MA1521 Finals Cheatsheet

~github/reidenong/cheatsheets~, AY23/24 S1

1. Limits

1.2 Continuity

f is continous at $x = c$ if (1) $\lim_{x\to c} f(x)$ exists and (2) $\lim_{x\to c} f(x) = f(c)$

- Differentiability implies continuity

1.4 Trigonometric identities

if $\lim_{x\to c} g(x) = 0$, for $\sin x$ and $\tan x$,
$\lim_{x\to c} \frac{g(x)}{\sin(g(x))} = \lim_{x\to c} \frac{\sin(g(x))}{g(x)} = 1$

2. Derivatives

2.1 Definition

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

2.2 Standard Derivatives

$f(x)$	$f'(x)$
$\tan(x)$	$\sec^2(x)$
$\sec(x)$	$\sec(x)\tan(x)$
$\csc(x)$	$-\csc(x)\cot(x)$
$\cot(x)$	$-\csc^2(x)$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\cot^{-1} x$	$-\frac{1}{1+x^2}$
$\sec^{-1} x$	$\frac{1}{\lvert x\rvert\sqrt{x^2-1}}, \lvert x\rvert > 1$
$\csc^{-1} x$	$-\frac{1}{\lvert x\rvert\sqrt{x^2-1}}, \lvert x\rvert > 1$
$a^x, a \in \mathbb{R}$	$a^x \ln(a)$

Rules of Differentiation

Quotient rule:

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

3. Applications of Differentiation

3.2 Increasing and decreasing Functions

$f'(x) > 0$ on $(a,b) \to f$ is increasing on (a,b)
$f'(x) < 0$ on $(a,b) \to f$ is decreasing on (a,b)

Critical points

a number c in function f is a critical point if

1. it is not a end point and
2. $f'(c) = 0$ or $f'(c)$ does not exist

3.8 Rolle's Theorem and Mean Value Theorem

Rolle's Theorem:
if f is continous on $[a,b]$ and differentiable on (a,b) and $f(a) = f(b)$,
then there exists c in (a,b) such that $f'(c) = 0$
Mean Value Theorem:
if f is continous on $[a,b]$ and differentiable on (a,b),
then there exists c in (a,b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$

4.Integrals

$\int \frac{1}{ax+b} dx$	$\frac{1}{a}\ln\lvert ax+b\rvert + C$
$\int \tan(ax+b)dx$	$\frac{1}{a}\ln\lvert\sec(ax+b)\rvert + C$
$\int \sec(ax+b)dx$	$\frac{1}{a}\ln\lvert\sec(ax+b)+\tan(ax+b)\rvert + C$
$\int \csc(ax+b)dx$	$-\frac{1}{a}\ln\lvert\csc(ax+b)+\cot(ax+b)\rvert + C$
$\int \cot(ax+b)dx$	$-\frac{1}{a}\ln\lvert\csc(ax+b)\rvert + C$
$\int \sec^2(ax+b)dx$	$\frac{1}{a}\tan(ax+b) + C$
$\int \csc^2(ax+b)dx$	$-\frac{1}{a}\cot(ax+b) + C$
$\int \sec(ax+b)\tan(ax+b)dx$	$\frac{1}{a}\sec(ax+b) + C$
$\int \csc(ax+b)\cot(ax+b)dx$	$-\frac{1}{a}\csc(ax+b) + C$
$\int \frac{1}{a^2+(x+b)^2}dx$	$\frac{1}{a}\tan^{-1}\frac{x+b}{a} + C$
$\int \frac{1}{\sqrt{a^2-(x+b)^2}}dx$	$\sin^{-1}\frac{x+b}{a} + C$
$\int -\frac{1}{\sqrt{a^2-(x+b)^2}}dx$	$\cos^{-1}\frac{x+b}{a} + C$
$\int \frac{1}{a^2-(x+b)^2}dx$	$\frac{1}{2a}\ln\left\lvert\frac{x+b+a}{x+b-a}\right\rvert + C$
$\int \frac{1}{(x+b)^2-a^2}dx$	$\frac{1}{2a}\ln\left\lvert\frac{x+b-a}{x+b+a}\right\rvert + C$
$\int \frac{1}{\sqrt{(x+b)^2+a^2}}dx$	$\ln\left\lvert(x+b)+\sqrt{(x+b)^2+a^2}\right\rvert + C$
$\int \frac{1}{\sqrt{(x+b)^2-a^2}}dx$	$\ln\left\lvert(x+b)+\sqrt{(x+b)^2-a^2}\right\rvert + C$
$\int \sqrt{a^2-x^2}dx$	$\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$
$\int \sqrt{x^2-a^2}dx$	$\frac{x}{2}\sqrt{x^2-a^2} + \frac{a^2}{2}\ln\lvert x+\sqrt{x^2-a^2}\rvert + C$

