ST2334 AY23/24 Sem 2 Midterms Cheat Sheet

Chapter 1: Probability

Sample Space	• The sample space, denoted by S, is the set of ALL possible outcomes of a statistical experiment. • The sample space depends on the problem of interest. • An **event** is a subset of a sample space.
Notation	For a finite set A, $\|A\|$ denotes the number of elements in A.
Equally Likely Probability	If S is a finite sample space in which all outcomes are equally likely and E is an event in S, then the **probability** of E, denoted $P(E)$, is $P(E) = \frac{\text{The number of outcomes in } E}{\text{The total number of outcomes in } S} = \frac{\|E\|}{\|S\|}$
Statistical Experiment	A Statistical Experiment is any procedure that produces data/ observations.
Sample Point	A sample point is an outcome (element) in the sample space
Event	An event is a subset of the sample space.

- The sample space is itself an event, and is called a sure event
- An event that contains NO ELEMENTS is the empty set, denoted by $\emptyset$, aka null event.

Event Operation & Relationship Laws

Basic		Distributive Law
$A \cap A' = \emptyset$	$A \cap \emptyset = \emptyset$	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
$A \cup A' = S$	$(A')' = A$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Set Union Law with Complement		**Absorption Law**
$A \cup B = A \cup (B \cap A')$		$A = (A \cap B) \cup (A \cup B')$
De Morgan's Law		
$(A_1 \cup A_2 \cup \ldots \cup A_N)' = A'_1 \cap A'_2 \cap \ldots \cap A'_N$ Note: $(A \cup B)' = A' \cap B$		$(A_1 \cap A_2 \cap \ldots \cap A_N)' = A'_1 \cup A'_2 \cup \ldots \cup A'_N$ Note: $(A \cap B)' = A' \cup B$

	Order Matters	Order Does Not Matter
Repetition Is Allowed	n^k	$\binom{k+n-1}{k}$
Repetition Is Not Allowed	$P(n,k)$	$\binom{n}{k}$

$$P(n,r) = \frac{n!}{(n-r)!} = n(n-1)(n-2)\ldots(n-r+1)$$

Probability Axioms

Let S be a sample space. A probability function P from the set of all events in S to the set of real numbers satisfies the following axioms: For all events A and B in S,

1. $0 \le P(A) \le 1$
2. $P(\emptyset) = 0$ and $P(S) = 1$
3. If A and B are disjoint events ($A \cap B = \emptyset$), then (i.e A & B are mutually exclusive events) $P(A \cup B) = P(A) + P(B)$

Basic Properties of Probabilities

Proposition 1: The probability of the empty set $\emptyset$ is $P(\emptyset) = 0$	
Proposition 2: If $A_1, A_2, \ldots, A_N$ are mutually exclusive events, that is $A_i \cap A_j = \emptyset$ for any $i \neq j$, then $P(A_1 \cup A_2 \cup \ldots \cup A_N) = P(A_1) + P(A_2) + \ldots + P(A_N)$	
Proposition 3: Complement Rule For any event A, we have: $P(A') = 1 - P(A)$	**Proposition 4:** For any 2 events A & B, $P(A) = P(A \cap B) + P(A \cap B')$
Proposition 5: General Union of 2 Events For any events A & B, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$	**Proposition 6:** If $A \subset B$, then $P(A) \le P(B)$

Independence, Mutual Exclusivity

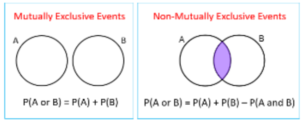
ME	• 2 events CANNOT occur at the same time • A, B mutually exclusive $\Leftrightarrow P(A \cap B) = \emptyset$
Independent Indep → $\perp$ dep → $\not\perp$	$P(A) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(A) \times P(B) = P(A \cap B)$ • A, B independent $\Leftrightarrow P(A \cap B) = P(A)P(B)$ • If independent & $P(A) \neq 0$ → $P(B\|A) = P(B)$ • If independent & $P(B) \neq 0$ → $P(A\|B) = P(A)$
Complement	$P(A') = 1 - P(A)$
Expected Value	$\sum_{k=1}^{n} a_k p_k = a_1 p_1 + a_2 p_2 + a_3 p_3 + \cdots + a_n p_n$

