Problem 1: Problem 3.45 in R1

Problem 2: Problem 4.22 in R1 ((c) and (e) only)

Problem 3: Problem 5.27 in R1

Problem 4: Problem 5.68 in R1 ($r_{xy}(l)$ only)

Problem 5: Problem 5.84 in R1

hw 4

3.45 Determine the zero-state response of the system

$$y(n) = \frac{1}{2}y(n-1) + 4x(n) + 3x(n-1)$$

to the input

$$x(n) = e^{j\omega_0 n}u(n)$$

What is the steady-state response of the system?

$$y(n) = \frac{1}{2}y(n-1) + 4x(n) + 3x(n-1)$$

$$Y(z) = \frac{1}{2}Y(z)z^{-1} + 4X(z) + 3X(z)z^{-1}$$

$$Y(z) - \frac{1}{2}Y(z)z^{-1} = 4X(z) + 3X(z)z^{-1}$$

$$Y(z)\left(1 - \frac{1}{2}z^{-1}\right) = X(z)\left(4 + 3z^{-1}\right)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{4+3z^{-1}}{1-\frac{1}{2}z^{-1}}$$

$$x(n) = e^{j\omega_0 n}u(n)$$

$$X(z) = \frac{1}{1 - e^{j\omega_0}z^{-1}}$$

$$Y(z) = X(z)H(z) = \frac{4+3z^{-1}}{(1-\frac{1}{2}z^{-1})(1-e^{j\omega_0}z^{-1})} = \frac{A}{1-\frac{1}{2}z^{-1}} + \frac{B}{1-e^{j\omega_0}z^{-1}}$$

$$A = \frac{5}{\frac{1}{2} - e^{j\omega_0}}$$

$$B = \frac{4e^{j\omega_0}+3}{e^{j\omega_0} - \frac{1}{2}}$$

$$\Rightarrow Y(z) = \frac{\frac{5}{\frac{1}{2} - e^{j\omega_0}}}{1-\frac{1}{2}z^{-1}} + \frac{\frac{4e^{j\omega_0}+3}{e^{j\omega_0}-\frac{1}{2}}}{1-e^{j\omega_0}}$$

$$y(n) = \left[\left(\frac{5}{\frac{1}{2}-e^{j\omega_0}}\right)\left(\frac{1}{2}\right)^n + \left(\frac{4e^{j\omega_0}+3}{e^{j\omega_0}-\frac{1}{2}}\right)e^{j\omega_0 n}\right]u(n)$$

$$\lim_{n\to\infty} y(n) = y_{\text{steady state}}(n) = \left(\frac{4e^{j\omega_0}+3}{e^{j\omega_0}-\frac{1}{2}}\right)e^{j\omega_0 n}$$

4.22 A signal $x(n)$ has the following Fourier transform:

$$X(\omega) = \frac{1}{1 - ae^{-j\omega}}$$

Determine the Fourier transforms of the following signals:

(a) $x(2n+1)$

(b) $e^{\pi n/2}x(n+2)$

(c) $x(-2n)$

(d) $x(n)\cos(0.3\pi n)$

(e) $x(n) * x(n-1)$

(f) $x(n) * x(-n)$

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Problem 5: Problem 5.84 in R1

c $X(w) = \dfrac{1}{1-ae^{-jw}}$ // $x(-2n)$

$$\sum_n x(-2n)e^{-jwn} = -\sum_j x(j)e^{-jw\left(\frac{j}{2}\right)} = -\sum_j x(j)\, e^{-j\left(\frac{w}{2}\right)j}$$

$$= X\left(-\frac{w}{2}\right) = \frac{-1}{1-ae^{-jw/2}}$$

e $x(n) * x(n-1)$

$$= X(w)\cdot X(w)e^{-jw}$$

$$= [X(w)]^2 e^{-jw} = \frac{e^{-jw}}{(1-ae^{-jw})^2}$$

Problem 1: Problem 3.45 in R1

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Problem 5: Problem 5.84 in R1

5.27 A digital filter is characterized by the following properties:

1. It is highpass and has one pole and one zero.
2. The pole is at a distance $r = 0.9$ from the origin of the z-plane.
3. Constant signals do not pass through the system.

(a) Plot the pole–zero pattern of the filter and determine its system function $H(z)$.

(b) Compute the magnitude response $|H(\omega)|$ and the phase response $\measuredangle H(\omega)$ of the filter.

