Risk-Based Indexation

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Seminar Applied Portfolio Theory

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Overview

Introduction

Risk-Based Indexation*

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Abstract

A capitalization-weighted index is the most common way to gain access to broad equity market performance. These portfolios are generally concentrated in a few stocks and present some lack of diversification. In order to avoid this drawback or to simply diversify market exposure, alternative indexation methods have recently prompted great interest, both from academic researchers and market practitioners. Fundamental indexation computes weights with regard to economic measures, while risk-based indexation focuses on risk and diversification criteria. This paper describes risk-based indexation methodologies, highlights potential practical issues when implemented, and illustrates these issues as it applies to the Euro Stoxx 50 universe.

Introduction

- Capitalization-weighted indices are the most common way to gain access to broad equity markets
- CAPM theory: market-capitalization weighting is efficient for asset allocation (market portfolio)
  - In practice, CAPM shows little validity
  - Plethora of evidence for low volatility/low beta anomaly
- Therefore, investors have shown great interest in alternative-weighted indexes
  - Fundamental indexes: computes weights with regard to economic measures
  - Risk-based indexes: focuses on risk and diversification criteria

Index taxonomy

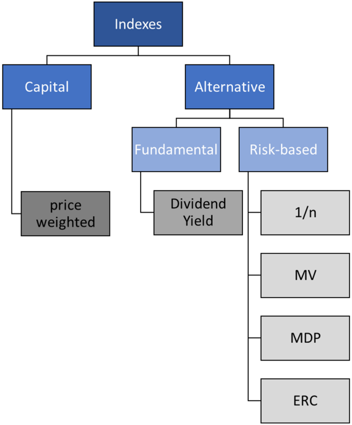

Figure: Index taxonomy

Risk-based optimization techniques

Overview

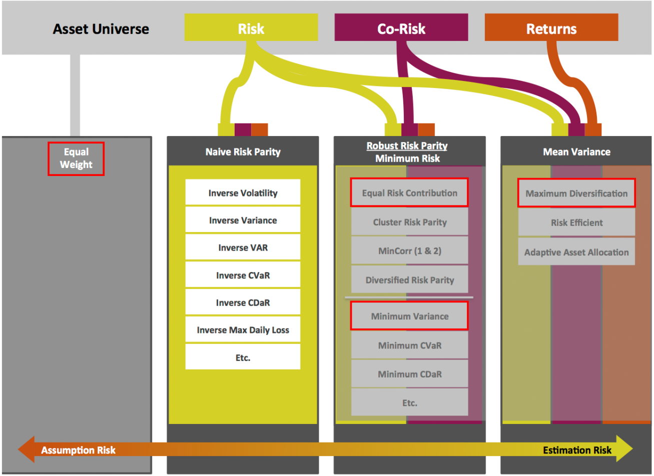

Figure: Risk-based optimization techniques ([ReSolve AM])

Capital-weighted Index

Capital Index

$$\omega_i(t) = \frac{N_i(t) \cdot P_i(t)}{\sum_{j=1}^{n} N_j(t) \cdot P_j(t)} \tag{1}$$

- Low trading costs because of low turnover
- Easily comprehensible and no measurement error
- Can easily be hedged and replicated because of high liquidity
- Many indices are capital-weighted

Capital-weighted Index

Issues

- If EMH holds, all investors would hold market portfolio = CWI
  - Clearly, this is not the case in practice, as many paper provide evidence
- Trend following, leading to bubble risks
- Growth bias, high value stocks weigh more
- High draw-down risk and limited diversification (e.g. think of sectors)

Risk-based portfolio construction I

- Fundamental indexation promises α
- Whereas a risk-based index offers risk-diversification
- Portfolio optimization techniques have different parameters ($\mathbb{E}[r]$, $\mathbf{\Sigma}$, etc.)
- Estimation to be made about these parameters and their predictability
  - Hard, especially for expected returns!

Risk-based portfolio construction II

Equal-weighted (1/N)

$$\omega_i = \frac{1}{N} \tag{2}$$

- Every asset has the same weight in the portfolio $(1/N)$
  - Extremely easy to calculate
- Turning blind eye to $\mathbb{E}[\mathrm{r}]$ and $\boldsymbol{\Sigma}$
- Can perform well, but potentially faces huge draw-downs
  - i.e. small, risky companies have same weight as stable, large companies
- Not used in practice often, merely useful as basic benchmark
- We have: $\sigma_{MV} \leq \sigma_{ERC} \leq \sigma_{1/N}$

Risk-based portfolio construction III

Minimum Variance

$$\omega^* = \arg\min \ \omega^T \Sigma \omega \quad (3)$$

- Choose weights so that the portfolio variance is minimized
- Construction is independent from $\mathbb{E}[r]$
- Shows robust performance out-of-sample
- By construction low volatility
- Tends to concentrate in low-volatility assets

Risk-based portfolio construction IV

Max. Diversification Portfolio (MDP) / Max. Sharpe Ratio (MSR)

$$\omega^* = \arg\max \frac{\mu^T\omega - r}{\sqrt{\omega^T\Sigma\omega}} = s\frac{\sigma^T\omega}{\sqrt{\omega^T\Sigma\omega}} \tag{4}$$

- Maximizes the Sharpe Ratio based on $\mathbb{E}[r]$ and Σ estimates
  - Main difficulty: estimation of $\mathbb{E}[r]$
- Most sensitive to parameter estimates of the techniques covered in this presentation
- We have: $\sigma_{MV} \leq \sigma_{MSR}$,
  - cannot make statement about relation to $\sigma_{1/n}$ or σ_{ERC}

Risk-based portfolio construction V

Portfolio Volatility: Equal Risk Contribution (ERC)

$$\sigma(\omega) = \Sigma_{i=1}^{n} RC_i = \Sigma_{i=1}^{n} \omega_i \frac{\partial \sigma(\omega)}{\partial \omega_i} \tag{5}$$

$$\omega_i \frac{\partial \sigma(\omega)}{\partial \omega_i} = \omega_j \frac{\partial \sigma(\omega)}{\partial \omega_j} \quad \forall i, j \tag{6}$$

- Chooses the weights such that the contribution of the respective asset to the portfolio volatility is the same for all portfolios
- It defines a portfolio that is well diversified in terms of risk and weights
- Does **not** depend on any expected returns hypothesis
- It is less sensitive to small changes in the covariance matrix than the MV and MSR techniques

Risk-based portfolio construction VI

- In an ERC portfolio, each asset contributes equally to the total risk
- How does this differ from the Global Minimum Variance portfolio?
- ERC ensures that some amount is actually invested in all assets
  - So we expect that ERC behaves more robust regarding diversification

MV equalizes: $$\frac{\partial\sigma(\omega)}{\partial\omega_i} = \frac{\partial\sigma(\omega)}{\partial\omega_j}$$

ERC equalizes: $$\omega_i \frac{\partial\sigma(\omega)}{\partial\omega_i} = \omega_j \frac{\partial\sigma(\omega)}{\partial\omega_j}$$

MSR equalizes: $$\frac{1}{\sigma_i}\frac{\partial\sigma(\omega)}{\partial\omega_i} = \frac{1}{\sigma_j}\frac{\partial\sigma(\omega)}{\partial\omega_j}$$

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Empirical Results

A short journey through selected portfolio optimization techniques

Outline

- Our goal was to implement, analyze and compare different risk-based portfolio optimization techniques:
  - **1/N** (for comparison)
  - **Minimum Variance**
  - **Maximum Sharpe Ratio**
  - **Equal Risk Contribution**
- The backtest hopefully allows us to gain new insights and develop some intuition about the techniques and their performance
- Software
  - Based on **Matlab R2019b**
  - Written from scratch using the built-in optimization functions `fmincon`, `quadprog` from the *Optimization Toolbox*

