ECE113: DSP

Homework 8 Solutions

Problem 1: Problem 3.45 in R1

Solution:

$$
\begin{aligned}
y(n) &= \frac{1}{2}y(n-1) + 4x(n) + 3x(n-1) \\
Y(z) &= \frac{4+3z^{-1}}{1-\frac{1}{2}z^{-1}}X(z) \\
x(n) &= e^{jw_0n}u(n) \\
X(z) &= \frac{1}{1-e^{jw_0}z^{-1}} \\
Y(z) &= \frac{4+3z^{-1}}{(1-\frac{1}{2}z^{-1})(1-e^{jw_0}z^{-1})} \\
Y(z) &= \frac{A}{1-\frac{1}{2}z^{-1}} + \frac{B}{1-e^{jw_0}z^{-1}} \\
\text{where } A &= \frac{5}{\frac{1}{2}-e^{jw_0}} \\
B &= \frac{4e^{jw_0}+3}{e^{jw_0}-\frac{1}{2}} \\
\text{Then } y(n) &= \left[A(\frac{1}{2})^n + Be^{jw_0n}\right]u(n) \\
\text{The steady state response is} & \\
lim_{n\to\infty}y(n) \equiv y_{ss}(n) &= Be^{jw_0n}
\end{aligned}
$$

Problem 2: Problem 4.22 in R1 ((c) and (e) only)

Solution:

(c)

$$
\begin{aligned}
X_3(w) &= \sum_n x(-2n)e^{-jwn} \\
&= -\sum_k x(k)e^{-jkw/2)} \\
&= X(-\frac{w}{2})
\end{aligned}
$$

(e) $X_5(w) = X(w)\left[X(w)e^{-jw}\right] = X^2(w)e^{-jw}$

Problem 3: Problem 5.27 in R1

Solution:

(a) $H(z) = k\frac{1-z^{-1}}{1+0.9z^{-1}}$. Refer to fig 5.27-1.

(b)

$$\begin{aligned} H(w) &= k\frac{1-e^{-jw}}{1+0.9e^{-jw}} \\ |H(w)| &= k\frac{2|sin\frac{w}{2}|}{\sqrt{1.81+1.8cosw}} \\ \Theta(w) &= tan^{-1}\frac{sinw}{1-cosw} + tan^{-1}\frac{0.9sinw}{1+0.9cosw} \end{aligned}$$

(c) $H(\pi) = k\frac{1-e^{-j\pi}}{1+0.9e^{-j\pi}} = k\frac{2}{0.1} = 20k = 1 \Rightarrow k = \frac{1}{20}$

(d) $y(n) = -0.9y(n-1) + \frac{1}{20}[x(n) - x(n-1)]$

(e)

$$\begin{aligned} H(\frac{\pi}{6}) &= 0.014e^{j\Theta(\frac{\pi}{6})} \\ y(n) &= 0.028cos(\frac{\pi}{6}n + 134.2^o) \end{aligned}$$

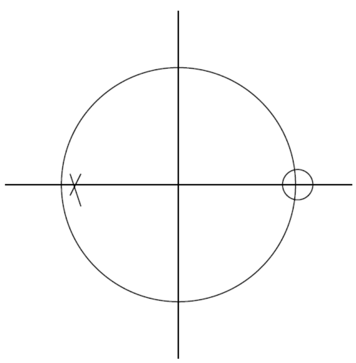


Figure 5.27-1:

Problem 4: Problem 5.68 in R1 ($r_{xy}(l)$ only)

Solution:

$$\begin{aligned}
y(n) &= \frac{1}{2}y(n-1) + x(n) \\
x(n) &= (\frac{1}{4})^n u(n) \\
H(z) &= \frac{Y(z)}{X(z)} \\
&= \frac{1}{1-\frac{1}{2}z^{-1}} \\
X(z) &= \frac{1}{1-\frac{1}{4}z^{-1}} \\
Y(z) &= \frac{1}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{2}z^{-1})}
\end{aligned}$$

$$\begin{aligned}
R_{xy}(z) &= X(z)Y(z^{-1}) \\
&= \frac{1}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{4}z)(1-\frac{1}{2}z)} \\
&= -\frac{16}{17}\frac{1}{1-2z^{-1}} + \frac{16}{15}\frac{1}{1-4z^{-1}} + \frac{128}{105}\frac{1}{1-\frac{1}{4}z^{-1}} \\
\text{Hence, } r_{xy}(n) &= \frac{16}{17}(2)^n u(-n-1) - \frac{16}{15}(4)^n u(-n-1) + \frac{128}{105}(\frac{1}{4})^n u(n)
\end{aligned}$$

