MPC-Friendly Symmetric Key Primitives

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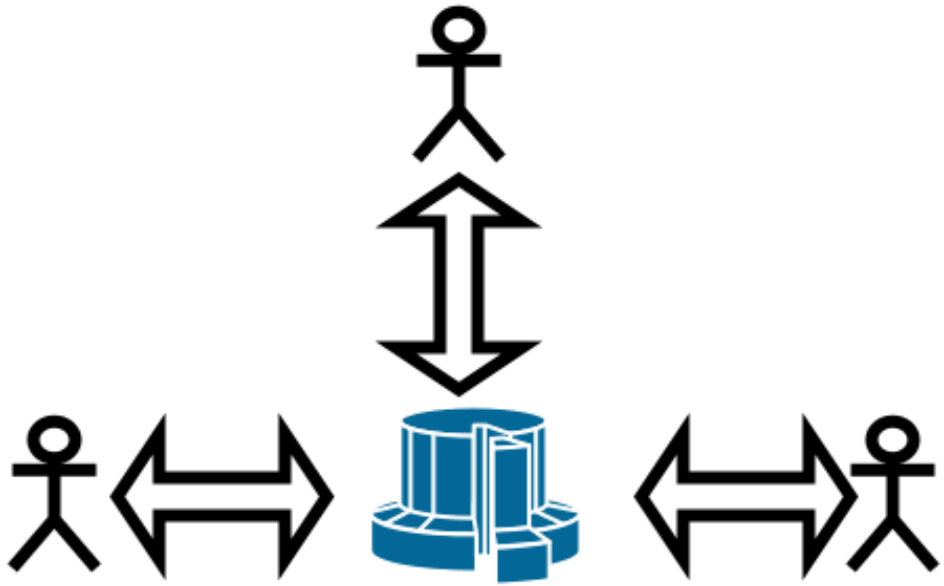
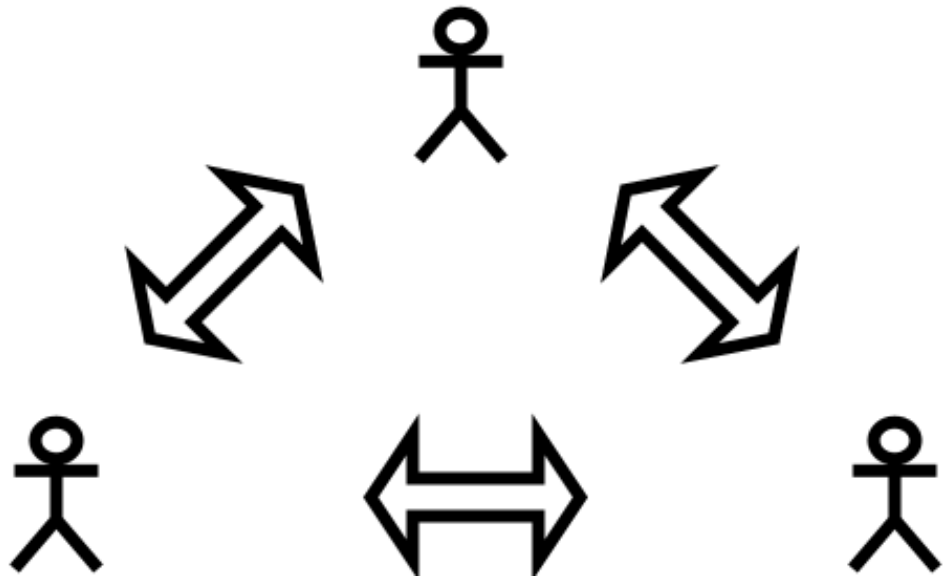
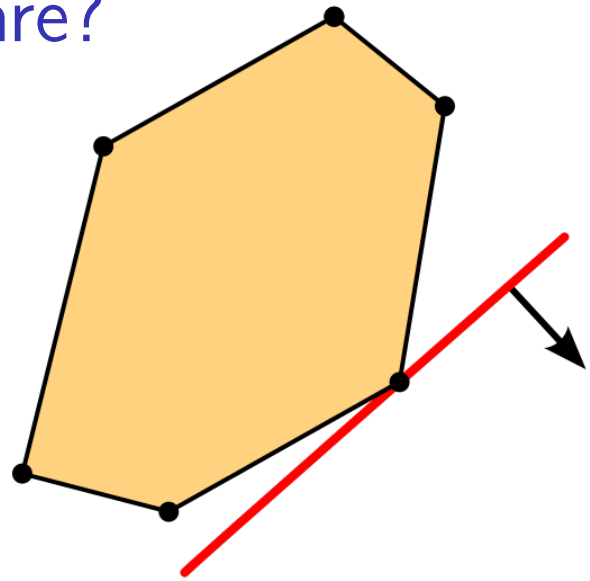
What is Multiparty Computation?



What is Multiparty Computation?

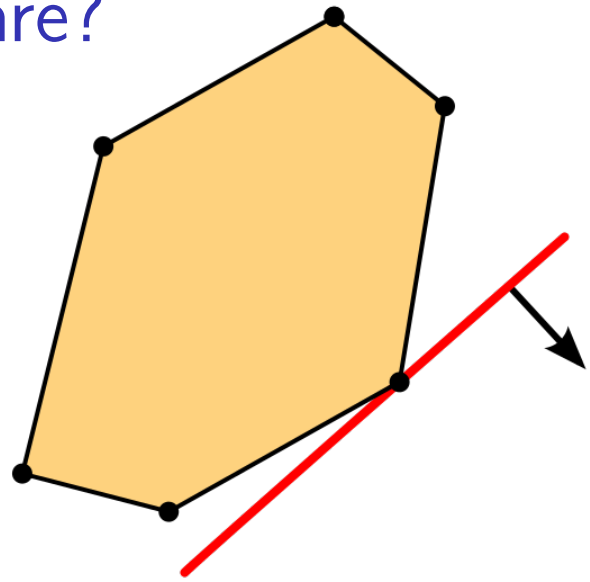


What do these share?



Linear Programming

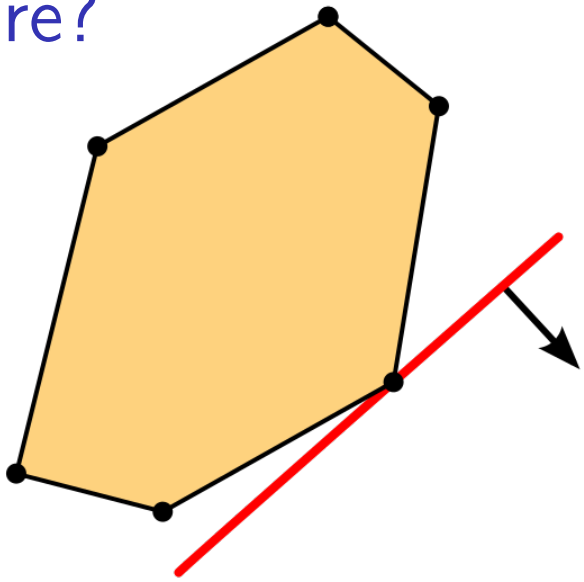
What do these share?



