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Hypotheses of interest

Example Palmer and Gough (P&G)

Palmer and Gough (2007) expect that

- non-offenders (1) attribute more importance to defective education for explaining crime than the other two offenders (2 & 3):
i.e., $\mu_1 > \mu_2$ and $\mu_1 > \mu_3$,
- property offenders (2) attribute more importance to defective education than person offenders (3):
i.e., $\mu_2 > \mu_3$.

This leads to the theory-based hypothesis:

$$H_1 : \quad \mu_1 > \mu_2 > \mu_3.$$

Note that ‘<’ denotes “smaller than” and ‘>’ denotes “larger than”.

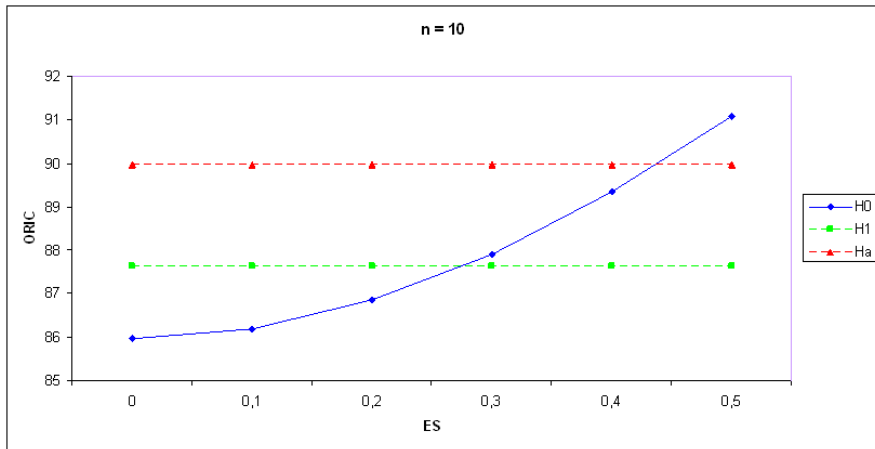
NHST

Example Palmer and Gough

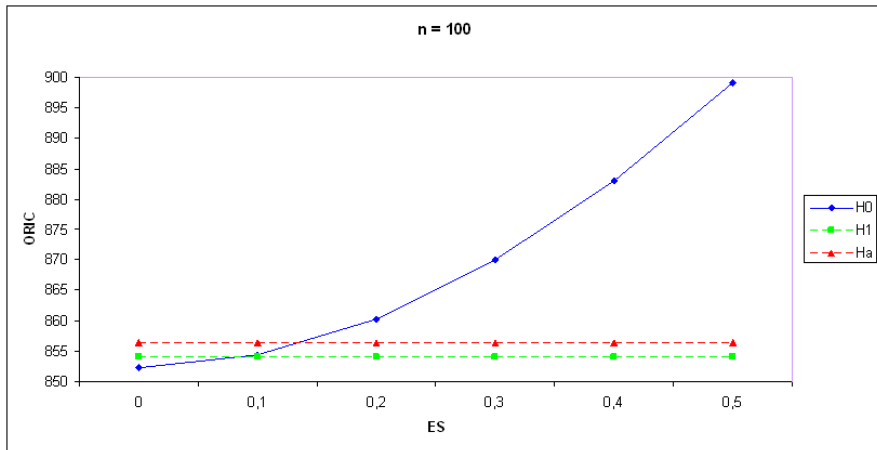
```
PandG_data <- read.table("Data_PalmerAndGough.txt",
                          header=TRUE)
PandG_data$group <- factor(PandG_data$group)
pairwise.t.test(PandG_data$Importance, PandG_data$group,
                p.adj = 'bonferroni')

##
## Pairwise comparisons using t tests with pooled SD
##
## data: PandG_data$Importance and PandG_data$group
##
##      1      2
## 2 0.191 -
## 3 0.011 1.000
##
## P value adjustment method: bonferroni
```


Confirmation more power: 1 data set. GORIC values for 3 groups, effect size ES , and $n = 10$ observations per group



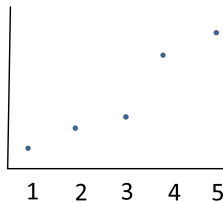
Confirmation more power: 1 data set. GORIC values for 3 groups, effect size ES , and $n = 100$ observations per group



