

Eine Woche, ein Beispiel

8.15 indecomposable representation of Affine quiver.

Task: give some examples to correct my misunderstanding of AR theory.

1: $\widehat{A}_2 \quad 1 \rightrightarrows 2$

Ind rep: $M_{n+1,n} : K^{n+1} \begin{matrix} \xrightarrow{(Id_n | 0)} \\ \xleftarrow{(0 | Id_n)} \end{matrix} K^n$

$$\text{End}(M_{n+1,n}) = K \quad [M_{n+1,n}, M_{n+1,n}]' = 0$$

They all corresponds to preproj/preinj.

$M_A : K^n \begin{matrix} \xrightarrow{A} \\ \xleftarrow{Id} \end{matrix} K^n \quad A \in M_n(K)$

$$M_A \sim M_{A'} \Leftrightarrow A \text{ conj to } A'$$

They're not all ind reps of dim vector (n, n) .

2. $\widehat{A}_3 \quad 1 \xrightarrow{2} 3$

$$C_P = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{pmatrix} \quad C_I = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Phi_A = -C_I C_P^{-1} = - \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 & -2 \\ 1 & 0 & -1 \\ 1 & 1 & -1 \end{pmatrix}$$

Ind rep: $M : K \begin{matrix} \xrightarrow{\begin{pmatrix} 1 \\ 0 \end{pmatrix}} K^2 \\ \xrightarrow{1} \end{matrix} K \begin{matrix} \xleftarrow{(1 \ 0)} \\ \end{matrix}$

$$\text{End}(M) = \begin{pmatrix} \alpha & 0 \\ \beta & \alpha \end{pmatrix} \quad [M, M]' = 2$$

$$\dim \tau(M) = \Phi_A \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

AR sequence: $0 \rightarrow \tau M \rightarrow ? \rightarrow M \rightarrow 0$

$$\begin{array}{ccccccc} & & \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} & & \begin{pmatrix} 0 & 0 & 0 \\ * & 1 & 0 \end{pmatrix} K^3 & & \begin{pmatrix} 1 \\ 0 \end{pmatrix} K^2 \\ \begin{pmatrix} 1 & 0 \end{pmatrix} K & \xrightarrow{\quad} & & \xrightarrow{\quad} & & \xrightarrow{\quad} & K^2 \\ \downarrow \text{Id} & & \downarrow \begin{pmatrix} 1 & 0 \end{pmatrix} & & \downarrow \text{Id} & & \downarrow \begin{pmatrix} 1 & 0 \end{pmatrix} \\ K^2 & \xrightarrow{\quad} & K^2 & \xrightarrow{\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}} & K^3 & \xrightarrow{\begin{pmatrix} 1 & 0 & 0 \\ * & * & 1 \end{pmatrix}} & K \\ & & \downarrow \text{Id} & & \downarrow \text{Id} & & \downarrow \text{Id} \\ & & K^2 & \xrightarrow{\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}} & K^3 & \xrightarrow{(1 \ 0 \ 0)} & K \end{array}$$

non-split sequence which is not AR sequence: $0 \rightarrow \tau M \rightarrow ? \rightarrow M \rightarrow 0$