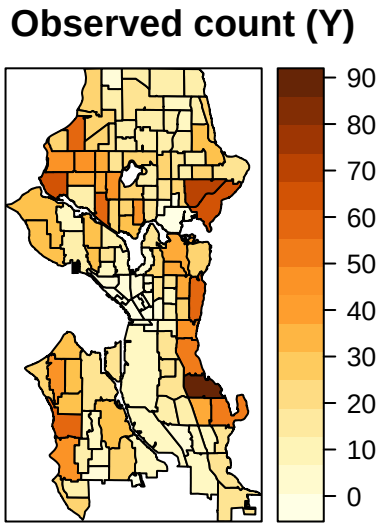
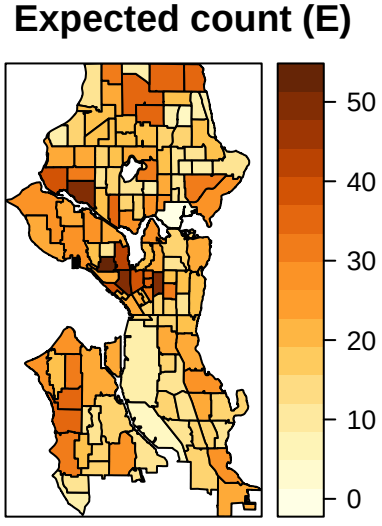
Statistical Methods for Spatial Data: INLA

Yohan Min

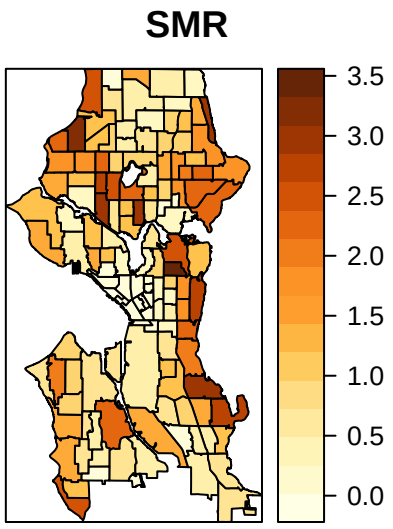
Tue Mar 12 00:34:31 2019

1. Seattle solar installation Poisson model





$$SMR_i = \frac{Y_i}{E_i}$$



2. Poisson-Lognormal-Spatial model

$$
\begin{aligned}
Y_i|\beta_0, S_i, \epsilon_i &\sim_{ind} \text{Poisson}(E_i \text{e}^{\beta_0+\beta_1 X_1+\beta_2 X_2} \text{e}^{S_i+\epsilon_i}),\\
\epsilon_i|\sigma^2_\epsilon &\sim_{iid} \text{N}(0, \sigma^2_\epsilon),\\
S_1, ..., S_n|\sigma^2_s &\sim \quad \text{ICAR}(\sigma^2_s).
\end{aligned}
$$

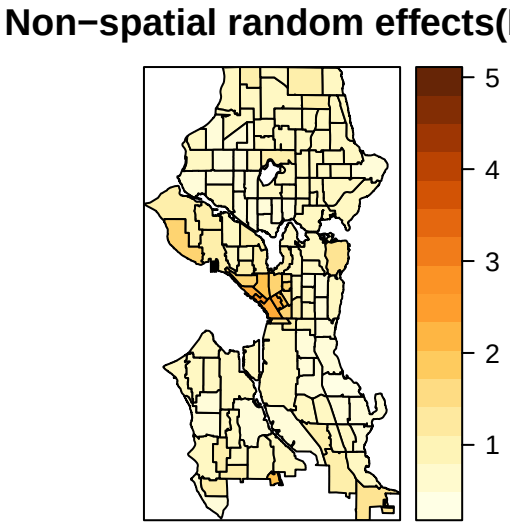
	mean	0.025quant	0.5quant	0.975quant
(Intercept)	-2.248	-2.6	-2.246	-1.906
I(single_unit)	1.642	0.77	1.641	2.52
I(hu_med_val)	2.489	1.415	2.486	3.577
I(hu_ex_1000)	0.8677	-0.6038	0.8687	2.332
SD for IID	0.007438	0.02815	0.008871	0.003894
SD for spatial	0.8086	0.9833	0.8165	0.6753

50% chance that the proportion of the spatial variance, ϕ is greater 0.5 and 1% chance that the total residual standard deviation is greater than 0.3.

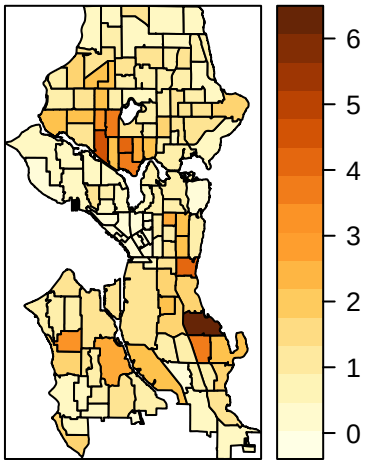
BYM2 Model

	mean	0.025quant	0.5quant	0.975quant
(Intercept)	-2.273	-2.613	-2.271	-1.942
I(single_unit)	1.636	0.7836	1.635	2.492
I(hu_med_val)	2.63	1.56	2.627	3.718
I(hu_ex_1000)	0.8481	-0.5934	0.8487	2.285
Total SD	0.5548	0.6783	0.5597	0.4637
Phi for ID	0.9424	0.7732	0.9624	0.9976

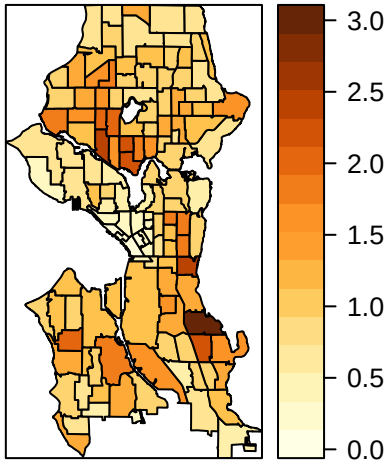
- 2.2645061 - 50% increase in the percentage in single unit may incur the amount increase in solar installation.
- 3.466117 - 50% increase in the percentage in house unit median value may incur the amount increase in solar installation.
- 1.5439506 - 50% increase in the percentage in homeownership with more than $1,000 expenditure may incur the amount increase in solar installation. ## (b) Relative risk estimates and comparison



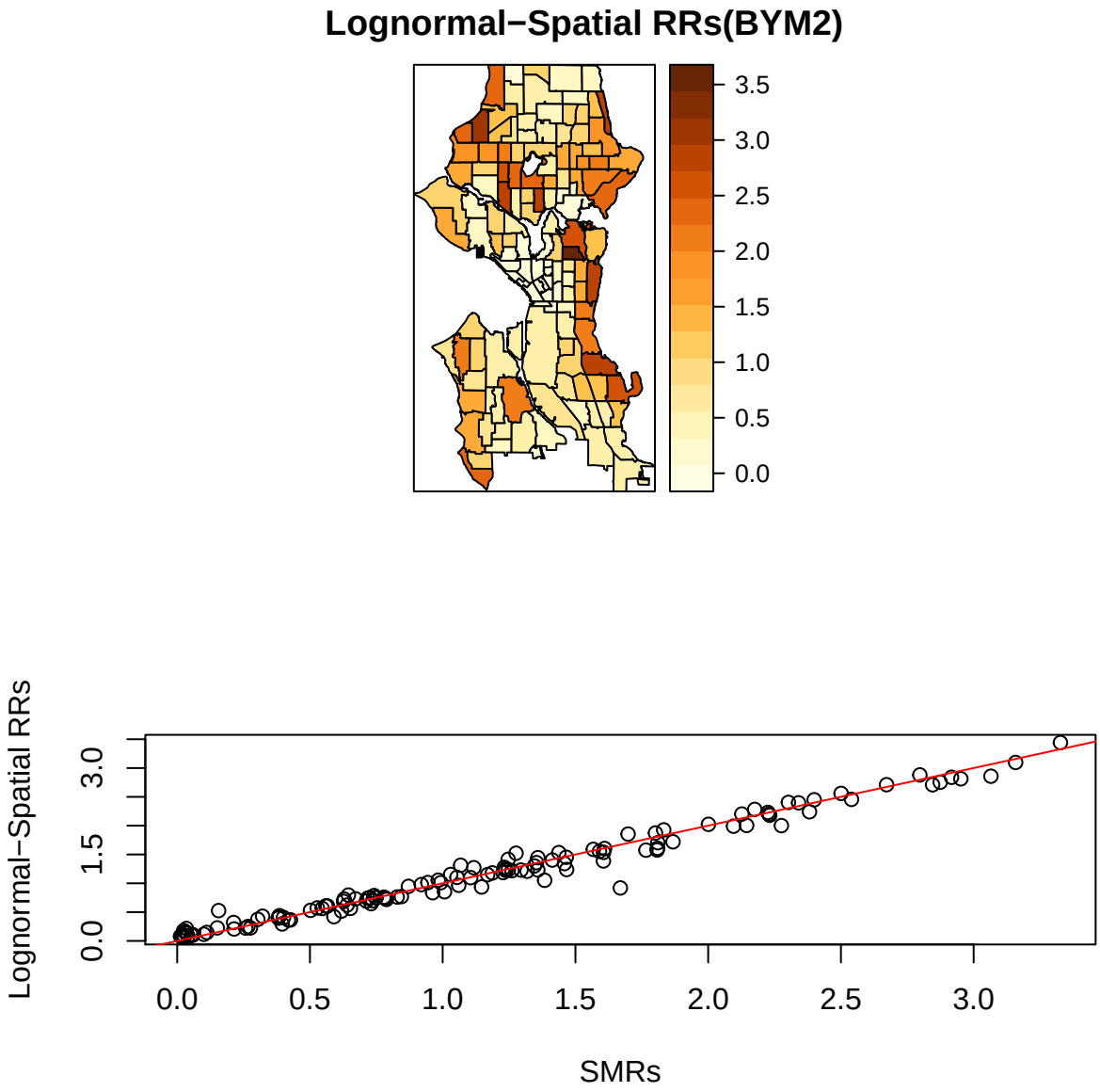
Spatial random effects(BYM2)



Total random effects or residual(BYM2)



Can I consider the total random effects to be residuals?



