# Optimizer Evaluation

(Version 0.06)

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## Preface

The program embedded in this document is intended to help determine the state of the art of optimization in production C compilers. I hope that the document itself will be useful to compiler writers, industrial and academic, and to programmers who are interested in learning what they can expect from a compiler.

This program should not be used to compare compilers. When comparison shopping for compilers, the following factors should be considered:

correctness The compiler shouldn't crash. Ought to generate correct code.

completeness cover the whole language? include extensions you want?

speed For most development, compilation speed is quite important.

space If it requires too much working space while compiling, you won't be able to run it on large routines.

#### error reporting

debugging support For correctness, but also for performance

cost A consideration for many individuals.

Certainly the engineers responsible for each compiler balance all these factors and make their design accordingly. It isn't fair to compare a quick development compiler (e.g., lcc) with a slower optimizing compiler (e.g., gcc) on the basis of optimization. They have different goals and strengths.

Of course, many people are interested in code quality – that is, the speed of the code generated for real applications (especially their applications). The examples in this program are not real; they are tiny, artificial cases, useful only because they help expose certain picky details of optimizer design. On the other hand, it can be argued that attention to the details tested here will pay off in terms of more complete optimization of real programs.

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## Chapter 1

# Introduction

[1]

Many people invent their own approaches to problems as they arise. While I applaud invention, reinvention is less interesting. I am particularly distressed by inferior approaches to well-understood problems.

[5, 8, 6, 7]

get all the details right.

"Best" versus "best, simple" [19].

[12] want optimizer that doesn't degrade in the face of inlining

#### 1.1 Goals

collect and record examples and counter-examples interested in finding out which algorithms are used in industry does anyone do better than published work? educates by showing what is (and is not) possible, weaknesses and strengths of different algorithms

#### 1.2 Related Work

[25]

[20]

less detailed, different goals; they're interested in how much optimization affects different programs. I want details about whether particular optimizations are done well.

## 1.3 Approach

discuss state of the art, and perhaps some common alternatives (and their weaknesses) a series of examples (in C for now), test code, result code, time when creating test cases, verify behaviour by examining object code try to show real-world examples

#### 1.3.1 Optimizations Covered

dead code elimination Not the same as elimination of unreachable code

constant propagation

value numbering elimination of redundant computations

code motion primarily moving invariant code out of loops

strength reduction

Well known, widely applicable, I understand them

#### 1.3.2 Other Possibilities

I haven't tried to cover every possible optimization. Important but neglected areas include:

- instruction scheduling
- register allocation
- instruction selection
- vectorization
- cache management
- pointer/structure/array/alias analysis

Nor have I tried to test every desirable feature of a compiler, or even the optimizer.

- correctness
- efficiency (time and space) I wish these were reported along with SPECmarks.

I haven't even tried to cover any language beyond C. Certainly a similar test would be practical, and I think desirable, for Fortran compilers.

## 1.4 Acknowledgements

Ken Kennedy, Keith Cooper, Linda Torczon, Cliff Click, Rob Shillner, Steve Carr, Brian Koblenz, Michael Lewis, John Elder, Robert Metzger

# Chapter 2

# **Experiments**

The plan is to conduct a large number of experiments to test the abilities of an optimizer.

## 2.1 Program Organization

```
"{\tt global.h"} {\equiv}
3a
                   \langle \text{Include files } 3d, \ldots \rangle
                   \langle \text{Types } 3e \rangle
           "main.c"≡
3b
                   #include "global.h"
                   \langle Constants 7a, \ldots \rangle
                   \langle \text{Variables } 9b \rangle
                   (Prototypes 4, ...)
           See 3c, 6b, 8cd, 9e, 10.
           "{\tt main.c}" \equiv
                   int main()
                      \langle Initialize 9c \rangle
                      puts("Begin tests, version 0.06");
                      (Evaluation drivers 11a, ...)
                      puts("Tests completed");
                      exit(0);
           See 3b, 6b, 8cd, 9e, 10.
           ⟨Include files⟩ ≡
3d
                   #include <stdlib.h>
                   #include <stdio.h>
           See 3a, 6a, 9a.
```

## 2.2 Controlling the Experiments

Each experiment will be controlled by test, a routine taking two function parameters, base\_case and test\_case, each of type FunPar.

```
\exists e \langle \text{Types} \rangle \equiv typedef void (*FunPar)(int *); \Diamond See 3a.
```

The test routine will repeatedly measure the time required to run each of the parameter functions. If the average times required for the two functions do not differ significantly, test will return 1, indicating that the

compiler was able to optimize the test case to match the (hand optimized) base case. If the time required for the base case is significantly less than the time required for the test case, test will return 0, indicating that the compiler did not perform the optimization in question.

Of course, there's a third possibility: The test case might require significantly less time than the base case. In this event, test will complain and halt, printing the id of the bungled test. This shouldn't happen during an actual run, though it might happen during development of the test cases.

The data parameter allows us to specify some arbitrarily-structured data to be passed as an argument to each of the routines.

#### 2.2.1 Theory

4

The question of whether the times required for the two functions differs significantly is hard. On most modern machines, there are at least three components contributing to measurement uncertainty:

- 1. timer quantization error,
- 2. jitter caused by context switches, and
- 3. cache pollution caused by competing processes.

In the next three sections, I'll outline approaches to each of these problems.

#### **Timer Quantization Errors**

The timers available on some common workstations are relatively coarse, typically 60 or 100Hz. Since a sample run may start anywhere within the interval of a single timer tick, the measured length of a run may vary from its true value by up to one tick. If our timing runs are too short, this error will dominate our measurements. The way around the problem is to ensure that tests run for many ticks. For example, if a single test runs for more than 100 ticks, we would expect that the quantization error would be less than 1%.

Since the tests will typically be quite short (perhaps a dozen instructions each), I'll wrap a loop around each one. By setting the number of iterations for each loop high enough, I can ensure that each test runs long enough (say, at least 100 ticks) to minimize quantization errors.

#### Context Switches

Context switches under a multitasking operating system exacerbate the quantization error described above. A process may be interrupted anywhere in a tick and control returned at some other point. Furthermore, a potentially unbounded number of interuptions may occur during a given run. The effect of each interruptions is introduce a new error of up to one tick, either or longer or shorter. While we cannot predict the number of interruptions that might occur during a single run, we can expect that the cummulative errors introduced would be normally distributed.

To control this source of error, we'll measure each routine several times and compare the results using statistical analysis based on Student's t distributions [23, Section 11–3]. This is the desired approach given the following assumptions:

• the size of the sample sets is small,

- they are normally distributed, and
- they have the same variance.

Minimizing the size of the sample sets (the number of times we have to measure each routine) is desirable, since we'd like the complete set of tests to finish is a reasonable time. The assumption of identical variance turns out not to matter; that is, under the conditions of our experiment (two sample sets of equal size), the test described below is equivalent to a variance test.

We begin by collecting a sample set for each routine x and y, where the number of samples in each set is identical and will be called n. (A reasonable number of samples might be 10.) Compute arithmetic means  $\bar{x}$  and  $\bar{y}$  for the sample sets, where

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Compute the variance  $\sigma_x$  and  $\sigma_y$  for each sample set, where

$$\sigma_x = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n - 1}$$

Compute an estimate of the pooled variance  $\hat{\sigma}$ , where

$$\hat{\sigma} = \frac{\sigma_x + \sigma_y}{2}$$

Finally, compute the statistic T, where

$$T = \frac{\bar{x} - \bar{y}}{\sqrt{2\hat{\sigma}/n}}$$

Large absolute values of T suggest that it is unlikely that  $\bar{x} = \bar{y}$ . Thus, if |T| > t(2n-2,p), we say that  $\bar{x}$  differs from  $\bar{y}$  at the p level (where t is a table of magic values copied from a statistics book [23, Table A-8]). For example, if n = 10 and T = 2.61, we can examine the row given by t(18,\*) in Table 2.1 and we see that 2.878 < T < 2.552, leading us to reject the hypothesis that  $\bar{x} = \bar{y}$ , with less than 1% chance of error.

#### Cache Pollution

We use the term "cache pollution" to encompass an entire set of difficulties introduced by the memory hierarchy. In the worst case, we might see effects from the data cache, the instruction cache, a second-level cache, and the TLB. There seem to be two sources of cache pollution:

self interference In the absence of context switches, we would expect that the possibility of self interference would be minimized since each test is quite small, certainly fitting into the I cache, and only references

Degrees of			Pr	obability			
freedom	0.005	0.01	0.025	0.05	0.1	0.15	
:							
;							
16	2.921	2.583	2.120	1.746	1.363	1.071	
17	2.898	2.567	2.110	1.740	1.333	1.069	
18	2.878	2.552	2.101	1.743	1.330	1.067	
19	2.861	2.539	2.093	1.729	1.328	1.066	
20	2.845	2.528	2.086	1.725	1.325	1.064	
:							

 Table 2.1
 Student's t Distributions

a tiny amount of data, which will easily fit into the D cache. A single, preliminary run of a test should provoke any initial cache misses, effectively warming them up.

Note that a machine with a combined cache (versus separate I and D caches) may provoke conflicts between instructions and data, particularly in a direct-mapped cache. Since the conflict may (unpredicably) hurt one case without harming another, it won't really be possible to ensure fair comparisons on such machines.

interference from other processes Besides the quantization errors discussed above, a context switch can also leave the cache in a non-deterministic state, typically provoking a string of cache misses.

Again, the tactic of using many runs comes to mind. Unfortunately, the errors introduced in this case are not normally distributed. Cache misses always add to the run time; they never decrease it. This suggests using the minimum time from several runs rather than computing the mean. Unfortunately, this notion conflicts with our earlier approach to minimizing the effects of quantization errors due to context switches.

I don't have a compelling solution to the problems raised here. It may be a topic for research. I'll have to talk to some statistics people about it. Things that come to mind:

- Obviously, try to get the machine in single-user mode, or at least lightly loaded.
- If the standard deviations seem "too high" (more than 2 clock ticks, maybe?), complain that the data is too noisy and ignore the results.

Is there any chance of bounding, statistically, the chance for error? I'm already unsure about the combination of running for a long time and running many times. Sure, the more/longer we run the thing, the more confidence we can have in our conclusion; but what is the real chance for error?

#### 2.2.2 Practice

When collecting samples, run once, ignoring the result, just to get the Instruction cache stable. Collect 10 times from one, then 10 timings from the other.