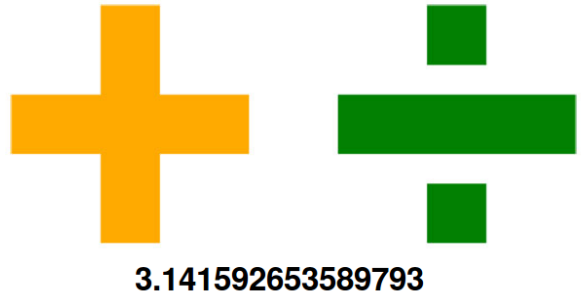
Linear Programming

Integer Comparison

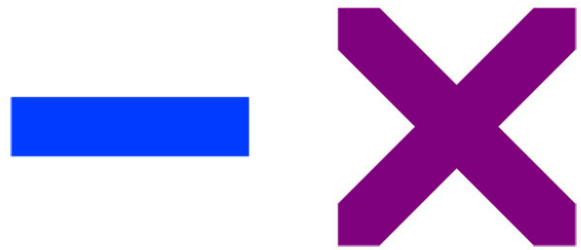
What do these share?



Linear Programming

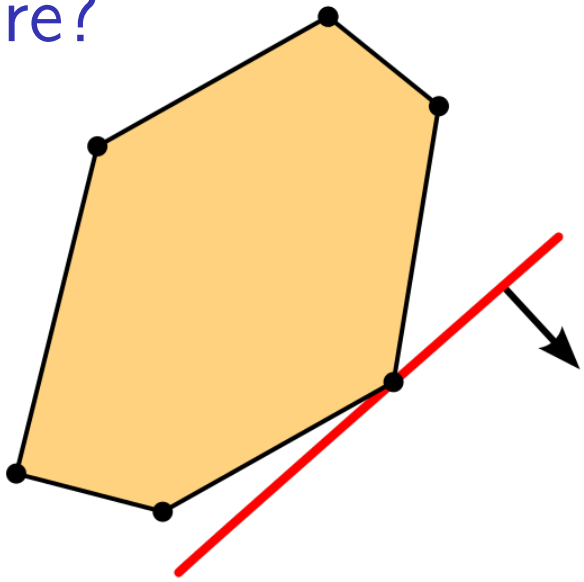


Integer Comparison



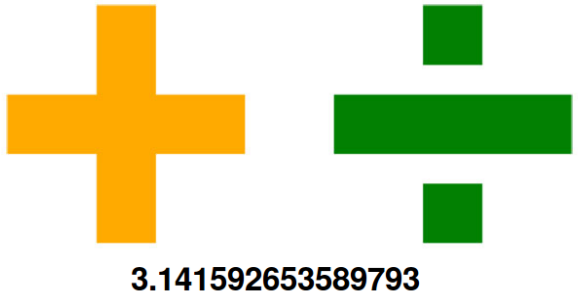
Fixed Point Arithmetic

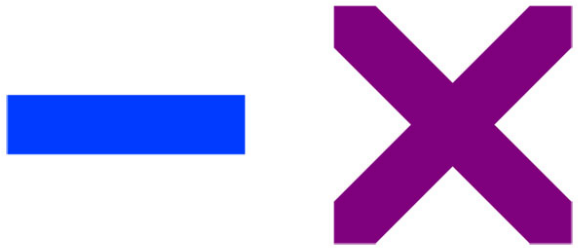
What do these share?



Linear Programming

Integer Comparison



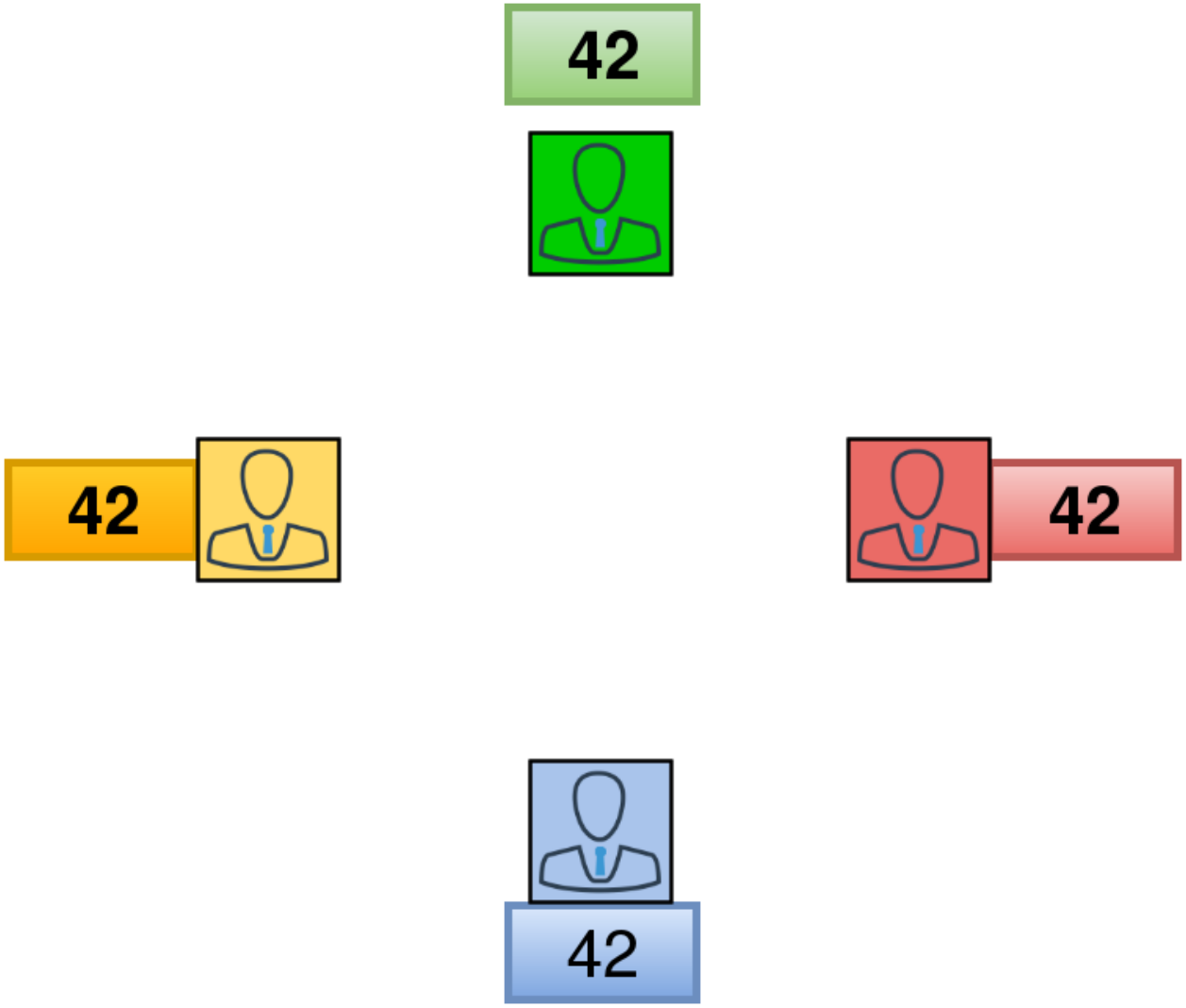


Fixed Point Arithmetic

What do these share?