3. Clustering

$$Y_i \sim \text{Poisson}(E_i e^{\beta_0})$$

(a) Clustering of the residual of the poisson model using Moran's

Table 3: Moran I test under randomisation: **seattleres**
weights: col.w (continued below)

Test statistic	P value	Alternative hypothesis	Moran I statistic
8.809	6.327e-19 * * *	greater	0.4706

Expectation	Variance
-0.007463	0.002945

How can I get residulas from INLA for the moran's I test?

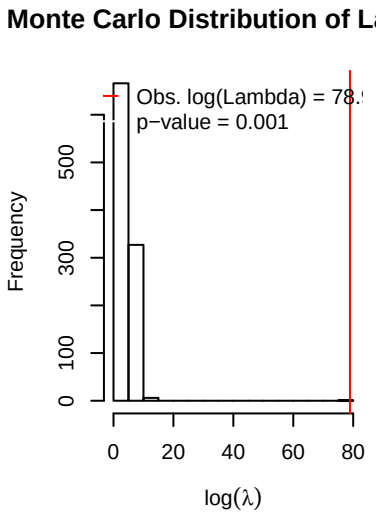
(b) Clustering of the residual using Geary's

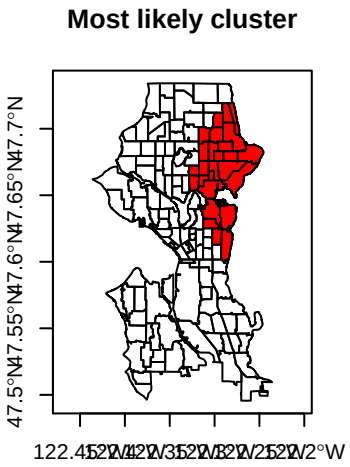
Table 5: Geary C test under randomisation: **seattleres**
weights: col.w (continued below)

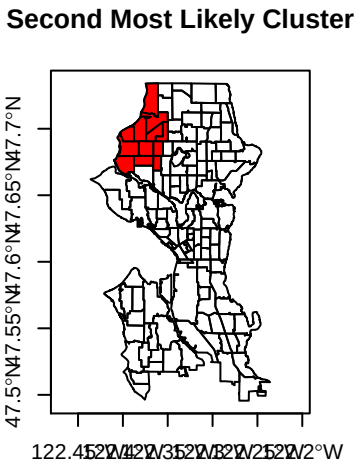
Test statistic	P value	Alternative hypothesis
8.759	9.867e-19 * * *	Expectation greater than statistic

Geary C statistic	Expectation	Variance
0.5052	1	0.003191

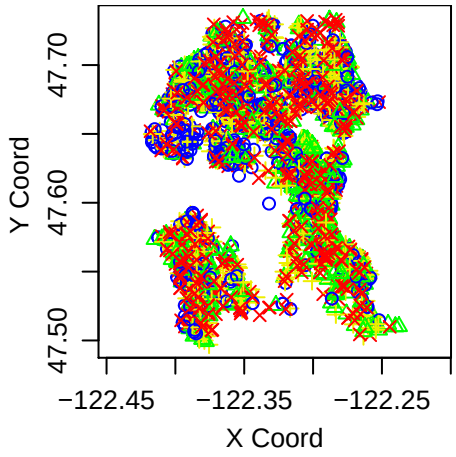
(c) Cluster detection based on Y_i count

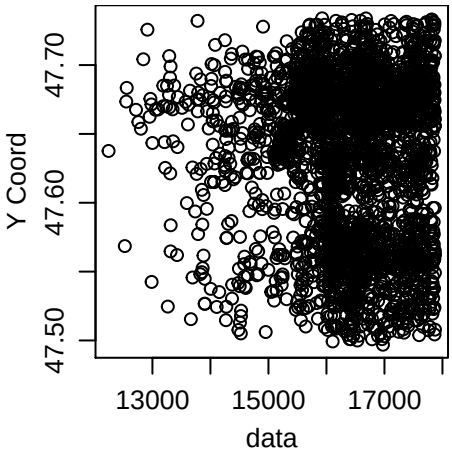


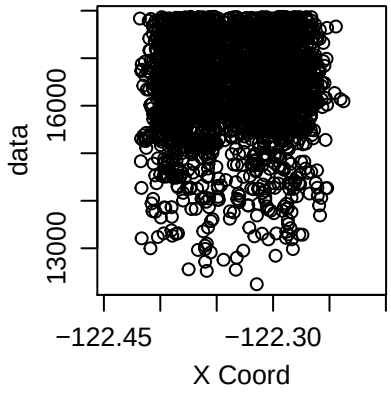


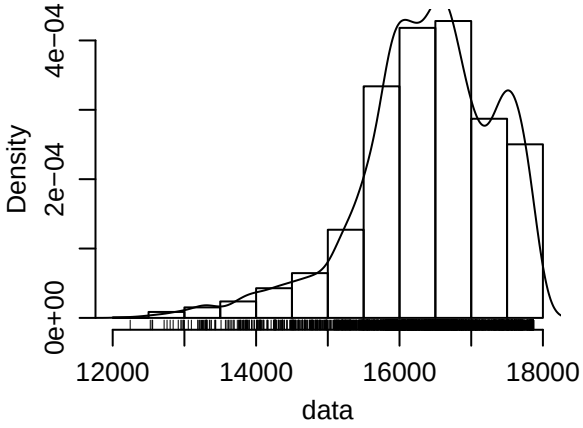


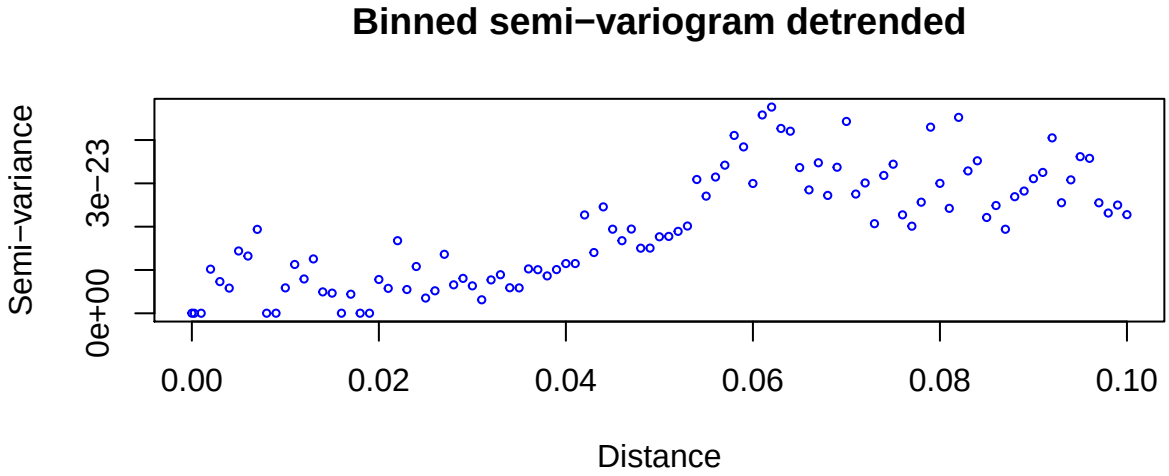
4. Point data analysis



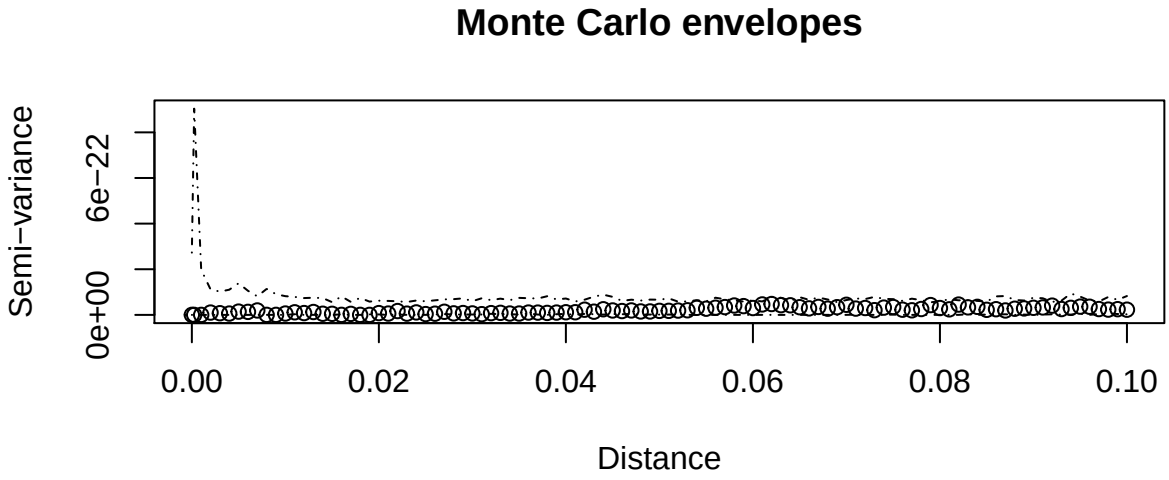








how to project to the different coordinates? how to address the very small variance cos I can't test any regressions on it such as MLE. ## (b) Monte Carlo intervals



