ECE113, Digital Signal Processing **Name:** ______________________

UCLA Spring 2021

Final Exam

06/07/2021

Time Limit: 48 hours

(a) This exam contains 12 pages (including this cover page) and 8 problems. Total of points is 100.

(b) You must scan and submit your solutions in a single PDF file together with your MATLAB m-file via the CCLE portal by **8:00am on 06/09/21 at the latest**.

(c) Please write your full name on your papers. Allso **include it as part of your file names**.

(d) Please fully justify your answers and clearly show ALL the intermediate steps and calculations in all your solutions. And, when appropriate, box your final answer.

(e) Feel free to use any calculators and computers, including MATLAB. But please always make sure to explain your approach and present all your intermediate steps.

(f) Note that some of the different parts in multi-part problems may be solved independently of the other parts.

(g) Open books and notes, but no collaboration please. You are expected to work on the solutions individually. Unreasonably similar write-ups would be heavily penalized on all parties suspected of collaboration.

(h) **Good Luck...**

Grade Table (for instructor use only)

Question	Points	Score
1	0	
2	10	
3	18	
4	10	
5	8	
6	16	
7	20	
8	18	
Total:	100	

1. **1-point Extra Credit on your total score**
   Please answer the following two questions before you submit your solutions.
   (a) Did you fill out the online survey? Yes, or No?
   (b) Approximately how many hours did you have to spend on this exam? Please try to exclude all the extra time you may spend on reviewing the lecture notes again during the exam period, or on double or triple checking your solutions again etc., and try to include the actual number of hours you think you would have to spend if, say, this were an in-person closed-book exam.

2. (10 points) **Quick Review**
   Carefully read each statement below and identify it as *True* or *False*, and briefly explain your reason in one or two sentences.

   1. An anti-causal LTI system (i.e., $h(n) = 0, \text{for}\, n \geq 0$) will be BIBO stable as long as all its poles reside inside the unit circle.
      True or False? Why?
   2. Rational, causal, and stable all-pass filters are always non-minimum-phase systems.
      True or False? Why?
   3. For the same filter specifications, Chebyshev IIR filters will typically lead to a lower order filter than Butterworth filters.
      True or False? Why?
   4. Digitizing a stable analog filter using either the Impulse Invariance method or the Bilinear transformation will always lead to a stable IIR filter.
      True or False? Why?
   5. In window-based FIR design, using a Hann window will result in a larger bandwidth for the filter transition band, when compared with a Blackman window.
      True or False? Why?
   6. In window-based FIR design, when using a rectangular window, increasing the order of the filter will lead to increased attenuation in the filter stopband.
      True or False? Why?
   7. Assuming N even, any real-valued finite sequence of length $M \leq N$ can be uniquely represented by the first $\frac{N}{2} + 1$ samples of its N-point DFT.
      True or False? Why?
   8. The magnitude response and the phase response of a causal filter can be designed independently.
      True or False? Why?
   9. Any symmetric or anti-symmetric FIR filter will always have a constant group delay.
      True or False? Why?
   10. A causal, stable, and rational IIR filter can have linear phase response as long as its impulse response is symmetric or anti-symmetric.
       True or False? Why?

3. (18 points) **LCCDE, Transfer Function, LTI System Response**:

Consider the LTI system shown in Figure 1 below.

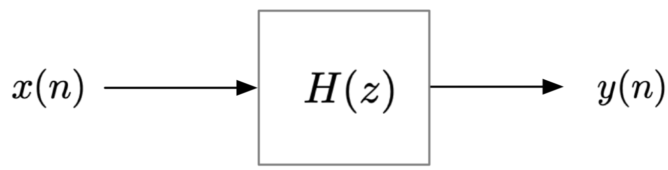

Figure 1: LTI System

Given the following input sequence:

$$x(n) = (\frac{1}{2})^n u(n) + 2^n u(-n-1)$$

we have obtained the zero-state output sequence as:

$$y(n) = 6(\frac{1}{2})^n u(n) - 6(\frac{3}{4})^n u(n)$$

where $u(n)$ is the unit step sequence.

(a) (1 point) Obtain $X(z) = \mathcal{Z}\{x(n)\}$ and determine its Region of Convergence (ROC).

(b) (2 points) Find the system Transfer Function $H(z)$ and determine its Region of Convergence (ROC).

(c) (1 point) Find the system Impulse Response $h(n)$.

(d) (2 points) Is this system BIBO stable? Is it causal? Is it minimum-phase? Please explain all your answers.

