

# Replication Report Rhemtulla et al 2012

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## Abstract

This documents the replication attempt of the simulation study reported in Rhemtulla, M., Brosseau-Liard, P. É., & Savalei, V. (2012). When can categorical variables be treated as continuous? A comparison of robust continuous and categorical SEM estimation methods under suboptimal conditions. *Psychological Methods*, 17(3), 354–373. <https://doi.org/10.1037/a0029315>. The study compared two different estimation methods (robust Maximum Likelihood (ML) and categorical least squares (cat-LS/ULSMV)) for fitting confirmatory factor analysis models in the context of categorical variables. Our replication involved writing simulation code based on the information provided in the manuscript and the corresponding supplemental material. Information provided in the original study was detailed and well structured, thus allowing us to reimplement the study to the best of our knowledge. Detailed result tables provided in the supplemental material allowed us to compare our replicated results to the original results.

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# 1 Introduction

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This replication report documents the replication attempt of the simulation study

Rhemtulla, M., Brosseau-Liard, P. É., & Savalei, V. (2012). When can categorical variables be treated as continuous? A comparison of robust continuous and categorical SEM estimation methods under suboptimal conditions. *Psychological Methods*, 17(3), 354–373. <https://doi.org/10.1037/a0029315>

Following the definition of Rougier et al. (2017) we understand the replication of a published study as writing and running new code based on the description provided in the original publication with the aim of obtaining the same results.

## 2 Method

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### 2.1 Information basis

The replication attempt was based on the information provided in the original manuscript as well as the supplemental material accompanying the publication. The main text provided a link to the supplements (<http://dx.doi.org/10.1037/a0029315.supp>) which referred to the website of the publisher where an additional pdf document with extensive result tables was freely available.

### 2.2 Data Generating Mechanism

The information provided indicated that the following simulation factors were systematically varied in a full-factorial design for generating the artificial data.

Simulation factor	No. levels	Levels
<i>Varied</i>		
CFA model size	2	10 indicators, 20 indicators
Underlying distribution	2	normal, non-normal
Number of categories	6	2,3,4,5,6,7
Threshold symmetry	5	symmetry, moderate asymmetry, moderate asymmetry alternative, extreme asymmetry
Sample Size	4	100, 150, 350, 600
<i>Fixed</i>		
factor loadings		0.3, 0.4, 0.5, 0.6, 0.7
factor correlation		0.3

This results in a total of 480 scenarios under which data is generated. Each of these conditions was simulated with 1000 repetitions.

Generating data consisted of two steps. (1) Data was generated based on the underlying distribution, CFA model and sample size. (2) The generated data was categorized based on the given category thresholds corresponding to a given number of categories and threshold symmetry.

### 2.2.1 CFA model

The CFA models underlying data generation were described as “Model 1 was a two-factor CFA model with five indicators per factor, for a total of 10 indicators. Factor loadings for the five indicators were .3, .4, .5, .6, .7. [...] The model was identified by fixing the variances of each latent variable to 1. Generated continuous variables had unit variance (prior to categorization). Model 2 was identical to Model 1, but with 10 indicators per factor.” (p.359) We translated this information into the following matrices:

$$\Lambda = \begin{bmatrix} 0.3 & 0 \\ 0.4 & 0 \\ 0.5 & 0 \\ 0.6 & 0 \\ 0.7 & 0 \\ 0 & 0.3 \\ 0 & 0.4 \\ 0 & 0.5 \\ 0 & 0.6 \\ 0 & 0.7 \end{bmatrix}$$

$$\Psi = \begin{bmatrix} 1 & 0.3 \\ 0.3 & 1 \end{bmatrix}$$

$$\Theta = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

We used these matrices as input for the `model()` function of the `simsem` package.

### 2.2.2 Underlying distribution, CFA model size and Sample Size

The original study indicated that data were generated using the Fleishman (1978) and Vale Maurelli (1983) method. We emulated this approach using the `generate()` function from the `simsem` package with the parameter `inDist` set to `NULL` in the normal case and to `simsem::bindDist(skewness = 2, kurtosis = 7)` in the non-normal case. The `model` parameter from the `generate()` was specified as detailed above. This constituted the first step of the data generation.

### 2.2.3 Number of categories and Threshold symmetry

After data was generated based on the given CFA model and the underlying distribution the resulting data was categorized into the number of categories for the scenario at hand. For each number of categories and each threshold symmetry, Z-scores for category thresholds could be obtained from the first table of the supplemental material. The sample covariance matrix of the resulting categorized data was tested for positive definiteness. In case it was found to be non-positive definite data was resampled with a different seed until it was. Additionally, it was ensured that none of the generated variables had zero variance. These measures are not documented in the original study. Hence, we do not know whether or at which point in the simulation pipeline these issues were dealt with.

## 2.3 Investigated Methods

The study compares the performance of robust normal theory maximum likelihood (ML) and robust categorical least squares (ULS) methodology for estimating confirmatory factor analysis (CFA) with ordinary variables. The underlying CFA model was fit using each of the two methods under investigation.

### 2.3.1 Robust normal theory maximum likelihood (ML)

CFA's were carried out using the `cfa()` function of the `lavaan` package. For the *Robust normal theory maximum likelihood* approach we set the `estimator` argument to `MLVM`.

### 2.3.2 Robust categorical least squares (ULS)

The *Robust categorical least squares (ULS)* approach was also implemented using the `cfa()` function from the `lavaan` package. In this case the `estimator` argument was set to `ULSMV`. Additionally, the `ordered` argument was set to `TRUE`.

<Describe how the second method is defined and implemented. You can include equations and or R code. If applicable, mention specialized R packages, their version as well as, parameters of specific functions.>

## 2.4 Performance measures

The models estimated using the two methods described above were compared on various performance measures.

### 2.4.1 Convergence Failures

The original article assessed the number of convergence failures. We implemented convergence failure via the `lavInspect()` function with the `what` argument set to "converged".

### 2.4.2 Improper solutions

The original study reports assessing the number of improper solutions. The paper defines improper solution as *"when cat-LS estimation produced a factor loading greater than 1 or continuous ML estimation produced a standardized factor loading greater than 1"* (p.361) We implemented convergence failure via the `lavInspect()` function with the `what` argument set to "post.check".

### 2.4.3 Parameter Estimates

We extracted parameter estimates from the fitted lavaan object using the `lavInspect()` function.