Analysis

- We will take a look at the following characteristics of the resulting portfolios
  - Weights distribution and behavior over time
  - Risk contribution and behavior over time
  - Summary statistics
  - Portfolio performance (Sharpe Ratio)
- Main focus lies on the composition and robustness of the portfolio weights, i.e. low turnover/stable weights

Backtest setup

- The following setup has been chosen for the backtest:
  - **Estimation window of 4 years** (moving window)
  - **Optimization interval of 6 months (biyearly)**
  - No rebalancing because of frequent re-optimization
  - Two (small) asset universes (homogeneous and heterogeneous)
  - **Long-only** (non-negative weights)
- These are hyper-parameters and their influence should be analyzed in further studies
  - We fixed them, because the relative results are relatively stable for the given datasets
- Expected returns and covariance estimation
  - **Expected returns** only relevant for MSR, hence simply chose the arithmetic mean
  - **Covariance matrix**
    - Crucial for our optimization, because they are risk-based
    - Analyzed sample covariance and shrinkage estimators (OAS)

Data

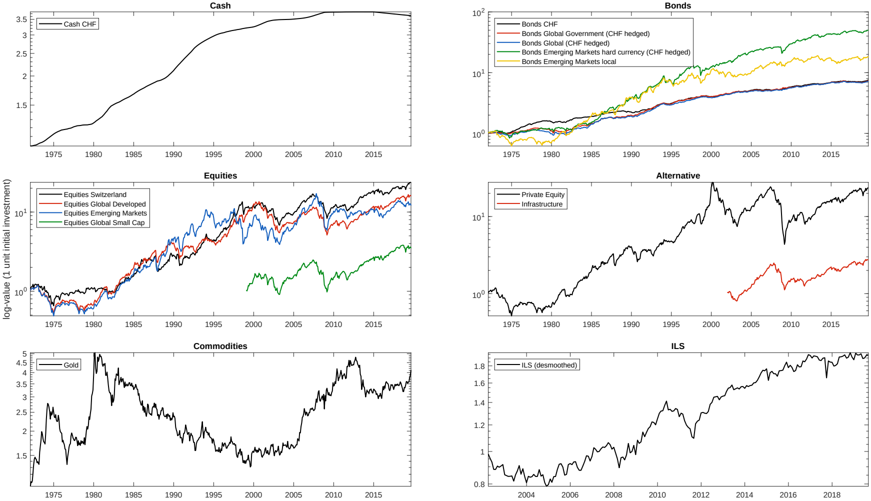
- Monthly returns from **31.01.1972** until **31.08.2019** (48 years, 572 months)
- Datasets
  - **Lecture dataset** consisting of 13 assets from different asset classes
    - heterogeneous (bonds, equities, commodities, alternatives)
    - 13 assets
  - **Dow Jones Industrial Average 30**
    - homogeneous (only equity asset class)
    - ≈30 assets
    - Price-index, but not relevant for us since we mostly care about relative performance and optimization robustness
    - Retrieved through the *Wharton Research Data Services* using the CRSP/Compustat database

Summary statistics
Lecture dataset

Entity	Years	SR	Ret p.a.	Worst	Best	Vol p.a.	Skew	Kurt
Cash CHF	48	3.43	2.74	[0]	0.01	[0.79]	0.78	2.91
Bonds CHF	48	1.44	4.35	-0.03	0.04	3.01	-0.02	4.16
Bonds Global Government ..	48	1.12	4.3	-0.05	0.06	3.83	0.28	6.26
Bonds Global (CHF hedged)	48	1.04	4.26	-0.04	0.06	4.11	0.37	5.39
Bonds Emerging Markets h..	48	0.83	8.54	-0.26	0.1	10.61	-1.64	15.6
Bonds Emerging Markets l..	48	0.52	6.25	-0.16	0.13	13.45	-0.19	4.14
Equities Switzerland	48	0.5	6.87	-0.26	0.21	16.08	-0.71	6.37
Equities Global Developed	48	0.43	5.98	-0.23	0.15	16.77	-0.65	4.39
Equities Emerging Markets	48	0.35	5.37	-0.37	0.19	22.33	-0.84	6.32
Equities Global Small Cap	21	0.43	6.24	-0.21	0.15	18	-0.78	4.59
Private Equity	48	0.43	6.84	-0.31	0.33	20.56	-0.26	7.7
Infrastructure	18	0.49	5.88	-0.17	0.09	13.71	-1.12	5.48
ILS (desmoothed)	18	0.39	3.84	-0.13	0.12	11.19	0.04	5.06
Gold	48	0.25	3.02	-0.18	0.27	18.17	0.55	5.43

Data plot

Lecture dataset



Summary statistics

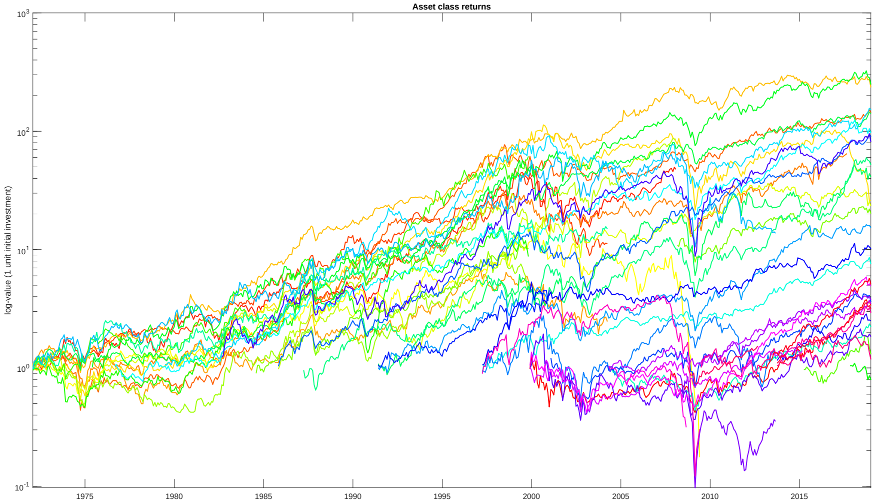
Dow Jones Industrial Average 30 (DJIA)

