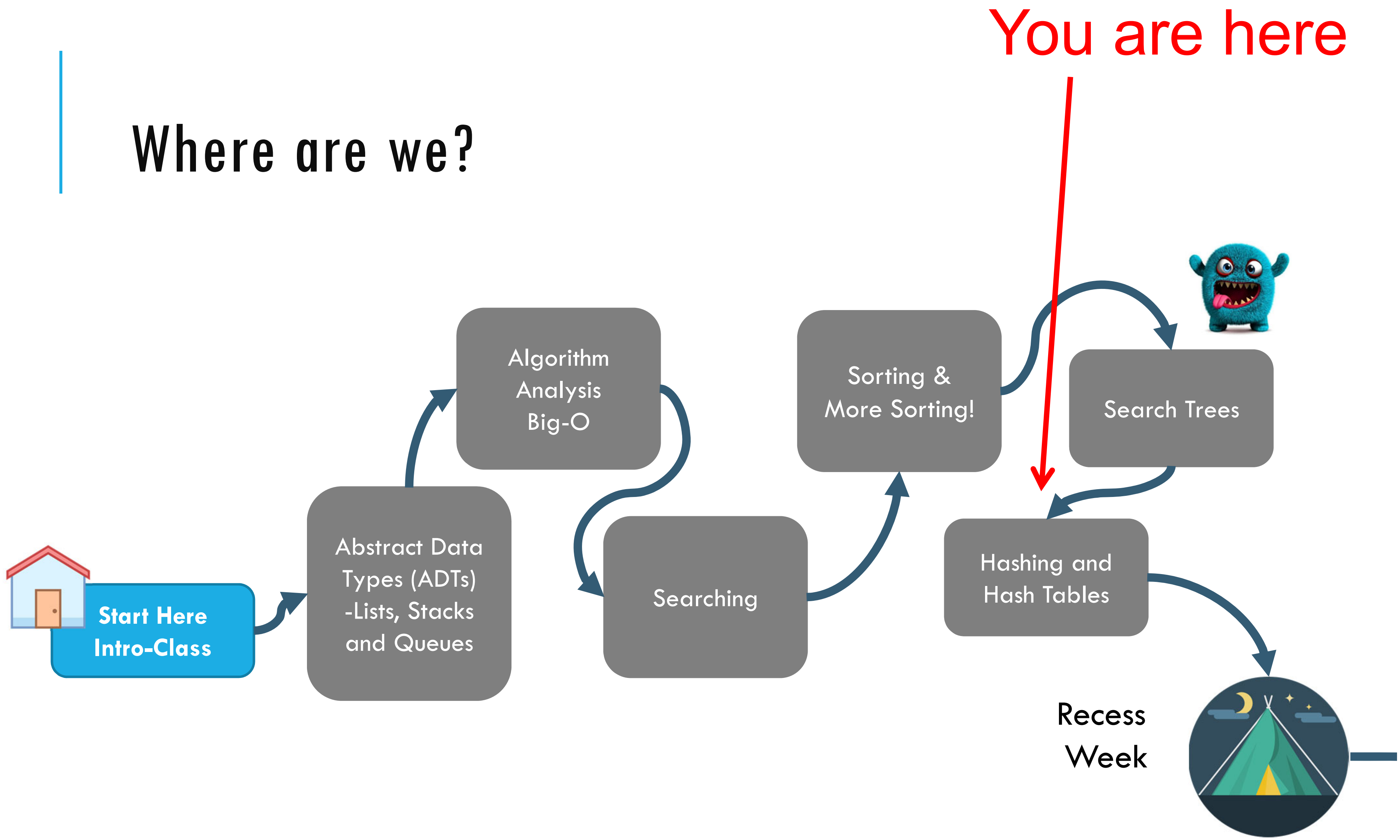
CS2040S
Data Structures and Algorithms

Hashing!

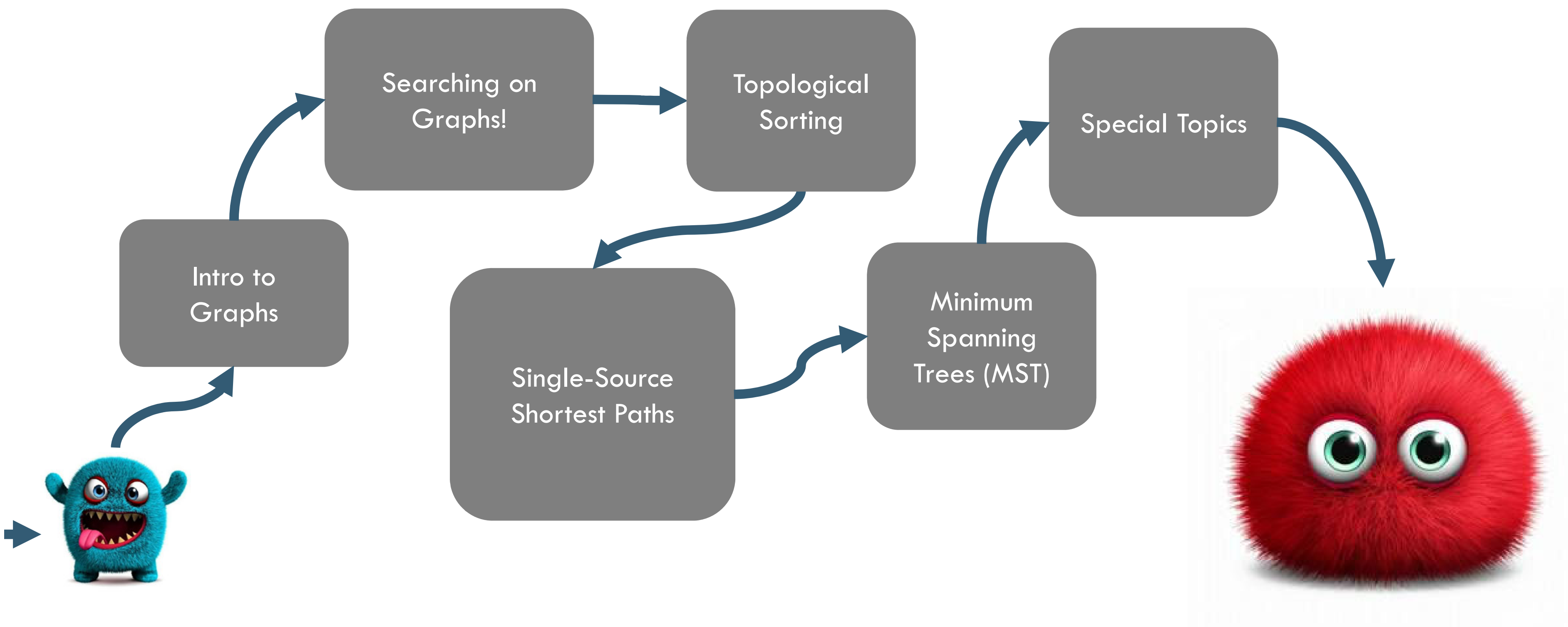
(Part 1)

Where are we?



Part I: Organizing your Data

Where are we?



Part II: Modelling and Solving Problems

Announcements

Midterm : Tuesday March 9, 4pm

(Note: In person, face-to-face)

Will post last year’s midterm next week…

Will discuss how to prepare after recess week…

(Hint: practice!)

Plan: this week and next

Three Days of Hashing

- Applications
- Basic theory
- Handling collisions
- (Hashing in Java)
- Amortized analysis (doubling/shrinking)
- Sets and Bloom filters

Topic of the Week: Hash Tables

Abstract Data Types

Symbol Table

public interface	**SymbolTable**	
void	insert(Key k, Value v)	*insert (k,v) into table*
Value	search(Key k)	*get value paired with k*
void	delete(Key k)	*remove key k (and value)*
boolean	contains(Key k)	*is there a value for k?*
int	size()	*number of (k,v) pairs*

Note: no successor / predecessor queries.

Symbol Table

Examples:

Dictionary:	key = word
	value = definition
Phone Book	key = name
	value = phone number
Internet DNS	key = website URL
	value = IP address
Java compiler	key = variable name
	value = type and value

Implement symbol table with an AVL tree:
(C_I= cost insert, C_S=cost search)

1. $C_I = O(1)$, $C_S = O(1)$
2. $C_I = O(1)$, $C_S = O(\log n)$
3. $C_I = O(1)$, $C_S = O(n)$
4. ✓ $C_I = O(\log n)$, $C_S = O(\log n)$
5. $C_I = O(n)$, $C_S = O(\log n)$
6. $C_I = O(n)$, $C_S = O(n)$

Symbol Table

Implement a symbol table with:

- $C_I = O(1)$
- $C_S = O(1)$

Fast, fast, fast….