Work on tests so that the differences are significant.

```
⟨Include files⟩ ≡
6 a
              #include <math.h>
              \Diamond
        See 3a, 3d, 9a.
        "main.c"≡
6h
              int test(int ok,
                       FunPar base_case,
                        FunPar test_case,
                        int *data,
                        int n,
                        char *id)
              {
                int ret_val = 0;
                printf("
                             %d) %s - ", n, id);
                if (1 /* always test, for now */ || ok) {
                  double base_times[SAMPLES], test_times[SAMPLES];
                  long int iterations = determine_iterations(base_case, data);
                  collect_timings(iterations, test_case, data, test_times);
                  collect_timings(iterations, base_case, data, base_times);
                  \langle Analyze timing results 7c \rangle
                else puts("skipping");
                return ret_val;
        See 3bc, 8cd, 9e, 10.
```

```
\langle Constants \rangle \equiv
\gamma_a
                #define SAMPLES 10
                                                                  for SPARC */
                /* #define CLOCKS_PER_SEC 1000000
         See 3b, 7d.
         \langle Prototypes \rangle \equiv
7h
                long int determine_iterations(FunPar base_case,
                                                      int *data);
                void collect_timings(long int iterations,
                                          FunPar routine,
                                          int *data,
                                          double *times);
                \Diamond
         See 3b, 4, 8e, 17, 25, 32b, 39, 45c.
```

Note that we've done a little algebraic manipulation, effectively rewriting the computation of T as

$$T = (\bar{x} - \bar{y})\sqrt{\frac{n}{\sigma_x + \sigma_y}}$$

which looks a little uglier, but should save some time and precision. Additionally, we take care to avoid division by 0. This is a real possibility, especially with relatively slow clocks. It doesn't imply any sort of problem; it only means that all the times measured for each were identical.

```
⟨Analyze timing results⟩ ≡
       double base_mean = mean(base_times);
       double test_mean = mean(test_times);
       double result = test_mean - base_mean;
       double base_variance = variance(base_mean, base_times);
       double test_variance = variance(test_mean, test_times);
       double sum = base_variance + test_variance;
       if (sum == 0.0) {
         if (result == 0.0)
                                 ret_val = 1;
         else if (result < 0.0) ret_val = -1;
       }
       else {
         result = result * sqrt(SAMPLES / sum);
         if (fabs(result) <= CUTOFF) ret_val = 1;</pre>
         else if (result < 0.0)
                               puts("yes");
       if (ret_val > 0)
       else if (ret_val == 0) puts("no");
                               puts("bad test");
       (Check for exceptionally noisy data 8a)
     }◊
See 6b.
```

We'll define CUTOFF to limit ourselves to a 1% chance of mistakenly claiming an optimization was not performed.

```
7d \langle \text{Constants} \rangle \equiv
#define CUTOFF 2.552
\Diamond
See 3b, 7a.
```

7c

```
(Check for exceptionally noisy data) ≡
8 a.
                int n = noisy(base_variance) || noisy(test_variance);
                  puts("\t(warning -- the timings were not very consistent)");
                if (1 /* always print, for now */ || n || ret_val == -1)
                   (Print times and statistical measures 8b)
              }◊
        See 7c.
        I've broken this out as a separate scrap so I can use it conveniently during development of the tests.
8b
        \langle Print times and statistical measures \rangle \equiv
                int i:
                puts("\nTimings\n\tbase-case\ttest-case");
                for (i=0; i<SAMPLES; i++)</pre>
                  printf("\t\%-9.3f\t\%-9.3f\n", base_times[i], test_times[i]);
                printf("means\t\"-9.3f\t\"-9.3f\n", base_mean, test_mean);
                printf("vars\t\%-9.6f\t\%-9.6f\n", base_variance, test_variance);
                printf("sds\t%-9.6f\t%-9.6f\n", sqrt(base_variance), sqrt(test_variance));
                printf("result\t%-9.6f\n\n", result);
              }◊
        See 8a.
        We'll assume the data was noisy if the standard deviation is more than 2 ticks.
8c
        "main.c"≡
              int noisy(double variance)
              {
                double std_dev = sqrt(variance);
                return std_dev > 2.0 / ticks_per_sec;
              }
              \Diamond
        See 3bc, 6b, 8d, 9e, 10.
        "main.c"≡
8d
              double mean(double *times)
                                                                  double variance(double mean, double *times)
                int i;
                                                                    int i;
                double sum = 0.0;
                                                                    double sum = 0.0;
                for (i=0; i<SAMPLES; i++)</pre>
                                                                    for (i=0; i<SAMPLES; i++) {</pre>
                  sum += times[i];
                                                                      double diff = mean - times[i];
                return sum / SAMPLES;
                                                                      sum += diff * diff;
              }
              \Diamond
                                                                    return sum / (SAMPLES - 1);
                                                                 }
                                                                  \Diamond
        See 3bc, 6b, 8c, 9e, 10.
8 e
        \langle Prototypes \rangle \equiv
              double mean(double *times);
              double variance(double mean, double *times);
        See 3b, 4, 7b, 17, 25, 32b, 39, 45c.
```

#### 2.2.3 Timing Routines

In the bad old days, timing routines were fairly machine dependent and programs like this had to be customized for each machine. Fortunately, there's now some standardization in the area and we are able to use routines from ANSI C's standard runtime library to avoid some of this porting effort.

```
g_a \langle \text{Include files} \rangle \equiv #include <time.h> \Diamond See 3a, \beta d, 6a.
```

One might think that the value of CLOCKS\_PER\_SEC would indicate the number of timer ticks per second, allowing convenient determination of the timer resolution. Unfortunately, this isn't the case. Some systems define CLOCKS\_PER\_SEC to be a very large value, despite a fairly coarse timer.

```
9b
        \langle Variables \rangle \equiv
              clock_t ticks_per_sec = 0;
        See 3b.
gc
        ⟨Initialize⟩ ≡
                clock_t start, diff;
                (Make sure clock() works on this system 9d)
                start = clock();
                diff = clock() - start;
                while (diff == 0)
                  diff = clock() - start;
                ticks_per_sec = CLOCKS_PER_SEC / diff;
                printf("Timer seems to run at %d ticks/second\n", ticks_per_sec);
              }◊
        See 3c.
```

Though clock is a standard routine, systems are not required to support it; as an alternative, they may return -1. On such systems, an alternative approach will be required (i.e., we're back to customizing the code for every machine).

```
(Make sure clock() works on this system) ≡
9d
               if (clock() == -1) {
                 fputs("\nThe routine 'clock' is not supported\n", stderr);
                 exit(-1);
             }◊
       See 9c.
       "main.c"≡
9 e
             long int determine_iterations(FunPar base_case,
                                             int *data)
             {
               long int iterations = 1;
               base_case(data); /* an initial, untimed invocation */
               while (1) {
                 long int i;
                 clock_t start = clock();
                 for (i=0; i<iterations; i++)</pre>
                   base_case(data);
                 if (((double) clock() - start) / CLOCKS_PER_SEC > 100.0 / ticks_per_sec)
                   return iterations;
                 iterations += iterations;
             }
             \Diamond
```

See 3bc, 6b, 8cd, 10.

```
"main.c"≡
10
              {\tt void\ collect\_timings(long\ int\ iterations,}
                                       {\tt FunPar\ routine}\,,
                                       int *data,
                                       double *times)
              {
                 int i;
                 \verb"routine(data)"; \quad /* \ \verb"an initial", untimed invocation" */
                 for (i=0; i<SAMPLES; i++) {</pre>
                   long int j;
                   clock_t start = clock();
                   for (j=0; j<iterations; j++)
                     routine(data);
                   times[i] = ((double) (clock() - start)) / CLOCKS_PER_SEC;
              }
               \Diamond
        See 3bc, 6b, 8cd, 9e.
```

## Chapter 3

## **Dead Code Elimination**

```
best approach is based on use-def chains [15, 16, 17]
using liveness analysis is inferior [9]
ref-counting is inferior
SSA-based version of use-def chains is cheaper [13, Section 7.1]
Allows removal of useless control flow, and therefore more useless expressions partially dead code
within block, across blocks, simple loop, complex loop case, dead control flow (if and loop)

(Evaluation drivers) \equiv \{
    int ok;
    puts ("Dead code elimination");
    \{Dead code elimination \( 11c, \ldots \) \}
See 3c, \( 18a, \( 26a, \) \( 33a, \) \( 40a. \)
```

#### 3.1 Local

#### 3.1.1 The Easiest Case

See 12ac, 13ac, 14ac, 15ac, 16ac.

This illustrates one of the easiest possible cases. The initialization of j is useless, so the add and the first two references to data may be removed, as shown in the result case.

This bit of code is typical of the way we'll invoke test. We initialize a small array of data, then call test, passing in the test data and the two routines to be compared. The first parameter is simply a flag indicating the test is to be performed (in other cases, we'll pass in a variable controlling whether the comparison is to be performed or skipped). The last two parameters are used to identify the test.

#### 3.1.2 Slightly More Complex

```
"dead.c"≡
12a
              void dead_test2(int *data)
                                                                                  void dead_result2(int *data)
                int j = data[0] + data[1];
                                                                                     int j;
                int k = j + data[2];
                                                                                     j = data[2];
                int m = k + data[3];
                                                                                     data[j] = 2;
                int n = m + data[4];
                                                                                  }
                j = data[2];
                data[j] = 2;
        See 11b, 12c, 13ac, 14ac, 15ac, 16ac.
12b
        \langle \text{Dead code elimination} \rangle \equiv
              {
                int data[5] = { 0, 1, 2, 3, 4 };
                ok = test(ok, dead_result2, dead_test2, data, 2, "basic block");
        See 11a, 11c, 12d, 13bd, 14bd, 15bd, 16bd.
```

#### 3.2 Global

#### 3.2.1 Across Basic Blocks

#### Easy

```
12c
        "dead.c"≡
              void dead_test3(int *data)
                                                                                   void dead_result3(int *data)
                int k = 0;
                                                                                     int k = 0;
                int j = data[1];
                                                                                     int j = data[1];
                if (j) {
                                                                                     if (j)
                  k++;
                                                                                        k++;
                  j = k + data[0] * j;
                                                                                      else
                }
                                                                                        k--;
                else
                                                                                      j = data[2];
                  k--;
                                                                                      data[j] = 2;
                j = data[2];
                                                                                     data[3] = k;
                data[j] = 2;
                                                                                   }
                                                                                    \Diamond
                data[3] = k;
        See 11b, 12a, 13ac, 14ac, 15ac, 16ac.
12d
        \langle \text{Dead code elimination} \rangle \equiv
              {
                int data[4] = { 0, 1, 2 };
                ok = test(ok, dead_result3, dead_test3, data, 3, "across basic blocks");
        See 11a, 11c, 12b, 13bd, 14bd, 15bd, 16bd.
```

```
More Interesting
```

See 11a, 11c, 12bd, 13b, 14bd, 15bd, 16bd.

"dead.c"≡

13a

```
void dead_test4(int *data)
                                                                                 void dead_result4(int *data)
                int k = 0;
                                                                                   int k = 0;
                int j = data[1];
                                                                                   int j = data[1];
                if (j) {
                                                                                   if (j)
                  k++;
                                                                                      k++;
                  j = k + j * data[0];
                                                                                   else
                                                                                      k--;
                                                                                   if (data[4] & 1)
                else
                  k--;
                                                                                      k++;
                if (data[4] & 1)
                                                                                      k--;
                                                                                   j = data[2];
                else {
                                                                                   data[j] = 2;
                  k--;
                                                                                   data[3] = k;
                  j++;
                                                                                 }
                                                                                 \Diamond
                j = data[2];
                data[j] = 2;
                data[3] = k;
        See 11b, 12ac, 13c, 14ac, 15ac, 16ac.
13b
        \langle \text{Dead code elimination} \rangle \equiv
                int data[5] = { 0, 1, 2, 3, 4 };
                ok = test(ok, dead_result4, dead_test4, data, 4, "across basic blocks");
        See 11a, 11c, 12bd, 13d, 14bd, 15bd, 16bd.
        3.2.2
                 Around Loops
        Simple
        "dead.c"≡
13c
              void dead_test5(int *data)
                                                                                 void dead_result5(int *data)
                int i, j;
                                                                                   int i, j;
                int k = 0;
                                                                                   int stop = data[0];
                                                                                   for (i=0; i<stop; i++) {
                int stop = data[0];
                for (i=0; i<stop; i++) {
                                                                                      j = data[2];
                  k = k * data[1];
                                                                                      data[j] = 2;
                  j = data[2];
                  data[j] = 2;
                                                                                 }
                }
             }
        See 11b, 12ac, 13a, 14ac, 15ac, 16ac.
13d
        \langle \text{Dead code elimination} \rangle \equiv
             {
                int data[3] = { 2, 1, 2 };
                ok = test(ok, dead_result5, dead_test5, data, 5, "around loops");
```