Entity	Years	SR	Ret p.a.	Worst	Best	Vol p.a.	Skew	Kurt
10107	19	0.36	6.53	-0.34	0.41	30.01	0.38	6.23
10145	36	0.52	11.3	-0.38	0.51	29.16	0.03	7.02
10401	32	0.43	7.79	-0.26	0.39	24.95	0.15	5.9
11308	47	0.6	11.14	-0.29	0.26	21.29	-0.12	5.32
11703	46	0.49	9.56	-0.24	0.32	24.83	0.17	4.5
11754	32	0.24	2.84	-0.34	0.24	26.01	-0.25	4.85
11850	47	0.76	12.32	-0.14	0.23	17.31	0.26	4.08
12060	47	0.41	7.19	-0.27	0.25	24.39	-0.07	4.38
12079	37	0.04	-4.53	-0.61	0.28	33.18	-0.85	7.88
12490	47	0.4	6.94	-0.26	0.35	24.58	0.22	5.17
13901	22	0.86	22.07	-0.27	0.34	27.88	-0.29	5.14
14322	28	0.34	5.54	-0.3	0.28	26.86	-0.01	4.06
14541	26	0.66	12.12	-0.18	0.24	20.5	0.14	3.77
14593	4	0.4	7.31	-0.18	0.2	26.4	0.01	3.17
15659	28	0.67	15.94	-0.26	0.3	28.21	0.14	3.61
16432	28	0.41	8.1	-0.33	0.39	29.25	0.23	5.16
16851	1	-0.66	-16.78	-0.16	0.08	23.63	-0.65	2.51
17830	44	0.65	13.46	-0.39	0.25	24.36	-0.48	6.11
18163	47	0.64	11.1	-0.36	0.25	19.35	-0.36	6.67
18542	28	0.59	14.37	-0.35	0.41	30.64	0.15	5.42
19561	32	0.6	13.22	-0.35	0.24	27.24	-0.48	4.27
21573	32	0.44	8.75	-0.28	0.27	28.33	0.26	3.65
21936	15	0.39	5.58	-0.18	0.15	18.49	-0.15	3.16
22111	22	0.61	9.69	-0.16	0.17	17.74	-0.04	4.05
22592	42	0.65	11.37	-0.28	0.26	19.67	-0.04	5.05
22752	40	0.66	13.53	-0.26	0.23	23.65	-0.15	3.61
24643	42	0.36	6.49	-0.49	0.51	32.55	-0.03	6.22
26403	28	0.53	10.37	-0.27	0.24	24.54	-0.15	4.23
27828	17	0.22	1.2	-0.32	0.35	38.33	0.16	3.6
43449	33	0.75	14.46	-0.26	0.18	20.92	-0.29	3.95
47896	18	0.39	7.39	-0.28	0.25	29.37	-0.2	4.38
48071	10	0.71	16.92	-0.26	0.25	27.38	0.1	4.44
55976	22	0.6	11.18	-0.21	0.26	21.74	0.24	4.53
57665	5	0.98	18.83	-0.11	0.16	19.62	0.12	2.83
59176	36	0.54	12.83	-0.32	0.86	30.79	1.55	22.24
59328	19	0.27	3.23	-0.44	0.34	33.78	-0.52	5.67
59408	6	0.04	-16.98	-0.53	0.73	64.85	0.47	5.91
59459	10	0.87	14.76	-0.1	0.2	17.72	0.55	4.01

[10 rows truncated]

Data plot

Dow Jones Industrial Average 30 (DJIA) dataset



Estimation method for ERC

How to derive the ERC portfolio

- As we saw before, the optimization objective is to have equal risk contributions for all weighted portfolio assets
- The total portfolio variance is defined as follows:

$$\sigma^2(R_P) = w^T \Sigma w$$

- We can decompose this expression to get a vector $RC(R_P)$ of risk contributions per asset (sum of elements equals portfolio variance):

$$RC(R_P) = w^T \odot (\Sigma w) \quad \iff \quad RC_i(R_P) = w_i \cdot (\Sigma w)_i$$

- Now, we have to optimize this expression such that all vector elements have the same value
  - We can use a simple trick; we minimize the variance of this expression – a variance of zero implies that all values are equal

Equal Risk Contribution optimization objective

$$w^*_{ERC} = \arg\min_{w} \; Var\left(w^T \odot (\Sigma w)\right) \;\; \text{s.t.} \;\; 0 < w \leq 1 \;\; \text{and} \;\; w^T \mathbb{1} = 1 \tag{7}$$

Estimation method for ERC

ERC optimization via `fmincon`

Nonlinear programming solver.

Finds the minimum of a problem specified by

$$\min_x f(x) \text{ such that } \begin{cases} c(x) \le 0 \\ ceq(x) = 0 \\ A \cdot x \le b \\ Aeq \cdot x = beq \\ lb \le x \le ub, \end{cases}$$

b and beq are vectors, A and Aeq are matrices, $c(x)$ and $ceq(x)$ are functions that return vectors, and $f(x)$ is a function that returns a scalar. $f(x)$, $c(x)$, and $ceq(x)$ can be nonlinear functions.

- Matlab offers a generalized solver (`fmincon`) for certain types of optimization problems
- We can use this infrastructure for the purpose of optimizing our ERC portfolio optimization problem
  - $f(x)$ will be our minimization function
  - lb and ub are vector of 0s and 1s to limit the weights between 0 and 1
  - Aeq and b is used to enforce the weights to sum up to 1

Estimation method for ERC

Matlab implementation

```matlab
function weights = EqualRiskContribution(optParams)
    % compute constraints
    constraints = optParams.ConstraintsFunc(optParams);

    % lower/upper bounds (lb <= x <= ub)
    lb = constraints.LowerBounds;
    ub = constraints.UpperBounds;

    % equality constraints (Aeq*x = beq)
    Aeq = constraints.Aeq;
    beq = constraints.beq;

    % inequality constraints (A*x < b)
    A = constraints.A;
    b = constraints.b;

    % initial weights (equal weighted)
    x0 = 1/optParams.N * ones(optParams.N, 1);

    % scale covariance matrix by large factor for increased optimization accuracy
    Sigma = optParams.CovMat * 10^14;

    % Sequential Quadratic Programming (SQP) algorithm
    fun = @(W) var(W.*(Sigma*W));
    opts = optimset('Display','off', 'Algorithm','sqp');
    weights = fmincon(fun, x0, A, b, Aeq, beq, lb, ub, [], opts);
end
```

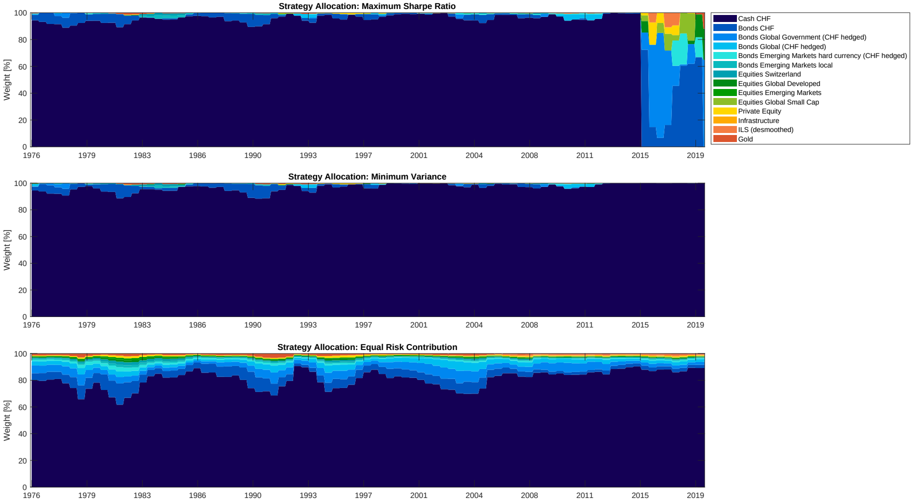
The naïve approach

Setup

- We start with the **lecture dataset**, and then repeat the analysis with the **DJIA** dataset
- We feed the optimization methods the **sample mean of the returns as expected returns** (if required), and the **sample covariance matrix**
- We do not set weight limits, only the condition that the portfolio weights must sum up to 100% and cannot be negative
  - no short-selling allowed

The naïve approach

Results: **Lecture dataset**



The naïve approach
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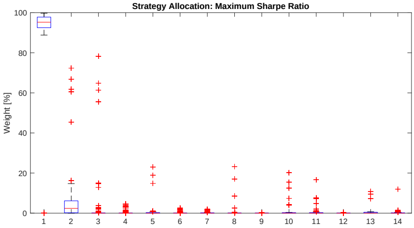
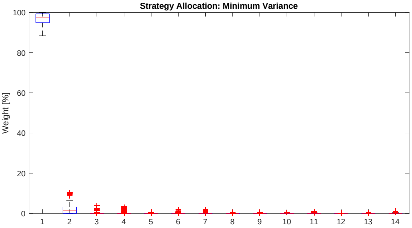
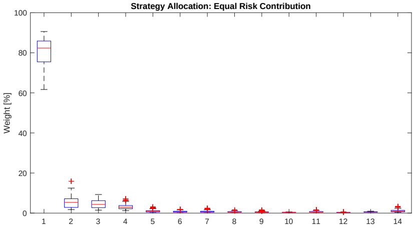






