firpm

Parks-McClellan optimal FIR filter design

Syntax

```
b = firpm(n,f,a)
b = firpm(n,f,a,w)
b = firpm(n,f,a,ftype)
b = firpm(n,f,a,lgrid)
[b,err] = firpm( ___ )
[b,err,res] = firpm( ___ )
b = firpm(n,f,fresp,w)
b = firpm(n,f,fresp,w,ftype)
```

Description

`b = firpm(n,f,a)` returns row vector `b` containing the `n+1` coefficients of an order-`n` FIR filter. The frequency and amplitude characteristics of the resulting filter match those given by vectors `f` and `a`. example

`b = firpm(n,f,a,w)` uses `w` to weigh the frequency bins. example

`b = firpm(n,f,a,ftype)` uses a filter type specified by `'ftype'`.

`b = firpm(n,f,a,lgrid)` uses the integer `lgrid` to control the density of the frequency grid.

`[b,err] = firpm( ___ )` returns the maximum ripple height in `err`. You can use this with any of the previous input syntaxes.

`[b,err,res] = firpm( ___ )` returns the frequency response characteristics as a structure `res`.

`b = firpm(n,f,fresp,w)` returns an FIR filter whose frequency-amplitude characteristics best approximate the response returned by function handle `fresp`.

`b = firpm(n,f,fresp,w,ftype)` designs antisymmetric (odd) filters, where `ftype` specifies the filter as a differentiator or Hilbert transformer. If you do not specify an `ftype`, a call is made to `fresp` to determine the default symmetry property.

Examples

collapse all

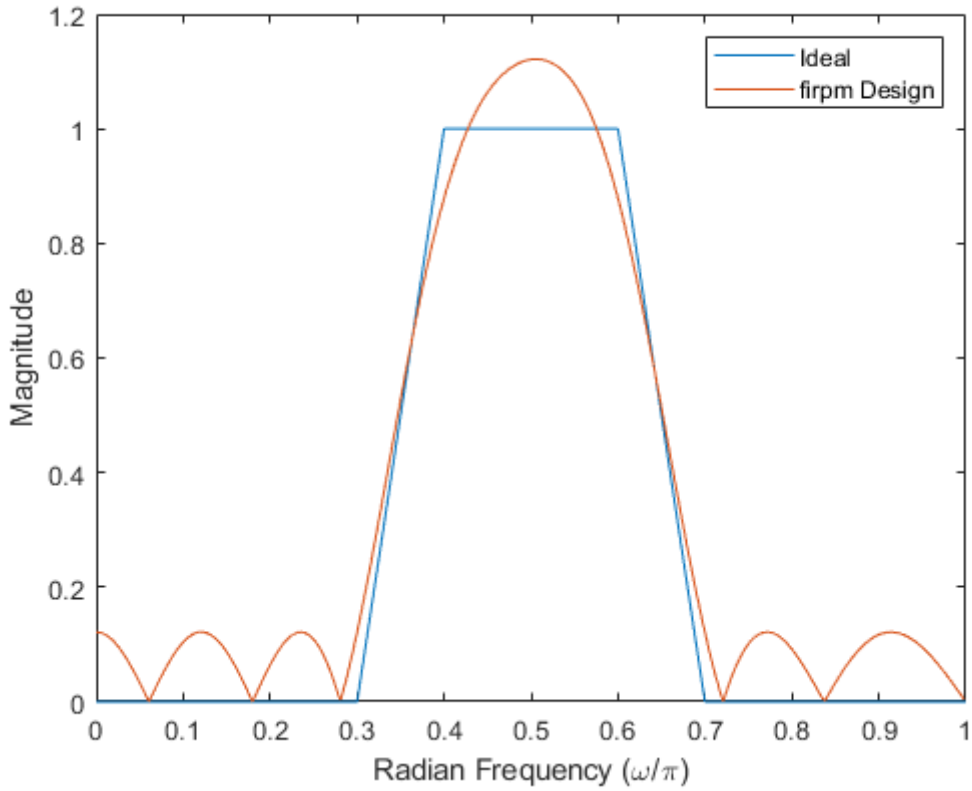
Parks-McClellan Bandpass Filter

Use the Parks-McClellan algorithm to design an FIR bandpass filter of order 17. Specify normalized stopband frequencies of 0.3π and 0.7π rad/sample and normalized passband frequencies of 0.4π and 0.6π rad/sample. Plot the ideal and actual magnitude responses.