Problem 5: Problem 5.84 in R1

Solution:

$$H(z) = (1-0.9\,e^{j0.4\pi}z^{-1})(1-0.9\,e^{-j0.4\pi}z^{-1}) \cdot (1-1.5\,e^{j0.6\pi}z^{-1})(1-1.5\,e^{-j0.6\pi}z^{-1})$$

$$H(z) = H_{min}(z) \cdot H_{ap}(z)$$

$$H_{ap}(z) = \frac{(1-1.5\,e^{j0.6\pi}z^{-1})(1-1.5\,e^{-j0.6\pi}z^{-1})}{(1-\frac{2}{3}e^{j0.6\pi}z^{-1})(1-\frac{2}{3}e^{-j0.6\pi}z^{-1})} \cdot \frac{4}{9}$$

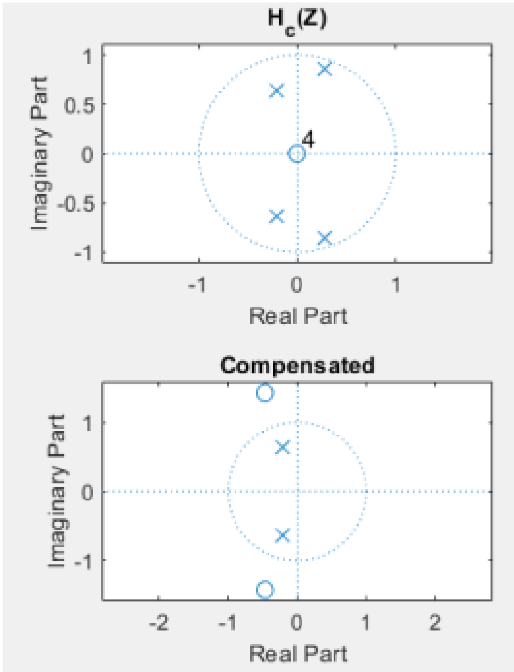
$$|H_{ap}(z)| = 1$$

$$\therefore H_{min}(z) = (1-0.9\,e^{j0.4\pi}z^{-1})(1-0.9\,e^{-j0.4\pi}z^{-1}) \cdot (1-\tfrac{2}{3}e^{j0.6\pi}z^{-1})(1-\tfrac{2}{3}e^{-j0.6\pi}z^{-1}) \cdot \frac{9}{4}$$

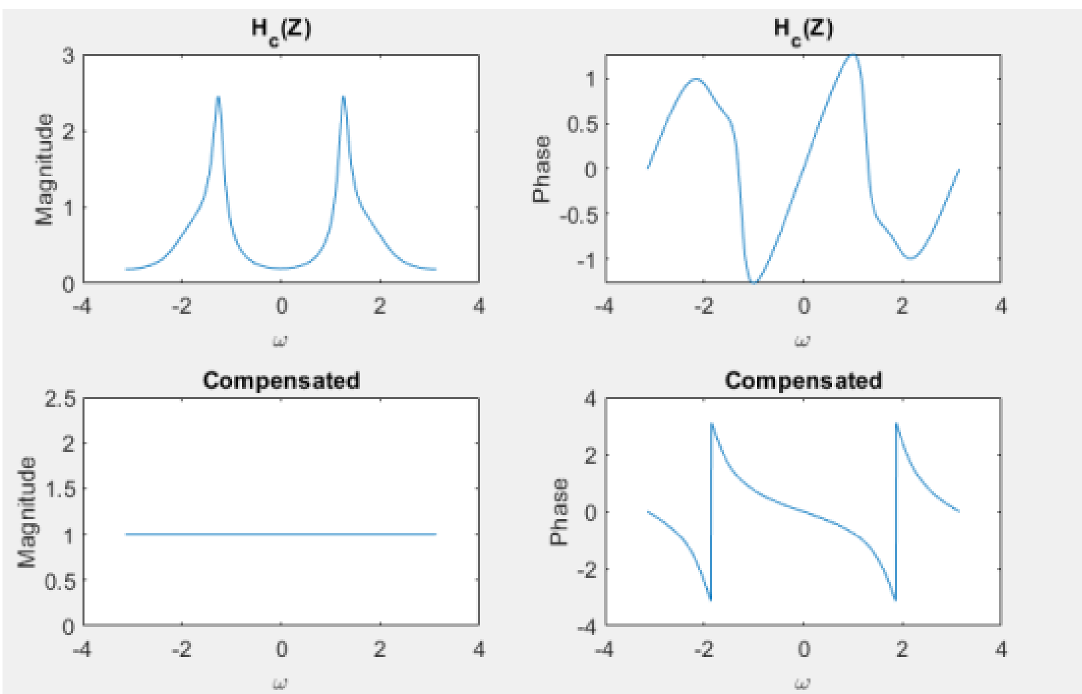
To get a flat magnitude response for the compensated system $H_{comp} = H(z)H_c(z) = H_{min}(z)H_{ap}(z)H_c(z)$

