BML lecture #4: Bayesian Nonparametrics

http://github.com/rbardenet/bml-course

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Outline

What comes to your mind when you hear "Bayesian Nonparametrics"?

KDE : Nonparam but not Bayes.

Dirichlet process

Parametric versus nonparametric

Parametric models

- Finite and fixed number of parameters
- Number of parameters is independent of the dataset

Nonparametric models

- Do have parameters
- Can be understood as having an infinite number of parameters
- Can be understood as having a random number of parameters
- Number of parameters can grow with the dataset

Parametric versus nonparametric

Complexity of the model $\{P_\theta : \theta \in \Theta\}$:

Models:	**Parametric**	**Nonparametric**
Dimension:	Finite dimensional Θ.	Infinite dimensional Θ.
Advantages:	Easier to handle and make interpretations of the results. Computationally faster.	Less chance for misspecifications. More flexible.
Disadvantages:	Without strong belief in the particular structure of the model not reliable.	Computationally and analytically challenging.
Examples:	Poisson (number of car crashes, typos in a book). Normal distribution (grades of students, height, weight, footsize of people).	Density, regression function estimation. Clustering (unknown cluster size and number).

Different flavours in Bayesian Nonparametrics

Two categories of priors depending on parameter spaces

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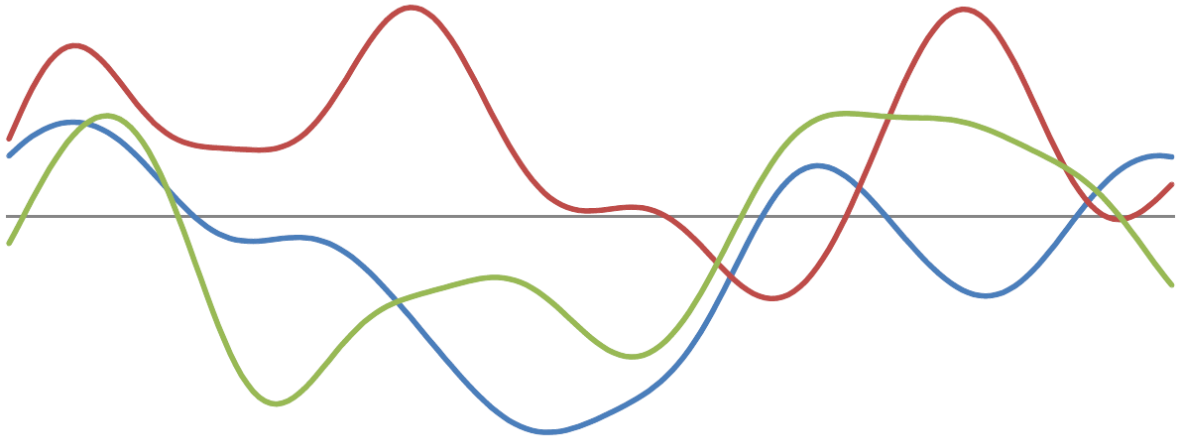
Spaces of functions
random functions

- Continuous stochastic processes (eg GP)
- Random basis expansions
- Random densities



[Wikipedia]

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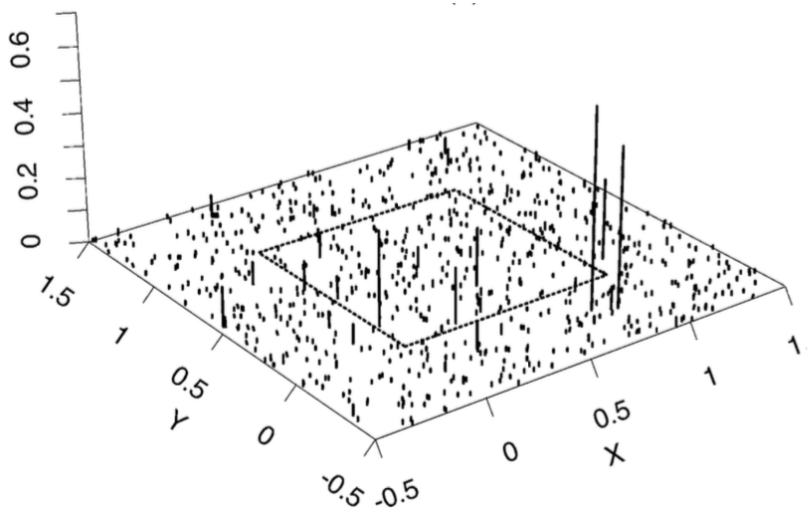
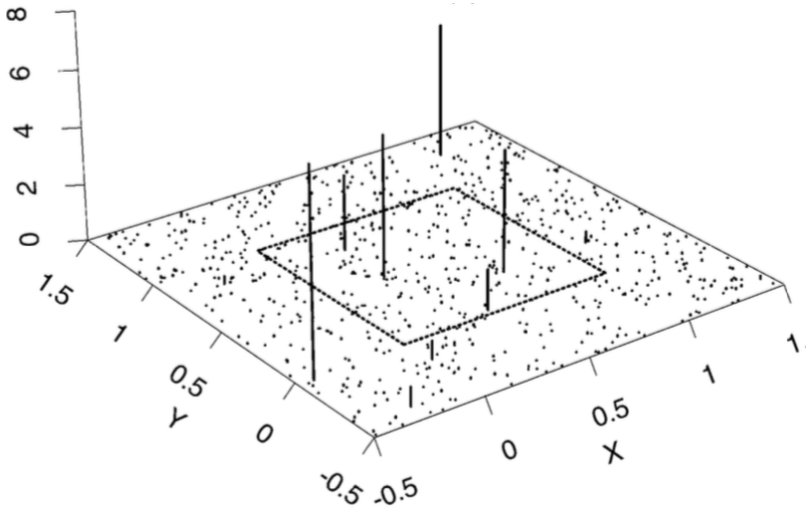
random probability measures (RPM)