Arithmetic circuits mod p

Where is the problem?



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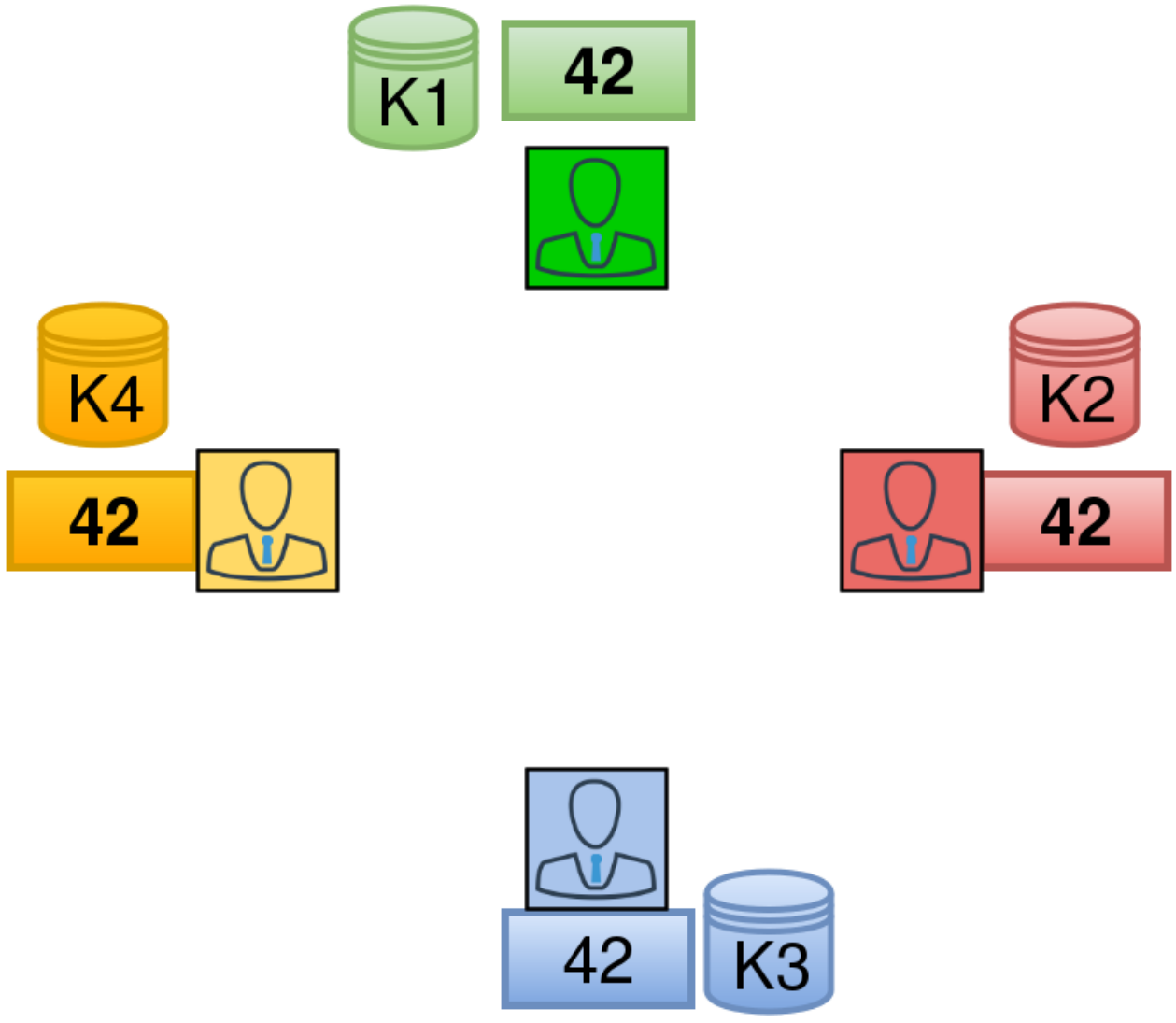

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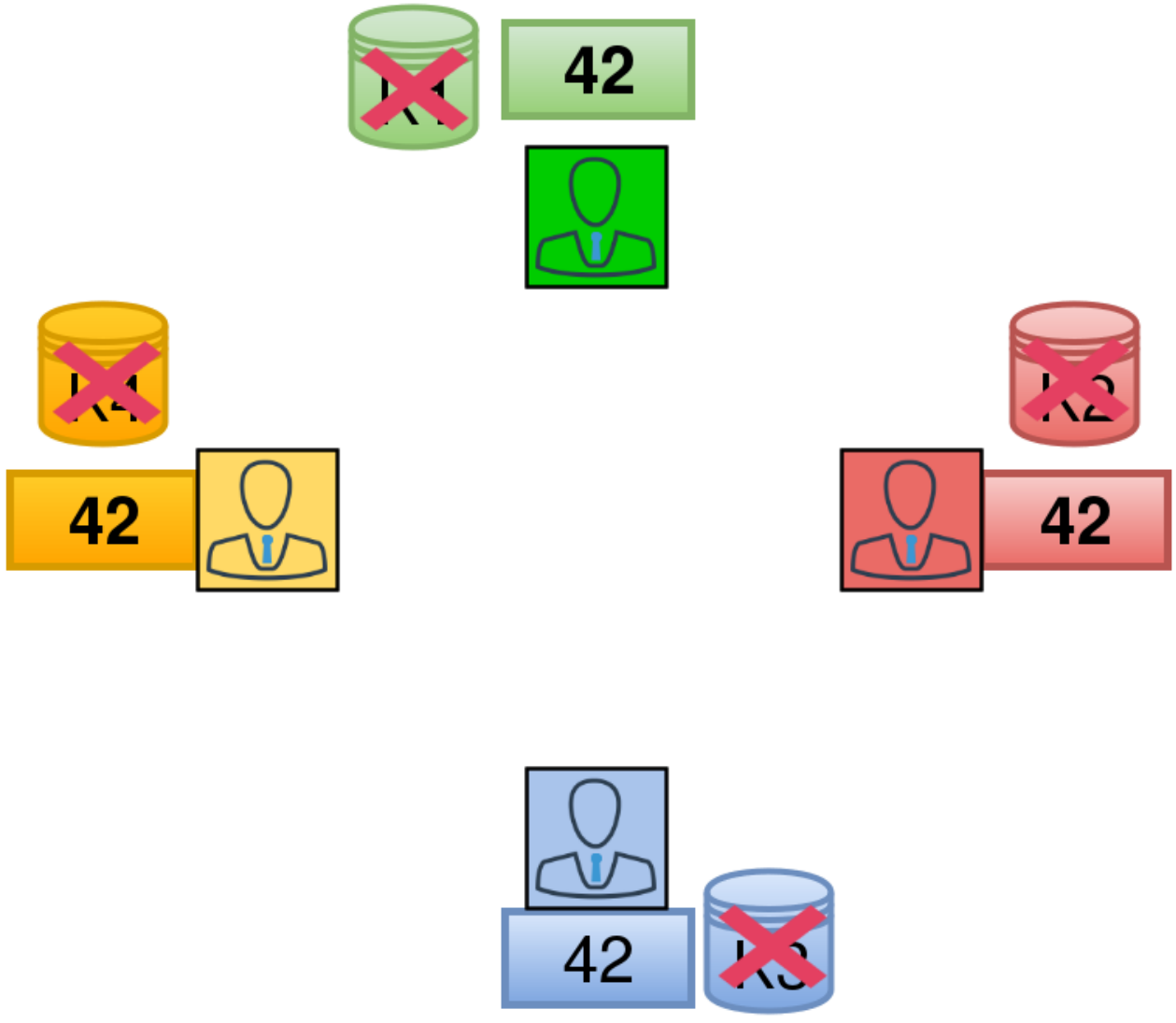