# Interesting "dead.c"≡

14a

```
void dead_test6(int *data)
                                                                                 void dead_result6(int *data)
              {
                int i;
                                                                                   int i;
                int k = 0;
                                                                                   int k = 0;
                int m = 0;
                                                                                   int m = 0;
                int n = 0;
                                                                                   int n = 0;
                int stop = data[0];
                                                                                   int stop = data[0];
                for (i=0; i<stop; i++) {
                                                                                   for (i=0; i<stop; i++) {
                  int j = data[1];
                                                                                     int j = data[1];
                  if (j) {
                                                                                     if (j)
                    n = j * stop + m;
                                                                                       k++;
                                                                                     else
                    k++;
                                                                                       k--;
                                                                                     j = data[2];
                  else {
                                                                                     data[j] = 2;
                    m = n * stop + k;
                    k--;
                                                                                   data[3] = k;
                  j = data[2];
                  data[j] = 2;
                data[3] = k;
              }
        See 11b, 12ac, 13ac, 14c, 15ac, 16ac.
14b
        \langle \text{Dead code elimination} \rangle \equiv
                int data[4] = { 2, 1, 2 };
                ok = test(ok, dead_result6, dead_test6, data, 6, "around loops");
        See 11a, 11c, 12bd, 13bd, 14d, 15bd, 16bd.
```

#### 3.3 Dead Control Flow

#### 3.3.1 Conditionals

#### Simple

```
"dead.c"≡
14c
              void dead_test7(int *data)
                                                                                      void dead_result7(int *data)
              {
                                                                                      {
                 int k = 0;
                                                                                        int j = data[2];
                 int j = data[2];
                                                                                        data[j] = 2;
                 data[j] = 2;
                                                                                     }
                 if (j) k++;
                                                                                      \Diamond
                 else k--;
              }
        See 11b, 12ac, 13ac, 14a, 15ac, 16ac.
        \langle \text{Dead code elimination} \rangle \equiv
14d
              {
                 int data[3] = { 0, 1, 2 };
                 ok = test(1, dead_result7, dead_test7, data, 7, "useless conditionals");
        See 11a, 11c, 12bd, 13bd, 14b, 15bd, 16bd.
```

# Interesting $"dead.c" \equiv$

15a

```
void dead_test8(int *data)
                                                                                   void dead_result8(int *data)
              {
                int i;
                                                                                      int i;
                int k = 0;
                int stop = data[0];
                                                                                      int stop = data[0];
                for (i=0; i<stop; i++) {
                                                                                      for (i=0; i<stop; i++) {
                  int j = data[1] + k;
                                                                                        int j;
                  if (j) k++;
                                                                                        j = data[2];
                                                                                        data[j] = 2;
                  else k--;
                  j = data[2];
                                                                                   }
                  data[j] = 2;
                                                                                    \Diamond
              }
        See 11b, 12ac, 13ac, 14ac, 15c, 16ac.
15b
        \langle \text{Dead code elimination} \rangle \equiv
              {
                int data[3] = { 2, 1, 2 };
                ok = test(ok, dead_result8, dead_test8, data, 8, "useless conditionals");
        See 11a, 11c, 12bd, 13bd, 14bd, 15d, 16bd.
        3.3.2 Loops
        Conservative
        "\mathtt{dead.c"} \equiv
15c
              void dead_test9(int *data)
                                                                                    void dead_result9(int *data)
                                                                                   {
              {
                int k = 0;
                                                                                     int j;
                                                                                     j = data[2];
                int j;
                for (j=0; j<5; j++) k++;
                                                                                     data[j] = 2;
                j = data[2];
                data[j] = 2;
              }
        See 11b, 12ac, 13ac, 14ac, 15a, 16ac.
15d
        \langle \text{Dead code elimination} \rangle \equiv
                int data[3] = \{0, 1, 2\};
                ok = test(1, dead_result9, dead_test9, data, 9, "useless loops (conservative)");
```

#### Aggressive

See 11a, 11c, 12bd, 13bd, 14bd, 15b, 16bd.

Removing a loop with unknown bounds (a potentially infinite loop) may change the behaviour of a non-terminating program.

```
16a
        "dead.c"≡
              void dead_test10(int *data)
                                                                                    void dead_result10(int *data)
                 int k = 0;
                                                                                      int j;
                 int j;
                                                                                      j = data[2];
                 for (j=0; j<data[0]; j++) k++;
                                                                                      data[j] = 2;
                                                                                    }
                 j = data[2];
                 data[j] = 2;
                                                                                    \Diamond
        See 11b, 12ac, 13ac, 14ac, 15ac, 16c.
16b
        \langle \text{Dead code elimination} \rangle \equiv
                 int data[3] = { 2, 1, 2 };
                 (void) test(ok, dead_result10, dead_test10, data, 10, "useless loops (aggressive)");
        See 11a, 11c, 12bd, 13bd, 14bd, 15bd, 16d.
```

### 3.4 Partially Dead Code

Removal of partially-dead code is actually an unrelated optimization; it's closer in spirit to partial redundancy elimination. 2 papers in PLDI '94. This case can be done by a simple forward propagation.

```
"\mathtt{dead.c}" \equiv
16c
               void dead_test11(int *data)
                                                                                     void dead_result11(int *data)
                 int i = data[0];
                                                                                       int i = data[0];
                 int j = data[1];
                                                                                       int j = data[1];
                 int k = i * j;
                                                                                       if (i & j)
                                                                                          data[0] = i * j;
                 if (i & j)
                                                                                        j = data[2];
                   data[0] = k;
                 j = data[2];
                                                                                        data[j] = 2;
                                                                                     }
                 data[j] = 2;
                                                                                     \Diamond
               }
        See 11b, 12ac, 13ac, 14ac, 15ac, 16a.
16d
        \langle \text{Dead code elimination} \rangle \equiv
                 int data[3] = { 0, 1, 2 };
                 (void) test(1, dead_result11, dead_test11, data, 11, "partially dead");
        See 11a, 11c, 12bd, 13bd, 14bd, 15bd, 16b.
```

## 3.5 Prototypes

```
17
        \langle Prototypes \rangle \equiv
             extern void dead_test1(int *data);
                                                                   extern void dead_result1(int *data);
             extern void dead_test2(int *data);
                                                                   extern void dead_result2(int *data);
                                                                   extern void dead_result3(int *data);
             extern void dead_test3(int *data);
             extern void dead_test4(int *data);
                                                                   extern void dead_result4(int *data);
             extern void dead_test5(int *data);
                                                                   extern void dead_result5(int *data);
                                                                   extern void dead_result6(int *data);
             extern void dead_test6(int *data);
             extern void dead_test7(int *data);
                                                                   extern void dead_result7(int *data);
             extern void dead_test8(int *data);
                                                                   extern void dead_result8(int *data);
             extern void dead_test9(int *data);
                                                                   extern void dead_result9(int *data);
             extern void dead_test10(int *data);
                                                                   extern void dead_result10(int *data);
             extern void dead_test11(int *data);
                                                                   extern void dead_result11(int *data);
       See 3b, 4, 7b, 8e, 25, 32b, 39, 45c.
```

# Chapter 4

Best is [21, 22]

# Constant Propagation

See 18a, 19bd, 20ace, 21bd, 22bd, 23ace, 24b.

```
combines constant prop with elimination of unreachable code
           ideal time to convert multiplies into shifts, adds, and subtracts [7]
           can take further advantage of conditionals (assertion stuff)
            basic block, extended basic block, dominance, global (pessimistic), global (optimistic), conditional con-
        stant, reassociation, through floats, intrinsics (but this is C)
            maybe check for overflows
        ⟨Evaluation drivers⟩ ≡
18a
                int ok;
                puts("Constant propagation");
                (Constant propagation 18c, ...)
        See 3c, 11a, 26a, 33a, 40a.
        4.1
                 Local
        4.1.1
                  Constant Folding
        \texttt{"cprop.c"} \equiv
18b
              void cprop_test1(int *data)
                                                                                 void cprop_result1(int *data)
                int j = 1 + 2 * 4;
                                                                                    int j = 9;
                data[0] = j;
                                                                                    data[0] = j;
                                                                                 }
        See 19ace,\ 20bd,\ 21ac,\ 22ace,\ 23bd,\ 24a.
18c
        \langle Constant propagation \rangle \equiv
                int data[1];
                ok = test(1, cprop_result1, cprop_test1, data, 1, "folding");
```

#### 4.1.2 Propagating through Expressions

```
19a
        "cprop.c"≡
              void cprop_test2(int *data)
                                                                                void cprop_result2(int *data)
                int j = 1;
                                                                                  int j;
                int k = 2;
                                                                                  i = 9;
                int m = 4;
                                                                                  data[0] = j;
                int n = k * m;
                                                                                }
                                                                                \Diamond
                j = n + j;
                data[0] = j;
        See 18b, 19ce, 20bd, 21ac, 22ace, 23bd, 24a.
19b
        ⟨Constant propagation⟩ ≡
             {
                int data[1];
                ok = test(ok, cprop_result2, cprop_test2, data, 2, "basic block");
        See 18a, 18c, 19d, 20ace, 21bd, 22bd, 23ace, 24b.
```

#### Global 4.2

#### 4.2.1 Extended Basic Blocks

```
19c
        "cprop.c"≡
              void cprop_test3(int *data)
                                                                                void cprop_result3(int *data)
                int j = 12345;
                                                                                  if (data[0])
                if (data[0])
                                                                                    data[1] = 11112;
                  data[1] = 1 + j - 1234;
                                                                                  else
                                                                                    data[2] = 12478;
                  data[2] = 123 + j + 10;
                                                                                }
                                                                                \Diamond
              }
              \Diamond
        See 18b, 19ae, 20bd, 21ac, 22ace, 23bd, 24a.
19d
        ⟨Constant propagation⟩ ≡
             {
                ok = test(ok, cprop_result3, cprop_test3, data, 3, "extended basic blocks");
        See 18a, 18c, 19b, 20ace, 21bd, 22bd, 23ace, 24b.
        This one exposes a problem I stumbled across in gcc (ok with -02).
19e
        "cprop.c"≡
             void cprop_test4(int *data)
                                                                                void cprop_result4(int *data)
                int j = 12345;
                                                                                  if (data[0])
                if (data[0])
                  data[1] = 1 + j - 1234;
                                                                                    data[1] = 11112;
                                                                                    data[2] = -12212;
                  data[2] = 123 - j + 10;
              }
                                                                               }
                                                                                \Diamond
        See 18b, 19ac, 20bd, 21ac, 22ace, 23bd, 24a.
```

```
20 a
        \langle Constant propagation \rangle \equiv
                int data[3] = { 1 };
                 (void) test(ok, cprop_result4, cprop_test4, data, 4, "extended basic blocks");
        See 18a, 18c, 19bd, 20ce, 21bd, 22bd, 23ace, 24b.
        4.2.2 Dominators
20b
        "cprop.c"≡
              void cprop_test5(int *data)
                                                                                  void cprop_result5(int *data)
                int j = 5;
                                                                                     if (data[0])
                if (data[0])
                                                                                       data[1] = 10;
                  data[1] = 10;
                                                                                     else
                                                                                       data[2] = 15;
                                                                                     data[3] = 26;
                  data[2] = 15;
                                                                                  }
                data[3] = j + 21;
                                                                                  \Diamond
        See 18b, 19ace, 20d, 21ac, 22ace, 23bd, 24a.
        \langle Constant propagation \rangle \equiv
20c
                int data[4] = { 1 };
                ok = test(ok, cprop_result5, cprop_test5, data, 5, "dominators");
        See 18a, 18c, 19bd, 20ae, 21bd, 22bd, 23ace, 24b.
        Still more odd problems with gcc. The multiplies don't seem to provoke the problem; it also appears with
        simple adds.
20 d
        "cprop.c"≡
              void cprop_test6(int *data)
                                                                                  void cprop_result6(int *data)
                                                                                     if (data[0])
                int j = 5;
                if (data[0])
                                                                                       data[1] = 50;
                  data[1] = j * 10;
                                                                                       data[2] = 75;
                  data[2] = j * 15;
                                                                                     data[3] = 105;
                data[3] = j * 21;
                                                                                  }
              }
                                                                                  \Diamond
        See 18b, 19ace, 20b, 21ac, 22ace, 23bd, 24a.
        \langle Constant propagation \rangle \equiv
20e
                int data[4] = { 1 };
                 (void) test(ok, cprop_result6, cprop_test6, data, 6, "dominators");
        See 18a, 18c, 19bd, 20ac, 21bd, 22bd, 23ace, 24b.
```

```
4.2.3 DAGs
```

