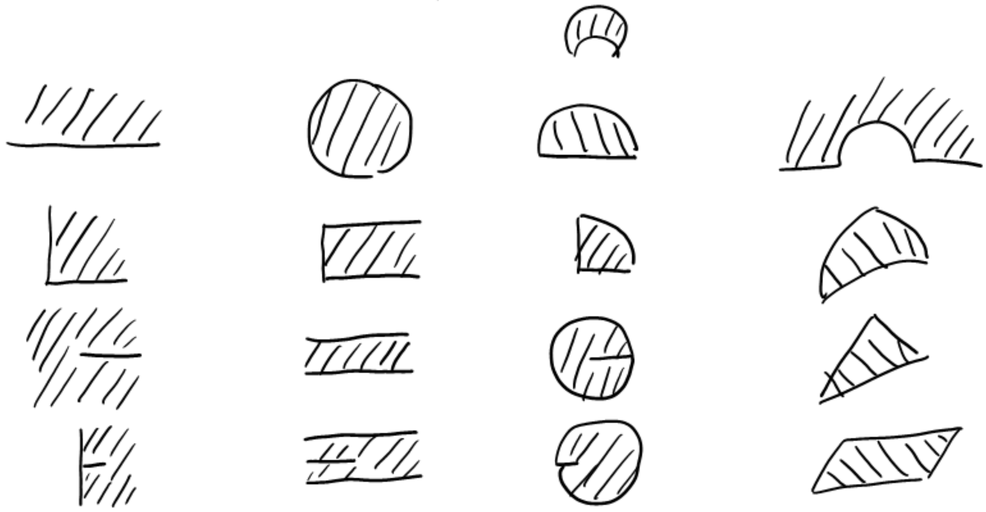
Modular form

0. small topics in complex analysis (prepared for Yilong Zhang)



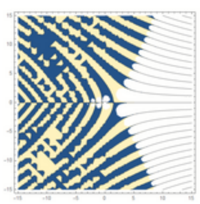
There are more examples in Ahlfors' book or Jihuai Shi's book, but I think it's enough for one-time show.

https://math.stackexchange.com/questions/585182/why-is-the-riemann-mapping-theorem-important

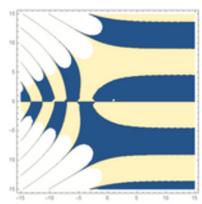
with sphere packing: https://scholarworks.calstate.edu/downloads/rn301358k
By 2.2.1, every simply connected proper open subset of C is not biholomorphic to C.

Liouville's theorem: every bounded entire function must be constant.
Cor1. For every entire function f, if Re f is bounded, then f is constant.
Cor2. For every entire function f, if f^{-1} ([0,\infty]) is empty, then f is constant.
Cor3. For every entire function f, if f is injective, then f is surjective.

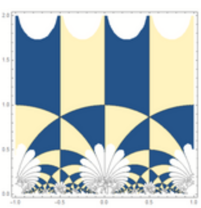
Little Picard Theorem is the strongest version of this type of results. For a statement, see wiki: Picard_theorem; for a proof, see [WWL, 例3.3.6].



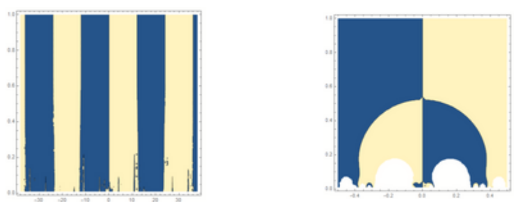
$\Gamma(z)$



$\zeta(z)$



$j(z)$



$\tau(z)$

Some concrete biholomorphic functions in the Riemann mapping theorem give us arithmetical informations.

RS. Task: give def & examples to motivate study of RS.

- Def
- Examples

  e.g. 0 $\quad \mathcal{U} \subseteq \mathbb{C}$ open

  e.g. 1. $\quad \mathbb{CP}^1$

  e.g. 2. $\quad \mathbb{C}/\Lambda$

  e.g. 3. $\quad f: \mathcal{U} \to \mathbb{C} \quad \rightsquigarrow \quad \Gamma_f$

  e.g. 4. $\quad F: \mathbb{C}^2 \to \mathbb{C}$ poly $\quad \rightsquigarrow \quad I(F) \subseteq \mathbb{C}^2$ $\qquad \frac{\partial F}{\partial x} = \frac{\partial F}{\partial y} = F = 0$ $\nexists$ solution

  e.g. 5. $\quad F: \mathbb{C}^3 \to \mathbb{C}$ homo poly $\quad \rightsquigarrow \quad I(F) \subseteq \mathbb{CP}^2$ $\qquad \frac{\partial F}{\partial x} = \frac{\partial F}{\partial y} = \frac{\partial F}{\partial z} = 0$ $\nexists$ solution

  genus?

- Claim: We know cpt conn RS of genus 0, 1 quite well.
- Hundreds of new questions waiting to answer.

  e.g. gp structure of EC?

  $E \rightsquigarrow \mathbb{C}/\Lambda$ which Λ?

  fct & map of RS? $\rightsquigarrow$ $\mathrm{trdeg}_{\mathbb{C}} \mathcal{M}(X) = ?$

  Why do we have categorical equiv as follows?

$$
\begin{tikzcd}
\text{RS } S \arrow[rr] & & \text{finite extensions of } \mathbb{C}(X) \arrow[dl, "\text{PET (primitive element theorem)}"] \\
& \text{Irr alg curves } C: F(X,Y)=0 \arrow[ul] &
\end{tikzcd}
$$