(c) Normalize the frequency response $H(\omega)$ so that $|H(\pi)| = 1$.

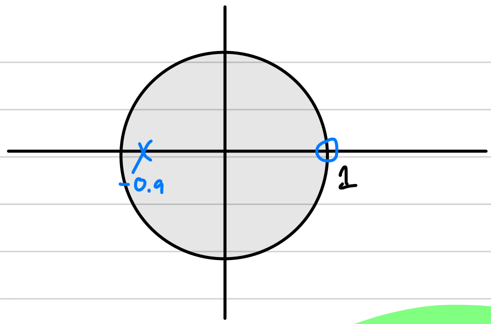
(d) Determine the input–output relation (difference equation) of the filter in the time domain.

(e) Compute the output of the system if the input is

$$x(n) = 2\cos\left(\frac{\pi}{6}n + 45°\right), \qquad -\infty < n < \infty$$

(You can use either algebraic or geometrical arguments.)

(a)



constant

$$H(z) = A \frac{1 - z^{-1}}{1 + 0.9 z^{-1}}$$

(b) $$H(\omega) = H(z)\Big|_{z=e^{j\omega}} = A \frac{1 - e^{-j\omega}}{1 + 0.9 e^{-j\omega}}$$

$$|H(\omega)| = \left| A \frac{1 - e^{-j\omega}}{1 + 0.9 e^{-j\omega}} \right| = \left| A \frac{1 - \cos\omega + j\sin\omega}{1 + 0.9\cos\omega - 0.9 j \sin\omega} \right| = A \frac{\sqrt{(1-\cos\omega)^2 + (\sin\omega)^2}}{\sqrt{(1 + 0.9\cos\omega)^2 + (-0.9\sin\omega)^2}}$$

$$= A \frac{\sqrt{1 - 2\cos\omega + \underbrace{\cos^2\omega + \sin^2\omega}_{1}}}{\sqrt{1 + 1.8\cos\omega + \underbrace{0.81\cos^2\omega + 0.81\sin^2\omega}_{0.81}}} = A \frac{\sqrt{-2\cos\omega + 2}}{\sqrt{1.81 + 1.8\cos\omega}}$$

$$\boxed{|H(\omega)| = \frac{2|\sin(\omega/2)|}{\sqrt{1.81 + 1.8\cos\omega}}}$$

$$\boxed{\measuredangle H(\omega) = \tan^{-1}\left(\frac{\sin\omega}{1 - \cos\omega}\right) + \tan^{-1}\left(\frac{0.9\sin\omega}{1 + 0.9\cos\omega}\right)}$$

(c) $|H(\pi)| = 1 \qquad H(\pi) = A\left(\frac{1 - e^{-j\pi}}{1 + 0.9 e^{-j\pi}}\right) = A \frac{2}{1/10} = 20A = 1 \Rightarrow A = 1/20$

$$H(\omega) = \frac{1}{20}\left(\frac{1 - e^{-j\omega}}{1 + 0.9 e^{-j\omega}}\right)$$

(d)

$$H(z) = \frac{1}{20} \frac{1 - z^{-1}}{1 + \frac{9}{10} z^{-1}} = \frac{Y(z)}{X(z)} \Rightarrow Y(z) + 0.9 z^{-1} Y(z) = X(z) - z^{-1} X(z)$$

$$y(n) + 0.9\, y(n-1) = \tfrac{1}{20} x(n) - \tfrac{1}{20} x(n-1) \Rightarrow y(n) = -0.9\, y(n-1) + \tfrac{1}{20} x(n) - \tfrac{1}{20} x(n-1)$$

(e) $x(n) = 2\cos\left(\frac{\pi}{6} n + \pi/4\right) \quad // \ \omega = \frac{\pi}{6}$

$$|H(\tfrac{\pi}{6})| = \frac{1}{20} \frac{2|\sin\frac{\pi}{12}|}{\sqrt{1.81 + 1.8\cos\pi/6}} = 0.014$$

$$\measuredangle H(\tfrac{\pi}{6}) = \tan^{-1}\left(\frac{\sin\pi/6}{1 - \cos\pi/6}\right) + \tan^{-1}\left(\frac{0.9\sin\pi/6}{1 + 0.9\cos\pi/6}\right)$$

$$\measuredangle H(\pi/6) = 89.192° + 45° = 134.2°$$

$$y(n) = (2 \cdot 0.014)\cos\left(\pi/6\, n + 134.2°\right)$$

$$y(n) = 0.028\cos\left(\pi/6\, n + 134.2°\right)$$

5.68 The system

$$y(n) = \frac{1}{2}y(n-1) + x(n)$$

is excited with the input

$$x(n) = (\frac{1}{4})^n u(n)$$

Determine the sequences $r_{xx}(l)$, $r_{hh}(l)$, $r_{xy}(l)$, and $r_{yy}(l)$.