View MATLAB Command

```
f = [0 0.3 0.4 0.6 0.7 1];
a = [0 0 1 1 0 0];
b = firpm(17,f,a);

[h,w] = freqz(b,1,512);
plot(f,a,w/pi,abs(h))
legend('Ideal','firpm Design')
xlabel 'Radian Frequency (\omega/\pi)', ylabel 'Magnitude'
```

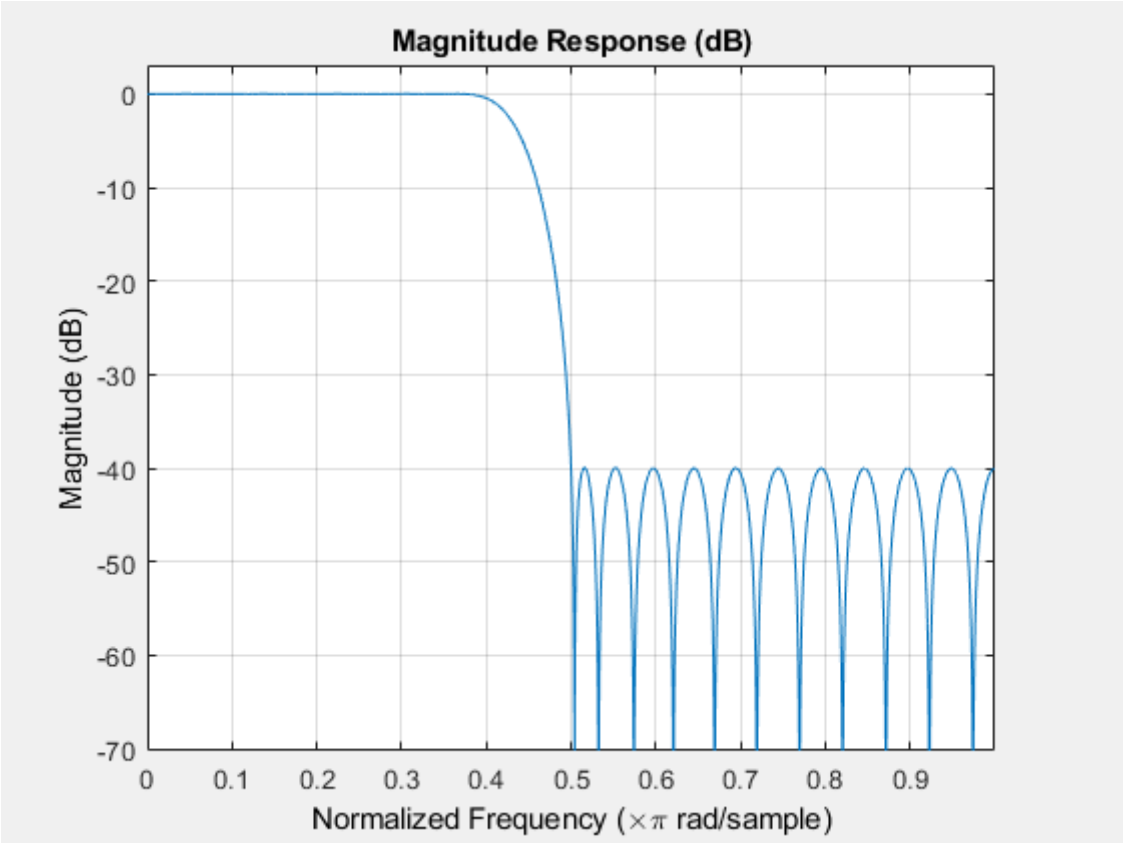


Parks-McClellan Lowpass Filter

View MATLAB Command

Design a lowpass filter with a 1500 Hz passband cutoff frequency and 2000 Hz stopband cutoff frequency. Specify a sampling frequency of 8000 Hz. Require a maximum stopband amplitude of 0.01 and a maximum passband error (ripple) of 0.001. Obtain the required filter order, normalized frequency band edges, frequency band amplitudes, and weights using `firpmord`.

```
[n,fo,ao,w] = firpmord([1500 2000],[1 0],[0.001 0.01],8000);
b = firpm(n,fo,ao,w);
fvtool(b,1)
```



