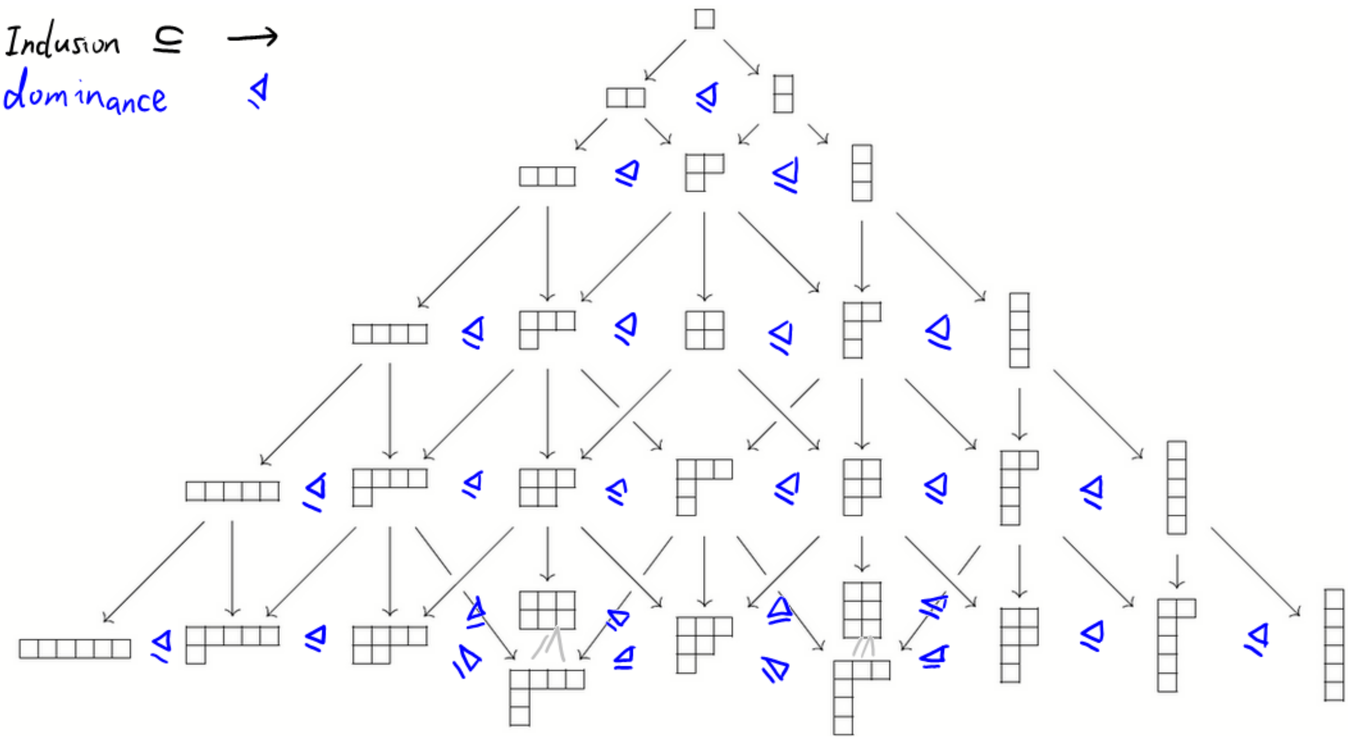
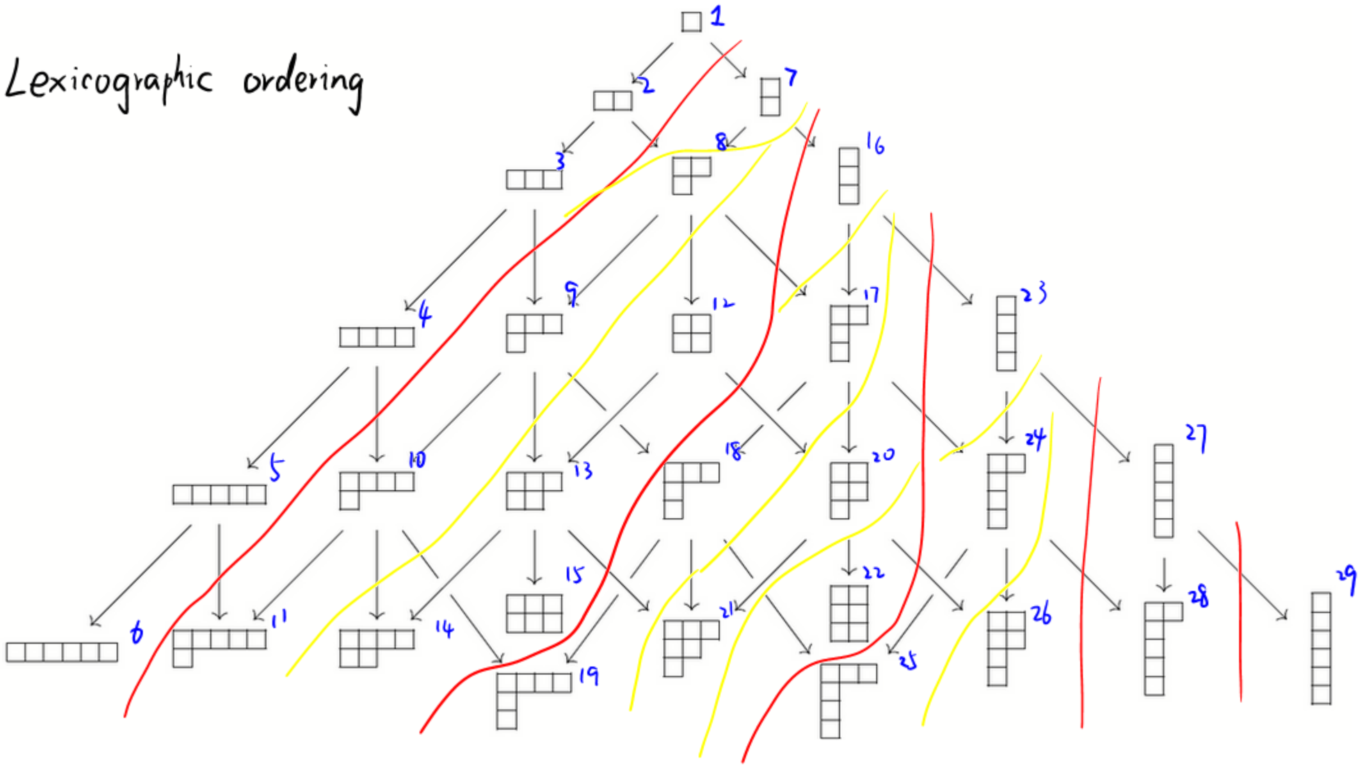
Inclusion $\subseteq$ $\longrightarrow$