Conditional Probability

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \quad -(1)$$

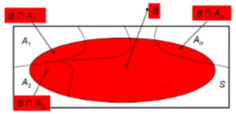
Multiplying both sides of (1) by P(A)	Dividing both sides of (2) by P(B|A)
$P(A \cap B) = P(B\|A) \cdot P(A) \quad -(2)$	$P(A) = \frac{P(A \cap B)}{P(B\|A)} \quad -(3)$

Multiplication Rule		Inverse Probability Formula	
$P(A \cap B) = P(B\|A) \cdot P(A), \text{if } P(A) \neq 0$ $P(A \cap B) = P(A\|B) \cdot P(B), \text{if } P(B) \neq 0$		$P(B\|A) = \frac{P(A \cap B)}{P(A)}$ Then inverse: $P(A\|B) = \frac{P(A)P(B\|A)}{P(B)}$	
False Positive	**False Negative**	**Sensitivity**	**Specificity**
P(+'ve | D^c)	P(-'ve | D)	P(+'ve | D)	P(-'ve | D^c)



Partition, Law of Total Probability

$$P(B) = \sum_{i=i}^{N} P(B \cap A_i) = \sum_{i=i}^{N} P(A_i)P(B|A_i)$$



For any events A & B, we have:

$$P(B) = P(A)P(B|A) + P(A')P(B|A')$$

Bayes Theorem:

K variables	$P(B_k\|A) = \frac{P(A\|B_k) \cdot P(A_k)}{\sum_{i=1}^{n} P(B_i)P(A\|B_i)}$ $P(B_k\|A) = \frac{P(A\|B_k) \cdot P(B_k)}{P(A\|B_1) \cdot P(B_1) + P(A\|B_2) \cdot P(B_2) + \cdots + P(A\|B_n) \cdot P(B_n)}$
2 variables	$P(B\|A) = \frac{P(A\|B) \cdot P(B)}{P(A)} = \frac{P(A\|B) \cdot P(B)}{P(A\|B) \cdot P(B) + P(A\|\bar{B}) \cdot P(\bar{B})} = \frac{P(A \cap B)}{P(A)}$

Pairwise Independent/ Mutually Independent

- Events are mutually independent IFF 4 conditions are satisfied:
  1. $P(A \cap B) = P(A) \bullet P(B)$
  2. $P(A \cap C) = P(A) \bullet P(C)$
  3. $P(B \cap C) = P(B) \bullet P(C)$
  4. $P(A \cap B \cap C) = P(A) \bullet P(B) \bullet P(C)$
- Events can be pairwise independent without satisfying the condition $P(A \cap B \cap C) = P(A) \bullet P(B) \bullet P(C)$
- Conversely, they can satisfy the condition $P(A \cap B \cap C) = P(A) \bullet P(B) \bullet P(C)$ without being pairwise independent.

Mutually Independent:

$$P(A_1 \cap A_2 \cap \ldots \cap A_n) = P(A_1) \bullet P(A_2) \bullet \ldots \bullet P(A_n)$$