FIR Bandpass Filter with Asymmetric Attenuation

Use the Parks-McClellan algorithm to create a 50th-order equiripple FIR bandpass filter to be used with signals sampled at 1 kHz.

View MATLAB Command

```
N = 50;
Fs = 1e3;
```

Specify that the passband spans the frequencies between 200 Hz and 300 Hz and that the transition region on either side of the passband has a width of 50 Hz.

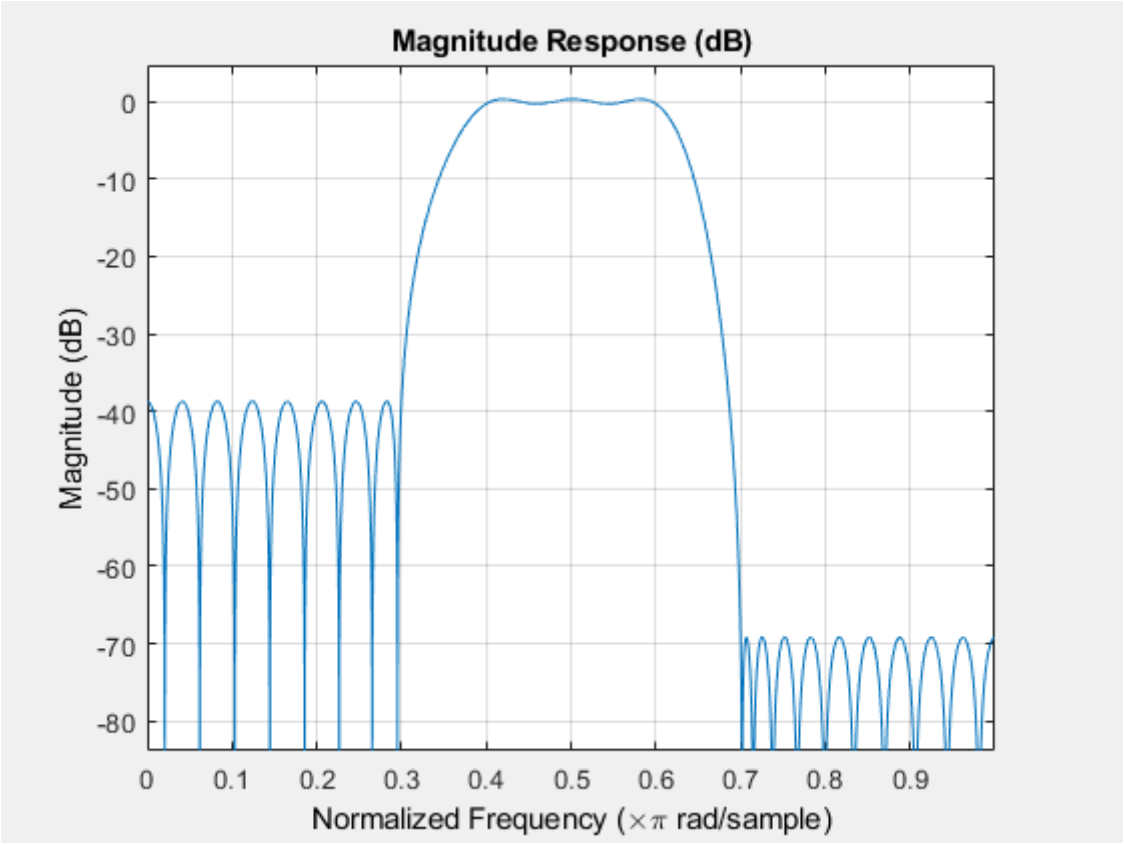
```
Fstop1 = 150;
Fpass1 = 200;
Fpass2 = 300;
Fstop2 = 350;
```