### 2.4.4 Parameter Bias

The parameter bias was calculated as the difference of the mean estimate per scenario and the true value.

### 2.4.5 Coverage

For each iteration of each scenario it was assessed whether the estimated parameter fell within 1.96 standard errors of the true value. We used robust standard errors from the estimated model for this assessment.

## 2.5 Power

In addition to the above mentioned analyses the study included a brief evaluation of the relative power of the ML-based and the ULS-based robust test statistics to detect a least major model misspecification. For this purpose the authors fit a *"one-factor model to the data generated by Model 1 (the 10-indicator, two factor model) for the subset of conditions in which the underlying distribution was normal and thresholds were symmetrically distributed."* (p.369). This subset corresponds to 60 of the 480 scenarios. We interpreted the above to indicate that the same generated data as for the rest of the simulation study was used. We hence filtered the generated data sets to only retain the scenarios including model 1, normally distributed variables and symmetrically distributed thresholds for categorization and fit a one-factor model to each of the data sets that fit these criteria.

A p-value < 0.05 of the robust  $\chi^2$  statistic was used to indicate a model misspecification.

## 2.6 Technical implementation

The original simulation study was carried out in EQS (Version 6.1) as well as Mplus (Version 6.11). The authors of the original study report that data generation was carried out in EQS and data analysis was conducted using both EQS as well as MPlus. However, only results from the Mplus analysis are reported. Our replication was implemented using the R programming environment (details regarding software versions can be obtained from the section Reproducibility Information). The corresponding R code can be obtained from <https://github.com/replisims/rhemtulla-2012>.

## 2.7 Replicator degrees of freedom

The following table provides an overview of replicator degrees of freedom, i.e. decisions that had to be made by the replicators because of insufficient or contradicting information. Issues were resolved by discussion among the replicators.

Issue	Replicator decision	Justification
Data basis fig 1&2, tab 1	Simulate just one variable	it seemed unlikely that dozens of variables from the models were collapsed
Factor loadings of Model 2	each factor loading is assumed to occur twice	most likely
Error handling	Case-wise deletion	Text indicated that “cases” were removed

### 2.7.1 Data basis for Figures 1 and 2

<More details on how the information provided was insufficient, unclear or vague> “Some weird quote from the original article that you could not make any sense of” (p.XY) The text indicated that the data underlying figures 1 and 2 as well as table 1 were generated for each “scenario” and a sample size of 1000000. We interpreted this to mean that one variable of length 1000000 was generated according to the specifications of each scenario although each scenario technically generated data according to an entire CFA model.

### 2.7.2 Factor loadings of model 2

The original article indicated that “Model 2 was identical to Model 1, but with 10 indicators per factor.”(p.359) No additional information regarding the factor loadings for these additional factor loadings was provided. We hence assumed that additional indicators reused the same set of factor loadings such that each loading occurred twice.

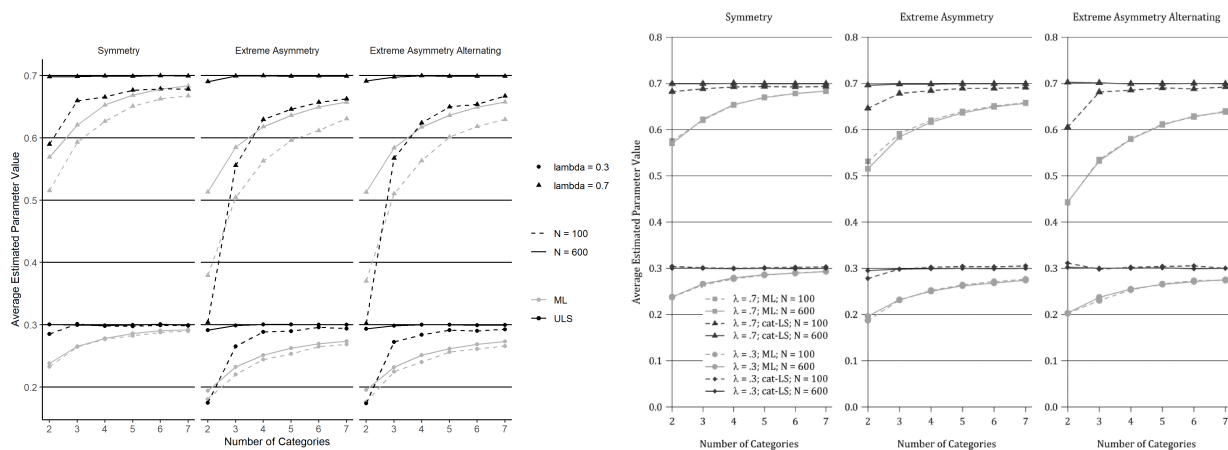


Figure 1: Parameter estimates (factor loadings, underlying distribution is normal). Values are averaged across model size and across all loadings for which the true parameter value was the same. Lines represent different estimators and different sample sizes (see legend). ML = robust continuous maximum likelihood estimation; cat-LS = robust categorical least squares estimation. The upper set of lines represents results for a true parameter value of .7. The lower set of lines represents results for a true parameter value of .3. Vertical panels represent different levels of threshold symmetry. Left figure: replication; right figure original study.

### 2.7.3 Error handling

<More details on how the information provided was insufficient, unclear or vague> “Some weird quote from the original article that you could not make any sense of” (p.XY)

## 3 Results

### 3.1 Replication of result figures

### 3.2 Simulation descriptives

<Describe the sampling distribution if any of the simulation parameters were sampled> The original study provides descriptives for the simulated data in two figures. Figure 1 and Figure 2 of the original manuscript