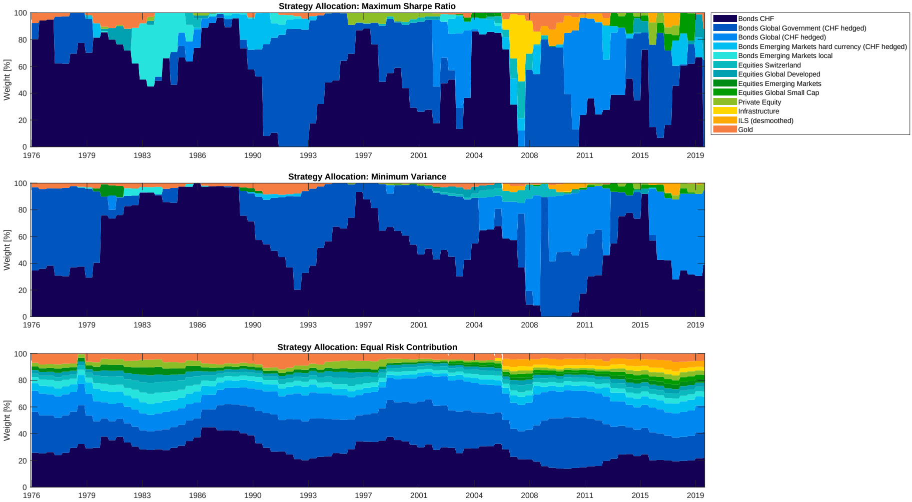
The naïve approach

Results: **Lecture dataset**

- For the lecture dataset, the **low volatility of Cash (0.79% p.a.)** causes the optimization algorithm to excessively overweigh Cash in all cases
- First, we will try again by dropping the *Cash CHF* asset to mitigate the volatility discrepancy...

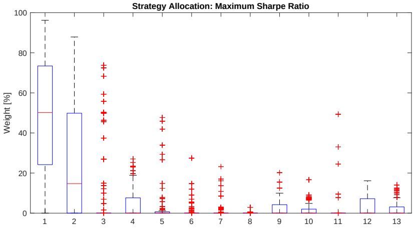
The naïve approach

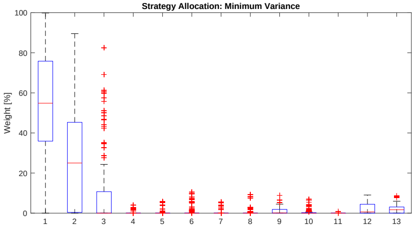
Results: **Lecture dataset** w/o *Cash CHF*

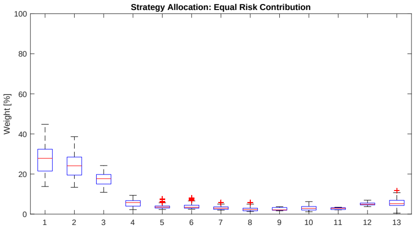


The naïve approach

Results: **Lecture dataset** w/o *Cash CHF*







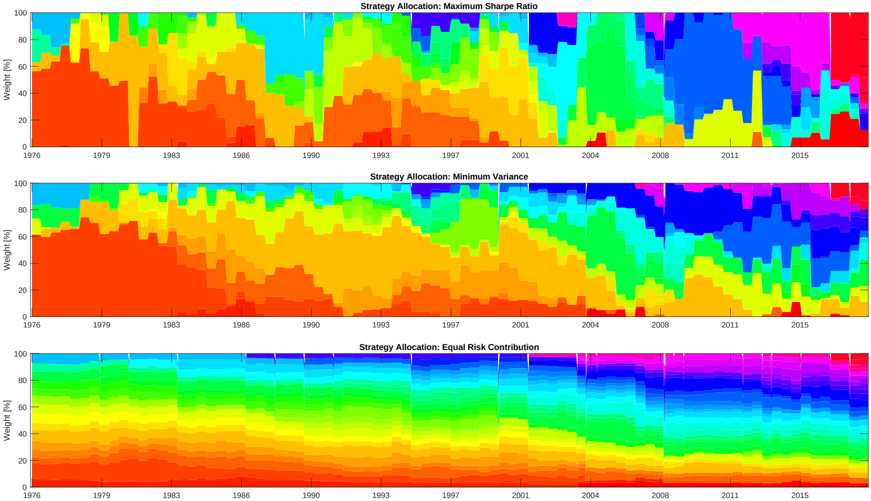
The naïve approach

Results: **Lecture dataset** w/o *Cash CHF*

- Removing Cash CHF shows a dramatically improved picture (still not good though)
- The weight concentration now moved to the other low-volatility assets (namely bonds)
- Allocation very erratic over time, except for ERC
- We will exclude Cash CHF for the rest of our analysis, because the Volatility discrepancy is large and its weight is mostly a strategic choice, not subject to optimization (debatable)

The naïve approach

Results: **Dow Jones Industrial Average 30 (DJIA)**



The naïve approach

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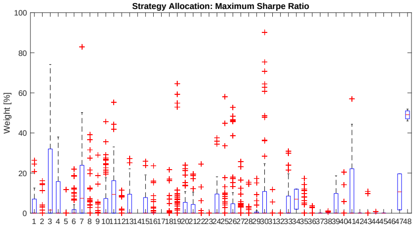
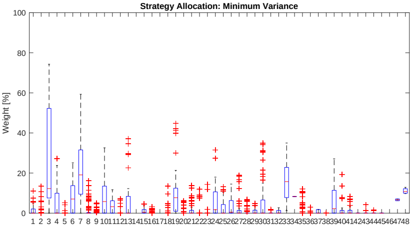
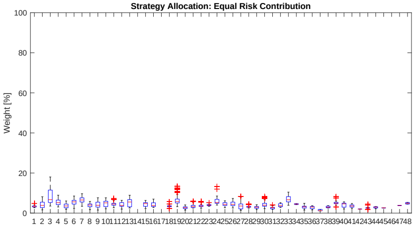






The naïve approach

Results: **Dow Jones Industrial Average 30 (DJIA)**

- The **DJIA** suffers less from the concentration problem, because the covariances of the assets are of the same magnitude
- Still, allocation very erratic over time
- Can we do better?
  - ...without having to exclude concentration-prone assets?
  - ...and with a more robust portfolio allocation?
- We can!
  - Expected returns are extremely difficult to estimate
    - That's why we leave this for another day and use the sample mean
  - Studies have shown that the sample covariance matrix is a bad estimator, especially for portfolio optimization
    - See [Ledoit, Wolf (2004)]
  - We will use the Shrinkage estimator for the covariance matrix to improve the optimization

From the abstract of [Ledoit, Wolf (2004)]:

The central message of this article is that no one should use the sample covariance matrix for portfolio optimization. It is subject to estimation error of the kind most likely to perturb a mean-variance optimizer. Instead, a matrix can be obtained from the sample covariance matrix through a transformation called shrinkage. This tends to pull the most extreme coefficients toward more central values, systematically reducing estimation error when it matters most.

- We will follow their advice and incorporate a shrinkage estimator for the covariance matrix!

