ECEN 671 Final Project

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1 Introduction

Over the last year one of my main projects I have been working on is the visualization of a propagating acoustic field in the ultrasound range throughout a medium. This is done using some specialized hardware and an MRI scanner. This project was undertaken as to provide further comprehension of acoustical propagation in a medium, more specifically through a human skull. The setup consists of a gradient coil with a piezo electric cell running through its' center, see Fig 1.

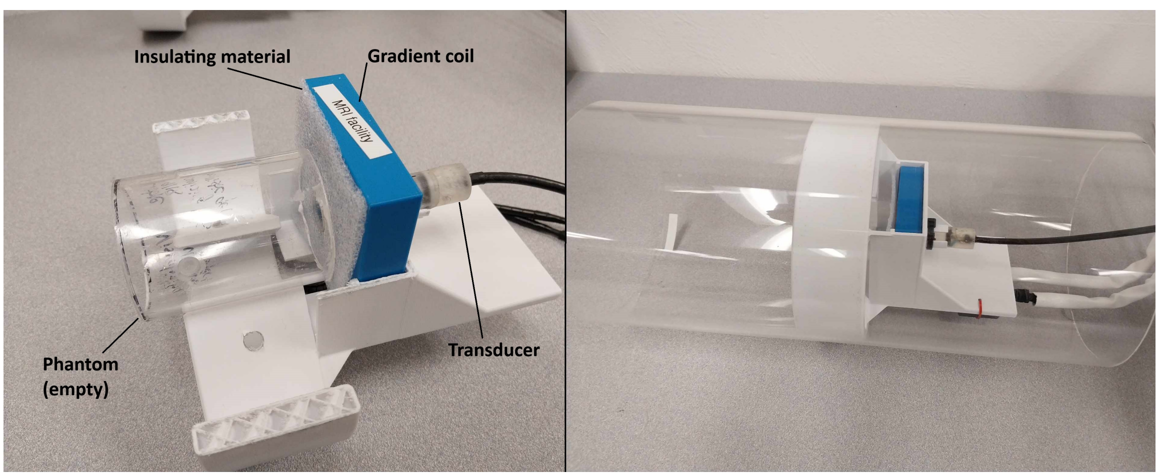

Figure 1: Full assembly of gradient coil, ultrasound transducer, and agar phantom. There are two pictures, one is a cutaway, and the second is the full assembly.

The idea behind the setup is that an oscillating gradient field is generated by a coil. This creates a varying gradient across the imaging plane. As particles are moved back and forth by the acoustic field, they lose or gain phase depending if they are pushed away or towards the coil. This result can be measured in the recreated phase image, see Fig 2.

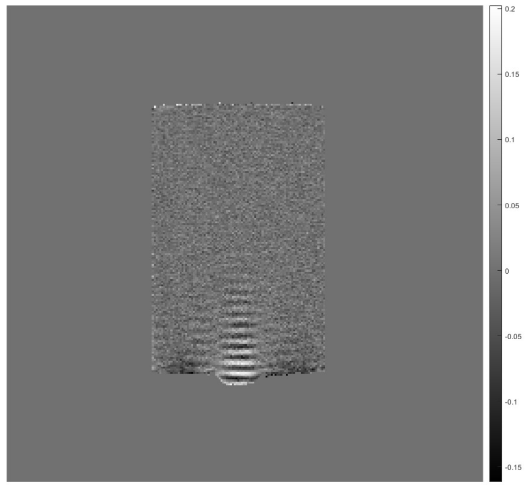

Figure 2: Resulting phase image with the acoustic field visibly present. The units are radians.

Using Eq. 1 the pressure of the acoustic field can be measured using the phase image. The variables needed are the density of the medium ρ, the gradient's frequency ω, the speed of sound c, the gyromagnetic ratio γ, the location dependent phase ϕ, and the location dependent gradient G_0. Since time is "held fixed", we don't need to consider how the system changes over time.

$$p(\mathbf{r}, t) = \frac{2\rho c \omega \phi(\mathbf{r}, t)}{\gamma G_0(\mathbf{r}, t) T} \tag{1}$$

The variable of interest G_0 is the main focus of this final project. This can be measured directly, but no instrumentation was available. Instead, the field map was acquired from a finite element analysis (FEA). This, however, poses a few problems. The first is that an FEA consist of modeling using tetrahedrons with the nodes being at the corners, meaning, the solution is very noisy. As seen in Fig. 3, sharp transitions can be seen from the resulting solution. Secondly, the image needs to be re-sampled. The acquired image has a 256x256 resolution and the FEA returned a 168x168.