5. Regression - Moran's test

Table 7: Global Moran I for regression residuals: **`model: lm(formula = SMR_s ~ single_unit + hu_med_val + hu_ex_1000, data = seattle@data) weights: col.w`** (continued below)

Test statistic	P value	Alternative hypothesis	Observed Moran I
4.779	1.764e-06 * * *	two.sided	0.2362

Expectation	Variance
-0.01732	0.002815

Table 9: Global Moran I for regression residuals: **model: glm(formula = n_s ~ single_unit + hu_med_val + hu_ex_1000 + offset(log(solar_E)), family = "poisson", data = seattle@data) weights: col.w** (continued below)

Test statistic	P value	Alternative hypothesis	Observed Moran I
6.343	2.257e-10 * * *	two.sided	0.3558

Expectation	Variance
-0.0009297	0.003163

Table 11: Moran I test under randomisation: **seattleres weights: col.w** (continued below)

Test statistic	P value	Alternative hypothesis	Moran I statistic
4.521	3.08e-06 * * *	greater	0.2362

Expectation	Variance
-0.007463	0.002905

Table 13: Moran I test under randomisation: **seattlegres weights: col.w** (continued below)

Test statistic	P value	Alternative hypothesis	Moran I statistic
6.166	3.498e-10 * * *	greater	0.3243

Expectation	Variance
-0.007463	0.002895

6. Spatial regression - Moran's test

lagsar

```
##
## Call:lagsarlm(formula = SMR_s ~ single_unit + hu_med_val + hu_ex_1000,
##     data = seattle@data, listw = col.w)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -1.360760 -0.298758 -0.035265  0.218166  1.524414
##
## Type: lag
## Coefficients: (asymptotic standard errors)
##             Estimate Std. Error z value  Pr(>|z|)
## (Intercept) -0.83317    0.13433 -6.2025 5.557e-10
## single_unit  0.59642    0.35302  1.6895  0.091122
## hu_med_val   2.02468    0.52699  3.8420  0.000122
## hu_ex_1000   1.85168    0.65000  2.8487  0.004389
##
## Rho: 0.31696, LR test value: 13.459, p-value: 0.0002438
## Asymptotic standard error: 0.083652
##     z-value: 3.789, p-value: 0.00015123
## Wald statistic: 14.357, p-value: 0.00015123
##
## Log likelihood: -87.7104 for lag model
## ML residual variance (sigma squared): 0.21012, (sigma: 0.45839)
## Number of observations: 135
## Number of parameters estimated: 6
## AIC: 187.42, (AIC for lm: 198.88)
## LM test for residual autocorrelation
## test value: 2.6691, p-value: 0.10231
```

Table 15: Moran I test under randomisation: **`residuals(spmod, type = "pearson")`** **`weights: col.w`** (continued below)

Test statistic	P value	Alternative hypothesis	Moran I statistic
1.288	0.09887	greater	0.06195

Expectation	Variance
-0.007463	0.002904

stsls

```
##
## Call:stsls(formula = SMR_s ~ single_unit + hu_med_val + hu_ex_1000,
##     data = seattle@data, listw = col.w)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -1.340087 -0.300259 -0.022753  0.219727  1.547746
##
## Coefficients:
##             Estimate Std. Error t value  Pr(>|t|)
## Rho          0.27406    0.11812  2.3202 0.0203299
## (Intercept) -0.83128    0.13735 -6.0523 1.428e-09
## single_unit  0.64770    0.37468  1.7287 0.0838637
## hu_med_val   2.12063    0.58536  3.6228 0.0002915
## hu_ex_1000   1.82263    0.66952  2.7223 0.0064833
##
## Residual variance (sigma squared): 0.21989, (sigma: 0.46893)

## Impact measures (lag, exact):
##                Direct  Indirect     Total
## single_unit 0.6100936 0.2630940 0.8731876
## hu_med_val  2.0710948 0.8931295 2.9642243
## hu_ex_1000  1.8941239 0.8168134 2.7109373
```

Table 17: Moran I test under randomisation: **`residuals(sp2mod, type = "pearson")`** **`weights: col.w`** (continued below)

Test statistic	P value	Alternative hypothesis	Moran I statistic
1.719	0.04281 *	greater	0.08516

Expectation	Variance
-0.007463	0.002904

errorsar

```
##
## Call:errorsarlm(formula = SMR_s ~ single_unit + hu_med_val + hu_ex_1000,
##     data = seattle@data, listw = col.w)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.23141 -0.25942 -0.03261  0.22694  1.56976
##
## Type: error
## Coefficients: (asymptotic standard errors)
##             Estimate Std. Error z value  Pr(>|z|)
## (Intercept) -0.67302    0.17159 -3.9222 8.774e-05
## single_unit  0.90357    0.37720  2.3955 0.0165993
## hu_med_val   2.07205    0.59388  3.4890 0.0004849
## hu_ex_1000   1.87189    0.65992  2.8365 0.0045604
##
## Lambda: 0.4769, LR test value: 16.945, p-value: 3.8483e-05
## Asymptotic standard error: 0.098309
##     z-value: 4.851, p-value: 1.2284e-06
## Wald statistic: 23.532, p-value: 1.2284e-06
##
## Log likelihood: -85.96762 for error model
```

```
## ML residual variance (sigma squared): 0.19853, (sigma: 0.44556)
## Number of observations: 135
## Number of parameters estimated: 6
## AIC: 183.94, (AIC for lm: 198.88)
```

Table 19: Moran I test under randomisation: **residuals(err, type = "pearson") weights: col.w** (continued below)

Test statistic	P value	Alternative hypothesis	Moran I statistic
-0.007851	0.5031	greater	-0.007886

Expectation	Variance
-0.007463	0.002906

GWR

```
##    ************************************************************************
##    *                       Package   GWmodel                                *
##    ************************************************************************
##    Program starts at: 2019-03-12 00:35:33
##    Call:
##    gwr.basic(formula = SMR_s ~ single_unit + hu_med_val + hu_ex_1000,
##     data = seattle, bw = bw.ans, kernel = "bisquare")
##
##    Dependent (y) variable:  SMR_s
##    Independent variables:  single_unit hu_med_val hu_ex_1000
##    Number of data points: 135
##    ************************************************************************
##    *                    Results of Global Regression                      *
##    ************************************************************************
##
##    Call:
##     lm(formula = formula, data = data)
```

```
##
##    Residuals:
##     Min      1Q  Median      3Q     Max
## -1.4752 -0.3174 -0.0517  0.2479  1.6968
##
##    Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
##    (Intercept)  -0.8192     0.1447  -5.661 9.08e-08 ***
##    single_unit   0.9753     0.3659   2.665  0.00866 **
##    hu_med_val    2.7336     0.5508   4.963 2.11e-06 ***
##    hu_ex_1000    1.6371     0.7009   2.336  0.02102 *
##
##    ---Significance stars
##    Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##    Residual standard error: 0.4944 on 131 degrees of freedom
##    Multiple R-squared: 0.6633
##    Adjusted R-squared: 0.6556
##    F-statistic: 86.02 on 3 and 131 DF,  p-value: < 2.2e-16
##    ***Extra Diagnostic information
##    Residual sum of squares: 32.02475
##    Sigma(hat): 0.4907011
##    AIC:  198.88
##    AICc:  199.3451
##    ***********************************************************************
##    *          Results of Geographically Weighted Regression              *
##    ***********************************************************************
##
##    *********************Model calibration information*********************
##    Kernel function: bisquare
##    Fixed bandwidth: 0.05907508
##    Regression points: the same locations as observations are used.
##    Distance metric: Euclidean distance metric is used.
##
##    ****************Summary of GWR coefficient estimates:******************
##                      Min.  1st Qu.   Median  3rd Qu.    Max.
##    Intercept   -2.36104 -1.24462 -0.62232 -0.34915  1.0219
```

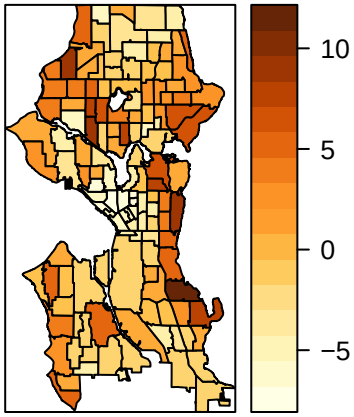
```
##    single_unit -6.55438  0.48397  1.32156  2.19765  6.5677
##    hu_med_val  -1.71636  0.96406  2.54644  4.78920 16.3447
##    hu_ex_1000  -9.47302  0.63698  1.24784  2.05757 11.9672
##    ************************Diagnostic information*************************
##    Number of data points: 135
##    Effective number of parameters (2trace(S) - trace(S'S)): 40.994
##    Effective degrees of freedom (n-2trace(S) + trace(S'S)): 94.006
##    AICc (GWR book, Fotheringham, et al. 2002, p. 61, eq 2.33): 180.6732
##    AIC (GWR book, Fotheringham, et al. 2002,GWR p. 96, eq. 4.22): 122.828
##    Residual sum of squares: 15.41753
##    R-square value:  0.8378957
##    Adjusted R-square value:  0.7664454
##
##    ************************************************************************
##    Program stops at: 2019-03-12 00:35:33
```