Shrinkage

Theory

- Empirical covariance is unbiased, but converges slowly (high MSE)
- Factor model covariance is biased, but converges faster (lower MSE)
- For portfolio optimization, we care more about lower MSE/faster convergence than low bias, because we usually work with sparse datasets

Shrinkage estimator

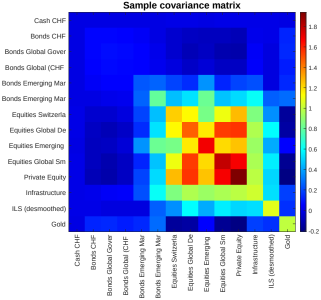
$$\arg\min_{\rho} \; \mathbb{E}\left[\|\hat{\boldsymbol{\Sigma}}_{\textbf{shrink}} - \boldsymbol{\Sigma}\|_F^2\right] \tag{8}$$

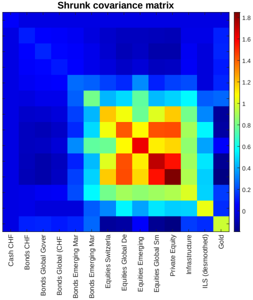
$$\text{s.t.} \quad \hat{\Sigma}_{shrink} = (1-\rho)\hat{\mathbf{S}} + \rho\hat{\mathbf{F}} \tag{9}$$

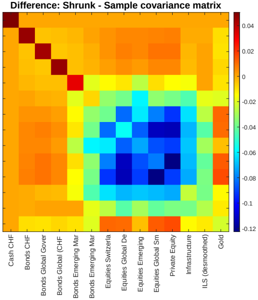
- We resort to the *Oracle Approximating Shrinkage* estimator (OAS), see [Chen (2010)] for details
  - It is an improved version of the Ledoit-Wolf shrinkage estimator

Shrinkage

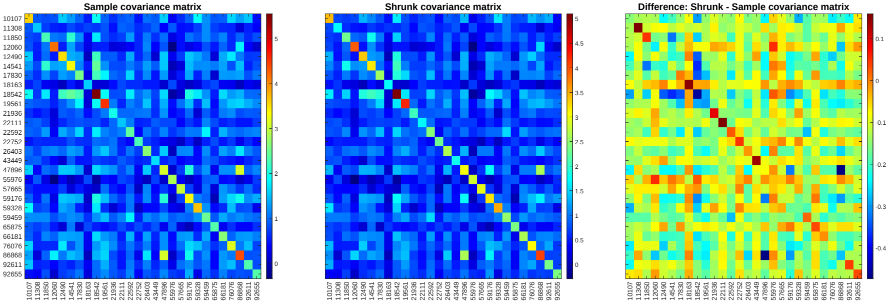
Sample covariance vs. Shrinkage for **Lecture dataset**







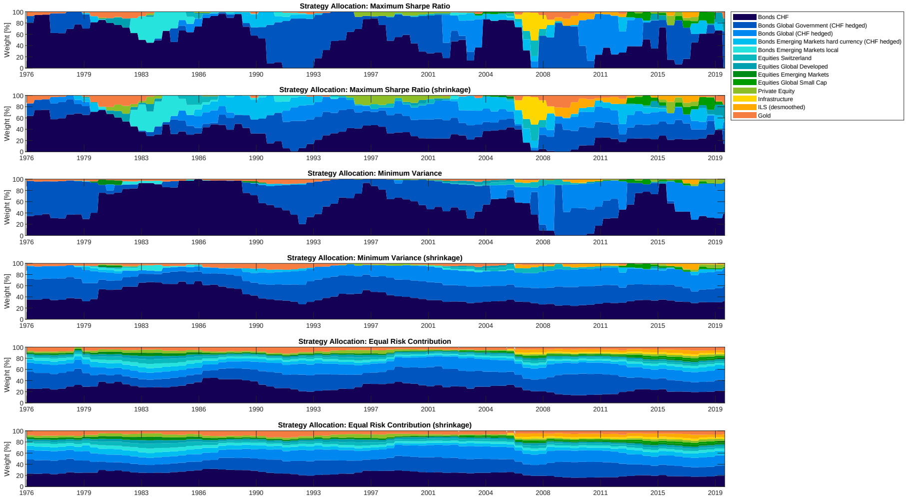
- The shrunk covariance matrix (OAS) does not greatly deviate from the sample covariance matrix at first sight
- The shrinking effect is strongest for the extreme values
  - The low variance of Cash/Bonds is increased, and the high variance of equities is decreased
- More sophisticated and domain-specific covariance modeling approaches should show even bigger deviations



- The Shrinkage target has non-zero diagonal elements and is zero everywhere else, hence the shrunk matrix gets diagonal-heavy

