

L = number of links

F = number of flows

D_f = number of multicast destinations in flow f

N = number of nodes

C is an $N \times L$ flow conservation matrix

- Link 2 is from Node N to Node 1 (+1 for destination of link; -1 for source)
- It has $2L$ non-zero entries

| | | Link Number | | | |
|-------------|-------|-------------|----|-----|-------|
| | | 0 | 2 | ... | $L-1$ |
| Node Number | 0 | | +1 | | |
| | 1 | | | | |
| | ... | | | | |
| | $N-1$ | | -1 | | |

X_f is a network coding matrix for flow f

- $D_f (N+L)$ rows
- $D_f L + L + 1$ columns
- Each C matrix handles node flow conservation for one of the destinations in the multicast flow.
- The last column is the multicast flow rate. There will be a +1 in rows where a node is a source and a -1 in rows where a node is a destination.
- The last $D_f L$ rows compute the maximum flow rate on a link over all the destinations in a flow (network coding assumption)

| | | L | L | | L | L | 1 | |
|-----|---------|-----|-----|-----|---------|-----|---|----------|
| | | 0 | 1 | ... | D_f-1 | | | |
| N | 0 | C | | | | | | =0 |
| N | 1 | | C | | | | | |
| ... | | | | ... | | | | |
| N | D_f-1 | | | | C | | | |
| L | 0 | -I | | | | I | | ≥ 0 |
| L | 1 | | | | | I | | |
| ... | ... | | | | | ... | | |
| L | D_f-1 | | | | | I | | |

Number of non-zero entries is $D_f (2L + 2)$ for equality constraints and $D_f (2L)$ for inequality constraints for a total of $2D_f (2L + 1)$.

System matrix looks like

| | | | | | | |
|-----------------|-----|--------------|--------------|-----|--------------|----------|
| | | $L(D_1+1)+1$ | $L(D_2+1)+1$ | ... | $L(D_F+1)+1$ | |
| | | 0 | 1 | ... | F-1 | |
| $D_0 (N+L)$ | 0 | X_0 | | | | |
| $D_1 (N+L)$ | 1 | | X_1 | | | |
| ... | ... | | | ... | | |
| $D_{F-1} (N+L)$ | F-1 | | | | X_{F-1} | |
| L | | LC_0 | LC_1 | ... | LC_{F-1} | $\leq C$ |

Where LC_f looks like

| | | | |
|---|-------------|---|---|
| | $D_f L + L$ | L | 1 |
| L | 0 | I | 0 |

LC_f is responsible for enforcing link capacity constraints (C) and sums over the multicast information flows (max over per destination within a multicast flow).

System matrix has

- $(D_0 + \dots + D_{F-1})(L+N) + L$ rows
- $(D_0 + \dots + D_{F-1})L + F(L+1)$ columns

Number of non-zero entries

- $2(D_0 + \dots + D_{F-1})(2L+1) + FL$

The variable vector looks like

| | |
|---|--|
| 0 to L-1 | Link flows for destination 0 in flow 0 |
| ... | |
| d L + (0 to L-1) | Link flows for destination d in flow 0 |
| ... | |
| (D ₀ -1) L + (0 to L-1) | Link flows for destination D ₀ (last one) in flow 0 |
| D ₀ L + (0 to L-1) | Max link flows over all destinations in flow 0 |
| D ₀ L + L | Rate for flow 0 |
| ... | |
| (D ₀ +...+D _{f-1}) L + f (L + 1) + d L + (0 to L-1) | Link flows for destination d in flow f |
| ... | |
| (D ₀ +...+D _f) L + f (L + 1) + (0 to L-1) | Max link flows over all destinations in flow f |
| (D ₀ +...+D _f) L + f (L+1) + L | Rate for flow f |
| ... | |
| (D ₀ +...+D _{F-2}) L + (F-1) (L + 1) + d L + (0 to L-1) | Link flows for destination d in flow F-1 (last flow) |
| ... | |
| (D ₀ +...+D _{F-1}) L + (F-1) (L + 1) + (0 to L-1) | Max link flows over all destinations in flow F-1 (last flow) |
| (D ₀ +...+D _{F-1}) L + (F-1) (L+1) + L | Rate for flow F-1 |

Row mapping

Flow conservation equation for node n, destination d in flow f

$$(D_0 + \dots + D_{f-1})(N + L) + dN + n \quad (\text{if } f = 0, \text{ then } dN + n)$$

Flow on link l for destination d in flow f is less than max flow on link l for multicast flow f

$$(D_0 + \dots + D_{f-1})(N + L) + D_f N + dL + l$$

Sum of multicast flows on link l are less than link capacity

$$(D_0 + \dots + D_f)(N + L) + l$$

Implementing Constraints

To test for feasibility of a set of flows, we may want to require each multicast flow rate be between some lower and upper bound. Alternatively, we may want to optimize network utility while supporting a pre-specified set of flows at a given rate. We can implement these alternatives as follows. We can impose lower and upper bounds on multicast flow rates by putting appropriate upper and lower limits on flow rate variables. E.g., if flow f must be between R_{lower} and R_{upper} , then bound variable $(D_0 + \dots + D_f) L + f(L+1) + L$

If we want to require that certain flows only use low latency links, we can require that they NOT use high latency links. This can be done as follows. We can impose requirement that a flow not use a subset of links by imposing an upper bound of zero on the max link flow rate (i.e., the maximum of the flows to different destinations in flow f) for the prohibited links. E.g., if flow f cannot use link l , then we impose an upper bound of zero on variable $(D_0 + \dots + D_f) L + f(L+1) + l$.