Dictionaries vs. Symbol Tables

What can you do with a dictionary but not a symbol table?

Dictionaries vs. Symbol Tables

Sorting with a dictionary:

1) Insert every item into the dictionary.
2) Search for the minimum item.
3) Repeat: find successor

Running time to implement sorting:

With an AVL tree/dictionary?

With a symbol table?

Dictionaries vs. Symbol Tables

Sorting with a dictionary:

1) Insert every item into the dictionary.

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With an AVL tree/dictionary? O(n log n)

With a symbol table?

Dictionaries vs. Symbol Tables

Sorting with a dictionary:

1) Insert every item into the dictionary.
2) Search for the minimum item.
3) Repeat: find successor

Running time to implement sorting:

With an AVL tree/dictionary? O(n log n)

With a symbol table? $O(n^2)$

Sorting (aside)

Isn't O(1) search/insert impossible?

- Sorting takes $\Omega(n \log n)$ comparisons.

Sorting (aside)

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Sorting takes $\Omega(n \log n)$ comparisons.

- How do you sort with a symbol table?
- Only search/insert/delete.

Sorting (aside)

Isn't O(1) search/insert impossible?

Sorting takes $\Omega(n \log n)$ comparisons.

- How do you sort with a symbol table?
- Only search/insert/delete.

(Binary) search takes $\Omega(\log n)$ comparisons.

- Impossible to search in fewer than $\log(n)$ comparisons.
- But a symbol table finds an item in O(1) steps!!
- Conclusion: symbol table is not ***comparison-based***.

Symbol Tables in Java

Symbol Tables in Java

java.util.Map

public interface java.util.Map<Key, Value>

void	clear()	*removes all entries*
boolean	containsKey(Object k)	*is k in the map?*
boolean	containsValue(Object v)	*is v in the map?*
Value	get(Object k)	*get value for k*
Value	put(Key k, Value v)	*adds (k,v) to table*
Value	remove(Object k)	*remove mapping for k*
int	size()	*number of entries*

Note: no successor / predecessor queries.

Symbol Tables in Java

java.util.Map

- Parameterized by key and value.
- Not necessarily comparable

`public interface java.util.Map<Key, Value>`

`void`	`clear()`	*removes all entries*
`boolean`	`containsKey(Object k)`	*is k in the map?*
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Note: no successor / predecessor queries.

Symbol Tables in Java

- Search by key.

java.util.Map

public interface	java.util.Map<Key, Value>	
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Value	remove(Object k)	*remove mapping for k*
int	size()	*number of entries*

Note: no successor / predecessor queries.

Symbol Tables in Java

java.util.Map

- Search by key.
- Search by value.

(May not be efficient.)

```
public interface  java.util.Map<Key, Value>

      void  clear()                      removes all entries
   boolean  containsKey(Object k)        is k in the map?
   boolean  containsValue(Object v)      is v in the map?
     Value  get(Object k)                get value for k
     Value  put(Key k, Value v)          adds (k,v) to table
     Value  remove(Object k)             remove mapping for k
       int  size()                       number of entries
```

Note: no successor / predecessor queries.

Symbol Tables in Java

java.util.Map

• Can use any Object as key?

```
public interface  java.util.Map<Key, Value>
```

void	clear()	*removes all entries*
boolean	containsKey(Object k)	*is k in the map?*
boolean	containsValue(Object v)	*is v in the map?*
Value	get(Object k)	*get value for k*
Value	put(Key k, Value v)	*adds (k,v) to table*
Value	remove(Object k)	*remove mapping for k*
int	size()	*number of entries*

Note: no successor / predecessor queries.

Symbol Tables in Java

java.util.Map

• Put new (key, value) in table.

`public interface java.util.Map<Key, Value>`

`void`	`clear()`	*removes all entries*
`boolean`	`containsKey(Object k)`	*is k in the map?*
`boolean`	`containsValue(Object v)`	*is v in the map?*
`Value`	`get(Object k)`	*get value for k*
`Value`	`put(Key k, Value v)`	*adds (k,v) to table*
`Value`	`remove(Object k)`	*remove mapping for k*
`int`	`size()`	*number of entries*

Note: no successor / predecessor queries.

Map Interface in Java

java.util.Map<Key, Value>

- No duplicate keys allowed.
- No *mutable* keys
  - If you use an *object* as a key, then you can’t modify that object later.

Symbol Table

Key Mutability

What time does this plane depart at?

```
SymbolTable<Time, Plane> t =
           new SymbolTable<Time, Plane>();

Time  t1 = new Time(9:00);
Time  t2 = new Time(9:15);

t.insert(t1, “SQ0001”);
t.insert(t2, “SQ0002”);

t1.setTime(10:00);

x = new Time(9:00);
t.search(x);
```

Symbol Table

Key Mutability

Moral: Keys should be immutable.

Examples: Integer, String

```
SymbolTable<Time, Plane> t =
          new SymbolTable<Time, Plane>();

Time  t1 = new Time(9:00);
Time  t2 = new Time(9:15);

t.insert(t1, “SQ0001”);
t.insert(t2, “SQ0002”);

t1.setTime(10:00);

x = new Time(9:00);
t.search(x);
```

Symbol Tables in Java

java.util.Map

public interface	**java.util.Map<Key, Value>**	
`Set<Map.Entry<Key, Value>`	`entrySet()`	*set of all mappings*
`Set<Key>`	`keySet()`	*set of all keys*
`Collection<Value>`	`values()`	*collection of all values*

Note: not sorted

not necessarily efficient to work with these sets/collections.

What is wrong here?

Example:

There is a bug here!

```
Map<String, Integer> ageMap = new Map<String, Integer>();

ageMap.put(“Alice”, 32);
ageMap.put(“Bernice”, 84);
ageMap.put(“Charlie”, 7);

Integer age = ageMap.get(“Alice”)
```

- Key-type: String
- Value-type: Integer

What is wrong here?

Example:

Map is an interface!
Cannot instantiate an interface.

```
Map<String, Integer> ageMap = new Map<String, Integer>();

ageMap.put("Alice", 32);
ageMap.put("Bernice", 84);
ageMap.put("Charlie", 7);

Integer age = ageMap.get("Alice")
```

- Key-type: String
- Value-type: Integer

Map Class in Java

Example: HashMap

```
Map<String, Integer> ageMap = new HashMap<String, Integer>();

ageMap.put("Alice", 32);
ageMap.put("Bernice", 84);
ageMap.put("Charlie", 7);

Integer age = ageMap.get("Alice");
System.out.println("Alice's age is: " + age + ".");
```

- Key-type: String
- Value-type: Integer

Map Class in Java

Example: HashMap

```
Map<String, Integer> ageMap = new HashMap<String, Integer>();

ageMap.put("Alice", 32);
ageMap.put("Bernice", null);
ageMap.put("Charlie", 7);

Integer age = ageMap.get("Bob");
if (age==null){
    System.out.println("Bob's age is unknown.");
}
```

- Returns "null" when key is not in map.
- Returns "null" when value is null.

Map Classes in Java

HashMap

Symbol Table

- containsKey
- containsValue
- entrySet
- get
- isEmpty
- keySet
- put
- putAll
- remove
- values

TreeMap

Dictionary

- containsKey
- containsValue
- entrySet
- get
- isEmpty
- keySet
- put
- putAll
- remove
- values

Map Classes in Java

HashMap

Symbol Table

TreeMap

Dictionary

- ceilingEntry
- ceilingKey
- descendingKeySet
- firstEntry
- firstKey
- floorEntry
- floorKey
- headMap
- higherEntry
- higherKey
- … (and more)

Lots of functionality

Wide Interfaces

vs.

Narrow Interfaces

Limited functionality

Lots of functionality
- Java
- No guarantee of efficiency.
- Easy to use (badly).

Wide Interfaces

vs.

Narrow Interfaces

Limited functionality
- Enforces proper use.
- Restricts usage.

Symbol Tables are Useful

Examples:

1. Spelling correction (key=misspelled word, data=word)
2. Scheme interpreter (key=variable, data=value)
3. Web server
   - Lots of simultaneous network connections.
   - When a packet arrives, give it to the right process to handle the connection.
   - key=ip address, data = connection handler

In this cases, O(log n) often isn't fast enough!

Symbol Tables are Useful

Example 1: Pilot Scheduling

1. Check to see if feasible to schedule at time *t*.

   No two airplanes can land with 15 minutes of each other.

2. Find schedule of pilot *p*.

   Get a list of all the planes that are being flown by a specified pilot.

