RESEARCH STATEMENT

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My research lies in algebraic geometry and geometric representation theory, with a particular focus on using geometric methods to reveal hidden combinatorial and representation-theoretic structures. Throughout my work—ranging from affine pavings of quiver varieties and equivariant K-theory of Steinberg varieties to the geometry of characteristic cycles of perverse sheaves—I have been motivated by understanding how rich algebraic and representation-theoretic phenomena naturally emerge from geometry. By employing tools from algebraic geometry, topology, and Auslander–Reiten theory in novel ways, I have bridged seemingly distinct areas and developed a unifying perspective that guides my current and future research.

1 Past and Current Research

Affine pavings of quiver partial flag varieties.

In my master's thesis, I studied quiver partial flag varieties for Dynkin quivers and constructed affine pavings for these varieties. The construction uses a systematic stratification combined with Auslander–Reiten combinatorics to reduce the geometry to explicit combinatorial data. These pavings give a transparent description of the cohomology groups of these varieties.

Equivariant K-theory of Steinberg varieties.

Another component of my master's work was the computation of the equivariant K-theory of Steinberg varieties. The K_0-groups acquire an algebra structure analogous to the affine Hecke algebra, where the action of generators can be described through combinatorial strand diagrams. This project sparked my interest in the geometrical representation theory.

Characteristic cycles of perverse sheaves on abelian varieties.

During my PhD, I shifted focus to the geometry of characteristic cycles of perverse sheaves on an abelian variety A, which can be expressed as formal sums of the conormal varieties of certain subvarieties in A. Building on Prof. Krämer's work, we developed a purely geometric construction of these subvarieties and studied their fundamental properties, including irreducibility, dimension, and homology classes.

We also gained new insights into the monodromy of the conormal Gauss maps in the curve case. In particular, we obtained a family of cases where the monodromy group $\mathrm{Gal}(\gamma_Z)$ is strictly smaller than the associated Weyl group, and we formulated a conjecture precisely characterizing when this occurs, stated in terms of the Gauss curvature of the corresponding curve.

2 Future Research Directions

My future research aims to deepen the geometric understanding of representation-theoretic structures and to connect my current work to broader themes in geometric representation theory, particularly those surrounding Hecke algebras, Schubert varieties, and Langlands duality.

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Algebraic structures in equivariant K-theory of Steinberg varieties.

I plan to revisit my earlier work on the equivariant K-theory of Steinberg varieties with the goal of identifying the "big algebra" in K_0. Beginning with the type A full flag variety, I aim to clarify the relationship between this algebra structure, convolution geometry, and the geometry of Schubert varieties. Understanding this interaction may shed new light on Hecke algebras from a geometric viewpoint, with potential relevance to the geometric Langlands program.

Characteristic cycles on Schubert varieties.

A second research direction is to extend my work on characteristic cycles from abelian varieties to Schubert varieties. Since the K-theory of the Steinberg variety and the category of perverse sheaves are related by the Bezrukavnikov equivalence, which is the categorification of the Kazhdan–Lusztig isomorphism, I plan to describe characteristic cycles of perverse sheaves on Schubert varieties and trace their images under the equivalence. This project connects naturally to the Springer correspondence, the geometry of conic Lagrangians, and Langlands correspondence.

3 Outlook

Across my projects, my goal is to develop geometric frameworks that reveal the underlying combinatorial and representation-theoretic structures. The Hausel group, with its emphasis on geometric representation theory, equivariant geometry, and Langlands correspondence, offers an ideal environment for advancing these ideas. I look forward to developing new geometric frameworks that illuminate deep connections between algebraic geometry, representation theory, and combinatorics, while contributing to and learning from the Hausel group's vibrant research environment.

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