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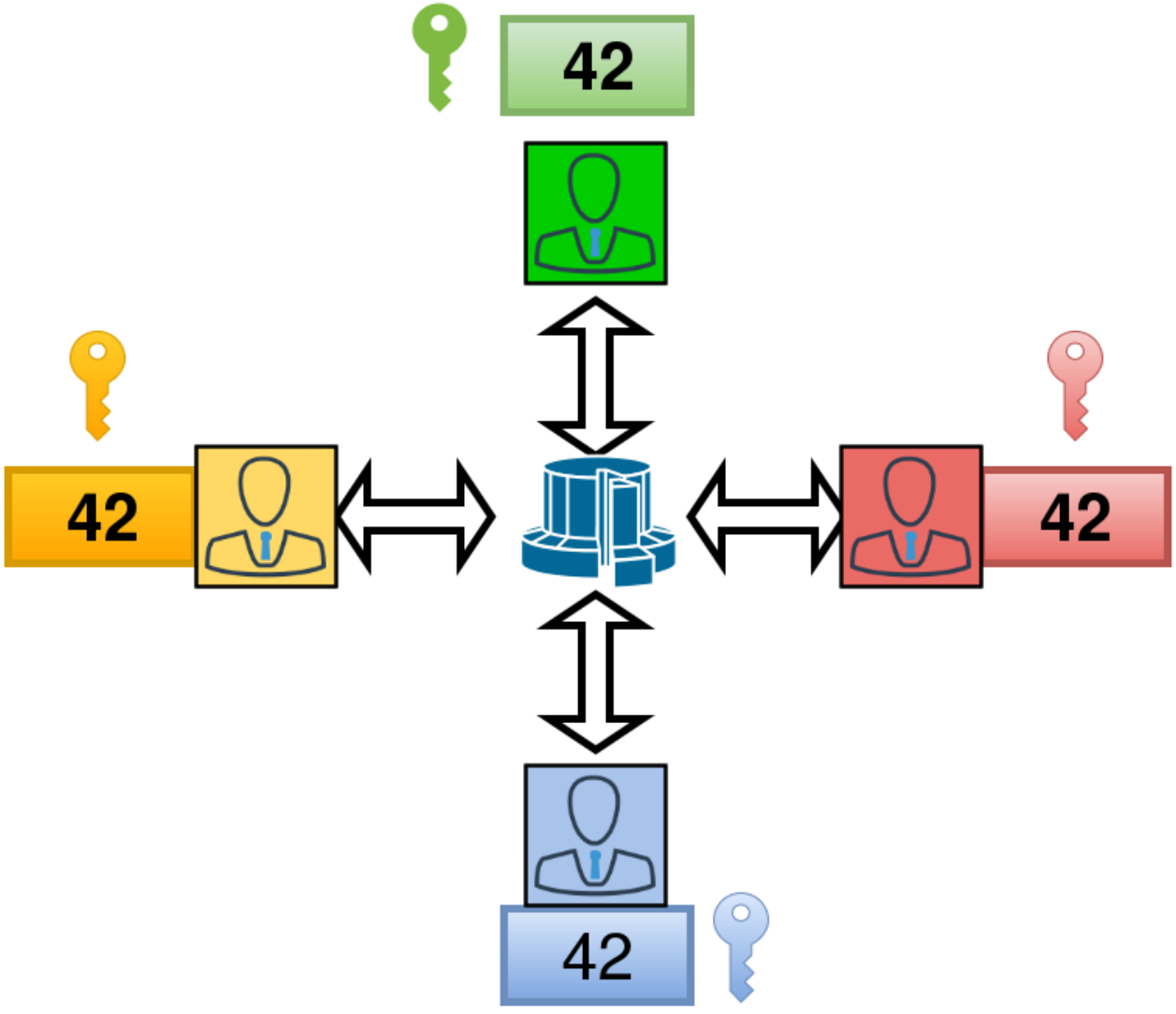


Why?

- Avoid the n fold database blowup by secret share the key and use a PRF mod p in MPC!
- Why mod p? Conversion between binary and arithmetic shares is expensive.

Other use cases for oblivious PRF's

- Secure database joins [LTW13]
- Oblivious RAM [LO13]
- Searchable symmetric encryption, order-revealing encryption [BCO'N11, BLRSZZ15, CLWW16, BBO'N07, CJJKRS13]

What we have done

Benchmark and "tune" different PRF's within SPDZ protocol.

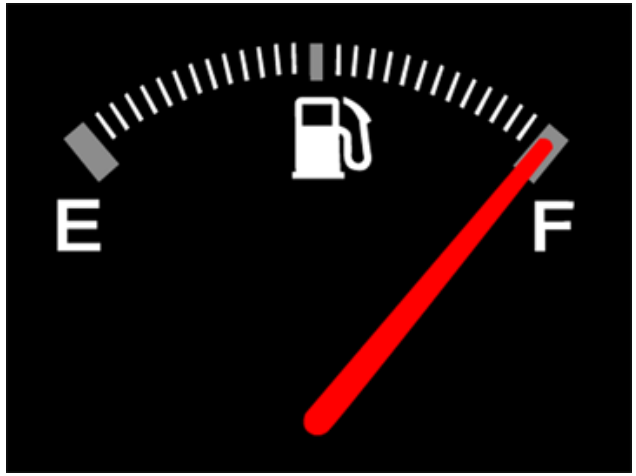
MPC with secret sharing 101

- Triples generation:
  $[a] = [b] \cdot [c]$
- Random bits and squares:
  $[b]$, $[s^2]$.

