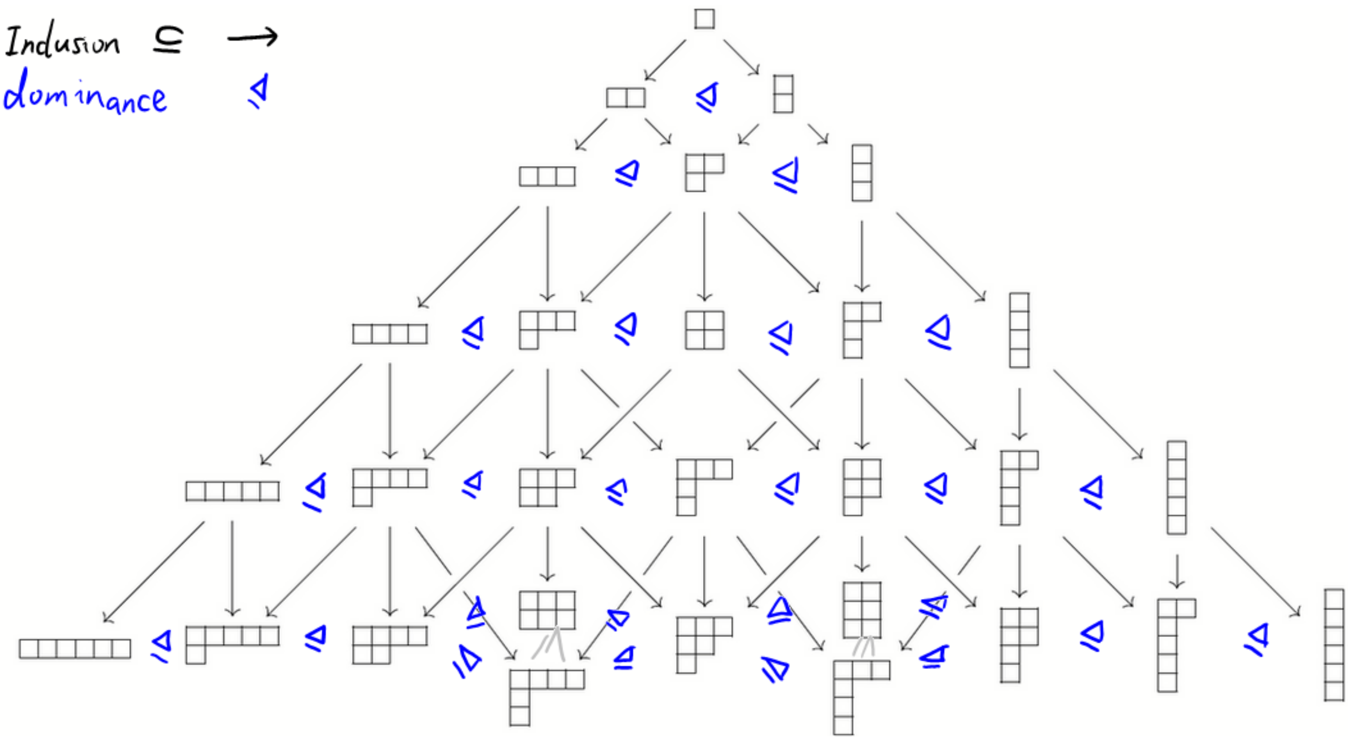
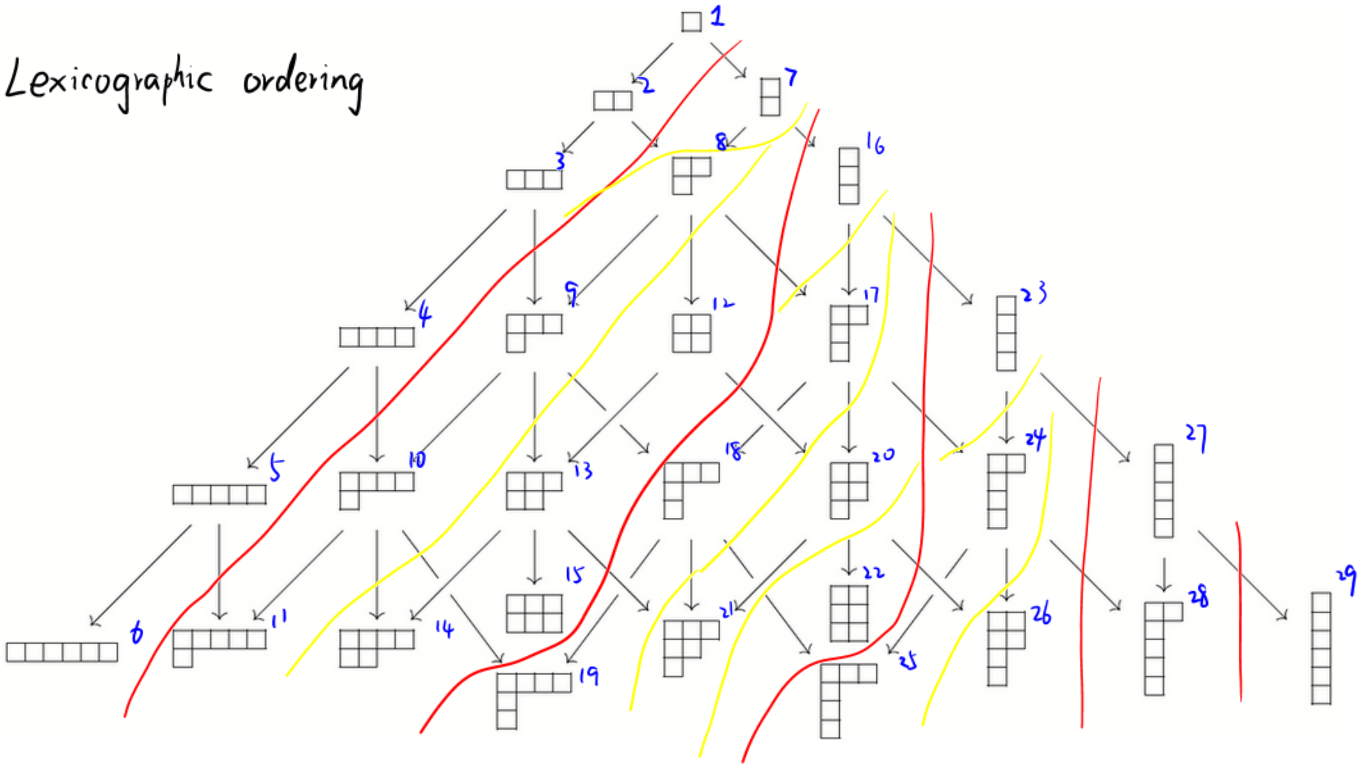
Inclusion $\subseteq$ $\longrightarrow$