#### 3.2.1 Figure 3 and 4 Parameter estimates (factor loadings)

#### 3.2.2 Figure 5 Parameter estimates (factor correlation)

#### 3.2.3 Figure 6 and 7 Coverage (factor loadings)

#### 3.2.4 Figure 8 Coverage (factor correlations)

#### 3.2.5 Type I error rate

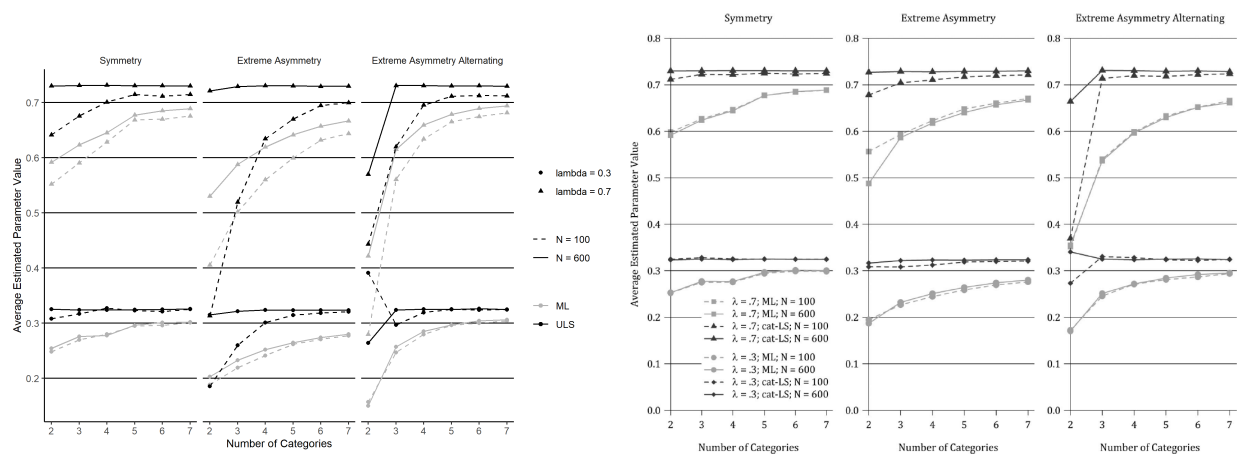


Figure 2: Parameter estimates (factor loadings, underlying distribution is nonnormal; skew 2, kurtosis 7). Values are averaged across model size and across all loadings for which the true parameter value was the same. Lines represent different estimators and different sample sizes (see legend). ML = robust continuous maximum likelihood estimation; cat-LS = robust categorical least squares estimation. The upper set of lines represents results for a true parameter value of .7. The lower set of lines represents results for a true parameter value of .3. Vertical panels represent different levels of threshold symmetry. Left figure: replication; right figure original study.

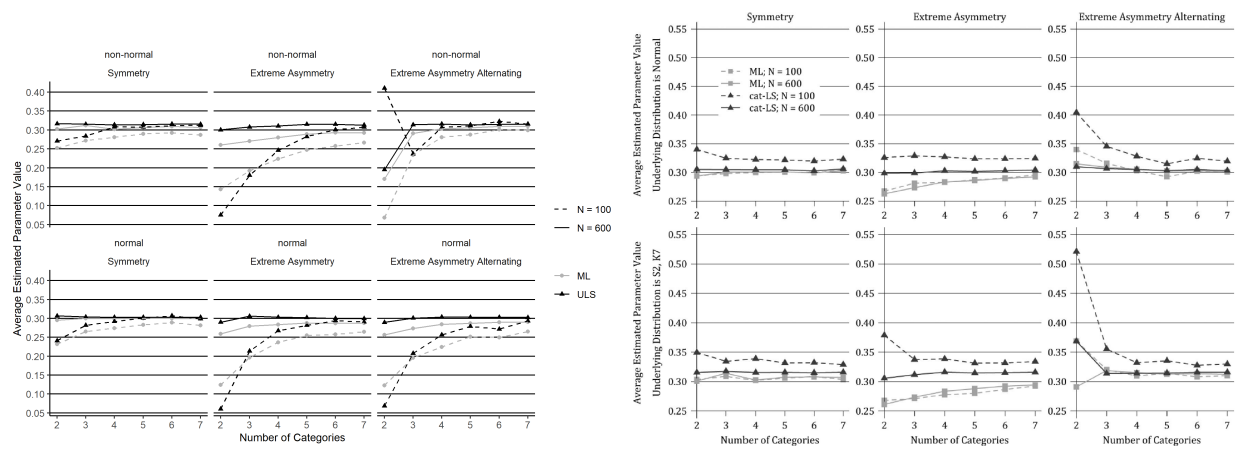


Figure 3: Parameter estimates (factor correlation, true value is .3). Values are averaged across model size. Lines represent different estimators and different sample sizes (see legend). ML = robust continuous maximum likelihood estimation; cat-LS = robust categorical least squares estimation. The upper panel corresponds to conditions in which the underlying distribution is normal; the lower panel corresponds to conditions in which the underlying distribution is nonnormal (skew 2, kurtosis 7). Vertical panels represent different levels of threshold symmetry. Left figure: replication; right figure original study.



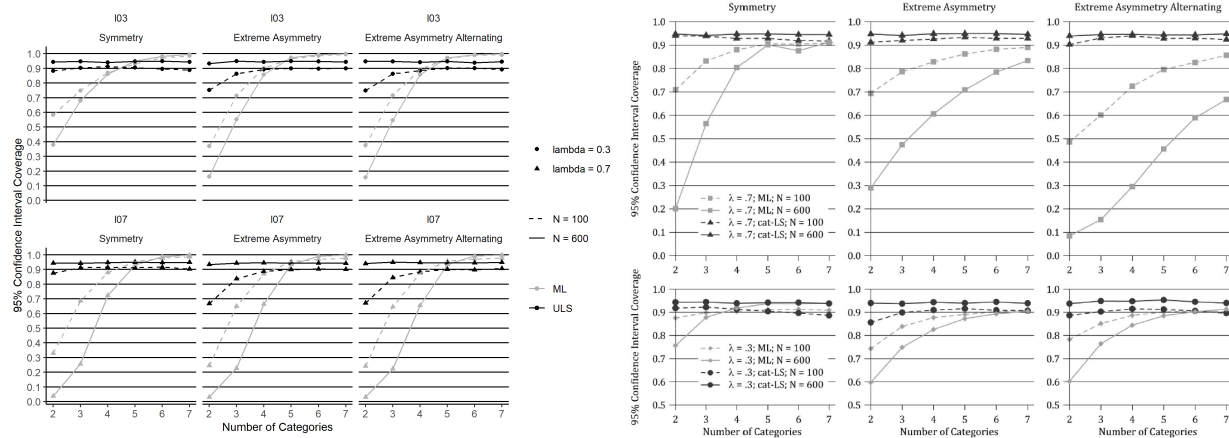


Figure 4: Coverage by number of categories (.7 and .3 factor loadings); underlying distribution is normal. Values are averaged across model size and across all loadings for which the true parameter value was the same. Lines represent different estimators and different sample sizes (see legend). ML = robust continuous maximum likelihood estimation; cat-LS = robust categorical least squares estimation. The upper panel represents results for a true parameter value of .7. The lower panel represents results for a true parameter value of .3. Vertical panels represent different levels of threshold symmetry. Left figure: replication; right figure original study.