Table 21: Moran I test under randomisation: **gwr.res$SDF@data$residual weights: col.w** (continued below)

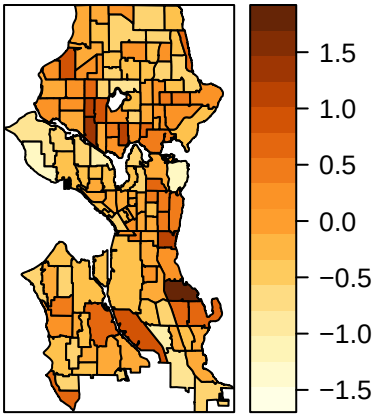
Test statistic	P value	Alternative hypothesis	Moran I statistic
1.554	0.06012	greater	0.07633

Expectation	Variance
-0.007463	0.002908

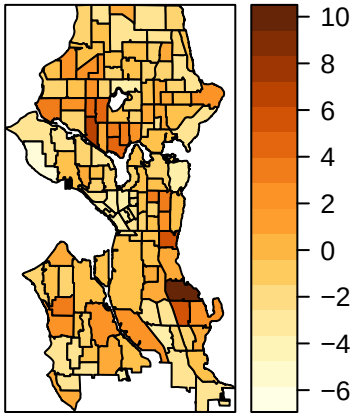
Residuals of poisson model
(very small P value of Moran's I)



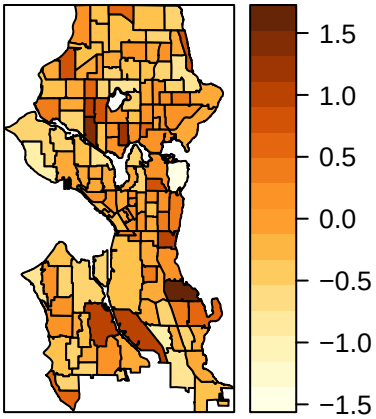
Residuals of OLS model with covariates
(very small P value of Moran's I)



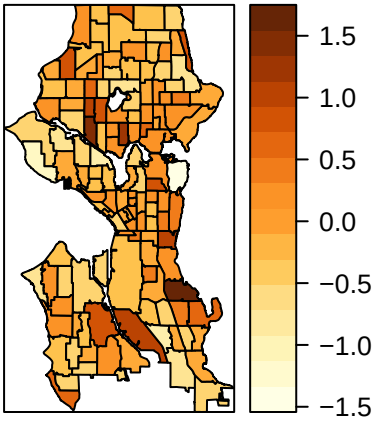
Residuals of Poission model with covariates
(very small P value of Moran's I)



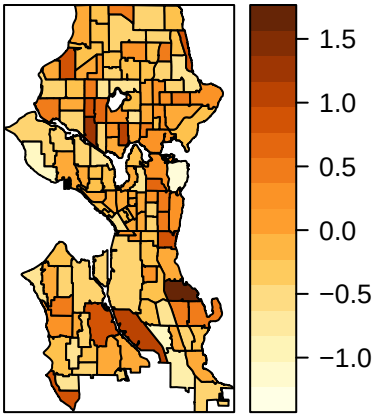
Residuals of spatial lagsar regression with covariates
(0.09887 P value of Moran's I)

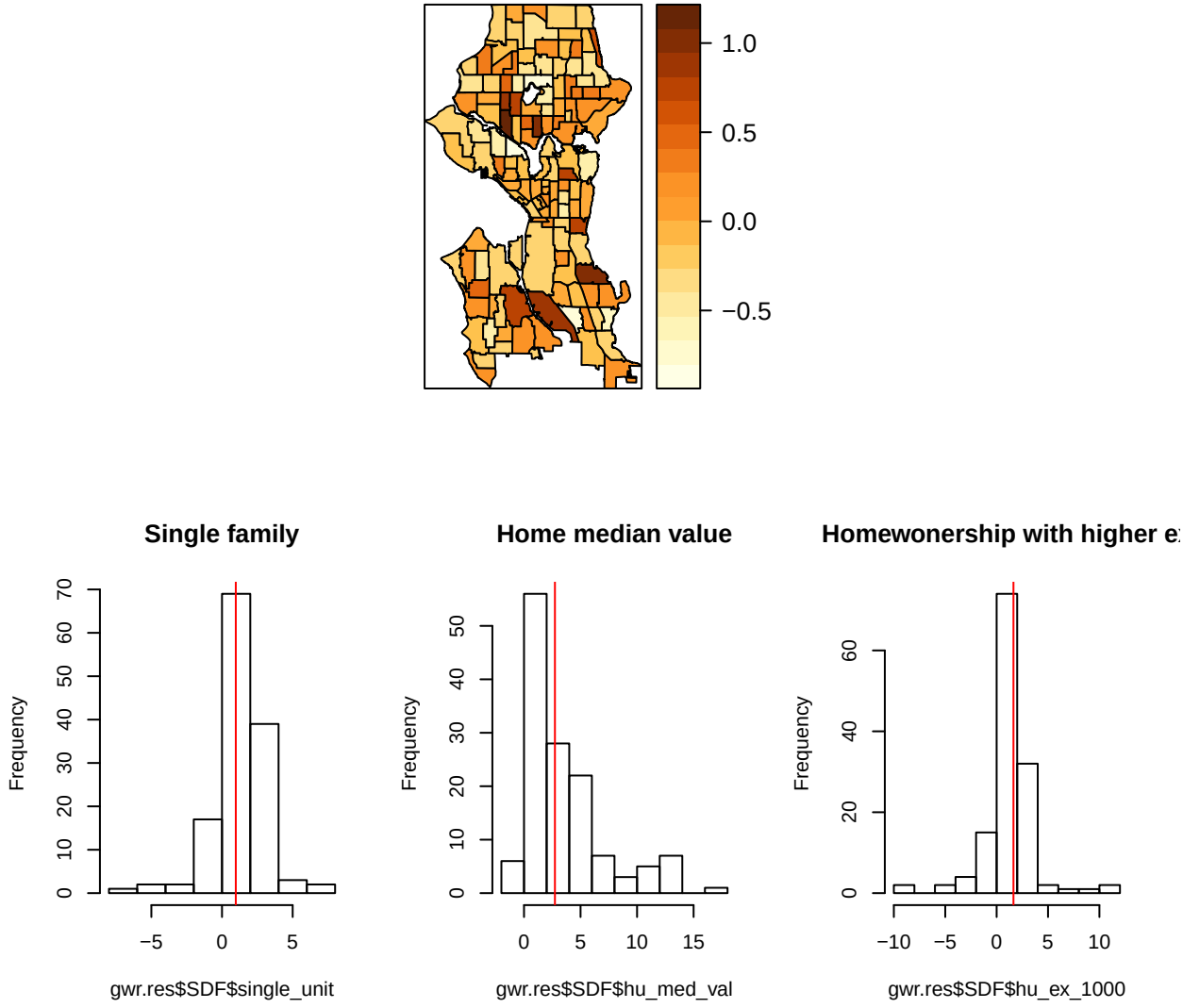


Residuals of spatial stsls regression with covariates (0.04 P value of Moran's I)

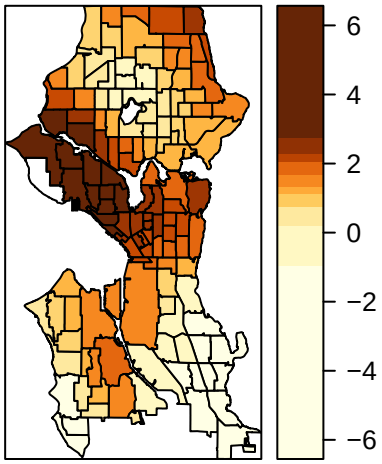


Residuals of spatial errorsar regression with covariates (0.50 P value of Moran's I)

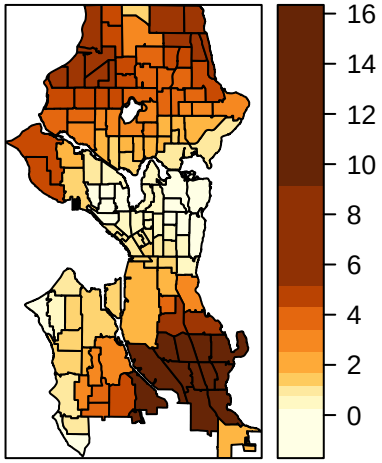


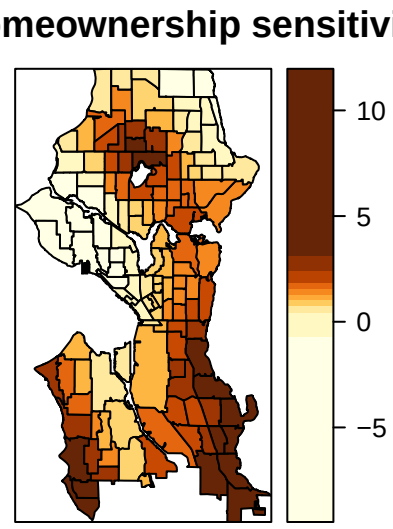

GWR residuals with covariates
(0.06 P value of Moran's I)
1.0
0.5
0.0
−0.5
Single family
Home median value
Homewonership with higher e
Frequency
gwr.resSDFsingle_unit
gwr.resSDFhu_med_val
gwr.resSDFhu_ex_1000

