

Formal Analysis: Bargaining

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1st February 2022

Roadmap

- ▶ Recap
- ▶ Introduction to extensive form games and SPNE
- ▶ Backwards induction technique
- ▶ Backwards induction in the real world?
- ▶ Legislative bargaining model

Last week

- ▶ Simultaneous move games: 2 (or more) players simultaneously choose an action
- ▶ Nash equilibrium: No player has an incentive to unilaterally deviate

“Given what other players are doing, each player is at least as well off with their strategy as with any other available to them”
— Kydd (Ch. 3, 2015)

Example: Entry game

An incumbent faces the possibility of entry by a challenger. If the challenger enters, the incumbent can choose to fight or make a truce. Incumbent prefers not to be challenged but also prefers not to fight. Challenger prefers to enter if there is a truce. Both players' worst outcome is when they fight.

Normal form representation:

| | | Incumbent | |
|------------|-----------------|--------------|--------------|
| | | <i>Fight</i> | <i>Truce</i> |
| Challenger | <i>Stay out</i> | (1, 3) | (1, 3) |
| | <i>Enter</i> | (0, 0) | (2, 2) |

Example: Entry game

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Normal form representation:

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| | | <i>Fight</i> | <i>Truce</i> |
| Challenger | <i>Stay out</i> | $(1^\diamond, 3^\diamond)$ | $(1, 3^\diamond)$ |
| | <i>Enter</i> | $(0, 0)$ | $(2^\diamond, 2^\diamond)$ |

There are 2 (pure) Nash equilibria:

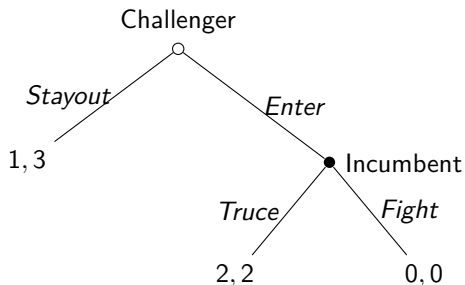
- ▶ (Enter, Truce)
- ▶ (Stay out, Fight).

This week: Extensive form games

- What is an extensive form game?

Simplest example: Players choose actions sequentially and observe each others' actions

Revisiting the entry game:



Why?

Why do we want (or need) to study extensive form games?

1) More realistic: Many important real-world interactions are sequential.

- ▶ Bills becoming law:

U.S. Congress passes bill → President decides whether to sign (presidential veto) → if President vetoes, Congress can override (with support of two thirds of both chambers).

- ▶ U.S. Senate voting:

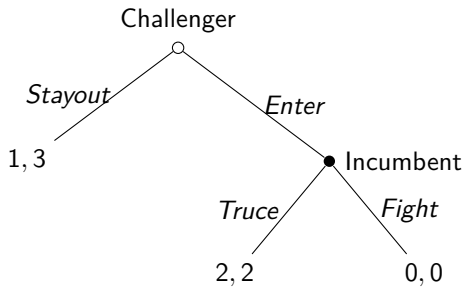
Rule XII says "*Senators shall be called alphabetically; and each Senator shall [...] declare his assent or dissent to the question [...]*"

House voting is also sequential but, in the modern era, no predetermined order.

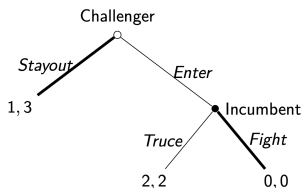
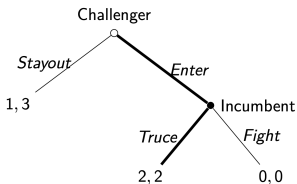
Why?

2) Misleading to study sequential interactions using a simultaneous move toolkit

Revisiting the entry game:



- Recall that there are 2 (pure) Nash equilibria: (Enter, Truce) and (Stay out, Fight).



- Is one more reasonable?

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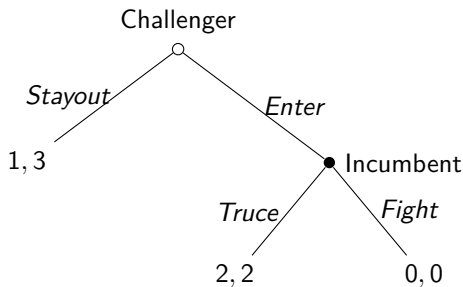
Formal details

Definition

A (complete information)¹ extensive form game consists of

- ▶ a set of players
- ▶ order of play (which players moves when)
- ▶ a set of actions available at each stage (or 'node') of the game
- ▶ for each player, preferences over complete sequence of actions.