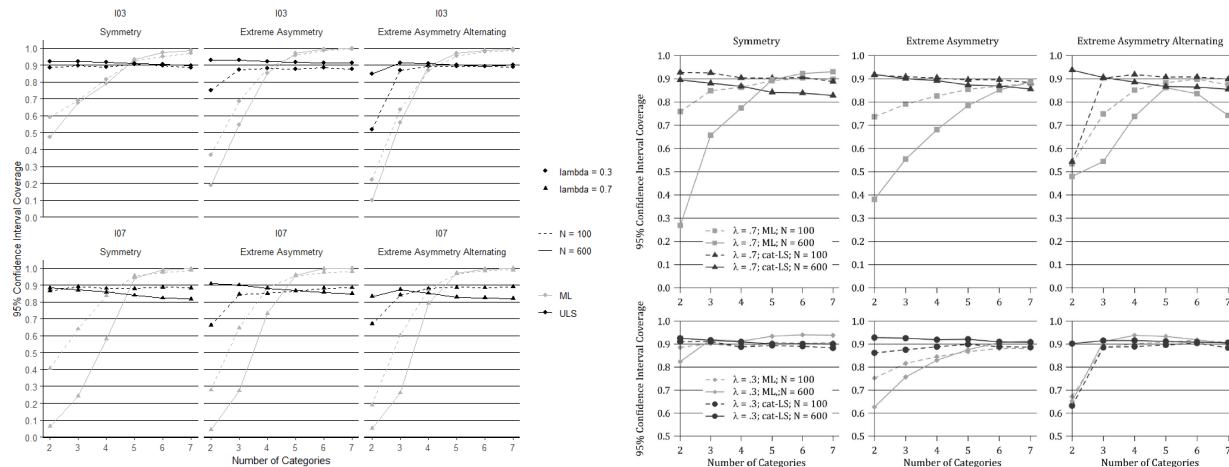


Figure 5: Coverage by number of categories (.7 and .3 factor loadings); underlying distribution is nonnormal (skew 2, kurtosis 7). Values are averaged across model size, and across all loadings for which the true parameter value was the same. Lines represent different estimators and different sample sizes (see legend). ML = robust continuous maximum likelihood estimation; cat-LS = robust categorical least squares estimation. The upper panel represents results for a true parameter value of .7. The lower panel represents results for a true parameter value of .3. Vertical panels represent different levels of threshold symmetry. Left figure: replication; right figure original study.

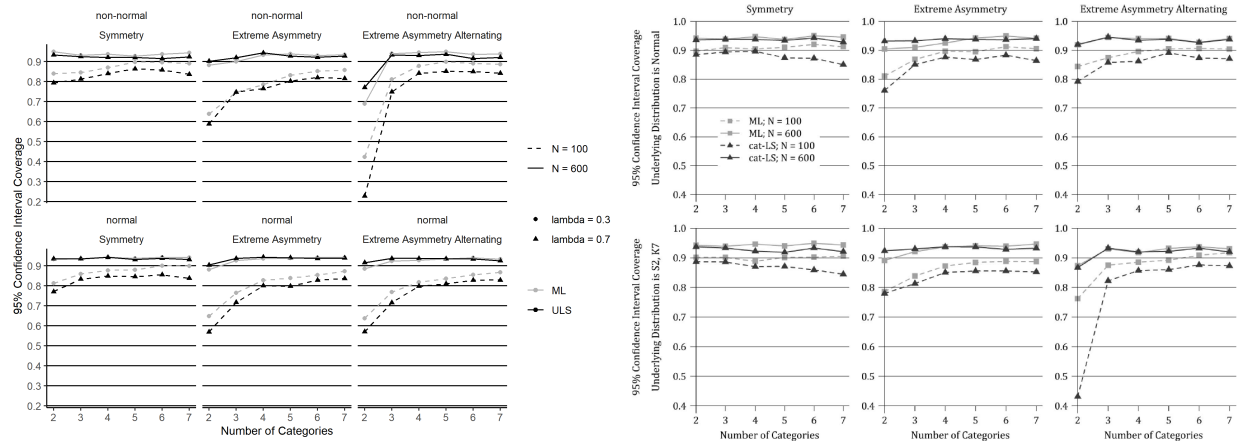


Figure 6: Coverage by number of categories (factor correlation). Values are averaged across model size. Lines represent different estimators and different sample sizes (see legend). ML = robust continuous maximum likelihood estimation; cat-LS = robust categorical least squares estimation. The upper panel corresponds to conditions in which the underlying distribution is normal; the lower panel corresponds to conditions in which the underlying distribution is nonnormal (skew 2, kurtosis 7). Vertical panels represent different levels of threshold symmetry. Left figure: replication; right figure original study.