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Problem 5: Problem 5.84 in R1

$$Y(z) = \frac{1}{2} Y(z) z^{-1} + X(z) \Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1-\frac{1}{2}z^{-1}}$$

$$X(z) = \frac{1}{1-\frac{1}{4}z^{-1}}$$

$$Y(z) = X(z)\, H(z) = \frac{1}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{2}z^{-1})}$$

$$R_{xy}(z) = X(z)\, Y(z^{-1}) = \left(\frac{1}{1-\frac{1}{4}z^{-1}}\right)\left(\frac{1}{(1-\frac{1}{4}z)(1-\frac{1}{2}z)}\right) = \frac{8z^{-2}}{(1-\frac{1}{4}z^{-1})(1-4z^{-1})(1-2z^{-1})}$$

$$R_{xy}(z) = \frac{A}{1-\frac{1}{4}z^{-1}} + \frac{B}{1-4z^{-1}} + \frac{C}{1-2z^{-1}}$$

$$A + B + C = 0$$
$$-6A - 9/2\, B - 17/4\, C = 0$$
$$8A + 1/2\, B + C = 8$$

$$A = 128/105$$
$$B = -16/15$$
$$C = 16/7$$

$$R_{xy}(z) = \frac{128/105}{1-\frac{1}{4}z^{-1}} + \frac{16/15}{1-4z^{-1}} + \frac{-16/7}{1-2z^{-1}}$$

$$\boxed{r_{xy}(l) = \frac{128}{105}\left(\frac{1}{4}\right)^l \cdot u(l) - \frac{16}{15} \cdot 4^l \cdot u(-l-1) + \frac{16}{7} \cdot 2^l \cdot u(-l-1)}$$

5.84 The system function of a communication channel is given by

$$H(z) = (1 - 0.9e^{j0.4\pi}z^{-1})(1 - 0.9e^{-j0.4\pi}z^{-1})(1 - 1.5e^{j0.6\pi}z^{-1})(1 - 1.5e^{-j0.6\pi}z^{-1})$$

Determine the system function $H_c(z)$ of a causal and stable compensating system so that the cascade interconnection of the two systems has a flat magnitude response. Sketch the pole–zero plots and the magnitude and phase responses of all systems involved into the analysis process. [*Hint:* Use the decomposition $H(z) = H_{ap}(z)H_{min}(z)$.]

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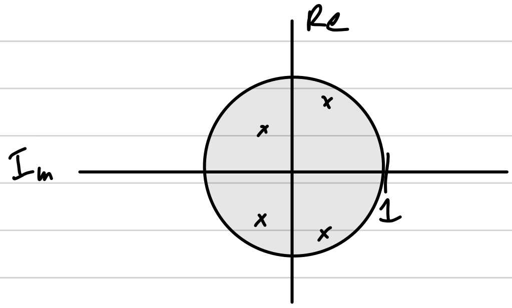
$$H(z) = \frac{z^4}{z^4} \qquad H(z) = \underbrace{\frac{1}{z^4}}_{X(z)}\underbrace{(z - 0.9e^{j0.4\pi})(z - 0.9e^{-j0.4\pi})}_{Y_1(z)}\underbrace{(z - 1.5e^{j0.6\pi})(z - 1.5e^{-j0.6\pi})}_{Y_2(z)}$$

$$H_{min}(z) = \frac{Y_1(z)\,Y_2(z)}{4(z)} = \frac{(z - 0.9e^{j0.4\pi})(z - 0.9e^{-j0.4\pi})(z - 1.5e^{j0.6\pi})(z - 1.5e^{-j0.6\pi})}{z^4}$$