(e) (2 points) Obtain the Linear Constant Coefficient Difference Equation (LCCDE) describing the time-domain input-output relationship for this system.

(f) (5 points) Now, suppose the system has an initial condition $y(-1) = 4$ and we have applied the following input sequence:

$$x(n) = n(\frac{1}{2})^n u(n)$$

Using unilateral Z-transform, obtain the system response $y(n)$, $n \geq 0$. Specifically, please identify the *zero-state* response $y_{zs}(n)$, and the *zero-input* response $y_{zi}(n)$ for this system. (Note: For Partial Fraction Expansion, you must write how you obtain the residues, even if you choose to check your result with MATLAB).

(g) (2 points) In the response you obtained in Part (f), identify the *forced response* as well as the *natural response*. Moreover, identify the separate contributions of the initial condition and the input signal on the system natural response.

(h) (3 points) Obtain the *steady-state* response of this system to the following input signal:

$$x(n) = \frac{1}{4} + 7(-1)^n + 5\sin(\frac{\pi}{4}n + \frac{\pi}{3})$$

4. (10 points) **Frequency Domain Analysis: DFS, DFT, Frequency Response**:

Consider an ideal LTI lowpass filter with the following frequency response:

$$H(\omega) = \begin{cases} 1 & \text{for} \quad |\omega| < \dfrac{3\pi}{N} \\ 0 & \text{for} \quad \dfrac{3\pi}{N} \leq |\omega| \leq \pi \end{cases}$$

where N is a given integer ($N \geq 3$). Suppose that the input sequence to this filter, $x_p(n)$, is an N-point periodic extension of an L-point pulse sequence ($L \leq N$), i.e.,

$$x_p(n) = \sum_{l=-\infty}^{\infty} x(n + lN)$$

where:

$$x(n) = \begin{cases} 1 & \text{for} \quad 0 \leq n \leq L-1 \\ 0 & \text{otherwise} \end{cases}$$

(a) (7 points) Given $x_p(n)$ as the input to the filter, what will be the sequence at the output of the filter, $y(n), -\infty < n < +\infty$?

(b) (3 points) Assume $L = 3$ and $N = 4$. Plot $y(n)$ over $0 \leq n \leq 10$. Feel free to use MATLAB or any other calculator to obtain the values for $y(n)$ using the equation you obtained in Part (a).

5. (8 points) **System Inverse**:

As shown in Figure 2 below, suppose we have access to the signal $y(n)$ which is the distorted version of the signal $x(n)$, and we have modeled the distortion by the LTI system $H(z)$ given below:

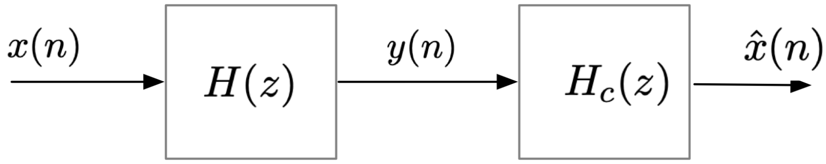

Figure 2: Distortion Compensation

$$H(z) = \frac{1 - 2z^{-1} + 2z^{-2}}{1 - 0.25z^{-2}}$$

Assume we only care about the magnitude response. Obtain the transfer function for a **stable** and **causal** magnitude distortion compensation system $H_c(z)$ such that: $|\hat{X}(\omega)| = |X(\omega)|$.

6. (16 points) **Upsampling, Downsampling, Z-Transforms**:

We have learned about *upsamplers* and *downsamplers*, as shown in Figure 3 below:

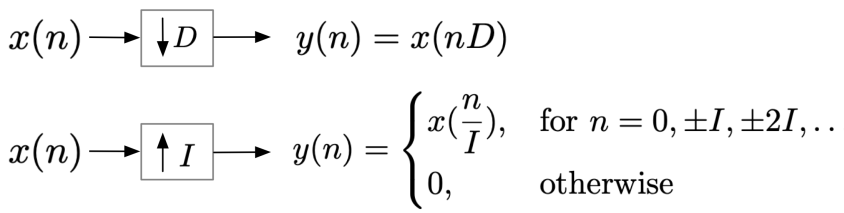

Figure 3: Downsampler and Upsampler

(a) (1 point) Are upsamplers and downsamplers linear systems? Why?

(b) (1 point) Are upsamplers and downsamplers time-invariant systems, or time-varying systems? Why?