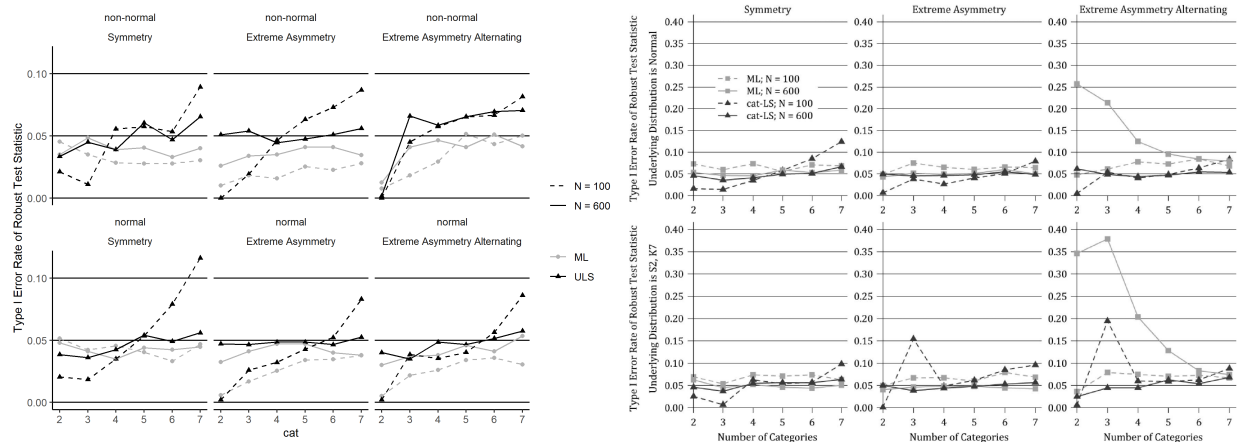


Figure 7: Type I error of mean-and-variance adjusted test statistic by number of categories. Values are averaged across model size. Lines represent different estimators and different sample sizes (see legend). ML = robust continuous maximum likelihood estimation; cat-LS = robust categorical least squares estimation. The upper panel corresponds to conditions in which the underlying distribution is normal; the lower panel corresponds to conditions in which the underlying distribution is nonnormal (skew 2, kurtosis 7). Vertical panels represent different levels of threshold symmetry.

### 3.3 Replication of result tables

<Compare any tabulated data to the original>

#### 3.3.1 Table 1

Table 1 presents the “*Skew and Kurtosis of Observed Categorical Variables by Threshold Distribution, Underlying Distribution, and Number of Categories*” (p.363). The “[v]alues in this table were obtained by generating samples of size  $N = 1,000,000$  for each condition and recording the skew and kurtosis of the observed distributions.” (p.363) As discussed above we understood “each condition” to only include underlying distribution, number of categories and threshold symmetry. We hence only simulated one variable of sample size 1,000,000 per condition in order to replicate figure 1, figure 2 as well as table 1.

#### 3.3.2 Observed Power (Table 2)

N	2 categories		3 categories		4 categories		5 categories		6 categories		7 categories	
	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS
100	0.398	0.408	0.602	0.667	0.713	0.806	0.769	0.890	0.809	0.927	0.849	0.955
150	0.654	0.702	0.840	0.889	0.936	0.960	0.971	0.988	0.976	0.990	0.979	0.993
350	0.994	0.996	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
600	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Note:

Type I error was assessed by fitting a one-factor model to two-factor simulated data. ML = robust continuous maximum likelihood estimation; ULS = robust categorical least squares estimation.



### 3.3.3 Number of nonconverged cases per 1000 replications

Distribution	Model	cats	Ext. Asym.						Ext. Asym.-Alt						Mod. Asym.						Mod. Asym.-Alt						Symmetric												
			N = 100		N = 150		N = 350		N = 600		N = 100		N = 150		N = 350		N = 600		N = 100		N = 150		N = 350		N = 600		N = 100		N = 150		N = 350		N = 600						
			ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS							
non-normal	1	2	159	51	70	32	3	1	0	0	405	3	337	3	118	9	14	7	33	9	6	1	0	0	0	0	2	0	0	0	0	0	0	0					
non-normal		3	40	30	9	3	0	0	0	0	21	10	3	1	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0						
non-normal		4	13	8	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	1	0	0	0	0	3	2	0	0	0	0	0	0						
non-normal		5	5	2	1	0	0	0	0	0	2	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0						
non-normal		6	4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	1	0	0						
non-normal		7	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0						
non-normal	2	2	8	0	0	0	0	0	0	0	149	0	79	0	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0						
non-normal		3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0						
non-normal		4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0						
non-normal		5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0						
non-normal		6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0						
non-normal		7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0						
normal	1	2	193	88	72	39	3	1	0	165	74	76	49	1	1	0	0	33	10	5	2	0	0	0	0	24	8	8	1	0	0	0	23	6	5	1	0	0	0
normal		3	32	15	3	1	0	0	0	40	24	3	3	0	0	0	0	1	0	2	2	0	0	0	0	0	0	0	0	0	0	4	2	0	0	0	0		
normal		4	11	1	1	1	0	0	0	12	4	2	0	0	0	0	0	4	0	0	0	0	0	0	0	3	0	0	0	0	0	1	0	0	0	0			
normal		5	6	4	1	0	0	0	0	2	0	1	0	0	0	0	0	3	0	0	0	0	0	0	0	2	1	0	0	0	0	1	0	0	0				
normal		6	1	1	0	1	0	0	0	2	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0				
normal		7	0	0	0	0	0	0	0	3	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0			
normal	2	2	16	2	0	0	0	0	0	11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0				
normal		3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0				
normal		4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0				
normal		5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0				
normal		6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0				
normal		7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0				

Note:  
 Mod Asym = Moderate Asymmetry;  
 Mod Asym-Alt = Moderate Asymmetry-Alternating;  
 Ext Asym = Extreme Asymmetry;  
 Ext Asym-Alt = Extreme Asymmetry-Alternating;  
 ML = robust normal-theory maximum likelihood;  
 ULS = robust categorical least squares.



### 3.3.4 Number of improper solutions per 1000 replications

Distribution	Model	Ext. Asym.												Ext. Asym.-Alt												Mod. Asym.												Mod. Asym.-Alt												Symmetric																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																				
		N = 100		N = 150		N = 350		N = 600		N = 100		N = 150		N = 350		N = 600		N = 100		N = 150		N = 350		N = 600		N = 100		N = 150		N = 350		N = 600		N = 100		N = 150		N = 350		N = 600																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																														
		ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML

Note:

Mod Asym = Moderate Asymmetry;

Mod Asym-Alt = Moderate Asymmetry-Alternating;

Ext. Asym = Extreme Asymmetry;

Ext. Asym-Alt = Extreme Asymmetry-Alternating;

ML = robust normal-theory maximum likelihood;

ULS = robust categorical least squares.