Single family sensitivity

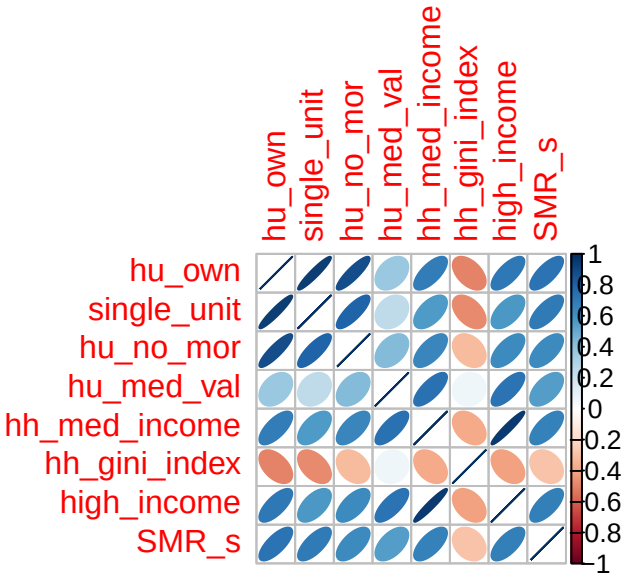


House median value sensitivity

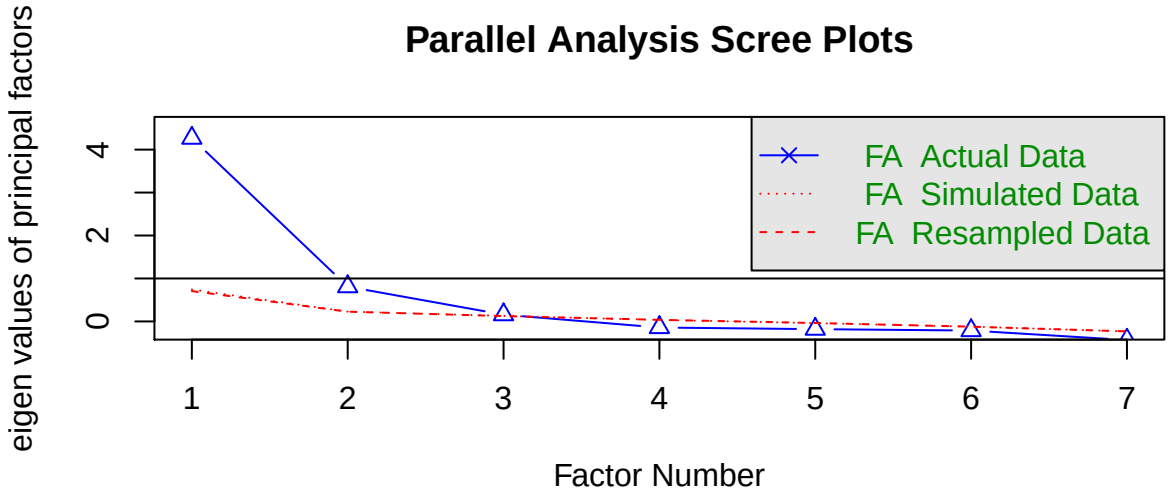




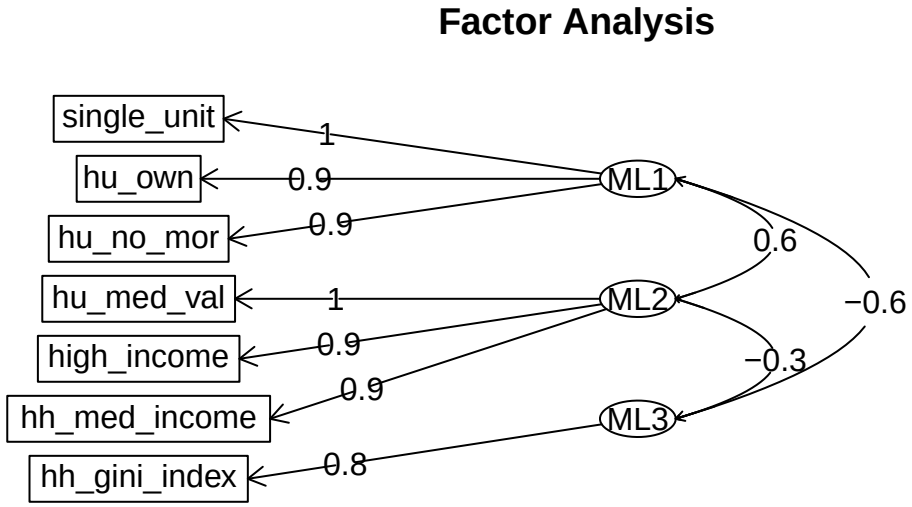
6. Analysis in detail



Factor analysis




```
## Parallel analysis suggests that the number of factors =  2  and the number of compone
```



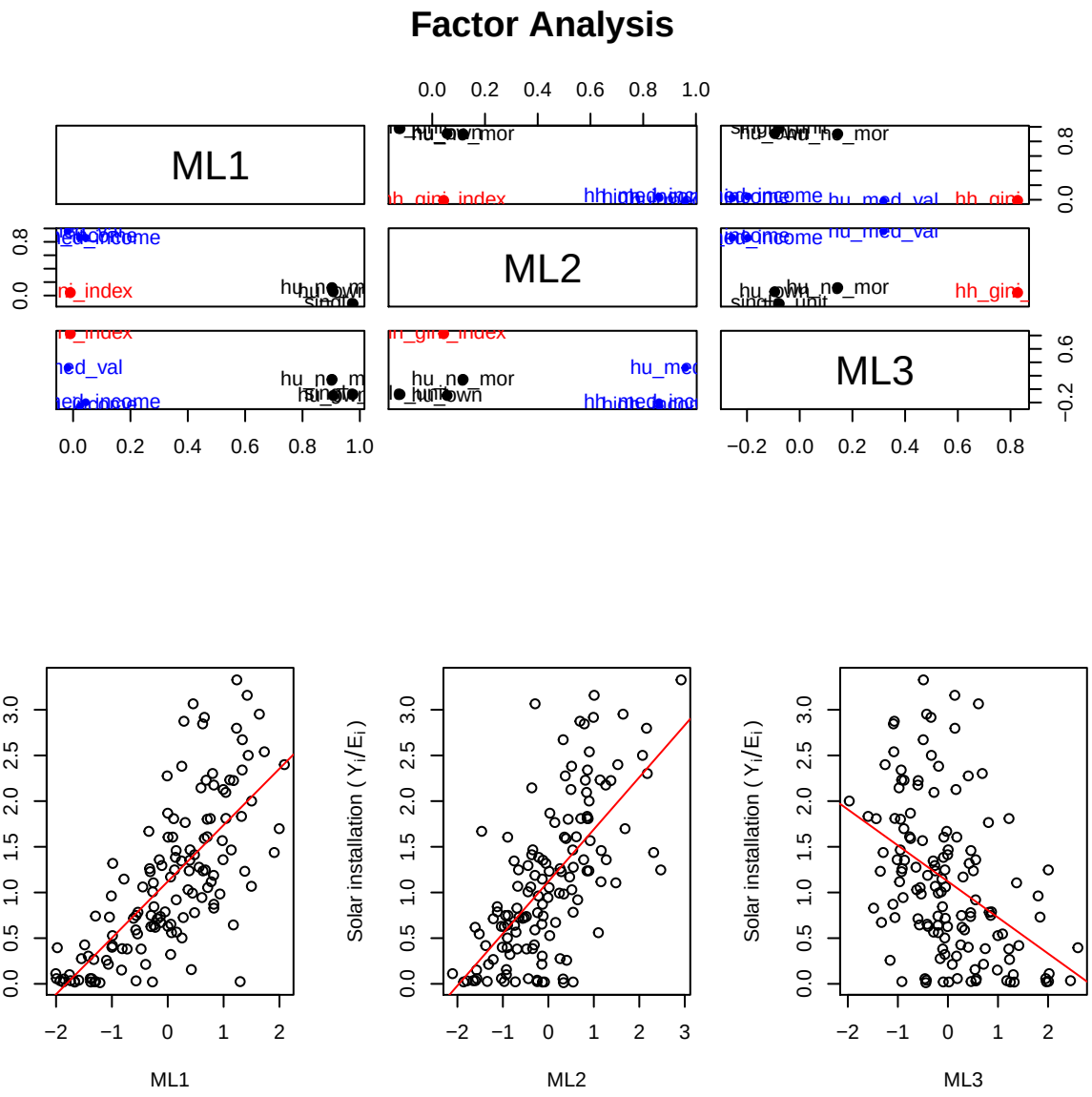


Table 23: Fitting linear model: seattle@data[[“SMR_s”]] ~ dat[, 1] + dat[, 2]

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	1.12	0.0452	24.79	1.682e-51
dat[, 1]	0.4363	0.05666	7.7	2.844e-12
dat[, 2]	0.3076	0.05727	5.371	3.431e-07

Table 24: Fitting generalized (poisson/log) linear model: seattle@data[["n_s"]] ~ offset(log(seattle@data[["solar_E"]])) + dat[, 1] + dat[, 2]

	Estimate	Std. Error	z value	Pr(>|z|)
(Intercept)	-0.1145	0.02038	-5.615	1.965e-08
dat[, 1]	0.5227	0.02451	21.33	6.585e-101
dat[, 2]	0.183	0.02243	8.157	3.424e-16

