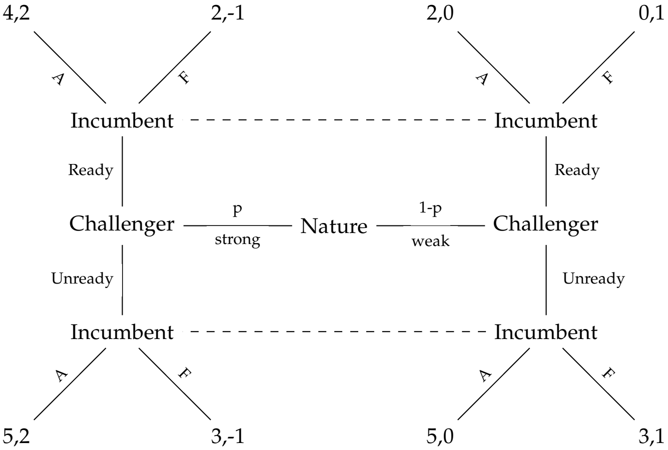
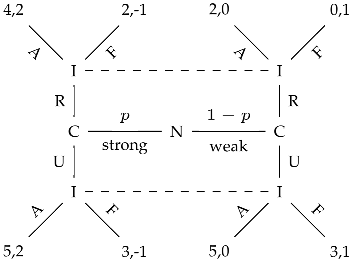
Signaling Games


4,2
2,-1
2,0
0,1
A
F
A
F
Incumbent
Incumbent
Ready
Ready
p
1-p
Challenger
Nature
Challenger
strong
weak
Unready
Unready
Incumbent
Incumbent
A
F
A
F
5,2
3,-1
5,0
3,1



4,2
2,-1
2,0
0,1
A
F
A
F
I
I
R
R
C
N
C
p
$1-p$
strong
weak
U
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5,2
3,-1
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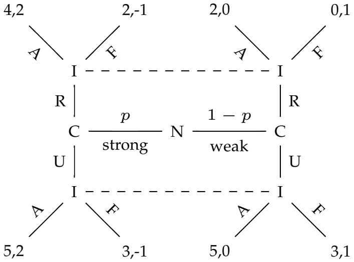
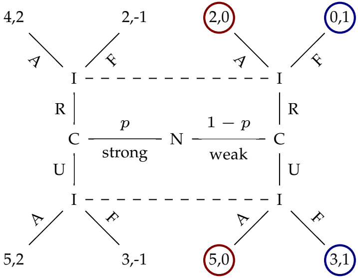
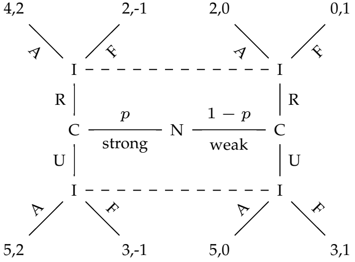


How do we find the equilibria?



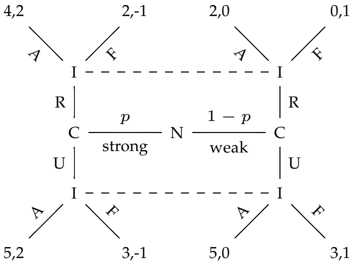
Weak challenger always prefers U



Let's suppose strong challengers choose R

⇒ Fully revealing strategies

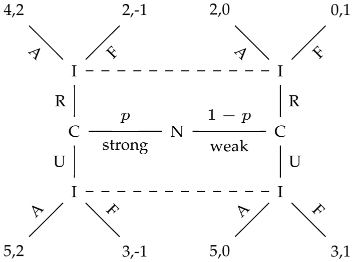
⇒ Incumbent chooses A if C strong, F if C weak



We have an assessment

Behaviorial strategies:
- –Strong: R
- –Weak: U
- –I: A following R, F following U

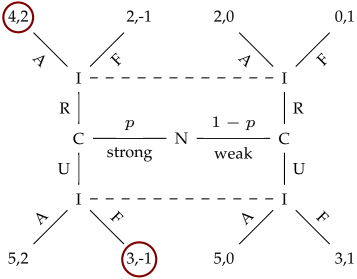
Belief System:
- –p(S)=1 if R
- –p(W)=0 if U



An assessment is a weak sequential equilibrium if:

i) Sequential rationality

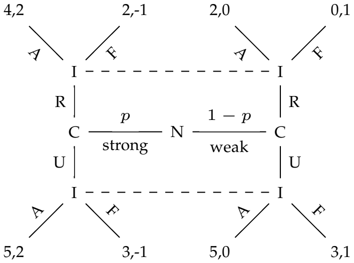
ii) Weak consistency of beliefs



Equilibrium?

