

Ecology Appendix S1  
Posterior distribution and Gibbs sampler  
*Modeling abundance, distribution, movement, and  
space use with camera and telemetry data*

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## Posterior distribution

The posterior distribution of the joint spatial capture-recapture movement model (with constant  $\sigma$  and data augmentation) is

$$\begin{aligned}
 p(p, \lambda_0, \sigma_{\text{det}}, \rho, \sigma, \{\mathbf{u}_{it}\}, \{\mathbf{s}_i\}, \{z_i\}, \psi | \mathbf{y}^{\text{cap}}, \mathbf{y}) \propto \\
 \left\{ \prod_{i=1}^M p(y_i^{\text{cap}} | z_i, p) \left\{ \prod_{t=1}^T \left\{ \prod_{j=1}^J p(y_{ijt} | z_i, \lambda_0, \sigma_{\text{det}}, \mathbf{u}_{it}) \right\} p(\mathbf{u}_{it} | \mathbf{u}_{i,t-1}, \mathbf{s}_i, \rho, \sigma) \right\} p(\mathbf{s}_i) p(z_i | \psi) \right\} \times \\
 p(p) p(\lambda_0) p(\sigma_{\text{det}}) p(\rho) p(\sigma) p(\psi)
 \end{aligned} \tag{1}$$

where

$$\begin{aligned}
 p(y_i^{\text{cap}} | z_i, p) &= \text{Bern}(y_i | z_i \times p) \\
 p(y_{ijt} | z_i, \lambda_0, \sigma_{\text{det}}, \mathbf{u}_{it}) &= \text{Pois}(y_{ijt} | z_i \times \lambda_{ijt}^{\text{det}}) \\
 p(\mathbf{u}_{it} | \mathbf{u}_{i,t-1}, \mathbf{s}_i, \rho, \sigma) &= \begin{cases} \text{Norm}(\mathbf{u}_{it} | \mathbf{s}_i + (\mathbf{u}_{i,t-1} - \mathbf{s}_i)\rho, \text{diag}(\sigma^2 - \sigma^2\rho^2)) & \text{for } t > 1 \\ \text{Norm}(\mathbf{u}_{it} | \mathbf{s}_i, \text{diag}(\sigma^2)) & \text{for } t = 1 \end{cases} \\
 p(\mathbf{s}_i) &= \text{Unif}(\mathcal{M}) \\
 p(z_i | \psi) &= \text{Bern}(z_i | \psi)
 \end{aligned}$$

and the other probability distributions are priors for the parameters. Note that some or all of the  $\mathbf{u}_{it}$  locations could be observed.

## Gibbs sampler

Sampling from the joint posterior is computationally challenging because of the latent movement paths for the  $M - n$  augmented individuals. The burden can be reduced by marginalizing these latent paths while retaining the activity centers  $\{\mathbf{s}_i\}$ . This is accomplished using Eq. 6 in the manuscript and the probability density:

$$p(0|\lambda_0, \sigma_{\text{det}}, \rho, \sigma, \mathbf{s}_i, z_i) = \text{Bern}(0|\tilde{p}_i)$$

The Gibbs sampler begins by initializing the unknown parameters and then sampling from the following full conditional distributions.

Use Metropolis-Hastings (MH) to sample from:

$$p(\rho, \sigma|\cdot) \propto \left\{ \prod_{i=1}^n \prod_{t=1}^T p(\mathbf{u}_{it}|\mathbf{u}_{i,t-1}, \mathbf{s}_i, \rho, \sigma) \right\} \left\{ \prod_{i=n+1}^M p(0|\lambda_0, \sigma_{\text{det}}, \rho, \sigma, \mathbf{s}_i, z_i) \right\} p(\rho)p(\sigma)$$

Use MH to sample from:

$$p(\lambda_0, \sigma_{\text{det}}|\cdot) \propto \left\{ \prod_{i=1}^n \prod_{j=1}^J \prod_{t=1}^T p(y_{ijt}|z_i, \lambda_0, \sigma_{\text{det}}, \mathbf{u}_{it}) \right\} \left\{ \prod_{i=n+1}^M p(0|\lambda_0, \sigma_{\text{det}}, \rho, \sigma, \mathbf{s}_i, z_i) \right\} p(\lambda_0)p(\sigma_{\text{det}})$$

Sample directly from

$$p(\psi|\mathbf{z}) = \text{Beta} \left( 1 + \sum_{i=1}^M z_i, 1 + M - \sum_{i=1}^M z_i \right)$$

For  $i = n + 1, \dots, M$ , use MH to sample from

$$p(z_i|\cdot) \propto p(y_i^{\text{cap}}|z_i, p) \left\{ \prod_{j=1}^J \prod_{t=1}^T p(y_{ijt}|z_i, \lambda_0, \sigma_{\text{det}}, \mathbf{u}_{it}) \right\} p(0|\lambda_0, \sigma_{\text{det}}, \rho, \sigma, \mathbf{s}_i, z_i) p(z_i|\psi)$$

Use MH (or direct draw from beta full conditional) to sample from

$$p(p|\cdot) \propto \prod_{i=1}^M p(y_i^{\text{cap}}|z_i \times p) p(p)$$

For  $i = 1, \dots, n$ , use MH to sample from

$$p(\mathbf{s}_i|\cdot) \propto p(\mathbf{u}_{it}|\rho, \sigma, \mathbf{s}_i) p(\mathbf{s}_i)$$

For  $i = n + 1, \dots, M$ , use MH to sample from

$$p(\mathbf{s}_i|\cdot) \propto p(0|\lambda_0, \sigma_{\text{det}}, \rho, \sigma, \mathbf{s}_i, z_i) p(\mathbf{s}_i)$$

For  $i = 1, \dots, n$  and for cases where  $\mathbf{u}_{it}$  is not observed, use MH to sample from

$$p(\mathbf{u}_{it}|\cdot) \propto p(\mathbf{u}_{i,t+1}|\mathbf{u}_{i,t}, \mathbf{s}_i, \rho, \sigma) p(\mathbf{u}_{i,t}|\mathbf{u}_{i,t-1}, \mathbf{s}_i, \rho, \sigma) \prod_{j=1}^J p(y_{ijt}|\lambda_0, \sigma_{\text{det}}, \mathbf{u}_{it})$$