Chapter 2: Random Variables

Probability Mass Function (PMF)	$f(x) = \begin{cases} P(X = x) & \text{for } x \in R_X \\ 0 & \text{for } x \notin R_X \end{cases}$ Properties of PMF: The pmf, $f(x)$ of a discrete random variable MUST satisfy these conditions: 1) $f(x_i) \ge 0$ for all $x_i \in R_X$ 2) $f(x_i) = 0$ for all $x_i \notin R_X$ 3) $\sum_{i=1}^{\infty} f(x_i) = 1 \text{ OR } \sum_{x_i \in R_X} f(x_i) = 1$ For any set $B \subset \mathbb{R}$, we have: $P(X \in B) = \sum_{x_i \in B \cap R_X} f(x_i)$
Probability Density Function (PDF)	1) $f(x) \ge 0$ for all $x \in R_X$; $f(x) = 0$ for $x \notin R_X$ 2) $\int_{R_X} f(x) dx = 1$ This is equivalent to: $\int_{-\infty}^{\infty} f(x)\, dx = 1$ Since $f(x) = 0$ for $x \notin R_X$ 3) For any a and b such that $a \le b$: $P(a \le X \le b) = \int_a^b f(x)\, dx$ Note: $P(a < X < b) = P(a < X \le b) = P(a \le X < b) =$ $P(a \le X \le b) = \int_a^b f(x)\, dx$. They all represent the area under the graph $f(x)$ between $x = a$ and $x = b$ To check if pdf: 1. $f(x) \ge 0$ for all $x \in R_X$; $f(x) = 0$ for $x \in R_X$ 2. $\int_{R_X} f(x) dx = 1$

Cumulative Distribution Function (CDF)

$$F(x) = P(X \le x)$$

Discrete	$F(x) = \sum_{t \in R_X; t \le x} f(t) = \sum_{t \in R_X; t \le x} P(X = t)$ • The cumulative distribution function of a DRV is a step function. • For any 2 numbers $a < b$, we have: $P(a \le X \le b) = P(X \le b) - P(X < a) = F(b) - F(a-)$ $F(a-) = \lim_{x \uparrow a} F(x)$
Continuous	$F(x) = \int_{-\infty}^{x} f(t)\, dt$ And $f(x) = \frac{dF(x)}{dx}$ Further: $P(a \le X \le b) = P(a < X < b) = F(b) - F(a)$

- Discrete → Summation; Continuous → Integrate
- The ranges of $F(x)$ and $f(x)$ satisfy the following conditions:
  1. $0 \le F(x) \le 1$
  2. For discrete distributions, $0 \le f(x) < 1$
  3. For continuous distributions, $0 \le f(x)$, but NOT NECESSARILY that $f(x) \le 1$

Expectation & Variance   **expectation = mean

Expectation for DRV	Expectation for CRV
$\mu x = E(X) = \sum_{x_i \in R_X} x_i f(x_i)$ $= \sum_{x_i \in R_X} x_i P(X = x) = \frac{\sum f(x)}{\sum f}$	$\mu x = E(X) = \int_{-\infty}^{\infty} x f(x)\, dx$ $= \int_{x \in R_X} x f(x)\, dx$
Let g(•) be an arbitrary function	
$E[g(X)] = \sum_{x \in R_X} g(x) f(x)$	$E[g(X)] = \int_{R_X} g(x) f(x)\, dx$

Variance

$$\sigma_x^2 = V(X) = E(X - \mu_x)^2 = E(X^2) - [E(X)]^2$$

Note:

- $V(X) \ge 0$ for any X.
- Equality holds iff $P(X = E(X)) = 1$, that is when X is a constant
- The positive root of the variance = standard deviation of X

$$\sigma_X = \sqrt{V(X)}$$

Variance for DRV	Variance for CRV
$V(X) = \sum_{x \in R_X} (x - \mu_x)^2 f(x)$	$V(X) = \int_{-\infty}^{\infty} (x - \mu_x)^2 f(x)\, dx$

Basic Properties of Expectations & Variance:

Expectation	Variance
a) $E(a) = a$	a) $V(a) = 0$
b) $E(aX) = aE(X)$	b) $V(aX) = a^2 V(X)$
c) $E(aX \pm b) = aE(X) \pm b$	c) $V(aX \pm b) = a^2 V(X)$
d) $E(aX \pm bY) = aE(X) \pm bE(Y)$	d) $V(aX \pm bY) = a^2 V(X) \pm b^2 V(Y)$
e) $E(x_1 + x_2 + \cdots + x_n) = nE(X)$ $E(a_1 X_1 + \ldots + a_k X_k) = a_1 E(X_1) + \ldots + a_k E(X_k)$	e) $V(x_1 + x_2 + \cdots + x_n) = nV(X)$

Chapter 3: Joint Distributions

Definition 2: 2D Random Vector

Let E be an experiment and S be a corresponding sample space.
Suppose X and Y are two functions each assigning a real number to each $s \in S$.
We call (X, Y) a **two-dimensional random vector**, or a **two-dimensional random variable**.

2D Discrete	(X, Y) is a **discrete 2D random variable** if the number of possible values of $(X(s), Y(s))$ are finite/countable. That is, the possible values of $(X(s), Y(s))$ may be represented by: $(x_i, y_j), \quad i = 1,2,3,\ldots; j = 1,2,3\ldots$
2D Continuous	(X, Y) is a **continuous 2D random variable** if the possible values of $(X(s), Y(s))$ can assume any value in some region of the Euclidean space $\mathbb{R}^2$.

Note:

- We can view X and Y separately to JUDGE whether (X, Y) is discrete or continuous.
  - If both X and Y are **discrete** random variables → (X, Y) is **discrete**.
  - If both X and Y are **continuous** random variables → (X, Y) is **continuous**.

Definition 3: n-Dimensional Random Vector

Let $X_1, X_2, \ldots, X_n$ be n functions each assigning a real number to all outcome $s \in S$.
We call $(X_1, X_2, \ldots, X_n)$ a **n-dimensional random vector**, or a **n-dimensional random variable**.

Discrete Joint Probability Function

$$f_{x,y}(x, y) = P(X = x, Y = y), \text{for } (x, y) \in R_{X,Y}$$

Properties of Discrete Joint Probability Function

1. $f_{x,y}(x, y) \ge 0$ for any $(x, y) \in R_{X,Y}$
2. $f_{x,y}(x, y) = 0$ for any $(x, y) \notin R_{X,Y}$
3. $\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} f_{x,y}(x_i, y_j) = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} P(X = x_i, Y = y_j) = 1$
   $\sum\sum_{(x,y) \in R_{X,Y}} f(x, y) = 1$
4. Let A be any subset of $R_{X,Y}$, then: $P((X, Y) \in A) = \sum\sum_{(x,y) \in A} f_{X,Y}(x, y)$

Continuous Joint Probability Function

$$P((X, Y) \in D) = \iint_{(x,y) \in D} f_{x,y}(x, y)\, dydx$$

For any $D \subset \mathbb{R}^2$, more specifically:

$$P(a \le X \le b, c \le Y \le d) = \int_a^b \int_c^d f_{x,y}(x, y)\, dydx$$

Properties of Continuous Joint Probability Function

1. $f_{x,y}(x, y) \ge 0$ for any $(x, y) \in R_{X,Y}$
2. $f_{x,y}(x, y) = 0$ for any $(x, y) \notin R_{X,Y}$
3. $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{x,y}(x, y)\, dydx = 1$
   $\iint_{(x,y) \in D} f_{x,y}(x, y)\, dydx = 1$

Marginal Probability Distribution

Y is DRV (Discrete)	For any x: $f_x(x) = \sum_y f_{X,Y}(x, y)$
Y is CRV (Continuous)	For any x: $f_x(x) = \int_{-\infty}^{\infty} f_{X,Y}(x, y)\, dy$

- Marginal distribution is like a "projection" of the 2D function $f_{X,Y}(x, y)$ onto the 1D function.
- The marginal distribution of X is the individual distribution of X ignoring the values of Y.
- $f_X(x)$ is a probability function; so it satisfies all the properties of the probability function

