

Eine Woche, ein Beispiel

7.25 irreducible representation of S_n / A_n









A000702





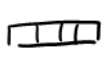



$$S_2/A_2$$

	$\begin{array}{ c } \hline \square \\ \hline \end{array}$	$\begin{array}{ c c } \hline \square & \square \\ \hline \end{array}$
	1^2	2^1
	1	(12)
$\begin{array}{ c } \hline \square \\ \hline \end{array}$	1	-1
$\begin{array}{ c c } \hline \square & \square \\ \hline \end{array}$	1	1












$$S_3 / A_3$$

			
	1^3	3^1	$2 \cdot 1$
	Id	(123)	(132)
	1	1	-1
	1	1	1
	1	1	1
	2	-1	0
	1	1	1
	1	1	1























S_4/A_4

					
	1^4 Id	2^2 (12)(34)	$3,1$ (123) (132)	$2,1^2$ (12)	$4,1$ (1234)
	1 1 1	1 1 1	1 1 1	-1 1	-1 1
	3 3 3	-1 -1 -1	0 0 0	-1 1	1 -1
	2 1 1	2 1 1	-1 $\frac{1+\sqrt{5}}{2}$ $\frac{1-\sqrt{5}}{2}$	0	0

S_5/A_5

							
	1^5 Id	$2^2,1$ (12)(34)	$3,1^2$ (123)	$5,1$ (12345) (12345)	$2,1^3$ (12)	$3,2$ (123)(45)	$4,1$ (1234)
	1 1 1 1 1	1 1 1 1 1	1 1 1 1 1	1 1 1 1 1	-1 1	-1 1	-1 1
	4 4 4	0 0 0	1 1 1	-1 -1 -1	-2 2	1 -1	0 0
	5 5 5	1 1 1	-1 -1 -1	0 0 0	-1 1	-1 1	1 -1
	6 3 3	-2 -1 -1	0 0 0	1 $\frac{1+\sqrt{5}}{2}$ $\frac{1-\sqrt{5}}{2}$	0	0	0

S_6/A_6
 ζ : outer automorphism action on representation.

												
	1^6	$2^2 \cdot 1^2$	$3 \cdot 1^3$	3^2	$4 \cdot 2$	$5 \cdot 1$		$2 \cdot 1^4$	2^3	$3 \cdot 2 \cdot 1$	$4 \cdot 1^2$	6
	Id	$(12)(34)$	(123)	$(123)(456)$	$(1234)(56)$	(12345)	(12354)	(12)	$(12)(34)(56)$	$(123)(45)$	(1234)	(123456)
	1	1	1	1	1	1	1	-1	-1	-1	-1	-1
	1	1	1	1	1	1	1	1	1	1	1	1
	1	1	1	1	1	1	1					
	5	1	2	-1	-1	0	0	-3	1	0	-1	1
	5	1	2	-1	-1	0	0	3	-1	0	1	-1
	5	1	2	-1	-1	0	0					
	9	1	0	0	1	-1	-1	-3	-3	0	1	0
	9	1	0	0	1	-1	-1	3	3	0	-1	0
	9	1	0	0	1	-1	-1					
	10	-2	1	1	0	0	0	-2	2	1	0	-1
	10	-2	1	1	0	0	0	2	-2	-1	0	1
	10	-2	1	1	0	0	0					
	5	1	-1	2	-1	0	0	-1	3	-1	1	0
	5	1	-1	2	-1	0	0	1	-3	1	-1	0
	5	1	-1	2	-1	0	0					
	16	0	-2	-2	0	1	1	0	0	0	0	0
	8	0	-1	-1	0	$\frac{1+\sqrt{5}}{2}$	$\frac{1-\sqrt{5}}{2}$					
	8	0	-1	-1	0	$\frac{1-\sqrt{5}}{2}$	$\frac{1+\sqrt{5}}{2}$					

We use Frobenius character formula to compute characters of S_n . For a clear statement of Frobenius character formula, see here: <https://mathoverflow.net/questions/323949/frobenius-formula>

After this, we can compute characters of A_n by using the theories in Fulton's book [Rep, Lec 5.1].

For a deeper research on the symmetry of the representation table, see here: <http://www.ma.rhul.ac.uk/~uvah099/Maths/labels.pdf>