Offline Phase

MPC with secret sharing 101

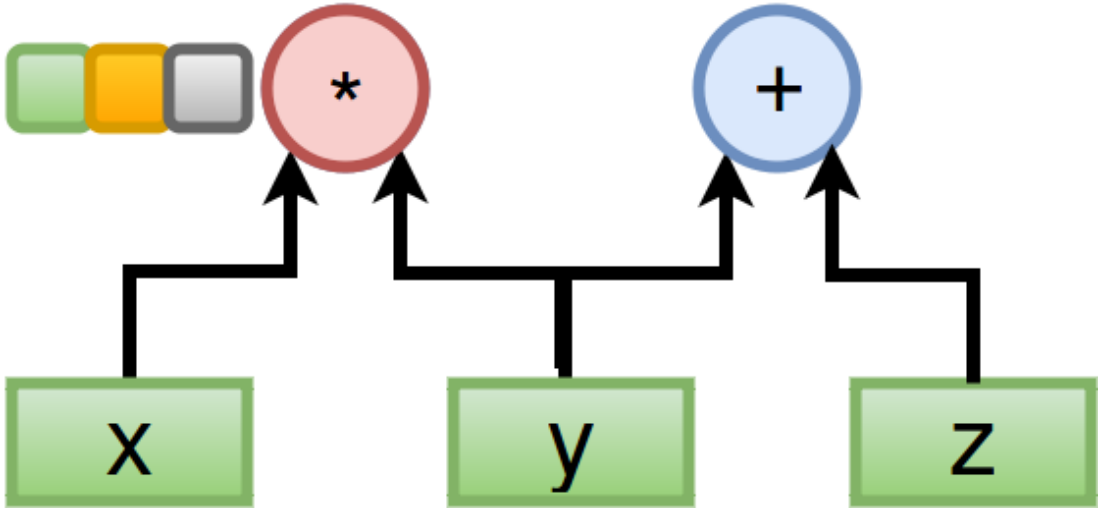
- Use 1 triple for each multiplication gate.
- Number of communcation rounds is given by the circuit depth.



Online Phase

Circuit Evaluation in SPDZ

Circuit Evaluation in SPDZ



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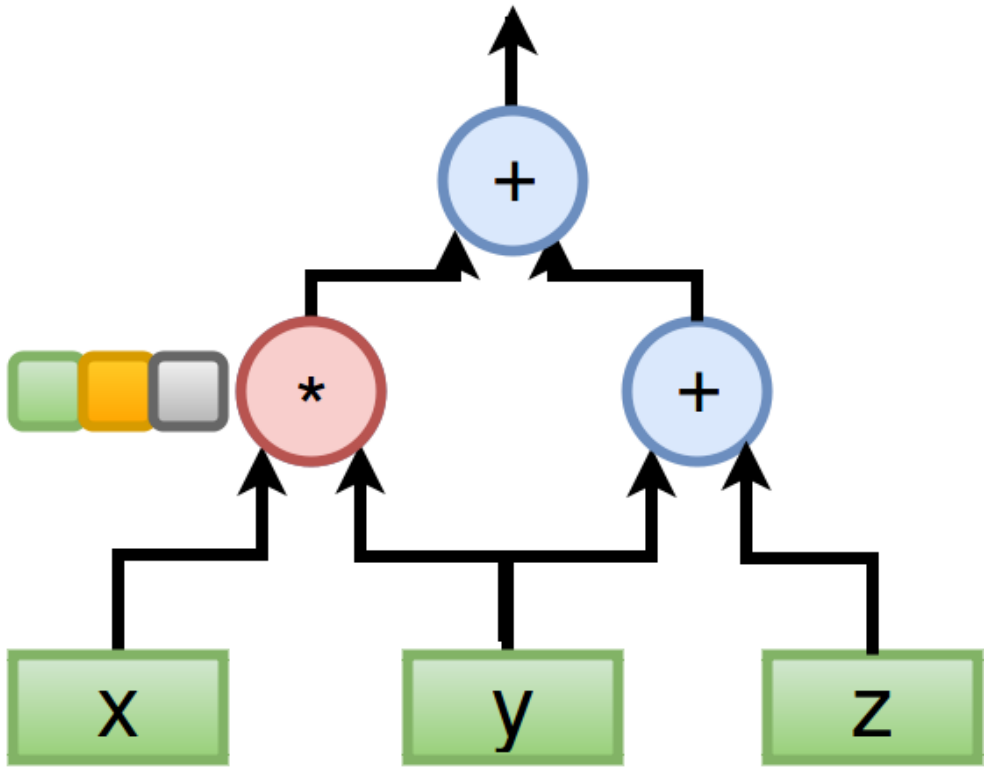

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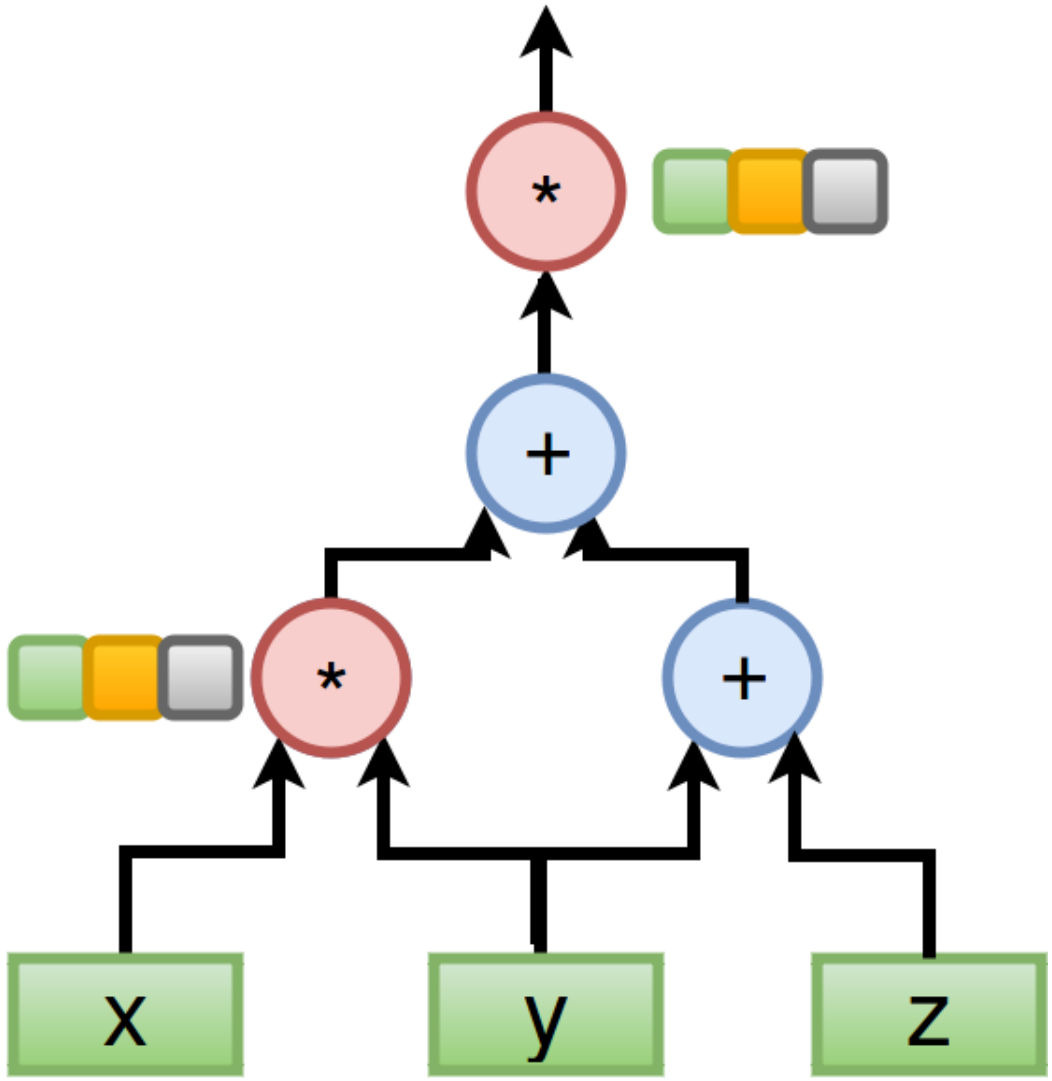