(c) (5 points) Prove the equivalences shown in Figure 4 below. These are known as *Noble Identities* in multirate signal processing, and can clearly be helpful when we have cascades of filters with upsamplers/downsamplers.

(*Hint:* There can be different approaches for proving these identities. A simple approach may be to use the definition of the Z-transform along with the convolution sum on both sides and show that you would get the same output).

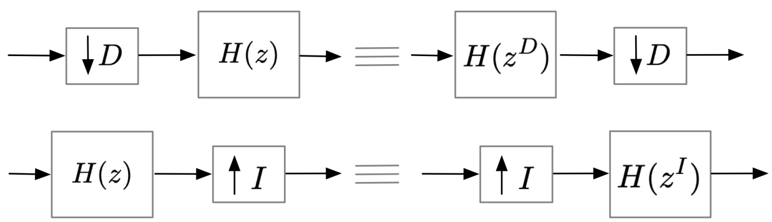

Figure 4: Noble Identities

(d) (2 points) Show that the order of consecutive upsampler/downsampler can be changed, as shown in Figure 5 ibelow:

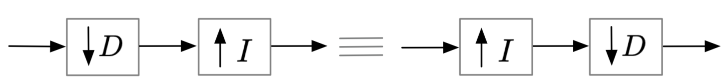

Figure 5: Swapping the order of upsampler/downsampler

(e) (6 points) Now, suppose we want to implement the multirate DSP system shown in Figure 6 below:

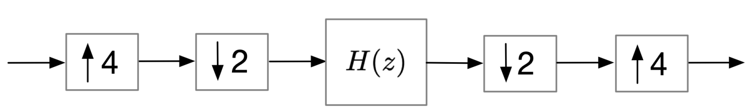

Figure 6: An example multirate DSP system

where:

$$H(z) = \frac{z^{-8}}{8 - 4z^{-8} + z^{-16}}$$

Obtain and draw the block diagram of an equivalent system (i.e., a system that would produce the same output sequence given the same input sequence) composed of a cascade of upsampler(s), downsampler(s), and LTI system(s), such that it would yield the *minimum* number of multiplications per output sample.

(f) (1 point) What will be the minimum number of multiplications per output sample?

7. (20 points) **Zero-Pole Plots and Filter Properties**:

In this problem, five different zero-pole plots for different filters are shown. All filters have real-valued coefficients. For each filter, you should answer the subsequent questions, with brief justifications for all of your answers.

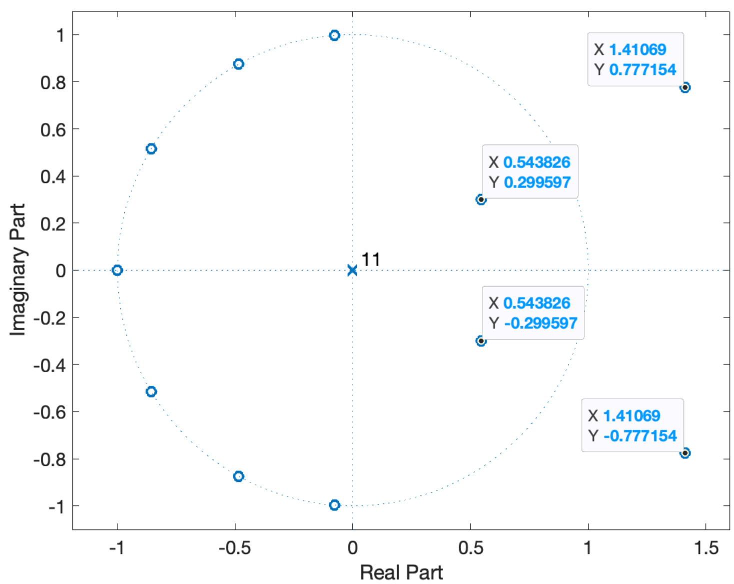

Figure 7: Zero-Pole Plot for Part (a)

(a) (4 points) For the zero-pole plot shown in Figure 7:

1. Is this filter lowpass, highpass, bandpass, bandstop, or allpass? why?
2. Is this filter FIR or IIR? Why? If it is FIR, is its impulse response symmetric ($h(n) = h(N - n)$) or anti-symmetric ($h(n) = -h(N - n)$)? Why?
3. Can this filter be causal and stable? Why?
4. Is this filter minimum-phase? Why?
5. Can this filter have a constant group delay? If yes, can you determine its value? If no, why not?

