

Eine Woche, ein Beispiel

10.6 Chow group notation

Ref:

Murre, Jacob P., Jan Nagel and Chris A. M. Peters. Lectures on the Theory of Pure Motives. Univ. Lect. Ser. Providence, RI: American Mathematical Society (AMS), 2013.

Setting: X : sm proj variety over k

$$\underbrace{\text{Pic } X \supset \text{Pic}^\tau X \supset \text{Pic}^0 X \supset 0}_{\substack{N^1(X) \\ \text{NS}(X)}}$$

$i=1$:

$$\begin{array}{ccc} \text{Div}(X) & Z^i(X) & \xrightarrow{\quad \cdot \quad} \text{Pic}(X) \quad CH^i(X) \\ \parallel & \downarrow & \\ \text{Div}^\tau(X) = \text{Div}_{\text{num}}(X) & Z_{\text{num}}^i(X) & \xrightarrow{\quad \cdot \quad} \text{Pic}^\tau(X) \quad CH_{\text{num}}^i(X) \quad \begin{array}{l} N^1(X) \\ N_m^i(X) \end{array} \\ \parallel & \downarrow & \\ \text{Div}_{\text{hom}}(X) & Z_{\text{hom}}^i(X) & \xrightarrow{\quad \cdot \quad} \text{Pic}^\tau(X) \quad CH_{\text{hom}}^i(X) \\ \parallel & \downarrow & \\ \text{Div}_{\text{alg}}(X) & Z_{\text{alg}}^i(X) & \xrightarrow{\quad \cdot \quad} \text{Pic}^0(X) \quad CH_{\text{alg}}^i(X) \quad \begin{array}{l} H^2(X; \mathbb{Z}) \\ \text{tor} \end{array} \\ \parallel & \downarrow & \\ & Z_{\text{rat}}^i(X) & \end{array}$$

$\text{NS}(X) \stackrel{k=\mathbb{C}}{=} H^2(X; \mathbb{Z}) \cap H^{1,1}(X)$

[algebraic geometry - Motive of a curve and its Jacobian - Mathematics Stack Exchange](https://math.stackexchange.com/questions/1274303/motive-of-a-curve-and-its-jacobian)

[algebraic geometry - What is this cycle on the Jacobian of a curve? - Mathematics Stack Exchange](https://math.stackexchange.com/questions/681492/what-is-this-cycle-on-the-jacobian-of-a-curve?rq=1)

[algebraic geometry - Chow motives of quadratic fields - Mathematics Stack Exchange](https://math.stackexchange.com/questions/854610/chow-motives-of-quadratic-fields?rq=1)

Kunneth is complicated for Chow ring
[ag.algebraic geometry - Chow ring of two varieties - MathOverflow](https://mathoverflow.net/questions/159959/chow-ring-of-two-varieties)
https://mathoverflow.net/questions/6834/kunneth-formula-for-motivic-cohomology

sheaf version of Chow group:
https://math.stackexchange.com/questions/3019937/chow-groups-with-coefficients-in-a-local-system

Examples:

<https://mathoverflow.net/questions/275674/jacobians-of-curves-with-maximal-picard-number>

<https://mathoverflow.net/questions/219312/what-is-the-formula-for-the-homology-class-represented-by-the-diagonal>

<https://mathoverflow.net/questions/74181/cohomology-class-of-the-diagonal>

<https://math.stackexchange.com/questions/1506182/pull-back-of-push-out-under-the-diagonal-embedding>

Computation with sage (for toric varieties)

[The Chow group of a toric variety - Schemes

(sagemath.org)](https://doc.sagemath.org/html/en/reference/schemes/sage/schemes/toric/chow_group.html)