Design the filter so that the optimization fit weights the low-frequency stopband with a weight of 3, the passband with a weight of 1, and the high-frequency stopband with a weight of 100. Display the magnitude response of the filter.

```
Wstop1 = 3;
Wpass = 1;
Wstop2 = 100;

b = firpm(N,[0 Fstop1 Fpass1 Fpass2 Fstop2 Fs/2]/(Fs/2), ...
    [0 0 1 1 0 0],[Wstop1 Wpass Wstop2]);

fvtool(b,1)
```



Parks-McClellan Lowpass Filter

View MATLAB Command

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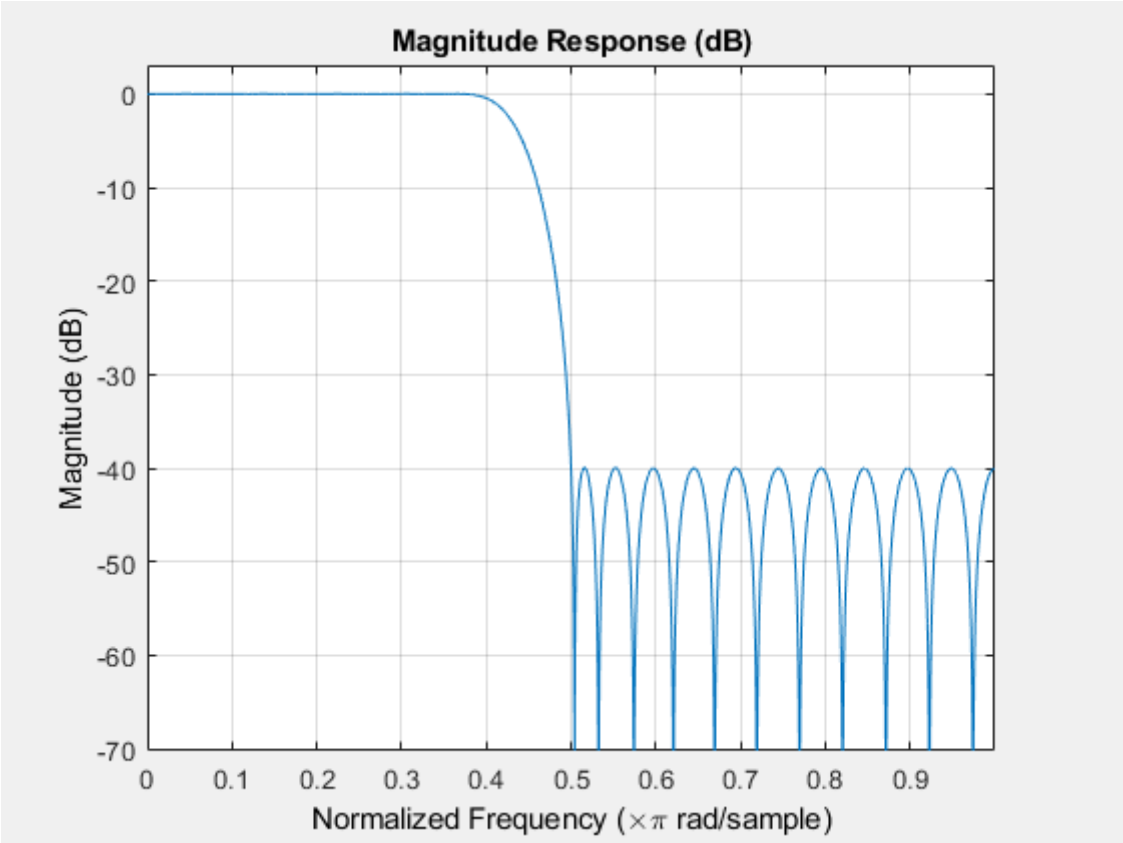
```
[n,fo,ao,w] = firpmord([1500 2000],[1 0],[0.001 0.01],8000);
b = firpm(n,fo,ao,w);
fvtool(b,1)
```



Input Arguments

collapse all

n — Filter order

real positive scalar

Filter order, specified as a real positive scalar.

f — Normalized frequency points

real-valued vector

Normalized frequency points, specified as a real-valued vector. The argument must be in the range [0, 1] , where 1 corresponds to the Nyquist frequency. The number of elements in the vector is always a multiple of 2. The frequencies must be in increasing order.

a — Desired amplitude

vector

Desired amplitudes at the points specified in `f`, specified as a vector. `f` and `a` must be the same length. The length must be an even number.

