Conflict Problems

Instructions: Answer each part separately.

1. The president of Benin, Patrice Talon, is angry at Algeria and starts a crisis. The presidents of both countries are rational and bargain over a good which is worth 1 to each. Benin makes an offer that gives $x \in [0, 1]$ to itself and $1 - x$ to Algeria. If Algeria rejects the offer, a war is fought where Benin wins with probability p_b and Algeria wins with probability p_a with $p_a + p_b = 1$. Algeria and Benin pay a cost of going to war of c_a and c_b respectively. Assume that $c_a < p_a$ and $c_b < p_b$.

   (a) Why is it necessary that $p_a + p_b = 1$?

   (b) Which offers x is Algeria willing to accept? **Hint:** Find the value of t such that if $x \leq t$ Algeria accepts and if $x > t$ Algeria rejects and starts a war.

   (c) Given Algeria's best response, what offer is Benin going to make?

   (d) State informally what happens to the actions and payoffs of both countries if the good is now worth 2 to both instead of 1?

   (e) Generally, players who have more bargaining power are better off. Which country has higher bargaining power in this case, Algeria or Benin, and why?

2. You have traveled back in time to 16th-century Mexico. Both the Aztecs and the Tlaxcaltecs have claimed ownership of a 500-square-mile piece of land; each square mile is worth 2,000 cocoa beans (one of the many currencies used at the time). In the event of a war, the Aztecs—the most powerful of the two groups— wins the land with probability $p = \frac{3}{4}$ and the Tlaxcaltecs win with probability $1 - p$. The costs of a war, measured in cocoa beans, are 60,000 for the Aztecs and 80,000 for the Tlaxcaltecs.

   (a) Suppose that the Aztecs and the Tlaxcaltecs are willing to divide the land. You are asked to split the land in order to prevent the war. You propose a division of land (x, y), where x is the area of the land that the Aztecs get and y is the area of the land that the Tlaxcaltecs get. What is the minimum value of x so that the Aztecs prefer dividing the land over fighting? What is the minimum value of y so that the Tlaxcaltecs prefer dividing the land over fighting? What is the bargaining range?

   (b) Suppose you wanted to charge a fee for preventing what seemed to be an imminent conflict between these groups. You do this by allocating some of the land to yourself. What is the area of the largest piece of land you could keep while still preventing a war?

(c) Now suppose the Aztecs and the Tlaxcaltecs are not willing to divide the land, but are willing to trade. The Tlaxcaltecs have huge savings of cocoa beans and plan to "buy" the land from the Aztecs. How many cocoa beans should they offer if they want to buy the land from the Aztecs and, at the same time, pay the lowest price for the land?

3. Consider a political party that is made up of two independent factions who meet for the party convention. One faction, the party leadership, L, is made up of political moderates whereas the other faction, R, is made up of the radical wing of the party. The party leadership proposes a party platform, denoted by s, where $0 \leq s \leq 1$. Moderate platforms are those that are closer to 1 and radical platforms are those that are closer to 0. The radical wing can accept the proposed policy platform, s, in which case the platform is adopted. In this case, L's payoff is s and R's payoff is $1 - s$. Otherwise, the radical wing can reject the platform proposal, which leads to an intraparty battle to determine the future of the party. An intraparty battle is costly to both factions of the party. Specifically, an interparty conflict costs the party leadership c_L and the radical wing c_R. The party leadership wins an intraparty struggle with probability p, and imposes the most moderate policy, 1. With probability $1 - p$, the radical wing wins the interparty conflict and adopts the most radical platform, 0. Together, this implies that the expected payoff of an intraparty conflict for the party leadership is $p - c_L$ and the expected payoff of an intraparty conflict for the radical wing is $1 - p - c_R$.

   (a) Draw a tree to represent the strategic interaction between the party leadership and the party's radical wing.

   (b) What are the set of platform proposals the party's radical wing will accept?

   (c) Given your answer to the previous part, what is the platform the party leadership will propose? Will it be accepted by the party's radical wing?

   (d) What happens to the party's platform as the cost of an intraparty conflict for the party's leadership, c_L, increases? What about if the cost of intraparty conflict for the party's radical wing, c_R, increases?

4. Consider a canonical deterrence scenario between two countries A and B. There are three stages: a provocation stage, a retaliation stage, and an escalation stage. First, in the provocation stage, country B can choose to take a provocative action, $m = 1$, (perhaps by seizing naval vessels of country A's ally) or country B can choose to leave the status quo unchanged, $m = 0$. For simplicity, assume that if the provocative action is not pursued (i.e. $m = 0$), the game ends. Otherwise, second, in the retaliation stage, country A can decide whether to retaliate against country B, $r = 1$, issuing some form of sanctions or military operations that are harmful to country B, or country A need not respond to the provocation, $r = 0$. Third, in the escalation stage, country B can choose to escalate the conflict, $x = 1$, which is only possible when country B has already taken provocative actions in the first stage (i.e. only when $m = 1$) or can desclate the international incident, $x = 0$.

   For country A the disutility of provocation is $-\lambda_1 < 0$ and the disutility of escalation is $-\lambda_2 < 0$. Four country A, retaliation costs c and for country B retaliation costs k

to endure. There are two types of country Bs. First, with probability p, country B can be a hawk, denoted by $\theta = 1$, and with probability $1-p$, country B can be a dove, $\theta = 0$. For a hawk, the benefit of provocation is $\pi_1 > 0$ and the benefit of escalation is $\pi_2 > 0$. Doves value the status quo at 1. Retaliation also has the effect of reducing the benefit of escalated conflict by q, where $0 < q < 1$.

To summarize, the utility of country B is

$$\theta \cdot \big(m\big(\pi_1 + (1 - rq)x\pi_2\big)\big) + (1 - \theta)(1 - m - x) - rk, \tag{0.1}$$

and the utility of country A is

$$-m\big(\lambda_1 + (1 - rq)x\lambda_2\big) - rc. \tag{0.2}$$

5. There are two parts:

   (i) Suppose that $p = 0$. Show that there is a unique subgame perfect Nash equilibrium where provocation does not occur. (20pts)

   (ii) Suppose that $p = 1$.

      (a) **The Credibility Problem**: Derive the condition (in terms of c, λ_2, and q) such that retaliation is credible, meaning that country A will retaliate whenever country B provokes. Why does this not depend on λ_1?

      (b) **The Capability Problem**: Derive the condition (in terms of k, π_1, π_2, and q) such that provocation is not optimal for a hawkish country B.