4.3 Partial Fractions

$$\frac{px+q}{(ax+b)(cx+d)} = \frac{A}{ax+b} + \frac{B}{cx+d}$$

$$\frac{px^2+qx+r}{(ax+b)(cx+d)^2} = \frac{A}{ax+b} + \frac{B}{cx+d} + \frac{C}{(cx+d)^2}$$

$$\frac{px^2+qx+r}{(ax+b)(x^2+c^2)} = \frac{A}{ax+b} + \frac{Bx+C}{x^2+c^2}$$

4.4 Integration by Substitution

Trigonometric Substitution

$\sqrt{a^2-x^2}$	$x = a\sin\theta$
$\sqrt{x^2+a^2}$	$x = a\tan\theta$
$\sqrt{x^2-a^2}$	$x = a\sec\theta$

4.5 Integration by Parts $\int u dv = uv - \int v du$

Logarithmic	differentiate it
Inverse Trigonometric	differentiate it
Algebraic	differentiate it
Trigonometric	differentiate/integrate
Exponential	integrate it

4.6 Riemann Sums and Definite Integrals

Formula for Riemann Sum:

$$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n}\frac{b-a}{n}\cdot f\left(a+(b-a)\frac{i}{n}\right)$$

5.2 Volumes of revolution

Disk method:
When the curve of $y = f(x)$ is rotated about the x-axis, the volume of the resulting solid is
$V = \pi\int_a^b (f(x))^2 dx$
Cylindrical Shell method
When the curve of $y = f(x)$ is rotated about the y-axis, the volume of the resulting solid is
$V = 2\pi\int_a^b x\lvert f(x)\rvert dx$

5.4 Arc length of a curve

The length of a curve $y = f(x), a \le x \le b$, is given by
$\int_a^b \sqrt{1+(f'(x))^2}dx$

6. Sequences and Series

6.2 Series Properties

Lemma 6.4:
If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

Theorem 6.5: n-th term test for divergence
If $\lim_{n\to\infty} a_n$ does not exist or is not equal to 0, then $\sum_{n=1}^{\infty} a_n$ diverges.

Theorem 6.7: Integral test
If f is a continuous, positive, decreasing function on $[1,\infty)$, then the series $\sum_{n=1}^{\infty} f(n)$ and the improper integral $\int_1^{\infty} f(x)dx$ either both converge or both diverge.

Theorem 6.8: p-series test
The series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \le 1$.

Theorem 6.9: Comparison Test
Suppose $0 \le a_n \le b_n$ for all $n \ge N$ and

1. $\sum_{n=1}^{\infty} b_n$ converges. Then $\sum_{n=1}^{\infty} a_n$ converges.
2. $\sum_{n=1}^{\infty} a_n$ diverges. Then $\sum_{n=1}^{\infty} b_n$ diverges.