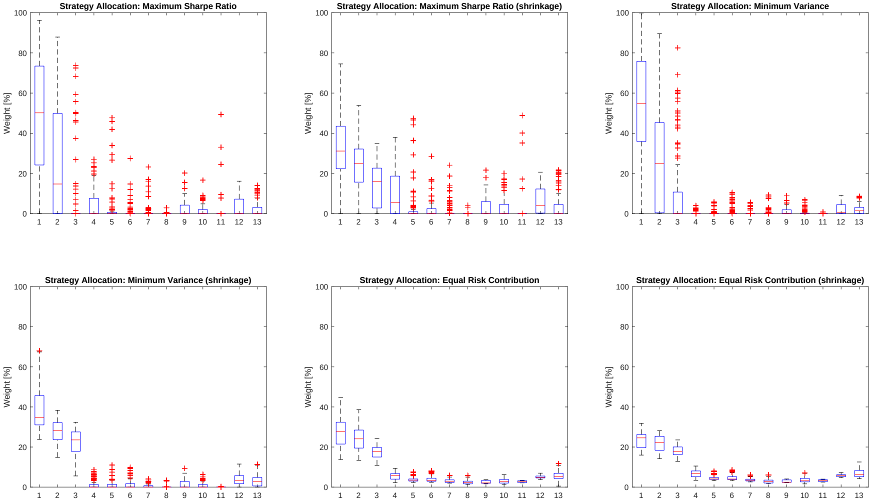
The shrinkage approach

Results: Weights distribution **Lecture dataset** w/o *Cash CHF*



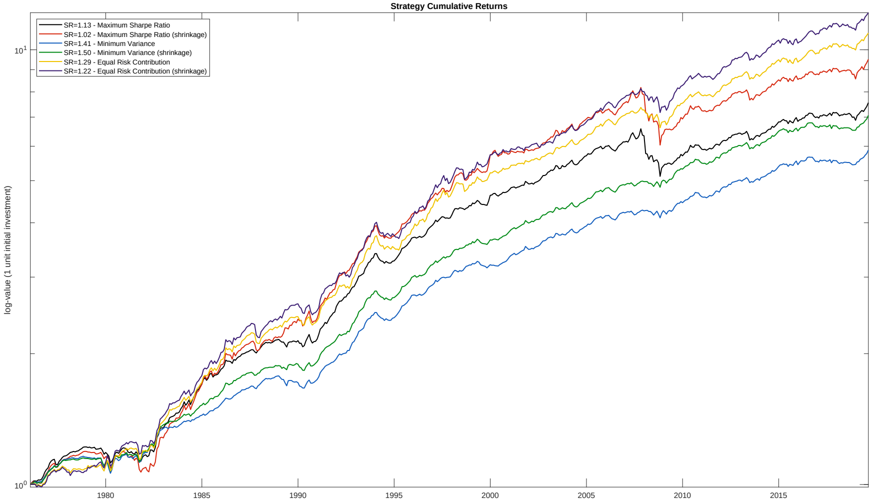
The shrinkage approach

Results: Weights distribution **Lecture dataset** w/o *Cash CHF*



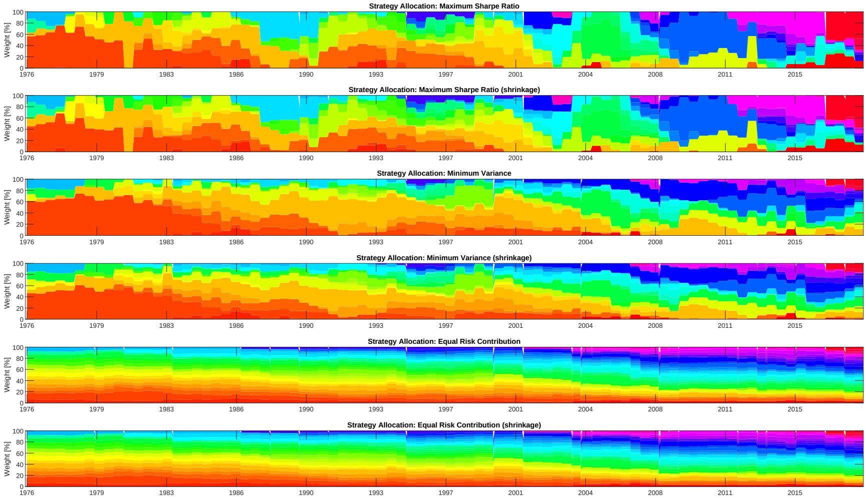
The shrinkage approach

Results: Performance **Lecture dataset** w/o *Cash CHF*



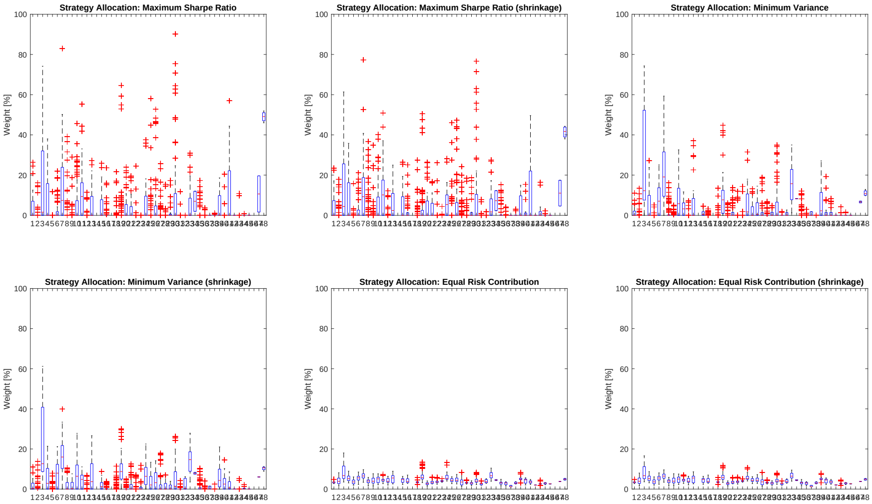
The shrinkage approach

Results: Weights distribution **DJIA**



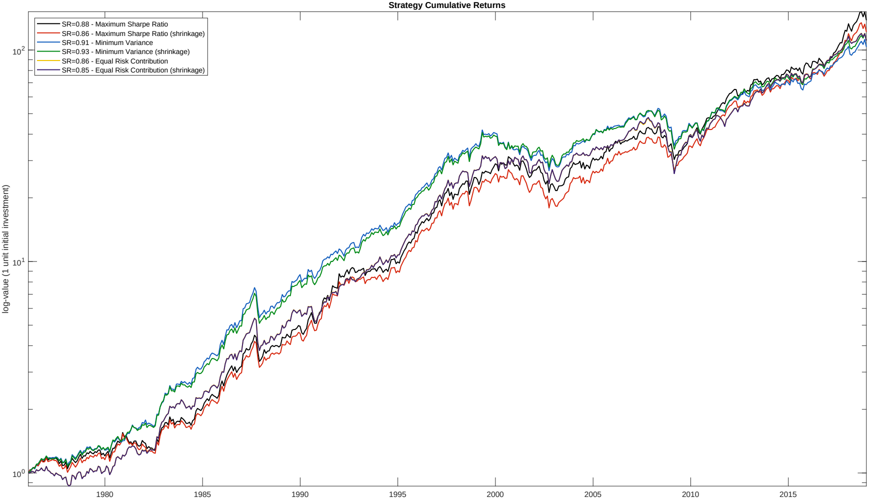
The shrinkage approach

Results: Weights distribution **DJIA**



The shrinkage approach

Results: Performance **DJIA**

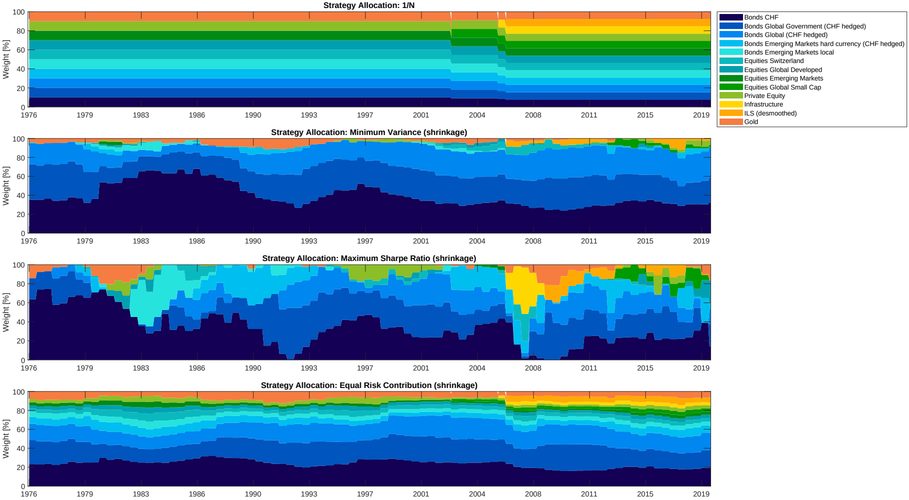


The shrinkage approach
Conclusion

- The shrinkage estimator shows a more robust weight distribution over time
  - This holds true for all three cases considered so far: **lecture dataset**, **lecture dataset w/o Cash CHF** and **DJIA**
  - For the **DJIA** dataset, the result is less apparent because of its homogeneous nature
- The evidence provided by our datasets and test cases shows that the Shrinkage estimator is superior to the sample covariance estimator in the context of portfolio optimization
- For a homogeneous asset universe, shrinkage has less influence (i.e. **DJIA**), compared to a heterogeneous one (i.e. **lecture dataset**)
- Verdict: Always use some form of shrinkage, or an alternative robust covariance estimator
  - There exist many other estimation techniques/Shrinkage estimators, or a combination thereof

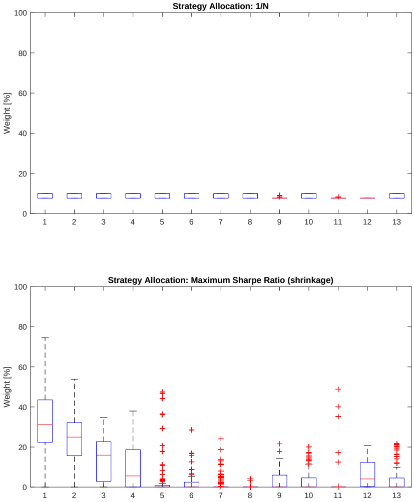
Comparison of risk-based optimization techniques

Results: Weights distribution **lecture dataset** w/o Cash (Shrinkage estimator)