Example:



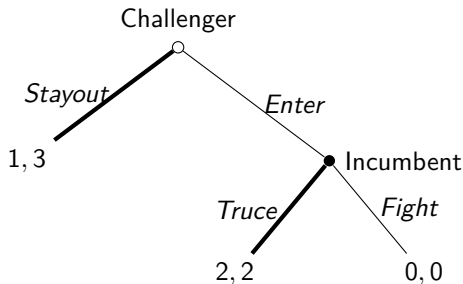
¹For a more general def. that includes uncertainty, see Kydd (Ch. 4, 2015).

Formal details

Definition

A strategy is a complete contingency plan of action: for every possible sequence of events, an action needs to be specified (regardless of whether the sequence of events is expected to happen).

Example: The Incumbent's strategy **MUST** specify their action if the Challenger were to choose "Enter" (even if we know that they won't!)



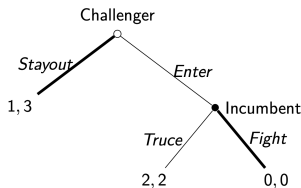
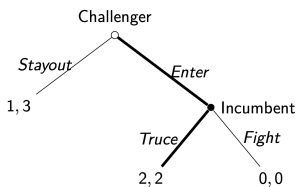
A new equilibrium concept

Informally, players should act optimally at every point of the game and everyone anticipates/knows this!

Definition (Subgame Perfect Nash equilibrium (SPNE))

A strategy profile such that players act optimally at all nodes of the game (regardless of whether these nodes are reached in equilibrium).²

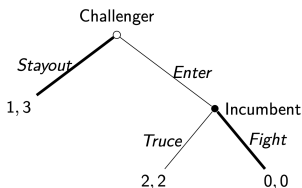
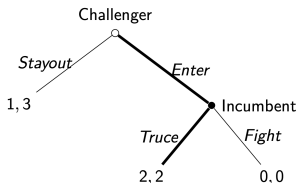
One last look at our previous example:



Is (Stay out, Fight) a SPNE?

²See, Kydd (Ch. 4, 2015) for a more general def. that can be applied to games of incomplete information.

A new equilibrium concept

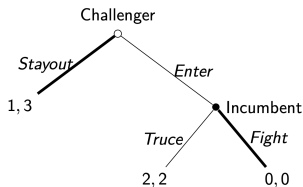
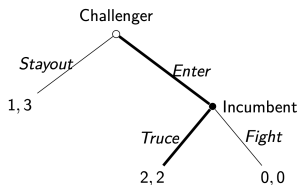


Is (Stay out, Fight) a SPNE?

Answer: No

- ▶ The incumbent's strategy *Fight* isn't credible
... it's not "*sequential rational*"
- ▶ SPNE requires "sequential rationality" even if we don't expect the node to be reached
(these 'unreached' nodes are said to be "off the equilibrium path" since, given the players' strategies, we don't expect to reach these nodes)

A new equilibrium concept



Is (Enter, Truce) a SPNE?

Answer: Yes

- ▶ at every node, players choose optimal strategies

General result. Every SPNE is a NE. But not every NE is a SPNE.

Roadmap

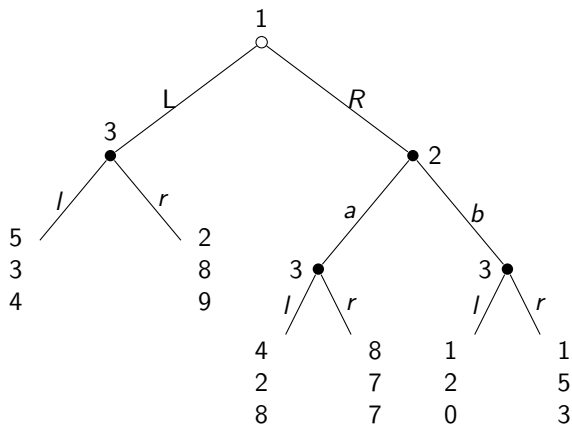
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Finding SPNE?