dominance $\trianglelefteq$

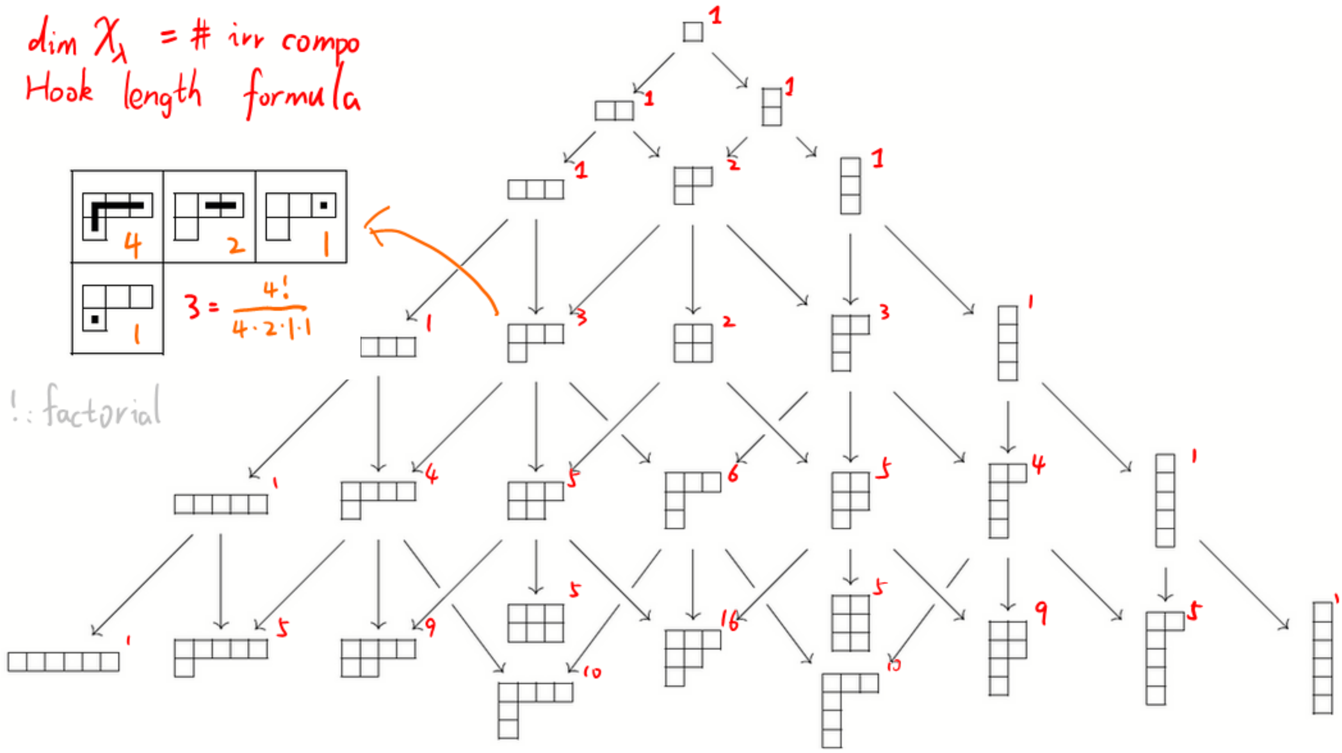


Lexicographic ordering



dim χ_λ = # irr compo

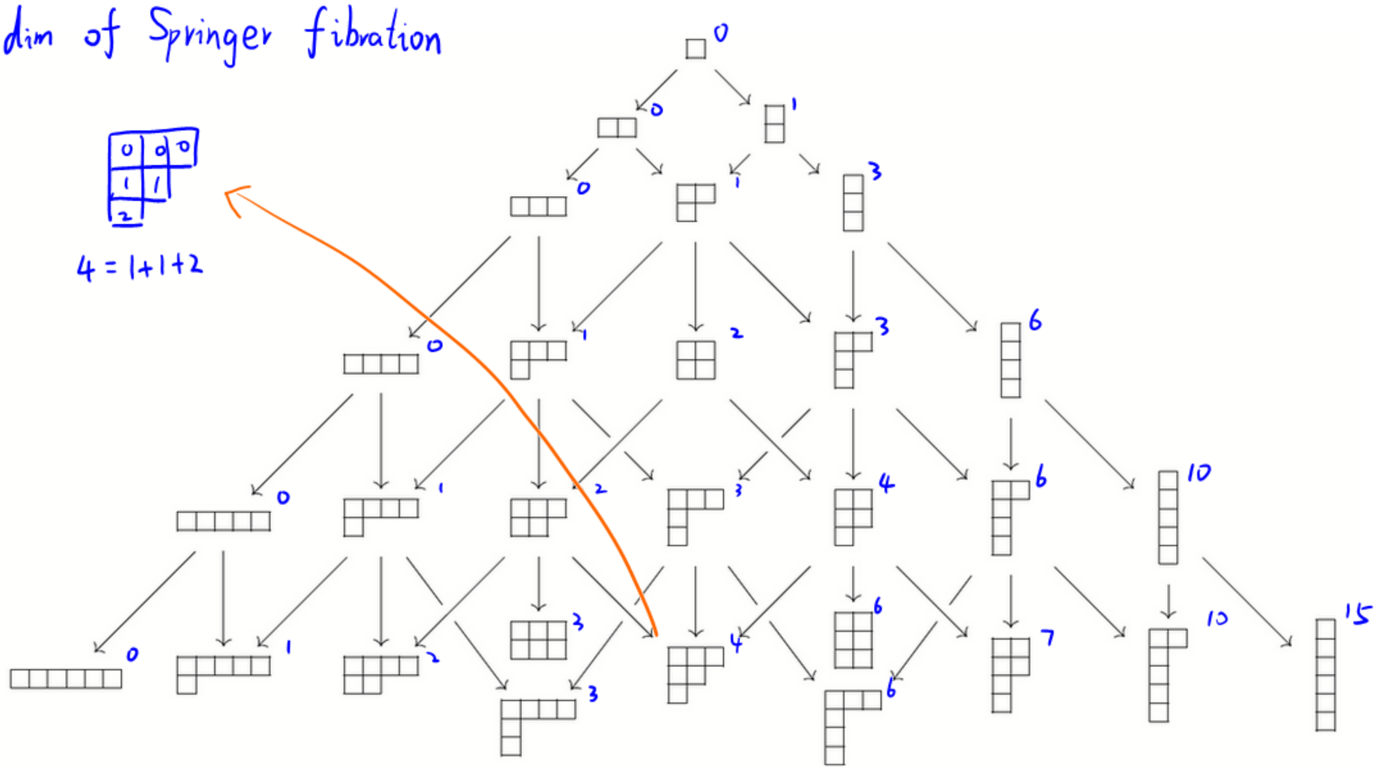
Hook length formula



$3 = \frac{4!}{4\cdot 2\cdot 1\cdot 1}$

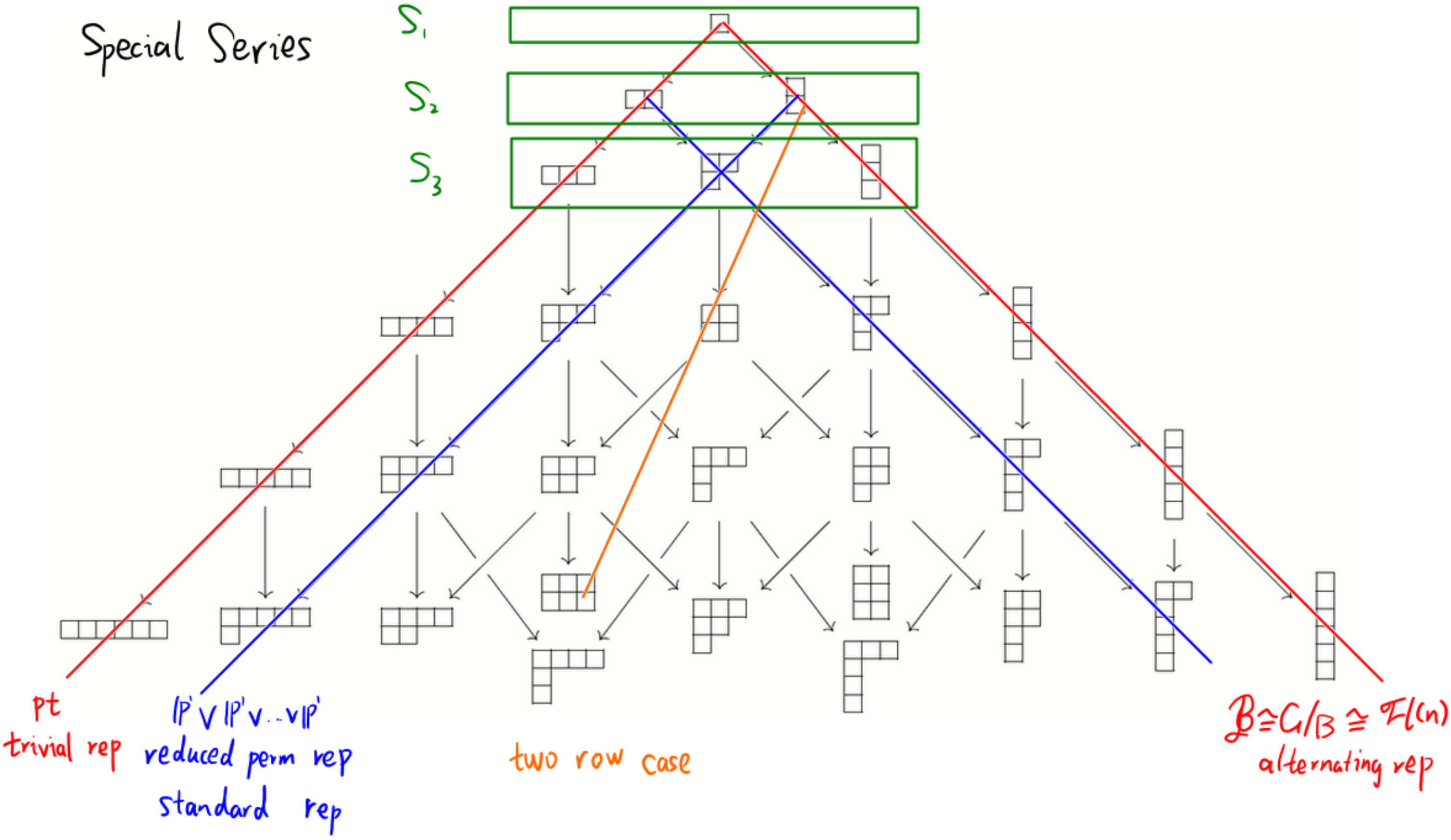
!: factorial

dim of Springer fibration



$4 = 1+1+2$

Special Series



Q: What's the dim of $\{\text{Cartan subalg of } \mathfrak{sl}_n(\mathbb{C})\}$?

$$
\begin{tikzcd}
\{\text{Borel subalg}\} \arrow[r, dashed, leftrightarrow] \arrow[d, "\text{finite}\ \text{surj?