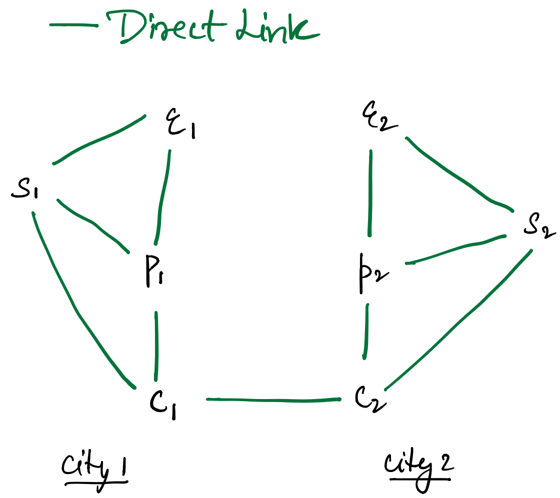
Hausman Instrument

Demand: $s_i = -\alpha p_i + \varepsilon_i, \ \alpha > 0$

Supply: $s_i = p_i + c_i$

and $c_i \perp\!\!\!\perp \varepsilon_j \quad \forall i,j$

City $i \in \{1,2\}$



$$\Rightarrow \quad p_i = \frac{-c_i}{1+\alpha} + \frac{\varepsilon_i}{1+\alpha} = \theta c_i + w_i$$

- $E(\varepsilon_i c_i) = 0, \ E(p_i c_i) \neq 0 \Rightarrow c_i$ valid instrument
- If c_i missing can we use p_{-i} as instrument?

① $E(p_{-i}\,\varepsilon_i) = E(\theta c_{-i} + w_{-i} \cdot \varepsilon_i)$

$$= \underbrace{E(\theta c_{-i} \cdot \varepsilon_i)}_{0 \text{ if } c_{-i} \perp\!\!\!\perp \varepsilon_i} + \underbrace{\frac{E(\varepsilon_{-i} \cdot \varepsilon_i)}{1+\alpha}}_{0 \text{ if } \varepsilon_{-i} \perp\!\!\!\perp \varepsilon_i}$$

② $E(p_{-i}\,p_i) = E((\theta c_i + w_i)(\theta c_{-i} + w_{-i}))$

$$= \theta^2 E(c_i c_{-i}) + \theta E(w_i c_{-i}) + \theta E(w_{-i} c_i) + E(w_i w_{-i})$$

$$= \theta^2 E(c_i c_{-i})$$

Conditions for validity of Hausman Instrument

1. $E(c_i \cdot \xi_j) = 0$ → costs are valid instrument
2. $E(\xi_i \cdot \xi_{-i}) = 0$ → demand shocks uncorrelated
3. $E(c_i \cdot c_{-i}) \neq 0$ → costs correlated.

BLP Instruments

- $X_t = (x_{1t}, x_{2t} \dots\ x_{Jt})$

$\Rightarrow$ characteristics of all products except price and advertising.

Instruments for p_{jt} :

(1) x_{jt} : instrument for itself (x_{jt})

(2) $\sum_{\substack{k \neq j \\ k \in J_f(j)}} x_{kt}$: sum of characteristics of other products produced by same firm (multiprod firm)

(3) $\sum_{\substack{k \neq j \\ k \notin J_f(j)}} x_{kt}$: sum of char of products of other firms.

Justification for 1

Is x_i an instrument for p_i ?

① $E(p_i \cdot x_i) \neq 0$

② $E(x_i \cdot \varepsilon_i) = 0$

characteristics do enter costs, so they will move price.

based on timing: firms set x_i, then see ε_i and then change p_i

" p_i changes more than x_i "

characteristics influence demand.

Model: D: $q = x_i - \alpha p_i + \varepsilon_i$

S: $q = p_i - \beta x_i$ characteristics influence costs

$\Rightarrow$ $E(p_i x_i)$ satisfied.

$\Rightarrow$ $E(\varepsilon_i x_i) = 0$ only if x_i fixed before ε_i.

That is we should not have a third equation, where x_i is on LHS and ϵ_i on RHS.

This is obviously not true if time horizon is say 10 years, then "demand shocks" will affect product characteristics.

Justification for 2

Firm produces 2 products $\rightarrow$ $i = 1, 2$

$$D: \quad q_i^d = -\alpha p_i + x_i + \epsilon_i$$

x_{-i} does not enter demand equation.

$$S: \quad q_i^s = p_i - \beta (x_i + x_{-i})$$

other products produced by same firm will influence the supply just like costs

$\Rightarrow$ $E(x_{-i} \cdot p_i) \neq 0$ plausible because x_{-i} is part of agg. costs of firm and p_i effects agg. revenue of firm.

$\Rightarrow$ $E(x_{-i}, \epsilon_i) = 0$ is more plausible than $E(x_i, \epsilon_i) = 0$ only when ϵ_i and ϵ_{-i} are uncorrelated and x_{-i} is set before ϵ_i and ϵ_{-i}.

$\Rightarrow$ But this is not so plausible when products are substitutes.

Justification for 3

Let $i = 1, 2$ be firms.

other characteristics matter for demand.

$$D: \quad q_i = -\alpha p_i + \beta(x_i - x_{-i}) + \epsilon_i$$

$$S: \quad q_i = p_i - x_i$$

promotion effects

Competitors product characteristics will certainly influence q^d and thus price. So $E(x_{-i} \cdot p_i) \neq 0$ should hold.

However to assume $E(\epsilon_i, x_{-i}) = 0$ we would have to assume that firm does not conduct promotions in response to competitors characteristics! This seems implausible!

For instance, it seems likely that competitors respond to promotions by changing their product characteristics.

Differentiation IVs

- Same logic for validity as BLP,

$$E(\xi_i\, x_{-i}) = 0$$

- Can be "stronger" than BLP because by only capturing "close competing products" we ensure that $Cov(p_i, x_{-i})$ is high.

For p_{jt} we create instruments:

let $D_{jkt} = x_{jt} - x_{kt}$ be distance in characteristic space we can then use,

$$\sum_k D_{jkt}^2 \,, \quad \sum_k \mathbb{1}(k_0 \le D_{jkt} \le k_1)$$

gives more weight to competitors far away.

captures no. of competitors "in range" of product.

By ignoring competitors that sell products very far away, we are able to ensure that our instruments are "strong" or have a large effect on own price.