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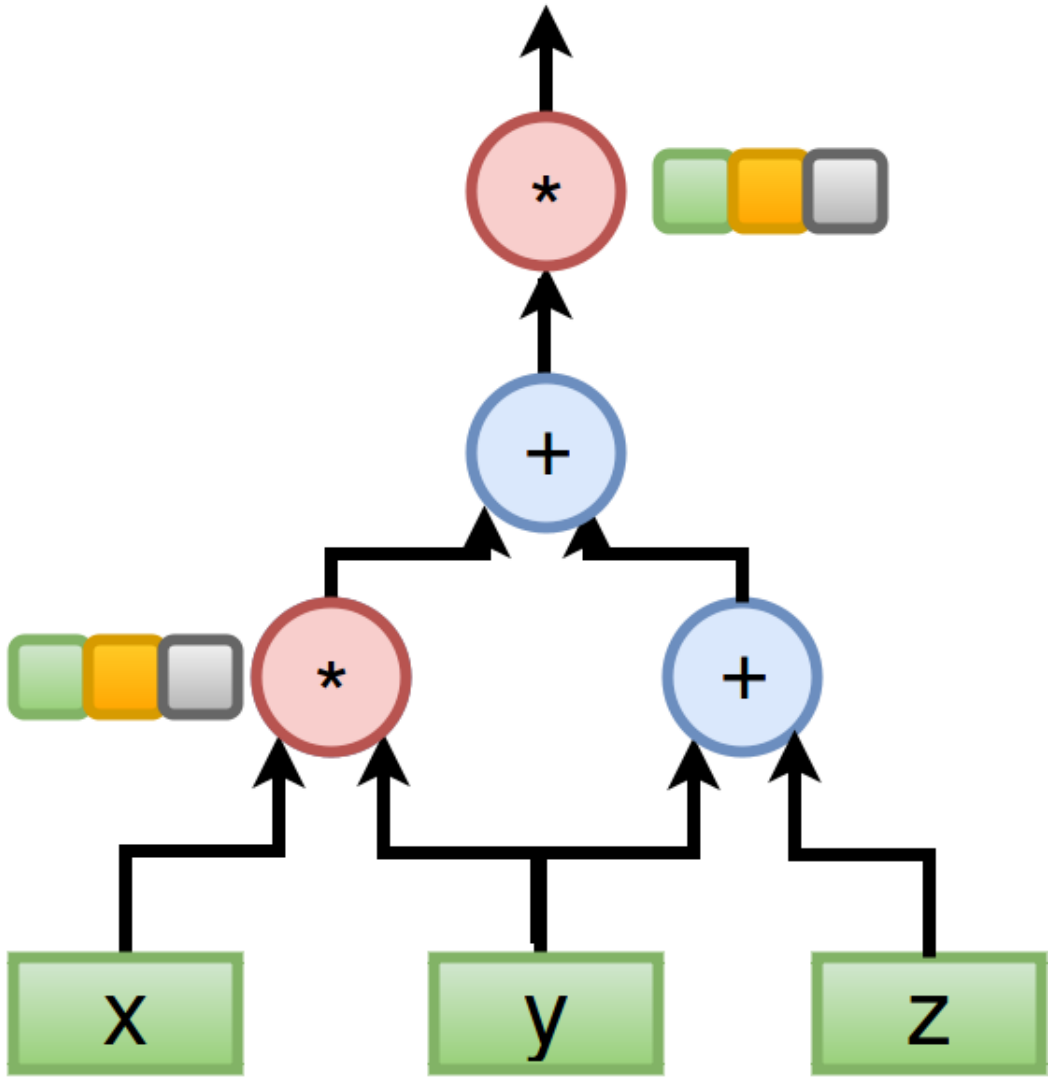

2 triples; 3 rounds.

What PRF's we looked at?

- AES [DR01]
- LowMC (Low Multiplicative Complexity) [ARS+15]
- Naor-Reingold PRF [NR04]
- MiMC (Minimum Multiplicative Complexity) [AGR+16]
- Legendre PRF [Dåm88]

Let's play a game

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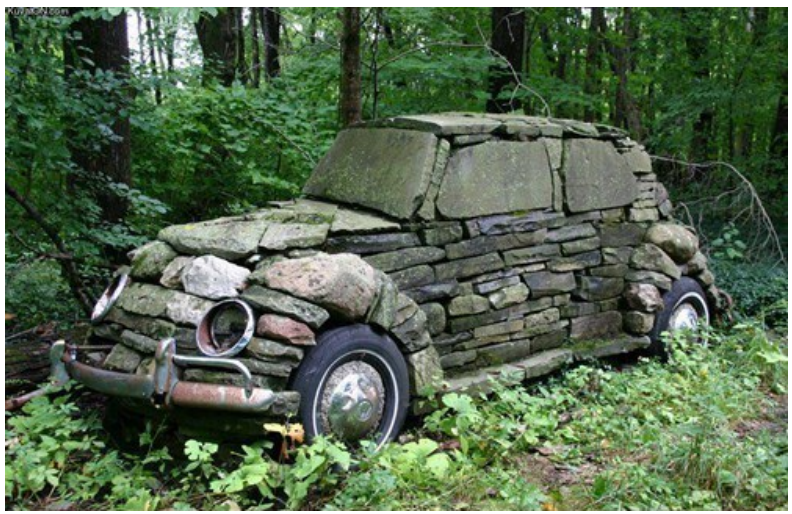
AES - de-facto benchmark

- 960 multiplications
- 50 rounds



PRF on blocks

AES - de-facto benchmark

- 960 multiplications
- 50 rounds

PRF on blocks

5 ops/s

AES - de-facto benchmark

- 960 multiplications
- 50 rounds

PRF on blocks

8ms latency

AES - de-facto benchmark

- 960 multiplications
- 50 rounds

PRF on blocks

530 ops/s throughput

LowMC - How does it work?

