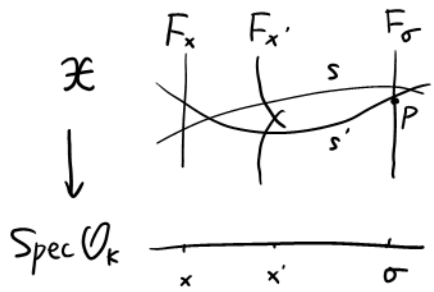
Eine Woche, ein Beispiel

11.21 Intersection on arithmetic surface.



Here K: number field
$\mathcal{O}_K$: intergral ring
$\mathfrak{X}$: arithmetic surface over $\mathcal{O}_K$ $(g \geq 1)$
Definition of ari surface?

Ref: EXPLICIT ARAKELOV GEOMETRY by R.S. de Jong

① divisor

$$\widehat{\mathrm{Div}}(\mathfrak{X}) = \mathrm{Div}(\mathfrak{X}) \oplus \bigoplus_{\sigma} \mathbb{Z} F_\sigma$$

where F_σ corresponds to the "Weil divisor" of

$$X_\sigma := \mathfrak{X} \times_{\mathrm{Spec}\,\mathcal{O}_K} \mathrm{Spec}\,\mathbb{C} \quad = (\mathfrak{X} \times_{\mathrm{Spec}\,\mathcal{O}_K} \mathrm{Spec}\,K) \times_{\mathrm{Spec}\,K} \mathrm{Spec}\,\mathbb{C}$$

I believe: X_σ is irreducible

② principal divisor $\mathrm{div}(f) \in \widehat{\mathrm{Div}}(\mathfrak{X})$ $\quad f \in K(\mathfrak{X})$

$$\mathrm{div}(f) = (f)_{fin} + (f)_{inf}$$

where

$$(f)_{fin} = \text{normal divisor} = \sum_C v_C(f) \cdot C$$

where v_C: normalized discrete val on $K(\mathfrak{X})$ defined by C.

$$(f)_{inf} = \text{divisor at inf place} = \sum_\sigma v_\sigma(f) \cdot F_\sigma$$

where $v_\sigma(f) = -\int_{X_\sigma} \log |f|_\sigma \cdot \mu_\sigma$ $\quad$ μ_σ: canonical measure

$$v_P(f) = -\log |f|_\sigma(P)$$

③ Intersection

normal: $(D_1, D_2)_{fin} = \sum_b (D_1, D_2)_b \log \# k(b)$

other cases: $(s, F_{\sigma'}) = \deg(s|_{X_{\sigma'}})$

$(s, s')_\sigma = -\log G_\sigma(s|_{X_\sigma}, s'|_{X_\sigma})$ $\quad$ G_σ: Green's fct on X_σ

	$F_{x'}$	s'	$F_{\sigma'}$
F_x	normal	normal	0
s	—	normal / $-\log G(\cdot,\cdot)$	$\deg(s\|_{X_{\sigma'}})$
F_σ	—	—	0