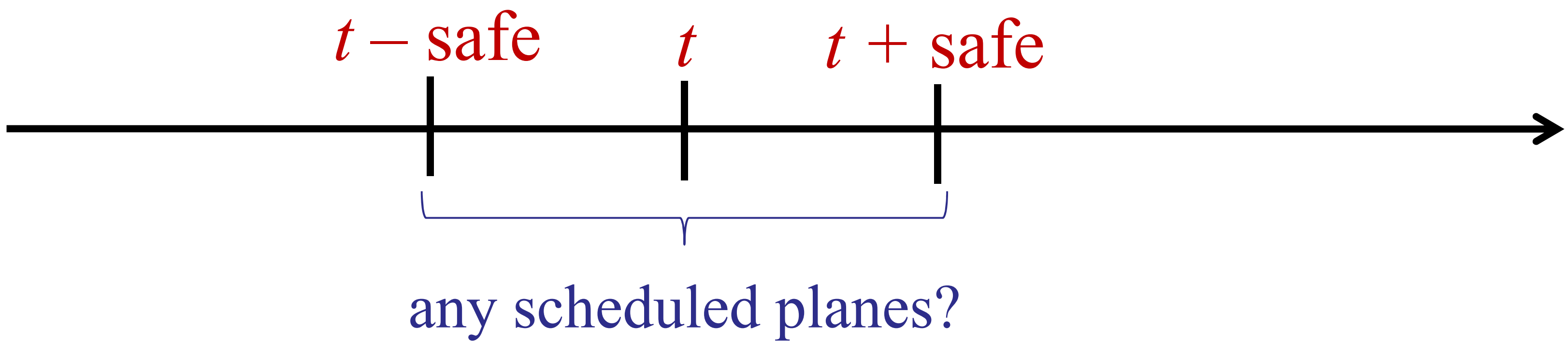
Which can be efficiently solved with a symbol table?

1. Both: scheduling and pilots info.
2. Only scheduling.
3. Only pilot info.
4. Neither.

Symbol Tables are Useful

Example 1: Pilot Scheduling

1. Check to see if feasible to schedule at time t.
    - Hard with a symbol table!
    - Need to find out if there are any planes scheduled in the interval $[t, t \pm \text{safe\_distance}]$



Symbol Tables are Useful

Example 1: Pilot Scheduling

1. Check to see if feasible to schedule at time *t*.
2. Find schedule of pilot *p*.
   - Perfect for a symbol table!
   - Can insert new pilots.
   - Can search for (and update) existing pilots.
   - Listing all pilots?

Symbol Tables are Useful

Example 2: Document Distance

- Given two documents A and B, how similar are they?
  - Two documents are *similar* if they have similar words in similar frequencies.
  - Formally, define each text as a vector with one entry per word.
  - The distance between the two texts is the angle between the two vectors.

Symbol Tables are Useful

Example 2: Document Distance

- Step 1: Read in each document
  - Read the file as a string.
  - Parse the file into words.
  - Sort the list of words.
  - Count the frequency of each word.
- Step 2: Compare the two documents
  - Calculate the norm $|A|$ and $|B|$ of each vector
  - Calculate the dot product AB.
  - Calculate the angle between A and B.

Symbol Tables are Useful

Example 2: Document Distance

– Step 1: Read in each document

$O(n)$ • Read the file as a string.

$O(n)$ • Parse the file into words.

$O(n \log n)$ • Sort the list of words.

$O(n)$ • Count the frequency of each word.

– Step 2: Compare the two documents

$O(n)$ • Calculate the norm $|A|$ and $|B|$ of each vector

$O(n)$ • Calculate the dot product AB.

$O(n)$ • Calculate the angle between A and B.

Performance Profiling (Sorting)

(Dracula vs. *Lewis & Clark)*

Step	Function	Running Time
Create vectors:	Read each file	1.03s
	Parse each file	1.23s
	Sort words in each file	2.04s
	Count word frequencies	0.41s
Dot product:		6.10s
Norm:		0.01s
Angle:		0.02s
Total:		**12.75s**

Performance Profiling (Symbol Table)

(Dracula vs. *Lewis & Clark)*

Step	Function	Running Time
Create vectors:	Read each file	1.19s
	Parse each file	1.37s
	Sort words in each file	0
	Count word frequencies	0
Dot product:		0.03s
Norm:		0.01s
Angle:		0.02s
Total:		**2.43s**

Symbol Tables are Useful

Example 2: Document Distance

– Step 1: Read in each document

  - Read and parse the file.
  - Put each (word, count) in a HashMap.

  Symbol Table:

  - key (String) = word
  - value (Integer) = count (# times in doc)

Document Distance

(Dracula vs. Lewis & Clark)

Version	Change	Running Time
Version 1		4,311.00s
Version 2	Better file handling	676.50s
Version 3	Faster sorting	6.59s
Version 4	Symbol Table	2.35s

Building a Symbol Table

Direct Access Tables

Attempt #1: Use a table, indexed by keys.

0	null
1	null
2	item1
3	null
4	null
5	item3
6	null
7	null
8	item2
9	null

Universe U={0..9} of size $m = 10$.

<u>(key, value)</u>

(2, item1)
(8, item2)
(5, item3)

Assume keys are distinct.

Direct Access Tables

Attempt #1: Use a table, indexed by keys.

0	null
1	null
2	item1
3	null
4	null
5	item3
6	null
7	null
8	item2
9	null

Example: insert(4, Seth)

Direct Access Tables

Attempt #1: Use a table, indexed by keys.

0	null
1	null
2	item1
3	null
4	Seth
5	item3
6	null
7	null
8	item2
9	null

Example: insert(4, Seth)

Time: O(1) / insert, O(1) / search

Direct Access Tables

Problems:

- Too much space
  - If keys are integers, then table-size > 4 billion
- What if keys are not integers?
  - Where do you put the key/value “**(hippopotamus, bob)**”?
  - Where do you put 3.14159…?

Direct Access Tables

Pythagoras said, “Everything is a number.”

“The School of Athens” by Raphael

Direct Access Tables

Pythagoras said, “Everything is a number.”

- Everything is just a sequence of bits.
- Treat those bits as a number.

- English:
  - 26 letters => 5 bits/letter
  - Longest word = 28 letters (antidisestablishmentarianism?)
  - 28 letters * 5 bits = 140 bits
  - So we can store any English word in a direct-access array of size 2^{140}.

Direct Access Tables

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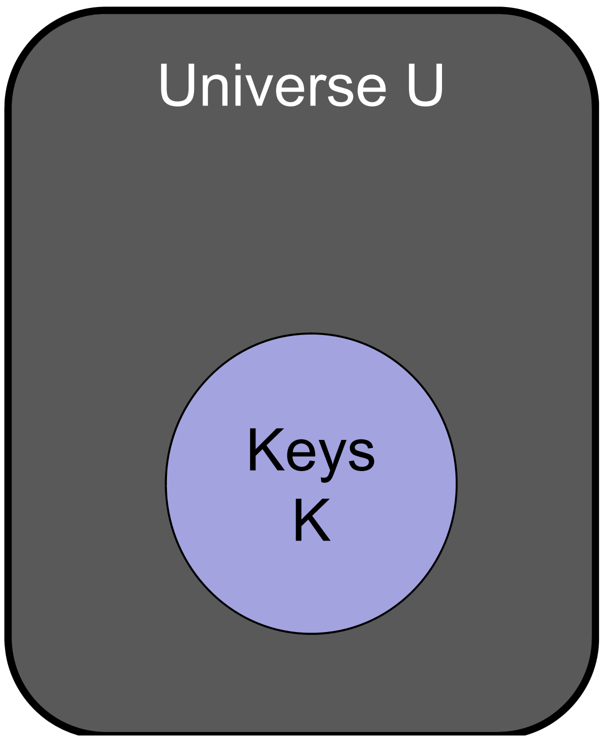
- English:
  - 26 letters => 5 bits/letter
  - Longest word = 28 letters (antidisestablishmentarianism?)
  - 28 letters * 5 bits = 140 bits
  - So we can store any English word in a direct-access array of size 2^{140}. ≈ number of atoms in observable universe

Hash Functions

Problem:

- Huge universe U of possible keys.
- Smaller number n of actual keys.

e.g., 2^{140}



Hash Functions

Problem:

- Huge universe U of possible keys.
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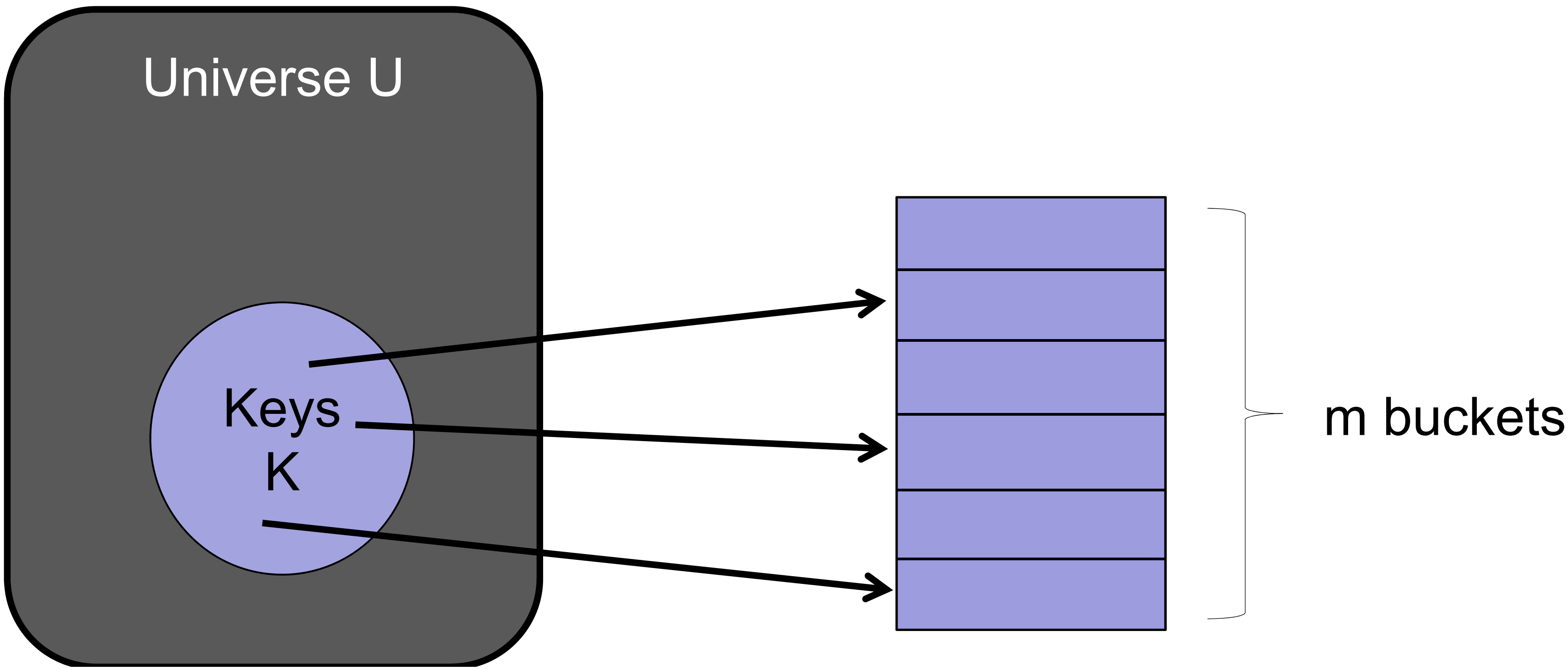
- How to map n keys to $m \approx n$ buckets?



Hash Functions

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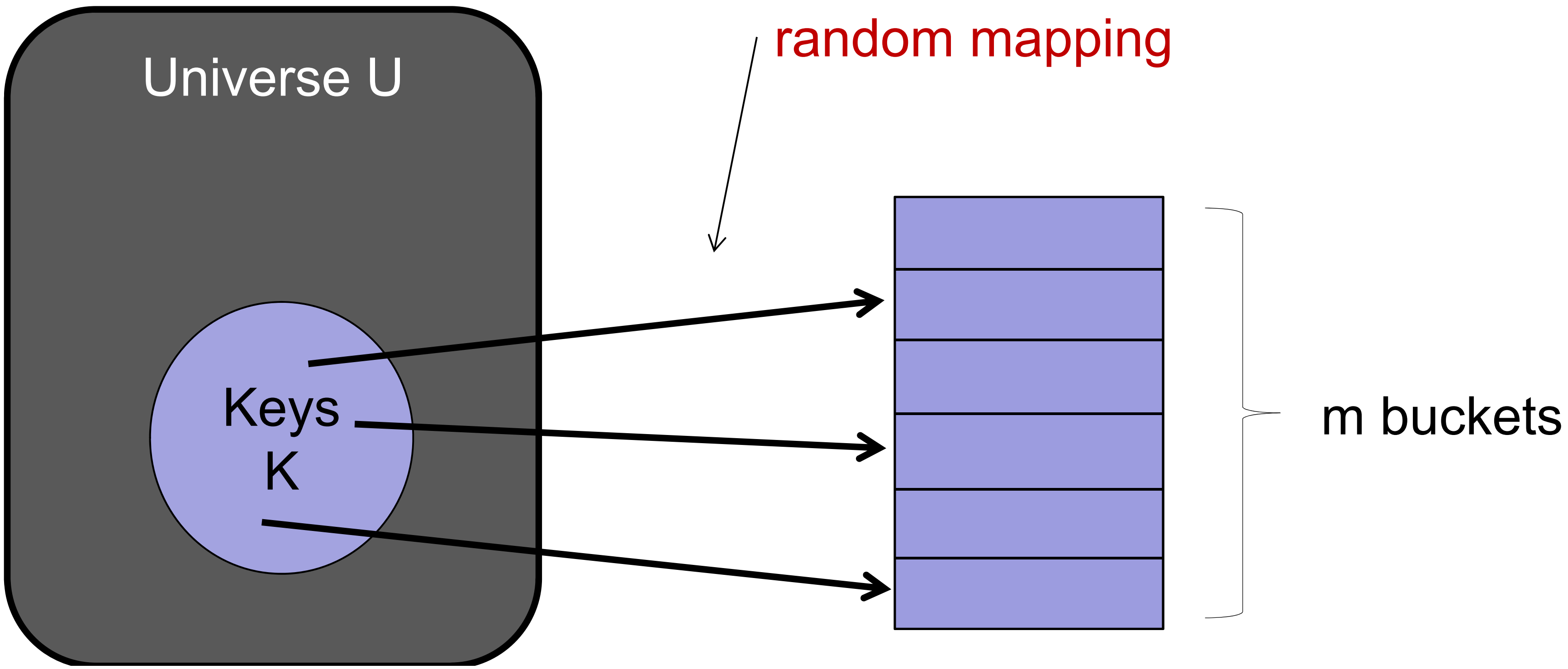
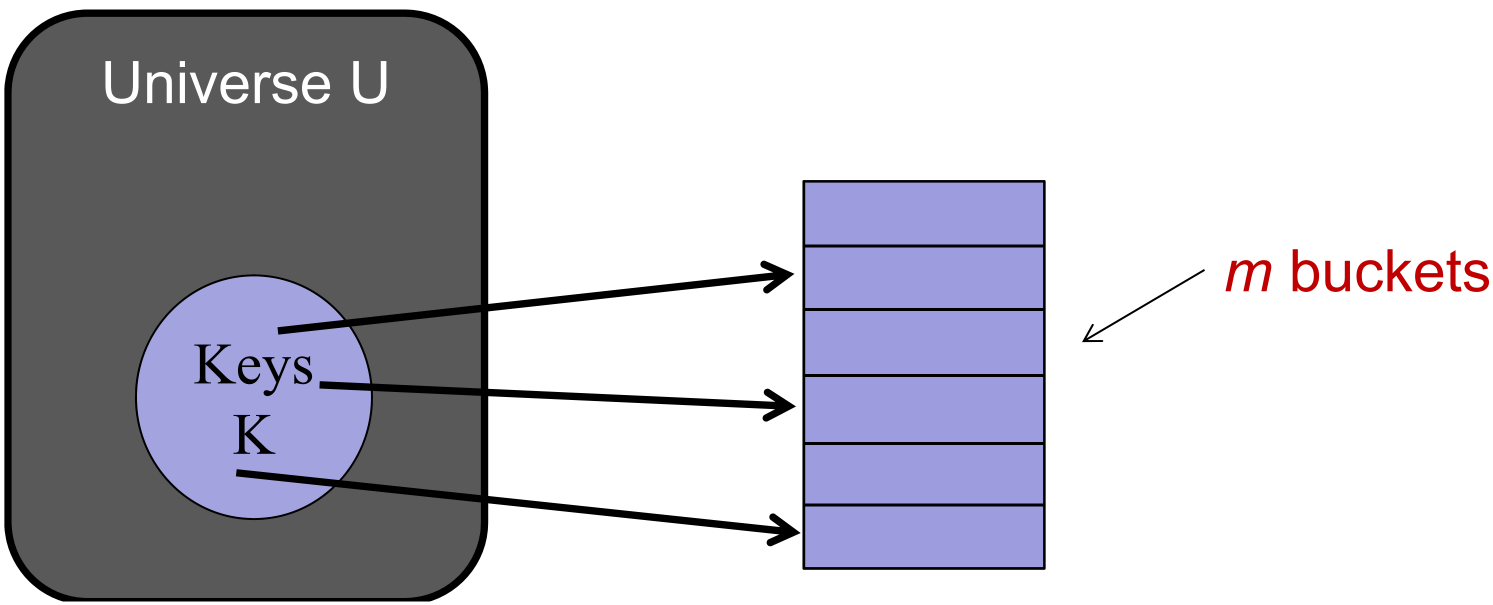