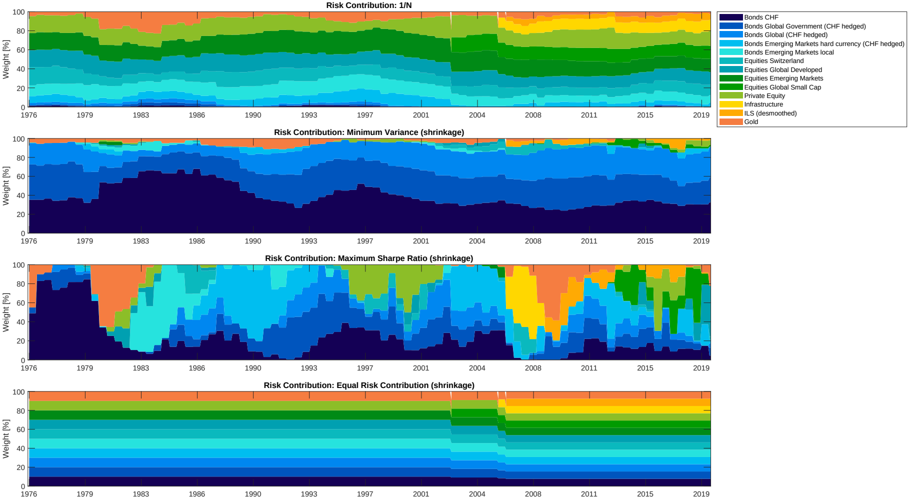
Comparison of risk-based optimization techniques

Results: Weights distribution **lecture dataset** w/o Cash (Shrinkage estimator)



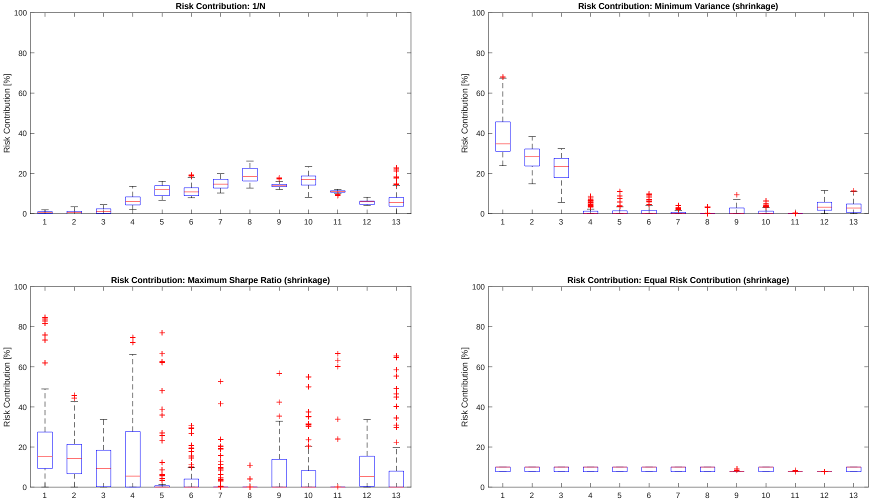
Comparison of risk-based optimization techniques

Results: Risk contribution **lecture dataset** w/o Cash (Shrinkage estimator)



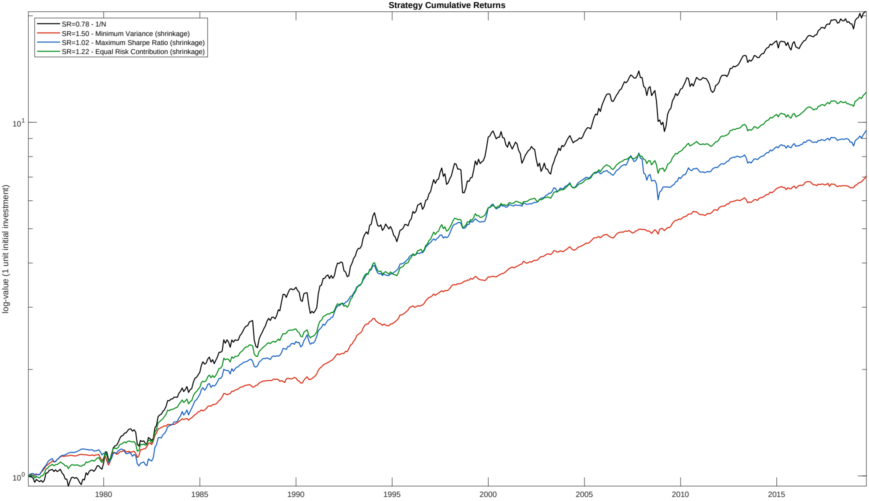
Comparison of risk-based optimization techniques

Results: Risk contribution **lecture dataset** w/o Cash (Shrinkage estimator)



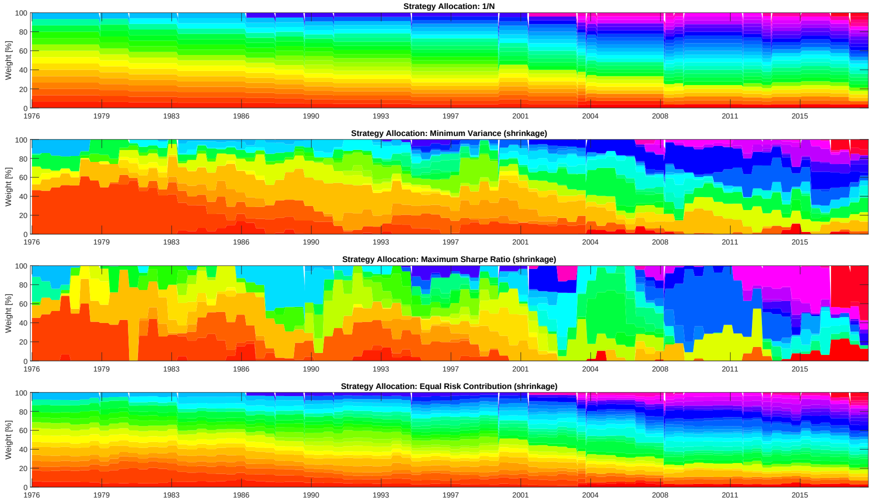
Comparison of risk-based optimization techniques

Results: Performance **lecture dataset** w/o Cash (Shrinkage estimator)



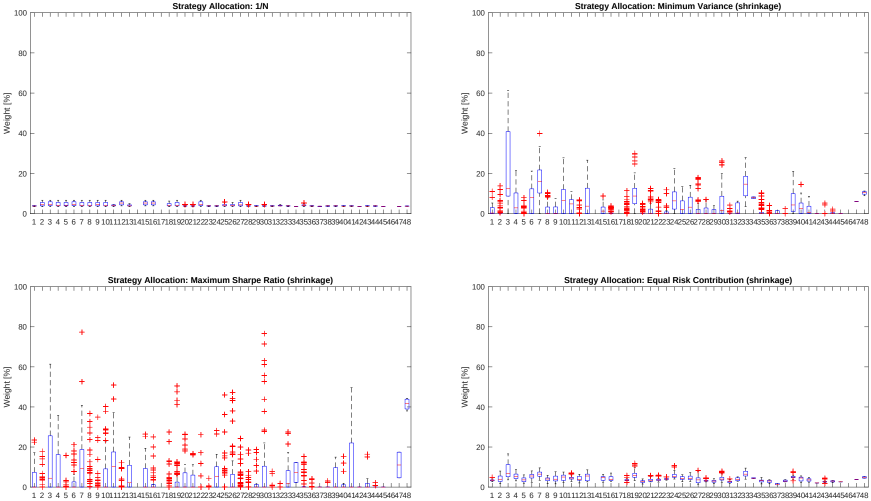
Comparison of risk-based optimization techniques

Results: Weights distribution **DJIA** (Shrinkage estimator)