C: R → U? I assumes weak, so F.

I: C's strategy reveals type, I's strategy clearly optimal



Now suppose strong challengers choose U

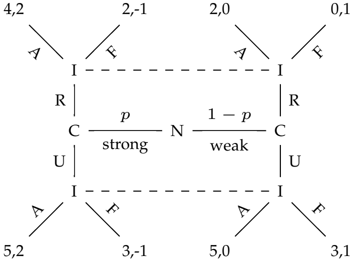
$\Rightarrow$ Pooling

No updating of beliefs

History is (S,U) with probability p, (W,U) w/probability $(1 - p)$

A if:
$p * 2 + (1 - p) * 0 > p * -1 + (1 - p) * 1$

$\Rightarrow p \geq 1/4$



Suppose $p \geq 1/4$ so I chooses A following U

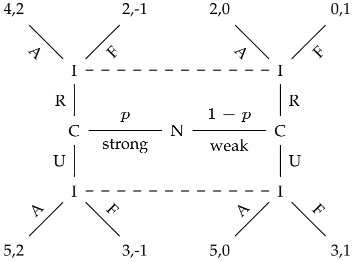
Is U optimal for (strong & weak) C?

Yes, gets a payoff of 5

What if I sees R?

I can hold any belief

Pooling equilibrium:
–All challengers choose U,
–I chooses A, &
–I's beliefs following U are same as priors and following R I can hold any belief



Suppose $p \leq 1/4$ so I chooses F following U

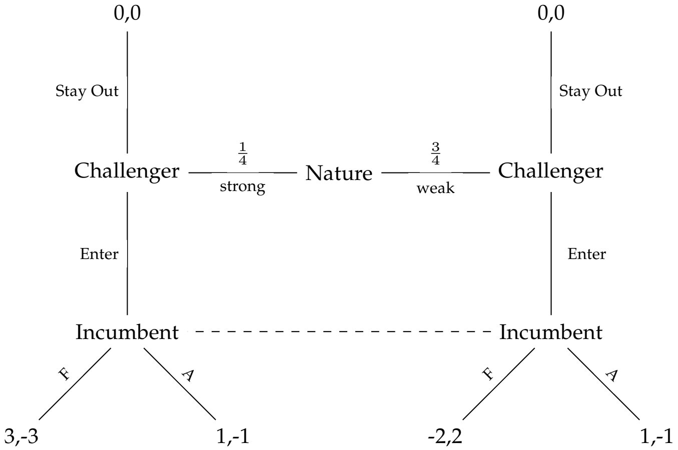
Is U optimal for (strong & weak) C?

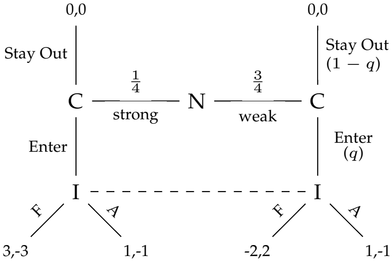
C gets a payoff of 3

C gets 4 (if A) or 2 (if F) if switching to R

U not optimal unless I responds to R with F:
$2q \leq 1 - 2q \Rightarrow q \leq 1/4$

Pooling equilibrium II:
–All challengers choose U,
–I chooses A after U & F after R
–I's beliefs after U same as priors &
–after R assigns a belief such that $q \leq 1/4$


0,0
0,0
Stay Out
Stay Out
Challenger
$\frac{1}{4}$
strong
Nature
$\frac{3}{4}$
weak
Challenger
Enter
Enter
Incumbent
Incumbent
F
A
F
A
3,-3
1,-1
-2,2
1,-1



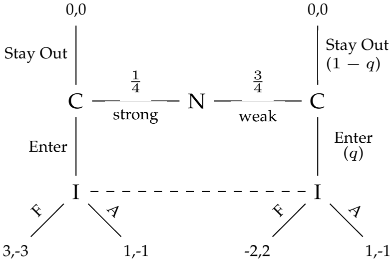
Separating Equilibrium?