}"] & \{\text{Borel subgroup}\} \arrow[r, "\overset{\text{fix } B}{\cong}", phantom] \arrow[d] & G/B \arrow[d, "\text{finite?}"] \\
\{\text{Cartan subalg}\} \arrow[r, dashed, leftrightarrow] & \{\text{Maximal torus}\} \arrow[r, "\overset{\text{fix } T}{\cong}", phantom] & G/N_G(T)
\end{tikzcd}
$$

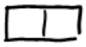
$\circlearrowleft G$ transitive?

structure?

Examples of Springer fiber. $E_{\lambda_1,\dots,\lambda_n}$ = Springer fiber of $\begin{bmatrix}J_{\lambda_1}&&\\&\ddots&\\&&J_{\lambda_n}\end{bmatrix}$ $J_\lambda=\begin{bmatrix}0&1&&\\&0&1&\\&&0&1\\&&&0\end{bmatrix}$

Sf 𝔽 𝓕 𝔅 X 𝕏 𝕍 𝕊 E $\lambda\times\lambda$

 $[0]$ $E_1=\{0\subseteq\mathbb{C}\}=\cdot$

 $\begin{bmatrix}0&1\\&0\end{bmatrix}$ $E_2=\{0\subseteq\langle v_1\rangle\subseteq\mathbb{C}^2\}=\cdot$

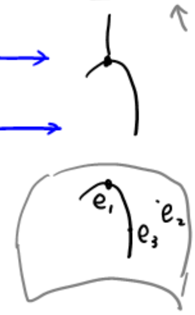
 $\begin{bmatrix}0&\\&0\end{bmatrix}$ $E_{1,1}=\{0\subseteq\langle ?\rangle\subseteq\mathbb{C}^2\}=\mathbb{P}^1$

 $\begin{bmatrix}0&1&\\&0&1\\&&0\end{bmatrix}$ $E_3=\{0\subseteq\langle v_1\rangle\subseteq\langle v_1,v_2\rangle\subseteq\mathbb{C}^3\}=\cdot$