### 3.3.5 Parameter Bias, Model1, Underlying Distribution = Normal

param.	Ext. Asym.												Mod. Asym.												Symmetric																																																																																																																																																																																																																																																																																																																																																																																																																																																																																															
	N = 100				N = 350				N = 600				N = 100				N = 350				N = 600				N = 100				N = 350				N = 600																																																																																																																																																																																																																																																																																																																																																																																																																																																																																							
	cats	ML	U.S.	ML	U.S.	ML	U.S.	ML	U.S.	ML	U.S.	ML	U.S.	ML	U.S.	ML	U.S.	ML	U.S.	ML	U.S.	ML	U.S.	ML	U.S.	ML	U.S.	ML	U.S.	ML	U.S.	ML	U.S.																																																																																																																																																																																																																																																																																																																																																																																																																																																																																							
lambda = 3	3	-0.071	-0.015	-0.072	-0.010	-0.072	-0.006	-0.068	-0.002	-0.066	-0.007	-0.075	-0.014	-0.070	-0.005	-0.070	-0.003	-0.034	0.000	-0.034	-0.000	-0.032	-0.002	-0.032	0.000	-0.032	-0.000	-0.037	-0.004	-0.035	-0.002	-0.032	0.001	-0.032	-0.001	-0.039	-0.001	-0.035	0.001	-0.035	0.000	-0.034	0.000	0.000																																																																																																																																																																																																																																																																																																																																																																																																																																																																												
	4	-0.060	-0.003	-0.049	-0.003	-0.052	-0.003	-0.047	0.003	-0.048	-0.010	-0.056	-0.007	-0.051	-0.002	-0.048	0.001	-0.032	-0.028	0.002	-0.028	0.002	-0.028	0.002	-0.028	0.000	-0.029	-0.001	-0.027	-0.000	-0.024	-0.002	-0.024	-0.001	-0.024	-0.001	-0.029	-0.001	-0.035	0.001	-0.035	0.000	-0.034	0.000	0.000																																																																																																																																																																																																																																																																																																																																																																																																																																																																											
	5	-0.047	-0.011	-0.035	-0.003	-0.039	-0.001	-0.037	0.001	-0.037	0.001	-0.041	-0.003	-0.039	-0.001	-0.042	-0.001	-0.023	-0.001	-0.023	-0.000	-0.023	-0.002	-0.017	0.003	-0.021	-0.001	-0.021	-0.002	-0.021	-0.002	-0.019	0.000	-0.019	0.000	-0.024	-0.002	-0.024	-0.001	-0.029	-0.001	-0.035	0.001	-0.035	0.000	-0.034	0.000	0.000																																																																																																																																																																																																																																																																																																																																																																																																																																																																								
	6	-0.047	-0.011	-0.035	-0.003	-0.039	-0.001	-0.037	0.001	-0.041	-0.003	-0.039	-0.002	-0.041	-0.002	-0.039	-0.001	-0.023	-0.001	-0.023	-0.000	-0.023	-0.002	-0.017	0.003	-0.021	-0.001	-0.021	-0.002	-0.021	-0.002	-0.019	0.000	-0.019	0.000	-0.024	-0.002	-0.024	-0.001	-0.029	-0.001	-0.035	0.001	-0.035	0.000	-0.034	0.000	0.000																																																																																																																																																																																																																																																																																																																																																																																																																																																																								
	7	-0.030	-0.006	-0.032	-0.000	-0.031	0.000	-0.031	-0.001	-0.042	-0.013	-0.032	-0.002	-0.033	-0.002	-0.030	0.000	-0.020	-0.003	-0.018	0.000	-0.015	0.002	-0.017	0.002	-0.017	-0.001	-0.020	-0.003	-0.018	0.001	-0.016	0.000	-0.008	0.004	-0.004	-0.001	-0.010	-0.001	-0.009	0.000	-0.008	0.000	0.000	0.000	0.000																																																																																																																																																																																																																																																																																																																																																																																																																																																																										
	8	-0.030	-0.006	-0.032	-0.000	-0.031	0.000	-0.031	-0.001	-0.042	-0.013	-0.032	-0.002	-0.033	-0.002	-0.030	0.000	-0.020	-0.003	-0.018	0.000	-0.015	0.002	-0.017	0.002	-0.017	-0.001	-0.020	-0.003	-0.018	0.001	-0.016	0.000	-0.008	0.004	-0.004	-0.001	-0.010	-0.001	-0.009	0.000	-0.008	0.000	0.000	0.000	0.000																																																																																																																																																																																																																																																																																																																																																																																																																																																																										
lambda = 4	2	-0.040	-0.021	-0.049	-0.021	-0.055	-0.047	-0.038	-0.031	-0.101	-0.062	-0.086	-0.041	-0.101	-0.062	-0.086	-0.041	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036	0.000	-0.036