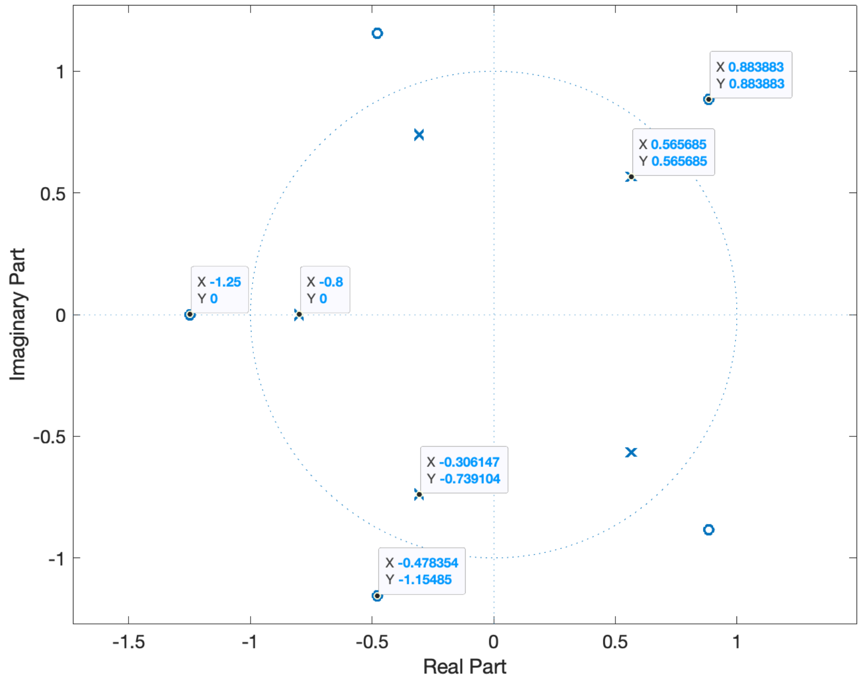

Figure 8: Zero-Pole Plot for Part (b)

(b) (4 points) For the zero-pole plot shown in Figure 8:

1. Is this filter lowpass, highpass, bandpass, bandstop, or allpass? why?
2. Is this filter FIR or IIR? Why? If it is FIR, is its impulse response symmetric ($h(n) = h(N - n)$) or anti-symmetric ($h(n) = -h(N - n)$)? Why?
3. Can this filter be causal and stable? Why?
4. Is this filter minimum-phase? Why?
5. Can this filter have a constant group delay? If yes, can you determine its value? If no, why not?

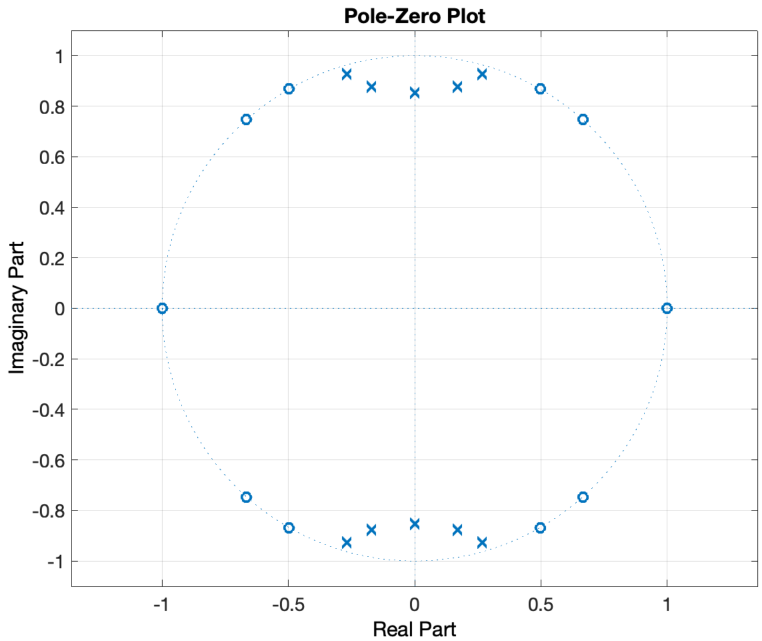

Figure 9: Zero-Pole Plot for Part (c)

(c) (4 points) For the zero-pole plot shown in Figure 9:

1. Is this filter lowpass, highpass, bandpass, bandstop, or allpass? why?
2. Is this filter FIR or IIR? Why? If it is FIR, is its impulse response symmetric ($h(n) = h(N-n)$) or anti-symmetric ($h(n) = -h(N-n)$)? Why?
3. Can this filter be causal and stable? Why?
4. Is this filter minimum-phase? Why?
5. Can this filter have a constant group delay? If yes, can you determine its value? If no, why not?