"cprop.c"≡

21a

```
void cprop_test7(int *data)
                                                                                  void cprop_result7(int *data)
                int j;
                                                                                     if (data[0])
                if (data[0]) {
                                                                                       data[1] = 10;
                  j = 5;
                                                                                       data[2] = 15;
                  data[1] = 10;
                                                                                     data[3] = 26;
                                                                                  }
                else {
                  data[2] = 15;
                                                                                  \Diamond
                  j = 5;
                data[3] = j + 21;
              \Diamond
        See 18b, 19ace, 20bd, 21c, 22ace, 23bd, 24a.
21b
        \langle Constant propagation \rangle \equiv
              {
                int data[4] = { 1 };
                ok = test(ok, cprop_result7, cprop_test7, data, 7, "dags");
        See 18a, 18c, 19bd, 20ace, 21d, 22bd, 23ace, 24b.
        Here's a similar looking case that I don't expect anyone to get, though it could be handled by first cloning
        the block containing the assignment to data[3].
        "cprop.c"≡
21c
                                                                                  void cprop_result8(int *data)
              void cprop_test8(int *data)
              {
                                                                                  {
                                                                                     if (data[0])
                int j, k;
                if (data[0]) {
                                                                                       data[1] = 4;
                  j = 4;
                                                                                     else
                  k = 6;
                                                                                       data[2] = 3;
                  data[1] = j;
                                                                                     data[3] = 210;
                                                                                  }
                                                                                  \Diamond
                else {
                  j = 7;
                  k = 3;
                  data[2] = k;
                data[3] = (j + k) * 21;
        See 18b, 19ace, 20bd, 21a, 22ace, 23bd, 24a.
21 d
        \langle Constant propagation \rangle \equiv
              {
                int data[4] = { 1 };
                 (void) test(ok, cprop_result8, cprop_test8, data, 8, "dags (hard)");
        See 18a, 18c, 19bd, 20ace, 21b, 22bd, 23ace, 24b.
```

#### 4.2.4 Loops

```
22a
        "cprop.c"≡
              void cprop_test9(int *data)
                                                                                    void cprop_result9(int *data)
                 int i;
                                                                                      int i;
                 int stop = data[0];
                                                                                       int stop = data[0];
                                                                                      for (i=1; i<stop; i++)
                 int j = 21;
                 for (i=1; i<stop; i++)</pre>
                   j = (j - 20) * 21;
                                                                                       data[1] = 21;
                 data[1] = j;
                                                                                      data[2] = i;
                                                                                    }
                 data[2] = i;
                                                                                    \Diamond
              }
        See 18b, 19ace, 20bd, 21ac, 22ce, 23bd, 24a.
22b
        \langle Constant propagation \rangle \equiv
                 int data[3] = { 2 };
                 ok = test(ok, cprop_result9, cprop_test9, data, 9, "loops");
              }\Diamond
        See 18a, 18c, 19bd, 20ace, 21bd, 22d, 23ace, 24b.
```

#### 4.3 Conditional Constants

```
22c
        "cprop.c"≡
              void cprop_test10(int *data)
                                                                                   void cprop_result10(int *data)
                                                                                     data[0] = 210 + data[1];
                 int j = 1;
                 if (j) j = 10;
                 else j = data[0];
                                                                                   \Diamond
                data[0] = j * 21 + data[1];
               \Diamond
        See 18b, 19ace, 20bd, 21ac, 22ae, 23bd, 24a.
22d
        \langle Constant propagation \rangle \equiv
              {
                 int data[2] = { 1, 2 };
                 ok = test(1, cprop_result10, cprop_test10, data, 10, "conditional constants");
        See 18a, 18c, 19bd, 20ace, 21bd, 22b, 23ace, 24b.
```

The previous test might have been passed successfully by running a relatively simple constant propagator repeatedly if unreachable code is removed between passes. The next case can never be handled by such an approach.

```
"cprop.c"≡
22e
             void cprop_test11(int *data)
                                                                             void cprop_result11(int *data)
                int i;
                                                                               int i;
                int stop = data[0];
                                                                               int stop = data[0];
               int j = 1;
                                                                               for (i=1; i<stop; i++)</pre>
                for (i=1; i<stop; i++)</pre>
                                                                               data[1] = 1;
                 if (!j) j = i;
                data[1] = j;
                                                                               data[2] = i;
                data[2] = i;
             }
```

See 18b, 19ace, 20bd, 21ac, 22ac, 23bd, 24a.

```
23a ⟨Constant propagation⟩ ≡
{
    int data[3] = { 2 };
    (void) test(ok, cprop_result11, cprop_test11, data, 11, "conditional constants");
} ♦

See 18a, 18c, 19bd, 20ace, 21bd, 22bd, 23ce, 24b.
```

#### 4.4 Conditional-Based Assertions

We need something expensive like the division; otherwise, the multiply can be hidden by scheduling on the RS/6000.

```
"cprop.c"≡
23b
              void cprop_test12(int *data)
                                                                                 void cprop_result12(int *data)
                int j = data[0];
                                                                                   int j = data[0];
                if (j == 5)
                                                                                   if (j == 5)
                  j = j * 21 + 25 / j;
                                                                                    j = 110;
                data[1] = j;
                                                                                  data[1] = j;
              }
                                                                                }
        See 18b, 19ace, 20bd, 21ac, 22ace, 23d, 24a.
        \langle Constant propagation \rangle \equiv
23c
                int data[2] = { 5 };
                ok = test(1, cprop_result12, cprop_test12, data, 12, "conditional-based assertions");
        See 18a, 18c, 19bd, 20ace, 21bd, 22bd, 23ae, 24b.
23d
        "cprop.c"≡
              void cprop_test13(int *data)
                                                                                void cprop_result13(int *data)
                int j = data[1];
                                                                                   int j = data[1];
                int k = data[0];
                                                                                   int k = data[0];
                if (j == 5 \&\& k == 10)
                                                                                   if (j == 5 && k == 10)
                  j = j * 21 + 100 / k;
                                                                                     j = 115;
                data[2] = j;
                                                                                  data[2] = j;
              }
                                                                                 \Diamond
        See 18b, 19ace, 20bd, 21ac, 22ace, 23b, 24a.
23e
        \langle Constant propagation \rangle \equiv
              {
                int data[3] = { 10, 5 };
                (void) test(ok, cprop_result13, cprop_test13, data, 13, "conditional-based assertions");
        See 18a, 18c, 19bd, 20ace, 21bd, 22bd, 23ac, 24b.
```

#### 4.5 Reassociation

```
"cprop.c"≡
24a
             void cprop_test14(int *data)
                                                                   void cprop_result14(int *data)
                int i = 10:
                                                                     int j = data[0];
                int j = data[0];
                                                                     int m = data[1];
                                                                     if (data[2])
                int k = 20;
                int m = data[1];
                                                                       data[3] = 60 + j + m;
                int n = 30;
                                                                       data[0] = 10 * (j - 20 * (m - 30));
                if (data[2])
                  data[3] = i + j + k + m + n;
                                                                   }
                                                                   \Diamond
                  data[0] = i * (j - k * (m - n));
             }
        See 18b, 19ace, 20bd, 21ac, 22ace, 23bd.
        ⟨Constant propagation⟩ ≡
24b
                int data[4] = { 0, 1, 2 };
                (void) test(ok, cprop_result14, cprop_test14, data, 14, "reassociation");
        See 18a, 18c, 19bd, 20ace, 21bd, 22bd, 23ace.
```

### 4.6 Algebraic Simplifications

```
\begin{array}{cccc} x+0 & \Rightarrow & x \\ x-0 & \Rightarrow & x \\ x\times0 & \Rightarrow & 0 \\ x\times1 & \Rightarrow & x \\ x/1 & \Rightarrow & x \\ x\cap \text{false} & \Rightarrow & \text{false} \\ x\cap \text{true} & \Rightarrow & x \\ x\cup \text{false} & \Rightarrow & x \\ x\cup \text{true} & \Rightarrow & \text{true} \\ x\ll0 & \Rightarrow & x \\ 0\ll x & \Rightarrow & 0 \\ \text{mod}(x,1) & \Rightarrow & 0 \end{array}
```

## 4.7 Floating-Point Operations

Sometimes propagation of floating-point constants at compile time can lead to loss of precision. In other cases, it's safe and accurate. Furthermore, sometimes users don't consider such losses important.

We should at least check to see how many of the safe cases are handled, and perhaps if they're doing some of the shakier cases (not so much to pass judgement as to show who does what).

conversions, rounding, truncation

## 4.8 Prototypes

See 3b, 4, 7b, 8e, 17, 32b, 39, 45c.

```
25
       \langle Prototypes \rangle \equiv
             extern void cprop_test1(int *data);
                                                                 extern void cprop_result1(int *data);
                                                                 extern void cprop_result2(int *data);
             extern void cprop_test2(int *data);
             extern void cprop_test3(int *data);
                                                                 extern void cprop_result3(int *data);
             extern void cprop_test35(int *data);
                                                                 extern void cprop_result4(int *data);
             extern void cprop_test4(int *data);
                                                                 extern void cprop_result5(int *data);
             extern void cprop_test5(int *data);
                                                                 extern void cprop_result6(int *data);
             extern void cprop_test6(int *data);
                                                                 extern void cprop_result7(int *data);
             extern void cprop_test7(int *data);
                                                                 extern void cprop_result8(int *data);
             extern void cprop_test8(int *data);
                                                                 extern void cprop_result9(int *data);
             extern void cprop_test9(int *data);
                                                                 extern void cprop_result10(int *data);
             extern void cprop_test10(int *data);
                                                                 extern void cprop_result11(int *data);
             extern void cprop_test11(int *data);
                                                                 extern void cprop_result12(int *data);
             extern void cprop_test12(int *data);
                                                                 extern void cprop_result13(int *data);
             extern void cprop_test13(int *data);
                                                                 extern void cprop_result14(int *data);
             extern void cprop_test14(int *data);
```