- Often discrete proba. measures
  Cornerstone: Dirichlet process
  Also: Pitman–Yor, Gibbs-type priors, etc





(Brix, 1999)

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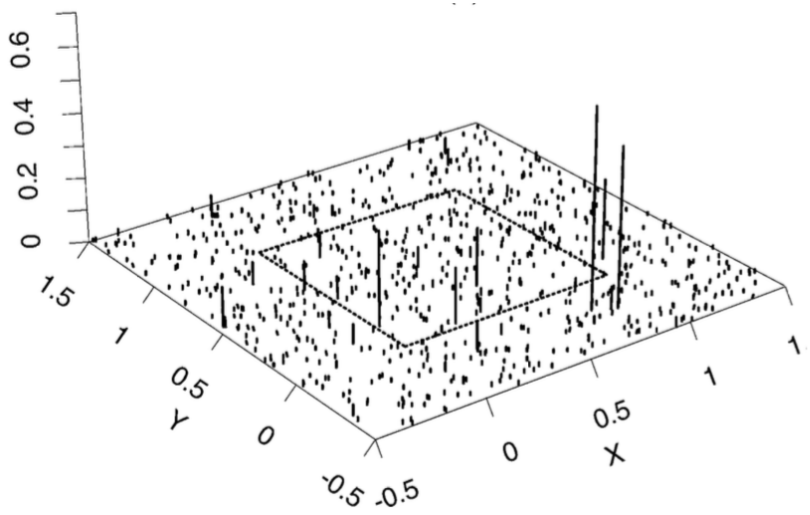

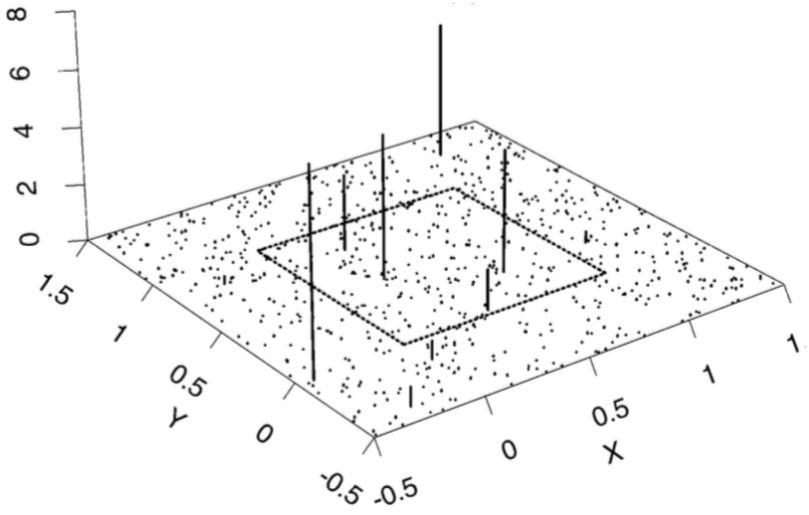

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1. Gaussian Processes (GPs)
2. Dirichlet process (DP) and other Random Probability Measures (RPMs)
3. Frequentist properties of Bayesian Nonparametric (BNP) models

What this chapter is not about:

- Not much about Bayesian Nonparametric Data Analysis – see Müller et al., 2015
- PAC-Bayes analysis – see Alquier, 2021

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References

- Asymptotics: J. K. Ghosh and R. V. Ramamoorthi. *Bayesian Nonparametrics*. New York: Springer, 2003
- RPMs: Nils Lid Hjort et al. *Bayesian nonparametrics*. Vol. 28. Cambridge University Press, Apr. 2010. URL: http://www.cambridge.org/us/academic/subjects/statistics-probability/statistical-theory-and-methods/bayesian-nonparametrics
- Asymptotics & RPMs: Subhashis Ghosal and Aad Van der Vaart. *Fundamentals of nonparametric Bayesian inference*. Vol. 44. Cambridge University Press, 2017
- GPs: C. E. Rasmussen and C. K. I. Williams. *Gaussian Processes for Machine Learning*. MIT Press, 2006

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[1] Pierre Alquier. *User-friendly introduction to PAC-Bayes bounds*. 2021. arXiv: 2110.11216 [stat.ML].

[2] Anders Brix. "Generalized gamma measures and shot-noise Cox processes". In: *Advances in Applied Probability* (1999), pp. 929–953.

[3] Subhashis Ghosal and Aad Van der Vaart. *Fundamentals of nonparametric Bayesian inference*. Vol. 44. Cambridge University Press, 2017.

[4] J. K. Ghosh and R. V. Ramamoorthi. *Bayesian Nonparametrics*. New York: Springer, 2003.

[5] Nils Lid Hjort, Chris Holmes, Peter Müller, and Stephen G Walker. *Bayesian nonparametrics*. Vol. 28. Cambridge University Press, Apr. 2010. URL: http://www.cambridge.org/us/academic/subjects/statistics-probability/statistical-theory-and-methods/bayesian-nonparametrics.

References II

[6] Peter Müller, Fernando Andrés Quintana, Alejandro Jara, and Tim Hanson. *Bayesian nonparametric data analysis*. English. Cham: Springer, 2015, pp. xiv + 193. ISBN: 978-3-319-18967-3/hbk; 978-3-319-18968-0/ebook. DOI: 10.1007/978-3-319-18968-0. URL: http://www.springer.com/us/book/9783319189673.

[7] C. E. Rasmussen and C. K. I. Williams. *Gaussian Processes for Machine Learning*. MIT Press, 2006.