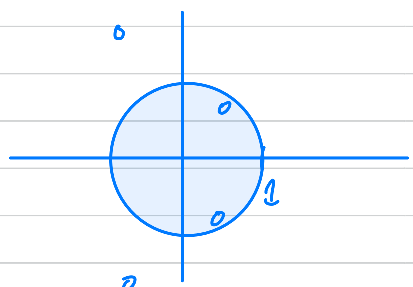
$$H_{ap}(z) = \frac{Y_2(z)}{Y_2(z^{-1})} = \frac{(z - 1.5e^{j0.6\pi})(z - 1.5e^{-j0.6\pi})}{(z^{-1} - 1.5e^{j0.6\pi})(z^{-1} - 1.5e^{-j0.6\pi})}$$

$$H_c(z) = \frac{1}{H_{min}(z)} = \frac{z^4}{(z - 0.9e^{j0.4\pi})(z - 0.9e^{-j0.4\pi})(z - 1.5e^{j0.6\pi})(z - 1.5e^{-j0.6\pi})}$$

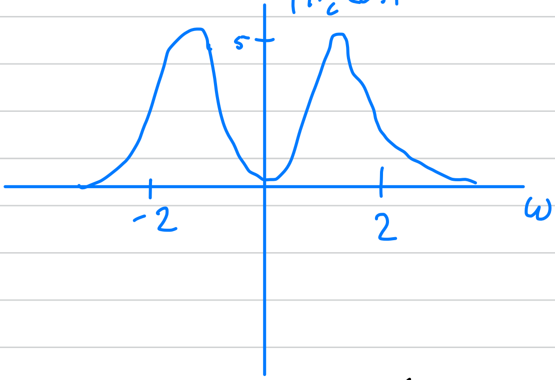
$H_c(z)$ // pole-zero



$H_{ap}(z)$ pole zero

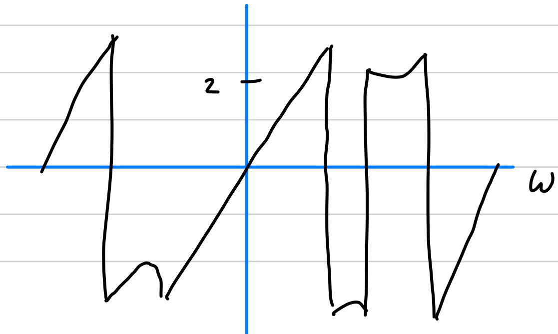


$|H_c(z)|$

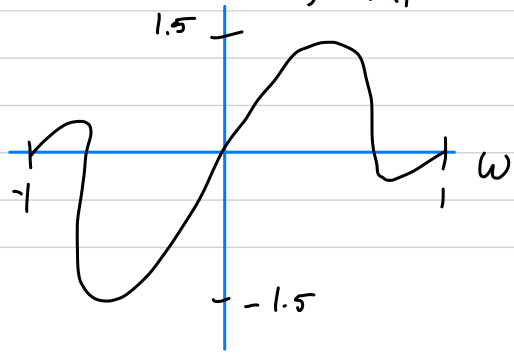


$|H_{ap}(z)|$

$\measuredangle H_c(z)$



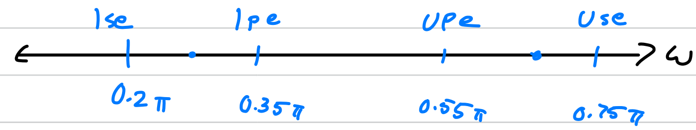
$\measuredangle H_{ap}(z)$



MATLAB:

Design a linear-phase bandpass filter using the Hann window design technique. The specifications are

lower stopband edge: 0.2π
upper stopband edge: 0.75π $\quad A_s = 40$ dB
lower passband edge: 0.35π
upper passband edge: 0.55π $\quad R_p = 0.25$ dB



Plot the impulse response and the magnitude response (in dB) of the designed filter.

Hints:

1) Use "fir1" function with proper arguments.
2) Note that the stopband attenuation A_s and the passband ripple R_p are defined as follows:

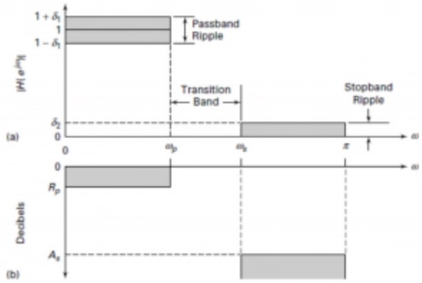


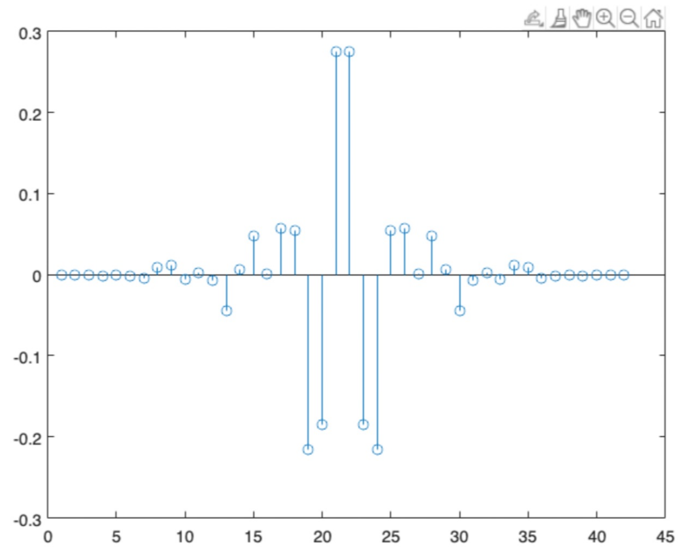
FIGURE 7.1 *FIR filter specifications: (a) absolute (b) relative*

$$R_p = -20\log_{10}\frac{1-\delta_1}{1+\delta_1} > 0 \ (\approx 0)$$

$$A_s = -20\log_{10}\frac{\delta_2}{1+\delta_1} > 0 \ (\gg 1)$$

3) But, remember that in most window-based designs, our only design parameters are the window type and the number of taps in the filter (or the filter order). We already know that the Hann window will meet ~44dB stopband attenuation, and as such, will meet our A_s spec above. Therefore, for this problem, you will NOT use the A_s and R_p specs above in your design explicitly. You will just need to find the minimum number of taps for your filter, as described below, and then once you have the filter, just confirm that it does meet your specs for both passband ripple and stopband attenuation.
4) Also remember that in window-based FIR design, we always get $\delta_1 = \delta_2$. So from A_s and R_p specs, you would have to find δ_1 and δ_2, and then once you have the filter, just confirm that it will indeed meet the minimum of the two.
5) The exact value of the main-lobe width, and therefore the filter transition bandwidth, associated with Hann window is $6.2\pi/M$ where M is the length of the window, or equivalently the number of taps in your FIR filter. So, given the specs, you can find the narrowest transition bandwidth you need to achieve, and based on that, find the minimum number of taps for your filter. Use an odd number so you end up with a Type I linear-phase FIR filter.
6) Use the center points of the transition bands as the edges of your passband, as passed to "fir1" function (i.e., [w₁ w₂]).

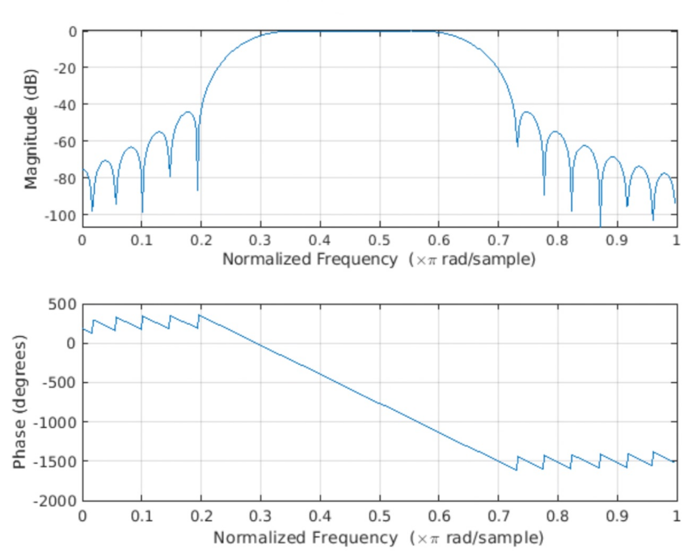
Impulse Response



hw8.m

```
clear variables; clc; close all;

% constants
% as = 40;
% rp = 0.25;
lse = 0.2;
use = 0.75;
lpe = 0.35;
upe = 0.55;

Wn = [(lse+lpe)/2 (use+upe)/2];
M = ceil(6.2/min(lpe-lse,use-upe)); % 42
order = M - 1; % 41
window = hann(M);

filter = fir1(order, Wn, window);

figure(1);
freqz(filter);

figure(2);
stem(filter);
```

Figure 1 Figure 2



Command Window