### 3.3.6 Parameter Bias, Model1, Underlying Distribution = Skew2, Kurtosis 7

param.	Ext. Asym.												Ext. Asym.-Alt												Mod. Asym.												Mod. Asym.-Alt												Symmetric											
	N = 100		N = 150		N = 350		N = 600		N = 100		N = 150		N = 350		N = 600		N = 100		N = 150		N = 350		N = 600		N = 100		N = 150		N = 350		N = 600		N = 100		N = 150		N = 350		N = 600																					
	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS	ML	ULS																				
lambda = 3	2	-0.086	-0.014	-0.098	-0.012	-0.104	0.001	-0.097	0.016	-0.078	-0.054	0.146	-0.138	0.021	-0.153	-0.019	-0.057	0.015	-0.062	0.017	-0.061	0.023	-0.060	0.025	-0.042	0.021	-0.043	0.025	-0.044	0.024	-0.041	0.027	-0.044	0.026	-0.050	0.020	-0.046	0.026	-0.046	0.025																				
	3	-0.069	-0.002	-0.069	0.010	-0.068	0.021	-0.067	0.022	-0.060	0.011	-0.045	0.022	-0.046	0.020	-0.043	0.025	-0.013	0.024	-0.009	0.027	-0.011	0.025	-0.016	0.024	-0.012	-0.012	-0.025	-0.033	0.020	-0.027	0.024	-0.026	0.024	-0.026	0.021	-0.026	0.024	-0.026	0.021																				
	4	-0.065	0.012	-0.061	0.018	-0.061	0.021	-0.047	0.024	-0.018	0.018	0.023	-0.017	0.024	-0.017	0.023	-0.014	0.027	-0.020	0.020	-0.014	0.026	-0.015	0.024	-0.009	0.030	-0.011	0.028	-0.010	0.026	-0.010	0.025	-0.017	0.031	-0.021	0.027	-0.022	0.025	-0.022	0.025																				
	5	-0.037	0.017	-0.038	0.019	-0.037	0.022	-0.036	0.022	-0.003	0.027	-0.005	0.024	-0.006	0.022	-0.003	0.026	-0.012	0.022	-0.014	0.022	-0.007	0.024	-0.007	0.025	-0.002	0.028	-0.003	0.026	-0.004	0.024	-0.005	0.023	0.000	0.027	0.002	0.027	0.001	0.025																					
	6	-0.029	0.021	-0.028	0.022	-0.025	0.024	-0.028	0.021	0.001	0.027	0.001	0.026	0.005	0.028	0.005	0.027	-0.024	0.021	-0.025	0.020	-0.019	0.024	-0.019	0.025	0.003	0.026	0.006	0.028	0.004	0.024	0.005	0.025	-0.003	0.023	0.000	0.027	0.002	0.027	0.001	0.025																			
	7	-0.020	0.024	-0.022	0.021	-0.022	0.021	-0.021	0.023	0.008	0.029	0.006	0.027	0.008	0.027	0.006	0.026	-0.019	0.019	-0.010	0.027	-0.012	0.024	-0.010	0.025	0.013	0.026	0.008	0.028	0.010	0.025	0.006	0.029	0.001	0.026	0.003	0.027	0.002	0.026																					
	2	-0.098	-0.052	-0.076	-0.009	-0.038	0.088	-0.022	0.118	-0.180	0.199	-0.164	0.134	-0.138	0.047	-0.113	0.038	-0.026	0.041	-0.007	0.065	0.022	0.128	0.021	0.127	0.010	0.064	0.025	0.110	0.042	0.131	0.040	0.129	0.028	0.087	0.050	0.116	0.063	0.127	0.064	0.128																			
lambda = 4	3	-0.041	0.025	-0.021	0.063	0.010	0.118	0.014	0.122	-0.009	0.046	0.020	0.092	0.045	0.129	0.046	0.130	0.062	0.108	0.076	0.124	0.081	0.127	0.084	0.130	0.060	0.113	0.072	0.123	0.079	0.128	0.080	0.129	0.083	0.129	0.084	0.123	0.089	0.128	0.072	0.131																			
	4	-0.006	0.059	0.017	0.099	0.040	0.127	0.039	0.127	0.058	0.108	0.071	0.122	0.079	0.130	0.079	0.129	0.059	0.105	0.074	0.122	0.081	0.130	0.081	0.129	0.063	0.110	0.077	0.128	0.082	0.129	0.083	0.129	0.049	0.103	0.064	0.123	0.069	0.128																					
	5	0.032	0.088	0.039	0.106	0.067	0.128	0.056	0.127	0.080	0.116	0.083	0.122	0.091	0.127	0.093	0.130	0.073	0.119	0.082	0.125	0.088	0.129	0.088	0.128	0.081	0.118	0.088	0.125	0.083	0.131	0.091	0.127	0.079	0.116	0.093	0.129	0.092	0.128																					
	6	0.038	0.086	0.080	0.119	0.068	0.127	0.066	0.125	0.089	0.119	0.088	0.123	0.101	0.129	0.100	0.128	0.049	0.114	0.081	0.124	0.078	0.132	0.074	0.128	0.092	0.122	0.096	0.128	0.101	0.128	0.102	0.128	0.094	0.114	0.091	0.122	0.099	0.129																					
	7	0.061	0.107	0.065	0.120	0.074	0.127	0.078	0.130	0.097	0.123	0.096	0.123	0.103	0.127	0.103	0.128	0.065	0.118	0.079	0.123	0.086	0.128	0.088	0.130	0.098	0.118	0.107	0.131	0.107	0.128	0.109	0.129	0.079	0.113	0.093	0.125	0.099	0.130																					
	2	-0.239	-0.197	-0.214	-0.146	-0.165	-0.022	-0.144	0.016	-0.360	0.036	-0.298	-0.011	-0.277	-0.078	-0.255	-0.088	-0.159	-0.083	-0.132	-0.026	-0.100	0.024	-0.064	0.031	-0.114	-0.033	-0.095	0.005	-0.075	0.030	-0.075	0.030	-0.120	-0.026	-0.101	-0.003	-0.080	0.027	-0.077																				
	3	-0.173	-0.109	-0.137	-0.044	-0.102	0.019	-0.096	0.029	-0.132	-0.069	-0.097	-0.015	-0.068	0.029	-0.064	0.032	-0.043	0.033	-0.049	0.010	-0.040	0.022	-0.030	0.031	-0.031	0.029	-0.093	-0.018	-0.060	0.020	-0.048	0.033	-0.048	0.033	-0.048	0.033	-0.048	0.030																					
lambda = 5	4	-0.128	-0.053	-0.098	-0.003	-0.072	0.027	-0.068	0.029	-0.053	0.006	-0.034	0.027	-0.030	0.028	-0.028	0.031	-0.060	-0.004	-0.037	0.021	-0.027	0.029	-0.026	0.030	-0.045	0.012	-0.035	0.025	-0.024	0.034	-0.025	0.032	-0.065	0.022	-0.046	0.025	-0.039	0.030																					
	5	-0.085	-0.014	-0.071	0.001	-0.049	0.032	-0.048	0.032	-0.032	0.009	-0.020	0.023	-0.012	0.029	-0.012	0.038	-0.034	0.019	-0.024	0.028	-0.014	0.033	-0.026	0.030	-0.031	0.015	-0.014	0.033	-0.013	0.032	-0.014	0.030	-0.033	0.012	-0.015	0.028	-0.014	0.029																					
	6	-0.074	-0.011	-0.051	0.019	-0.038	0.027	-0.036	0.029	-0.015	0.023	-0.008	0.028	-0.004	0.031	-0.002	0.031	-0.068	0.002	-0.045	0.025	-0.031	0.031	-0.029	0.032	-0.016	0.017	-0.004	0.031	-0.002	0.031	-0.001	0.032	-0.022	0.015	-0.008	0.029	-0.005	0.032																					
	7	-0.068	-0.008	-0.035	0.023	-0.028	0.031	-0.027	0.031	-0.018	0.012	-0.003	0.026	0.001	0.029	0.002	0.031	-0.057	0.012	-0.031	0.018	-0.018	0.030	-0.017	0.029	-0.008	0.017	0.001	0.029	0.007	0.032	0.006	0.031	-0.019	0.018	-0.004	0.033	-0.004	0.030																					
	2	-0.285	-0.254	-0.245	-0.183	-0.186	-0.033	-0.160	0.018	-0.452	-0.044	-0.379	-0.080	-0.313	-0.122	-0.277	-0.104	-0.184	-0.107	-0.144	-0.030	-0.112	0.025	-0.113	0.026	-0.139	-0.054	-0.109	0.004	-0.089	0.031	-0.091	0.029	-0.148	-0.047	-0.122	-0.014	-0.094	0.028	-0.094																				
	3	-0.166	-0.138	-0.152	-0.056	-0.112	0.018	-0.104	0.029	-0.161	-0.094	-0.115	-0.027	-0.081	0.027	-0.079	0.027	-0.057	-0.002	-0.042	0.022	-0.034	0.031	-0.034	0.030	-0.066	-0.001	-0.047	0.025	-0.044	0.027	-0.040	0.030	-0.113	-0.029	-0.075	-0.010	-0.054	0.023	-0.049																				
	4	-0.148	-0.073	-0.112	-0.018	-0.078	0.026	-0.077	0.029	-0.069	-0.006	-0.046	0.018	-0.036	0.029	-0.034	0.031	-0.067	-0.011	-0.040	0.019	-0.028	0.031	-0.028	0.032	-0.058	0.004	-0.045	0.023	-0.037	0.030	-0.037	0.031	-0.075	-0.010	-0.054	0.023	-0.049	0.029	-0.047																				
lambda = 6	5	-0.096	-0.029	-0.083	0.000	-0.056	0.029	-0.057	0.029	-0.037	0.012	-0.020	0.028	-0.018	0.030	-0.017	0.038	-0.046	0.010	-0.030	0.024	-0.020	0.031	-0.022	0.030	-0.041	0.012	-0.025	0.028	-0.023	0.028	-0.020	0.033	-0.040	0.004	-0.019	0.029	-0.019	0.031	-0.019	0.029																			
	6	-0.074	-0.010	-0.050	0.020	-0.040	0.027	-0.040	0.029	-0.022	0.018	-0.012	0.026	-0.008	0.030	-0.007	0.030	-0.074	0.002	-0.048	0.024	-0.039	0.027	-0.037	0.030	-0.028	0.012	-0.012	0.027	-0.008	0.029	-0.010	0.028	-0.020	0.015	-0.012	0.027	-0.009	0.032																					
	7	-0.068	-0.006	-0.043	0.019	-0.033	0.027	-0.030	0.030	-0.017	0.014	-0.005	0.027	-0.004	0.029	-0.002	0.030	-0.058	0.008	-0.028	0.024	-0.018	0.031	-0.020	0.030	-0.009	0.017	-0.005	0.025	0.004	0.033	0.003	0.031	-0.023	0.014	-0.013	0.024	-0.008	0.031																					
	2	-0.333	-0.316	-0.283	-0.234	-0.200	-0.045	-0.173	0.016	-0.483	-0.122	-0.416	-0.146	-0.359	-0.174	-0.320	-0.147	-0.215	-0.151	-0.177	-0.089	-0.133	-0.015	-0.104	0.029	-0.06	0.029	-0.171	-0.089	-0.133	-0.015	-0.104	0.029	-0.06	0.029	-0.171	-0.089	-0.133	-0.015	-0.104	0.029																			
	3	-0.229	-0.185	-0.175	-0.088	-0.119	0.019	-0.112	0.030	-0.173	-0.116	-0.133	-0.049	-0.090	0.025	-0.084	0.032	-0.069	-0.013	-0.051	0.016	-0.040	0.031	-0.042	0.031	-0.080	-0.012	-0.062	0.016	-0.049	0.031	-0.049	0.031	-0.132	-0.051	-0.094	0.004	-0.074	0.033	-0.075																				
	4	-0.180	-0.108	-0.117	-0.026	-0.083	0.027	-0.080	0.032	-0.081	-0.025	-0.055	0.011	-0.041	0.029	-0.041	0.038	-0.077	-0.027	-0.048	0.012	-0.035	0.029	-0.035	0.030	-0.074	-0.012	-0.055	0.016	-0.045	0.031	-0.045	0.032	-0.089	-0.026	-0.082	0.018	-0																						