Table 25: Moran I test under randomisation: **`residuals(falm, type = "pearson")`** **`weights: col.w`** (continued below)

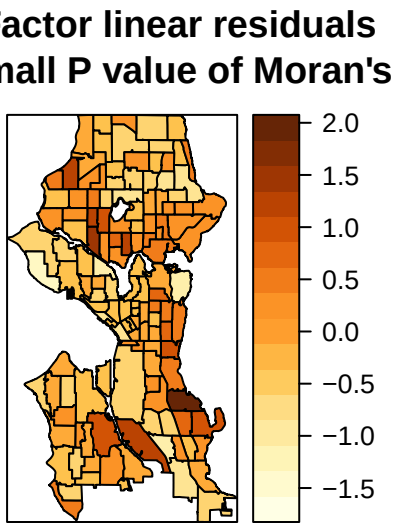
Test statistic	P value	Alternative hypothesis	Moran I statistic
5.356	4.254e-08 * * *	greater	0.2814

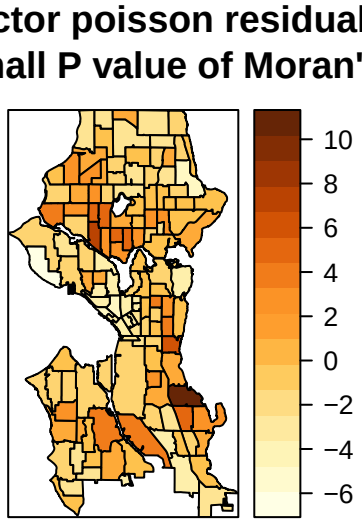
Expectation	Variance
-0.007463	0.002908

Table 27: Moran I test under randomisation: **`residuals(faglm, type = "pearson")`** **`weights: col.w`** (continued below)

Test statistic	P value	Alternative hypothesis	Moran I statistic
7.181	3.462e-13 * * *	greater	0.3794

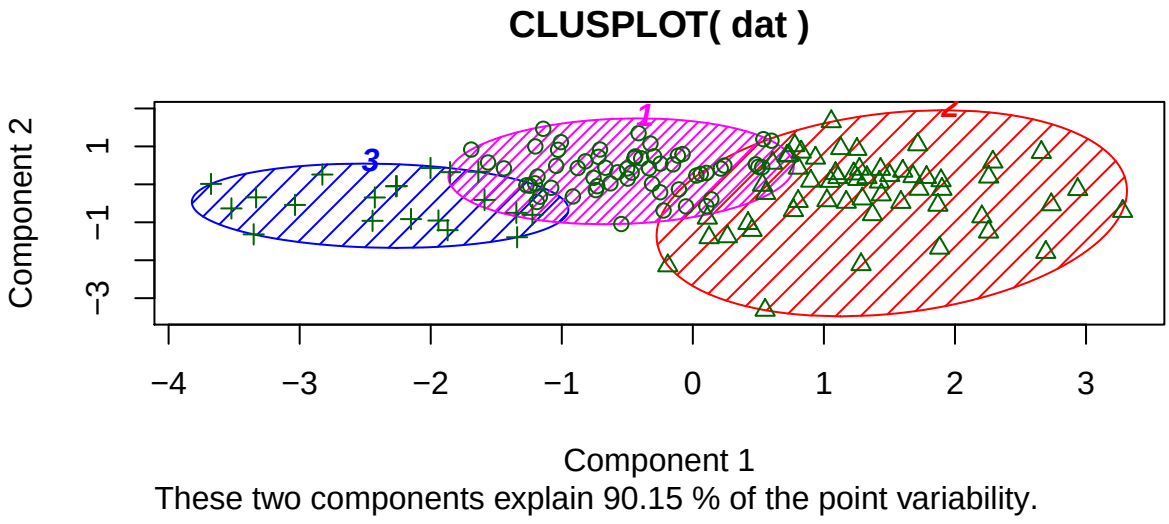
Expectation	Variance
-0.007463	0.002903





How can I use lagsar, stsls, errorsar and gwr for poisson model?

cluster analysis



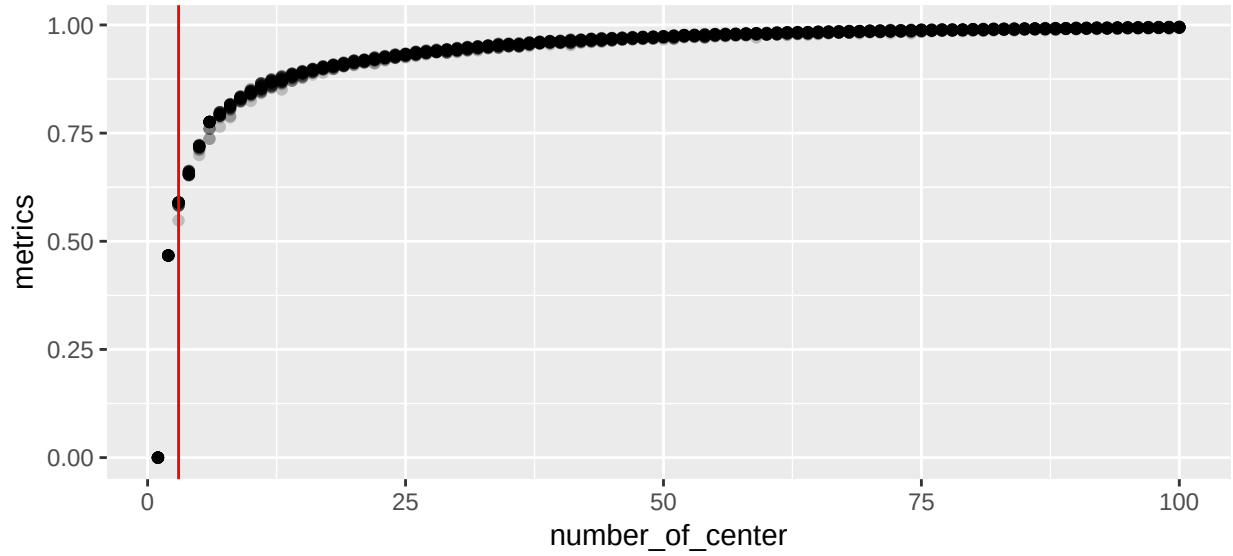
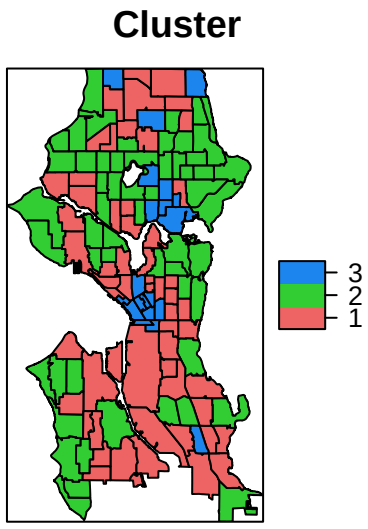


Table 29: Fitting linear model: regrs[["SMR_s"]] ~ single_unit + hu_med_val + hu_ex_1000

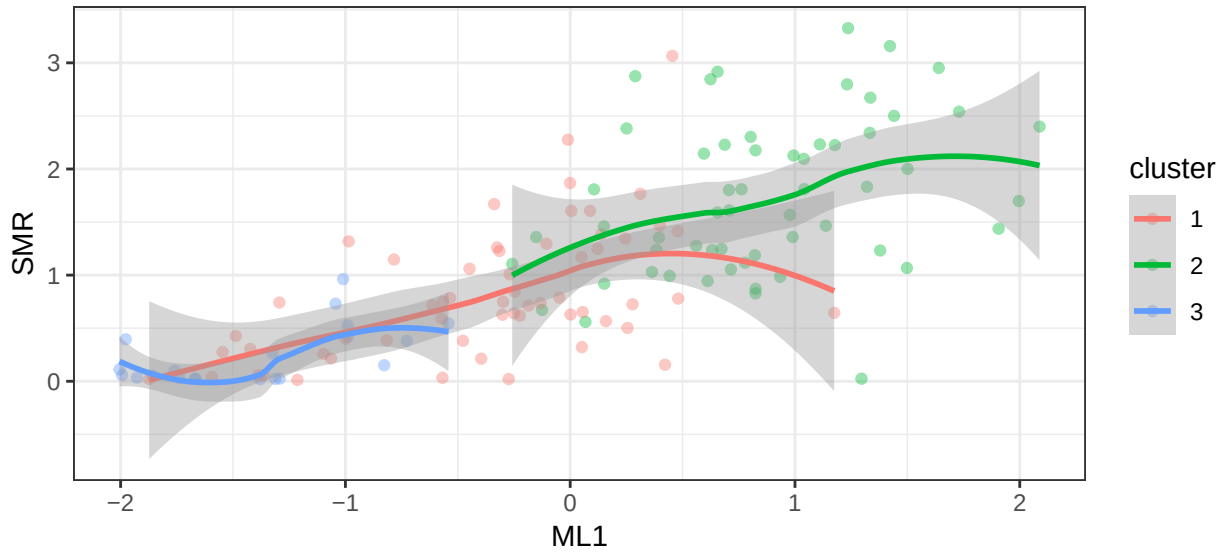
	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-0.8192	0.1447	-5.661	9.085e-08
single_unit	0.9753	0.3659	2.665	0.00866
hu_med_val	2.734	0.5508	4.963	2.115e-06
hu_ex_1000	1.637	0.7009	2.336	0.02102