	Baseball	Basketball	Football	Total
Male	13	15	20	48
Female	23	16	13	52
Total	36	31	33	100

Marginal distribution of gender

Marginal distribution of sports

Conditional Distribution

Given $f_X(x) > 0$, cond prob fn:	Given $f_Y(y) > 0$, cond prob fn:
$f_{Y\|X}(y\|x) = \frac{f_{X,Y}(x, y)}{f_X(x)}$	$f_{X\|Y}(x\|y) = \frac{f_{X,Y}(x, y)}{f_Y(y)}$

$If\ f_X(x) > 0, f_{X,Y}(x, y) = f_X(x) f_{Y|X}(y|x),\ \ If\ f_Y(y) > 0, f_{X,Y}(x, y) = f_Y(y) f_{X|Y}(x|y)$

Discrete	$P(Y = y\|X = x) = \frac{P(X = x, Y = y)}{P(X = x)} = \frac{f_{X,Y}(x, y)}{f_X(x)}$
Continuous	$P(Y \le y\|X = x) = \int_{-\infty}^{y} f_{Y\|X}(y\|x)\, dy$ $E(Y\|X = x) = \int_{-\infty}^{\infty} y\, f_{Y\|X}(y\|x)\, dy$

$$\text{conditional distribution} = \frac{\text{joint density}}{\text{marginal distribution}}$$

Independent Random Variables

- Random variables X and Y are independent IFF for any x and y:
  $$f_{X,Y}(x, y) = f_X(x) f_Y(y)$$
- Random variables $X_1, X_2, \ldots, X_n$ are independent IFF for any $x_1, x_2, \ldots, x_n$:
  $$f_{X_1, X_2, \ldots, X_n}(x_1, x_2, \ldots, x_n) = f_{X_1}(x_1) f_{X_2}(x_2) \ldots f_{X_n}(x_n)$$
- Just check if their joint probability = product of their individual probabilities
  - If $R_{X,Y}$ is NOT a product space → X and Y are NOT independent

$$Exponent\ Integral: \int_b^a x^n e^{x^{n+1}}\,dx = \left[\frac{1}{n+1} e^{x^n}\right]_b^a$$

Properties of Independent Random Variables

Suppose X, Y are independent random variables:

1. If A and B are arbitrary subsets of $\mathbb{R}$, the events $X \in A$ and $Y \in B$ are independent events in S. As such:
$$P(X \in A; Y \in B) = P(X \in A)P(Y \in B)$$
For any real numbers x, y:
$$P(X \le x; Y \le y) = P(X \le x)P(Y \le y)$$

2. For arbitrary functions $g_1(\bullet)$ and $g_2(\bullet)$, $g_1(X)$ and $g_2(Y)$ are independent. For example:
   - X^2 and Y are independent.
   - $sin(X)$ and $cos(Y)$ are independent.
   - e^X and $log(Y)$ are independent.

3. Independence is connected with conditional distribution.
   - If $f_X(x) > 0$, then $f_{Y|X}(y|x) = f_Y(y)$
   - If $f_Y(y) > 0$, then $f_{X|Y}(x|y) = f_X(x)$

CHECKING INDEPENDENCE

We have a handy way to check independence.

X and Y are independent if and only if both of the following hold:

(a) $R_{X,Y}$, the range where the probability function is positive, is a product space.

(b) For any $(x, y) \in R_{X,Y}$, we have

$$\boxed{f_{X,Y}(x, y) = C \times g_1(x) \times g_2(y).}$$

That is, $f_{X,Y}(x, y)$ can be "factorized" as the product of two functions g_1 and g_2, where g_1 **depends on x only**, g_2 **depends on y only**, and C is a constant not depending on both x and y.

Note: $g_1(x)$ and $g_2(y)$ on their own NEED NOT be probability functions.

