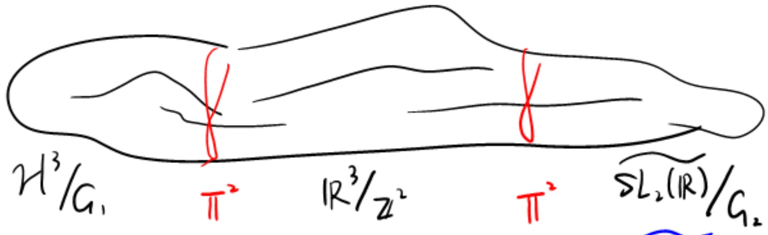
X



$X = H^3/G_1 \sqcup \mathbb{T}^2 \sqcup \mathbb{R}^3/\mathbb{Z}^2 \sqcup \mathbb{T}^2 \sqcup \widetilde{SL_2(\mathbb{R})}/G_2$

还不能确认有G_1 s.t $\partial(H^3/G_1) \cong \mathbb{T}^2$. 对$G_2$也有问题.

$\chi(\mathcal{O}_X)\ \chi(\Omega_X)\ \chi(\omega_X)$

$e := \chi_{top}(X)$

		1			b_4	$h^{2,2}$
	q		q		b_3	$h^{1,2}\ h^{2,1}$
p_g		$h^{1,1}$		p_g	b_2	$h^{0,2}\ h^{1,1}\ h^{2,0}$
	q		q		b_1	$h^{0,1}\ h^{1,0}$
		1			b_0	$h^{0,0}$

p_a

$c_2 = e$

$c_1^2 = K^2 = 12\chi(\mathcal{O}_X) - e$

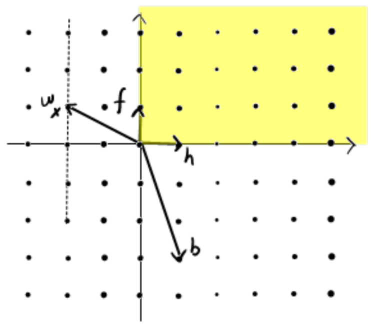
Filtration: $\underbrace{\mathrm{Pic}\,X \overset{\mathbb{Z}^{\rho(X)}}{\supseteq}}_{N^1(X)} \mathrm{Pic}^\tau X \overset{\text{torsion}}{\supseteq} \mathrm{Pic}^0 X \overset{\text{scheme}}{\supseteq} 0$

$NS(X)$

κ	$-\infty$			0		1	2
	ruled surface over C			K3	abelian		surfaces of general type
	$g(C)=0$: rational ($\mathbb{P}^2$, Hirzebruch)	$g(C)=1$	$g(C)\geq 2$	Enrique	bielliptic		

elliptic surface

toric variety



nef cone

effective cone

intersection matrix:
$$\begin{array}{c|cc} & h & f \\ \hline h & 3 & 1 \\ f & 1 & 0 \end{array}$$

$\omega_X = -2f - h - b$

$$\mathbb{C}[x_{1/0}, x_{2/0}] \underset{\cdot\times x_{1/0}^{n} = x_{0/1}^{-n}}{\overset{\cdot\times x_{0/1}^{n} = x_{1/0}^{-n}}{\rightleftarrows}} \mathbb{C}[x_{0/1}, x_{2/1}]$$

例子 $\boxed{\mathbb{P}^2 \quad \mathbb{P}^1\times\mathbb{P}^1 \quad \text{Hirzebruch 曲面} \quad \mathbb{P}^3_3\text{: 3次曲面}}$ } 有理曲面

理论 Pic X, 相交矩阵 直纹面 双有理等价 blow up 极小曲面模型

$$C \leftarrow C_1 \to C_2 \to C_3 \leftarrow \cdots \quad \cdots \to C'$$

$$\begin{array}{ccccc} & & 1 & & \\ & 0 & & 0 & \\ 1 & & 20 & & 1 \\ & 0 & & 0 & \\ & & 1 & & \end{array} \qquad \begin{array}{ccccc} & & 1 & & \\ & 0 & & 0 & \\ 0 & & 10 & & 0 \\ & 0 & & 0 & \\ & & 1 & & \end{array} \qquad \begin{array}{ccccc} & & 1 & & \\ & 2 & & 2 & \\ 1 & & 4 & & 1 \\ & 2 & & 2 & \\ & & 1 & & \end{array} \qquad \begin{array}{ccccc} & & 1 & & \\ & 1 & & 1 & \\ 0 & & 2 & & 0 \\ & 1 & & 1 & \\ & & 1 & & \end{array}$$

K3 Enrique abelian bielliptic

$$\begin{array}{cc} & H \\ H & \begin{bmatrix} 1 \end{bmatrix} \end{array} \qquad \begin{array}{cc} & \begin{array}{cc} h & f \end{array} \\ \begin{array}{c} h \\ f \end{array} & \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \end{array} \qquad \begin{array}{cc} & \begin{array}{cc} h & f \end{array} \\ \begin{array}{c} h \\ f \end{array} & \begin{bmatrix} n & 1 \\ 1 & 0 \end{bmatrix} \end{array}$$

$\mathbb{P}^2$ $\mathbb{P}^1\times\mathbb{P}^1$ $\mathbb{F}_n$

$$\begin{array}{ccccc} & & 1 & & \\ & 0 & & 0 & \\ 0 & & 1 & & 0 \\ & 0 & & 0 & \\ & & 1 & & \end{array} \qquad \begin{array}{ccccc} & & 1 & & \\ & 0 & & 1 & \\ 0 & & 0 & & 0 \\ & 1 & & 0 & \\ & & 1 & & \end{array}$$

Hopf surface