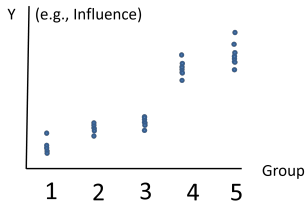
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Example 5 means

Data for 5 groups – Oversimplified representation

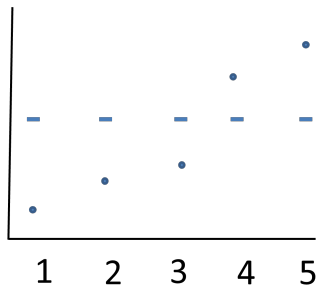


Data for 5 groups – Better representation (scatter of data points)



Intermezzo: Balance Fit and Complexity (2/6)

Example 5 means

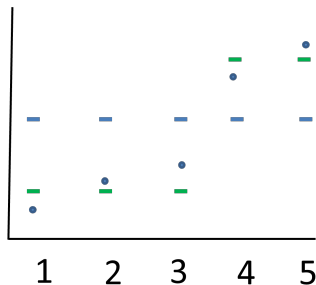


$$\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 = \mu$$

parameters: 1 mean parameter (i.e., μ)
Fit: Bad fit

Intermezzo: Balance Fit and Complexity (3/6)

Example 5 means



$$\mu_1 = \mu_2 = \mu_3 = \mu_{\text{low}}$$

$$\mu_4 = \mu_5 = \mu_{\text{high}}$$

parameters:

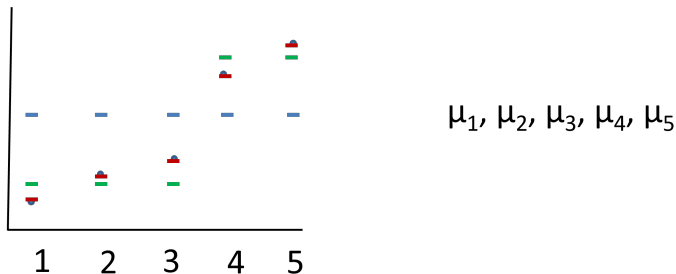
2 mean parameters (i.e., μ_{low} and μ_{high})

Fit:

Better than with one parameter, even good fit.

Intermezzo: Balance Fit and Complexity (4/6)

Example 5 means



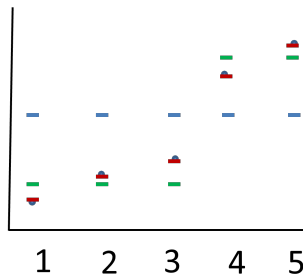
parameters: 5 mean parameter (complex)

Fit: Better than with two parameters, even best possible fit.

Thus: Best fit (= highest likelihood), but also most complex (= highest penalty).

Intermezzo: Balance Fit and Complexity (5/6)

Example 5 means



$$\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 = \mu$$

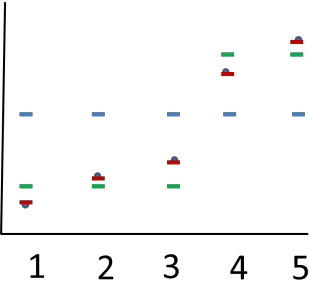
$$\mu_1 = \mu_2 = \mu_3 = \mu_{\text{low}} \text{ \& } \mu_4 = \mu_5 = \mu_{\text{high}}$$

$\mu_1, \mu_2, \mu_3, \mu_4, \mu_5$

In this example: **Two means** may be best trade-off between fit & complexity.

Intermezzo: Balance Fit and Complexity (6b/6)

Order-restrictions: From “Example 5 means’ to ‘Example 2 means’



In theory-based hypotheses, we also incorporate **order-restrictions** (e.g., $\mu_1 < \mu_2$). Then, helpful to look at likelihood using **contour plots** (not scatter plot, as done here).

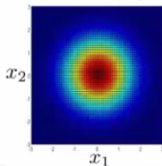
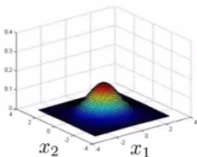
For ease, I will next use **2 means** (instead of 5).