# Chapter 5

# Value Numbering

See 26b, 27bd, 28ac, 29ac, 30bd, 31ac.

```
[11]
           [4]
           {\it reassociation}
           basic blocks, extended basic blocks, dominance regions, global, gated SSA
           combined with constant prop, unreachable code due to cprop, unreachable code due to value numbering
           improvements possible via conditionals (assertions)
        \langle \text{Evaluation drivers} \rangle \equiv
26 a
                int ok;
                puts("Value numbering");
                (Value numbering 26c, \ldots)
        See 3c, 11a, 18a, 33a, 40a.
        5.1
                 Local
        [11]
                  Expressions
        5.1.1
        "valnum.c"≡
26b
              void vnum_test1(int *data)
                                                                                  void vnum_result1(int *data)
                data[0] = data[1] * data[3] - data[1] * data[3];
                                                                                    data[0] = 0;
        See 26d, 27bd, 28ac, 29ac, 30bd, 31ac.
        ⟨Value numbering⟩ ≡
26c
                int data[4] = {0, 1, 2, 3 };
                ok = test(1, vnum_result1, vnum_test1, data, 1, "expressions");
        See 26a, 27ace, 28bd, 29b, 30ace, 31b, 32a.
        "{\tt valnum.c}" \equiv
26 d
              void vnum_test2(int *data)
                                                                                  void vnum_result2(int *data)
                data[0] = data[1] * data[3] - data[3] * data[1];
                                                                                    data[0] = 0;
                                                                                  }
```

```
27a
        ⟨Value numbering⟩ ≡
                int data[4] = { 0, 1, 2, 3 };
                (void) test(ok, vnum_result2, vnum_test2, data, 2, "expressions");
        See 26a, 26c, 27ce, 28bd, 29b, 30ace, 31b, 32a.
        5.1.2 Basic Blocks
27b
        "valnum.c"≡
              void vnum_test3(int *data)
                                                                                 void vnum_result3(int *data)
                int n;
                                                                                   int k = data[2];
                int j = data[1] * data[3];
                                                                                   data[k] = 2;
                int i = data[3];
                                                                                   data[0] = 0;
                                                                                 }
                int m = data[1];
                                                                                  \Diamond
                int k = data[2];
                data[k] = 2;
                n = m * i;
                data[0] = n - j;
        See 26bd, 27d, 28ac, 29ac, 30bd, 31ac.
27c
        \langle \text{Value numbering} \rangle \equiv
                int data[4] = { 0, 1, 2, 3 };
                ok = test(ok, vnum_result3, vnum_test3, data, 3, "basic block");
        See 26a, 26c, 27ae, 28bd, 29b, 30ace, 31b, 32a.
        Check commutativity since some compilers seem to miss it.
27d
        "valnum.c"≡
              void vnum_test4(int *data)
                                                                                 void vnum_result4(int *data)
                                                                                   int k = data[2];
                int n;
                int j = data[1] * data[3];
                                                                                   data[k] = 2;
                int i = data[3];
                                                                                   data[0] = 0;
                int m = data[1];
                                                                                 }
                int k = data[2];
                data[k] = 2;
                n = i * m;
                data[0] = n - j;
        See 26bd, 27b, 28ac, 29ac, 30bd, 31ac.
27e
        \langle Value\ numbering \rangle \equiv
                int data[4] = { 0, 1, 2, 3 };
                (void) test(ok, vnum_result4, vnum_test4, data, 4, "basic block");
        See 26a, 26c, 27ac, 28bd, 29b, 30ace, 31b, 32a.
```

#### 5.1.3 Extended Basic Blocks

```
"valnum.c"≡
28a
              void vnum_test5(int *data)
                                                                                 void vnum_result5(int *data)
                int j = data[1] * data[3];
                                                                                   int j = data[1] * data[3];
                if (data[3] == 3) {
                                                                                   if (data[3] == 3) {
                  int n;
                                                                                     int k = data[2];
                  int i = data[3];
                                                                                     data[k] = 2;
                                                                                     data[0] = 0;
                  int m = data[1];
                  int k = data[2];
                  data[k] = 2;
                                                                                   else if (data[0] & 1) {
                  n = m * i;
                                                                                     j = 3 + data[2] - j;
                  data[0] = n - j;
                                                                                     data[j] = 2;
                else if (data[0] & 1) {
                                                                                }
                  j = 3 + data[2] - j;
                                                                                 \Diamond
                  data[j] = 2;
        See 26bd, 27bd, 28c, 29ac, 30bd, 31ac.
28b
        \langle Value\ numbering \rangle \equiv
                int data[4] = \{ 0, 1, 2, 3 \};
                ok = test(ok, vnum_result5, vnum_test5, data, 5, "extended basic block");
        See 26a, 26c, 27ace, 28d, 29b, 30ace, 31b, 32a.
```

#### 5.2 Dominators

Can be handled pretty easily using SSA. Not published – due to Jonathan Brezin.

```
28c
        "valnum.c"≡
              void vnum_test6(int *data)
                                                                                   void vnum_result6(int *data)
              {
                int n;
                                                                                     int j = data[1] * data[3];
                int j = data[1] * data[3];
                                                                                     if (data[0])
                int m = data[1];
                                                                                       j = j + 3;
                int k = j;
                                                                                     else
                if (data[0])
                                                                                       j = j - 3;
                                                                                     j = data[2] + j;
                  j = j + 3;
                else
                                                                                     data[j] = 2;
                                                                                     data[4] = 0;
                  j = j - 3;
                n = data[3];
                j = data[2] + j;
                data[j] = 2;
                data[4] = k - m * n;
              }
              \Diamond
        See 26bd, 27bd, 28a, 29ac, 30bd, 31ac.
28d
        \langle \text{Value numbering} \rangle \equiv
                int data[5] = { 0, 1, 2, 3, 4 };
                ok = test(ok, vnum_result6, vnum_test6, data, 6, "dominators");
        See 26a, 26c, 27ace, 28b, 29b, 30ace, 31b, 32a.
```

#### 5.3 Global

[4]

#### 5.3.1 DAGs

```
29a
        "valnum.c"≡
             void vnum_test7(int *data)
                                                                             void vnum_result7(int *data)
                                                                             {
             {
               int i, j, k;
                                                                               int i;
                                                                               if (data[0]) {
               int m = data[1];
               int n = data[3];
                                                                                 i = data[2];
               if (data[0]) {
                                                                                 data[i] = 2;
                 j = m * n;
                 i = data[2];
                                                                               data[0] = 0;
                                                                             }
                 data[i] = 2;
                                                                              \Diamond
                 k = m * n;
               else {
                j = 5;
                 k = 5;
               data[0] = k - j;
       See 26bd, 27bd, 28ac, 29c, 30bd, 31ac.
       ⟨Value numbering⟩ ≡
29b
               int data[5] = { 0, 1, 2, 3, 4 };
               ok = test(1, vnum_result7, vnum_test7, data, 7, "global DAGs");
       See 26a, 26c, 27ace, 28bd, 30ace, 31b, 32a.
        5.3.2 Loops
        "valnum.c"≡
29c
             void vnum_test8(int *data)
                                                                             void vnum_result8(int *data)
             {
                                                                             {
               int i;
                                                                               int i;
               int stop = data[3];
                                                                               int stop = data[3];
               int m = data[4];
                                                                               for (i=0; i<stop; i++) {
               int n = m;
                                                                                 int k = data[2];
                                                                                  data[k] = 2;
               for (i=0; i<stop; i++) {
                 int k = data[2];
                                                                                  data[0] = 0;
                 data[k] = 2;
                 data[0] = m - n;
                                                                             }
                 k = data[1];
                 m = m + k;
                 n = n + k;
               }
             }
       See 26bd, 27bd, 28ac, 29a, 30bd, 31ac.
```

```
30 a
        ⟨Value numbering⟩ ≡
                int data[5] = { 0, 1, 2, 3, 4 };
                ok = test(ok, vnum_result8, vnum_test8, data, 8, "global loops");
              }◊
        See 26a, 26c, 27ace, 28bd, 29b, 30ce, 31b, 32a.
```

#### Conditional-Based Assertions 5.4

#### **5.4.1** Simple

```
306
        "valnum.c"≡
              void vnum_test9(int *data)
                                                                                     void vnum_result9(int *data)
                 int i = data[0];
                                                                                       int i = data[0];
                 int j = data[1];
                                                                                       int j = data[1];
                 if (i == j)
                                                                                       if (i == j)
                   data[2] = (i - j) * 21;
                                                                                          data[2] = 0;
                                                                                       else
                   data[2] = i + j;
                                                                                          data[2] = i + j;
              }
                                                                                     }
               \Diamond
                                                                                     \Diamond
        See 26bd, 27bd, 28ac, 29ac, 30d, 31ac.
30c
        \langle \text{Value numbering} \rangle \equiv
              {
                 int data[3] = \{ 0, 0 \};
                 ok = test(1, vnum_result9, vnum_test9, data, 9, "conditional-based assertions");
        See 26a, 26c, 27ace, 28bd, 29b, 30ae, 31b, 32a.
```

#### 5.4.2 Derived

This one is quite hard.

```
"\mathtt{valnum.c"} \equiv
30d
              void vnum_test10(int *data)
                                                                                 void vnum_result10(int *data)
                int i = data[0];
                                                                                   int i = data[0];
                int m = i + 1;
                                                                                   int m = i + 1;
                int j = data[1];
                                                                                   int j = data[1];
                                                                                   int n = j + 1;
                int n = j + 1;
                data[2] = m + n;
                                                                                   data[2] = m + n;
                if (i == j)
                                                                                   if (i == j)
                  data[3] = (m - n) * 21;
                                                                                      data[3] = 0;
              }
                                                                                 }
              \Diamond
                                                                                 \Diamond
        See 26bd, 27bd, 28ac, 29ac, 30b, 31ac.
30e
        ⟨Value numbering⟩ ≡
              {
                int data[4] = { 0, 0 };
                ok = test(ok, vnum_result10, vnum_test10, data, 10, "conditional-based assertions");
        See 26a, 26c, 27ace, 28bd, 29b, 30ac, 31b, 32a.
```

#### 5.5 Conditional Value Numbers

Analogous to the conditional constants found by Wegman and Zadecks's constant propagation algorithm [22], combines value numbering and removal of unreachable code to find more equalities than could be found by running the passes separately. This example is due to Cliff Click, who has an algorithm to handle these cases.

```
"valnum.c"≡
31a
              void vnum_test11(int *data)
                                                                               void vnum_result11(int *data)
                                                                               {
                                                                                 int n;
                int n:
                int stop = data[3];
                                                                                 int stop = data[3];
                int j = data[1];
                                                                                 for (n=0; n \le top; n++)
                int k = j;
                                                                                   data[data[2]] = 2;
                int i = 1;
                                                                                 data[1] = 1;
                                                                               }
                for (n=0; n < stop; n++) {
                  if (i != k) i = 2;
                                                                               \Diamond
                  if (i != 1) k = 2;
                  data[data[2]] = 2;
                data[1] = i;
              }
        See 26bd, 27bd, 28ac, 29ac, 30bd, 31c.
31b
        ⟨Value numbering⟩ ≡
                int data[4] = { 0, 1, 2, 3 };
                ok = test(1, vnum_result11, vnum_test11, data, 11, "conditional value numbers");
        See 26a, 26c, 27ace, 28bd, 29b, 30ace, 32a.
```

## 5.6 Combining Value Numbering and Constant Propagation

This one is quite hard. Combines global value numbering, constant propagation, and elimination of unreachable code. Again, Click's approach should handle this sort of case.

```
"valnum.c"≡
31c
              void vnum_test12(int *data)
                                                                               void vnum_result12(int *data)
                int n;
                                                                                 int n;
                int stop = data[3];
                                                                                 int stop = data[3];
                int j = data[1];
                                                                                 for (n=0; n \le top; n++)
                int k = j;
                                                                                    data[data[2]] = 2;
                int i = 1;
                                                                                 data[1] = 1;
                for (n=0; n < stop; n++) {
                                                                               }
                                                                               \Diamond
                  if (j != k) i = 2;
                  i = 2 - i;
                  if (i != 1) k = 2;
                  data[data[2]] = 2;
                }
                data[1] = i;
             }
        See 26bd, 27bd, 28ac, 29ac, 30bd, 31a.
```

```
32a ⟨Value numbering⟩ ≡
{
    int data[4] = { 0, 1, 2, 3 };
    ok = test(ok, vnum_result12, vnum_test12, data, 12, "cprop + vnum");
    }♦

See 26a, 26c, 27ace, 28bd, 29b, 30ace, 31b.
```

#### 5.7 Reassociation

- 5.7.1 Expressions
- **5.7.2** Local
- 5.7.3 Global

### 5.8 Algebraic Simplifications

```
x = x \Rightarrow \text{true}
x \neq x \Rightarrow \text{false}
x > x \Rightarrow \text{false}
x \geq x \Rightarrow \text{true}
x \leq x \Rightarrow \text{false}
x \leq x \Rightarrow \text{true}
x \cap x \Rightarrow x
x \cup x \Rightarrow x
x - x \Rightarrow 0
```

### 5.9 Prototypes

```
326
       \langle Prototypes \rangle \equiv
             extern void vnum_test1(int *);
                                                                        extern void vnum_result1(int *);
             extern void vnum_test2(int *);
                                                                        extern void vnum_result2(int *);
             extern void vnum_test3(int *);
                                                                        extern void vnum_result3(int *);
             extern void vnum_test4(int *);
                                                                        extern void vnum_result4(int *);
                                                                        extern void vnum_result5(int *);
             extern void vnum_test5(int *);
             extern void vnum_test6(int *);
                                                                        extern void vnum_result6(int *);
                                                                        extern void vnum_result7(int *);
             extern void vnum_test7(int *);
             extern void vnum_test8(int *);
                                                                        extern void vnum_result8(int *);
             extern void vnum_test9(int *);
                                                                        extern void vnum_result9(int *);
             extern void vnum test10(int *);
                                                                        extern void vnum_result10(int *);
             extern void vnum_test11(int *);
                                                                        extern void vnum_result11(int *);
             extern void vnum_test12(int *);
                                                                        extern void vnum_result12(int *);
       See 3b, 4, 7b, 8e, 17, 25, 39, 45c.
```

# Chapter 6

## Code Motion

Redundancies that can't simply be removed. The examples I'll test for are partial redundancy elimination, loop unswitching, and the possibility of hoisting entire invariant control structures out of loops.