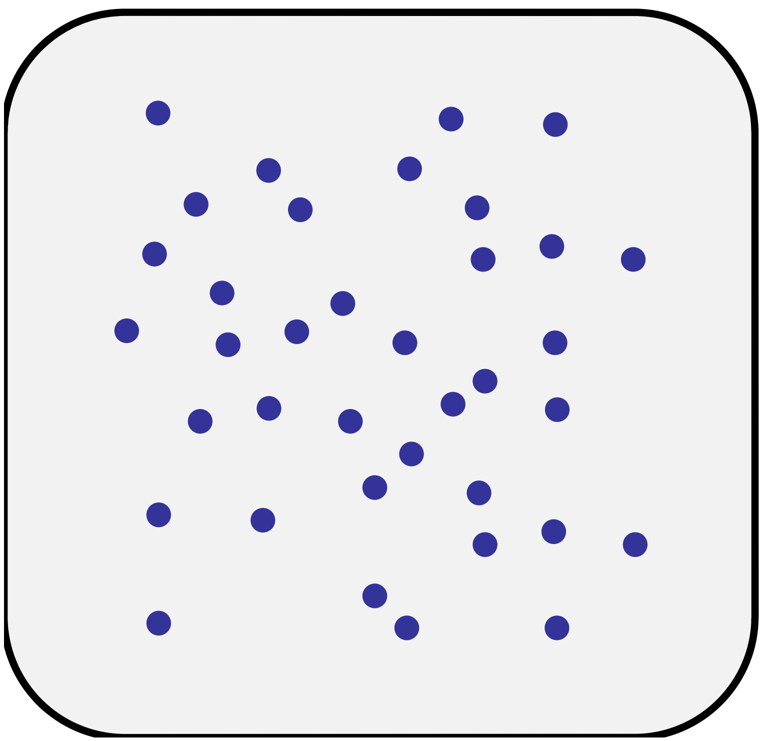
Hash Functions

Define hash function $h : U \rightarrow \{1..m\}$

- Store key k in bucket $h(k)$.
- Time complexity:
  - Time to compute h + Time to access bucket
- For now: assume hash function has cost 1 to compute.