Intermezzo: Contour plot (2/2)

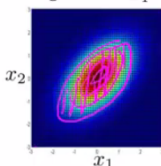
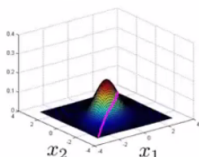
Example 2 means

Multivariate Gaussian (Normal) examples

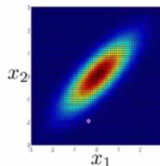
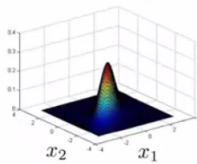
$$\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



$$\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$



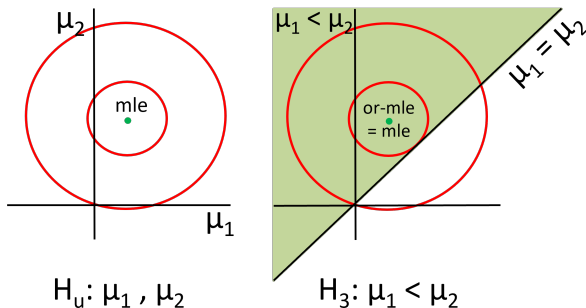
$$\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 1 & 0.8 \\ 0.8 & 1 \end{bmatrix}$$



Andreas

Note: if in agreement, or-mle = mle

Example 2 means



mle in H_3 : highest likelihood is in allowable (= green) space.
Hence, or-mle = mle.

Idea fit

Example Palmer and Gough, that is, 3 means

$$H_0 : \mu_1 = \mu_2 = \mu_3,$$

$$H_1 : \mu_1 > \mu_2 > \mu_3,$$

$$H_2 : \mu_1 > \mu_2 < \mu_3,$$

$$H_3 : \mu_1 < \mu_2 < \mu_3,$$

$$H_u : \mu_1, \mu_2, \mu_3.$$

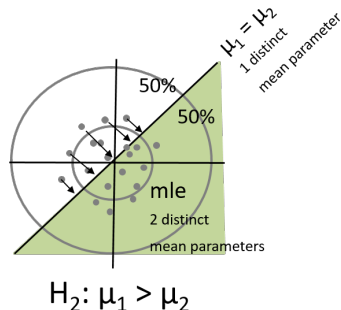
```
## group1 group2 group3
## 11.95  9.75  8.77
```

GORIC

Model	Fit	Complexity	GORIC	GORIC weights
H_0	-196.36	2.00	396.71	0.01
H_1	-191.89	2.81	389.41	0.56
H_2	-192.34	3.19	391.05	0.25
H_3	-196.36	2.81	398.34	0.01
H_u	-191.89	4.00	391.79	0.17

Idea complexity: penalty (PT)

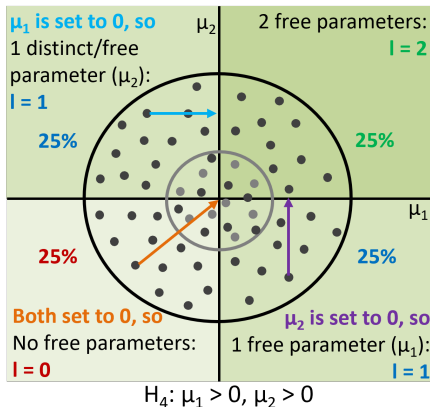
Example 2 means



Note: sampling from distribution with $\mu_1 = \mu_2 = 0$.
 complexity $H_2 = PT_2 = 1 + 0.5 \times 1 + 0.5 \times 2 = 2.5$.
 PT = number of expected distinct/free parameters.

Extra: Levels and level probabilities

Example 2 means



Level (l): number of distinct/free parameters.

Level probability (LP_l): probability that there are l levels.

$$PT_4 = 1 + 0.25 \times 0 + (0.25 + 0.25) \times 1 + 0.25 \times 2 = 2.$$

Idea complexity

Example Palmer and Gough, that is, 3 means

$$H_0 : \mu_1 = \mu_2 = \mu_3,$$

$$H_1 : \mu_1 > \mu_2 > \mu_3,$$

$$H_2 : \mu_1 > \mu_2 < \mu_3,$$

$$H_3 : \mu_1 < \mu_2 < \mu_3,$$

$$H_u : \mu_1, \mu_2, \mu_3.$$

GORIC

Model	Fit	Complexity	GORIC	GORIC weights
H_0	-196.36	2.00	396.71	0.01
H_1	-191.89	2.81	389.41	0.56
H_2	-192.34	3.19	391.05	0.25
H_3	-196.36	2.81	398.34	0.01
H_u	-191.89	4.00	391.79	0.17