### 6.1 Invariant Expressions

[18]

#### 6.1.1 DAGs

both of these examples could be done via cloning followed by value numbering (or some other form of simple redundancy elimination)

#### The Prototypical Case

```
"motion.c"≡
336
                                                                                 void motion_result1(int *data)
              void motion_test1(int *data)
                int i;
                                                                                  int i;
                if (data[1])
                                                                                   int j;
                  i = data[0] + data[3];
                                                                                   if (data[1]) {
                else {
                                                                                     j = data[0] + data[3];
                                                                                     i = j;
                  data[data[2]] = 2;
                  i = 5;
                                                                                   else {
                data[3] = data[0] + data[3];
                                                                                     data[data[2]] = 2;
                data[4] = i;
                                                                                     i = 5;
              }
                                                                                     j = data[0] + data[3];
              \Diamond
                                                                                   data[4] = j;
                                                                                   data[5] = i;
                                                                                }
                                                                                 \Diamond
        See 34ac, 35ac, 36ac, 37ac, 38ac.
        \langle Code\ motion \rangle \equiv
33c
                int data[5] = { 0, 1, 2, 3 };
                ok = test(1, motion_result1, motion_test1, data, 1, "DAGs");
        See 33a, 34bd, 35bd, 36bd, 37bd, 38bd.
```

#### **Edge Placement**

"motion.c"≡

34a

```
void motion_test2(int *data)
                                                                                void motion_result2(int *data)
                int j;
                                                                                  int j;
                                                                                  int i = 1;
                int i = 1;
                if (data[1]) {
                                                                                  if (data[1]) {
                  data[data[2]] = 2;
                                                                                    data[data[2]] = 2;
                  j = data[0] + data[3];
                                                                                    j = data[0] + data[3];
                  i = i + j;
                                                                                    i = i + j;
                }
                                                                                  }
                data[4] = data[0] + data[3];
                                                                                  else
                                                                                    j = data[0] + data[3];
                data[5] = i;
              }
                                                                                  data[4] = j;
              \Diamond
                                                                                  data[5] = i;
                                                                                \Diamond
        See 33b, 34c, 35ac, 36ac, 37ac, 38ac.
34b
        \langle \text{Code motion} \rangle \equiv
                int data[6] = { 0, 1, 2, 3 };
                ok = test(ok, motion_result2, motion_test2, data, 2, "DAGs (edge placement)");
        See 33a, 33c, 34d, 35bd, 36bd, 37bd, 38bd.
        6.1.2 Loops
34c
        \texttt{''motion.c''} \equiv
              void motion_test3(int *data)
                                                                                void motion_result3(int *data)
              {
                int i = 0;
                                                                                  int i = 0;
                int k = data[2];
                                                                                  int k = data[2];
                                                                                  int j = 21 * data[0];
                int j = data[0];
                  i = 21 * j + i + 1;
                                                                                    i = j + i + 1;
                while (i < k);
                                                                                  while (i < k);
                data[4] = i;
                                                                                  data[4] = i;
              }
                                                                                \Diamond
        See 33b, 34a, 35ac, 36ac, 37ac, 38ac.
34d
        ⟨Code motion⟩ ≡
                int data[5] = { 0, 1, 2, 3 };
                ok = test(1, motion_result3, motion_test3, data, 3, "loops");
        See 33a, 33c, 34b, 35bd, 36bd, 37bd, 38bd.
        See if they're smart enough to hoist a divide (in this case, a safe move).
```

```
35a
        \texttt{''motion.c''} \equiv
               void motion_test4(int *data)
                                                                                   void motion_result4(int *data)
               {
                 int i = 0;
                                                                                     int i = 0;
                 int j = data[2];
                                                                                     int j = 2 / data[2];
                 do {
                                                                                     do {
                   data[i] = 2 / j + i - 1;
                                                                                       data[i] = j + i - 1;
                   i++;
                                                                                       i++;
                 } while (i < data[2]);</pre>
                                                                                     } while (i < data[2]);</pre>
                                                                                   \Diamond
               \Diamond
        See 33b, 34ac, 35c, 36ac, 37ac, 38ac.
35b
        \langle Code\ motion \rangle \equiv
              {
                 int data[5] = { 0, 1, 2, 3 };
                 (void) test(ok, motion_result4, motion_test4, data, 4, "loops (hoisting divide)");
        See 33a, 33c, 34bd, 35d, 36bd, 37bd, 38bd.
        Irreducible Loops
35c
        "motion.c"≡
               {\tt void motion\_test5(int *data)}
                                                                                   void motion_result5(int *data)
                 int i = 0;
                                                                                     int i = 0;
                 int j = data[0];
                                                                                     int j = data[0];
                 if (data[1])
                                                                                     if (data[1]) {
                   goto here;
                                                                                       j = j * 21;
                 j = data[3] + j;
                                                                                       goto here;
                                                                                     }
                 do {
                   i++;
                                                                                     j = data[3] + j;
              here:
                                                                                     j = j * 21;
                   data[i] = 21 * j + i;
                                                                                     do {
                 } while (i < data[2]);</pre>
                                                                                       i++;
               }
                                                                                   here:
               \Diamond
                                                                                       data[i] = j + i;
                                                                                     } while (i < data[2]);</pre>
                                                                                   \Diamond
        See 33b, 34ac, 35a, 36ac, 37ac, 38ac.
35d
        \langle Code\ motion \rangle \equiv
              {
                 int data[5] = { 0, 1, 2, 3 };
                 (void) test(ok, motion_result5, motion_test5, data, 5, "irreducible loops");
        See 33a, 33c, 34bd, 35b, 36bd, 37bd, 38bd.
```

#### 6.1.3 Reassociation

See 33a, 33c, 34bd, 35bd, 36b, 37bd, 38bd.

```
"motion.c"≡
36 a
             void motion_test6(int *data)
                                                                             void motion_result6(int *data)
                int j = data[1];
                                                                                int j = data[1];
                int k = data[2];
                                                                                int k = data[2];
                int i = data[0];
                                                                                int i = data[0];
                int n = data[3];
                                                                                int n = data[3];
                                                                                int m = j + k;
                 i = j + i + k;
                while (i < n);
                                                                                 i = m + i;
                data[4] = i;
                                                                                while (i < n);
             }
                                                                               data[4] = i;
             \Diamond
       See 33b, 34ac, 35ac, 36c, 37ac, 38ac.
36b
        \langle Code\ motion \rangle \equiv
                int data[5] = { 0, 1, 0, 3 };
                (void) test(ok, motion_result6, motion_test6, data, 6, "reassociation");
        See 33a, 33c, 34bd, 35bd, 36d, 37bd, 38bd.
        Aggressive Code Motion
        moving an expression onto a path where it may not have been executed, possibly slowing the code.
        "motion.c"≡
36c
             void motion_test7(int *data)
                                                                      void motion_result7(int *data)
             {
                int i = data[1];
                                                                        int i = data[1];
                int j = data[0];
                                                                        int j = data[0] * 21;
                do {
                                                                        do {
                  if (i & 1) data[data[2]] = 21 * j + i;
                                                                          if (i & 1) data[data[2]] = j + i;
                } while (i < data[3]);</pre>
                                                                        } while (i < data[3]);</pre>
```

#### Loop Rotation

```
"motion.c"≡
37a
             void motion_test8(int *data)
                                                                             void motion_result8(int *data)
                int i = 0;
                                                                                int i = 0;
                                                                                int j = data[0];
                int j = data[0];
                while (i < data[2]) {
                                                                                if (i < data[2]) {
                  data[i] = 21 * j + i;
                                                                                  i = i * 21;
                  i++;
                                                                                  do {
               }
                                                                                    data[i] = j + i;
             }
                                                                                    i++;
              \Diamond
                                                                                  } while (i < data[2]);</pre>
                                                                             }
```

See 33b, 34ac, 35ac, 36ac, 37c, 38ac.

If they're doing aggressive code motion, then we won't be able to distinguish the conservative approach of using loop rotation.

The test above might have been handled by the front end simply translating the while loop into an if containing a do-while (which I think of as the correct approach). Here we try the same test, but using a loop built with goto statements.

```
37c
        "motion.c"≡
              void motion_test9(int *data)
                                                                                  void motion_result9(int *data)
                                                                                    int i = 0;
                 int i = 0;
                 int j = data[0];
                                                                                    int j = data[0];
                                                                                    if (i >= data[2]) return;
                 if (i >= data[2]) return;
                                                                                    j = j * 21;
                   data[i] = 21 * j + i;
                                                                                  loop:
                   i++:
                                                                                      data[i] = j + i;
                   goto loop;
                                                                                      i++;
              }
                                                                                    if (i < data[2]) goto loop;</pre>
               \Diamond
                                                                                  \Diamond
        See 33b, 34ac, 35ac, 36ac, 37a, 38ac.
37d
        \langle Code\ motion \rangle \equiv
                 int data[5] = { 0, 1, 2, 3 };
                 (void) test(ok, motion_result9, motion_test9, data, 9, "loop rotation");
        See 33a, 33c, 34bd, 35bd, 36bd, 37b, 38bd.
```

#### 6.2 Invariant Control Flow

#### 6.2.1 Hoisting Invariant Control Flow

[14]. Hmmm. This one could also be done via loop unswitching.

```
38a
        \texttt{''motion.c''} \equiv
              void motion_test10(int *data)
                                                                                void motion_result10(int *data)
              {
                int j;
                                                                                  int j;
                int p = data[1];
                                                                                  int p = data[1];
                                                                                  int i = data[0];
                int i = data[0];
                do {
                                                                                  if (p)
                   if (p)
                                                                                     j = 1;
                     j = 1;
                                                                                  else
                  else
                                                                                     j = 2;
                     j = 2;
                                                                                  do {
                  i = i + j;
                                                                                     i = i + j;
                  data[data[2]] = 2;
                                                                                     data[data[2]] = 2;
                } while (i < data[3]);</pre>
                                                                                  } while (i < data[3]);</pre>
                                                                                }
                                                                                \Diamond
        See 33b, 34ac, 35ac, 36ac, 37ac, 38c.
38b
        \langle \text{Code motion} \rangle \equiv
                int data[5] = { 0, 1, 2, 10 };
                (void) test(ok, motion_result10, motion_test10, data, 10, "invariant control structures");
        See 33a, 33c, 34bd, 35bd, 36bd, 37bd, 38d.
        6.2.2 Loop Unswitching
           opportunity arises when using nested DO loops in Fortran 77 and sometimes with nested for loops in C.
38c
        "motion.c"≡
              void motion_test11(int *data)
                                                                                void motion_result11(int *data)
                                                                                {
                int p = data[1];
                                                                                  int p = data[1];
                int i = data[0];
                                                                                  int i = data[0];
                do {
                                                                                  if (p)
                   if (p)
                                                                                     do {
                     i = i + 1;
                                                                                       i = i + 1;
                   else
                                                                                       data[data[2]] = 2;
                     i = i + 2;
                                                                                     } while (i < data[3]);</pre>
                  data[data[2]] = 2;
                                                                                  else
                } while (i < data[3]);</pre>
                                                                                     do {
              }
                                                                                       i = i + 2;
              \Diamond
                                                                                       data[data[2]] = 2;
                                                                                     } while (i < data[3]);</pre>
                                                                                }
        See 33b, 34ac, 35ac, 36ac, 37ac, 38a.
38d
        \langle \text{Code motion} \rangle \equiv
                int data[5] = \{ 0, 1, 2, 10 \};
                 (void) test(ok, motion_result11, motion_test11, data, 11, "loop unswitching");
```

See 33a, 33c, 34bd, 35bd, 36bd, 37bd, 38b.