- Strong C: E
- Weak C: S

$\Rightarrow$ I: A following E

but then a weak C would enter

Other separating equilibrium?



Pooling Equilibrium?

- Strong & Weak C: E

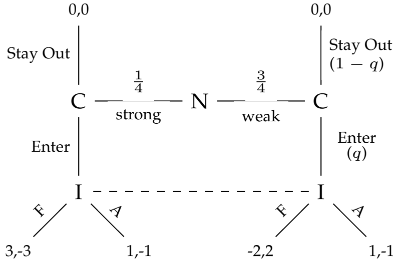
$\Rightarrow$ I: No new info

I choose F if

$\frac{1}{4} * -3 + \frac{3}{4} * 2 \geq \frac{1}{4} * -1 + \frac{3}{4} * -1$

$\Rightarrow$ I chooses F

$\Rightarrow$ Weak C prefers staying out

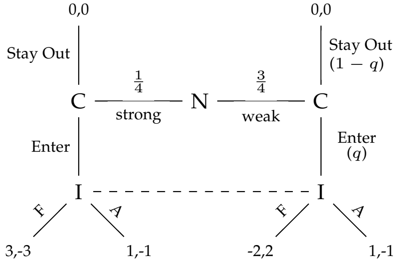


Semi-separating/partial pooling equilibrium

Not separating, not pooling $\Rightarrow$ Some type uses a mixed strategy.

Strong C has a dominant strategy to enter

Weak C must be mixing.

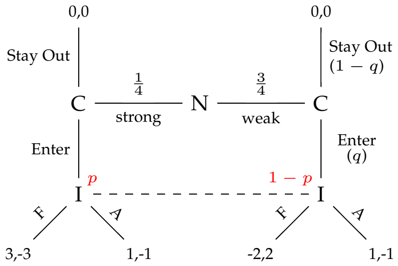


Semi-separating/partial pooling equilibrium

Weak C only mixes if indifferent

Expected utility of entering?

⇒ I must mix with probability so that the weak type is indifferent



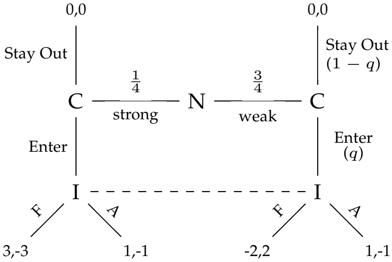
So $E[u_I(F)] = p * -3 + (1-p) * 2$
& $E[u_I(A)] = p * -1 + (1-p) * -1$

$\Rightarrow p = \frac{3}{5}$

But I's beliefs must be consistent, i.e., p must be consistent with Bayes' rule

$\Rightarrow \frac{3}{5} = \frac{\frac{1}{4} * 1}{\frac{1}{4} * 1 + \frac{3}{4} q}$

$\Rightarrow q = \frac{2}{9}$



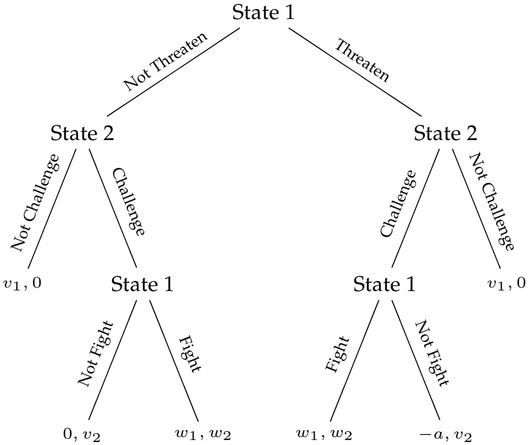
Equilibrium

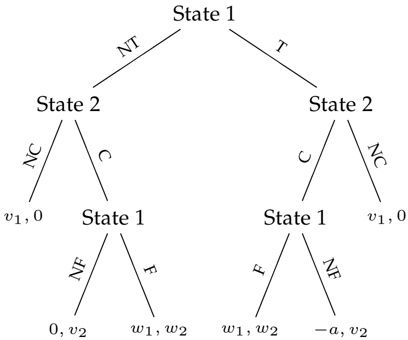
Behavioral strategies:

- Strong: E
- Weak: E w/prob. $q = \frac{2}{9}$
- Incumbent: F w/prob. $r = \frac{1}{3}$

Beliefs:

- After S: Pr(strong)=0
- After E: Pr(strong)=$q = \frac{3}{5}$

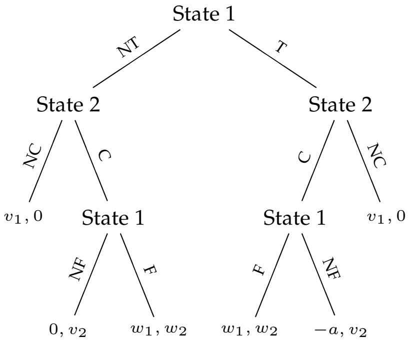

State 1
Not Threaten
Threaten
State 2
State 2
Not Challenge
Challenge
Challenge
Not Challenge
$v_1, 0$
State 1
State 1
$v_1, 0$
Not Fight
Fight
Fight
Not Fight
$0, v_2$
w_1, w_2
w_1, w_2
$-a, v_2$



Audience Costs

State 1 chooses whether to make a public threat — backing down is costly (a)

- v_i: value of good to state i
- w_i: war payoff: $p_i v_i - c_i$
- p_i: i's prob. of winning war
- c_i: i's cost of fighting war



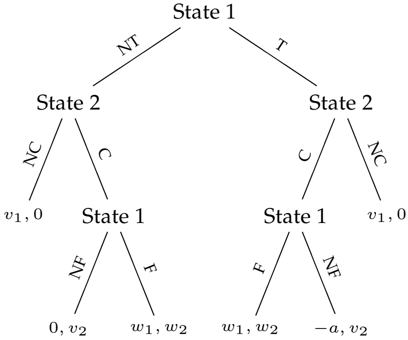
Assume there is uncertainty about the states' valuation of the good

v_i is drawn from distribution, f on $\mathbb{R}^+$, continuous and strictly positive

So, $F(x)$ is the probability of drawing a value of x or lower

This is a signaling game (not cheap talk) because the threat involves a (potential) cost

fight?



A separating equilibrium?

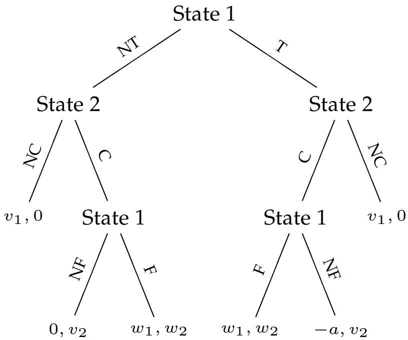
Consider State 2 following a threat: C if $p_2 v_2 \geq c_2$

$\Rightarrow v_2^* = \frac{c_2}{p_2}$ separates fighters & lovers

Now consider State 1's initial move. It might face a lover or it might face a fighter so expected payoff:

$$F(v_2^*) * v_1 + (1 - F(v_2^*))(p_1 v_1 - c_1)$$

$$\Rightarrow v_1^* = \frac{(1-F(v_2^*))c_1}{F(v_2^*)+(1-F(v_2^*))p_1}$$
$$= \frac{c_1}{\frac{1}{\frac{1}{F(v_2^*)}-1}+p_1}$$



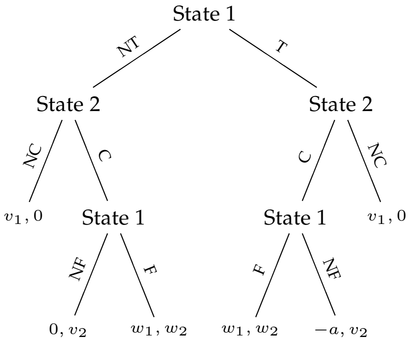
A pooling equilibrium?

