Noname manuscript No.
(will be inserted by the editor)

Multifractal characterization and classification of bread crumb digital images

Rodrigo Baravalle · Claudio Delrieux · Juan Carlos Gómez

Received: date / Accepted: date

Abstract An adequate model of bread crumb structure can be critical for understanding flow and transport processes in bread, creating synthetic bread crumb images for photo-realistic rendering, and evaluating similarity of crumbs of different breads.

In this article multifractal analysis employing the multifractal spectrum (MFS) has been used to study the structure of the bread crumb for four varieties of breads (*baguette*, *lactal*, *bran*, and *sandwich*). The extracted dimensions could be used to discriminate among bread crumbs from different types. Also, high correlations were found between some of these parameters and the porosity, coarseness, and heterogeneity of the samples. These results demonstrate that the MFS is an appropriate tool for characterizing the internal structure of the bread crumb and thus may be used to establish important quality properties that it should have.

The MFS has shown to provide local and global image features that are both robust and low-dimensional. In this work we also apply the MFS for bread crumb classification based on color scans of slices of different bread types. Results show that MFS based classification is able to distinguish different bread crumbs with very high accuracy. The multifractal modeling of the bread crumb structure can be an efficient method for parameterizing and simulating the appearance of different bread crumbs.

Keywords Fractal · Multifractal · Image analysis · Image classification · Bread crumb

Rodrigo Baravalle and Juan Carlos Gómez
Laboratorio de Sistemas Dinámicos y Procesamiento de Información
FCEIA, Universidad Nacional de Rosario, - CIFASIS - CONICET
Riobamba 250 bis, 2000, Rosario, Argentina.
Tel.: +54-341-4237248 int. 301
Fax: +54-341-4821772 int. 3
E-mail: baravalle@cifasis-conicet.gov.ar

Claudio Delrieux
DIEC, Universidad Nacional del Sur - IIIE-CONICET
Avenida Colón 80 - Bahía Blanca (8000FTN)
Provincia de Buenos Aires - República Argentina

1 Introduction

Among other important factors, the quality of a bread loaf is related to its crumb structure. Close examination of different slices reveals considerable variation in the cell size even within a single sample of the same bread type.

Fractal and multifractal analysis of images have proved to capture useful properties of the underlying material being represented. Characterization of images using these features have been successfully applied in different areas, such as medicine ([1,2]) and texture classification ([3]). Through several procedures, it is possible to obtain different Fractal Dimensions (FD), each of them capturing a different property of the material (*e.g.*, porosity, rugosity).

For each material, the results obtained in the classification process and in the characterization analysis are useful in quality measurements of real samples and also in the validation of synthetic representations of them. In other words, these processes are useful to determine if a given image presents the observed features in that material, allowing to associate quality measure parameters to the material. In [4], a quality bread crumb test based on Gabor filters was performed in that paper, obtaining good results. Nevertheless, a small database was used (30 images). In [5] several fractal features were obtained for one type of bread, showing that a vector comprising them would be capable of obtaining key features of its crumb texture.

In this work we propose the application of the MFS [6] to describe and discriminate among different bread types. One of the main features of the MFS is its bi-Lipschitz invariance [?], which includes perspective transforms (viewpoint changes) and smooth texture surface deformations. It is shown that the MFS is also locally invariant to affine changes in illumination.

The proposed method is compared to other classifiers that uses state-of-the-art features for texture classification. The results of this feature extraction procedure show that the classifier is robust and presents good discrimination properties to distinguish between different types of bread and also non bread images. The objectives of this study were: (1) to evaluate if the MFS can be applied to characterize and discriminate the bread crumb structure from different bread types from digital images, and (2) to investigate the efectiveness of the method in the classification of these structures.

In section 2 we briefly introduce the theory underlying fractal sets. In section 3 we describe the materials and methods employed in the classification. In section 4 we show the results obtained in the characterization and classification and the results are discussed. In section 5 the conclusions are summarized, and some possible future works are posed.