dominance $\trianglelefteq$



Lexicographic ordering

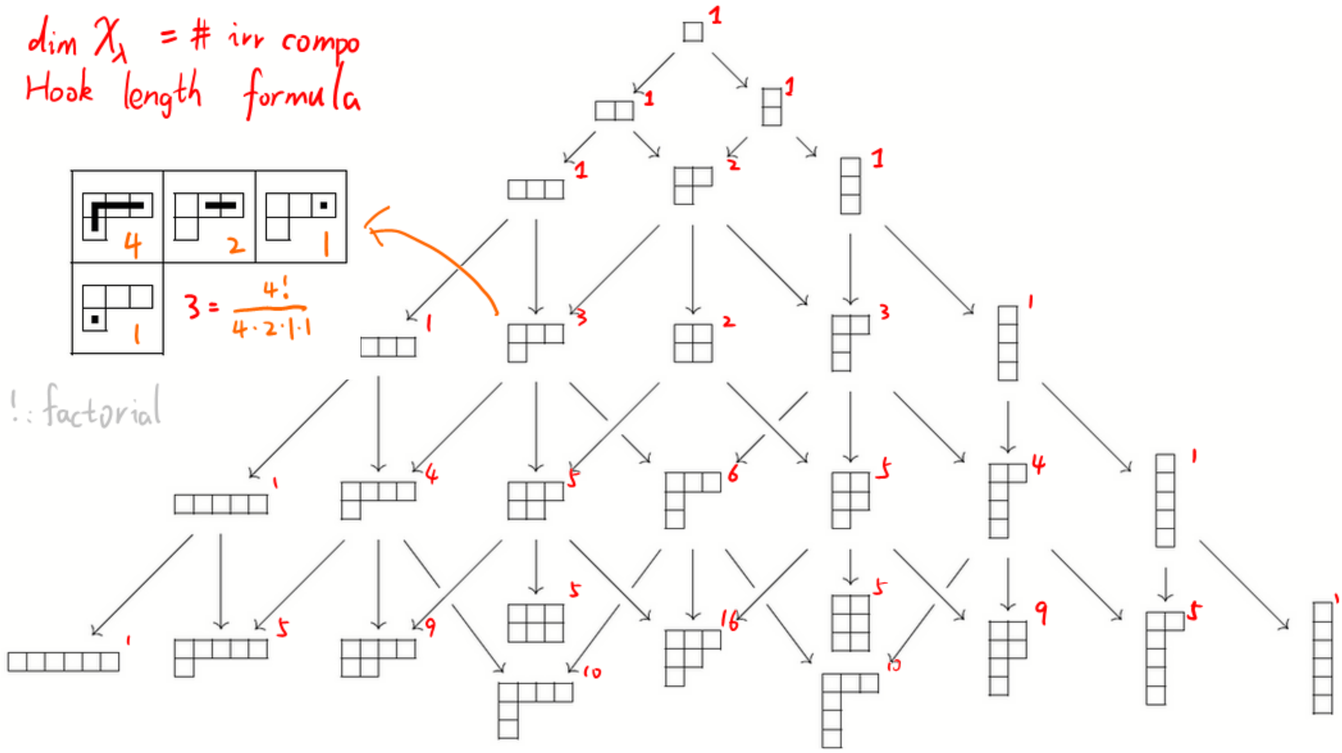


dim χ_λ = # irr compo

Hook length formula

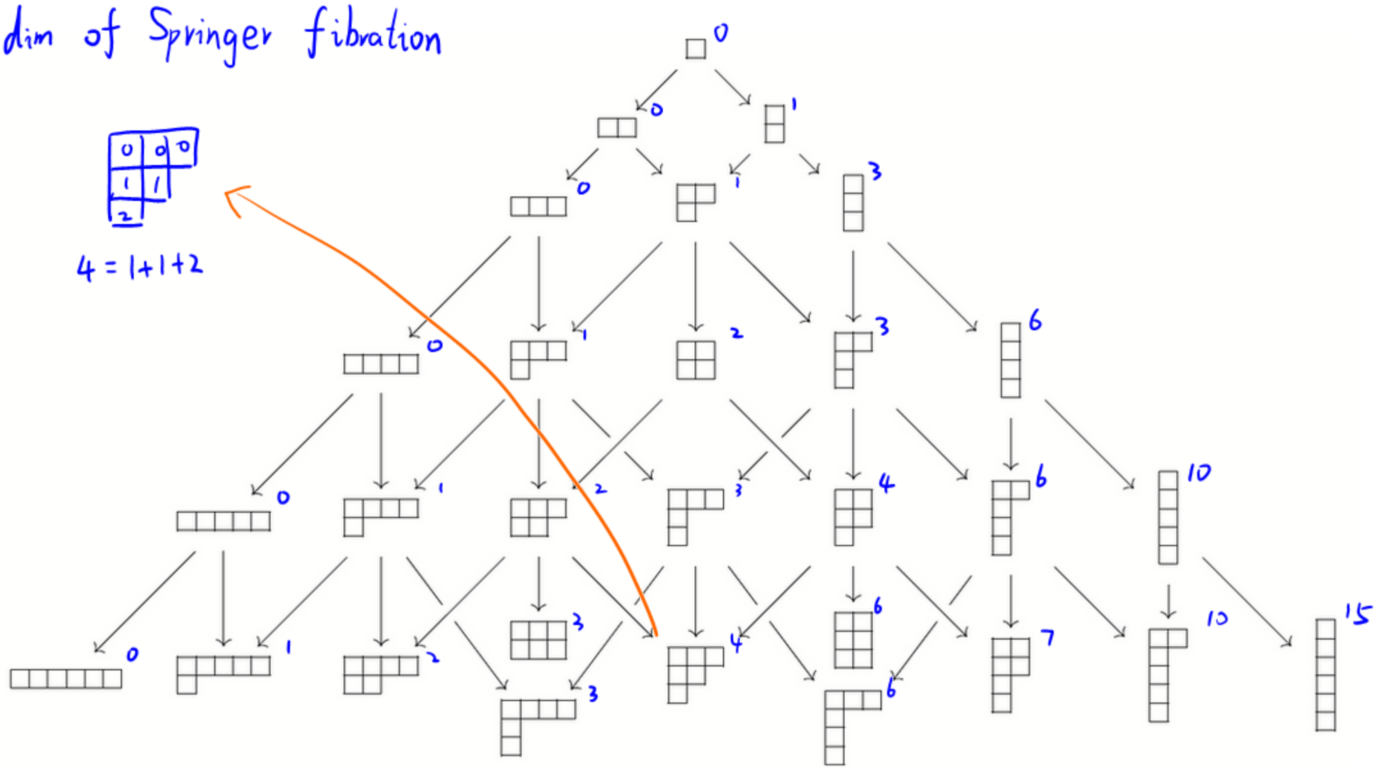
$3 = \frac{4!}{4\cdot 2\cdot 1\cdot 1}$

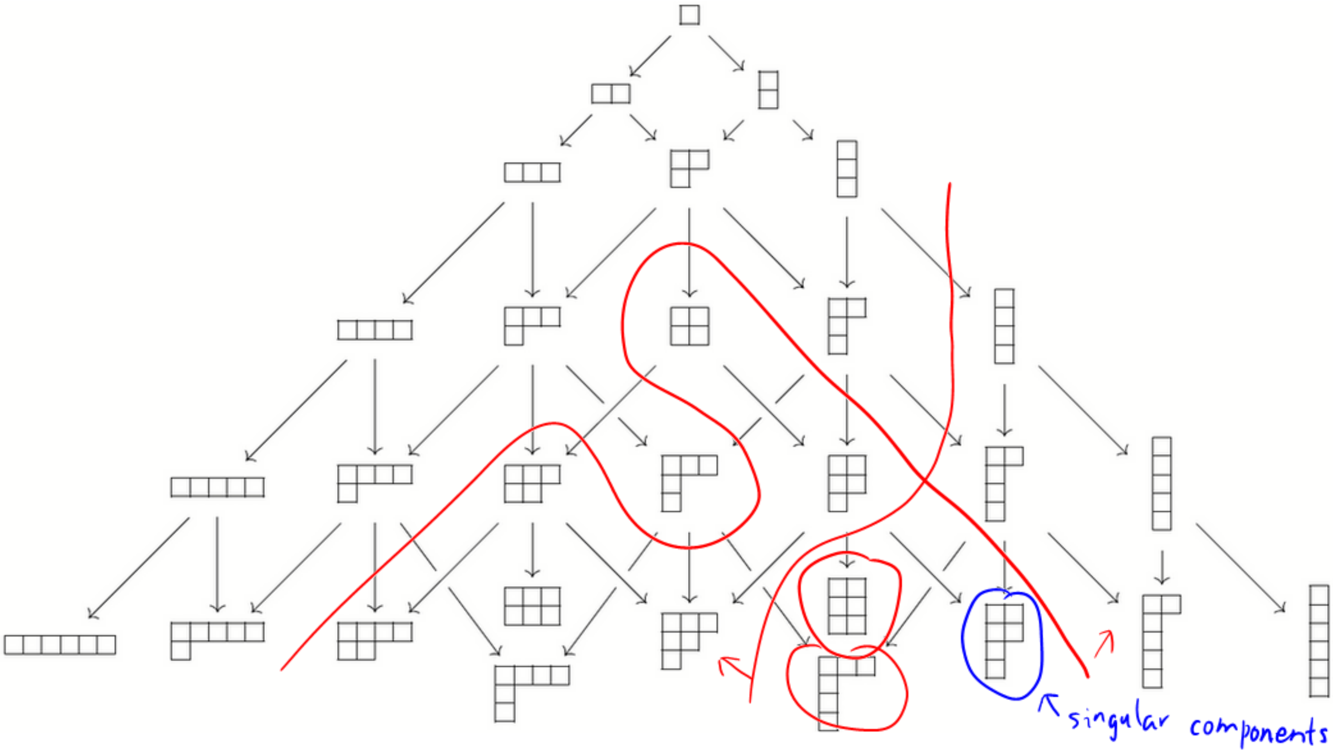
!: factorial



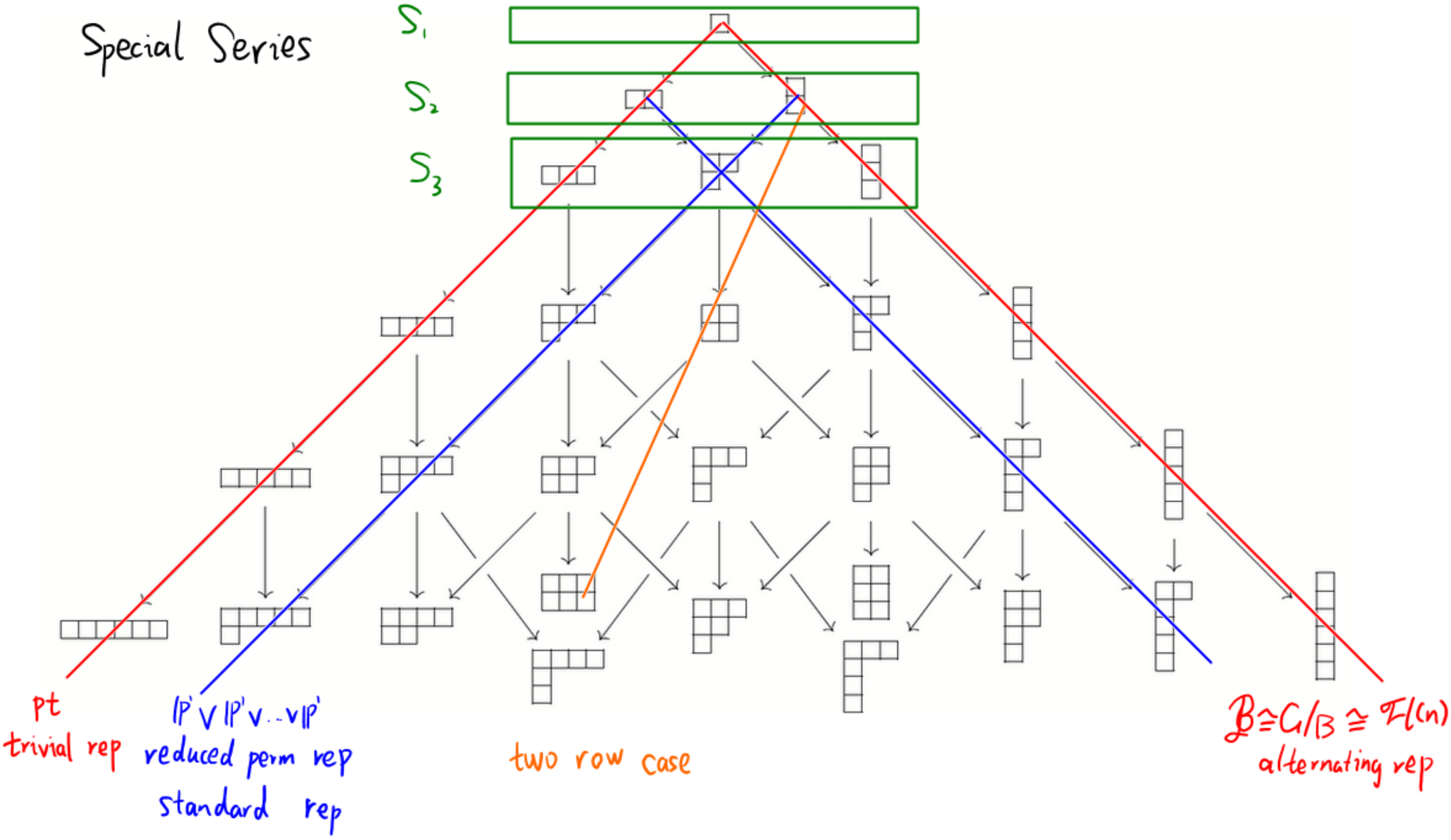
dim of Springer fibration

$4 = 1+1+2$




singular components

Special Series



Q: What's the din of $\{$Cartan subalg of $sl_n(\mathbb{C})\}$?

$$\begin{tikzcd}
\{\text{Borel subalg}\} \arrow[r, dashed, leftrightarrow] \arrow[d, "\text{finite}" ', "\text{surj?}"] & \{\text{Borel subgroup}\} \arrow[r, "\cong", "\text{fix } B" above] \arrow[d] & G/B \arrow[d, "\text{finite?}"] \\
\{\text{Cartan subalg}\} \arrow[r, dashed, leftrightarrow] & \{\text{Maximal torus}\} \arrow[r, "\cong", "\text{fix } T" above] & G/N_G(T)
\end{tikzcd}$$

$\circlearrowleft G$ transitive?

structure?

