Eine Woche, ein Beispiel

5.5 Beilinson gluing

This title is a bit cheating. In fact, I'm computing the cohomology of the intersection complex on A^1.

Notation

$A \in GL_n(\mathbb{Q})$

$\mathcal{L}_A :=$ local system on $\mathbb{C}^\times$ with monodromy A

$\mathcal{L}_A$ crspds to $(\rho_A, V) \in \mathrm{rep}_{\mathbb{Q}}(\pi_1(\mathbb{C}^\times))$

$\gamma := \rho_A(1)$ 1: generator of $\pi_1(\mathbb{C}^\times)$

Step 1. Compute $Rj_*\mathcal{L}_A$.

$$j^* Rj_* \mathcal{L}_A = \mathcal{L}_A$$
$$i^! Rj_* \mathcal{L}_A = 0$$
$$\begin{aligned} i^* Rj_* \mathcal{L}_A &\cong R\Gamma(\mathbb{C}, Rj_*\mathcal{L}_A) && \text{global coh} \\ &\cong R\pi_* \mathcal{L}_A && \text{pushforward} \\ &\cong H^\cdot(\mathbb{C}^\times; \mathcal{L}_A) && \text{local system coh} \\ &\cong H^\cdot([*/\mathbb{Z}]; V) && \text{stack coh} \\ &\cong H^\cdot(\mathbb{Z}; V) && \text{gp coh} \\ &\cong V^\gamma \oplus V_\gamma[-1] && \mathbb{Q}\text{-v.s.} \end{aligned}$$

$Rj_*\mathcal{L}_A[1]$

	n	-2	-1	0	1
$\mathcal{U}$	j^*	0	$\mathcal{L}_A$	0	0
$\{0\}$	i^*	0	V^γ	V_γ	0
	$i^!$	0	0	0	0
	$R^n\Gamma$	0	V^γ	V_γ	0

Step 2. Compute $IC(\mathbb{C}, \mathcal{L}_A)$.

One has triangle

$$IC(\mathbb{C}, \mathcal{L}_A) \longrightarrow Rj_*\mathcal{L}_A[1] \longrightarrow i_*V_\gamma \xrightarrow{+1}$$

$IC(\mathbb{C}, \mathcal{L}_A)$

	n	-2	-1	0	1
$\mathcal{U}$	j^*	0	$\mathcal{L}_A$	0	0
$\{0\}$	i^*	0	V^γ	0	0
	$i^!$	0	0	0	V_γ
	$R^n\Gamma$	0	V^γ	0	0

Step 3. compute NMD!

$\mathcal{F}^\bullet$	$NMD(\mathcal{F}^\bullet, \{0\})$	$\mathcal{F}^\bullet_x$	$R\Gamma(L_x, \mathcal{F}^\bullet)$
$IC(\mathbb{C}^\times, \mathcal{L}_A)$	V/V^γ	$V^\gamma[1]$	$V[1]$
$Rj_* \mathcal{L}_A[1]$	$V/V^\gamma \oplus V_\gamma$	$V^\gamma[1] \oplus V_\gamma$	$V[1]$
$j_! \mathcal{L}_A[1]$	V	0	$V[1]$
	$\phi_f \mathcal{F}^\bullet[-1]$	$i^*\mathcal{F}^\bullet$	$\psi_f \mathcal{F}^\bullet$