

## SIX-FUNCTOR FORMALISMS ORAL EXAM

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The course name is “Six Functor Formalisms”, and the examiner is Prof. Scholze.

I went to the office with high pressure, since that is my first time to take an exam of Prof. Scholze. The exam goes pretty nice, actually.

We begin with the six functors. I said that these are pullbacks, pushforward,..., I said that the relations about these functors. I was asked about one example for this formalism. I said that  $X$  may be some nice topological space, but then I got confused what  $D(X)$  should be. I thought that it should be a derived category, and then I state what a derived category is. But Prof. Scholze are still not quite satisfied. Actually, I should say that  $D(X)$  should be a derived category of abelian sheaves. I was asked about the definition of sheaf, which I get quite confused.

After that thing goes better. I need to say what the 3-functor formalism is, and then I need to say how that symmetric monoidal functor realize these three functors.

After that, I said something about the constructions. I recalled all the conditions for constructing the proper pushforward, and I asked if I need to do those diagram argument. He said that we have no enough time, and he had one problem to ask me, but he forget.

Finally, he asked something about the Poincaré duality. I said the definition of cohomologically smooth, and said how it is related with the classical Poincaré duality. Since we have no time, he said that he have no more questions and I can say anything if I want. I said the structure of the last section, see [https://github.com/ramified/personal\\_handwritten\\_collection/blob/main/weeklyupdate/2023.02.05\\_six\\_functors.pdf](https://github.com/ramified/personal_handwritten_collection/blob/main/weeklyupdate/2023.02.05_six_functors.pdf). He was quite happy and told me that the further results in this topic can be seen in an article posted on arXiv recently.

After the exam, he was quite satisfied and give me quite nice notes. I told him that my master thesis is also related with the 6-functors formalism, and he replied that he is always confused with those  $K$ -groups. He also answered my questions concerning about invertible objects and Fourier-Mukai functors.

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