we have $H_c = \dfrac{1}{H_{min}}$

$$= \frac{4}{9} \cdot \frac{1}{(1-0.9e^{j0.4\pi}z^{-1})(1-0.9e^{-j0.4\pi}z^{-1})(1-\frac{2}{3}e^{j0.6\pi}z^{-1})(1-\frac{2}{3}e^{-j0.6\pi}z^{-1})}$$



$H_c(Z)$
Imaginary Part
Real Part
4
Compensated
Imaginary Part
Real Part



$H_c(Z)$
Magnitude
ω
$H_c(Z)$
Phase
ω
Compensated
Magnitude
ω
Compensated
Phase
ω

MATLAB:

Design a linear-phase bandpass filter using the Hann window design technique. The specifications are

$$\begin{array}{rl} \text{lower stopband edge:} & 0.2\pi \\ \text{upper stopband edge:} & 0.75\pi \end{array} \quad A_s = 40 \text{ dB}$$

$$\begin{array}{rl} \text{lower passband edge:} & 0.35\pi \\ \text{upper passband edge:} & 0.55\pi \end{array} \quad R_p = 0.25 \text{ dB}$$

Plot the impulse response and the magnitude response (in dB) of the designed filter.

Hints:

1) Use "fir1" function with proper arguments.
2) Note that the stopband attenuation A_s and the passband ripple R_p are defined as follows:

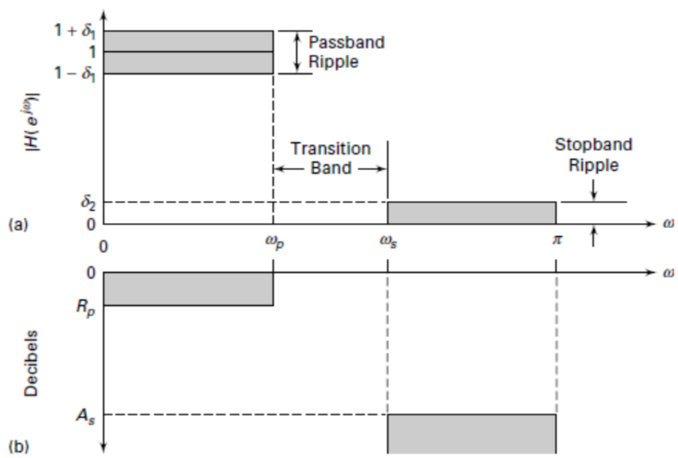


FIGURE 7.1 *FIR filter specifications: (a) absolute (b) relative*

$$R_p = -20\log_{10}\frac{1-\delta_1}{1+\delta_1} > 0\ (\approx 0)$$

$$A_s = -20\log_{10}\frac{\delta_2}{1+\delta_1} > 0\ (\gg 1)$$

3) But, remember that in most window-based designs, our only design parameters are the window type and the number of taps in the filter (or the filter order). We already know that the Hann window will meet ~44dB stopband attenuation, and as such, will meet our A_s spec above. <u>Therefore, for this problem, you will NOT use the A_s and R_p specs above in your design explicitly</u>. You will just need to find the minimum number of taps for your filter, as described below, and then once you have the filter, just confirm that it does meet your specs for both passband ripple and stopband attenuation.
4) Also remember that in window-based FIR design, we always get $\delta_1 = \delta_2$. So from A_s and R_p specs, you would have to find δ_1 and δ_2, and then once you have the filter, just confirm that it will indeed meet the minimum of the two.