Hands-on/Demo (1a): GORIC

- Go to <https://github.com/rebeccakuiper/Tutorials>:
 - Click on green button called Code.
 - Download zip (last option in list).
 - Unzip it on your machine (that folder is now your working dir.).
- Start Rstudio. Optional: make project.
- Open 'Tutorial_GORIC_restriktor_ANOVA.html',
'PalmerAndGough_and_Lucas.R', and/or
'**Hands-on_1_GORIC**_Unc_ANOVA_restriktor.R' ('Hands-on files').
- Install packages and load them.
- Read and inspect data.
Use **Data_PalmerAndGough.txt** and/or Data_Lucas.txt.
- Run model (lm()).
- Specify hypotheses (make up your own).
Note: Use names used in the model.
- Run goric().
- Inspect and interpret output.
Note: 'GORIC weights' will be explained next.



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Illustration of the GORIC weights (w_m)

Example Palmer and Gough

$H_0 : \mu_1 = \mu_2 = \mu_3$ (include only when of interest)

$$H_1 : \mu_1 > \mu_2 > \mu_3$$
$$H_2: \mu_2 > \mu_1 > \mu_3 \text{ (if competing theory/hypothesis)}$$
$$H_\mu : \quad \mu_1, \mu_2, \mu_3.$$

GORIC

Model	Fit	Complexity	GORIC	GORIC weights
H_1	-191.89	2.81	389.41	0.68
H_2	-193.70	2.81	393.03	0.11
H_u	-191.89	4.00	391.79	0.21

H_1 is $0.68 / 0.11 \approx 6.11$ times more supported than H_2 .

Another illustration: one-way ANOVA

Lucas: 5 groups

Lucas (2003) investigated difference between female and male leadership w.r.t. influence of the leader.

Five experimental groups:

1. a randomly selected male leader
2. a randomly selected female leader
3. male leader selected via task
4. female leader selected via task
5. female leader selected via task + institutionalized female leadership via movie

R code: Data and fit object

Lucas

```
# Data
lucas_data <- read.table("Data_Lucas.txt", header=TRUE)
lucas_data$group <- factor(lucas_data$group)

# fit object (needed as input for goric function)
fit.lucas <- lm(Influence ~ group - 1, data = lucas_data)
```


Use of H_u

Palmer & Gough

$$H_1 : \mu_1 > \mu_2 > \mu_3,$$

$$H_2 : \mu_2 > \mu_1 > \mu_3$$

$$H_u : \mu_1, \mu_2, \mu_3.$$

GORIC

Model	Fit	Complexity	GORIC	GORIC weights
H_1	-191.89	2.81	389.41	0.68
H_2	-193.70	2.81	393.03	0.11
H_u	-191.89	4.00	391.79	0.21

If at least one informative hypothesis not weak (i.e., $w_m > w_u$ or $w_m/w_u > 1$), then compare informative hypotheses.

Hence: H_u is only a failsafe, not another hypothesis of interest.

Alternative failsafe: Complement of H_m

Alternatively (in the case of one hypothesis of interest)

Evaluate hypothesis of interest against its complement; that is, all other possible hypotheses.

More powerful than against the unconstrained if H_m has maximum fit.

Reference:

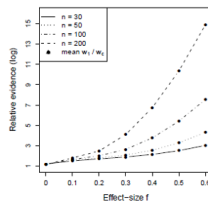
Vanbrabant, L., Van Loey, N., and Kuiper, R. M. (2020). Evaluating a Theory-Based Hypothesis Against Its Complement Using an AIC-Type Information Criterion With an Application to Facial Burn Injury. *Psychological Methods*, 25(2), 129-142.

<https://doi.org/10.1037/met0000238>

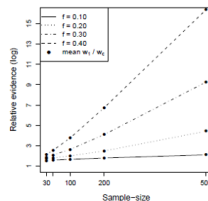
Alternative failsafe: Complement of H_m

Based on simulation

vs complement

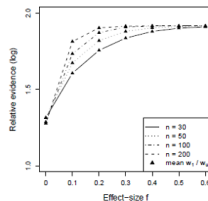


(a)

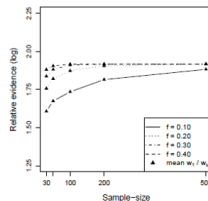


(b)

vs unconstrained



(c)



(d)

H_1 vs H_c

Palmer & Gough

H_1 : $\mu_1 > \mu_2 > \mu_3$,
 H_c : not H_1 ,
 complement of H_1 .