2 Materials and Methods

2.1 Fractals and Multifractals

2.1.1 Box dimension

Box FD is a simplification of the Hausdorff (originally Minkowski - Bouligand) dimension for non strictly self-similar objects ([7]). Given a binarised image, it is subdivided in a grid of size $M \times M$ where the side of each box formed is ϵ. If N_ϵ represents the amount of boxes that contains at least one pixel in the binarisation of the set for that ϵ, then the box dimension D_b is defined as

$$D_b \triangleq \lim_{\epsilon \to 0} \frac{\log(N_\epsilon)}{\log(1/\epsilon)}. \tag{1}$$

The algorithm computes a binarised image from the original one and then selects different values of ϵ in it, making a count of the boxes that contains pixels in each case (to avoid numerical instabilities, a mean of cases is computed, establishing different positions in the grid over the image). Finally, a linear regression adjustment is made with the obtained data, in the $\log - \log$ space, and the slope of the straight line is by definition the box dimension of the image.

2.2 Multifractal analysis

Some elements in nature show fractal features or auto similarity. The fractal dimension is an exponent which relates the statistical auto similarity of the object at different scales. On the one hand, deterministic fractals are characterized by the same FD at all scales. They are called *monofractals* (for instance, Koch Curve, Sierpinsky triangle). On the other hand, *multifractals* ([8]) are characterized by a set of FDs depending on the scale. It is assumed that these structures are composed by different fractals coexisting simultaneously.

2.2.1 Hölder exponent

Informally, the way to proceed with multifractal analysis is to examine, in the limit, the local behavior of a measure μ at each point of the set under study. This means, to find the Hölder exponent α in that point. The *multifractal spectrum* $f(\alpha)$ is obtained applying this procedure to the entire set, in this case, an image.

Let E be an structure divided in disjoint substructures E_i of size ϵ in such a way that

$$\bigcup_i E_i = E. \tag{2}$$

Each substructure E_i is characterized by a measure $\mu(E_i)$. From the point of view of multifractal analysis, it is useful to define this value as a function of ϵ, *i.e.*

$$\alpha_i = \frac{ln(\mu(E_i))}{ln(\epsilon)}, \tag{3}$$

and to take the limit when ϵ tends to 0. The limit represents the value of the Hölder exponent at a point in the structure, that is

$$\alpha = \lim_{\epsilon \to 0} \alpha_i. \tag{4}$$

The exponent characterizes the local regularity of the structure at a point. To obtain a global characterization of its regularity it is necessary to obtain the distribution of α in E. For this, a counting N_ϵ must be done for each α_i, related to the value of ϵ, *i.e.*

$$f_\epsilon(\alpha_i) = -\frac{ln(N_\epsilon(\alpha_i))}{ln(\epsilon)}. \tag{5}$$

When ϵ tends to 0, the limiting value is the FD of the structure E characterized by α, the Hausdorff dimension of the α distribution, also known as the *multifractal spectrum* $f(\alpha)$ ([9]), *i.e.*

$$f(\alpha) = \lim_{\epsilon \to 0} f_\epsilon(\alpha). \tag{6}$$

2.2.2 Procedure for the MFS

As illustrated in ([6]), the domain is partitioned into non-overlapping boxes of length r. The q-th moment of a measure μ is defined as

$$M_r(q) = \sum \mu(B(x,r))^q, \tag{7}$$

where the sum is over the r mesh squares for which $\mu(B(x,r)) > 0$. To denote the power law behavior of $M_r(q)$, $\beta(q)$ is defined as a straight line fit of the values $M_r(q)$ with respect to r, for r in $1,..,n$. It is shown in ([10]), that the MFS and $\beta(q)$ are related to each other by a Legendre transform as

$$f(\alpha(q)) = q\alpha(q) - \beta(q), \tag{8}$$

where

$$\alpha(q) = \frac{d\beta(q)}{dq}. \tag{9}$$

Using equations 6 and 7 the MFS is estimated. In this paper, the author's implementation, with the default parameters (except for the number of FDs) is used.

2.2.3 Multifractal Measures

Defining different μ functions led to different image features. The first approach is to define μ in the intensity domain, *i.e.*

$$\mu(B(x,r)) = \int_{B(x,r)} (G_r * I)dx, \tag{10}$$

where $*$ is the $2D$ convolution operator and G_r is a Gaussian smoothing kernel with variance r, *i.e.,* μ is the average intensity value in the disk of radius r centered at x ($B(x,r)$). This is the density of the intensity function, and it describes how the intensity at a point changes over scale.

