REPRESENTATION II ORAL EXAM

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The course name is "Representation II", and the examiner is Prof. Stroppel.
Like anyone else, I was asked to begin with one topic. I begin with the Coxeter group.

- definition;
- define (W_P, S_P). Is that obvious that (W_P, S_P) is a Coxeter group? (No)
- show that by geometrical representation. (define geometrical representation) Key ingredient is to distinct two different W_P.
- give an example of Coxeter group which is not finite. Can you see that group?

Then we work on Hecke algebras.

- define generic Hecke algebra $\mathcal{H}_v(W)$ for a Coxeter group;
- find a basis, show the well-defineness, and why it is a basis;
- for the linear independent part, define Hecke algebra $H(\mathrm{GL}_n(\mathbb{F}_q), B)$;
- state the Bruhat decomposition and sketch a proof;
- state the KL-basis;
- compute $T_s * T_s$; (explains the relation in $\mathcal{H}_v(W)$)

We switch to the finite irreducible representation of $\mathrm{GL}_n(\mathbb{C})$.

- Do we have a classification for that? (rep <-> dominant weight)
- Why is that surjective? (construction, some cases to check)
- How to find a highest weight? (reduced to finding a weight, reduced to finding a common eigenvectors, which uses the Borel fixed point theorem)
- State the Borel fixed point theorem, and give a sketsch of proof. (Use one fact concerning about solvable group and complete varieties)

Since I mentioned about G/G_x in the last question, we go to part of varieties.

- For an affine algebraic group, is any closed subgroup also affine?
- Show that affine algebraic group is linear.
- The same method can be applied to what problem? (The existence of Chevalley quotient)
- Does proper apply projective?
- Is there any example of affine and proper variety? (points) Other examples? (I believe no)
- By using the idea of projective apply proper, show that $\mathbb{P}^1$ is not affine. (I didn't understand that question, and said that we just show that by computing the global sections. The final answer is "to shoot a mosquito with an anti-aircraft gun", and she expected me to say the fact that affine and proper variety can be only finite many points.)

Finally we come back to Borel-Weil theorem.

- State that theorem and give a sketch of proof quickly. (I give the geometrical proof, which uses the Hartdog's lemma.)

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Date: March 14, 2023.