

Eine Woche, ein Beispiel

10.29. hermitian metric

Goal. Give examples of

- hermitian metric of $O(n)$ on $\mathbb{P}_{\mathbb{C}}^1$ } Fubini-study metric
- hermitian metric of $\Omega_{\mathbb{P}_{\mathbb{C}}^n/\mathbb{C}}$ on $\mathbb{P}_{\mathbb{C}}^n$ }
- hermitian metric of curves of genus ≥ 1 Arakelov Kähler form
- hermitian metric of proj space,
eg. Klein quartic curve, cubic surface, etc.

Def of hermitian metric:

https://en.wikipedia.org/wiki/Hermitian_manifold

Def of Fubini-study metric:

https://en.wikipedia.org/wiki/Fubini%E2%80%93Study_metric

<https://math.stackexchange.com/questions/2541216/fubini-study-metric-with-respect-to-a-specific-coordinate-system>

(from $S^{\wedge}\{2n-1\}$ is also equivalent) <https://math.stackexchange.com/questions/3037197/computing-fubini-study-metric-from-the-formal-definition>

Computation/application:

<https://math.stackexchange.com/questions/2360629/could-we-compute-integral-of-z-i-barz-j-sum-z-k2-on-mathbbcpn-with-noredirect=1&lq=1>

<https://math.stackexchange.com/questions/808049/volume-of-projective-space-textvol-mathbb-cpn>

Others:

<https://math.stackexchange.com/questions/3746233/fubini-study-metric-on-the-complex-projective-space-mathbbcpm>

我看到每一个Hermitte度量取实部都是一个Riemann度量，所以现在我们其实有 \mathbb{CP}^n 上的Riemann度量，我们又可以算测地线，两点之间的距离，体积(stackexchange)，曲率了！(特别有趣的是 \mathbb{CP}^1 的case,它应该是等距同构于 S^2 ,那么点 $[x,y]$ 的对径点是啥呢？它上面的测地线都是闭的，可以把它们写下来吗？测地线是不是某个代数方程组决定的子流形？)

This is for future use.

E.g. on $\mathbb{P}_{\mathbb{C}}^2$

$$U = [1 : z_1 : z_2]$$

$$[g_{i,j}] = \frac{1}{(1 + |z_1|^2 + |z_2|^2)^2} \begin{bmatrix} 1 + |z_2|^2 & -\bar{z}_1 z_2 \\ -\bar{z}_2 z_1 & 1 + |z_1|^2 \end{bmatrix}$$

$$\begin{aligned} & \partial_i \bar{\partial}_j (\log (|z_0|^2 + |z_1|^2 + |z_2|^2)) \\ &= \partial_i \frac{z_j}{|z_0|^2 + |z_1|^2 + |z_2|^2} d\bar{z}_j \\ &= \frac{\delta_{ij} (|z_0|^2 + |z_1|^2 + |z_2|^2) - \bar{z}_i z_j}{(|z_0|^2 + |z_1|^2 + |z_2|^2)^2} dz_i d\bar{z}_j \end{aligned}$$