The definition of μ could serve to specific purposes. For instance, if robustness to illumination changes is needed, one choice is to define $\mu(B(x,r))$ on the energy of the gradients. Let $f_k, k = 1,2,3,4$ be four directional differential operators along the vertical, horizontal, diagonal and anti-diagonal directions. Then we define the measurement function $\mu(B(x,r))$ for the image I as in Equation 11.

$$\mu(B(x,r)) = \left(\int_{B(x,r)} \sum_k (f_k * (G_r * I))^2 dx\right)^{1/2}. \tag{11}$$

Another choice is to define $(B(x,r))$ as the sum of the Laplacians of the image inside $B(x,r)$ (12).

$$\mu(B(x,r)) = \int_{B(x,r)} |\nabla^2(G_r * I)|dx. \tag{12}$$

2.3 Image acquisition

20 images of four different bread types (*lactal*, *baguette*, *bran* and *sandwich*), counting 80 images, were obtained using an electric slicer. The images were digitalized using an HP PSC 1210 scanner and they were saved in TIFF format. Images showed a resolution of 380×380 pixels (the maximum possible area for the four bread types) and 350 dpi (1 pixel = $0.00527mm^2$). Then the images were converted to gray scale (8 bits). In addition, 20 images of each bread type were acquired with a digital camera, using the same spatial resolution, counting 80 images. The illumination conditions of these images were different from that of the scanner in order to test for the robustness of the method. In Fig. 3 four examples of bread images from the camera are shown. We also employed one hundred randomly selected images from the CalTech101 ([11]) dataset in order to test the method's performance with non-bread images. In Fig. 2 four examples of non-bread images from this dataset are shown.

The void fraction (VF), mean cell area (MCA) and standard deviation of mean cell area (stCA), were calculated in order to study the relationship of the MFS with the porosity, coarseness and heterogeneity of the bread crumbs. For this purpose, the image should be binarized. The algorithm presented in [12] was used. This algorithm applies a local thresholding schema, which showed better results than using a global thresholding schema. Particularly, the algorithm presented in [13] and used in [5], showed poor results when the illumination conditions vary in the image. In Fig. 1 an image of each bread type used in this work (top row) and its resulting binarisation using the proposed algorithm (bottom row) is shown. Small elements of one and two pixels were eliminated by an opening operation (erosion and dilation) using a 2×2 structuring element. The method showed good results even for different illumination conditions varying in the image.

In order to determine the result of the binarization at a pixel, the algorithm obtains some average from the gray levels in a window surrounding the pixel, and compares it to a threshold determined by the actual gray level of the pixel and a bias which is multiplied by this value. Two parameters must be set in the algorithm: the size of the window and the bias. It was found that different values are needed for better results when different capturing methods are used. The values for the scanner samples were 80 for the window and 1.15 for the bias. In the case of digital camera samples, the values found were 100 and 1 for the bias. These differences appear due to the different illumination conditions present in the images resulting from these different capturing