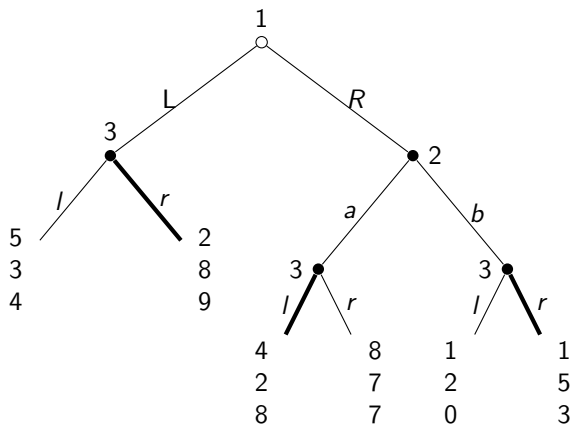
Guaranteed method for finding SPNE: Backwards induction.

- ▶ Start by determining the optimal action(s) at the final decision node
- ▶ Proceed to the next to last decision node and determine optimal actions by players who correctly anticipate optimal actions in the final decision node
- ▶ Repeat until you reach the initial node of the game

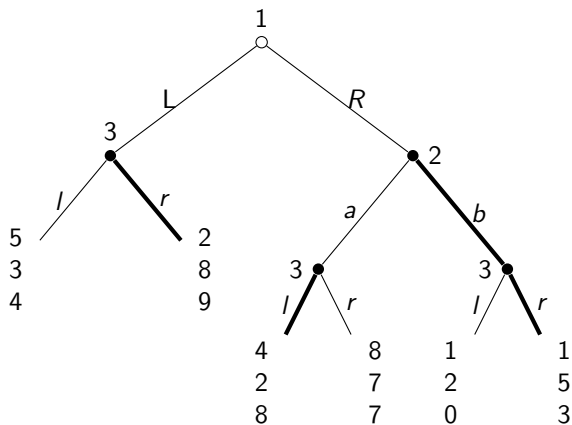
Example



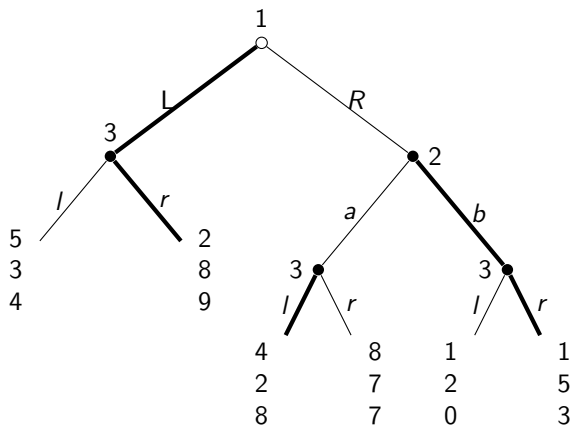
Example



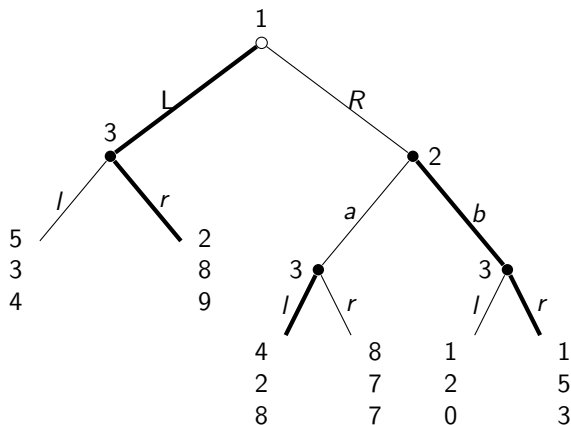
Example



Example



Example



The unique SPNE is (L, b, rlr).

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Do people backward induct in real life?

- ▶ Individuals in the lab typically fail to backward induct!

(see, e.g., McKelvey and Palfrey, 1992. "An experimental study of the Centipede game." *Econometrica*.)

- ▶ Deviations from theoretical predictions are rarer when individuals have a lot of experience

(see, e.g., List, 2003. "Does Market Experience Eliminate Market Anomalies?" *Quarterly Journal of Economics*; Palacios-Huerta, 2003. "Professionals Play Minimax." *Review of Economic Studies*.)