- The desired amplitude at frequencies between pairs of points ($f(k)$, $f(k+1)$) for k odd is the line segment connecting the points ($f(k)$, $a(k)$) and ($f(k+1)$, $a(k+1)$).
- The desired amplitude at frequencies between pairs of points ($f(k)$, $f(k+1)$) for k even is unspecified. The areas between such points are transition regions or regions that are not important for a particular application.

`w` — Weights
real-valued vector

Weights used to adjust the fit in each frequency band, specified as a real-valued vector. The length of `w` is half the length of `f` and `a`, so there is exactly one weight per band.

`ftype` — Filter type
`'hilbert'` | `'differentiator'`

Filter type for linear-phase filters with odd symmetry (type III and type IV), specified as either `'hilbert'` or `'differentiator'`:

- `'hilbert'` — The output coefficients in `b` obey the relation $b(k) = -b(n + 2 - k)$, $k = 1, \ldots, n + 1$. This class of filters includes the Hilbert transformer, which has a desired amplitude of 1 across the entire band.

  For example,

  ```
  h = firpm(30,[0.1 0.9],[1 1],'hilbert');
  ```

  designs an approximate FIR Hilbert transformer of length 31.
- `'differentiator'` — For nonzero amplitude bands, the filter weighs the error by a factor of $1/f$ so that the error at low frequencies is much smaller than at high frequencies. For FIR differentiators, which have an amplitude characteristic proportional to frequency, these filters minimize the maximum relative error (the maximum of the ratio of the error to the desired amplitude).

`lgrid` — Density of frequency grid
16 (default) | 1-by-1 cell array with integer value

Control the density of the frequency grid, which has roughly `(lgrid*n)/(2*bw)` frequency points, where `bw` is the fraction of the total frequency band interval [0,1] covered by `f`. Increasing `lgrid` often results in filters that more exactly match an equiripple filter, but that take longer to compute. The default value of `16` is the minimum value that should be specified for `lgrid`.

`fresp` — Frequency response
function handle

Frequency response, specified as a function handle. The function is called from within `firpm` with this syntax:

```
[dh,dw] = fresp(n,f,gf,w)
```

The arguments are similar to those for `firpm`:

- `n` is the filter order.
- `f` is the vector of normalized frequency band edges that appear monotonically between 0 and 1, where 1 is the Nyquist frequency.
- `gf` is a vector of grid points that have been linearly interpolated over each specified frequency band by `firpm`. `gf` determines the frequency grid at which the response function must be evaluated, and contains the same data returned by `cfirpm` in the `fgrid` field of the `opt` structure.
- `w` is a vector of real, positive weights, one per band, used during optimization. `w` is optional in the call to `firpm`; if not specified, it is set to unity weighting before being passed to `fresp`.
- `dh` and `dw` are the desired complex frequency response and band weight vectors, respectively, evaluated at each frequency in grid `gf`.

Output Arguments

collapse all

b — Filter coefficients

row vector

Filter coefficients, returned as a row vector of length n + 1. The coefficients are in increasing order.

err — Maximum ripple height

scalar

Maximum ripple height, returned as a scalar.

res — Frequency response characteristics

structure

Frequency response characteristics, returned as a structure. The structure `res` has the following fields:

`res.fgrid`	Frequency grid vector used for the filter design optimization
`res.des`	Desired frequency response for each point in `res.fgrid`
`res.wt`	Weighting for each point in `opt.fgrid`
`res.H`	Actual frequency response for each point in `res.fgrid`
`res.error`	Error at each point in `res.fgrid` (`res.des-res.H`)
`res.iextr`	Vector of indices into `res.fgrid` for extremal frequencies
`res.fextr`	Vector of extremal frequencies

Tips

If your filter design fails to converge, the filter design might not be correct. Verify the design by checking the frequency response.