Theorem 6.10/6.11: Ratio/Root Test

$$\lim_{n\to\infty}\left\lvert\frac{a_{n+1}}{a_n}\right\rvert = L, or \lim_{n\to\infty}(\lvert a_n\rvert)^{\frac{1}{n}} = L$$

1. If $0 \le L < 1$, then $\sum_{n=1}^{\infty} a_n$ converges absolutely ($\sum_{n=1}^{\infty}\lvert a_n\rvert$ converges).
2. If $L > 1$ then $\sum_{n=1}^{\infty} a_n$ diverges.
3. If $L = 1$, then the test is inconclusive.

Theorem 6.12: Alternating Series Test
If $a_n \ge 0$ for all n and $\lim_{n\to\infty} a_n = 0$ and a_n is decreasing, then the alternating series $\sum_{n=1}^{\infty}(-1)^{n+1}a_n$ converges.

6.10 Power Series

Theorem 6.14: Characteristics of Power Series
For a given power series $\sum_{n=0}^{\infty} c_n(x-a)^n$, either

1. The series converges only when $x = a$.
2. The series converges for all x.
3. There exists a positive number R such that the series converges absolutely if $\lvert x-a\rvert < R$ and diverges if $\lvert x-a\rvert > R$, where R is the *Radius of Convergence*

- The interval of convergence is then $[a-R, a+R]$.

Theorem 6.15: Calculating the Radius of Convergene
Consider the power series $\sum_{n=0}^{\infty} c_n(x-a)^n$.

$$\lim_{n\to\infty}\left\lvert\frac{c_{n+1}}{c_n}\right\rvert = L, or \lim_{n\to\infty}(\lvert c_n\rvert)^{\frac{1}{n}} = L$$

Then the radius of convergence $R = \frac{1}{L}$

6.12 Taylor and Maclaurin Series

If f has a power series representation at $x = a$, then it has a Taylor Series of the form

$$f(x) = \sum_{n=0}^{\infty}\frac{f^n(a)}{n!}(x-a)^n$$

The Maclaurin series is the Taylor series at $x = 0$.

$$f(x) = \sum_{n=0}^{\infty}\frac{f^n(0)}{n!}x^n$$

Common Expansions

e^x	$\sum_{n=0}^{\infty}\frac{x^n}{n!}$
$\sin x$	$\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$
$\cos x$	$\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}$
$\ln(1+x)$	$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n}$
$\frac{1}{1-x}$	$\sum_{n=0}^{\infty}x^n$
$\frac{1}{1+x}$	$\sum_{n=0}^{\infty}(-1)^n x^n$
$\frac{1}{1+x^2}$	$\sum_{n=0}^{\infty}(-1)^n x^{2n}$
$(1+x)^n, \lvert x\rvert < 1$	$\sum_{n=0}^{\infty}\frac{n(n-1)\dots(n-r+1)}{r!}x^r$
$(a+b)^n, n > 0$	$a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + b^n$

7. Vectors and Geometry of Space

Theorem 7.2: Equation of a Sphere

The equation of a sphere with center (h,k,l) and radius r is $(x-h)^2+(y-k)^2+(z-l)^2=r^2$

7.4 Projections

Projection of b onto a :

$$comp_{\boldsymbol{a}}\boldsymbol{b}=\|\boldsymbol{b}\|\cos\theta=\frac{\boldsymbol{a}\cdot\boldsymbol{b}}{\|\boldsymbol{a}\|}$$

$$\text{projected area}=\text{original area}\times\cos\theta$$

Distance from a point to a plane :

for point $P(x_0,y_0,z_0)$ to the plane $ax+by+cz=d$ is

$$\frac{|ax_0+by_0+cz_0-d|}{\sqrt{a^2+b^2+c^2}}$$

7.5 Dot Product and Cross Product

$$\boldsymbol{a}\cdot\boldsymbol{b}=\|\boldsymbol{a}\|\|\boldsymbol{b}\|\cos\theta \text{ and } \boldsymbol{a}\times\boldsymbol{b}=\|\boldsymbol{a}\|\|\boldsymbol{b}\|\sin\theta$$

$$\boldsymbol{a}=\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix},\boldsymbol{b}=\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix},\boldsymbol{a}\times\boldsymbol{b}=\begin{pmatrix}a_2b_3-a_3b_2\\a_3b_1-a_1b_3\\a_1b_2-a_2b_1\end{pmatrix}$$