Expectation & Covariance

Definition 9: Expectation of 2-Dimensional Random Variables	
Consider any 2-variable function $g(x, y)$	
If (x, y) is DRV (Discrete)	$E[g(X,Y)] = \sum_x \sum_y g(x,y) f_{X,Y}(x,y)$
If (x, y) is CRV (Continuous)	$E[g(X,Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y) f_{X,Y}(x,y)\,dydx$
Note: If we let $g(X,Y) = (X - E(X))(Y - E(Y)) = (X - \mu_x)(Y - \mu_y)$ The expectation $E[g(X,Y)]$ leads to the covariance of X and Y.	

Definition 10: Covariance	
The covariance of X and Y is defined to be: $cov(X,Y) = E[(X - E(X))(Y - E(Y))]$	
If X and Y are DRV (Discrete)	$cov(X,Y) = \sum_x \sum_y (x - \mu_x)(y - \mu_y) f_{X,Y}(x,y)$
If X and Y are CRV (Continuous)	$cov(X,Y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x - \mu_x)(y - \mu_y) f_{X,Y}(x,y)\,dydx$

Properties of the Covariance

1. $cov(X,Y) = E(XY) - E(X)E(Y)$

2. If X and Y are independent, then $cov(X,Y) = 0$
   However, $cov(X,Y) = 0$ does not imply independence (1 way relation). i.e:
   i) $X \perp Y \Rightarrow cov(X,Y) = 0$ (X &Y independent → cov = 0)
      - Since $E(XY) = E(X)E(Y) \rightarrow cov(X,Y) = 0$
   ii) $cov(X,Y) = 0 \nRightarrow X \perp Y$ (cov = 0 does not imply independence)

3. $cov(aX + b, cY + d) = ac \bullet cov(X,Y)$
   i) $cov(X,Y) = cov(Y,X)$
   ii) $cov(X + b, Y) = cov(X,Y)$
   iii) $cov(aX, Y) = a \bullet cov(X,Y)$

4. $V(aX + bY) = a^2V(X) + b^2V(Y) + 2ab \bullet cov(X,Y)$
   i) $V(aX) = a^2V(X)$
   ii) $V(X + Y) = V(X) + V(Y) + 2cov(X,Y)$

Properties of Variance and Covariance

Using $V(X + Y) = V(X) + V(Y) + 2cov(X,Y)$, we can derive the following:

1. For random variables X and Y that are independent, we have:
$$V(X \pm Y) = V(X) \pm V(Y)$$

2. For random variables $X_1, X_2, \ldots, X_n$, we have: (Not independent)
$$V(X_1 + X_2 + \ldots + X_n) = V(X_1) + V(X_2) + \ldots + V(X_n) + 2\sum_{j>i} cov(X_i, X_j)$$

3. For random variables $X_1, X_2, \ldots, X_n$ that are independent, we have:
$$V(X_1 \pm X_2 \pm \ldots \pm X_n) = V(X_1) \pm V(X_2) \pm \ldots \pm V(X_n)$$

$$\mathbf{VAR(X)} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - E(X))^2 \quad (1)$$

$$\mathbf{COV(X, Y)} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - E(X))(y_i - E(Y)) \quad (2)$$

$$\mathbf{COR(X, Y)} = \frac{\mathbf{COV(X, Y)}}{\sqrt{\mathbf{VAR(X)VAR(Y)}}} \quad (3)$$

$$\mathbf{R}^2 = 1 - \frac{\mathbf{VAR(X, Y)}_{FittedLine}}{\mathbf{VAR(X, Y)}_{Mean}} \quad (4)$$