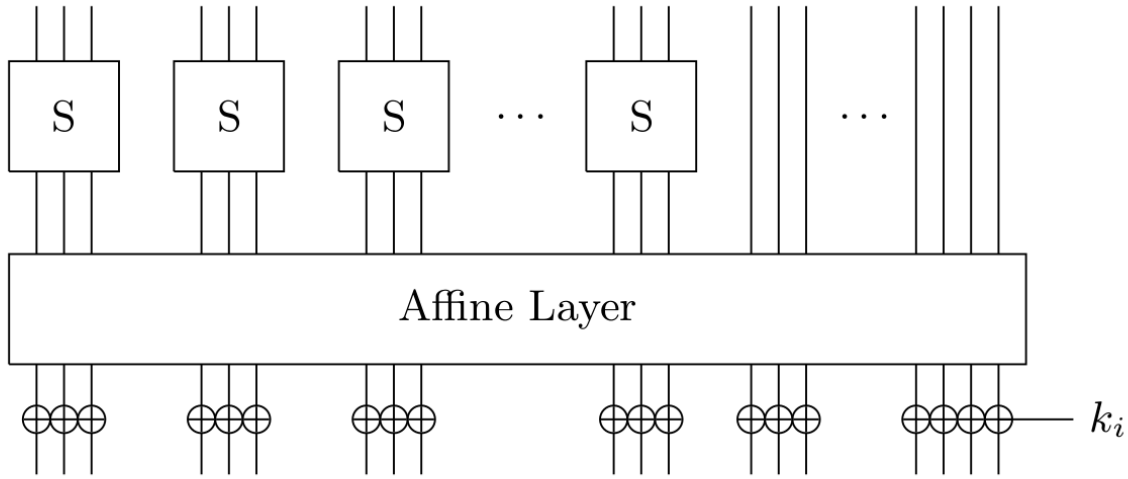
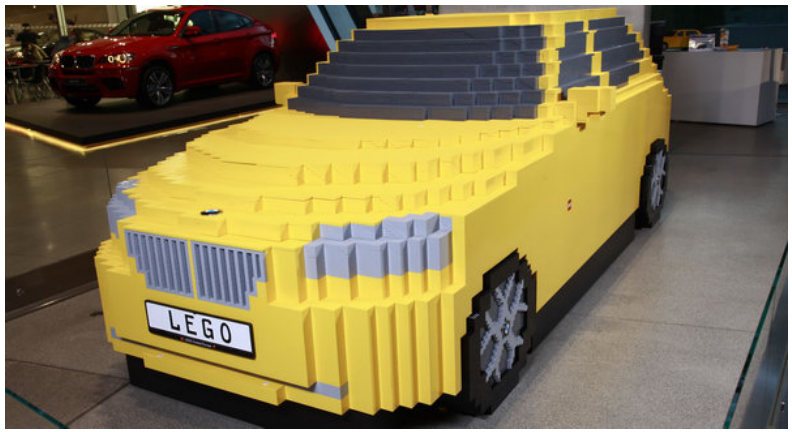


Fig. 1. Depiction of one round of encryption with LowMC.

[ARS+15]

LowMC - PRF for MPC and FHE

- 1911 multiplications
- 13 rounds



PRF on blocks

LowMC - PRF for MPC and FHE

- 1911 multiplications
- 13 rounds

PRF on blocks

2.5 ops/s - 2x slower than AES

LowMC - PRF for MPC and FHE

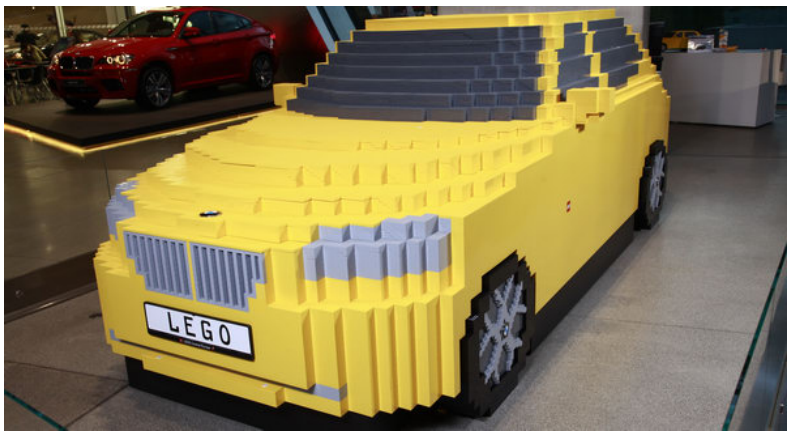
- 1911 multiplications
- 13 rounds

PRF on blocks

4ms latency - 2x faster than AES

LowMC - PRF for MPC and FHE

- 1911 multiplications
- 13 rounds



PRF on blocks

590 ops/s throughput

Naor-Reingold PRF

$$F_{\mathrm{NR}(n)}(\mathbf{k}, \mathbf{x}) = \mathrm{encode}(g^{k_0 \cdot \prod_{i=1}^{n} k_i^{x_i}})$$

where $\mathbf{k} = (k_0, \ldots, k_n) \in \mathbb{F}_p^{n+1}$ is the key.

Naor-Reingold PRF

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where $\mathbf{k} = (k_0, \ldots, k_n) \in \mathbb{F}_p^{n+1}$ is the key.

Fortunately, in some applications the output must be public!

Naor-Reingold PRF

- Active security version for public output.
- $2 \cdot n$ multiplications.
- $3 + \log n + 1$ rounds.

EC based PRF

Naor-Reingold PRF

- Active security version for public output.
- $4n + 2$ multiplications.
- 7 rounds.

EC based PRF in constant round

Naor-Reingold PRF

- Active security version for public output.
- $4n + 2$ multiplications.
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EC based PRF in constant round

5 ops/s

Naor-Reingold PRF

- Active security version for public output.
- $4n + 2$ multiplications.
- 7 rounds.

EC based PRF in constant round

4.3ms latency

Naor-Reingold PRF

- Active security version for public output.
- $4n + 2$ multiplications.
- 7 rounds.

EC based PRF in constant round

370 ops/s throughput

Naor-Reingold PRF

- Active security version for public output.
- $4n + 2$ multiplications.
- 7 rounds.

EC based PRF in constant round

Results have shown that over 70% of the time was spent on EC computations.
Computation is the bottleneck, not communication!

Security of Naor-Reingold PRF

Is it secure?

Security of Naor-Reingold PRF

Is it secure?

Yes!.

With probability 2^{-128} an adversary can pass the check with an incorrect share.

MiMC - How does it work?

