CS 224 – Foundations of Computer Science

Spring 2023

Assignment 2

Topics covered: Digital Logic Circuits, Number Systems, Predicates, Quantified Statements, Methods of Proof, Sequences, Mathematical Induction, Algorithms, and Recurrence Relations

Assignment Due: February 17, 2023 at 11.59 PM EST

Objective:

The objective of this assignment is to practice concepts of Digital Logic Circuits, Number Systems, Predicates, Quantified Statements, Methods of Proof, Sequences, Mathematical Induction, Algorithms, and Recurrence Relations. This assignment comprises 7 questions for a total of 100 points. **Please read the entire document and the associated instructions carefully to familiarize yourself with the questions, submission instructions, and grading rubric.**

Submission Instructions:

Your submission on Canvas should consist of only 1 .pdf file. Name the file as Assignment2.pdf. You are encouraged to type your answers and create a .pdf file for submission. You may also complete your assignments using handwriting, and then scan your written work as a .pdf file. Your work should be organized, carefully thought out, and well written or typed. Problems written out of order, illegible handwriting, work scanned upside down or sideways, or messy work with scribbled out scratch work will result in grade deductions. **There is a 25% deduction for each day late, so make sure to submit your assignment by the deadline.**

For this assignment, collaboration is allowed. You are welcome to brainstorm and work together on this assignment. However, everyone must write up their own submission. Put the names of anyone that you worked with at the beginning of your work. These lists should be "symmetric".

Your work must adhere to the provisions of the Honor Code. See the following: http://catalog.college.emory.edu/academic/policies-regulations/honor-code.html

Problem 1

State whether the following statements are True or False. There is no need to show or explain your work. Just write True or False for each part.

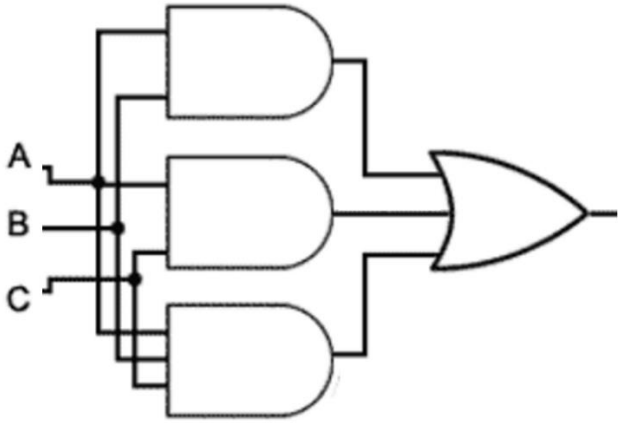
a. $1001_2 < 5_{10}$
b. $0111_2 = 111_{10}$
c. $0011_2 > 2_{10}$
d. $1001_2 > 1101_2$
e. $1011_2 = 11_{10}$
f. $1111_2 = 15_{10}$
g. $0000_2 < 0_{10}$
h. $1101_2 > 1010_2$

Problem 2

For the given circuit, perform the following:

(a) Obtain a Boolean Expression that represents the output of this circuit. Show the steps for obtaining this Boolean Expression.

(b) Simplify your Boolean Expression from 2(a) and present the simplified expression by using 3 or fewer logic gates. You may use truth tables or any of the laws from Theorem 2.1.1 (Slide #37 from Lecture 2) for simplification. Show/Discuss the steps of simplification.



[Note: The symbols for logic gates have their usual meanings. If you do not remember how a specific logic gate looks like, refer to the slides of Lecture 3]

Problem 3
Prove the statement below directly from the definitions of the terms. Do not use any other facts proved in the class or from the book or from any of the exercises in the book. Present all the steps of the proof. [Note: an integer can be an odd number or even number, so for full points your answer should present the analysis of the odd case and the even case separately]

For all integers n,
$n^2 + n + 1$ is odd.

Problem 4
Prove the following statements by contradiction. Present all the steps of the proof. Explanation of the steps is not necessary.
(a) For all integers n, if n^3 is even then n is even
(b) For all integers n, if n^3 is odd then n is odd

Problem 5
Use mathematical induction to prove that for all integers $n \geq 3$. Present all the steps of the proof.

$$2 \cdot 3 + 3 \cdot 4 + \cdots + (n-1) \cdot n = \frac{(n-2)(n^2+2n+3)}{3}$$

Problem 6
Calculate the loop invariant for this code snippet. Assume no syntax errors. Show at least 1 tracing table with integer inputs of your choice (for example: the tracing table for mystery5(3,4)) to justify your answer.

```
function mystery5(int a, int n)
{
int r = 1
```

```
int b = a
int i = n
  while (i > 0)
  {
    if (i % 2 == 0)
    {
      b = b * b
      i = i / 2
    }
    else
    {
      r = r * b
      i = i - 1
    }
  }
  return r
}
```

Problem 7

Consider that the number of bacteria in a colony triples every hour.

(a) Set up a recurrence relation for the number of bacteria after n hours have elapsed.

(b) If 1000 bacteria are used to begin a new colony, how many bacteria will be in the colony in 10 hours? Show the necessary calculations to support your answer.

Grading Rubric:

Question	Total Points
1	8
2	20
3	16
4	16
5	15
6	15
7	10