Formal Methods DPIR Hilary 2021

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Transitive Preferences

Definition 2.1 *A state has rational preferences, $\succcurlyeq$, over the outcomes in set X if they are:*

- *complete, that is, for all x_i, $x_j \in X$, we have either $x_i \succcurlyeq x_j$, or $x_j \succcurlyeq x_i$, or both:*
- *transitive, that is, for all x_i, x_j, and $x_k \in X$, if $x_i \succcurlyeq x_j$ and $x_j \succcurlyeq x_k$, then $x_i \succcurlyeq x_k$.*

Utility

Definition 2.2 *A function $u : X \to \mathbb{R}$ is a utility function representing the preferences $\succcurlyeq$ if, for all $x_i, x_j \in X$, $u(x_i) \geq u(x_j) \Leftrightarrow x_i \succcurlyeq x_j$.*

Lottery

Definition 2.3 *A lottery associated with a finite set of outcomes, X, with number of elements equal to $|X| = n$ is a vector $L = (p_1, \ldots, p_n)$, where $p_i \in [0, 1]$ is interpreted as the probability that outcome i occurs, so that $\sum_i p_i = 1$.*

Expected Utility

Definition 2.4 *The expected utility of a lottery L based on a finite outcome set X is defined as the expected value of the utilities of the outcomes*

$$EU(L) \equiv \sum_{i=1}^{n} p_i u(x_i)$$

Theorem 2.2 *Given a preference ordering* $\succcurlyeq$ *over the set of lotteries L defined on an outcome space X, a utility function over the outcomes* $u(x)$ *exists such that the expected utility of any lottery,* $EU(L)$*, reflects the preference ordering, that is,* $L_1 \succcurlyeq L_2 \Leftrightarrow EU(L_1) \geq EU(L_2)$ *if the following conditions hold:*

1. *The preference ordering* $\succcurlyeq$ *is complete and transitive.*
2. *Different lotteries that assign the same value to the outcomes are equivalent.*
3. *If* $L_1 \succ L_2$*, then all lotteries sufficiently close to* L_1 *are also preferred to* L_2*.*
4. *If* $L_1 \succ L_2$*, then adding an equal chance of obtaining* L_3 *to both sides does not alter the preference.*

Efficiency

Definition 2.5 *Given a set of actors with utility functions u_i defined over an outcome space X, an outcome $x^{'} \in X$ is* efficient *if for any other outcome $x^{''} \in X$ that makes some player i better off, $u_i(x^{''}) > u_i(x^{'})$, there must be some other actor j that is worse off, $u_j(x^{'}) > u_j(x^{''})$.*

Zero Sum

Definition 2.6 *A set of outcomes X and utility functions u_i defined over it is* zero sum *if every $x \in X$ is efficient*

Utility Functions

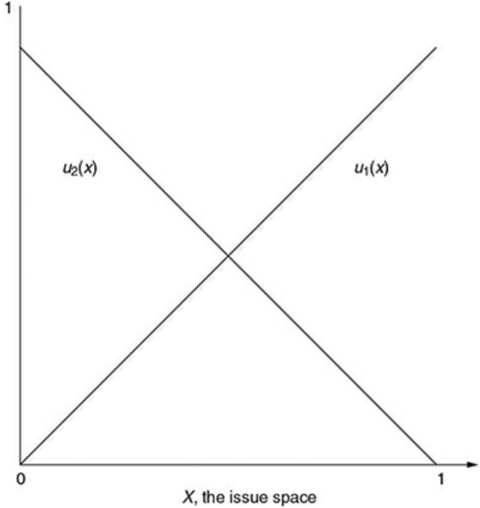

Figure 2.1 Utility functions

Multi-Dimensions

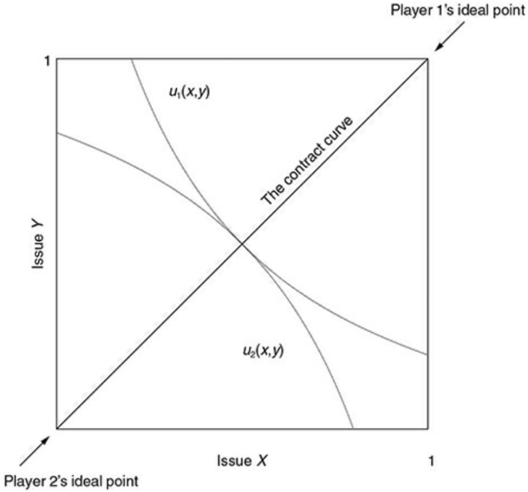

Figure 2.2 The Edgeworth Box

Risk Attitudes

Definition 2.7 *An actor is:*

- risk neutral *if* $pu(x') + (1 - p)u(x'') = u(px' + (1 - p)x'')$,
- risk averse *if* $pu(x') + (1 - p)u(x'') < u(px' + (1 - p)x'')$,
- risk acceptant *if* $pu(x') + (1 - p)u(x'') > u(px' + (1 - p)x'')$.

Risk Attitudes

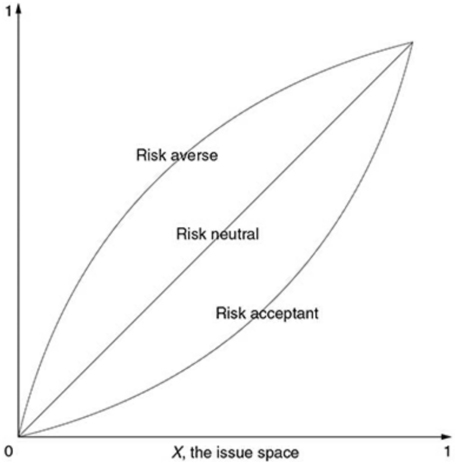

Figure 2.3 Risk attitudes

Prospect Theory

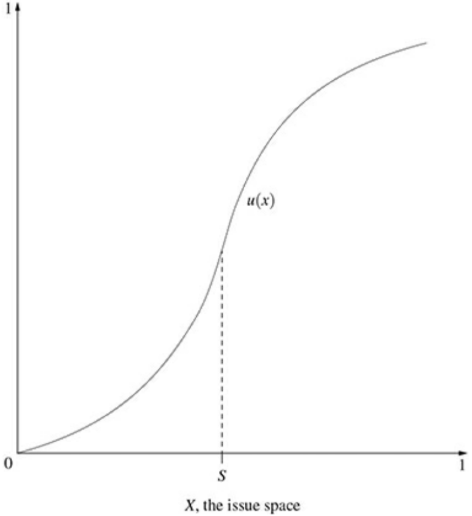

Figure 2.4 Prospect theory utility function