Examples of Springer fiber. $E_{\lambda_1,\dots,\lambda_n} =$ Springer fiber of $\begin{bmatrix} J_{\lambda_1} & & \\ & \ddots & \\ & & J_{\lambda_n} \end{bmatrix}$ $J_\lambda = \begin{bmatrix} 0 & 1 & & \\ & 0 & 1 & \\ & & 0 & 1 \\ & & & 0 \end{bmatrix}$

$\mathcal{S}f \quad \mathbb{F} \quad \mathcal{F} \quad \mathcal{B} \quad X \quad \mathbb{X} \quad \mathbb{V} \quad \mathbb{S} \quad E^{\lambda\times\lambda}$

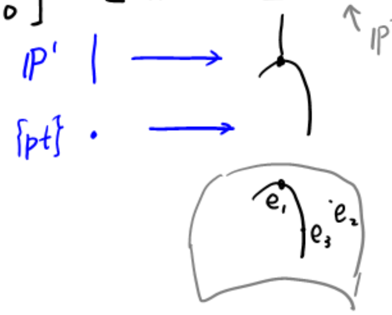
□ $[0]$ $E_1 = \{ 0 \subseteq \mathbb{C} \} = \cdot$

□□ $\begin{bmatrix} 0 & 1 \\ & 0 \end{bmatrix}$ $E_2 = \{ 0 \subseteq \langle v_1 \rangle \subseteq \mathbb{C}^2 \} = \cdot$

$\begin{bmatrix} 0 & \\ & 0 \end{bmatrix}$ $E_{1,1} = \{ 0 \subseteq \langle ? \rangle \subseteq \mathbb{C}^2 \} = \mathbb{P}^1$

□□□ $\begin{bmatrix} 0 & 1 & \\ & 0 & 1 \\ & & 0 \end{bmatrix}$ $E_3 = \{ 0 \subseteq \langle v_1 \rangle \subseteq \langle v_1, v_2 \rangle \subseteq \mathbb{C}^3 \} = \cdot$

$\begin{bmatrix} 0 & 1 & \\ & 0 & \\ & & 0 \end{bmatrix}$ $E_{2,1} = \{ 0 \subseteq \underset{\uparrow \mathbb{P}^2}{\langle ? \rangle} \subseteq \langle ?, ? \rangle \subseteq \mathbb{C}^3 \} = \mathbb{P}^1 \vee \mathbb{P}^1$



$$\begin{array}{ccc} E_{1,1} & \rightarrow & E_{2,1} \\ E_2 & \rightarrow & \\ & & \downarrow \\ & & \mathbb{P}^1 \subseteq \mathbb{P}^2 \end{array}$$

$F_{e_1} = \{ 0 \subseteq \langle e_1 \rangle \subseteq \langle e_1, ? \rangle \subseteq \mathbb{C}^3 \} \circlearrowleft \begin{bmatrix} 0 & 1 & \\ & 0 & \\ & & 0 \end{bmatrix}$

$\Updownarrow$

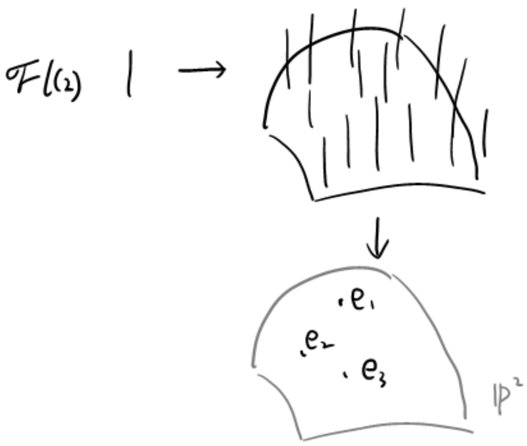
$\mathbb{P}^1 = E_{1,1} = \{ 0 \subseteq \langle ? \rangle \subseteq \mathbb{C}^2 \} \circlearrowleft \begin{bmatrix} 0 & \\ & 0 \end{bmatrix}$