GORIC

Model	Fit	Complexity	GORIC	GORIC weights
H1	-191.89	2.81	389.41	0.79
complement	-192.34	3.69	392.05	0.21

H_1 is $0.79 / 0.21 \approx 3.73$ times more supported than its complement,
 that is, any other hypothesis (i.e., any other ordering).

GORICA

Similarities with GORIC

- Form: $GORICA_m = -2 \text{ fit} + 2 \text{ complexity}$.
- Broad type of restrictions.

Differences compared to GORIC

- Uses asymptotic expression of the likelihood (is a normal):
can therefore be easily applied to all types of statistical models.
Disadvantage: might work less well in case of small samples.
- Does not need data set; mle's and their covariance matrix suffice.
- Can leave out nuisance parameters (i.e., not part of hypotheses).

Note

In case of normal linear models and/or not too small samples:
GORICA weights = GORIC weights.

R code: GORICA

Palmer & Gough

$$H_1 : \mu_1 > \mu_2 > \mu_3,$$

$$H_c : \text{not } H_1.$$

GORICA: type = "gorica" (not default for lm objects)

```

H1 <- 'group1 > group2 > group3'
# vs its complement (default in case of one hypothesis)
#
# GORICA (using goric function in restriktor package)
library(restriktor)
set.seed(123) # Set seed value
gorica.PandG_C <- goric(fit.PandG,
                        hypotheses = list(H1),
                        type = "gorica") # needed if lm object
    
```


Hands-on/Demo (3): GORICA

Let's practice.

- If needed: Start Rstudio again (optional: make project) and then also load packages again.
- Optional: Open 'Tutorial_GORIC_restriktor_ANOVA.html', 'PalmerAndGough_and_Lucas.R', and/or '**Hands-on_3_GORICA_UncAndCompl_ANOVA_restriktor.R**'.
- Read and inspect data.
Use Data_PalmerAndGough.txt and/or **Data_Lucas.txt**.
- Run model (lm()).
- Specify hypotheses (make up your own). E.g., for Lucas:

$$H_1 : \mu_5 = \mu_3 > \{\mu_1, \mu_4\} > \mu_2$$

Note: Use names used in the model (or overwrite those).

- Run goric(); now, add (since lm object):
`type = "gorica"`
- Inspect and interpret output.

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GORIC in JASP: screenshot

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ANOVA

Mixed Models

Regression

Frequencies

Factor

Bain

SEM

R (Beta)

+

▼ Order Restricted Hypotheses

Enter each restriction of one hypothesis on a new line, e.g.,
factorLow == factorMid
factorMid < factorHigh
where 'factor' is the factor (or covariate) name and 'Low'/'Mid'/'High' are the factor level names.
Click on the 'plus' icon to add more hypotheses.
Click the information icon for more examples.

Syntax settings

☐ Include intercept
☐ Show available coefficients

Set for all models

☐ Model summary
☐ Marginal means
☐ Informed hypothesis tests

Model 1

+

group1 > group2 > group3

Ctrl + Enter to apply. Click on the blue button above for help on the restriction syntax

☐ Summary for Model 1
☐ Marginal means for Model 1
☐ Informed hypothesis tests for Model 1

Results

ANOVA

ANOVA - Importance

Cases	Sum of Squares	df	Mean Square	F	p
group	123.960	2	61.980	4.554	0.014
Residuals	925.418	68	13.609		

Note. Type III Sum of Squares

Order Restricted Hypotheses

Model Comparison

Model Comparison Table

Model	Log-likelihood	Penalty	GORIC	Weight	Weights ratio
Model 1	-191.893	2.814	389.415	0.789	3.731
Complement	-192.338	3.686	392.048	0.211	1.000

Note. Weights ratios indicate the relative weight for each model against the "complement" model. GORIC = Generalized Order-Restricted Information Criterion (Kuiper, Hoijtink, & Silvapulle, 2011).

