Ecology Appendix S1
Posterior distribution and Gibbs sampler
Modeling abundance, distribution, movement, and space use with camera and telemetry data

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Posterior distribution

The posterior distribution of the joint spatial capture-recapture movement model (with constant σ and data augmentation) is

$$
\begin{gathered}
p(p, \lambda_0, \sigma_{det}, \rho, \sigma, \{\mathbf{u}_{it}\}, \{\mathbf{s}_i\}, \{z_i\}, \psi | \boldsymbol{y}^{\text{cap}}, \boldsymbol{y}) \propto \\
\left\{ \prod_{i=1}^{M} p(y_i^{\text{cap}} | z_i, p) \left\{ \prod_{t=1}^{T} \left\{ \prod_{j=1}^{J} p(y_{ijt} | z_i, \lambda_0, \sigma_{\text{det}}, \mathbf{u}_{it}) \right\} p(\mathbf{u}_{it} | \mathbf{u}_{i,t-1}, \mathbf{s}_i, \rho, \sigma) \right\} p(\mathbf{s}_i) p(z_i | \psi) \right\} \times \\
p(p) p(\lambda_0) p(\sigma_{\text{det}}) p(\rho) p(\sigma) p(\psi)
\end{gathered} \tag{1}
$$

where

$$
p(y_i^{\text{cap}} | z_i, p) = \text{Bern}(y_i | z_i \times p)
$$

$$
p(y_{ijt} | z_i, \lambda_0, \sigma_{\text{det}}, \mathbf{u}_{it}) = \text{Pois}(y_{ijt} | z_i \times \lambda_{ijt}^{\text{det}})
$$

$$
p(\mathbf{u}_{it} | \mathbf{u}_{i,t-1}, \mathbf{s}_i, \rho, \sigma) = \begin{cases} \text{Norm}(\mathbf{u}_{it} | \mathbf{s}_i + (\mathbf{u}_{i,t-1} - \mathbf{s}_i)\rho, \text{diag}(\sigma^2 - \sigma^2\rho^2)) & \text{for} \quad t > 1 \\ \text{Norm}(\mathbf{u}_{it} | \mathbf{s}_i, \text{diag}(\sigma^2)) & \text{for} \quad t = 1 \end{cases}
$$

$$
p(\mathbf{s}_i) = \text{Unif}(\mathcal{M})
$$

$$
p(z_i | \psi) = \text{Bern}(z_i | \psi)
$$

and the other probability distributions are priors for the parameters. Note that some or all of the $\mathbf{u}_{it}$ locations could be observed.

Gibbs sampler

Sampling from the joint posterior is computationally challenging because of the latent movement paths for the $M-n$ augmented individuals. The burden can be reduced by marginalizing these latent paths, while retaining the activity centers $\{\mathbf{s}_i\}$. This is accomplished using Eq. 6 in the manuscript and the probability density:

$$p(0|\lambda_0, \sigma_{\text{det}}, \rho, \sigma, \mathbf{s}_i, z_i) = \text{Bern}(0|\tilde{p}_i)$$

The Gibbs sampler begins by initializing the unknown parameters and then sampling from the following full conditional distributions.

Use Metropolis-Hastings (MH) to sample from:

$$p(\rho, \sigma|\cdot) \propto \left\{ \prod_{i=1}^{n} \prod_{t=1}^{T} p(\mathbf{u}_{it}|\mathbf{u}_{i,t-1}, \mathbf{s}_i, \rho, \sigma) \right\} \left\{ \prod_{i=n+1}^{M} p(0|\lambda_0, \sigma_{\text{det}}, \rho, \sigma, \mathbf{s}_i, z_i) \right\} p(\rho)p(\sigma)$$

Use MH to sample from:

$$p(\lambda_0, \sigma_{\text{det}}|\cdot) \propto \left\{ \prod_{i=1}^{n} \prod_{j=1}^{J} \prod_{t=1}^{T} p(y_{ijt}|z_i, \lambda_0, \sigma_{\text{det}}, \mathbf{u}_{it}) \right\} \left\{ \prod_{i=n+1}^{M} p(0|\lambda_0, \sigma_{\text{det}}, \rho, \sigma, \mathbf{s}_i, z_i) \right\} p(\lambda_0)p(\sigma_{\text{det}})$$

Sample directly from

$$p(\psi|\boldsymbol{z}) = \text{Beta}\left(1 + \sum_{i=1}^{M} z_i, 1 + M - \sum_{i=1}^{M} z_i\right)$$

For $i = n+1, \ldots, M$, use MH to sample from

$$p(z_i|\cdot) \propto p(y_i^{\text{cap}}|z_i, p) \left\{ \prod_{j=1}^{J} \prod_{t=1}^{T} p(y_{ijt}|z_i, \lambda_0, \sigma_{\text{det}}, \mathbf{u}_{it}) \right\} p(0|\lambda_0, \sigma_{\text{det}}, \rho, \sigma, \mathbf{s}_i, z_i)p(z_i|\psi)$$

Use MH (or direct draw from beta full conditional) to sample from

$$p(p|\cdot) \propto \prod_{i=1}^{M} p(y_i^{\text{cap}}|z_i \times p)p(p)$$

For $i = 1, \ldots, n$, use MH to sample from

$$p(\mathbf{s}_i|\cdot) \propto p(\mathbf{u}_{it}|\rho, \sigma, \mathbf{s}_i)p(\mathbf{s}_i)$$

For $i = n+1, \ldots, M$, use MH to sample from

$$p(\mathbf{s}_i|\cdot) \propto p(0|\lambda_0, \sigma_{\text{det}}, \rho, \sigma, \mathbf{s}_i, z_i)p(\mathbf{s}_i)$$

For $i = 1, \ldots, n$ and for cases where $\mathbf{u}_{it}$ is not observed, use MH to sample from

$$p(\mathbf{u}_{it}|\cdot) \propto p(\mathbf{u}_{i,t+1}|\mathbf{u}_{i,t}, \mathbf{s}_i, \rho, \sigma)p(\mathbf{u}_{i,t}|\mathbf{u}_{i,t-1}, \mathbf{s}_i, \rho, \sigma) \prod_{j=1}^{J} p(y_{ijt}|\lambda_0, \sigma_{\text{det}}, \mathbf{u}_{it})$$