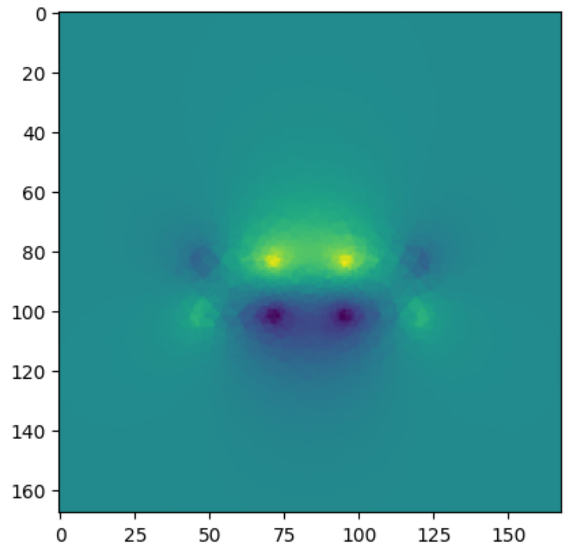

Figure 3: FEA resulting image, notice the sharp transitions and the triangular layout.

2 Solution

The proposed method for resampling would be to fit the data using a polynomial regression and resample the function at the correct intervals. The regression would require fitting Eq. 2 to be of the form Eq. 3. This also inherently low-pass filters the image as well.

$$z(x,y) = a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 y^2 + \cdots + a_{2n-1} x^n + a_{2n} y^n \tag{2}$$

$$Ax = b \tag{3}$$

The data from the FEA returned an X, Y location and the corresponding B field value which is represented as z. Each data sample, from 0 to i, is used to solve a single row of the A matrix for n orders, see Eq. 4.

$$\begin{bmatrix} x_0 & y_0 & x_0^2 & y_0^2 & \cdots & x_0^n & y_0^n \\ x_1 & y_1 & x_1^2 & y_1^2 & \cdots & x_1^n & y_1^n \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ x_i & y_i & x_i^2 & y_i^2 & \cdots & x_i^n & y_i^n \end{bmatrix} \cdot \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_i \end{bmatrix} = \begin{bmatrix} z_0 \\ z_1 \\ \vdots \\ z_i \end{bmatrix} \tag{4}$$

The final solution x was obtained by normalizing both sides, see Eq. 5, and performing LU decomposition.

$$A^T A x = A^T b \tag{5}$$

3 Results

The curve fit proved to be problematic, see Fig. 4. The fitting was fairly well for the y set of data, but the x axis did not work. All coefficients of x were 4 orders of magnitude smaller than their y counterparts. The convergence of the solution did not settle to a good value, see Fig 5. After the 20th order curve fit, the R^2 value does not grow and stays at 0.16. Localizing the curve data to a smaller data set of the overall graph yields much better results with a convergence occurring at an R^2 value of 0.68, see Fig 6 and 7.

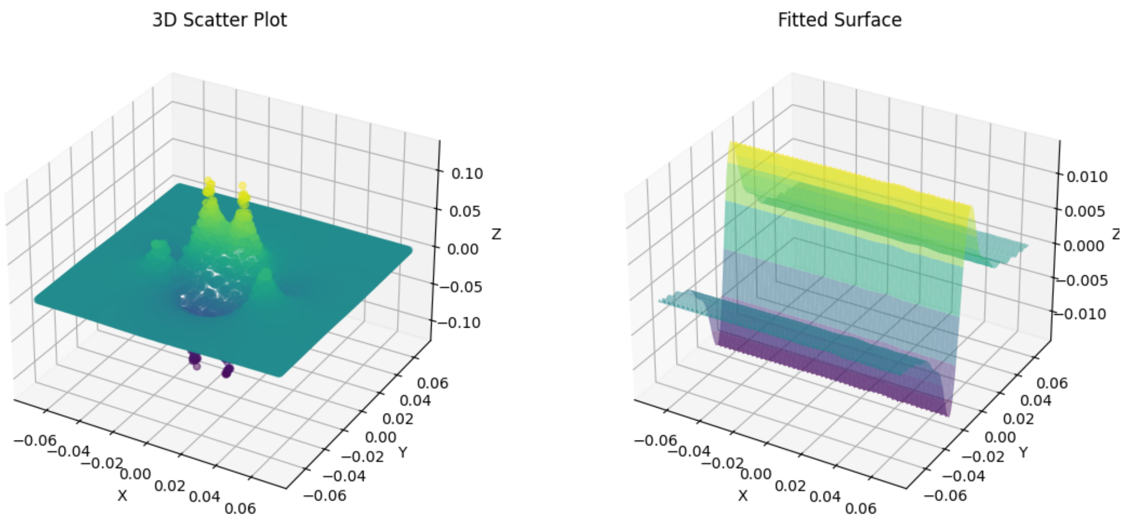

Figure 4: 60th order polynomial regression. The left figure is the actual field data, the on the right is the curve fit. The R^2 value is 0.16.