GORIC and GORICA in JASP

GORIC or GORICA in the following statistical models (as part of that model)

The screenshot displays the JASP software interface with three statistical models: ANOVA, Repeated Measures ANOVA, and ANCOVA. In each model's left-hand menu, the 'Order Restricted Hypotheses' option is circled in blue. The ANOVA and Repeated Measures ANOVA models show the 'ANOVA' and 'Repeated Measures ANOVA' options respectively, also circled in blue. The ANCOVA model shows the 'ANCOVA' option circled in blue. The right-hand panel of the ANCOVA model shows the 'Order Restricted Hypotheses' option circled in red. The 'Dependent Variable' is 'id', and the 'Fixed Factors' are 'sex', 'age', 'weight', 'setting', 'treat', 'prebody', 'preind', 'preumb', 'preval', 'preclaf', 'postbody', 'postel', 'postform', 'postnumb', 'postnatel', 'postclaf', and 'postmedu'. The 'Covariates' section is empty. The 'WLS Weights' section is empty. The 'Display' section shows 'Descriptive statistics', 'Estimates of effect size', 'Partial of', and 'Work-Selfie maximum p-ratio'. The 'Model' section shows 'Assumption Checks' and 'Contrasts'. The 'Order Restricted Hypotheses' option is circled in red in the bottom right panel.

ANOVA

Cases	Sum of Squares	df	Mean Square	F	p
Note: Type III Sum of Squares					

Repeated Measures ANOVA

Cases	Sum of Squares	df	Mean Square	F	p
Note: Type III Sum of Squares					

ANCOVA

Cases	Sum of Squares	df	Mean Square	F	p
Note: Type III Sum of Squares					

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The End

GORIC(A)

Thanks for listening!

Are there any questions?

Websites

<https://github.com/rebeccakuiper/Tutorials>

www.uu.nl/staff/RMKuiper/Extra1

www.uu.nl/staff/RMKuiper/Extra2

informative-hypotheses.sites.uu.nl/software/goric/

E-mail

r.m.kuiper@uu.nl

Notes

Note on possibilities multiple studies

- Update GORIC(A) values & weights.
More data collected: (re-)calculate.
- Update hypotheses.
First data set (or a part of it) generates one or more hypotheses.
Other data set (or part) used to determine evidence / support.
Download 'Tutorial_GORIC_restriktor_UpdateHypo.html' and/or
'Hands-on_4_GORIC_UpdateHypo_restriktor.R' from
<https://github.com/rebeccakuiper/Tutorials>.
- Aggregate evidence for hypotheses.
Aggregate the support for theories (diverse designs allowed).
Bear in mind: Meta-analysis aggregates parameter estimates or
effect sizes which need to be comparable (often same designs
required).
Download 'Tutorial_GORIC_restriktor_evSyn.html' and/or
'Hands-on_4_GORIC_evSyn_restriktor.R' from
<https://github.com/rebeccakuiper/Tutorials>.

Extra material (1/2)

- Logistic Regression Modeling
 - Article: doi.org/10.1037/met0000406
- GORICA on SEM
 - Article: www.tandfonline.com/doi/full/10.1080/10705511.2020.1836967.
 - R scripts: github.com/rebeccakuiper/GORICA_in_SEM.
- GORICA on cross-lagged panel model (CLPM)
 - Article: doi.org/10.1111/bjep.12455.
 - R scripts: github.com/rebeccakuiper/GORICA_in_CLPM.
- GORICA on Random-Intercept CLPM (RI-CLPM)
 - Article: Sukpan, C. and Kuiper, R.M. (2023). How to evaluate causal dominance hypotheses in lagged effects models.
 - R scripts: github.com/rebeccakuiper/GORICA%20in%20RI-CLPM.
- GORICA for EffectLiteR
 - R scripts: <https://github.com/rebeccakuiper/Tutorials/tree/main/GORICA%20for%20EffectLiteR>.

Note: On github site, go to Code (green button) and download zip.

Note on conditional error probability (1/3)

using PMPs and GORIC(A) weights

H_m	weights
H_1 : Sex Match	.04
H_2 : Gender Role Match	.81
H_3 : Sex Mismatch	.01
H_4 : Gender Role Mismatch	.00
H_u :	.14

The conditional error probability for preferred hypotheses H_2 is $1 - .81 = .19$.

Note on conditional error probability (2/3)

using PMPs and GORIC(A) weights

What if we compare many hypotheses?

H_m	weights
H_1 : Sex Match	.013
H_2 : Gender Role Match	.270
H_3 : Sex Mismatch	.003
H_4 : Gender Role Mismatch	.000
H_5 : Lets try this one too	.180
...	...
H_{12} : Don't miss something	.040
H_u :	.047

The conditional error probability for H_2 becomes $1 - .27 = .77$ (was .19).