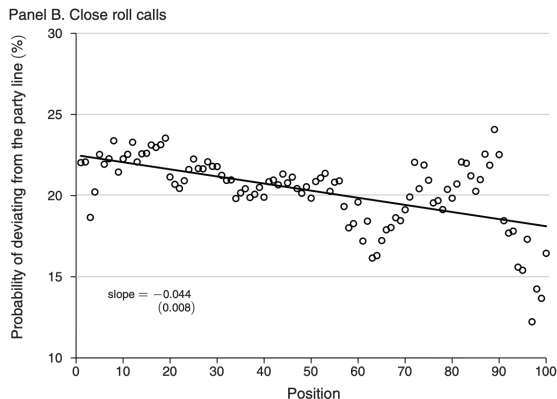
What about political settings?

Evidence of backward induction in Senate voting:

- ▶ Spenkuch, Montagnes, and Magleby, 2018. "Backward Induction in the Wild? Evidence from Sequential Voting in the US Senate." *American Economic Review*.

Spenkuch et al. (2018, AER)

- ▶ U.S. Senate, roll calls are held in alphabetical order (Rule XII)
- ▶ Observation: Senators earlier in the order are more likely deviate from the 'party line'



Can we explain this?

The authors propose a simple extensive form game

The (simplified) model:

- ▶ 3 Democrats left to vote (ordered by last name)
- ▶ Democrats need 2 more votes for a bill to pass
- ▶ All three are conflicted: they want to vote "No" but only if the bill will pass
- ▶ Payoffs: +2 (if bill passes), -1 (if vote YES).

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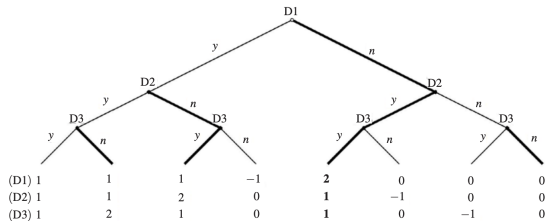


FIGURE 2. EXAMPLE OF SEQUENTIAL VOTING GAME

Prediction of the model: Senators earlier in the order are more likely to deviate from the party line!

With a slightly more complicated version of the model:

1. majority party is much more likely to barely win a roll-call vote than just lose it
2. vote-order effects only arises on roll calls that end up being close

The authors find support for all of these predictions.

Paper also includes empirical analysis and estimation:

- ▶ “about 32 percent of senators appear to explicitly account for the expected behavior of those who have not yet voted,”
- ▶ “behavior of another 17 percent is best described by a heuristic that crudely resembles the backward induction strategy.”
- ▶ “remaining senators do not engage in forward-looking, strategic play.”
- ▶ **Experience matters:** more likely to be strategic after participating in a few hundred roll calls.

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Legislative bargaining: Take-it-or-leave-it bargaining³

There are 2 parties (Party L and Party R). Bargaining over what policy to enact. Policies are represented by points along the real line.

Party L prefers policies closer to -1 and party R prefers policies closer $+1$, i.e., for a given policy outcome z , payoffs are:

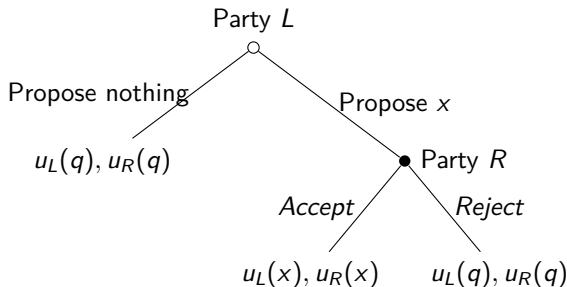
$$u_L(z) = -|-1 - z| \quad \text{and} \quad u_R(z) = -|1 - z|.$$

Actions and timing.

- ▶ L has agenda setting power and can propose any policy $x \in \mathbb{R}$ (or propose nothing)
- ▶ Given a proposal x , R can choose to accept or reject x
- ▶ If R accepts, then the policy x is enacted; otherwise, a status-quo policy q prevails.

³Attributed to Romer and Rosenthal, 1978. "Political resource allocation, controlled agendas, and the status quo" *Public Choice* and Krehbiel, 1998. "Pivotal politics."