Table 30: Fitting generalized (poisson/log) linear model: n_s ~ single_unit + hu_med_val + hu_ex_1000 + offset(log(solar_E))

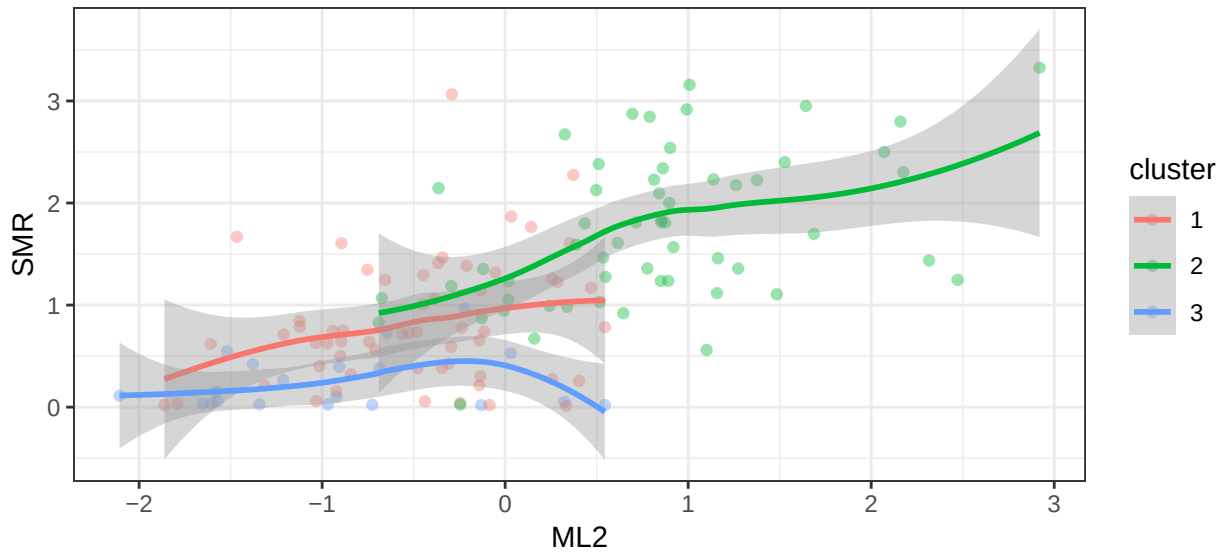
	Estimate	Std. Error	z value	Pr(>|z|)
(Intercept)	-1.996	0.07035	-28.37	4.925e-177
single_unit	1.962	0.1784	11	3.799e-28
hu_med_val	2.324	0.2168	10.72	8.004e-27
hu_ex_1000	0.2955	0.3177	0.9302	0.3523