If your filter design fails to converge and the resulting filter design is not correct, attempt one or more of the following:

- Increase the filter order.
- Relax the filter design by reducing the attenuation in the stopbands and/or broadening the transition regions.

Algorithms

`firpm` designs a linear-phase FIR filter using the Parks-McClellan algorithm [2]. The Parks-McClellan algorithm uses the Remez exchange algorithm and Chebyshev approximation theory to design filters with an optimal fit between the desired and actual frequency responses. The filters are optimal in the sense that the maximum error between the desired frequency response and the actual frequency response is minimized. Filters designed this way exhibit an equiripple behavior in their frequency responses and are sometimes called equiripple filters. `firpm` exhibits discontinuities at the head and tail of its impulse response due to this equiripple nature.

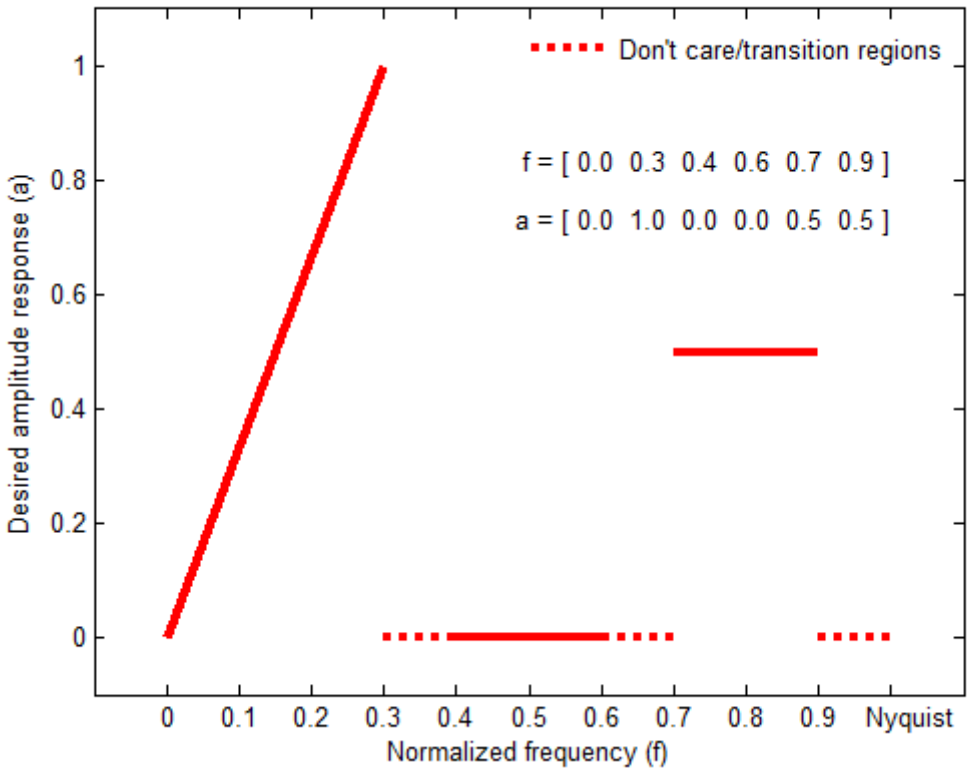
These are type I (`n` odd) and type II (`n` even) linear-phase filters. Vectors `f` and `a` specify the frequency-amplitude characteristics of the filter:

- `f` is a vector of pairs of frequency points, specified in the range between 0 and 1, where 1 corresponds to the Nyquist frequency. The frequencies must be in increasing order. Duplicate frequency points are allowed and, in fact, can be used to design a filter exactly the same as those returned by the `fir1` and `fir2` functions with a rectangular (`rectwin`) window.
- `a` is a vector containing the desired amplitude at the points specified in `f`.