8. Functions of Several Variables

8.4 Arc Length of a Space Curve

for a curve C given by $r(t)=\langle f(t),g(t),h(t)\rangle, a\le t\le b$, the length of the curve l is

$$l=\int_a^b\|r'(t)\|dt=\int_a^b\sqrt{(f'(t))^2+(g'(t))^2+(h'(t))^2}dt$$

if f',g',h' are continuous on $[a,b]$

8.6 Cylinders and Quadric Surfaces

Definition 8.5: Cylinder

A surface is a cylinder if there is a plane P such that all the planes parallel to P intersect the surface in the same curve. Any equation in x, y, z where one of the variables is missing is a cylinder.

Definition 8.7: Elliptic Paraboloid

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z}{c}$, symmetric about the z-axis, if c is +ve then it opens up, if c is -ve then it opens down.

Definition 8.8: Ellipsoid

$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

Theorem 8.4: Clairaut's Theorem

Suppse f is defined on a disk D that conains (a,b). If the functions f_{xy} and f_{yx} are both continuous on D, then $f_{xy}=f_{yx}$

Theorem 8.5: Equation of Tangent Plane

A normal vector to the tangent plane at $(a,b,f(a,b))$ to the surface $z=f(x,y)$ is

$$\langle f_{x(a,b)},f_{y(a,b)},-1\rangle$$

Theorem 8.6: Chain Rule

Case 1: Suppose that $z=f(x,y)$ is a differentiable function of x and y, where $x=g(t)$ and $y=h(t)$ are both differentiable functions of t. Then z is a differentiable function of t and

$$\frac{dz}{dt}=\frac{\partial f}{\partial x}\frac{dx}{dt}+\frac{\partial f}{\partial y}\frac{dy}{dt}$$

Case 2: Suppose that $z=f(x,y)$ is a differentiable function of x and y, where $x=g(s,t)$ and $y=h(s,t)$ are both differentiable functions of s and t. Then,

$$\frac{\partial z}{\partial s}=\frac{\partial f}{\partial x}\frac{\partial x}{\partial s}+\frac{\partial f}{\partial y}\frac{\partial y}{\partial s}, \text{ and} \frac{\partial z}{\partial t}=\frac{\partial f}{\partial x}\frac{\partial x}{\partial t}+\frac{\partial f}{\partial y}\frac{\partial y}{\partial t}$$

Theorem 8.9: Implicit Differentiation on Two independent variables

Suppose that $F(x,y,z)=0$, where F is differentiable, defines z implicitly as a differentiable function of x and y. Then, $\frac{\partial z}{\partial x}=-\frac{F_x(x,y,z)}{F_z(x,y,z)}$ and $\frac{\partial z}{\partial y}=-\frac{F_y(x,y,z)}{F_z(x,y,z)}$ provided $F_z(x,y,z)$ is not equal to 0.

Increments and Differentials

Let $z=f(x,y)$, and suppose Δx and Δy are increments. The increment in z is defined

$\Delta z=f(x+\Delta x,y+\Delta y)-f(x,y)$. The differentials $dx=\Delta x, dy=\Delta y$, and the total differential is $dz=f_x(x,y)dx+f_y(x,y)dy$.

Theorem 8.10: Total Differential

If $z=f(x,y)$ is differentiable, then the total differential dz is given by

$$\Delta z\approx dz=f_x(x,y)\Delta x+f_y(x,y)\Delta y$$

Definition 8.15: Directional Derivative

The directional derivative of $f(x,y)$ at (x_0,y_0) in the direction of *unit vector* $u=\langle a,b\rangle$ is

$$D_uf(x_0,y_0)=\lim_{h\to 0}\frac{f(x_0+ha,y_0+hb)-f(x_0,y_0)}{h}$$

Theorem 8.11: Computing Directional Derivatives

If f is differentiable, then f has a directional derivative in the direction of the unit vector $u=\langle a,b\rangle$ and

$$D_uf(x,y)=f_x(x,y)a+f_y(x,y)b=\nabla f(x,y)\cdot u$$

where $\nabla f(x,y)=\langle f_x,f_y\rangle$ is the gradient vector of f at (x,y).