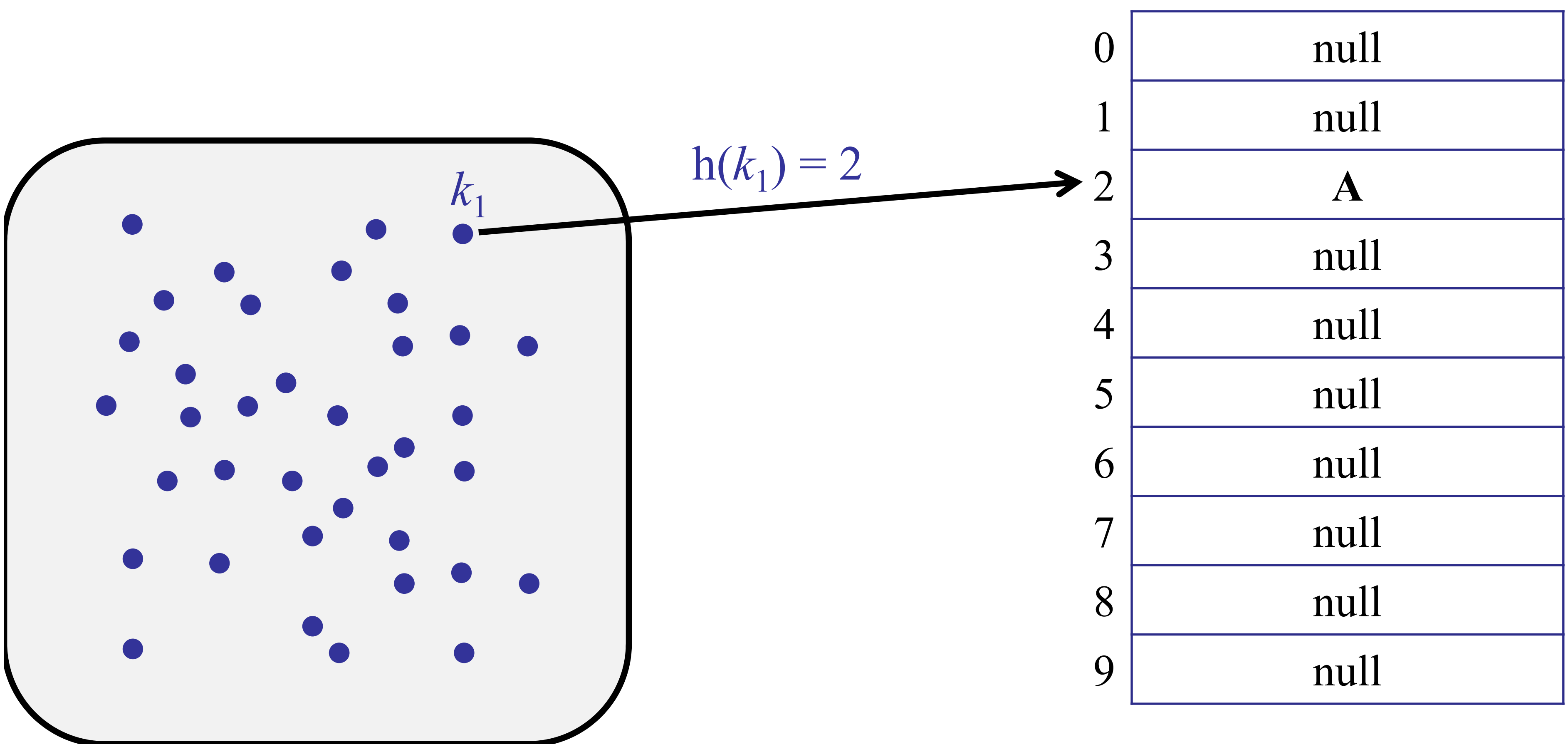
Hash Functions



0	null
1	null
2	null
3	null
4	null
5	null
6	null
7	null
8	null
9	null

Hash Functions

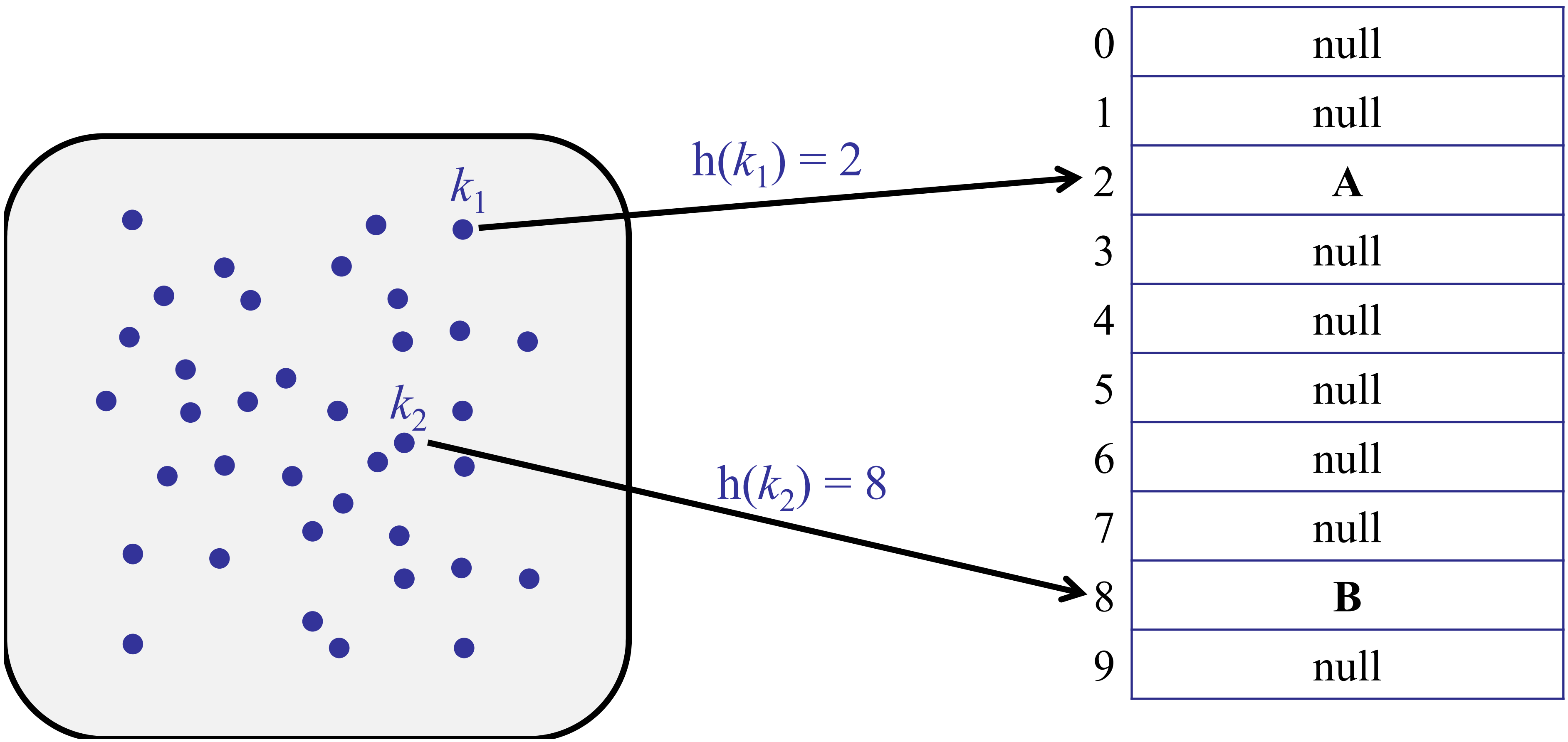
insert(k_1, A)



Hash Functions

insert(k_1, A)

insert(k_2, B)



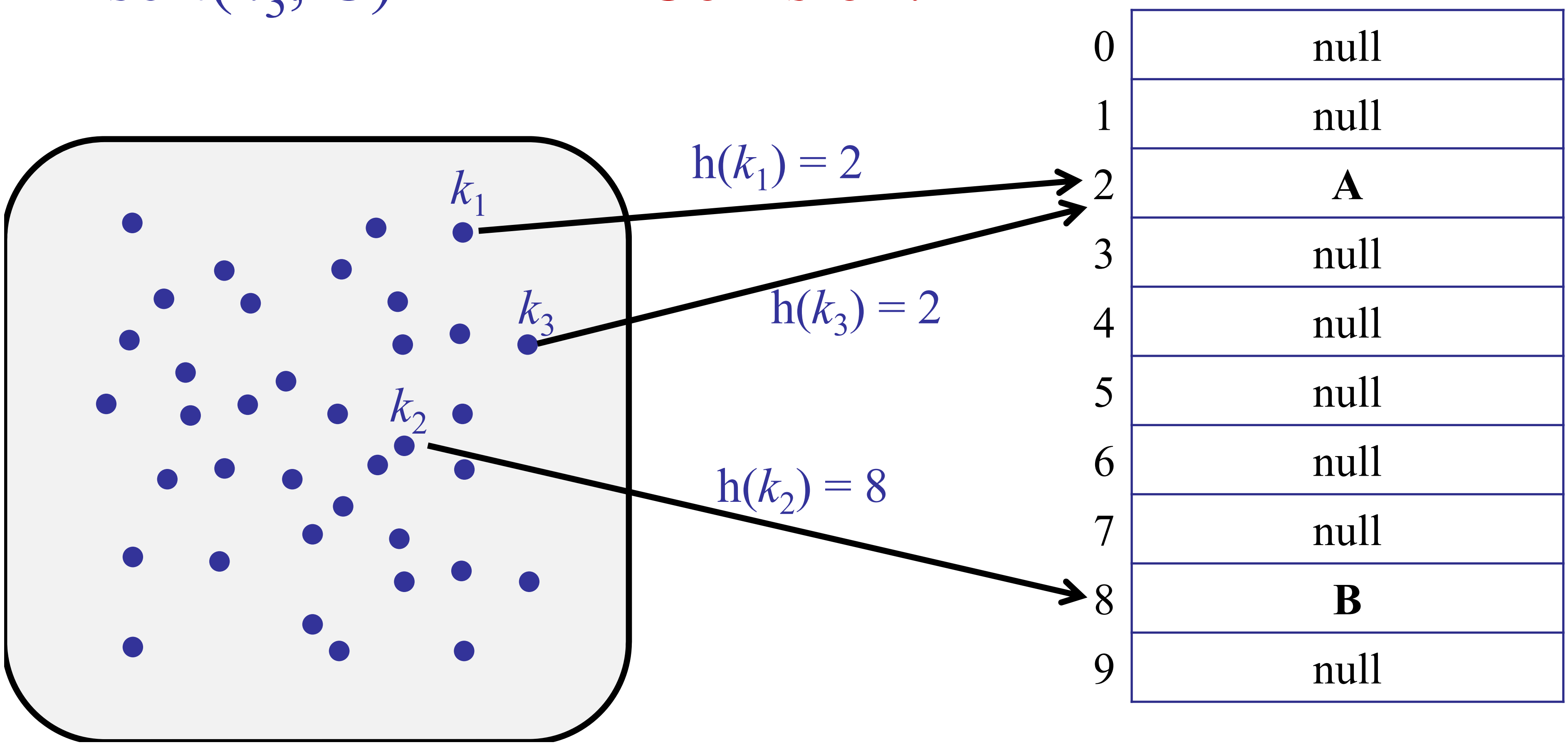
Hash Functions

insert(k_1, A)

insert(k_2, B)

insert(k_3, C)

Collision!



Hash Functions

Collisions:

- We say that two distinct keys k_1 and k_2 collide if:

$$h(k_1) = h(k_2)$$

Can we choose a hash function with no collisions?

1. Yes
2. Sometimes, if we choose carefully
3. No, impossible

Hash Functions

Collisions:

- We say that two distinct keys k_1 and k_2 collide if:

$$h(k_1) = h(k_2)$$

- Unavoidable!
  - The table size is smaller than the universe size.
  - The pigeonhole principle says:
    - There must exist two keys that map to the same bucket.
    - Some keys must collide!

Coping with Collision

Idea: choose a new, better hash functions

Coping with Collision

Idea: choose a new, better hash functions

- Hard to find.
- Requires re-copying the table.
- Eventually, there will be another collision.

Coping with Collision

Idea: choose a new, better hash functions

- Hard to find.
- Requires re-copying the table.
- Eventually, there will be another collision.

Idea: chaining (today)