$F_{e_3} = \{ 0 \subseteq \langle e_3 \rangle \subseteq \langle e_3, ? \rangle \subseteq \mathbb{C}^3 \} \circlearrowleft \begin{bmatrix} 0 & 1 & \\ & 0 & \\ & & 0 \end{bmatrix}$

$\Updownarrow$

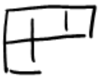
$\{pt\} = E_2 = \{ 0 \subseteq \langle ? \rangle \subseteq \mathbb{C}^2 \} \circlearrowleft \begin{bmatrix} 0 & 1 \\ & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & & \\ & 0 & \\ & & 0 \end{bmatrix}$ $E_{1,1,1} = \{ 0 \subseteq \underset{\uparrow \mathbb{P}^2}{\langle ? \rangle} \subseteq \langle ?, ? \rangle \subseteq \mathbb{C}^3 \} = \mathcal{F}l(3)$



$$\begin{array}{ccccccccc} \mathcal{F}l(1) & - & \mathcal{F}l(2) & - & \mathcal{F}l(3) & - & \mathcal{F}l(4) & - & \mathcal{F}l(5) \\ & & | & & | & & | & & | \\ & & \mathbb{P}^1 & & \mathbb{P}^2 & & \mathbb{P}^3 & & \mathbb{P}^4 \end{array}$$

$$\begin{array}{ccccccccc} E_1 & - & E_{1,1} & - & E_{1,1,1} & - & E_{1,1,1,1} & - & E_{1,1,1,1,1} \\ & & | & & | & & | & & | \\ & & \cdot & & \cdot & & \cdot & & \cdot \end{array}$$



$$\begin{bmatrix} 0 & 1 & & \\ & 0 & 1 & \\ & & 0 & \\ & & & 0 \end{bmatrix}$$

$$E_{3,1} = \{0 \subseteq \langle ? \rangle \subseteq \langle ? \rangle \subseteq \langle ? \rangle \subseteq \mathbb{C}^4\} = \mathbb{P}^1 \vee \mathbb{P}^1 \vee \mathbb{P}^1$$

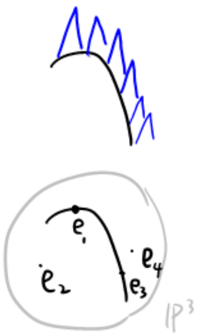
$E_{2,1}$

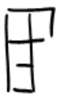
E_3

$$\begin{bmatrix} 0 & 1 & & \\ & 0 & & \\ & & 0 & 1 \\ & & & 0 \end{bmatrix}$$

$$E_{2,2} = \{0 \subseteq \langle ? \rangle \subseteq \langle ? \rangle \subseteq \langle ? \rangle \subseteq \mathbb{C}^4\} = \mathbb{P}^1\text{-bundle} \vee_{\mathbb{P}^1} \mathbb{P}^1\text{-bundle}$$

$E_{2,1}$





$$\begin{bmatrix} 0 & 1 & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{bmatrix}$$

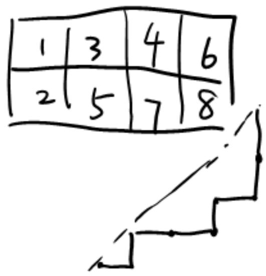
$$E_{2,1,1} = \{0 \subseteq \langle ? \rangle \subseteq \langle ? \rangle \subseteq \langle ? \rangle \subseteq \mathbb{C}^1\}$$

$\mathcal{F}(\mathbb{C}^3) = E_{1,1,1}$

$\mathbb{P}^1 \vee \mathbb{P}^1 = E_{2,1}$

Q: Can we construct an affine paving from this kind of fibration?

Q: How to understand the Weyl group action on Springer fibration?



"Bertrand's ballot theorem"

"bracket pairing"

Define M^λ to be the complex vector space with basis the tabloids $\{T\}$ of shape λ, with λ a partition of n.