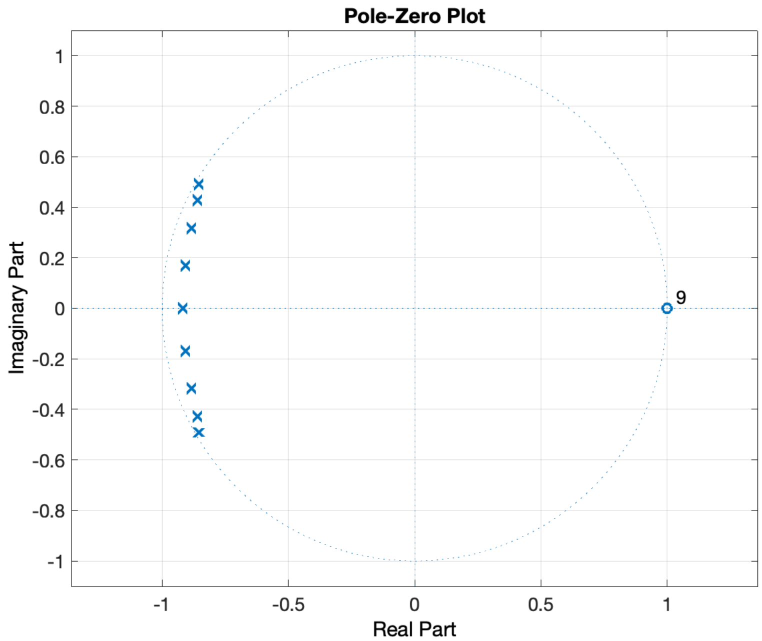

Figure 10: Zero-Pole Plot for Part (d)

(d) (4 points) For the zero-pole plot shown in Figure 10:

1. Is this filter lowpass, highpass, bandpass, bandstop, or allpass? why?
2. Is this filter FIR or IIR? Why? If it is FIR, is its impulse response symmetric ($h(n) = h(N - n)$) or anti-symmetric ($h(n) = -h(N - n)$)? Why?
3. Can this filter be causal and stable? Why?
4. Is this filter minimum-phase? Why?
5. Can this filter have a constant group delay? If yes, can you determine its value? If no, why not?

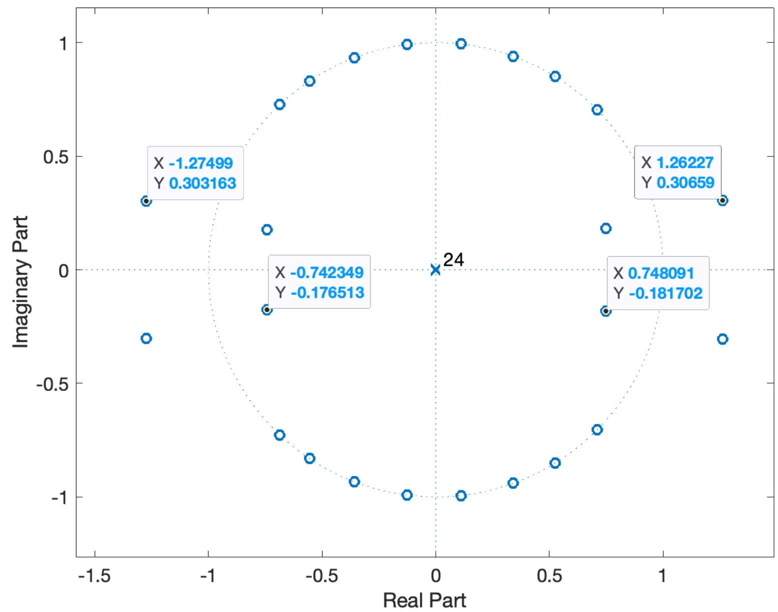

Figure 11: Zero-Pole Plot for Part (e)

(e) (4 points) For the zero-pole plot shown in Figure 11:

1. Is this filter lowpass, highpass, bandpass, bandstop, or allpass? why?
2. Is this filter FIR or IIR? Why? If it is FIR, is its impulse response symmetric ($h(n) = h(N - n)$) or anti-symmetric ($h(n) = -h(N - n)$)? Why?
3. Can this filter be causal and stable? Why?
4. Is this filter minimum-phase? Why?
5. Can this filter have a constant group delay? If yes, can you determine its value? If no, why not?

