# Eine Woche, ein Beispiel
## 7.3. le programme de Fontain

This document is full of mistakes. Don't believe me! I just record something from my schoolmate and the "p-adic Hodge theory" course, and of course it's totally uncompleted.

The motivation of p-adic Hodge theory (as well as these period rings) can be seen here(in Chinese):
https://www.zhihu.com/question/30816753/answer/1442810702
https://www.zhihu.com/question/51273075/answer/1505155046

Adjoint $W_{\mathbb{Z}_p} \dashv (-)^\flat$

$W_{\mathbb{Z}_p}$ : p-complete p-torsion free $\mathbb{Z}_p$-Alg $\longrightarrow$ $\mathbb{F}_p$-Alg

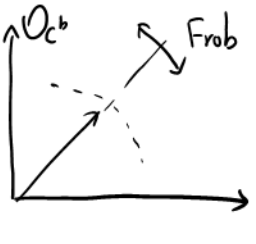
$(-)^\flat$ : tilting

$E/\mathbb{Q}_p$ $\quad F \in \mathbb{F}_p\text{-Alg} \quad W_{\mathcal{O}_E}(F) := W(F) \otimes_{\mathbb{Z}_p} \mathcal{O}_E \quad C = \mathbb{C}_p$

$A_{inf} := W_{\mathbb{Z}_p}(\mathcal{O}_C^\flat)$ Guess: $\mathcal{O}_C^\flat = \mathcal{O}_{C^\flat}$

$$\begin{array}{ccccccccc} & & & & A_{inf} & \xrightarrow{\theta} & \mathcal{O}_C & & \\ & & & & \downarrow & & \downarrow & & \\ 0 & \longrightarrow & (t) & \longrightarrow & A_{inf}[\tfrac{1}{p}] & \twoheadrightarrow & C & \longrightarrow & 0 \end{array}$$

where $t = [\omega] - p$, and




$$\begin{array}{ccc} \mathcal{O}_{C^\flat} & \longrightarrow & W(\mathcal{O}_{C^\flat}) \\ \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} \\ \omega & \longmapsto & [\omega] \end{array}$$

$A_{crys}$ = pd hull of $A_{inf}$

$= A_{inf}[\frac{t^n}{n!}]^\wedge$ completion at $(t)$

$B_{dR}^+ = A_{inf}[\frac{1}{p}]^\wedge$ completion at $(t)$

Cohen structure thm $C[[t]]$

$B_{dR} = B_{dR}^+[\frac{1}{t}]$

$B_{crys}^+ = A_{crys}[\frac{1}{p}]$

$B_{crys} = B_{crys}^+[\frac{1}{t}]$

Only $B_{dR}$ and $B_{crys}$ are fields. All rings are completed.

Even falser, but easier to remember:

$A_{inf} = \mathcal{O}_C[[t]]$

$A_{crys} = \mathcal{O}_C[[t]][\frac{t^n}{n!}]$ $\rightsquigarrow$ Crys coh

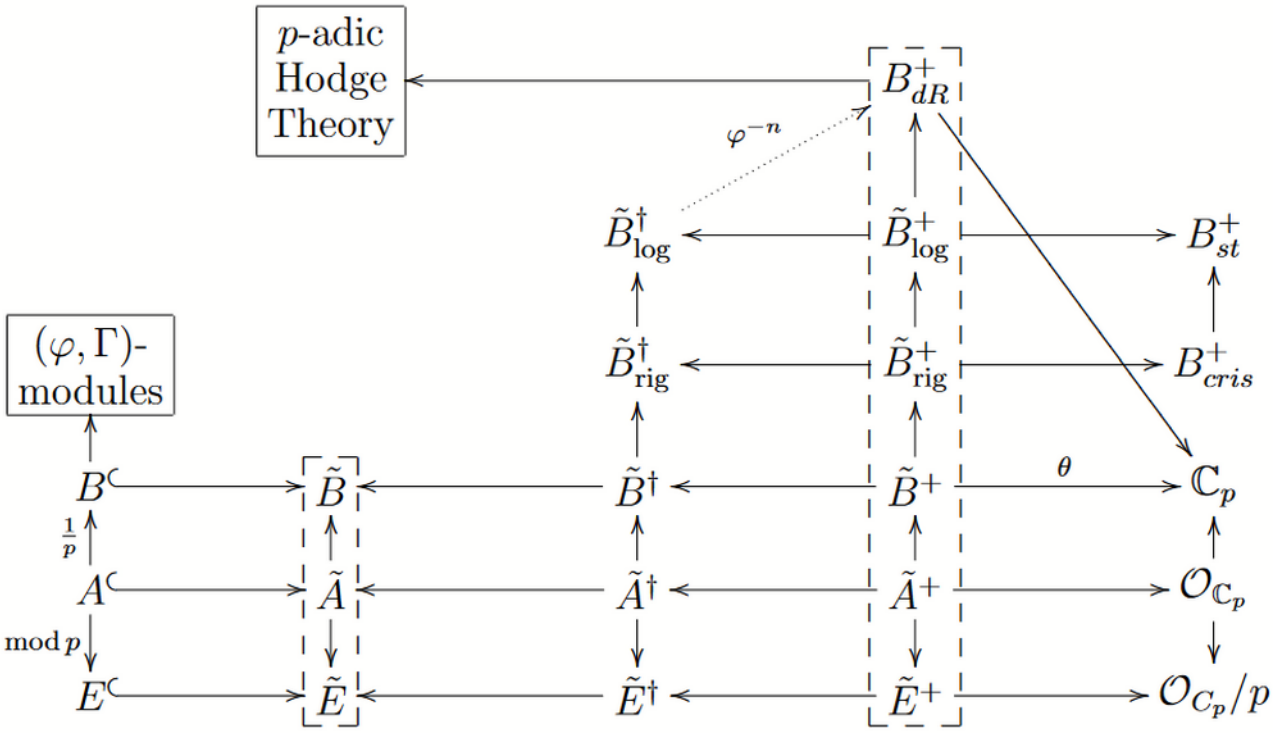
$B_{dR}^+ = C[[t]]$

$B_{dR} = C((t))$ $\rightsquigarrow$ de Rham coh

$B_{crys}^+ = C[[t]][\frac{t^n}{n!}]$

$B_{crys} = C((t))[\frac{t^n}{n!}]$

The following picture is copied from p149 of the following lecture note:
Colmez, Pierre. 《Fontaine's rings and p-adic L-functions》. Lecture notes 32 (2004): 33.
https://www.mathi.uni-heidelberg.de/~ariedel/plfun/colmez2.pdf




where

$$\tilde{B}^+_{\mathrm{rig}} = \bigcap_n \varphi^n(B^+_{cris}), \quad \tilde{B}^+_{\log} = \bigcap_n \varphi^n(B^+_{st}).$$