Consider a State 1 type that is going to bluff, i.e., it doesn't actually want to fight

Payoff from threatening is $F(v_2^*)v_1 + (1 - F(v_2^*))(-a)$

Payoff from not threatening is 0

$\Rightarrow v_1^* = \left(\frac{1}{F(v_2^*)} - 1\right) a$



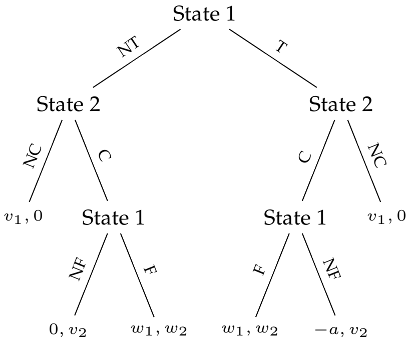
Now consider State 2 that hears a threat. The types making threats include bluffers as well as straight-talkers. Probability of straight-talker: P(F | T)

Payoff from challenging:

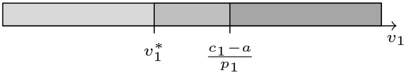
$$(1 - P(F|T))v_2 + P(F|T)(p_2 v_2 - c_2)$$

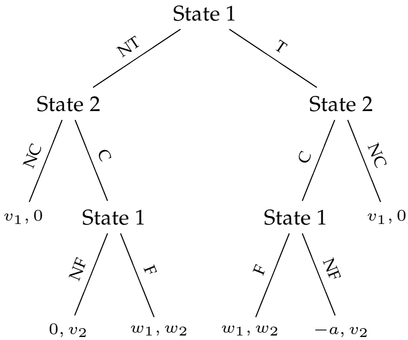
Payoff from not challenging: 0

$$\Rightarrow v_2^* = \frac{c_2}{\frac{1}{P(F|T)} - 1 + p_2}$$

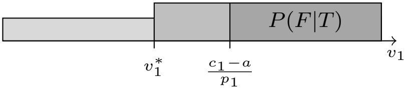


Almost there — just don't know $P(F|T)$, the belief that State 1 is a fighter, yet.

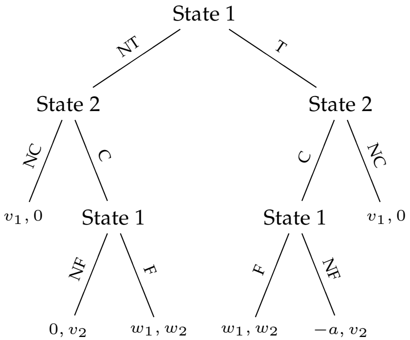




Almost there — just don't know $P(F|T)$, the belief that State 1 is a fighter, yet.



Bayes' rule: $P(F|T) = \frac{1-F(\frac{c_1-a}{p_1})}{1-F(v_1^*)}$

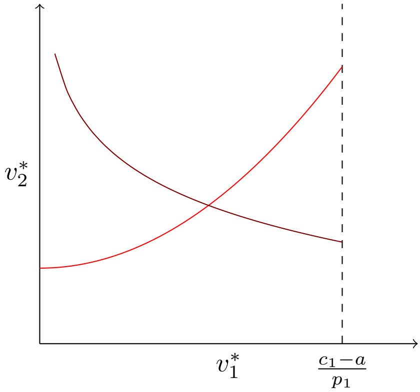


We can see that $v_1^*, v_2^*, P(F|T)$ are functions of one another

Given explicit forms of the distributions, we could solve for the equilibrium

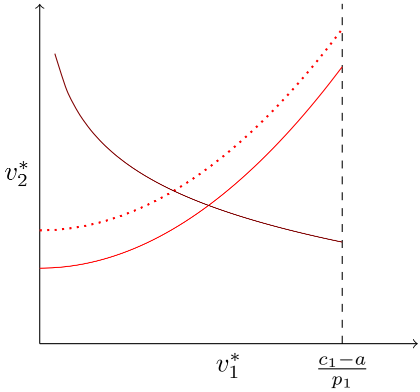
...but we don't need to in order obtain comparative statistics

$$v_1^* = \left(\frac{1}{F(v_2^*)} - 1\right) a \qquad v_2^* = \frac{c_2}{\frac{1}{P(F|T)} - 1 + p_2} = \frac{c_2}{\frac{1-F(v_1^*)}{1-F\left(\frac{c_1-a}{p_1}\right)} - 1 + p_2}$$