Both equations for directional derivatives 8.15, 8.11 can be extrapolated for nth-D variables.

Theorem 8.13: Level Curve vs ∇f

Suppose $\nabla f(x_0,y_0)\neq\mathbf{0}$. Then $\nabla f(x_0,y_0)$ is perpendicular to the level curve $f(x,y)$ at (x_0,y_0) where $f(x_0,y_0)=k$

Theorem 8.14: Level Surface vs ∇f

Let $\boldsymbol{r}(t)$ be a parametric equation of curve C which lies on *level surface* S such that $\boldsymbol{r}(t_0)=(x_0,y_0,z_0)$. Suppose $\nabla f(x_0,y_0,z_0)\neq\mathbf{0}$. Then $\nabla f(x_0,y_0,z_0)$ is perpendicular to the level surface $f(x,y,z)$ at (x_0,y_0,z_0) where $f(x_0,y_0,z_0)=k$, ie. $\nabla f(x_0,y_0,z_0)\cdot\boldsymbol{r}'(t_0)=0$

Theorem 8.16: Maximizing Rate of Increase/Decrease f

Suppose f is a differentiable function of two or three variables. Let P denote a given point. Assuming that $\nabla f(P)\neq\mathbf{0}$, letting u be a unit vector making a angle θ with ∇f. Then

1. $D_uf(P)=\|\nabla f(P)\|\cos\theta$
2. $\nabla f(P)$ points in the direction of maximum rate of increase of f and P, where maximum value of $D_uf(P)$ is $\|\nabla f(P)\|$
3. $-\nabla f(P)$ points in the direction of maximum rate of decrease of f and P, where minimum value of $D_uf(P)$ is $-\|\nabla f(P)\|$

Extrema of multivariable functions

If f has a local maximum or minimum at (a,b) and the first-order partial derivatives of f exist there, then $f_x(a,b)=f_y(a,b)=0$.

Definition 8.20: Critical Point

A critical point of f is a point (a,b) in the domain of f such that $f_x(a,b)=f_y(a,b)=0$ or one of the partial derivatives does not exist.

Theorem 8.18: Second Derivative Test

Suppose $f(x,y)$ has continuous second-order partial derivatives on a disk that contains the point (a,b) and $f_x(a,b)=f_y(a,b)=0$.

$$\text{Discriminant, } D(a,b)=f_{xx}(a,b)f_{yy}(a,b)-\left[f_{xy}(a,b)\right]^2$$

1. D > 0 and $f_{xx}(a,b)>0$, then $f(a,b)$ is a local minimum.
2. D > 0 and $f_{xx}(a,b)<0$, then $f(a,b)$ is a local maximum.
3. D < 0, then $(a,b))$ is a saddle point.
4. D = 0, then the test is inconclusive.

9. Double Integrals

Theorem 9.2: Fubini's Theorem

If f is continuous on the rectangle $R=[a,b]\times[c,d]$, then

$$\iint_R f(x,y)dA=\int_a^b\int_c^d f(x,y)dydx=\int_c^d\int_a^b f(x,y)dxdy$$

Theorem 9.3: Special case of Fubini

If $f(x,y)$ can be factored into a product of two functions, one of x and one of y, then

$$\iint g(x)\cdot h(y)dA=\left(\int_a^b g(x)dx\right)\left(\int_c^d h(y)dy\right)$$

Double Integral over General Region

Type 1 Region is a plane region D lies between the graphs of two continuous functions, and a Type 2 Region is a plane region D lies between the graphs of two continuous functions of y.