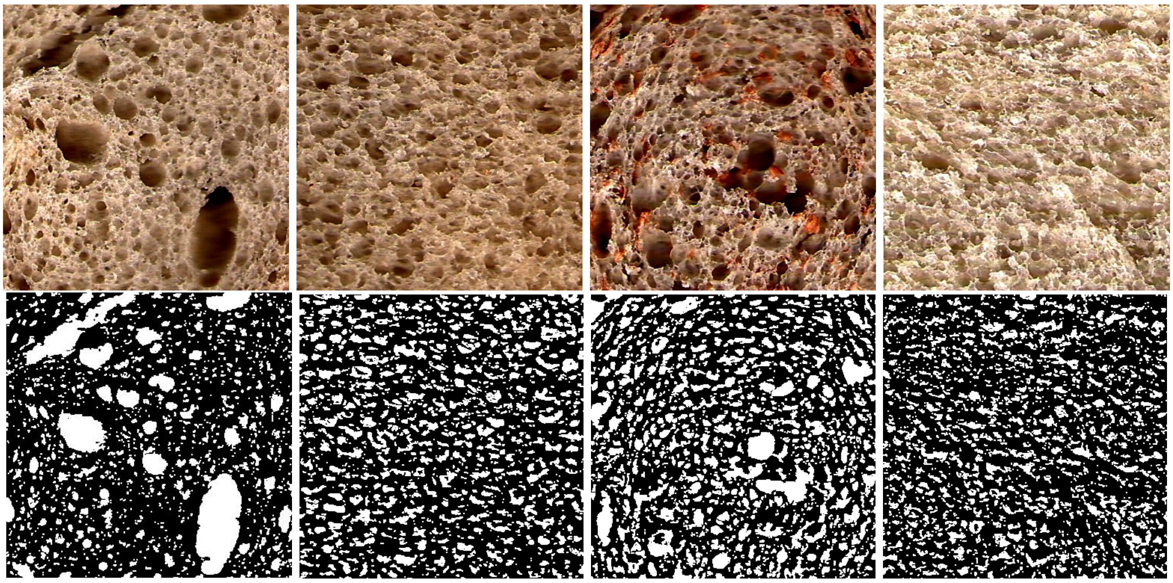

Fig. 1 Digitalized images of *baguette*, *lactal*, *bran* and *sandwich* bread types with its binarisations

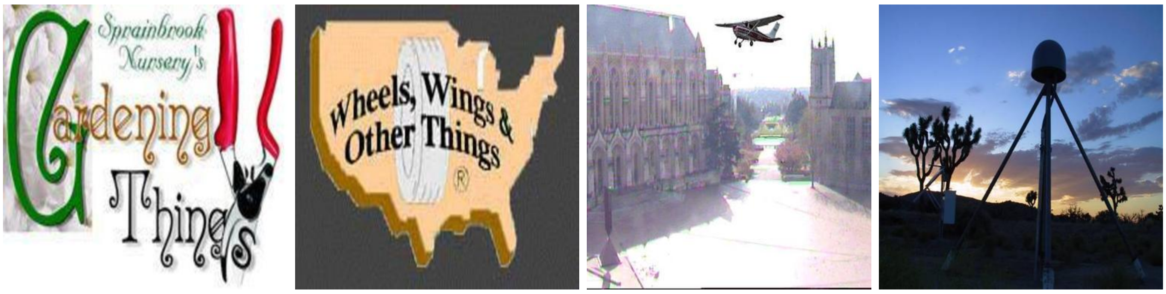

Fig. 2 Images from the dataset CalTech101

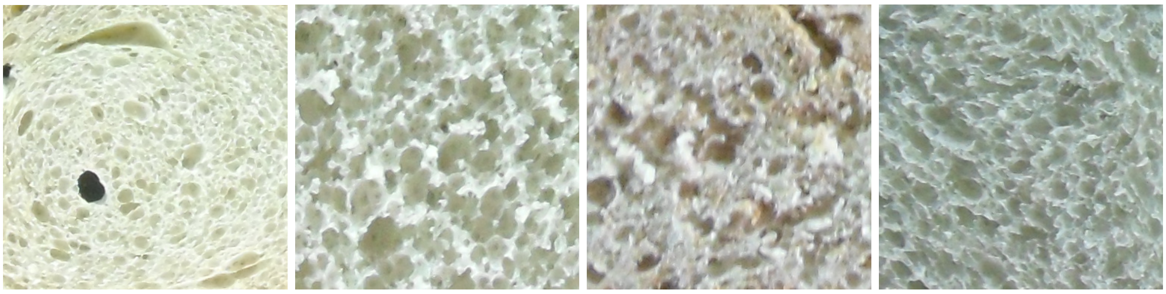

Fig. 3 Digitalized images from a digital camera

conditions. Further research is required in order to determine automatic values for these parameters.

3 Results and discussion

3.1 Bread Classification

In order to test the discriminative capability of the method, a classification experiment is made. Five classes are defined, *e.g.*, *b*aguette, *l*actal, *b*ran, *s*andwich and *n*on-bread, assigning 40 images to each class. A comparison is made between the MFS and state of the art features in the computer vision literature. It is worth to note that this intra class problem is harder to solve than that of an inter-class one.

