Springer Fibers for $SL_n(\mathbb{C})$

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In this talk, we use two methods to understand representations of S_n, and find connections/analogs between them.

methods	objects
combinatorial	Young diagram, Young tableu
geometrical	Springer fiber of $SL_n(\mathbb{C})$, irreducible components

char.

Recap: representation theory of finite groups

Restrict to **complex** representations, we have a nice theory:

- Any representation can be written of direct sum of **irreducible representation**;
- We can extract information of irreducible representations from the **character table**:

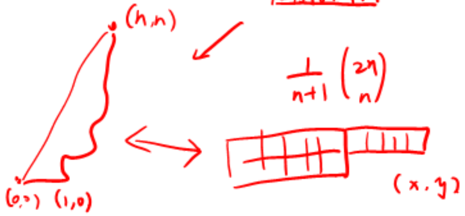
$$\#\{\text{irreducible representations}\} = \#\{\text{conjugation classes}\}$$
$$\sum_{\chi:\text{irr}} (\dim \chi)^2 = \# G$$

However, in general,

- NO standard way finding an **explicit construction** of all irreducible representations;
- NO **one-to-one correspondence** between irreducible representations and conjugation classes.

Goal of the Part 1

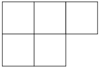
- Explicitly construct irreducible representations of S_n by Young diagram;
- Compute the charecter table;
  - $\dim \chi_i$ by recursion / Hook length formula
  - charecter by Frobenius formula
- Compute other representations.
  - e.g. $\otimes$, Sym^m, Λ^m;
  - e.g. M_λ.



Notation

For boxes:

(Young) diagram



filling

11	78	11
6	8	

standard filling

3	5	4
1	2	

tableau

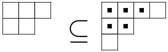
6	11	11
8	78	

standard tableau

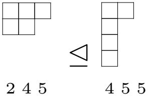
1	3	4
2	5	

Order of Young diagram:

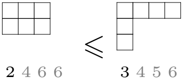
inclusion



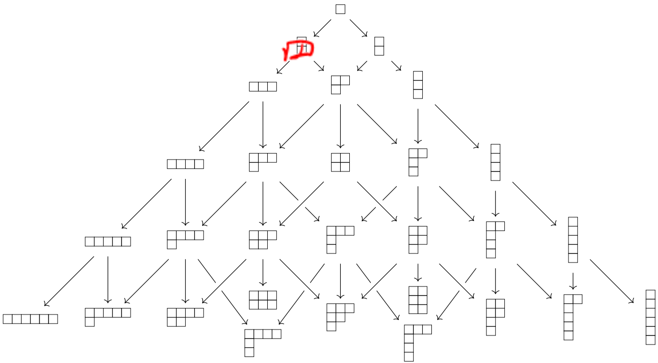
dominance



Lexicographic ordering

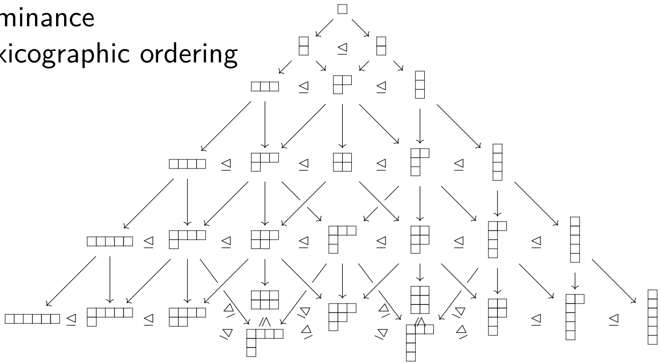


tree of Young diagram



Order

- $\subseteq$ inclusion
- $\trianglelefteq$ dominance
- $\leqslant$ Lexicographic ordering



S_n & Young diagram

The construction of $S^\lambda \subseteq M^\lambda$

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Main theorem of S^λ

Proof: basis

linear ordering

Proof: part 2&3

Proof: part 2&3

Proof: part 2&3

Example: trivial representation

Example: alternating representation

Example: standard representation

Goal

Example: dimension of irreducible representation

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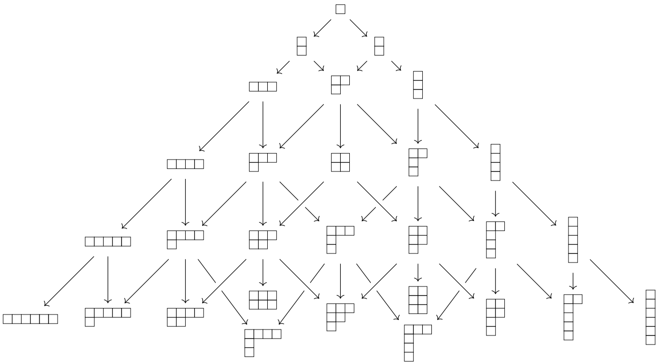
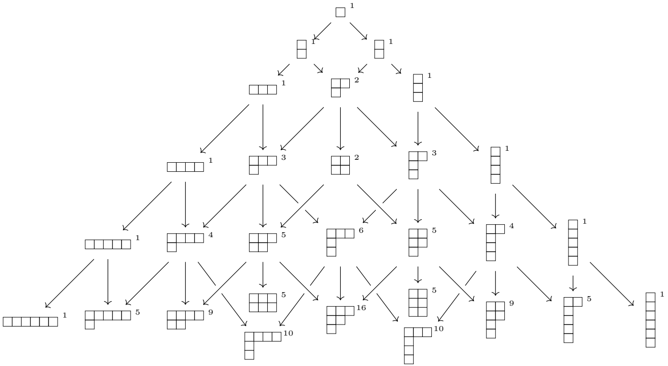


Example: dimension of irreducible representation



Hook length formula

Special case: $(n, 1)$ and (n, n)