134%

$\cdot \to \cdot \leftarrow \cdot \leftarrow \cdot$ $\qquad$ $\begin{tikzcd} \cdot \arrow[loop right] \end{tikzcd}$ $\qquad$ $\begin{tikzcd} \cdot \arrow[r, shift left] \arrow[r, shift right] & \cdot \end{tikzcd}$

Quiver of type A_3 $\qquad$ 1-loop quiver $L(1)$ $\qquad$ 2-Kronecker quiver $K(2)$

$Q:$ $\begin{tikzcd} 1 \arrow[r, shift left] \arrow[r, shift right] & 2 \end{tikzcd}$ $\qquad$ $M_\lambda:$ $\begin{tikzcd} K \arrow[r, shift left, "\lambda"] \arrow[r, shift right, "1"'] & K \end{tikzcd}$ $(\lambda \in K)$

$\mathrm{Hom}_{KQ}(P(1), M_1) = ?$ $\qquad$ $\mathrm{Hom}_{KQ}(S(1), P(1)) = ?$ $\qquad$ $\mathrm{Hom}_{KQ}(M_\lambda, M_\mu) = ?$

Let V be a k-linear space of finite dimension,

$\Phi(V):\ V \xrightarrow{Id_V} V \xrightarrow{Id_V} V$ be a rep of quiver $Q: \cdot \to \cdot \to \cdot$

What is the subrepresentaion of $\Phi(V)$?

$\mathcal{F}lag_3(V) = \{0 \subseteq V_1 \subseteq V_2 \subseteq V_3 \subseteq V \mid V_1, V_2, V_3 \text{ subspace}\}$

$Q:$ $\begin{tikzcd} & 1 \arrow[dl, "a"'] \arrow[dr, "c"] & \\ 2 \arrow[dr, "b"'] & & 3 \arrow[dl, "d"] \\ & 4 & \end{tikzcd}$ $\qquad$ $M \in \mathrm{mod}(KQ/I):$ $\begin{tikzcd} & M_1 \arrow[dl, "a"'] \arrow[dr, "c"] & \\ M_2 \arrow[dr, "b"'] & & M_3 \arrow[dl, "d"] \\ & M_4 & \end{tikzcd}$

where $b \circ a = d \circ c$

$Q:$ $\begin{tikzcd} & 1 \arrow[dl, "a"'] \arrow[dr, "c"] & \\ 2 \arrow[dr, "b"'] & & 3 \arrow[dl, "d"] \\ & 4 & \end{tikzcd}$ $\qquad$ $I = (ab - cd)$ $\qquad$ $P(1) = I(4)$

$$\mathrm{rep}(Q) \longrightarrow \mathrm{mod}(KQ)$$
$$(V_i, V_a) \longmapsto \oplus V_i$$
$$a \in Q_1 \subset KQ:\ \oplus V_i \xrightarrow{\pi_{s(a)}} V_{s(a)} \xrightarrow{V_a} V_{t(a)} \xrightarrow{\iota_{t(a)}} \oplus V_i$$

$$(e_i W,\ a\cdot{}:\ e_{s(a)}W \to e_{t(a)}W) \longleftarrow\!\shortmid\ W$$