Comparative Statics:

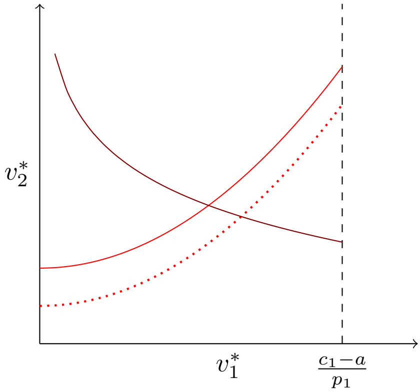
$$v_1^* = \left(\frac{1}{F(v_2^*)} - 1\right) a \qquad v_2^* = \frac{c_2}{\frac{1}{P(F|T)} - 1 + p_2} = \frac{c_2}{\frac{1-F(v_1^*)}{1-F\left(\frac{c_1-a}{p_1}\right)} - 1 + p_2}$$



Comparative Statics:

$\uparrow p_1$?

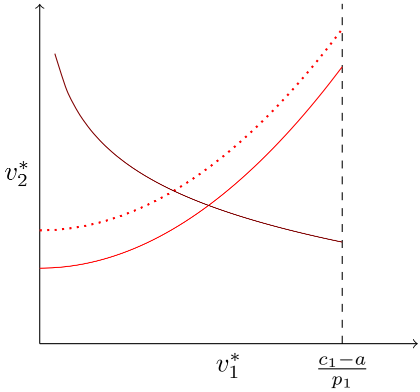
$$v_1^* = \left(\frac{1}{F(v_2^*)} - 1\right) a \qquad v_2^* = \frac{c_2}{\frac{1}{P(F|T)} - 1 + p_2} = \frac{c_2}{\frac{1-F(v_1^*)}{1-F\left(\frac{c_1-a}{p_1}\right)} - 1 + p_2}$$



Comparative Statics:

$\uparrow c_1$?

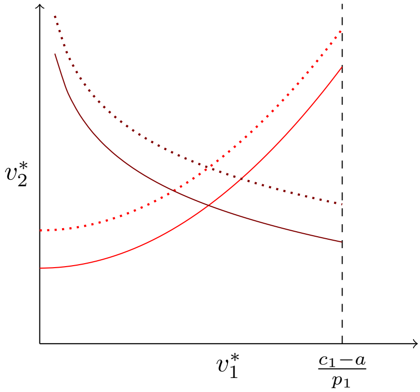
$$v_1^* = \left(\frac{1}{F(v_2^*)} - 1\right) a \qquad v_2^* = \frac{c_2}{\frac{1}{P(F|T)} - 1 + p_2} = \frac{c_2}{\frac{1-F(v_1^*)}{1-F\left(\frac{c_1-a}{p_1}\right)} - 1 + p_2}$$



Comparative Statics:

$\uparrow c_2$?

$$v_1^* = \left(\frac{1}{F(v_2^*)} - 1\right) a \qquad v_2^* = \frac{c_2}{\frac{1}{P(F|T)} - 1 + p_2} = \frac{c_2}{\frac{1 - F(v_1^*)}{1 - F\left(\frac{c_1 - a}{p_1}\right)} - 1 + p_2}$$



Comparative Statics:

$\uparrow a$?

To fight or not to fight?

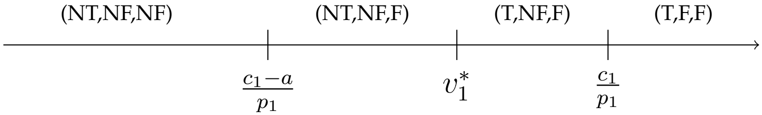
$$\begin{aligned} p_1 v_1 - c_1 &\geq 0 \\ p_1 v_1 &\geq \frac{c_1}{p_1} \end{aligned}$$

Fight after threatening?

$$\begin{aligned} p_1 v_1 - c_1 &\geq -a \\ p_1 v_1 &\geq \frac{c_1 - a}{p_1} \end{aligned}$$

back

No bluffing



Bluffing