 $\begin{bmatrix}0&1&\\&0&\\&&0\end{bmatrix}$ $E_{2,1}=\{0\subseteq\underbrace{\langle ?\rangle}_{\mathbb{P}^2}\subseteq\langle ?,?\rangle\subseteq\mathbb{C}^3\}=\mathbb{P}^1\vee\mathbb{P}^1$

$\mathbb{P}^1$ | $\longrightarrow$

$\{pt\}$ $\cdot$ $\longrightarrow$



$$\begin{array}{l} E_{1,1}\longrightarrow \\ E_2\longrightarrow \end{array} E_{2,1}$$
$$\downarrow$$
$$\mathbb{P}^1\subseteq\mathbb{P}^2$$

$F_{e_1}=\{0\subseteq\langle e_1\rangle\subseteq\langle e_1,?\rangle\subseteq\mathbb{C}^3\}\circlearrowleft\begin{bmatrix}0&1&\\&0&\\&&0\end{bmatrix}$

$\Updownarrow$

$\mathbb{P}^1=E_{1,1}=\{0\subseteq\langle ?\rangle\subseteq\mathbb{C}^2\}\circlearrowleft\begin{bmatrix}0&\\&0\end{bmatrix}$

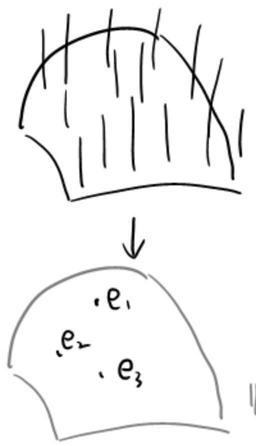
$F_{e_3}=\{0\subseteq\langle e_3\rangle\subseteq\langle e_3,?\rangle\subseteq\mathbb{C}^3\}\circlearrowleft\begin{bmatrix}0&1&\\&0&\\&&0\end{bmatrix}$

$\Updownarrow$

$\{pt\}=E_2=\{0\subseteq\langle ?\rangle\subseteq\mathbb{C}^2\}\circlearrowleft\begin{bmatrix}0&1\\&0\end{bmatrix}$

 $\begin{bmatrix}0&&\\&0&\\&&0\end{bmatrix}$ $E_{1,1,1}=\{0\subseteq\underbrace{\langle ?\rangle}_{\mathbb{P}^2}\subseteq\langle ?,?\rangle\subseteq\mathbb{C}^3\}=\mathcal{F}l(3)$

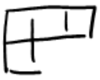
$\mathcal{F}l(2)$ | $\longrightarrow$



$\mathbb{P}^2$

$$\begin{array}{ccccccccc}\mathcal{F}l(1) & - & \mathcal{F}l(2) & - & \mathcal{F}l(3) & - & \mathcal{F}l(4) & - & \mathcal{F}l(5)\\ & | & & | & & | & & | & \\ & \mathbb{P}^1 & & \mathbb{P}^2 & & \mathbb{P}^3 & & \mathbb{P}^4 & \end{array}$$

$$\begin{array}{ccccccccc}E_1 & - & E_{1,1} & - & E_{1,1,1} & - & E_{1,1,1,1} & - & E_{1,1,1,1,1}\\ & | & & | & & | & & | & \\ & \cdot & & \cdot & & \cdot & & \cdot & \end{array}$$

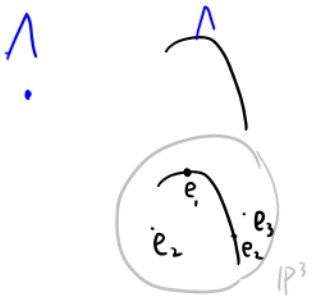


$$\begin{bmatrix} 0 & 1 & & \\ & 0 & 1 & \\ & & 0 & \\ & & & 0 \end{bmatrix}$$

$$E_{3,1} = \{ 0 \subseteq \langle ? \rangle \subseteq \langle ? \rangle \subseteq \langle ? \rangle \subseteq \mathbb{C}^4 \} = \mathbb{P}^1 \vee \mathbb{P}^1 \vee \mathbb{P}^1$$