Extensive form representation



In the SPNE, what will R accept?

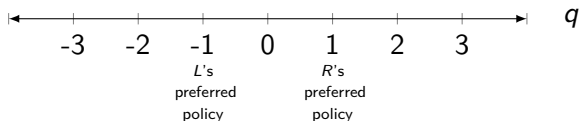
R accepts a proposal x if and only if $u_R(x) \geq u_R(q)$, i.e.,

$$-|1 - x| \geq -|1 - q|.$$

In other words, x must be closer to R 's ideal policy (+1) compared to the status-quo q .

E.g., If $q = 0$, then R will accept any proposal $x \in [0, 2]$.

What will L propose?

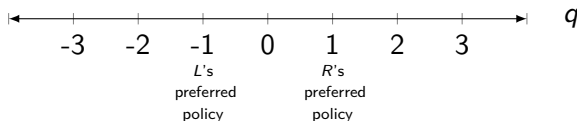


4 cases to consider:

1. $q < -1$: R will accept any $q \leq x \leq 2 - q$.

This includes L 's preferred policy; thus, L proposes $x = -1$ and $z^* = -1$

What will L propose?



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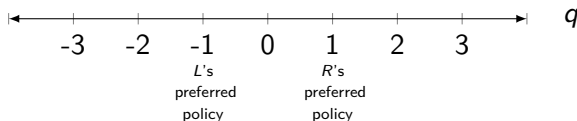
1. $q < -1$: R will accept any $q \leq x \leq 2 - q$.

This includes L 's preferred policy; thus, L proposes $x = -1$ and $z^* = -1$

2. $-1 \leq q \leq 1$: R will accept any $q \leq x \leq 2 - q$.

The best L can do is $z^* = q$ (propose any $x \notin (q, 2 - q]$).

What will L propose?



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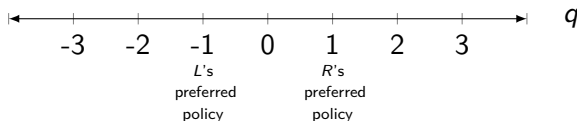
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3. $1 < q < 3$: R will accept any $2 - q \leq x \leq q$.

L 's most-preferred and acceptable proposal is $x = 2 - q$ ($z^* = 2 - q$)

What will L propose?



4 cases to consider:

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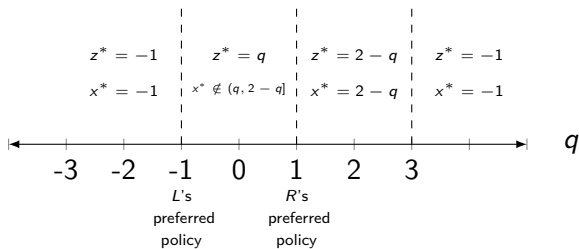
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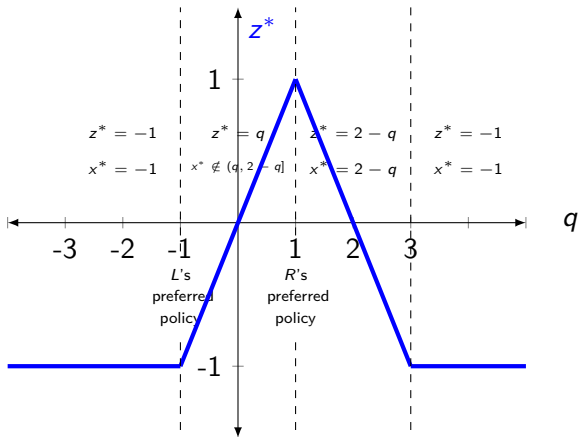
L 's most-preferred and acceptable proposal is $x = 2 - q$ ($z^* = 2 - q$)

4. $q \geq 3$: R will accept any $2 - q \leq x \leq q$.

This includes L 's preferred policy; thus, L proposes $x = -1$ ($z^* = -1$).

