RESEARCH STATEMENT

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1 Introduction

My master thesis consists of two parts. The first part computes the equivariant K-theory of Steinberg varieties, while the second part focuses on the geometry of certain varieties known as (partial) flag quiver varieties. Since the precise statements are already summarized in the introduction of the thesis, I will instead focus on the creativity and novel ideas in this document.

Part I: K-theory of the Steinberg variety.

For the first part, we fix a quiver Q (without loops and cycles) and dimension vector $\mathbf{d}$. The Steinberg variety $\mathcal{Z}_{\mathbf{d}}$ is an incidence variety, consisting of triples of a representation of Q and two complete flags fixed by this representation.

With such natural definition, $\mathcal{Z}_{\mathbf{d}}$ possesses a natural stratification which allows us to extract the geometric information through direct methods. For example, by the cellular fibration theorem, the stratification gives us a basis of the K-theory (or, any other cohomology theory) of $\mathcal{Z}_{\mathbf{d}}$.

The Steinberg variety $\mathcal{Z}_{\mathbf{d}}$ has a convolution structure which induces an algebraic structure of the K-theory. Another basis is introduced in order to compute this structure. The new basis corresponds to T-fixed points which "are more concentrated than the stratification", so we are able to get explicit convolution product formulas by applying the excess intersection formula. Finally, we deduce the Demazure operator and conclude our computations in the diagram of strands.

Part II: affine paving of quiver flag variety.

This part is already posted on arXiv. It focuses on the affine paving of the quiver (partial) flag varieties. We use basic cohomological algebraic tools to construct the affine pavings by induction, and the problem finally reduces to combinatorics related to the Auslander–Reiten theory.

2 What's new?

There have been many literature concerning the K-theory of (classical) Steinberg variety, and the method of calculation may be well-known to experts. In spite of this, I try to write down computations clearer than other literature, and point out some technicalities which are not mentioned due to the confusion of notations.

Moreover, I realized that those nontrivial results boil down to just a small amount of results from K-theory. The formalism for the K-theory differs from the usual 6-functor formalism due to the base change formula, where a twist by Euler class is needed. Therefore, to understand other cohomology theories of $\mathcal{Z}_{\mathbf{d}}$, one only needs to substitute the expressions of Euler class (and the cohomology ring of a point).

Characteristic classes also play a role in my thesis. It looks like that the Chern class measures the differences between two different cohomology theories, while the Euler class and Todd class measure the failure of commutativity for some diagrams. Under this interpretation, these classes fit in the language of (K-theoretical) 6-functor formalism. They are quite explicit in our setting but hard to connect with their original geometric meaning.

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3 Further plans

The K-theory of the Steinberg variety lies in the one side of the Kazhdan–Lusztig isomorphism, where the other side, the affine Hecke algebra, can be also explicitly computed. An immediate step is to explicitly construct this isomorphism. I also need to learn the Bezrukavnikov equivalence, which is the categorification of the Kazhdan–Lusztig isomorphism. An understanding of their representation theory is also needed during the process.

After that, I want to see the application of K-theory in the Langlands program. I want to understand how the Bezrukavnikov equivalence contributes to the geometric Langlands program. In particular, I would like to see how the Steinberg variety play a role, and if we can generalize some results in the quiver version.

Finally, I want to study the similarities between Shimura varieties and flag varieties. They are defined by group quotients, and their structures hold information about classical objects and representations. [1]

In my bachelor study, I was fascinated by the concreteness of the elliptic modular form. In my master study, I got surprised by the deep understandings and fruitful applications of representations. Now I see how modular form is viewed as automorphic form, and understand Hecke algebra much better than before. It is so intriguing to connect these two objects (Shimura varieties and flag varieties), and study how this connection gives us better understanding for both sides.

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[1] For example, One can read deformation informations of one abelian variety from Shimura varieties.