K-fold cross validation is applied to the entire set (with $K = 4$), employing three different classifiers: Support Vector Machines (SVM), Random Forests (RF), and Nearest Neighbors (NN). Results show that the MFS show good performance regardless of the classifier employed. The libsvm implementation [14] was used for the SVM classifier. In the case of the RF (100 trees) and the NN (1 neighbor) classifiers, the scikit-learn python

Table 1 classification results with different number of FDs for the MFS

Number of FDs	10	20	30
SVM	**96**	94.5	95.5
RF	91.5	**93.5**	93
NN	88.5	**90.5**	90

Table 2 classification results using different combinations of the MFS

Method	MFS	MFS+L	MFS+G	CIELab
SVM	94.5	95.5	97.5	**97.5**
RF	93.5	**96**	95	**96**
NN	90.5	90	87	**92**

library was employed. In Table 1, classification accuracy of the method is tested using different number of FDs. Using the MFS with 20 FDs shows good performance, so this number is used in the following calculations. Since the goal of this work is to show the impact of the features, the Nearest Neighbor result is considered the most important among the three classifiers, since it is the simplest of them, showing the effectiveness of the features rather than that of the classifier.

In Table 2, several combinations of the MFS and their performance is shown. Particularly, results when combining the MFS of the density of the intensity with: the Laplacian of the intensity, and the gradient of the intensity. Another test is made, using the CIELab color space. The key advantage of this color space is that it tends to reduce the dependency of the resulting image color on the dispositive used in the capture. The intensity of the images is transformed to the CIELab space, and the MFS of the three separated channels are combined together, forming a vector of 60 FDs. This combination showed the best performance for the three classifiers. It means that adding color information in the a and b channels is useful for better characterization and classification of different types of bread crumbs, when different capturing dispositives are used (in this case, a scanner and a digital camera).

In Table 3, other state-of-the-art features: Haralick features, Local Binary Pattern (LBP) and SIFT features are computed for the images. The best performance is obtained using the SIFT features, but it requires 128 features and it has to build a dictionary to work. In any case, the performance of the MFS overpass them, showing that the MFS has better performance than other state-of-the-art features for the bread crumb database.

Table 3 classification results for different features

Method	Haralick	Lbp	SIFT
SVM	94	78.5	**96.5**
RF	91	71.5	**92**
NN	79	70	**86**

3.2 Data Analysis

In Figure 4, the mean values for the MFS, using 20 FDs, for the four different bread types are shown. The image shows that the MFS could potentially characterize and classify the different bread crumb types. The standard deviations for the four different bread types are shown in Table **??**.

The correlation coefficients for the four bread types with the void fraction, mean cell area and standard deviation of mean cell area (in mm^2) are shown in Figures 5, 6 and 7, respectively. It is clear that the coefficients behave similarly for the first dimensions in all the bread types, but differently for the FD 9 and above. It could be concluded that the first dimensions are highly (negatively) correlated with the void fraction (porosity), mean cell area (coarseness) and the standard deviation of mean cell area (heterogeneity) of the bread types. It means that the first FDs increase when the mentioned features decrease. Other FDs also have a high (positive or negative) correlation, but it depends on the bread type which dimension is correlated. From the graphs of the correlation coefficients of the MCA and stCA, it could also be pointed out that the MCA has a higher correlation than the stCA with the FDs of the MFS. It means that the coarseness of the bread crumb structure could be better determined by the features than its heterogeneity, using the MFS.

4 Conclusions

The visual appearance of different types of bread crumbs can be successfully characterized by the fractal dimensions of its digital image. The first fractal dimensions obtained from the MFS method method negatively measured bread crumb coarseness, porosity and coarseness.

The use of multifractal features in bread crumb texture classification showed good performance. The MFS demonstrated to be accurate enough to perform a classification of different bread types and also to discriminate non bread from bread images. The use of the MFS of the a and b channels of the CIElab space color provided the method with classification capabilities and obtained the best classification performance. The MFS is comparable with other state-of-the-art features in texture