  The desired amplitude function at frequencies between pairs of points ($f(k)$, $f(k+1)$) for k odd is the line segment connecting the points ($f(k)$, $a(k)$) and ($f(k+1)$, $a(k+1)$).

  The desired amplitude function at frequencies between pairs of points ($f(k)$, $f(k+1)$) for k even is unspecified. These are transition or “don’t care” regions.
- `f` and `a` are the same length. This length must be an even number.

The figure below illustrates the relationship between the `f` and `a` vectors in defining a desired amplitude response.



`firpm` always uses an even filter order for configurations with even symmetry and a nonzero passband at the Nyquist frequency. The reason for the even filter order is that for impulse responses exhibiting even symmetry and odd orders, the frequency response at the Nyquist frequency is necessarily 0. If you specify an odd-valued `n`, `firpm` increments it by 1.

`firpm` designs type I, II, III, and IV linear-phase filters. Type I and type II are the defaults for `n` even and `n` odd, respectively, while type III (`n` even) and type IV (`n` odd) are specified with `'hilbert'` or `'differentiator'`, respectively, using the `ftype` argument.. The different types of filters have different symmetries and certain constraints on their frequency responses. (See [3] for more details.)

Linear Phase Filter Type	**Filter Order**	**Symmetry of Coefficients**	**Response $H(f), f = 0$**	**Response $H(f)$, $f = 1$ (Nyquist)**
Type I	Even	even: $b(k) = b(n+2-k), \quad k = 1, ..., n+1$	No restriction	No restriction

Linear Phase Filter Type	Filter Order	Symmetry of Coefficients	Response $H(f), f = 0$	Response $H(f)$, $f = 1$ (Nyquist)
Type II	Odd	even: $b(k) = b(n+2-k), \quad k = 1, ..., n+1$	No restriction	$H(1) = 0$ `firpm` increments the filter order by 1 if you attempt to construct a type II filter with a nonzero passband at the Nyquist frequency.
Type III	Even	odd: $b(k) = -b(n+2-k), \quad k = 1, ..., n+1$	$H(0) = 0$	$H(1) = 0$
Type IV	Odd	odd: $b(k) = -b(n+2-k), \quad k = 1, ..., n+1$	$H(0) = 0$	No restriction

You can also use `firpm` to write a function that defines the desired frequency response. The predefined frequency response function handle for `firpm` is `@firpmfrf`, which designs a linear-phase FIR filter.

Note

`b = firpm(n,f,a,w)` is equivalent to `b = firpm(n,f,{@firpmfrf,a},w)`, where, `@firpmfrf` is the predefined frequency response function handle for `firpm`. If desired, you can write your own response function. Use `help private/firpmfrf` and see Create Function Handle (MATLAB) for more information.

References

[1] Digital Signal Processing Committee of the IEEE Acoustics, Speech, and Signal Processing Society, eds. *Selected Papers in Digital Signal Processing*. Vol. II. New York: IEEE Press, 1976.

[2] Digital Signal Processing Committee of the IEEE Acoustics, Speech, and Signal Processing Society, eds. *Programs for Digital Signal Processing*. New York: IEEE Press, 1979, algorithm 5.1.

[3] Oppenheim, Alan V., Ronald W. Schafer, and John R. Buck. *Discrete-Time Signal Processing*. Upper Saddle River, NJ: Prentice Hall, 1999, p. 486.

[4] Parks, Thomas W., and C. Sidney Burrus. *Digital Filter Design*. New York: John Wiley & Sons, 1987, p. 83.

[5] Rabiner, Lawrence R., James H. McClellan, and Thomas W. Parks. "FIR Digital Filter Design Techniques Using Weighted Chebyshev Approximation." *Proceedings of the IEEE®*. Vol. 63, Number 4, 1975, pp. 595–610.

Extended Capabilities

C/C++ Code Generation

Generate C and C++ code using MATLAB® Coder™.

See Also

`butter` | `cfirpm` | `cheby1` | `cheby2` | `ellip` | `fir1` | `fir2` | `fircls` | `fircls1` | `firls` | `firpmord` | `rcosdesign` | `yulewalk`

Introduced before R2006a