition of norm/seminorms
7. $HK \neq H \times K$ $HK = \{g \in G \mid g = hk \text{ for some } h \in H, k \in K\}$