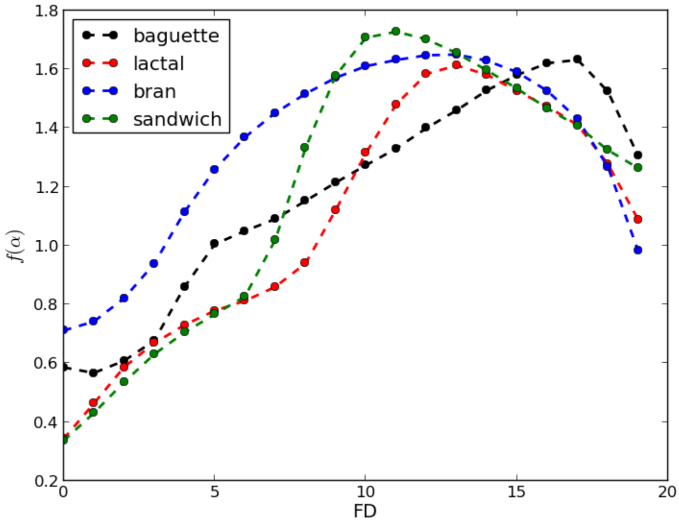

Fig. 4 Mean MFS for the four different bread types

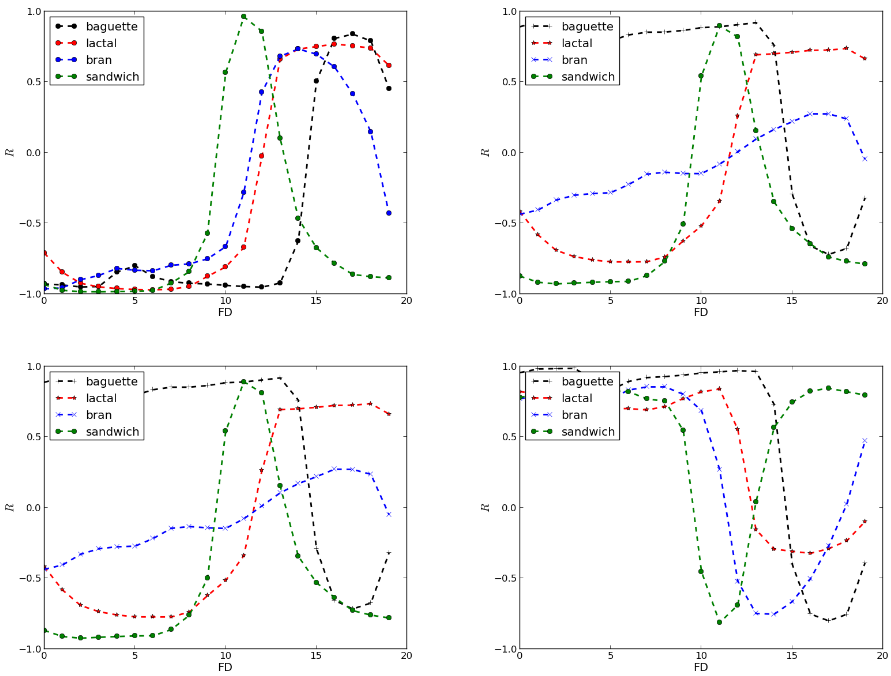

Fig. 5 Correlation coefficients for the FDs and the void fraction of the samples

classification like SIFT features, and in the case of the bread crumb database, it overpass their classification performance.

The results found can be applied to validate synthetic samples, *i.e.*, the latter should have similar features to the bread type that is trying to simulate. The features obtained will be used to determine particle system parameters (*e.g.*, lifetime of particles, color). These results can be extended to be used as quality parameters for these products.

Acknowledgements We would like to thank Gustavo Grieco and Pablo Speciale for technical discussions and for their support in the development of the present work. We would also like to thank Dra. Ursula Gonzales-Barron for the received images.