$E_{2,1}$

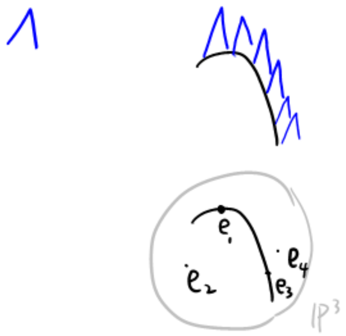
E_3



$$\begin{bmatrix} 0 & 1 & & \\ & 0 & & \\ & & 0 & 1 \\ & & & 0 \end{bmatrix}$$

$$E_{2,2} = \{ 0 \subseteq \langle ? \rangle \subseteq \langle ? \rangle \subseteq \langle ? \rangle \subseteq \mathbb{C}^4 \} = \mathbb{P}^1\text{-bundle} \vee_{\mathbb{P}^1} \mathbb{P}^1\text{-bundle}$$

$E_{2,1}$



$$\begin{bmatrix} 0 & 1 & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{bmatrix}$$

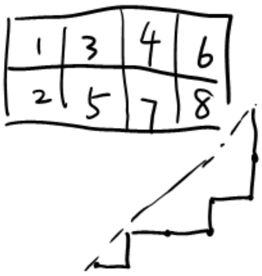
$$E_{2,1,1} = \{ 0 \subseteq \langle ? \rangle \subseteq \langle ? \rangle \subseteq \langle ? \rangle \subseteq \mathbb{C}' \}$$

$\mathcal{F}(\mathbb{C}^3) = E_{1,1,1}$

$\mathbb{P}^1 \vee \mathbb{P}^1 = E_{2,1}$

Q: Can we construct an affine paving from this kind of fibration?

Q: How to understand the Weyl group action on Springer fibration?



"Bertrand's ballot theorem"

"bracket pairing"

Define M^λ to be the complex vector space with basis the tabloids $\{T\}$ of shape λ, with λ a partition of n.

(3) $$a_T = \sum_{p \in R(T)} p, \quad b_T = \sum_{q \in C(T)} \operatorname{sgn}(q) q. \qquad v_T = b_T \cdot \{T\} = \sum_{q \in C(T)} \operatorname{sgn}(q)\{q \cdot T\}.$$

These elements, and the product

$$c_T = b_T \cdot a_T,$$

Define the ***Specht module*** S^λ to be the subspace of M^λ spanned by the elements v_T, as T varies over all numberings of λ.

Young tabloid

Proposition 1 *For each partition λ of n, S^λ is an irreducible representation of S_n. Every irreducible representation of S_n is isomorphic to exactly one S^λ.*

整体数矩阵.

Proposition 2 *The elements v_T, as T varies over the standard tableaux on λ, form a basis for S^λ.*

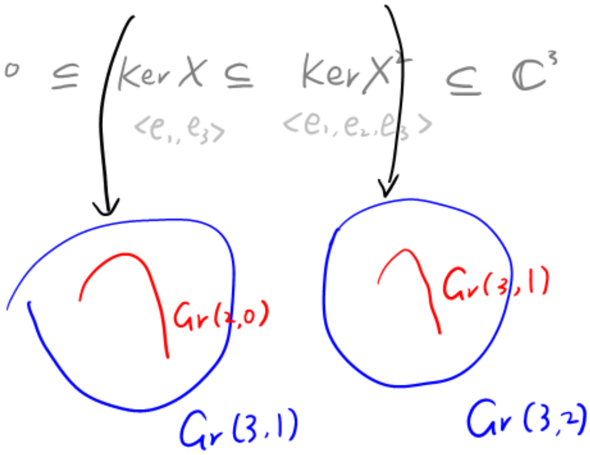
$\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 \\ \cline{1-1}\end{array}$ $\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 \\ \cline{1-1}\end{array}$ $\begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline 3 \\ \hline\end{array}$ $\begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline\end{array}$