Namely, included additional hypotheses may obtain some support.
Hence, the conditional error probabilities becomes larger.

Results Confirmation ($k = 3$ and $n = 50$)

$$H_{30C} : \mu_1 = \mu_2 = \mu_3$$

$$H_{31C} : \mu_1 < \mu_2 < \mu_3$$

$$H_{32C} : \mu_1 = \mu_2 < \mu_3$$

$$H_{33C} : \mu_1 < \mu_2 > \mu_3$$

$$H_{34C} : \mu_1, \mu_2, \mu_3$$

1 – Type I error (here):

preferring the true null hypothesis,
given the set of hypotheses (!).

'ES'	Method	'Prior'	H_{30C}	H_{31C}	H_{32C}	H_{33C}	H_{34C}
0	ORIC		0.662	0.065	0.119	0.120	0.034
0	BMS	3	0.865	0.012	0.075	0.036	0.012
0	BMS	2	0.774	0.022	0.124	0.058	0.022
0	BMS	1	0.656	0.047	0.147	0.120	0.033
2	ORIC		0.148	0.506	0.311	0.029	0.006
2	BMS	3	0.299	0.299	0.371	0.031	0.001
2	BMS	2	0.252	0.328	0.381	0.040	0.000
2	BMS	1	0.125	0.494	0.321	0.063	0.000
5	ORIC		0.000	0.955	0.044	0.001	0.000
5	BMS	3	0.000	0.887	0.112	0.001	0.000
5	BMS	2	0.001	0.890	0.107	0.002	0.000
5	BMS	1	0.000	0.928	0.070	0.002	0.000

Results Confirmation ($k = 3$ and $n = 50$)

$$H_{30C} : \mu_1 = \mu_2 = \mu_3$$

$$H_{31C} : \mu_1 < \mu_2 < \mu_3$$

$$H_{32C} : \mu_1 = \mu_2 < \mu_3$$

$$H_{33C} : \mu_1 < \mu_2 > \mu_3$$

$$H_{34C} : \mu_1, \mu_2, \mu_3$$

1 – Type II error (here):
preferring the true hypothesis,
given the set of hypotheses (!).

'ES'	Method	'Prior'	H_{30C}	H_{31C}	H_{32C}	H_{33C}	H_{34C}
0	ORIC		0.662	0.065	0.119	0.120	0.034
0	BMS	3	0.865	0.012	0.075	0.036	0.012
0	BMS	2	0.774	0.022	0.124	0.058	0.022
0	BMS	1	0.656	0.047	0.147	0.120	0.033
2	ORIC		0.148	0.506	0.311	0.029	0.006
2	BMS	3	0.299	0.299	0.371	0.031	0.001
2	BMS	2	0.252	0.328	0.381	0.040	0.000
2	BMS	1	0.125	0.494	0.321	0.063	0.000
5	ORIC		0.000	0.955	0.044	0.001	0.000
5	BMS	3	0.000	0.887	0.112	0.001	0.000
5	BMS	2	0.001	0.890	0.107	0.002	0.000
5	BMS	1	0.000	0.928	0.070	0.002	0.000

Extra: Prior choice for BMS

Prior

$$P(\mu, \sigma^2) = P(\mu_1) \times \dots \times P(\mu_k) \times P(\sigma^2),$$

where $P(\mu_i) = P(\mu) \forall i = 1, \dots, k$.

$P(\mu)$ is a data-based normal distribution, with parameters β_0 and τ_0^2 , and $P(\sigma^2)$ an inverse chi-squared distribution (see Klugkist et al., 2005)

β_0 and τ_0^2 data-based hyperparameters

For each μ_i ($i = 1, \dots, k$) a credibility interval is computed:

estimated group mean \pm $PV \times$ *the group standard error*.

The credibility interval for μ_i is represented by " $[LB_i, UB_i]$ ".

Lowest lower bound: LB_{min} .

Highest upper bound: UB_{max} .

Then, $\beta_0 = \frac{LB_{min} + UB_{max}}{2}$ and $\tau_0^2 = \left[\frac{UB_{max} - LB_{min}}{2} \right]^2$.

Three types of prior vagueness: BMS PV 1, BMS PV 2 and BMS PV 3