Finding Area using double Integral

Theorem 9.10: Area of plane region

$$\text{Area of plane region D}=\iint_D 1dA$$

$$\text{Surface Area}=\iint_D dS=\iint_D\sqrt{(f_x^2+f_y^2+1)}dA$$

Theorem 9.12: Changing to Polar Coordinates

Polar coordinates: $r^2=x^2+y^2, x=r\cos\theta, y=r\sin\theta$.

A circle $x^2+y^2=\alpha^2$ is now described as $R=\{(r,\theta)\mid 0\le r\le\alpha, 0\le\theta\le 2\pi\}$ with

$$\iint_D f(x,y)dA=\int_0^\alpha\int_0^{2\pi}f(r\cos\theta,r\sin\theta)\cdot r\cdot d\theta dr$$

10. Ordinary Differential Equations

First Order Ordinary DEs

1. Separable ODEs:

$$\frac{\boldsymbol{dy}}{\boldsymbol{dx}}=\boldsymbol{f(x)\cdot g(y)}\Rightarrow\int\frac{1}{g(y)}dy=\int f(x)dx+C$$

Reduction to Separable ODEs can be done by substitution of $v=\frac{y}{x}$ or $u=ax+by$.

2. Linear ODEs:

$$\frac{\boldsymbol{dy}}{\boldsymbol{dx}}+\boldsymbol{P(x)y}=\boldsymbol{Q(x)}\Rightarrow y\cdot I(x)=\int Q(x)\cdot I(x)dx$$

Where integrating factor $I(x)=e^{\int P(x)dx}$.

Bernoulli Equation

$$\frac{\boldsymbol{dy}}{\boldsymbol{dx}}+\boldsymbol{P(x)y}=\boldsymbol{Q(x)\cdot y^n}\Rightarrow\text{use substitution } u=y^{1-n}$$

Appendix

Useful Trigonometric Identities:

$$\sec^2x-1\equiv\tan^2x$$
$$\csc^2x-1\equiv\cot^2x$$
$$\sin(A\pm B)\equiv\sin A\cos B\pm\cos A\sin B$$
$$\cos(A\pm B)\equiv\cos A\cos B\mp\sin A\sin B$$
$$\tan(A\pm B)\equiv\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$
$$\sin 2A\equiv 2\sin A\cos A$$
$$\cos 2A\equiv\cos^2A-\sin^2A\equiv 2\cos^2A-1\equiv 1-2\sin^2A$$
$$\tan 2A\equiv\frac{2\tan A}{1-\tan^2A}$$
$$\sin P+\sin Q\equiv 2\sin\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$$
$$\sin P-\sin Q\equiv 2\cos\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right)$$
$$\cos P+\cos Q\equiv 2\cos\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$$
$$\cos P-\cos Q\equiv -2\sin\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right)$$
$$\sin A\cos B\equiv\left(\frac{1}{2}\right)(\sin(A+B)+\sin(A-B))$$
$$\cos A\sin B\equiv\left(\frac{1}{2}\right)(\sin(A+B)-\sin(A-B))$$
$$\cos A\cos B\equiv\left(\frac{1}{2}\right)(\cos(A+B)+\cos(A-B))$$
$$\sin A\sin B\equiv\left(\frac{1}{2}\right)(\cos(A-B)-\cos(A+B))$$

More on Limits

1.3 Leading terms

$$\lim_{x\to\pm\infty}\frac{P(x)}{Q(x)}=\lim_{x\to\pm\infty}\frac{Ax^\alpha}{Bx^\beta}=\begin{cases}0,\alpha<\beta\\\frac{A}{B},\alpha=\beta\\\pm\infty,\alpha>\beta\end{cases}$$

1.6 Squeeze theorem

if $f(x)\le g(x)\le h(x)$ and $\lim_{x\to c}f(x)=\lim_{x\to c}h(x)=L$, then $\lim_{x\to c}g(x)=L$

1.7 Intermediate value theorem

if $f(x)$ is continous on $[a,b]$ and $f(a)<0<f(b)$, then there exists c in $[a,b]$ such that $f(c)=0$