(3) $$a_T = \sum_{p \in R(T)} p, \quad b_T = \sum_{q \in C(T)} \operatorname{sgn}(q) q. \qquad v_T = b_T \cdot \{T\} = \sum_{q \in C(T)} \operatorname{sgn}(q)\{q \cdot T\}.$$

These elements, and the product

$$c_T = b_T \cdot a_T,$$

Define the ***Specht module*** S^λ to be the subspace of M^λ spanned by the elements v_T, as T varies over all numberings of λ.

Young tabloid

Proposition 1 *For each partition λ of n, S^λ is an irreducible representation of S_n. Every irreducible representation of S_n is isomorphic to exactly one S^λ.*

整体数矩阵.

Proposition 2 *The elements v_T, as T varies over the standard tableaux on λ, form a basis for S^λ.*

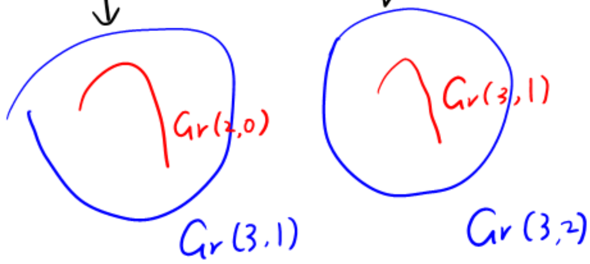
$\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & \\ \cline{1-1}\end{array}$ $\quad \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & \\ \cline{1-1}\end{array}$ $\quad \begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline 3 \\ \hline\end{array}$ $\quad \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline\end{array}$

$$X = \begin{bmatrix} 0 & 1 & \\ & 0 & \\ & & 0 \end{bmatrix}$$

$$0 \subseteq \underset{0}{\operatorname{Im} X^2} \subseteq \underset{\langle e_1 \rangle}{\operatorname{Im} X} \subseteq \mathbb{C}^3$$

$$B_{2,1} = \{ 0 \subseteq \langle ? \rangle \subseteq \langle ?, ? \rangle \subseteq \mathbb{C}^3 \}$$

$$0 \subseteq \underset{\langle e_1, e_3 \rangle}{\ker X} \subseteq \underset{\langle e_1, e_2, e_3 \rangle}{\ker X^2} \subseteq \mathbb{C}^3$$



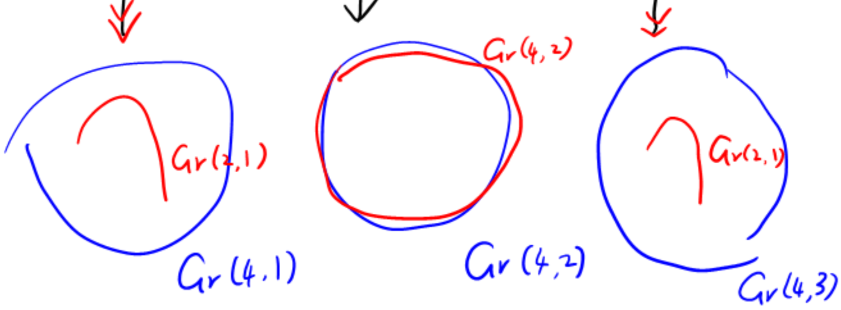
$$X = \begin{bmatrix} 0 & 1 & & \\ & 0 & & \\ & & 0 & 1 \\ & & & 0 \end{bmatrix}$$

$$0 \subseteq 0 \subseteq \langle e_1, e_3 \rangle$$

$$B_{2,1} = \{ 0 \subseteq \langle ? \rangle \subseteq \langle ?, ? \rangle \subseteq \langle ?, ?, ? \rangle \subseteq \mathbb{C}^4 \}$$

$$\langle e_1, e_3 \rangle \subseteq \mathbb{C}^4 \subseteq \mathbb{C}^4$$

π_1 π_2 not surj ! π_3



$$\operatorname{Im} \pi_2 = \{ V \subseteq \mathbb{C}^4 \mid \dim_{\mathbb{C}} V = 2,\ XV \subseteq V \} \subseteq Gr(4,2)$$

↲ not smooth!

$\operatorname{Im} \pi_2$

$\uparrow \pi_2$

↑ the other component.

$\xrightarrow{\pi_3} \mathbb{P}^1$

$\swarrow \pi_1$

$\mathbb{P}^1$