$$
\begin{array}{c}
M \\
| \; ① \\
L \\
| \; ② \\
K
\end{array}
\left.\vphantom{\begin{array}{c}M\\|\\L\\|\\K\end{array}}\right\} ③
\qquad\qquad
\begin{tikzcd}
 & LM \arrow[dl, dash] \arrow[dr, dash] \arrow[d, dash, "⑥"] & \\
L \arrow[r, dash, "④"'] & K & M \arrow[l, dash, "⑤"]
\end{tikzcd}
$$

normal:	③ ⇒ ①	③ ⇏ ②	① + ② ⇏ ③ ; $① \xRightarrow{③\ \text{pure ins}} ③$	⑥ ⇏ ④	④ + ⑤ ⇒ ⑥
+					
seperable:			① + ② = ③		④ + ⑤ = ⑥
=					
Galois:	③ ⇒ ①	③ ⇏ ②	① + ② ⇏ ③	⑥ ⇏ ④	④ + ⑤ ⇒ ⑥
purely inseparable			① + ② = ③		④ + ⑤ = ⑥

only 1 root for minimal poly

[GTM 167, Thm 4.13] char $F = p$. then

F perfect $\Leftrightarrow F^p = F$

$$
\begin{array}{lll}
\bar{K} & & \\
| & \text{closed subgroup} & \\
L & & \\
\text{(finite)} \; | & \text{quotient group} & \\
K & &
\end{array}
\qquad
\begin{array}{c}
\bar{\mathbb{F}}_p \\
| \; \mathbb{Z}_l \\
\bigcup_{l \nmid a} \mathbb{F}_{p^a} \\
| \; \prod_{p \nmid l} \mathbb{Z}_p \\
\mathbb{F}_p
\end{array}
\qquad
\begin{array}{c}
\bar{\mathbb{F}}_p \\
| \; \prod_{p \nmid l} \mathbb{Z}_p \\
\bigcup_n \mathbb{F}_{p^{l^n}} \\
| \; \mathbb{Z}_l \\
\mathbb{F}_p
\end{array}
\qquad
\begin{array}{c}
\bar{\mathbb{F}}_p \\
| \; d\hat{\mathbb{Z}} \\
\mathbb{F}_q \\
| \; \mathbb{Z}/d\mathbb{Z} \\
\mathbb{F}_p
\end{array}
\quad \hat{\mathbb{Z}} = \prod_p \mathbb{Z}_p \quad (q = p^d)
$$

open subgroup $\overset{\{1\} \subseteq \mathbb{Z}_p}{\subsetneq}$ closed subgroup $= \{\mathrm{Gal}(\bar{K}/L) \mid L/k \text{ ext}\} \overset{\mathbb{Z} \subseteq \mathbb{Z}_p}{\subsetneq}$ subgroup

Lem. A subgroup of a profinite group is open iff it's closed and has finite index.