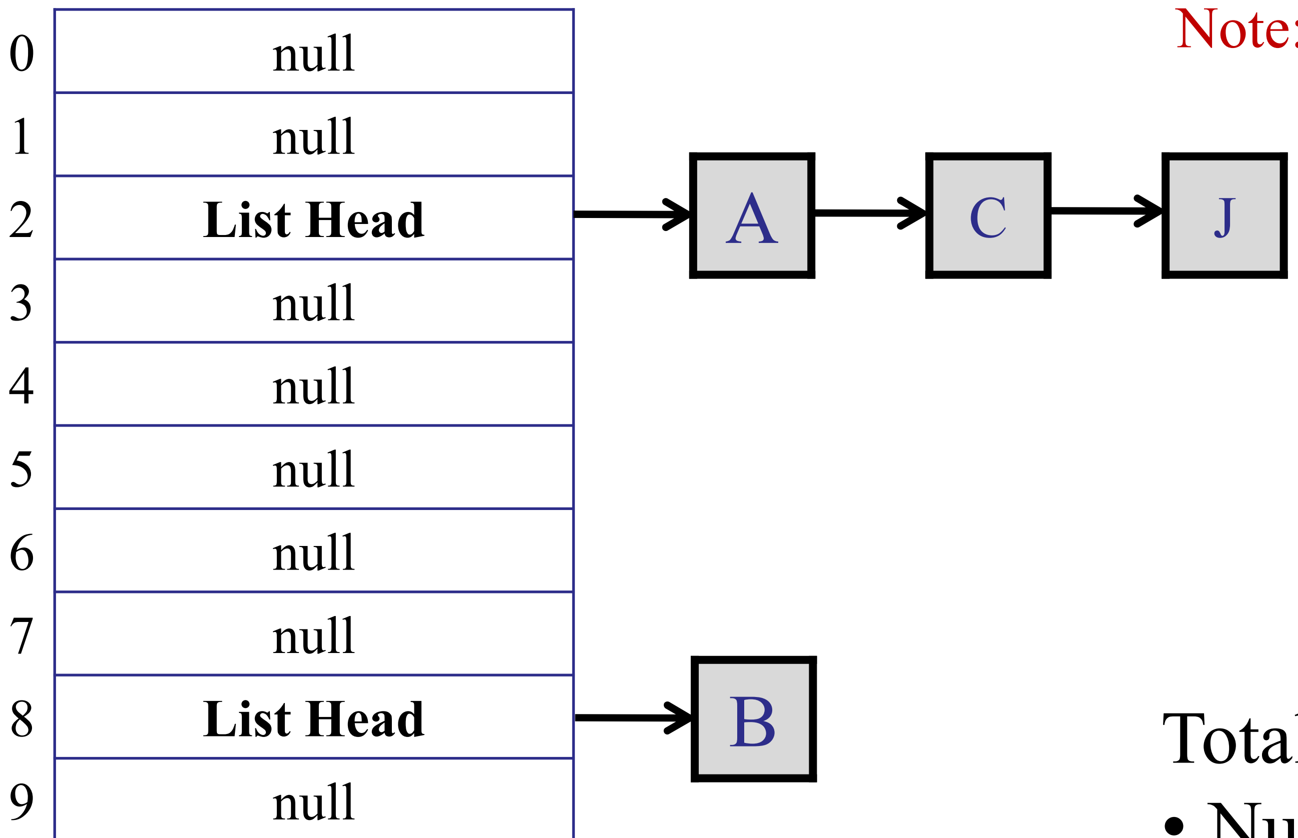
- Put both items in the same bucket!

Idea: open addressing (next week)

- Find another bucket for the new item.

Chaining

Each bucket contains a linked list of items.



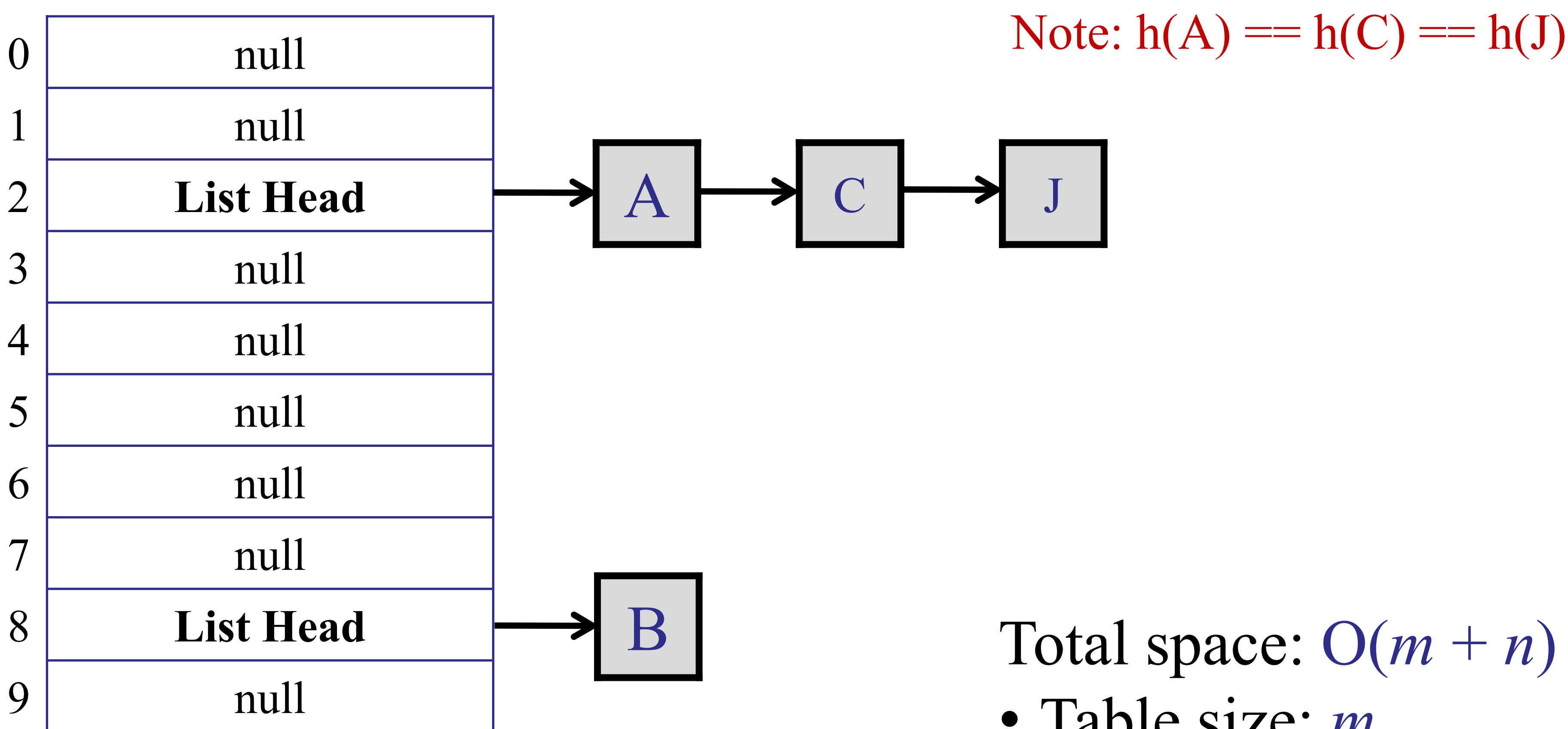
Note: h(A) == h(C) == h(J)

Total space:

- Number buckets: m
- Number entries: n

Chaining

Each bucket contains a linked list of items.



Total space: $O(m + n)$

- Table size: m
- Linked list size: n

Hashing with Chaining

Operations:

- insert(key, value)
  - Calculate h(key)
  - Lookup h(key) and add (key,value) to the linked list.

- search(key)
  - Calculate h(key)
  - Search for (key,value) in the linked list.

What is the worst-case cost of inserting a (key, value)?

✓ 1. $O(1 + \text{cost}(h))$
2. $O(\log n + \text{cost}(h))$
3. $O(n + \text{cost}(h))$
4. $O(n\ \text{cost}(h))$
5. $O(n^2)$.

Hashing with Chaining

Operations:

Always O(1).

- insert(key, value)
  - Calculate h(key)
  - Lookup h(key) and add (key,value) to the linked list.

- search(key)
  - Calculate h(key)
  - Search for (key,value) in the linked list.

What is the worst-case cost of searching a (key, value)?

1. $O(1 + \text{cost}(h))$
2. $O(\log n + \text{cost}(h))$
3. $O(n + \text{cost}(h))$
4. $O(n * \text{cost}(h))$
5. We cannot determine it without knowing h.

Hashing with Chaining

Operations:

