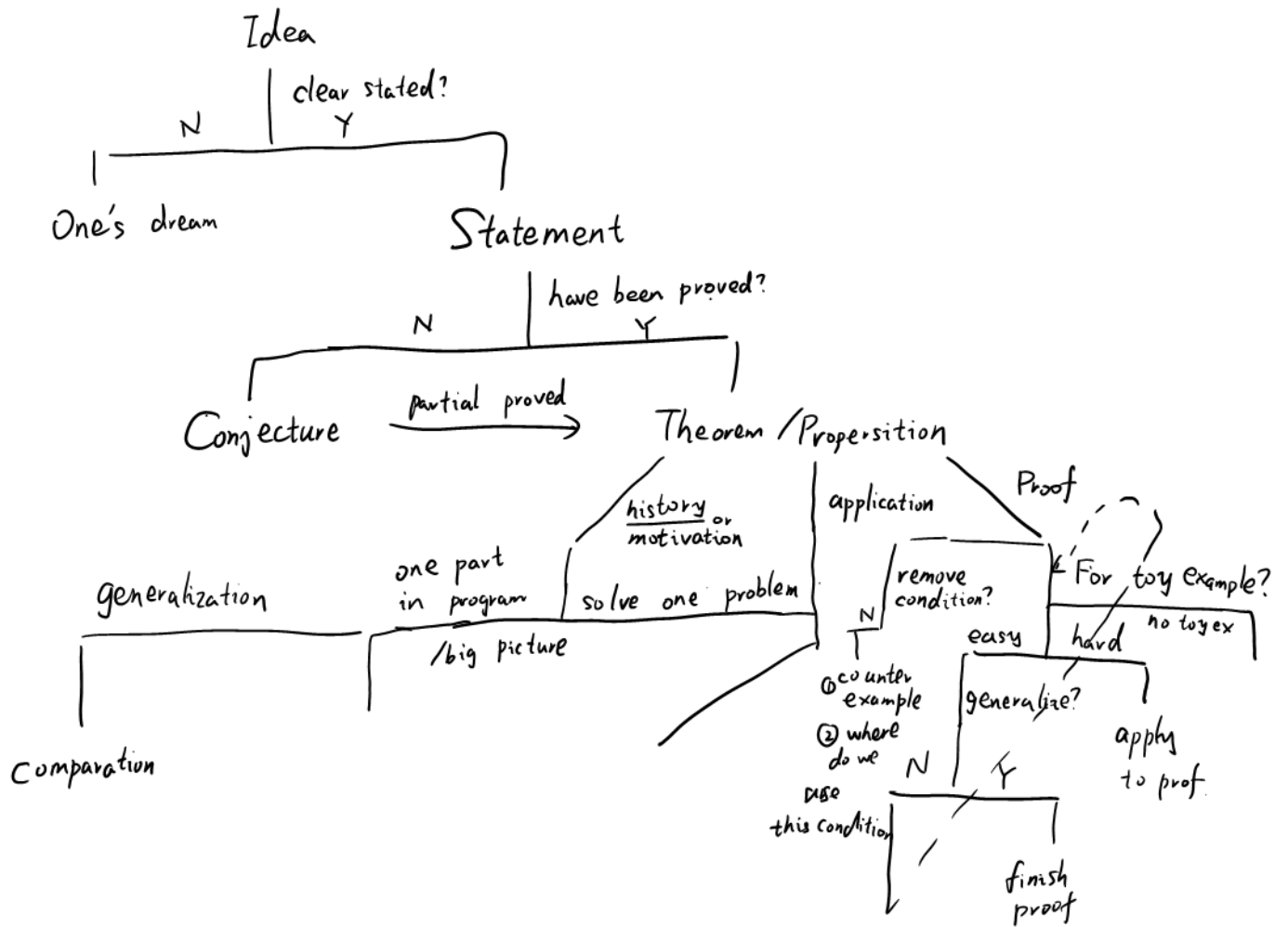


词典的重要性,词典的完备性

例子:

把几何对象翻译成代数对象(交换代数中屠老师的词典)

把新的对象翻译成旧对象(Grothendieck topology...)



从Grothendieck拓扑不是拓扑谈起
 数学概念的命名往往带有无意的误导性。
 同名不同义：平坦(flat), separated scheme/map and separated presheaf
 同源不同义：reduced & reducible 约化的不可约概形
 同名多类(支持变量多个类型)：拓扑空间的基本群，根系的基本群，概形的étale基本群
 affine scheme, affine map, affine scheme over S : 是affine scheme + over S 还是scheme over S + affine map?
 exact functor(of abelian category/triangulated category)

历史遗留问题：

presheaf, separated presheaf, sheaf(我们按照现代观点，或者说Vakil的note)
 quasi-compact and compact(代数几何+Class AT2使用quasi-compact,日常用compact)

概念的含混性：Borel-Moore同调是同调吗？紧支上同调是上同调吗？

推广而非限制：Grothendieck拓扑和étale拓扑都不是拓扑

含混义：descent

我大致知道的同调上同调：

type:

Borel-Moore homology

compact supported cohomology

real cohomology:

de Rham cohomology

cellular (co)homology

simplicial (co)homology

singular (co)homology

我大致不知道的同调上同调：

Cêch cohomology

sheaf cohomology (derived Hom functors)

Étale cohomology

Galois cohomology

Group (co)homology

Hochschild (co)homology

l-adic cohomology

crystalline cohomology

intersection (co)homology

elliptic cohomology

flat cohomology

infinitesimal cohomology

代数拓扑需要学啥？

six functors:

-Poincaré duality

-pullback and pushforward

Lefschetz trace formula

Lefschetz hyperplane theorem

Coefficients		cohomology groups	
$D_c^b(X; \mathbb{Q}_\ell)$ constructible ℓ -adic sheaves		ℓ -adic cohomology	✓
$D_c^b(X(\mathbb{C}); \mathbb{Z})$ constructible analytic sheaves		Betti cohomology	✓
$D_h^b(\mathcal{D}_X)$ holonomic \mathcal{D}-modules		de Rham cohomology	✓
$D^b(\text{Coh}(X))$ coherent sheaves	✓	coherent cohomology	✓
$D^b(\text{MHM}(X))$ mixed Hodge modules		absolute Hodge cohomology	
$\text{DM}(X)$ Voevodsky motivic sheaves		(weight-0) motivic cohomology	
$\text{SH}(X)$ stable motivic homotopy sheaves		stable motivic (weight-0) cohomotopy groups	

Ex. of Sheaves on $X_{\text{ét}}$

Let G : finite abelian group

① $\underline{G}_{\text{pre}}: - \mapsto G$ is a presheaf over $X_{\text{ét}}$

② $\underline{G}: \text{Hom}_{\text{Sch}/X}(-, \underline{G} \otimes_{\text{Spec } \mathbb{Z}} X)$ is a sheaf over $X_{\text{ét}}$

③ $\underline{G} = \underline{G}_{\text{pre}}$

③ reduced to $\underline{G}_p = G$ for $\forall p \in X$ i.e.

$$\lim_{\substack{p \in U \\ U \rightarrow X \text{ ét + lfp}}} \text{Hom}_{\text{Sch}/X}(U, \prod_{g \in G} X) = G$$