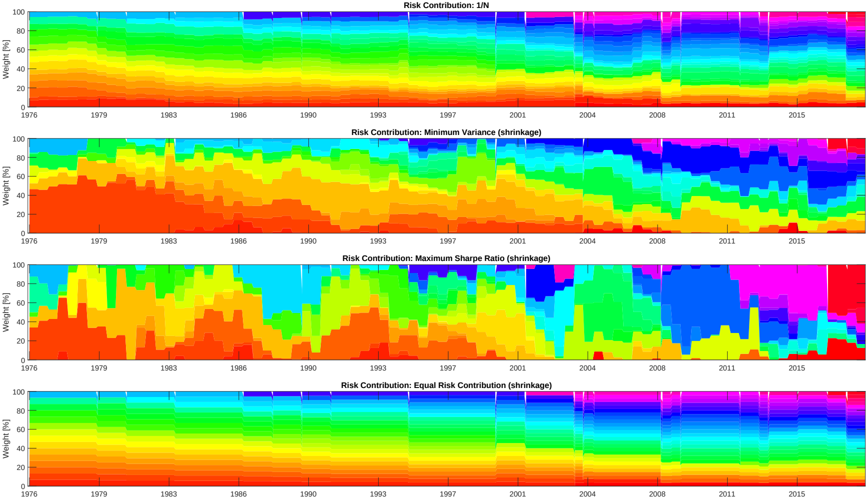
Comparison of risk-based optimization techniques

Results: Weights distribution **DJIA** (Shrinkage estimator)



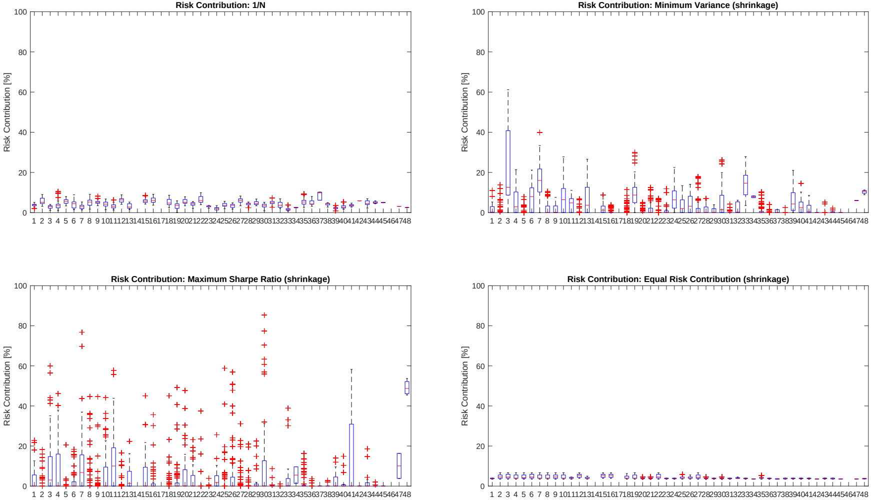
Comparison of risk-based optimization techniques

Results: Risk contribution **DJIA** (Shrinkage estimator)



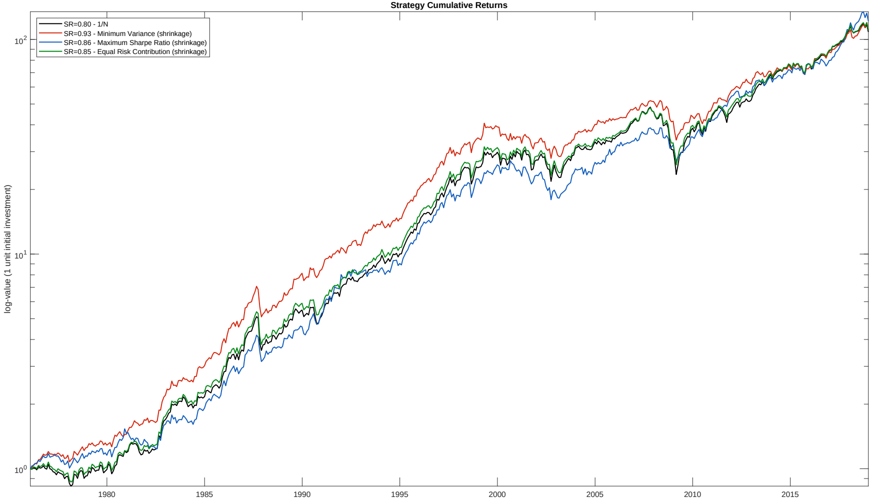
Comparison of risk-based optimization techniques

Results: Risk contribution **DJIA** (Shrinkage estimator)



Comparison of risk-based optimization techniques

Results: Performance **DJIA** (Shrinkage estimator)



Comparison of risk-based optimization techniques

Results: Summary statistics based on Shrinkage estimator

Lecture w/o Cash

Optimization technique	Years	SR	Ret p.a.	Worst	Best	Vol p.a.	Skew	Kurt
1/N	44	0.78	7.17	-0.14	0.09	9.46	-0.8	6.09
Minimum Variance (shrink..	44	1.5	4.58	-0.03	0.04	3.01	0.03	5
Maximum Sharpe Ratio (sh..	44	1.02	5.29	-0.1	0.07	5.2	-1.07	10.94
Equal Risk Contribution ..	44	1.22	5.89	-0.05	0.05	4.81	-0.35	5.17

Dow Jones Industrial Average 30

Optimization technique	Years	SR	Ret p.a.	Worst	Best	Vol p.a.	Skew	Kurt
1/N	43	0.8	11.51	-0.23	0.14	15.02	-0.52	5.57
Minimum Variance (shrink..	43	0.93	11.57	-0.17	0.13	12.69	-0.32	4.84
Maximum Sharpe Ratio (sh..	43	0.86	11.81	-0.14	0.17	14.12	-0.15	4.03
Equal Risk Contribution ..	43	0.85	11.57	-0.22	0.14	14.04	-0.52	5.55

Conclusion

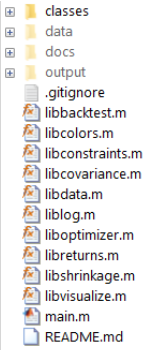
- The presented portfolio optimization techniques are sensitive to input parameters ($\mathbb{E}[r]$, Σ)
  - Some form of Shrinkage estimation necessary for robust portfolio allocation
- Equal Risk Contribution portfolios have attractive properties:
  - Focus on risk contribution to portfolio instead of returns
  - Robust weights, good diversification
  - Performs well for both, homogeneous and heterogeneous asset universes
  - No estimation of expected returns required!
- MV and MSR portfolios require weight constraints to circumvent concentration issue, not so much $1/N$ and ERC
- Choice of portfolio optimization technique depends on asset universe and strategy/diversification goals, no one-size-fits-all methodology

Outlook

- Research different covariance estimators
- Introduce weight constraints, especially important for MSR and MV portfolios
- Use sampling techniques to further manifest findings and reduce the influence of hyper-parameters
  - especially estimation windows, estimation intervals and returns frequencies (intraday, daily, weekly, etc.)
- Investigate other/more advanced risk-based optimization techniques
  - Cluster Risk Parity ([GitHub, JacobXPX (2018)])
  - Diversified Risk Parity ([Lohre et al. (2012)])

Code Repository

- GitHub repository: `https://github.com/rbeeli/portfolio_optimization_risk_indexation`
- Modular structure
- Parts easily extensible
  - datasets
  - expected returns estimators
  - covariance estimators (shrinkage)
  - (portfolio) optimizer
  - etc.
- Requires **Matlab R2019b** or higher, with *Optimization Toolbox* and *Financial Toolbox* installed



Q&A

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