Equilibrium outcome

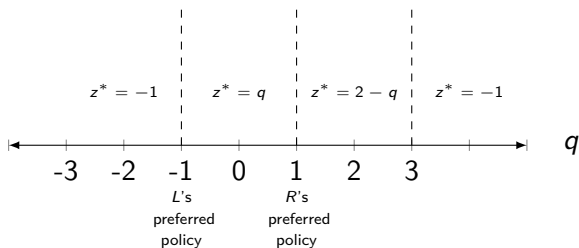




Takeaway messages

- ▶ Agenda setting power is useful but has limits
- ▶ Pareto-inefficient policy can't persist
- ▶ There is a “gridlock” interval (i.e., no policy passes)
- ▶ Unless $-1 \leq q \leq 1$, Party R never uses their veto in equilibrium—but this doesn't mean Party R is irrelevant!

Equilibrium outcome



Comparative statics: What happens as L and R 's preferred policies diverge (i.e., polarization)?

Empirical implications

All else equal, more polarization implies larger gridlock interval;
under some natural assumptions, we would expect more gridlock.

Is it true?

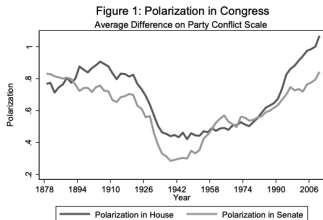


Figure: Barber and McCarty, 2015. “Causes and consequences of polarization”

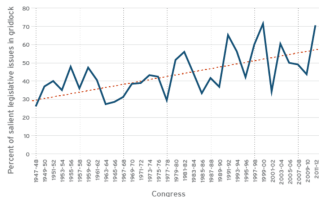


Figure: Binder, 2014. “Polarized we govern?”
Brookings Institution.

Highly debated (lots of methodological challenges⁴) :

- ▶ Mayhew's (1991) provocative book *Divided We Govern* argues that unified chambers (less polarization between chambers) does not lead to more legislative productivity.
- ▶ Binder (1999):⁵ Within-chamber polarization leads to gridlock.
- ▶ Suggestive evidence that, at the state level, within-chamber polarization doesn't cause gridlock⁶

⁴What is "gridlock"? How do we measure it? What's the denominator?
What's the numerator?

⁵"The dynamics of legislative gridlock, 1947–96" *American Political Science Review*

⁶See references within Masket (2019) "Political polarization is not a driver of gridlock at the state level" *Pacific Standard*.

Improving the model: What's missing?

Forward direction: Add extra layers of realism.

- ▶ Multiple legislators (not just 2 parties)
- ▶ 2 chambers and a president
- ▶ Supermajority rules in the Senate (filibuster)
- ▶ Coercion by party leaders
- ▶ Party loyalty

These extra layers are easy to add into the model. See, e.g.,

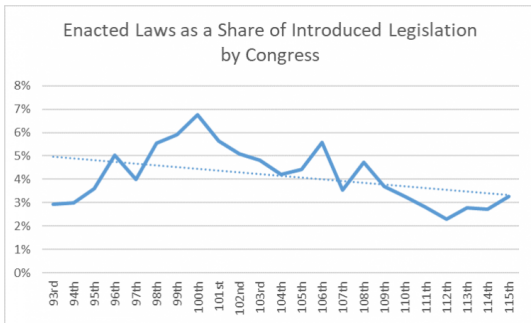
- ▶ Chiou and Rothenberg, 2003. "When pivotal politics meets partisan politics" *American Journal of Political Science*.

Backward direction: What's wrong with the model's predictions?

- ▶ how can we tweak the model to get them right?

In reality:

- ▶ Lots of gridlock! Policy proposals are frequently rejected.



In contrast, model suggests no gridlock—unless policy is in the gridlock interval.

Improving the model: What's missing?

How can we get failed proposals?

Some ideas:

- ▶ Payoff of vetoing party aren't perfectly known.
- ▶ Parties obtain extra benefits from rejecting policies (make the majority party look bad!)
- ▶ Finding the 'right' policy is hard

Further work

- ▶ Blame game politics

Groseclose and McCarty, 2001. "The politics of blame: Bargaining before an audience" *American Journal of Political Science*.

- ▶ Uncertainty about payoffs (and reputation building)

McCarty, 2003. "Presidential reputation and the veto" *Economics & Politics*.

- ▶ Dynamic bargaining and strategic polarization

Dziuda and Loeper, 2016 "Dynamic collective choice with endogenous status quo" *Journal of Political Economy*.

Survey of formal models of legislative bargaining:

Cameron and McCarty, 2004. "Models of vetoes and veto bargaining" *Annual Review of Political Science*.