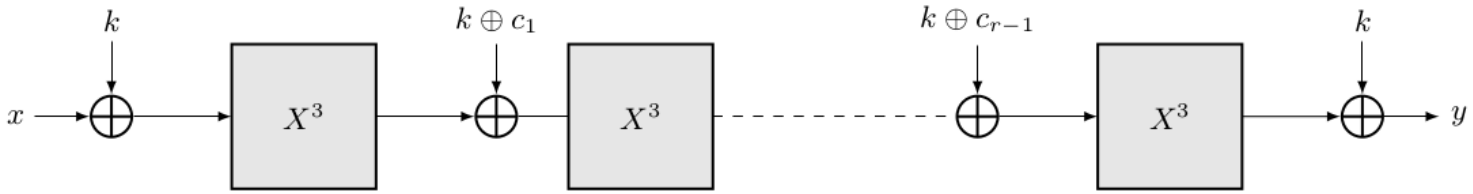


Fig. 1: r rounds of MiMC-n/n

[AGR+16]

MiMC PRF

- 146 multiplications
- 73 rounds
- 1 variant optimized for latency, other for throughput.

MiMC PRF - works in both worls

MiMC PRF

- 146 multiplications
- 73 rounds
- 1 variant optimized for latency, other for throughput.

MiMC PRF - works in both worls

34 ops/s

MiMC PRF

- 146 multiplications
- 73 rounds
- 1 variant optimized for latency, other for throughput.

MiMC PRF - works in both worls

6ms latency

MiMC PRF

- 146 multiplications
- 73 rounds
- 1 variant optimized for latency, other for throughput.

MiMC PRF - works in both worls

9000 ops/s throughput - **16x** AES

Security of MiMC

Is it secure?

Security of MiMC

Is it secure?

Yes, for now!

Legendre PRF

In 1988, Damgård conjectured that this sequence is pseuodarandom starting from a random seed k.

$$\left(\frac{k}{p}\right), \left(\frac{k+1}{p}\right), \left(\frac{k+2}{p}\right), \ldots$$

Legendre PRF

- $\log p$ multiplications.
- $\log p$ rounds.

Legendre PRF - old version

Legendre PRF

- ~~$\log p$~~ 2 multiplications.
- ~~$\log p$~~ 3 rounds.

Legendre PRF - new version

Legendre PRF

- ~~$\log p$~~ 2 multiplications.
- ~~$\log p$~~ 3 rounds.

Legendre PRF - new version

1225 ops/s - **250x** AES

Legendre PRF

- ~~log p~~ 2 multiplications.
- ~~log p~~ 3 rounds.

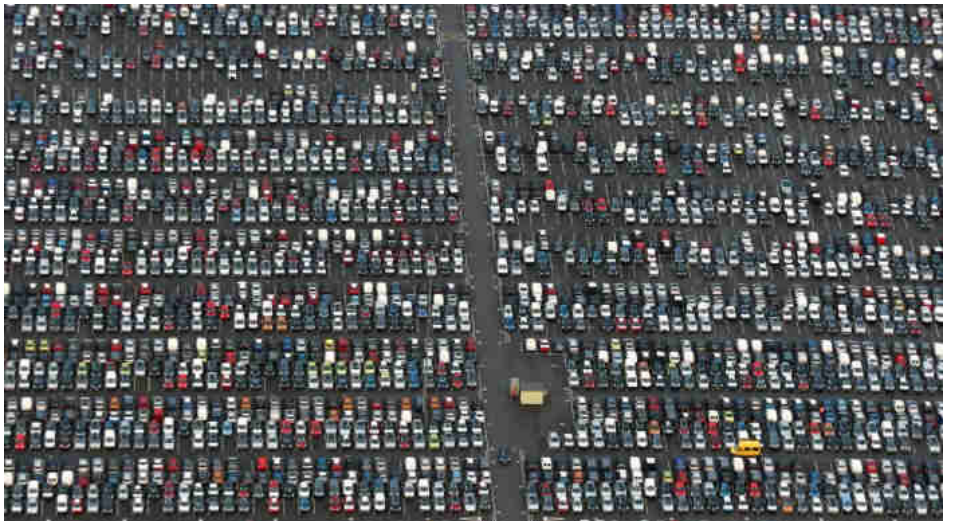
Legendre PRF - new version

0.3ms latency - **25x** faster AES

Legendre PRF

- ~~$\log p$~~ 2 multiplications.
- ~~$\log p$~~ 3 rounds.

Legendre PRF - new version



202969 ops/s throughput - **380x** AES

Security of Legendre PRF

Is it secure?

Security of Legendre PRF

Is it secure?

Yes, even against Quantum Computers! - for now

How does it work?

Protocol Π_{Legendre}

Let α be a fixed, quadratic non-residue modulo p.

Eval: To evaluate $F_{\text{Leg(bit)}}$ on input $[x]$ with key $[k]$:

1. Take a random square $[s^2]$ and a random bit $[b]$
2. $[t] \leftarrow [s^2] \cdot ([b] + \alpha \cdot (1 - [b]))$
3. $u \leftarrow \text{Open}([t] \cdot ([k] + [x]))$
4. Output $[y] \leftarrow (\left(\frac{u}{p}\right) \cdot (2[b] - 1) + 1)/2$

Securely computing the $F_{\text{Leg(bit)}}$ PRF with secret-shared output

Summary

- Oblivious PRF's mod p are fast! Can you find other applications built on top of these?
- Using special preprocessed data can help you often in SPDZ online phase.
- For proofs, WAN timings, other details, check out our paper!

Thank you!

Thank you!
Questions?

LAN results

PRF	Best latency	Best throughput		Prep. (ops/s)	Cleartext (ops/s)
	(ms/op)	Batch size	ops/s		
AES	8.378	2048	552	5.097	106268670
$F_{\mathsf{NR}(128)}(\mathsf{log})$	4.375	1024	370	4.787	1359
$F_{\mathsf{NR}(128)}(\mathsf{const})$	4.549	256	281	2.384	1359
$F_{\mathsf{Leg}(\mathsf{bit})}$	0.393	1024	163400	1225	17824464
$F_{\mathsf{Leg}(1)}$	1.418	64	1331	9.574	115591
$F_{\mathsf{MiMC}}(\mathsf{basic})$	14.053	1024	6561	33.575	189525
$F_{\mathsf{MiMC}}(\mathsf{cube})$	7.267	2048	5536	33.575	189525

Table 3. Performance of the PRFs in a LAN setting

WAN results

PRF	Best latency	Best throughput		Prep. (ops/s)
	(ms/op)	Batch size	ops/s	
AES	2640	1024	31.947	0.256
$F_{\mathsf{NR}(128)}(\mathsf{log})$	713	1024	59.703	0.2359
$F_{\mathsf{NR}(128)}(\mathsf{const})$	478	1024	30.384	0.1175
$F_{\mathsf{Leg}(\mathsf{bit})}$	202	1024	2053	60.241
$F_{\mathsf{Leg}(1)}$	210	512	68.413	0.4706
$F_{\mathsf{MiMC}}(\mathsf{basic})$	7379	512	59.04	1.650
$F_{\mathsf{MiMC}}(\mathsf{cube})$	3691	512	79.66	1.650

Table 4. Performance of the PRFs in a simulated WAN setting