### 6.3 Prototypes

```
39
        \langle Prototypes \rangle \equiv
             extern void motion_test1(int *);
                                                                     extern void motion_result1(int *);
             extern void motion_test2(int *);
                                                                     extern void motion_result2(int *);
             extern void motion_test3(int *);
                                                                     extern void motion_result3(int *);
             extern void motion_test4(int *);
                                                                     extern void motion_result4(int *);
             extern void motion_test5(int *);
                                                                     extern void motion_result5(int *);
             extern void motion_test6(int *);
                                                                     extern void motion_result6(int *);
             extern void motion_test7(int *);
                                                                     extern void motion_result7(int *);
             extern void motion_test8(int *);
                                                                     extern void motion_result8(int *);
             extern void motion_test9(int *);
                                                                     extern void motion_result9(int *);
             extern void motion_test10(int *);
                                                                     extern void motion_result10(int *);
             extern void motion_test11(int *);
                                                                     extern void motion_result11(int *);
       See 3b, 4, 7b, 8e, 17, 25, 32b, 45c.
```

# Chapter 7

# Strength Reduction

```
[10, 3]

[9] and other versions based on partial redundancy elimination are weaker linear function test replacement, odd induction variables, control flow in loop, goto loops, nested loops, irreducible loops, reduction of mod, div, etc.

straight-line code, loop invariants (vs. constants)

40a ⟨Evaluation drivers⟩ ≡

{
    int ok;
    puts ("Strength reduction");
    ⟨Strength reduction 40 c, ...⟩
} ♦

See 3c, 11a, 18a, 26a, 33a.
```

#### 7.1 Induction Variable and Constant

Testing a while loop might also be interesting; however, the proper method of handling them would be to perform loop rotation first, which is already covered elsewhere.

```
40b
        "strength.c" \equiv
               void strength_test1(int *data)
                                                                                  void strength_result1(int *data)
                                                                                  {
                 int i = 0;
                                                                                    int i = 0;
                 do {
                                                                                    do {
                   data[data[2]] = 2;
                                                                                      data[data[2]] = 2;
                   i = i + 1;
                                                                                      i = i + 21;
                 } while (i * 21 < data[1]);</pre>
                                                                                    } while (i < data[1]);</pre>
               }
                                                                                 }
                                                                                  \Diamond
        See 41ac, 42ac, 43ac, 44ac, 45a.
40c
        \langle Strength reduction \rangle \equiv
                 int data[3] = { 0, 22, 2 };
                 ok = test(1, strength_result1, strength_test1, data, 1, "iv * constant");
        See 40a, 41bd, 42bd, 43bd, 44bd, 45b.
```

### 7.2 Induction Variable and Region Constant

```
"strength.c" \equiv
41a
              void strength_test2(int *data)
                                                                              void strength_result2(int *data)
                                                                             {
                int k = data[0];
                                                                                int k = data[0];
                int i = 0;
                                                                                int i = 0;
                do {
                                                                                do {
                  data[data[2]] = 2;
                                                                                  data[data[2]] = 2;
                  i = i + 1;
                                                                                  i = i + k;
                } while (i * k < data[1]);</pre>
                                                                                } while (i < data[1]);</pre>
                                                                             }
                                                                              \Diamond
        See 40b, 41c, 42ac, 43ac, 44ac, 45a.
41b
        ⟨Strength reduction⟩ ≡
                int data[3] = { 1, 3, 2 };
                (void) test(ok, strength_result2, strength_test2, data, 2, "iv * rc");
              }◊
        See 40a, 40c, 41d, 42bd, 43bd, 44bd, 45b.
```

### 7.3 Multiplying Two Induction Variables

```
"strength.c" \equiv
41c
              void strength_test3(int *data)
                                                                              void strength_result3(int *data)
                                                                              {
                int i = data[0];
                                                                                int i = data[0];
                int j = data[1];
                                                                                int j = data[1];
                do {
                                                                                int k = i * j;
                  data[data[2]] = 2;
                                                                                do {
                  i = i + 1;
                                                                                  data[data[2]] = 2;
                  j = j + 1;
                                                                                  i = i + 1;
                } while (i * j < data[3]);</pre>
                                                                                  k = k + j;
              }
                                                                                  j = j + 1;
                                                                                  k = k + i;
              \Diamond
                                                                                } while (k < data[3]);</pre>
                                                                              }
                                                                              \Diamond
        See 40b, 41a, 42ac, 43ac, 44ac, 45a.
41d
        ⟨Strength reduction⟩ ≡
              {
                int data[4] = { 0, 1, 2, 100 };
                (void) test(ok, strength_result3, strength_test3, data, 3, "iv * iv");
        See 40a, 40c, 41b, 42bd, 43bd, 44bd, 45b.
```

### 7.4 Irreducible Loops

```
"strength.c" \equiv
42a
              void strength_test4(int *data)
                                                                             void strength_result4(int *data)
              {
                                                                             {
                int i;
                                                                                int i:
                if (data[1]) {
                                                                                if (data[1]) {
                  i = 2;
                                                                                  i = 42;
                  goto here;
                                                                                  goto here;
                i = 0;
                                                                                i = 0;
                do {
                                                                                do {
                  i = i + 1;
                                                                                 i = i + 21;
                                                                             here:
              here:
                  data[data[2]] = 2;
                                                                                  data[data[2]] = 2;
                } while (i * 21 < data[3]);</pre>
                                                                                } while (i < data[3]);</pre>
                                                                             }
                                                                              \Diamond
        See 40b, 41ac, 42c, 43ac, 44ac, 45a.
42b
        (Strength reduction) ≡
                int data[4] = { 0, 1, 2, 100 };
                (void) test(ok, strength_result4, strength_test4, data, 4, "irreducible loop");
        See 40a, 40c, 41bd, 42d, 43bd, 44bd, 45b.
```

### 7.5 Control Flow in the Loop

```
42c
        "strength.c"≡
              void strength_test5(int *data)
                                                                             void strength_result5(int *data)
                int i = 0;
                                                                                int i = 0;
                while (1) {
                                                                                while (1) {
                  i = i + 1;
                                                                                  i = i + 21;
                  if (data[1] && i * 21 > data[3])
                                                                                  if (data[1] && i > data[3])
                    break;
                                                                                    break;
                  data[data[2]] = 2;
                                                                                  data[data[2]] = 2;
              }
                                                                             }
                                                                              \Diamond
        See 40b, 41ac, 42a, 43ac, 44ac, 45a.
42d
        \langle Strength reduction \rangle \equiv
                int data[4] = { 0, 1, 2, 100 };
                (void) test(ok, strength_result5, strength_test5, data, 5, "control flow in loop");
              }◊
        See 40a, 40c, 41bd, 42b, 43bd, 44bd, 45b.
```

### 7.6 More Complex Induction Variables

There are many possibilities [24]. I'm only covering a few cases.

### 7.6.1 Increment by a Region Constant

```
43a
        "strength.c"≡
             void strength_test6(int *data)
                                                                           void strength_result6(int *data)
                int j = data[0];
                                                                             int j = data[0] * 21;
                int i = data[1];
                                                                             int i = data[1] * 21;
                                                                             do {
                 data[data[2]] = 2;
                                                                               data[data[2]] = 2;
                 i = i + j;
                                                                               i = i + j;
                                                                             } while (i < data[3]);</pre>
                } while (i * 21 < data[3]);</pre>
                                                                           }
                                                                           \Diamond
        See 40b, 41ac, 42ac, 43c, 44ac, 45a.
43b
        (Strength reduction) ≡
             {
                int data[4] = { 1, 1, 2, 100 };
                (void) test(ok, strength_result6, strength_test6, data, 6, "inc by rc");
        See 40a, 40c, 41bd, 42bd, 43d, 44bd, 45b.
        7.6.2 Monotonic Induction Variables
43c
        "strength.c"≡
             void strength_test7(int *data)
                                                                           void strength_result7(int *data)
                int i = 0;
                                                                             int i = 0;
                do {
                                                                             do {
                  if (data[1])
                                                                               if (data[1])
                                                                                 i = i + 21;
                   i = i + 1;
                  else
                                                                               else
                    i = i + 2;
                                                                                 i = i + 42;
                  data[data[2]] = 2;
                                                                               data[data[2]] = 2;
                } while (i * 21 < data[3]);</pre>
                                                                             } while (i < data[3]);</pre>
             }
                                                                           }
                                                                           \Diamond
        See 40b, 41ac, 42ac, 43a, 44ac, 45a.
43d
        (Strength reduction) ≡
                int data[4] = { 0, 1, 2, 100 };
                (void) test(ok, strength_result7, strength_test7, data, 7, "monotonic iv");
             }◊
        See 40a, 40c, 41bd, 42bd, 43b, 44bd, 45b.
```

#### 7.6.3 Mutual Induction Variables

```
"strength.c"≡
44a
              void strength_test8(int *data)
                                                                               void strength_result8(int *data)
                 int i = 0;
                                                                                  int i = 0;
                 do {
                                                                                 int n = 0;
                   int j = i + 1;
                                                                                 do {
                  data[data[2]] = j;
                                                                                    int j = i + 1;
                                                                                   n = n + 21;
                   i = j + 1;
                } while (i * 21 < data[3]);</pre>
                                                                                    data[data[2]] = j;
                                                                                    i = j + 1;
              \Diamond
                                                                                   n = n + 21;
                                                                                 } while (n < data[3]);</pre>
                                                                               }
        See 40b, 41ac, 42ac, 43ac, 44c, 45a.
44b
        \langle Strength reduction \rangle \equiv
                 int data[4] = { 0, 1, 1, 100 };
                 (void) test(ok, strength_result8, strength_test8, data, 8, "mutual iv's");
        See 40a, 40c, 41bd, 42bd, 43bd, 44d, 45b.
        7.7
                 Multiple Strides
        "strength.c" \equiv
44c
              void strength_test9(int *data)
                                                                               void strength_result9(int *data)
                                                                               {
                 int i = 0;
                                                                                 int i = 0;
                                                                                 int j = 0;
                 do {
                  data[data[2]] = i;
                                                                                 do {
                  i = i + 1;
                                                                                    data[data[2]] = i;
                 } while (i * 21 < data[3]);</pre>
                                                                                   i = i + 1;
              }
                                                                                   j = j + 21;
              \Diamond
                                                                                 } while (j < data[3]);</pre>
                                                                               }
                                                                               \Diamond
        See 40b, 41ac, 42ac, 43ac, 44a, 45a.
44d
        \langle Strength reduction \rangle \equiv
                 int data[4] = { 0, 1, 1, 100 };
                 (void) test(ok, strength_result9, strength_test9, data, 9, "multiple strides");
        See 40a, 40c, 41bd, 42bd, 43bd, 44b, 45b.
```