5) The exact value of the main-lobe width, and therefore the filter transition bandwidth, associated with Hann window is $6.2\pi/M$ where M is the length of the window, or equivalently the number of taps in your FIR filter. So, given the specs, you can find the narrowest transition bandwidth you need to achieve, and based on that, find the minimum number of taps for your filter. Use an odd number so you end up with a Type I linear-phase FIR filter.
6) Use the center points of the transition bands as the edges of your passband, as passed to "fir1" function (i.e., [w_1 w_2]).

Solution:

```
function [Rp,As] = delta2db(delta1,delta2)
% Conversion from Absolute delta specs to Relative dB specs
% [Rp,As] = delta2db(delta1,delta2)
% Rp = Passband ripple
% As = Stopband attenuation
% d1 = Passband tolerance
% d2 = Stopband tolerance
Rp = -20*log10((1-delta1)/(1+delta1));
As = -20*log10(delta2/(1+delta1));
```

```
function [d1,d2] = db2delta(Rp,As)
% Conversion from Relative dB specs to Absolute delta specs.
% [d1,d2] = db2delta(Rp,As)
% d1 = Passband tolerance
% d2 = Stopband tolerance
% Rp = Passband ripple
% As = Stopband attenuation
K = 10^(Rp/20);
d1 = (K-1)/(K+1); d2 = (1+d1)*(10^(-As/20));
```

```
clc; close all;
%% Specifications:
ws1 = 0.2*pi;  % lower stopband edge
wp1 = 0.35*pi; % lower passband edge
wp2 = 0.55*pi; % upper passband edge
ws2 = 0.75*pi; % upper stopband edge
 Rp = 0.25;    % passband ripple
 As = 40;      % stopband attenuation
%
% Select the min(delta1,delta2) since delta1=delta2 in windodow design
[delta1,delta2] = db2delta(Rp,As);
if (delta1 < delta2)
   delta2 = delta1; disp('Delta1 is smaller than delta2')
   [Rp,As] = delta2db(delta1,delta2)
end
%
tr_width = min((wp1-ws1),(ws2-wp2));
M = ceil(6.2*pi/tr_width); M = 2*floor(M/2)+1, % choose odd M
M =
    43
n = 0:M-1; w_han = (hann(M))';
wc1 = (ws1+wp1)/2; wc2 = (ws2+wp2)/2; hd = ideal_lp(wc2,M)-ideal_lp(wc1,M);
h = fir1(M-1,[wc1,wc2]/pi,'bandpass',w_han);
[db,mag,pha,grd,w] = freqz_m(h,1); delta_w = pi/500;
Rpd = -min(db((wp1/delta_w)+1:(wp2/delta_w)+1)), % Actual passband ripple
Rpd =
    0.1030
Asd = floor(-max(db(1:(ws1/delta_w)+1))),        % Actual Attn
Asd =
    44
```

```
%
%% Filter Response Plots
Hf_1 = figure('Units','inches','position',[1,1,6,4],'color',[0,0,0],...
    'paperunits','inches','paperposition',[0,0,6,4]);
set(Hf_1,'NumberTitle','off','Name','P7.17');
subplot(2,2,1); Hs_1= stem(n,hd,'filled'); set(Hs_1,'markersize',3);
title('Ideal Impulse Response'); set(gca,'XTick',[0;M-1],'fontsize',8)
axis([-1,M,min(hd)-0.1,max(hd)+0.1]); xlabel('n'); ylabel('h_d(n)')
subplot(2,2,2); Hs_1 = stem(n,w_han,'filled'); set(Hs_1,'markersize',3);
axis([-1,M,-0.1,1.1]); xlabel('n'); ylabel('w_{han}(n)'); title('Hann Window');
set(gca,'XTick',[0;M-1],'fontsize',8); set(gca,'YTick',[0;1],'fontsize',8)
subplot(2,2,3); Hs_1 = stem(n,h,'filled'); set(Hs_1,'markersize',3);
title('Actual Impulse Response'); set(gca,'XTick',[0;M-1],'fontsize',8)
axis([-1,M,min(hd)-0.1,max(hd)+0.1]); xlabel('n'); ylabel('h(n)')
subplot(2,2,4); plot(w/pi,db,'linewidth',1); title('Magnitude Response in dB');
axis([0,1,-As-30,5]); xlabel('\omega/\pi'); ylabel('Decibels')
set(gca,'XTick',[0;0.2;0.35;0.55;0.75;1])
set(gca,'XTickLabel',[' 0  ';'0.2 ';'0.35';'0.55';'0.75';' 1  '],'fontsize',8)
set(gca,'YTick',[-40;0]); set(gca,'YTickLabel',[' 40';' 0 ']);grid
```