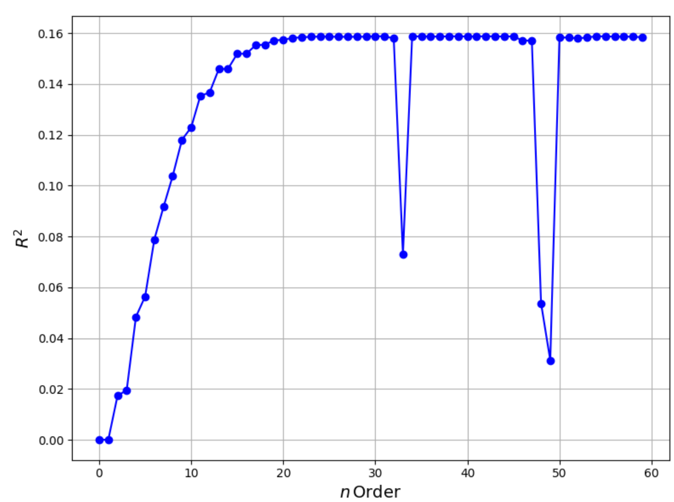

Figure 5: R^2 Value as a function of the order of the curve fit

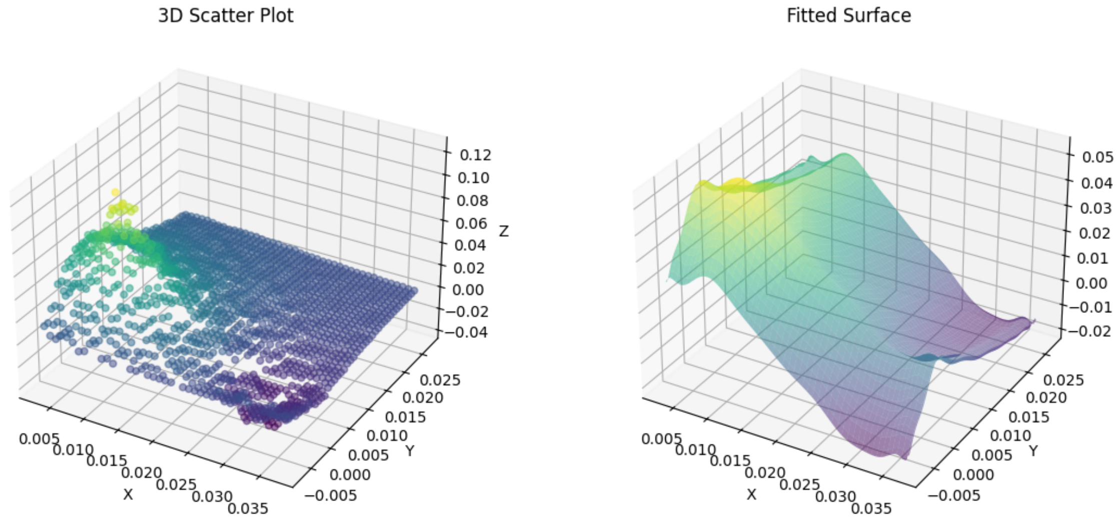

Figure 6: 20th order polynomial regression. The left figure is the actual field data, the on the right is the curve fit. The R^2 value is 0.68.

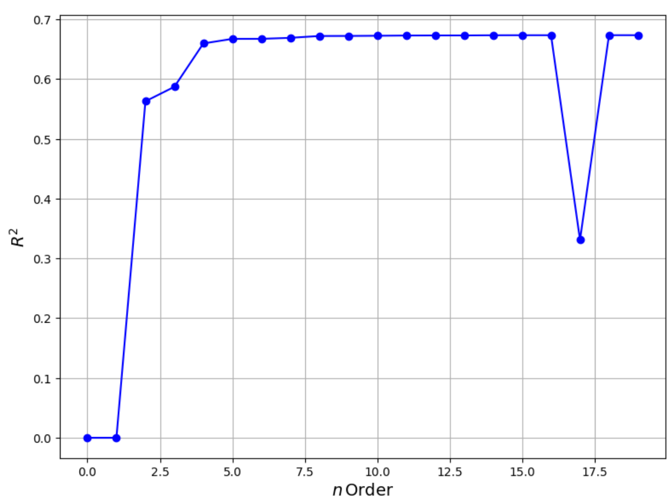

Figure 7: R^2 Value as a function of the order of the curve fit

4 Conclusion

The results proved to be unpractical as the polynomial regression could only converge when the data range was localized to a single peak. A combination of localized regression and stitching of those could be combined. However, this goes beyond the reach of this project. The overall result is insufficient to give a good solution to the issue of resampling the field mapping of the gradient.