LATEX TEMPLATE

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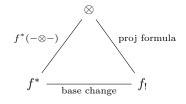
1. A small toolkit

$$\begin{split} f: Y &\longrightarrow \text{pt } f: p \hookrightarrow X \\ f^* \text{ constant sheaf } \mathcal{F}_p \\ Rf_* \text{ cohomology } \text{sky}_p(\mathbb{Q}) \\ Rf_! \text{ cpt supp cohomology } \text{sky}_p(\mathbb{Q}) \\ f^! \text{ orientation sheaf } [n] \ \mathcal{F}_p[-n] \\ \text{For } f^!, \text{ assume } Y, X \text{ are manifolds of dimension } n. \\ j_! j^* \mathcal{F} \ \mathcal{F} \ i_! i^* \mathcal{F} \end{split}$$

$$Z \stackrel{i}{\longleftarrow} X \stackrel{j}{\longleftarrow} U$$

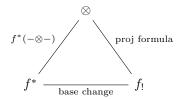
$$D(Z) \xrightarrow[i_*=i_!]{i_*=i_!} D(X) \xrightarrow[Rj_*]{j^*=j^!} D(U)$$

$$j_! j^* \mathcal{F} \longrightarrow \mathcal{F} \longrightarrow i_! i^* \mathcal{F} \stackrel{+1}{\longrightarrow}$$



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	$f: Y \longrightarrow \mathrm{pt}$	$f:p\hookrightarrow X$
f^*	constant sheaf	\mathcal{F}_p
Rf_*	cohomology	$\mathrm{sky}_p(\mathbb{Q})$
$Rf_!$	cpt supp cohomology	$\operatorname{sky}_p(\mathbb{Q})$
f!	orientation sheaf	$[n] \mathcal{F}_p[-n]$



2. A Short List of Applications

Assuming the six-functor formalism (and everything derived), let X be a smooth manifold of dimension n.

1. Define four types of cohomology and the relative cohomology. Verify that:

$$\begin{aligned} & \mathbf{H}_{\mathbf{c}}^{i}(X;\mathbb{Q}) \cong \mathbf{H}^{i}\left(\bar{X}, \{\infty\}; \mathbb{Q}\right) \\ & \mathbf{H}_{i}^{\mathrm{BM}}(X;\mathbb{Q}) \cong \mathbf{H}^{n-i}(X; \mathrm{Or}_{X}) \\ & \mathbf{H}_{i}(X;\mathbb{Q}) \cong \mathbf{H}_{\mathbf{c}}^{n-i}(X; \mathrm{Or}_{X}) \end{aligned}$$

Also, define the cup and cap product structures.

2. Using the projection formula, show Poincaré duality:

$$\mathrm{H}^i_\mathrm{c}(X;\mathbb{Q})^* \cong \mathrm{H}^{n-i}(X;\mathrm{Or}_X)$$

 $\mathrm{H}^i(X;\mathbb{Q}) \cong \mathrm{H}^{n-i}_\mathrm{c}(X;\mathrm{Or}_X)^*$

3. Derive the Gysin sequence for any oriented S^k -bundle $\pi: E \longrightarrow B$:

$$H^n(B) \xrightarrow{\pi^*} H^n(E) \xrightarrow{\pi_*} H^{n-k}(B) \xrightarrow{eu_\pi}^{+1}$$

Derive the Mayer-Vietoris sequence and the relative cohomology sequence, and verify the equivalence of different cohomology groups.

4. Compute the upper shriek for singular spaces.

$$\begin{array}{lll} \mathrm{H}^{i}(Y,\mathbb{Q}) &= \mathrm{H}^{i}(Y,\underline{\mathbb{Q}}_{Y}) &= f_{*}\underline{\mathbb{Q}}_{Y} &= f_{*}f^{*}\mathbb{Q} \\ \mathrm{H}^{i}_{c}(Y,\mathbb{Q}) &= \mathrm{H}^{i}_{c}(Y,\underline{\mathbb{Q}}_{Y}) &= f_{!}\underline{\mathbb{Q}}_{Y} &= f_{!}f^{*}\mathbb{Q} \\ \mathrm{H}_{-i}(Y,\mathbb{Q}) &= \mathrm{H}^{n+i}_{c}(Y,\mathrm{Or}_{Y}) &= f_{!}\,\mathrm{Or}_{Y}[n] &= f_{!}f^{!}\mathbb{Q} \\ \mathrm{H}^{\mathrm{BM}}_{-i}(Y,\mathbb{Q}) &= \mathrm{H}^{n+i}(Y,\mathrm{Or}_{Y}) &= f_{*}\,\mathrm{Or}_{Y}[n] &= f_{*}f^{!}\mathbb{Q} \end{array}$$

six functor formalism \approx cohomology theory

References

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