ECE113: DSP

Homework 1

Due 04/09/2021, 11:59pm

Problem 1:

1.1 Classify the following signals according to whether they are (1) one- or multi-dimensional; (2) single or multichannel, (3) continuous time or discrete time, and (4) analog or digital (in amplitude). Give a brief explanation.

(a) Closing prices of utility stocks on the New York Stock Exchange.
(b) A color movie.
(c) Position of the steering wheel of a car in motion relative to car's reference frame.
(d) Position of the steering wheel of a car in motion relative to ground reference frame.
(e) Weight and height measurements of a child taken every month.

Problem 2:

1.3 Determine whether or not each of the following signals is periodic. In case a signal is periodic, specify its fundamental period.

(a) $x_a(t) = 3\cos(5t + \pi/6)$
(b) $x(n) = 3\cos(5n + \pi/6)$
(c) $x(n) = 2\exp[j(n/6 - \pi)]$
(d) $x(n) = \cos(n/8)\cos(\pi n/8)$
(e) $x(n) = \cos(\pi n/2) - \sin(\pi n/8) + 3\cos(\pi n/4 + \pi/3)$

Problem 3:

1.5 Consider the following analog sinusoidal signal:

$$x_a(t) = 3\sin(100\pi t)$$

(a) Sketch the signal $x_a(t)$ for $0 \le t \le 30$ ms.
(b) The signal $x_a(t)$ is sampled with a sampling rate $F_s = 300$ samples/s. Determine the frequency of the discrete-time signal $x(n) = x_a(nT)$, $T = 1/F_s$, and show that it is periodic.
(c) Compute the sample values in one period of $x(n)$. Sketch $x(n)$ on the same diagram with $x_a(t)$. What is the period of the discrete-time signal in milliseconds?
(d) Can you find a sampling rate F_s such that the signal $x(n)$ reaches its peak value of 3? What is the minimum F_s suitable for this task?

Problem 4:

2.1 A discrete-time signal $x(n)$ is defined as

$$x(n) = \begin{cases} 1 + \dfrac{n}{3}, & -3 \le n \le -1 \\ 1, & 0 \le n \le 3 \\ 0, & \text{elsewhere} \end{cases}$$

(a) Determine its values and sketch the signal $x(n)$.

(b) Sketch the signals that result if we:

(1) First fold $x(n)$ and then delay the resulting signal by four samples.

(2) First delay $x(n)$ by four samples and then fold the resulting signal.

(c) Sketch the signal $x(-n+4)$.

(d) Compare the results in parts (b) and (c) and derive a rule for obtaining the signal $x(-n+k)$ from $x(n)$.

(e) Can you express the signal $x(n)$ in terms of signals $\delta(n)$ and $u(n)$?

Problem 5:

2.5 Show that the energy (power) of a real-valued energy (power) signal is equal to the sum of the energies (powers) of its even and odd components.

Problem 6:

2.10 The following input–output pairs have been observed during the operation of a time-invariant system:

$$x_1(n) = \{\underset{\uparrow}{1}, 0, 2\} \xleftrightarrow{\mathcal{T}} y_1(n) = \{\underset{\uparrow}{0}, 1, 2\}$$

$$x_2(n) = \{\underset{\uparrow}{0}, 0, 3\} \xleftrightarrow{\mathcal{T}} y_2(n) = \{\underset{\uparrow}{0}, 1, 0, 2\}$$

$$x_3(n) = \{\underset{\uparrow}{0}, 0, 0, 1\} \xleftrightarrow{\mathcal{T}} y_3(n) = \{1, \underset{\uparrow}{2}, 1\}$$

Can you draw any conclusions regarding the linearity of the system. What is the impulse response of the system?

Problem 7:

2.21 Compute the convolution $y(n) = x(n) * h(n)$ of the following pairs of signals.

(a) $x(n) = a^n u(n)$, $h(n) = b^n u(n)$ when $a \ne b$ and when $a = b$

(b) $x(n) = \begin{cases} 1, & n = -2, 0, 1 \\ 2, & n = -1 \\ 0, & \text{elsewhere} \end{cases}$

$h(n) = \delta(n) - \delta(n-1) + \delta(n-4) + \delta(n-5)$

Problem 8:

2.23 Express the output *y(n)* of a linear time-invariant (LTI) system with impulse response *h(n)* in terms of its step response *s(n)=h(n)*u(n)* and the input *x(n)*.

Problem 9:

(a) Let $x[n]$ and $y[n]$ be real-valued sequences both of which are even-symmetric: $x[n] = x[-n]$ and $y[n] = y[-n]$. Under these conditions, prove that $r_{xy}[\ell] = r_{yx}[\ell]$ for all ℓ.

(b) Express the autocorrelation sequence $r_{zz}[\ell]$ for the complex-valued signal $z[n] = x[n] + jy[n]$ where $x[n]$ and $y[n]$ are real-valued sequences, in terms of $r_{xx}[\ell]$, $r_{xy}[\ell]$, $r_{yx}[\ell]$, and $r_{yy}[\ell]$.

MATLAB Exercises:

Please submit your MATLAB script source code along with any necessary plots and discussion.

P2.8 The operation of *signal dilation* (or *decimation* or *down-sampling*) is defined by

$$y(n) = x(nM)$$

in which the sequence $x(n)$ is down-sampled by an integer factor M. For example, if

$$x(n) = \{\ldots, -2, 4, \underset{\uparrow}{3}, -6, 5, -1, 8, \ldots\}$$

then the down-sampled sequences by a factor 2 are given by

$$y(n) = \{\ldots, -2, \underset{\uparrow}{3}, 5, 8, \ldots\}$$

1. Develop a MATLAB function **dnsample** that has the form

```
function [y,m] = dnsample(x,n,M)
% Downsample sequence x(n) by a factor M to obtain y(m)
```

   to implement the above operation. Use the indexing mechanism of MATLAB with careful attention to the origin of the time axis $n = 0$.
2. Generate $x(n) = \sin(0.125\pi n), \quad -50 \leq n \leq 50$. Decimate $x(n)$ by a factor of 4 to generate $y(n)$. Plot both $x(n)$ and $y(n)$ using **subplot** and comment on the results.
3. Repeat the above using $x(n) = \sin(0.5\pi n), \quad -50 \leq n \leq 50$. Qualitatively discuss the effect of down-sampling on signals.

P2.16 Let $x(n) = (0.8)^n u(n)$, $h(n) = (-0.9)^n u(n)$, and $y(n) = h(n) * x(n)$. Use 3 columns and 1 row of subplots for the following parts.

1. Determine $y(n)$ analytically. Plot first 51 samples of $y(n)$ using the **stem** function.
2. Truncate $x(n)$ and $h(n)$ to 26 samples. Use **conv** function to compute $y(n)$. Plot $y(n)$ using the **stem** function. Compare your results with those of part 1.
3. Using the **filter** function, determine the first 51 samples of $x(n) * h(n)$. Plot $y(n)$ using the **stem** function. Compare your results with those of parts 1 and 2.