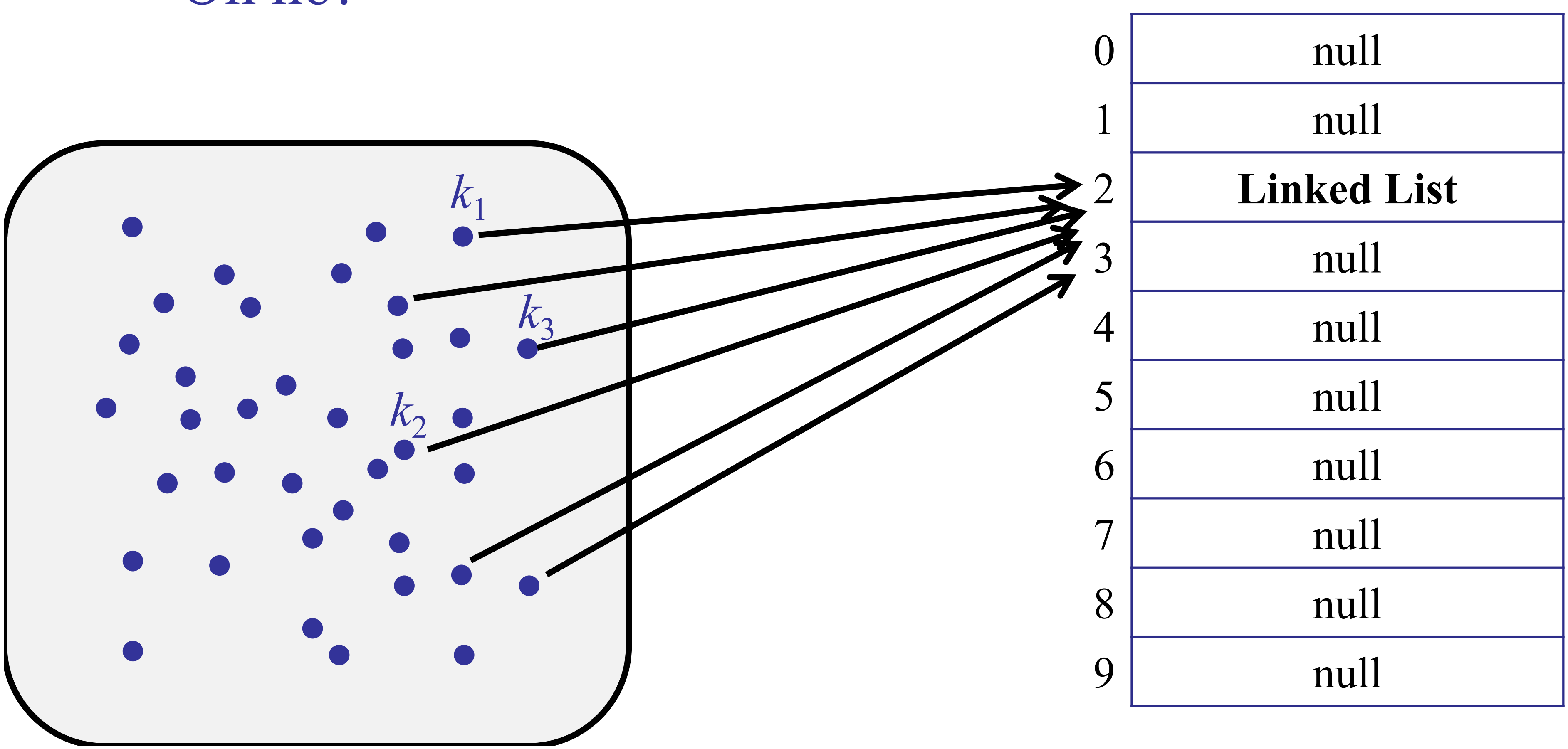
- insert(key, value)
  - Calculate h(key)
  - Lookup h(key) and add (key,value) to the linked list.

- search(key) → time depends on length of linked list
  - Calculate h(key)
  - Search for (key,value) in the linked list.

Hashing with Chaining

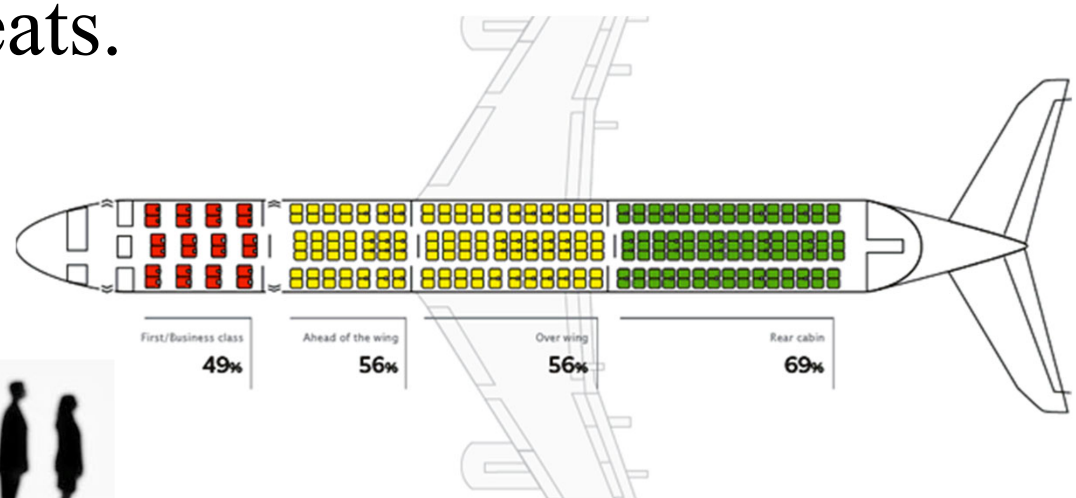
Assume all keys hash to the same bucket!

- Search costs O(n)
- Oh no!



Puzzle Break

An airplane has 100 seats.



100 passengers board the airplane in a random order.

Puzzle Break

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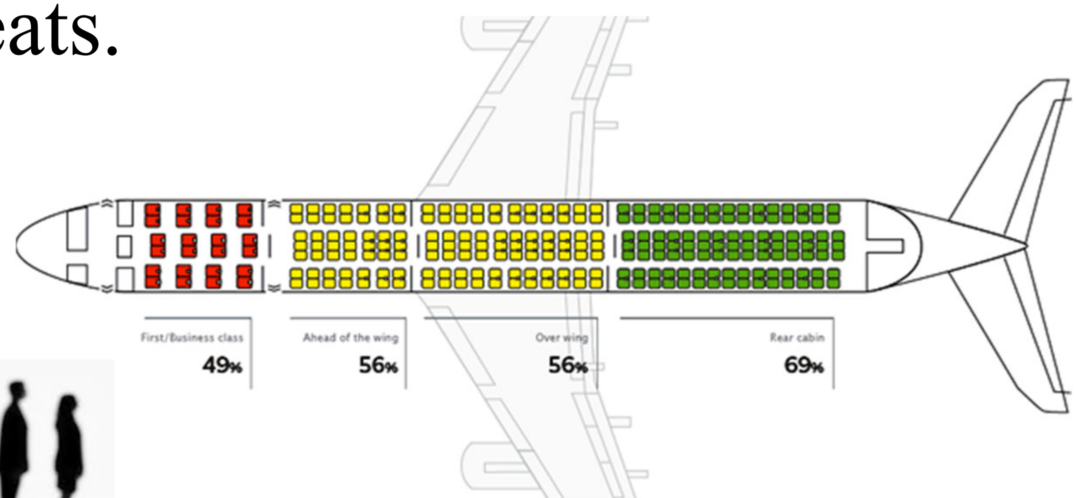


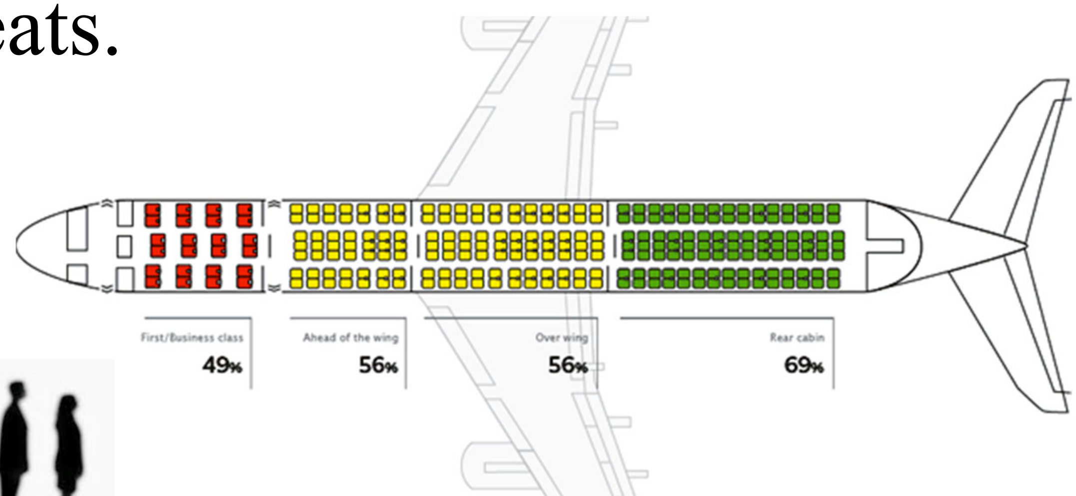
100 passengers board the airplane in a random [illegible]er.

Passenger 1 is Mr. Burns.

Mr. Burns sits in a random seat.

Puzzle Break

An airplane has 100 seats.



Every other passenger:

- Sits in their assigned seat, if it is free.
- Otherwise, sits in a random seat.

Puzzle Break

An airplane has 100 seats.

You are passenger #100.

What is the probability your seat is free when you board?

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Problem Solving techniques:

Try a plane with 2 seats. Try a plane with 3 seats.

Summary

Symbol Tables are pervasive

- You find them everywhere!

Hash tables are fast, efficient symbol tables.

- Under optimistic assumptions, provably so.
- In the real world, often so.
- But be careful!

Beats BSTs:

- Operate directly on keys (i.e., indexing)
- Gave up: successor/predecessor/etc.