Simple Derivation & Integration Formulas

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

Function	Derivative	Function	Derivative
Elementary Functions		**Elementary Functions**	
x^n	nx^{n-1}	$(f(n))^n$	$nf'(x)(f(n))^{n-1}$
e^x	e^x	$e^{f(x)}$	$f'(x)e^{f(x)}$
$\ln(x)$	$\frac{1}{x}$	$\ln(f(x))$	$\frac{f'(x)}{f(x)}$
Trigonometric Functions		**Trigonometric Functions**	
$\cos(x)$	$-\sin(x)$	$\cos(f(x))$	$-f'(x)\sin(f(x))$
$\sin(x)$	$\cos(x)$	$\sin(f(x))$	$f'(x)\cos(f(x))$
$\tan(x)$	$sec^2(x)$	$\tan(f(x))$	$f'(x)sec^2(f(x))$
$\sec(x)$	$\sec(x)\tan(x)$	$\sec(f(x))$	$f'(x)\sec(f(x))tan(f(x))$
$\csc(x)$	$-\csc(x)\tan(x)$	$\csc(f(x))$	$-f'(x)\csc(f(x))tan(f(x))$
$\cot(x)$	$-csc^2(x)$	$\cot(f(x))$	$-f'(x)csc^2(f(x))$
Inverse Trigonometric Functions		**Inverse Trigonometric Functions**	
$sin^{-1}(x)$	$\frac{1}{\sqrt{1-x^2}}$	$sin^{-1}(f(x))$	$\frac{f'(x)}{\sqrt{1-f(x)^2}}$
$cos^{-1}(x)$	$-\frac{1}{\sqrt{1-x^2}}$	$cos^{-1}(f(x))$	$-\frac{f'(x)}{\sqrt{1-f(x)^2}}$
$tan^{-1}(x)$	$\frac{1}{1+x^2}$	$tan^{-1}(f(x))$	$\frac{f'(x)}{1+f(x)^2}$
$cot^{-1}(x)$	$-\frac{1}{1+x^2}$	$cot^{-1}(f(x))$	$-\frac{f'(x)}{1+f(x)^2}$
$sec^{-1}(x)$	$\frac{1}{\lvert x\rvert\sqrt{x^2-1}}, \lvert x\rvert > 1$	$sec^{-1}(f(x))$	$\frac{f'(x)}{\lvert f(x)\rvert\sqrt{f(x)^2-1}}, \lvert f(x)\rvert > 1$
$csc^{-1}(x)$	$-\frac{1}{\lvert x\rvert\sqrt{x^2-1}}, \lvert x\rvert > 1$	$csc^{-1}(f(x))$	$-\frac{f'(x)}{\lvert f(x)\rvert\sqrt{f(x)^2-1}}, \lvert f(x)\rvert > 1$

Rules of Differentiation: Let u and v be differentiable functions of x and c be a constant

Constant Rule	$\frac{d}{dx}(c) = 0$	**Sum Rule**	$\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$
Constant Multiple	$\frac{d}{dx}(cu) = c\frac{du}{dx}$	**Product Rule**	$\frac{d}{dx}(uv) = \frac{du}{dx}v + u\frac{dv}{dx}$
Quotient Rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\frac{du}{dx}v - u\frac{dv}{dx}}{v^2}$	**Chain Rule**	$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$

Implicit Differentiation:

- Differentiate both sides wrt x and solving the resultant equation for $\frac{dy}{dx}$
  - Obtain $\frac{dy}{dx}$ by differentiating every term by x
- When differentiating a function in y wrt x: $\frac{d}{dx}g(y) = g'(y) \times \frac{dy}{dx}$