The filter response plots are shown in Figure 7.10.

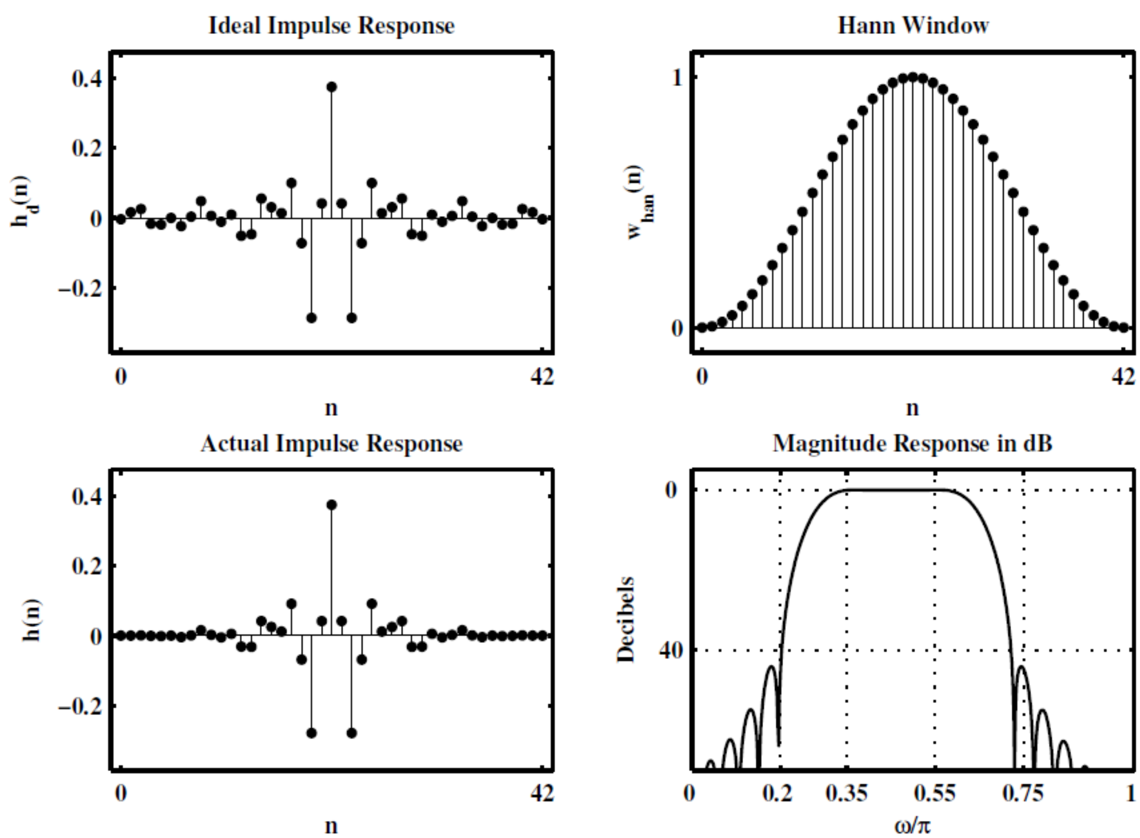


Figure 7.10: Filter design plots in Problem 7.17