$X = \begin{bmatrix} 0 & 1 & \\ & 0 & \\ & & 0 \end{bmatrix}$

$$\overset{0}{0 \subseteq \operatorname{Im} X^2} \subseteq \overset{\langle e_1 \rangle}{\operatorname{Im} X} \subseteq \mathbb{C}^3$$

$$B_{2,1} = \{ 0 \subseteq \langle ? \rangle \subseteq \langle ?, ? \rangle \subseteq \mathbb{C}^3 \}$$

$$0 \subseteq \underset{\langle e_1, e_3 \rangle}{\operatorname{Ker} X} \subseteq \underset{\langle e_1, e_2, e_3 \rangle}{\operatorname{Ker} X^2} \subseteq \mathbb{C}^3$$

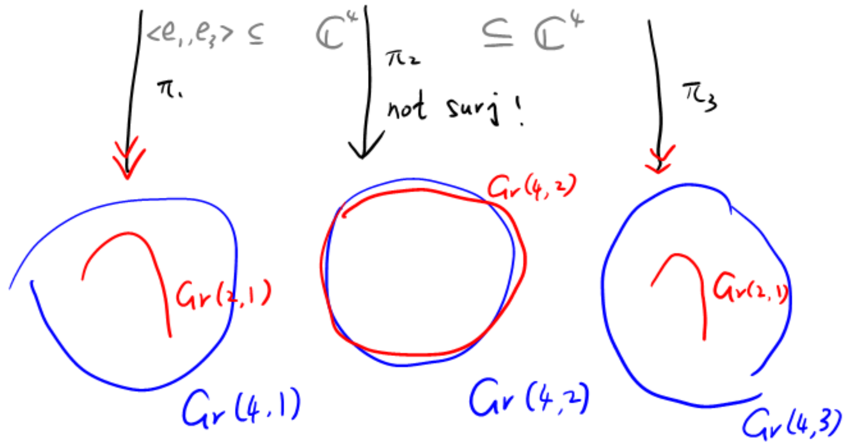


$X = \begin{bmatrix} 0 & 1 & & \\ & 0 & & \\ & & 0 & 1 \\ & & & 0 \end{bmatrix}$

$$0 \subseteq 0 \subseteq \langle e_1, e_3 \rangle$$

$$B_{2,1} = \{ 0 \subseteq \langle ? \rangle \subseteq \langle ?, ? \rangle \subseteq \langle ?, ?, ? \rangle \subseteq \mathbb{C}^4 \}$$

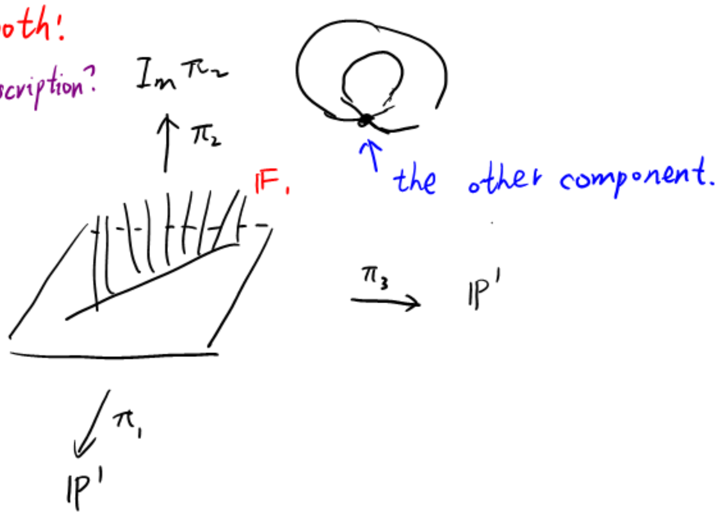
$$\langle e_1, e_3 \rangle \subseteq \mathbb{C}^4 \subseteq \mathbb{C}^4$$



$$\operatorname{Im} \pi_2 = \{ V \subseteq \mathbb{C}^4 \mid \dim_{\mathbb{C}} V = 2,\ XV \subseteq V \} \subseteq Gr(4,2)$$

not smooth!

Global description? $\operatorname{Im} \pi_2$



Cohomology of B_λ