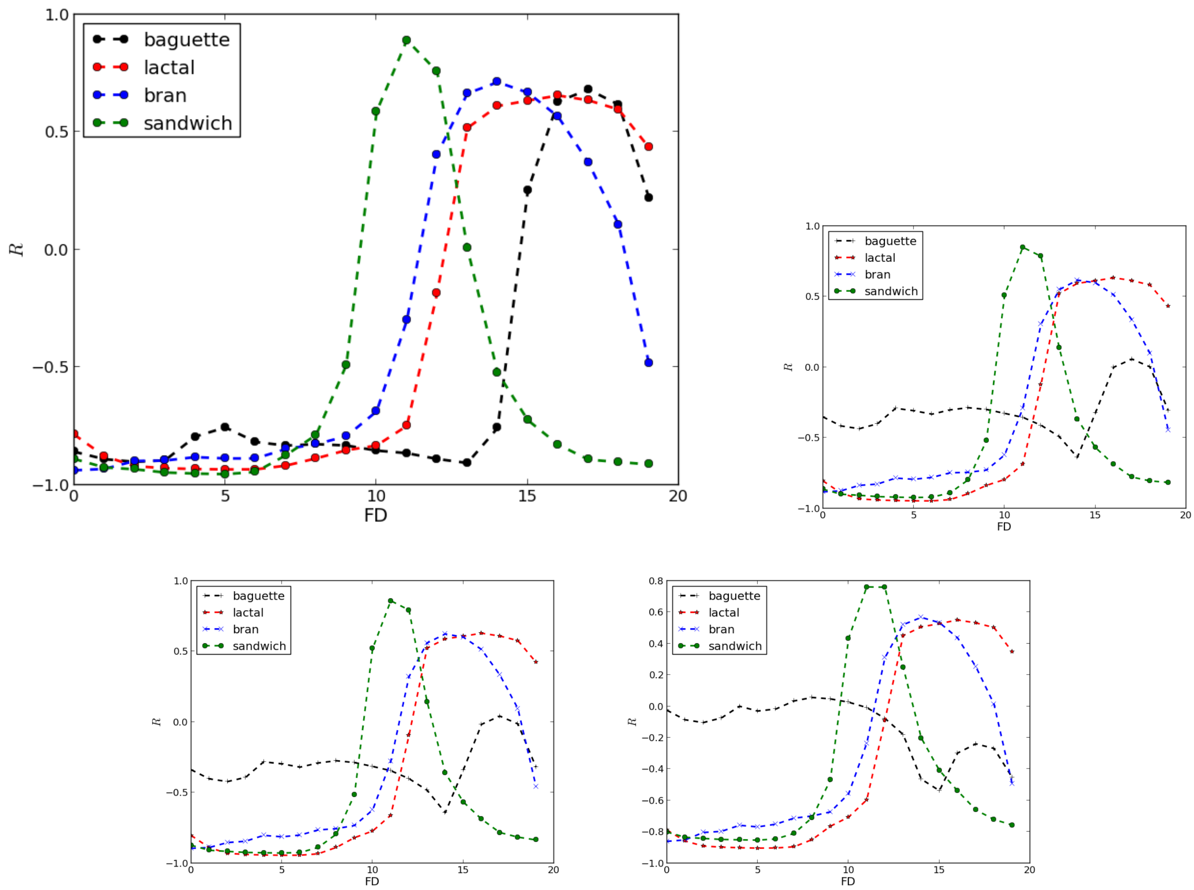

Fig. 6 Correlation coefficients for the FDs and the mean cell area (in mm^2) of the samples

References

1. J. Andjelkovic, N. Zivic, B. Reljin, V. Celebic, I. Salom, in *Proceedings of the 8th conference on Signal, Speech and image processing* (World Scientific and Engineering Academy and Society (WSEAS), Stevens Point, Wisconsin, USA, 2008), SSIP'08, pp. 88–92
2. L. Yu, D. Qi, in *Control and Decision Conference (CCDC), 2011 Chinese* (2011), pp. 2040 –2045
3. H. Wendt, P. Abry, S. Jaffard, H. Ji, Z. Shen, in *16th IEEE International Conference Image Processing (ICIP)* (2009), pp. 3829 –3832
4. Y. Fan, H. Zhang, in *Mechatronics and Automation, Proceedings of the 2006 IEEE International Conference on* (2006), pp. 1498 –1502
5. U. Gonzales-Barron, F. Butler, European Food Research and Technology **226**, 721 (2008). 10.1007/s00217-007-0582-3
6. Y. Xu, H. Ji, C. Fermuller, in *Computer Vision and Pattern Recognition, 2006 IEEE Computer Society Conference on*, vol. 2. vol. 2, pp. 1932–1939
7. H.O. Peitgen, H. Jürgens, D. Saupe, (2004)
8. B.B. Mandelbrot, Pure and Applied Geophysics **131**, 5 (1989)
9. S. A., D. C., (2010), Actas de la trigésimo novena Jornada Argentina de Informática e Investigación Operativa (JAIIO), pp. 1575 – 1581
10. K.J. Falconer, *Techniques in Fractal Geometry* (John Wiley, 1997)
11. L. Fei-Fei, R. Fergus, P. Perona, in *Proceedings of the Workshop on Generative-Model Based Vision* (Washington, DC, 2004)
12. J.M. White, G.D. Rohrer, IBM J. Res. Dev. **27**(4), 400 (1983)
13. L.K. Huang, M.J.J. Wang, Pattern Recognition **28**(1), 41 (1995)
14. C.C. Chang, C.J. Lin, ACM Transactions on Intelligent Systems and Technology **2**, 27:1 (2011)

