Example of alg closed field

① $\bar{\mathbb{Q}} \overset{\pi}{\underset{\bar{\mathbb{Z}}}{\subsetneq}} \mathbb{C} \overset{t}{\subsetneq} \bigcup_n \mathbb{C}((t^{\frac{1}{n}})) = \overline{\mathbb{C}((t))}$ $\mathbb{C}[[t]]$

Puiseux series

$\sum_{n=0}^{+\infty} p^n$ $\quad \bar{\mathbb{Q}} \cap \dots$

$\bar{\mathbb{Q}}_p \underset{\bar{\mathbb{Z}}_p}{\subsetneq} \mathbb{C}_p$

$\text{"}\mathbb{Q}_p \cdot \bar{\mathbb{Q}}\text{"}$

② char $K = p$: $\bar{\mathbb{F}}_q = \bar{\mathbb{F}}_p$ $\quad (Gal(\bar{\mathbb{F}}_q / \mathbb{F}_q) = \hat{\mathbb{Z}})$

Task. ① Prove they are alg closed. $(\mathbb{C}, \bigcup_n \mathbb{C}((t^{\frac{1}{n}})), \mathbb{C}_p)$

② Find an element in each "$\subsetneq$".

$Q_p \cap \bar{Q}$: https://math.stackexchange.com/questions/1280053/explicit-description-of-bbb-q-p-cap-bar-bbb-q

$C_p \setminus \bar{Q_p}$: https://math.stackexchange.com/questions/123925/is-the-algebraic-closure-of-a-p-adic-field-complete

https://math.stackexchange.com/questions/2430665/algebraic-closure-of-q-p-is-composite-of-bar-mathbbq-and-mathbbq-p

https://math.stackexchange.com/questions/2153580/transcendental-numbers-in-mathbbq-p