Integration of Elementary Functions		
1.	$\int (ax + b)^n\,dx$	$\frac{(ax+b)^{n+1}}{(n+1)a} + C \quad (n \ne -1)$
2.	$\int \frac{1}{ax + b}\,dx$	$\frac{1}{a}\ln\lvert ax + b\rvert + C$
3.	$\int e^{ax+b}\,dx$	$\frac{1}{a}e^{ax+b} + C$
Integration of Trigonometric Functions		
4.	$\int \sin(ax + b)\,dx$	$-\frac{1}{a}\cos(ax + b) + C$
5.	$\int \cos(ax + b)\,dx$	$\frac{1}{a}\sin(ax + b) + C$
6.	$\int \tan(ax + b)\,dx$	$\frac{1}{a}\ln\lvert\sec(ax + b)\rvert + C$
7.	$\int \sec(ax + b)\,dx$	$\frac{1}{a}\ln\lvert\sec(ax + b) + \tan(ax + b)\rvert + C$
8.	$\int \csc(ax + b)\,dx$	$-\frac{1}{a}\ln\lvert\csc(ax + b) + \cot(ax + b)\rvert + C$
9.	$\int \cot(ax + b)\,dx$	$-\frac{1}{a}\ln\lvert\csc(ax + b)\rvert + C$
10.	$\int \sec^2(ax + b)\,dx$	$\frac{1}{a}\tan(ax + b) + C$
11.	$\int \csc^2(ax + b)\,dx$	$-\frac{1}{a}\cot(ax + b) + C$
12.	$\int \sec(ax + b)\tan(ax + b)\,dx$	$\frac{1}{a}\sec(ax + b) + C$
13.	$\int \csc(ax + b)\cot(ax + b)\,dx$	$-\frac{1}{a}\csc(ax + b) + C$
Integration of Polynomials (Partial Fractions)		
14.	$\int \frac{1}{a^2 + (x + b)^2}\,dx$	$\frac{1}{a}\tan^{-1}\left(\frac{x+b}{a}\right) + C$
15.	$\int \frac{1}{\sqrt{a^2 - (x + b)^2}}\,dx$	$\sin^{-1}\left(\frac{x+b}{a}\right) + C$
16.	$\int \frac{-1}{\sqrt{a^2 - (x + b)^2}}\,dx$	$\cos^{-1}\left(\frac{x+b}{a}\right) + C$
17.	$\int \frac{1}{a^2 - (x + b)^2}\,dx$	$\frac{1}{2a}\ln\left\lvert\frac{x+b+a}{x+b-a}\right\rvert + C$
18.	$\int \frac{1}{(x + b)^2 - a^2}\,dx$	$\frac{1}{2a}\ln\left\lvert\frac{x+b-a}{x+b+a}\right\rvert + C$
19.	$\int \frac{1}{\sqrt{(x + b)^2 + a^2}}\,dx$	$\ln\left\lvert(x + b) + \sqrt{(x+b)^2 + a^2}\right\rvert + C$
20.	$\int \frac{1}{\sqrt{(x + b)^2 - a^2}}\,dx$	$\ln\left\lvert(x + b) + \sqrt{(x+b)^2 - a^2}\right\rvert + C$
21.	$\int \sqrt{a^2 - x^2}\,dx$	$\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$
22.	$\int \sqrt{x^2 - a^2}\,dx$	$\frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\ln\left\lvert x + \sqrt{x^2 - a^2}\right\rvert + C$
Modulus		
23.	$\int_{-a}^{a} \lvert bx^n\rvert\,dx$	$\left[\frac{x^{n+1}}{b(n+1)}\right]_0^a - \left[\frac{x^{n+1}}{b(n+1)}\right]_{-a}^0$
Result from tutorials (prob only works with trigonometry)		
24.	$\int_a^b f(x)\,dx$	$bf(b) - af(a) - \int_{f(a)}^{f(b)} f^{-1}(x)\,dx$
25.	$\int_0^a f(x)\,dx$	$\int_0^a f(a - x)\,dx$

By Parts	By Substitution
$\int u\frac{dv}{dx} = uv - \int v\frac{du}{dx}\,dx$	$u = f(x)$

$f(x, y)$		x = 2	x = 4	$f_Y(y)$
y	1	0.10	0.15	0.25
	3	0.20	0.30	0.50
	5	0.10	0.15	0.25
$f_X(x)$		0.40	0.60	1

- To test if X and Y are independent:
  - Check $f_{X,Y}(x, y) = f_X(x) \bullet f_Y(y)$ for all possible combinations.