8. (18 points) **Filter Design**:

In this problem, you are given the specifications for four different filters, and, using MATLAB, you will be designing both FIR and IIR filters that meet the given specifications, and you will be comparing the various aspects of your filters.

Please do submit your completed MATLAB m-file alongside your PDF.

- Filters 1 and 2 have the same passband cutoff frequencies but different stopband cutoff frequencies and hence different transition bandwidths (sharpness).
- Filters 3 and 4 have different passband cutoff frequencies but the same transition bandwidths (sharpness).
- All four filters have the same specifications for passband ripple and stopband attenuation.
- You will first design these filters as linear-phase FIR filters. Specifically, you will first use a window-based design technique with a Kaiser window, and then use the Parks-McClellan optimal equiripple design technique.
- You will then realize Filter 3 as an IIR filter using both Chebyshev Type II and Elliptic filters.
- Please use the MATLAB template file posted on CCLE. Please feel free to introduce your own variables, as needed, but do not change the variable names already defined. This is just to ensure that the code for plotting the frequency responses will work correctly.
- Please answer the questions below and include your MATLAB plots in your PDF. Also please make sure to submit your completed MATLAB m-file alongside your PDF.

The sampling rate is $F_s = 1\,\text{MHz}$, and the filter specifications for the four filters are given in Table 1 below:

Table 1: FIR Filter Specifications

Specification	Filter 1	Filter 2	Filter 3	Filter 4
Passband Edge Frequency (KHz)	100	100	100	300
Stopband Edge Frequency (KHz)	150	250	200	400
Max Passband Ripple (dB)	0.1	0.1	0.1	0.1
Min Stopband Attenuation (dB)	60	60	60	60

(a) (1 point) Even before you design the filters, which filter among Filters 1, 2, and 3 would you expect to have the smallest order? Why? Please explain.

(b) (1 point) Even before you design the filters, how would you expect the orders of Filter 3 and 4 to compare (using either design technique)? Why? Please explain.

Table 2: FIR Filter Orders

Design Technique	Filter 1	Filter 2	Filter 3	Filter 4
Kaiser Window-Based Design	$N_{11} =$	$N_{12} =$	$N_{13} =$	$N_{14} =$
Parks-McClellan Optimal Equiripple Design	$N_{21} =$	$N_{22} =$	$N_{23} =$	$N_{24} =$

(c) (3 points) Use `kaiserord` and `firpmord` commands in the provided MATLAB template file (posted on CCLE) to obtain the minimum order required for the four filters using the two different techniques and fill out Table 2 with your results.

(d) (2 points) Use your results from `kaiserord` command in Part (c) and design the four filters using Kaiser window-based design technique using `fir1` command in the provided template. Obtain the frequency responses for the four filters using `freqz` command. Plot the filter magnitude and phase responses for the four filters (the full code for plotting is provided in the template).

(e) (2 points) Use your results from `firpmord` command in Part (c) and design the four filters using Parks-McClellan optimal equiripple design technique using `firpm` command in the provided template. Obtain the frequency responses for the four filters using `freqz` command. Plot the filter magnitude and phase responses for the four filters (the full code for plotting is provided in the template).

(f) (2 points) We now want to design **Filter 3** above, as an IIR filter. Use `cheb2ord` command to find the minimum Chebyshev Type II filter order given the specifications for Filter 3 in Table 1 above. Then use `cheby2` command to design Filter 3 as a Chebyshev Type II filter, and find out the numerator and denominator coefficients in its transfer function $H(z)$. Obtain its frequency response using `freqz` command and plot the magnitude and phase responses for this filter. (Again, please follow the template. The code for plotting is already included).

(g) (2 points) Finally, lets design Filter 3 as an Elliptic IIR filter. Use `ellipord` command to find the minimum Elliptic filter order given the specifications for Filter 3. Then use `ellip` command to design Filter 3 as an Elliptic filter, and find out the numerator and denominator coefficients in its transfer function $H(z)$. Obtain its frequency response using `freqz` command and plot the magnitude and phase responses for this filter. (Again, please follow the template. The code for plotting is already included).

(h) (2 points) As you shall notice, the template will plot the frequency responses for Filter 3 and will compare Chebyshev Type II IIR, Elliptic IIR, and Parks-McClellan FIR implementations. Which filter yields the smallest filter order? Which filter has the most *nonlinear* phase response? Please discuss.

(i) (3 points) Realize (i.e., show the signal flow graph for) the Elliptic IIR filter you designed in Part (g) in *Transposed Direct Form II.*