

Eine Woche, ein Beispiel

1.12 monodromy of the Gauss map

Ref:

[Kr16, cubic threefold]: Krämer, Thomas. Cubic Threefolds, Fano Surfaces and the Monodromy of the Gauss Map. Manuscripta Mathematica 149,

This is written for presentations. I may forget some important hints, so I collect the process here.

$$\begin{aligned}
 A &\cong \mathbb{C}^g / \Lambda \\
 Z &\subset A \quad \text{sm of dim } r \\
 \leadsto \text{Gauss map} & \\
 \phi: \begin{array}{c} \text{---} TZ \\ \text{---} Z \\ \cup \\ \{p_1, \dots, p_d\} \end{array} &\longrightarrow \begin{array}{c} \text{---} \mathcal{S} \\ \text{---} Gr(r, g) \\ \cup \\ Gr(r, g-1) \end{array} & z \longmapsto [T_z Z]
 \end{aligned}$$

e.g. $\mathcal{C} \subset \text{Jac}(\mathcal{C})$

tautological bd

$$d = \deg \Delta_Z = \phi^* \sigma_{\text{cor}} = (-1)^r \phi^* c_r(\mathcal{S}) = (-1)^r c_r(TZ)$$

$$\begin{aligned}
 I_{\text{mon}} &\subset Z \times Gr(q-1, g) \xrightarrow{\subset A \times Gr(q-1, g)} Gr(r, g) \times Gr(q-1, g) \\
 &\downarrow \\
 &Gr(q-1, g)
 \end{aligned}$$

\star

\leadsto monodromy gp

$$\text{Gal}(\gamma) := (\text{mon gp of } I_{\text{mon}} \longrightarrow Gr(q-1, g))$$

Q: How can we compute $\text{Gal}(\gamma)$?

E.g. 1. $Z = \mathcal{C}$ non-hyperelliptic, $Gr(1, g) = \mathbb{P}^{g-1}$,

$$\begin{array}{ccc} \phi = |\omega_{\mathcal{C}}| : & \mathcal{C} & \longrightarrow \mathbb{P}^{g-1} \\ \cup & & \cup \\ \{p_1, \dots, p_{2g-2}\} & \longrightarrow & H \end{array}$$

$$Gal(\gamma) = S_{2g-2} = W(A_{2g-3})$$

E.g. 2. $Z = \mathcal{C}$ hyperelliptic, $Gr(1, g) = \mathbb{P}^{g-1}$,

$$\begin{array}{ccccc} \phi = |\omega_{\mathcal{C}}| : & \mathcal{C} & \longrightarrow & \mathbb{P}^1 & \longrightarrow \mathbb{P}^{g-1} \\ \cup & & \cup & & \cup \\ \{p_1, \dots, p_{2g-2}\} & \longrightarrow & \{z_1, \dots, z_{g-1}\} & \longrightarrow & H \end{array}$$

$$Gal(\gamma) = S_{g-1} \rtimes (\mathbb{Z}/2\mathbb{Z})^{g-1} = W(C_{g-1})$$

E.g. 3. X : cubic threefold $\subseteq \mathbb{P}^4$
 $F = F(X)$: Fano surface $\subseteq Gr(2, 5)$
 $F \subset Alb(F) \leftarrow \dim = 5$

$$\begin{array}{ccc} \leadsto \phi : & F & \longrightarrow Gr(2, 5) \\ \cup & & \cup \\ & F(X \cap H) & \longrightarrow Gr(2, 4) \\ & \cup & \\ & \{p_1, \dots, p_7\} & \end{array}$$

$$Gal(\gamma) = Aut(\text{Schläfli graph}) = W(E_6)$$

Q: Can we find $Z \subset A$ sm s.t. $Gal(\gamma) = W(E_7)$?