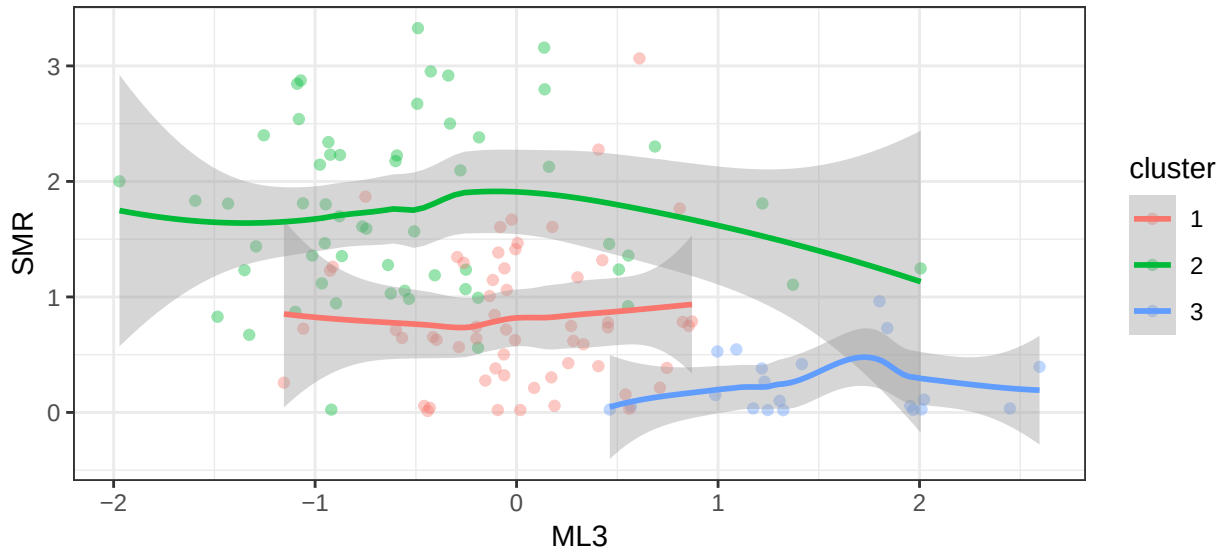

```
## [[1]]
## [[1]][[1]]
```



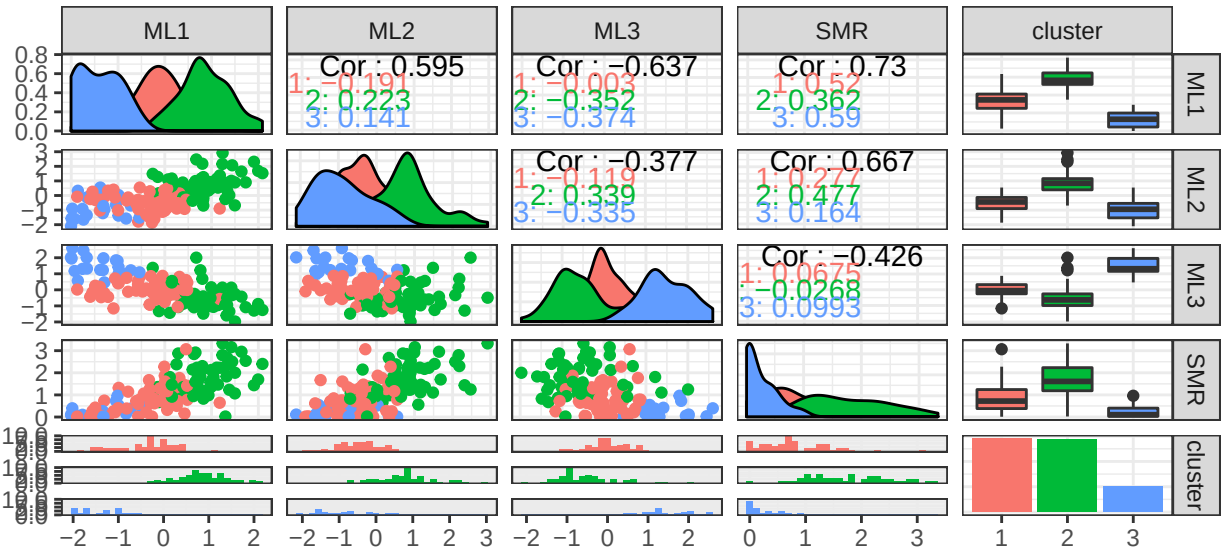

```
##
## [[1]][[2]]
```



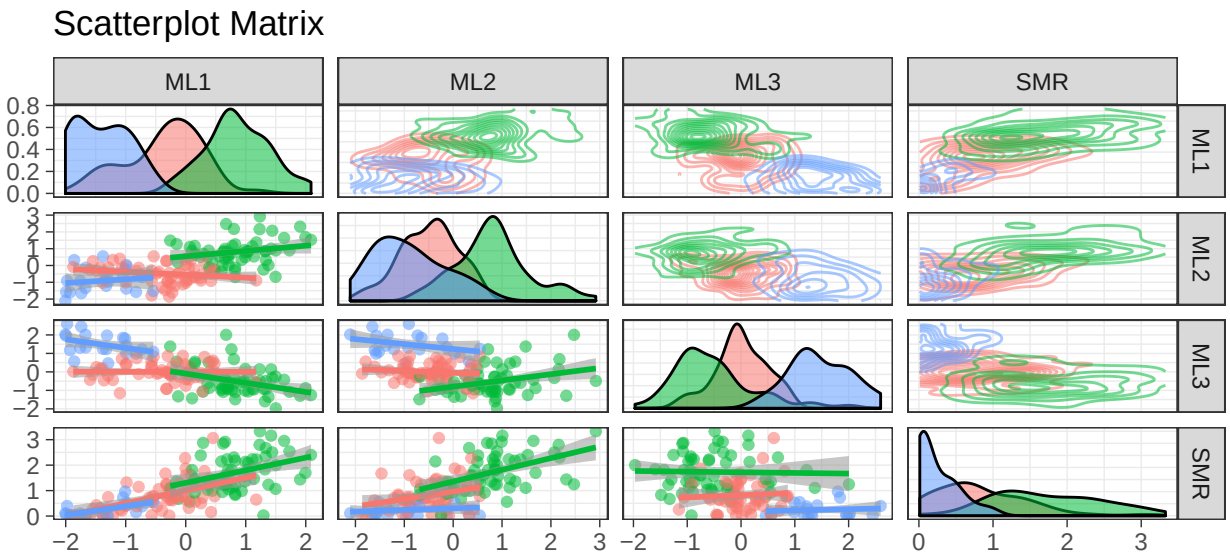

```
##
## [[1]][[3]]
```


SMR
ML3
cluster
1
2
3

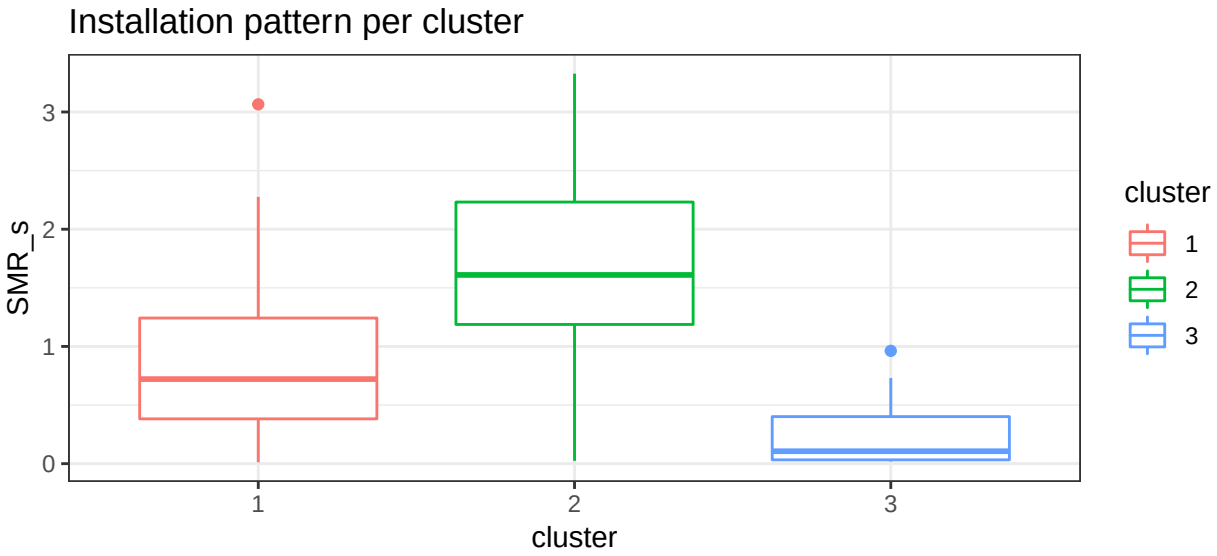
```
##
##
## [[2]]
```


ML1
ML2
ML3
SMR
cluster
Cor : 0.595
Cor : −0.637
Cor : 0.73
Cor : −0.377
Cor : 0.667
Cor : −0.426

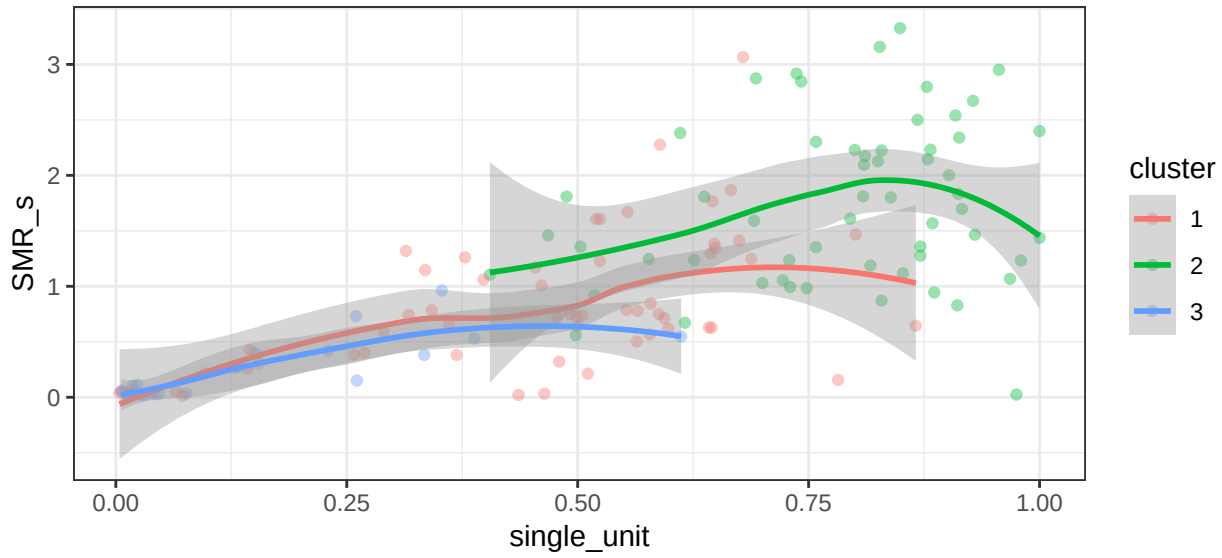
```
##
## [[3]]
```


Scatterplot Matrix
ML1
ML2
ML3
SMR

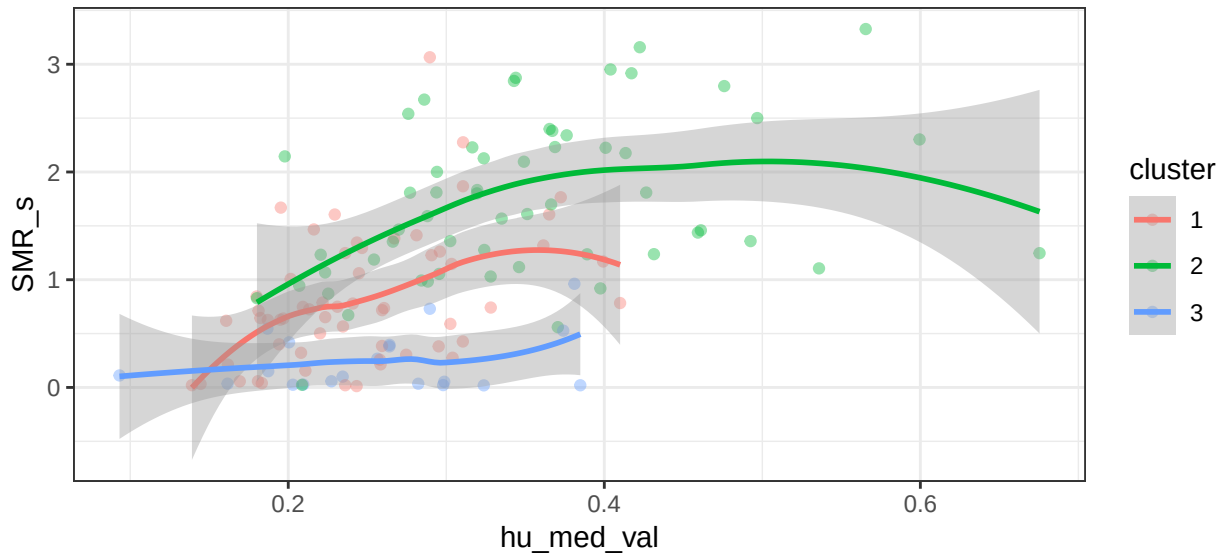
```
## [[1]]
```


Installation pattern per cluster
SMR_s
cluster
cluster
1
2
3

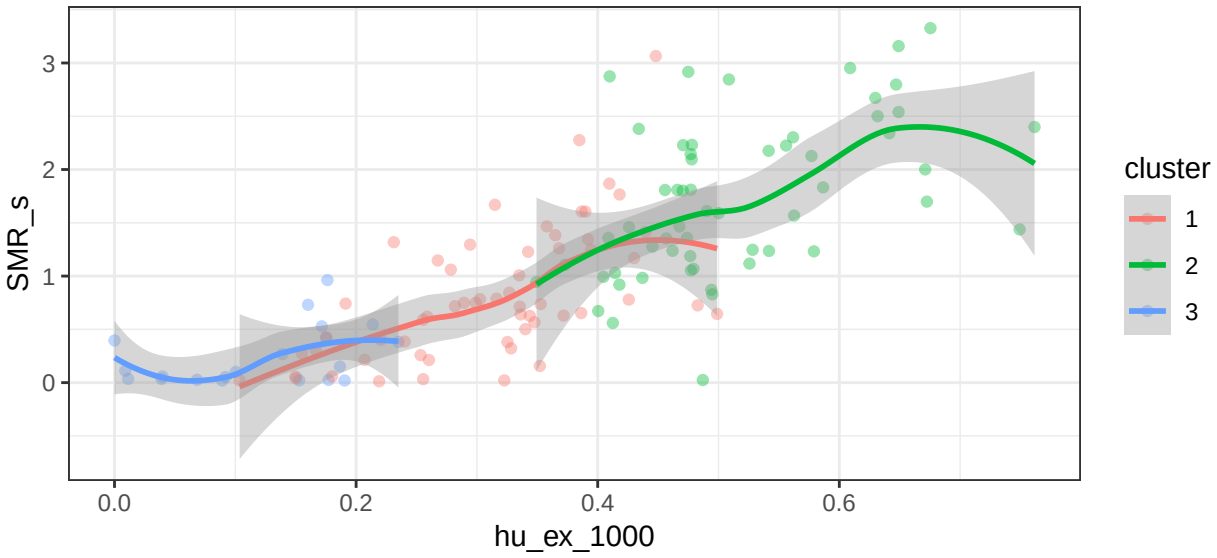
```
##
## [[2]]
## [[2]][[1]]
```


SMR_s
single_unit
0.00
0.25
0.50
0.75
1.00
0
1
2
3
cluster
1
2
3

```
##
## [[2]][[2]]
```


SMR_s
hu_med_val
0.2
0.4
0.6
0
1
2
3
cluster
1
2
3

```
##
## [[2]][[3]]
```


SMR_s
3
2
1
0
0.0
0.2
0.4
0.6
hu_ex_1000
cluster
1
2
3