### 7.8 Linear Function Test Replacement

See 3b, 4, 7b, 8e, 17, 25, 32b, 39.

```
"strength.c" \equiv
45a
             void strength_test10(int *data)
                                                                         void strength_result10(int *data)
               int stop = data[3];
                                                                           int stop = data[3] * 21;
               int i = 0;
                                                                           int i = 0;
               do {
                                                                          do {
                 data[data[2]] = 21 * i;
                                                                             data[data[2]] = i;
                  i = i + 1;
                                                                             i = i + 21;
               } while (i < stop);</pre>
                                                                          } while (i < stop);</pre>
                                                                        }
                                                                         \Diamond
        See 40b, 41ac, 42ac, 43ac, 44ac.
45b
        (Strength reduction) ≡
               int data[4] = { 0, 1, 1, 20 };
                (void) test(ok, strength_result10, strength_test10, data, 10, "test replacement");
             }◊
        See 40a, 40c, 41bd, 42bd, 43bd, 44bd.
                Prototypes
        7.9
45c
        \langle Prototypes \rangle \equiv
             extern void strength_test1(int *);
                                                                    extern void strength_result1(int *);
             extern void strength_test2(int *);
                                                                    extern void strength_result2(int *);
             extern void strength_test3(int *);
                                                                    extern void strength_result3(int *);
             extern void strength_test4(int *);
                                                                    extern void strength_result4(int *);
             extern void strength_test5(int *);
                                                                    extern void strength_result5(int *);
             extern void strength_test6(int *);
                                                                    extern void strength_result6(int *);
             extern void strength_test7(int *);
                                                                    extern void strength_result7(int *);
             extern void strength_test8(int *);
                                                                    extern void strength_result8(int *);
             extern void strength_test9(int *);
                                                                    extern void strength_result9(int *);
             extern void strength_test10(int *);
                                                                    extern void strength_result10(int *);
```

# Appendix A

## **Indices**

### A.1 Files

```
"cprop.c" 18b, 19ace, 20bd, 21ac, 22ace, 23bd, 24a.
"dead.c" 11b, 12ac, 13ac, 14ac, 15ac, 16ac.
"global.h" 3a.
"main.c" 3bc, 6b, 8cd, 9e, 10.
"motion.c" 33b, 34ac, 35ac, 36ac, 37ac, 38ac.
"strength.c" 40b, 41ac, 42ac, 43ac, 44ac, 45a.
"valnum.c" 26bd, 27bd, 28ac, 29ac, 30bd, 31ac.
```

#### A.2 Macros

```
\langle \text{Analyze timing results} \rangle 6 \text{ b}, 7c.
(Check for exceptionally noisy data) 7c, 8a.
(Code motion) 33a, 33c, 34bd, 35bd, 36bd, 37bd, 38bd.
(Constant propagation) 18a, 18c, 19bd, 20ace, 21bd, 22bd, 23ace, 24b.
(Constants) 3b, 7ad.
(Dead code elimination) 11a, 11c, 12bd, 13bd, 14bd, 15bd, 16bd.
(Evaluation drivers) 3c, 11a, 18a, 26a, 33a, 40a.
(Include files) 3a, 3d, 6a, 9a.
\langle \text{Initialize} \rangle 3c, 9c.
(Make sure clock() works on this system) 9c, 9d.
(Print times and statistical measures) 8a, 8b.
(Prototypes) 3b, 4, 7b, 8e, 17, 25, 32b, 39, 45c.
(Strength reduction) 40a, 40c, 41bd, 42bd, 43bd, 44bd, 45b.
\langle \text{Types} \rangle 3a, 3e.
(Value numbering) 26a, 26c, 27ace, 28bd, 29b, 30ace, 31b, 32a.
(Variables) 3b, 9b.
```

### A.3 Identifiers

```
clock: 9a, 9cde, 10.
CLOCKS_PER_SEC: 7a, 9a, 9ce, 10.
clock_t: 9a, 9bce, 10.
collect_timings: 6b, 7b, 10.
cprop_result1: 18b, 18c, 25.
cprop_result10: 22c, 22d, 25.
cprop_result11: 22e, 23a, 25.
cprop_result12: 23b, 23c, 25.
cprop_result13: 23d, 23e, 25.
cprop_result14: 24a, 24b, 25.
cprop_result2: 19a, 19b, 25.
cprop_result3: 19c, 19d, 25.
cprop_result4: 19e, 20a, 25.
cprop_result5: 20b, 20c, 25.
```

```
cprop_result6: 20d, 20e, 25.
cprop_result7: 21a, 21b, 25.
cprop_result8: 21c, 21d, 25.
cprop_result9: 22a, 22b, 25.
cprop_test1: 18b, 18c, 25.
cprop_test10: 22c, 22d, 25.
cprop_test11: 22e, 23a, 25.
cprop_test13: 23d, 23e, 25.
cprop_test14: 24a, 24b, 25.
cprop_test2: 19a, 19b, 25.
cprop test3: 19c, 19d, 25.
cprop_test4: 19e, 20a, 25.
cprop_test5: 20b, 20c, 25.
cprop_test6: 20d, 20e, 25.
cprop_test7: 21a, 21b, 25.
cprop_test8: 21c, 21d, 25.
cprop_test9: 22a, 22b, 25.
CUTOFF: 7c. 7d.
dead_result1: 11b, 11c, 17.
dead_result10: 16a, 16b, 17.
dead_result11: 16c, 16d, 17.
dead_result2: 12a, 12b, 17.
dead_result3: 12c, 12d, 17.
dead result4: 13a, 13b, 17.
dead_result5: 13c, 13d, 17.
dead_result6: 14a, 14b, 17.
dead_result7: 14c, 14d, 17.
dead_result8: 15a, 15b, 17.
dead_result9: 15c, 15d, 17.
dead_test1: 11b, 11c, 17.
dead test10: 16a, 16b, 17.
dead_test11: 16c, 16d, 17.
dead_test2: 12a, 12b, 17.
dead_test3: 12c, 12d, 17.
dead_test4: 13a, 13b, 17.
dead_test5: 13c, 13d, 17.
dead_test6: 14a, 14b, 17.
dead_test7: 14c, 14d, 17.
dead_test8: 15a, 15b, 17.
dead_test9: 15c, 15d, 17.
determine_iterations: 6b, 7b, 9e.
exit: 3c, 3d, 9d.
fabs: 6a, 7c.
fputs: 3d, 9d.
FunPar: 3e, 4, 6b, 7b, 9e, 10.
main: 3c.
mean: 7c, 8de.
motion_result1: 33b, 33c, 39.
motion_result10: 38a, 38b, 39.
motion_result11: 38c, 38d, 39.
motion_result2: 34a, 34b, 39.
motion_result3: 34c, 34d, 39.
motion_result4: 35a, 35b, 39.
motion_result5: 35c, 35d, 39.
motion_result6: 36a, 36b, 39.
motion_result7: 36c, 36d, 39.
motion_result8: 37a, 37b, 39.
motion_result9: 37c, 37d, 39.
motion_test1: 33b, 33c, 39.
```

```
motion_test10: 38a, 38b, 39.
motion_test11: 38c, 38d, 39.
motion_test2: 34a, 34b, 39.
motion_test3: 34c, 34d, 39.
motion_test4: 35a, 35b, 39.
motion_test5: 35c, 35d, 39.
motion_test6: 36a, 36b, 39.
motion_test7: 36c, 36d, 39.
motion_test8: 37a, 37b, 39.
motion_test9: 37c, 37d, 39.
noisy: 7c, 8a, 8c.
printf: 3d, 6b, 8b, 9c.
\mathtt{puts} \colon 3\mathtt{c}, \ 3d, \ 6\mathtt{b}, \ 7\mathtt{c}, \ 8\mathtt{ab}, \ 11\mathtt{a}, \ 18\mathtt{a}, \ 26\mathtt{a}, \ 33\mathtt{a}, \ 40\mathtt{a}.
SAMPLES: 6b, 7a, 7c, 8bd, 10.
sqrt: 6a, 7c, 8bc.
stderr: 3d, 9d.
strength_result1: 40b, 40c, 45c.
strength_result10: 45a, 45bc.
strength_result2: 41a, 41b, 45c.
strength_result3: 41c, 41d, 45c.
strength_result4: 42a, 42b, 45c.
strength_result5: 42c, 42d, 45c.
strength_result6: 43a, 43b, 45c.
strength_result7: 43c, 43d, 45c.
strength_result8: 44a, 44b, 45c.
\mathtt{strength\_result9} \colon \textit{44}\,\textit{c},\, 44\mathrm{d},\, 45\mathrm{c}.
strength_test1: 40b, 40c, 45c.
strength_test10: 45a, 45bc.
strength_test2: 41a, 41b, 45c.
strength_test3: 41c, 41d, 45c.
strength_test4: 42a, 42b, 45c.
strength_test5: 42c, 42d, 45c.
strength_test6: 43a, 43b, 45c.
strength_test7: 43c, 43d, 45c.
strength_test8: 44a, 44b, 45c.
strength_test9: 44c, 44d, 45c.
test: 4, 6b, 7c, 11c, 12bd, 13bd, 14bd, 15bd, 16bd, 18c, 19bd, 20ace, 21bd, 22bd, 23ace, 24b, 26c, 27ace, 28bd, 29b,
      30ace, 31b, 32a, 33c, 34bd, 35bd, 36bd, 37bd, 38bd, 40c, 41bd, 42bd, 43bd, 44bd, 45b.
ticks_per_sec: 8c, 9b, 9ce.
variance: 7c, 8c, 8de.
vnum_result1: 26b, 26c, 32b.
vnum_result10: 30d, 30e, 32b.
vnum_result11: 31a, 31b, 32b.
vnum_result12: 31c, 32ab.
vnum_result2: 26d, 27a, 32b.
vnum_result3: 27b, 27c, 32b.
vnum_result4: 27d, 27e.
vnum_result5: 28a, 28b, 32b.
vnum_result6: 28c, 28d, 32b.
vnum_result7: 29a, 29b, 32b.
vnum_result8: 29c, 30a, 32b.
vnum_result9: 30b, 30c, 32b.
vnum_test1: 26b, 26c, 32b.
vnum_test10: 30d, 30e, 32b.
vnum_test11: 31a, 31b, 32b.
vnum_test12: 31c, 32ab.
vnum_test2: 26d, 27a, 32b.
vnum_test3: 27b, 27c, 32b.
vnum_test4: 27d, 27e, 32b.
```

# Appendix B

## Results

Try to report dispassionately. Just give the facts, with some interpretation when helpful; but don't accuse of laziness or stupidity. Similarly, don't brag on anyone.

Need version number of compiler can we get Cray Research, KSR, Paragon (Portland Group), other supers? design a form for reporting results electronically

### B.1 Apple

MPW

- B.2 Borland
- B.3 Convex
- **B.4** Cray Computer
- B.5 Cray Research
- B.6 DEC
- B.6.1 Alpha

 $\operatorname{Rob}$ 

#### **B.6.2 DECstation**

MIPS, Rob

### B.7 Gnu CC

need report by machine (since SPARC version seems fairly inferior)

### B.8 Hewlett-Packard

Steve

- B.9 IBM
- B.9.1 RS/6000
- B.10 Microsoft
- **B.11 Silicon Graphics**

 $\operatorname{Rob}$ 

- B.12 Sun
- B.12.1 Solaris
- B.13 Tera

Brian

### **B.14** Texas Instruments

 $\operatorname{Reid}$ 

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