RESEARCH STATEMENT

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Contents

1. Introduction

My work mainly consists of two parts. The first part computes the equivariant K-theory of Steinberg varieties, while the second part focuses on the geometry of certain varieties known as (partial) flag quiver varieties. Since the precise statements are already summarized in the introduction of the thesis, I will instead focus on the creativity and novel ideas in this document.

2. Part I: K-theory of the Steinberg variety

For the first part, we fix a quiver Q (without loops and cycles) and dimension vector $\mathbf{d}$. The Steinberg variety $\mathcal{Z}_{\mathbf{d}}$ is an incidence variety, consisting of triples of a representation of Q and two complete flags fixed by this representation.

With such natural definition, $\mathcal{Z}_{\mathbf{d}}$ possesses a natural stratification which allows us to extract the geometrical information throughout direct methods. For example, by the cellular fibration theorem, the stratification gives us a basis of the K-theory (or, any other cohomology theory) of $\mathcal{Z}_{\mathbf{d}}$.

The Steinberg variety $\mathcal{Z}_{\mathbf{d}}$ has a convolution structure which induces an algebraic structure on the K-theory. Another basis is introduced in order to compute this structure. The new basis corresponds to T-fixed points which "are more concentrated than the stratification", so we are able to get explicit convolution product formulas by applying the excess intersection formula. Finally, we deduce the Demazure operator and conclude our computations in the diagram of strands.

3. Part II: affine paving of quiver flag variety

This part is already posted on arXiv. It focuses on the affine paving of the quiver (partial) flag varieties. We use basic cohomological algebraic tools to construct the affine pavings by induction, and the problem finally reduces to combinatorics related to the Auslander–Reiten theory.

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4. What's new

There have been literature concerning the K-theory of (classical) Steinberg variety, and the method of calculation may be well-known to experts. In spite of this, I try to write down computations more clear than other literature, and point out some technicalities which are not mentioned due to the confusion of notations.

Moreover, I realized that those nontrivial results boil down to just a small amount of results about K-theory. The formalism for the K-theory differs from the usual 6-functor formalism due the base change formula, where a twist by Euler class is needed.

Character classes also plays a role in my thesis. It looks like that the Chern class measure the differences between two different cohomologies, while the Euler class and Todd class measure the failure of commutativity for some diagrams. These classes fit in the language of 6-functor formalisms, they are quite explicit in our setting but hard to connect with their original geometrical meaning.

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