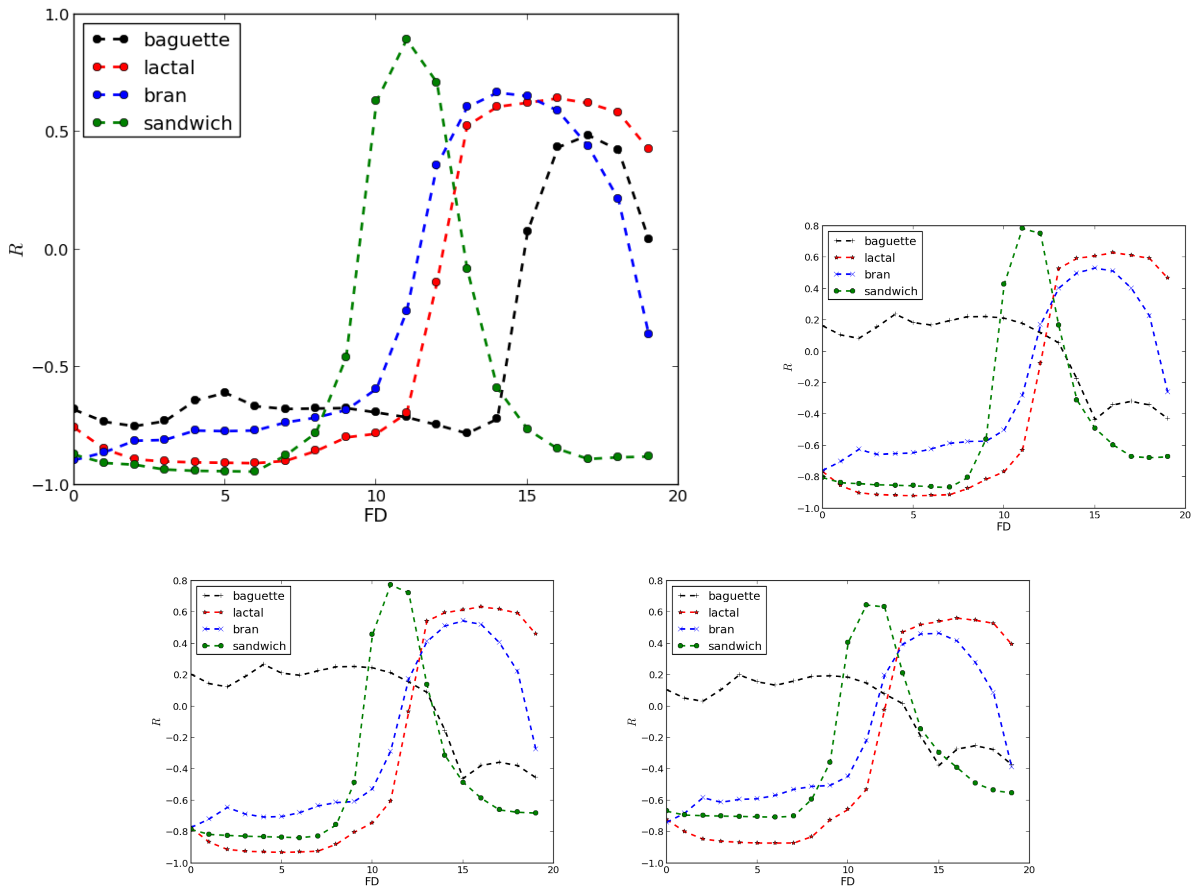

Fig. 7 Correlation coefficients for the FDs and the standard deviation of the mean cell area (in mm^2) of the samples