LaTeX TEMPLATE

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Contents

1. A small toolkit

$f : Y \longrightarrow \mathrm{pt}$ $f : p \hookrightarrow X$
f^* constant sheaf $\mathcal{F}_p$
Rf_* cohomology $\mathrm{sky}_p(\mathbb{Q})$
$Rf_!$ cpt supp cohomology $\mathrm{sky}_p(\mathbb{Q})$
$f^!$ orientation sheaf $[n]$ $\mathcal{F}_p[-n]$
For $f^!$, assume Y, X are manifolds of dimension n.
$j_!j^*\mathcal{F}$ $\mathcal{F}$ $i_!i^*\mathcal{F}$
Certainly! Here is the corrected version of your list:
—

2. A Short List of Applications

Assuming the six-functor formalism (and everything derived), let X be a smooth manifold of dimension n.

1. Define four types of cohomology:
...
Verify that:
...
Also, define the cup and cap product structures.
2. Using the projection formula, show Poincaré duality:
...
3. Define the relative cohomology (using only six functors) so that:
...
4. Derive the Gysin sequence for any oriented S^k-bundle $\pi : E \longrightarrow B$:
...

Derive the Mayer-Vietoris sequence and the relative cohomology sequence, and verify the equivalence of different cohomology groups.

5. Compute the upper shriek for singular spaces.
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Let me know if you need further adjustments!

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$$\begin{array}{lllll}
\mathrm{H}^i(Y,\mathbb{Q}) & = \mathrm{H}^i(Y,\underline{\mathbb{Q}}_Y) & = f_*\underline{\mathbb{Q}}_Y & = f_*f^*\mathbb{Q} \\
\mathrm{H}^i_c(Y,\mathbb{Q}) & = \mathrm{H}^i_c(Y,\underline{\mathbb{Q}}_Y) & = f_!\underline{\mathbb{Q}}_Y & = f_!f^*\mathbb{Q} \\
\mathrm{H}_{-i}(Y,\mathbb{Q}) & = \mathrm{H}^{n+i}_c(Y,\mathrm{Or}_Y) & = f_!\,\mathrm{Or}_Y[n] & = f_!f^!\mathbb{Q} \\
\mathrm{H}^{\mathrm{BM}}_{-i}(Y,\mathbb{Q}) & = \mathrm{H}^{n+i}(Y,\mathrm{Or}_Y) & = f_*\,\mathrm{Or}_Y[n] & = f_*f^!\mathbb{Q}
\end{array}$$

six functor formalism $\approx$ cohomology theory

References

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