### 3.4 Replication of results presented in text form

While the vast majority of results is presented in the form of figures, a few outcomes regarding outliers, relative bias of parameter estimates as well as relative bias of robust standard errors are only communicated in text form. <If the text describes any results using words describe how that relates to your findings.>

#### 3.4.1 Outliers

The original study reports the frequency of outliers in the text. There was one outlier in the original study. In our replication we found ...

#### 3.4.2 Relative bias

Figures and tables report absolute bias. Results pertaining to relative bias are only summarized in a more qualitative manner in text form. *"As the number of categories increases, ML estimates gradually become less biased and by five categories relative bias is always less than 10%."*(p.362) *"When the underlying distribution is non-normal, all cat-LS parameter estimates take on a slightly positive bias (around 4%), except when there are just two categories."* (p.364) *"[B]ias is almost never greater than 5% with either method."*

#### 3.4.3 Relative bias for robust standard error estimates

"ML standard errors are from 8% to 30% (average = 15%) smaller than empirical standard errors when the sample size is small, and cat-LS standard errors are from 3% to 37% (average 13%) smaller than empirical standard errors when the sample size is small." "Cat-LS produces better robust standard errors for factor loadings, and ML produces better robust standard errors for factor correlations. This finding is consistent across number of categories.

## 4 Discussion

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### 4.1 Replicability

Due to the high amount of details in the original publication and the corresponding supplemental materials the replication was straight forward. The largest amount of time was spent ensuring that the methods used for data generation and analysis did indeed correspond to what was used in the original study. This is, however, in no way the fault of the authors but rather due to limited documentation of the R packages used for replication. On the contrary the detailed description of the implementation allowed for a close correspondence of methodology which would have otherwise been left to guesswork.

A feature that deserves special praise with regards to facilitating replicability is the high amount of documentation that the authors dedicated to the generation of the simulated data as well as the descriptives of the same. The ability to closely monitor the data generation process and compare features of the simulated

data to the original study instilled a great deal of confidence in the replicators and ensured that any potential deviations of results could not be attributed to faulty interpretation and implementation of the data generating mechanism.

Another feature that increased reproducibility was the structure of the manuscript. The very first element of the method section was an overview of the simulation factors. Readability was increased by listing each factor as a separate bullet point. Subsequent sections detailed the implementation of each simulation factor. A separate subheading for each simulation factor made it easy to locate relevant information.

The detailed description of error handling procedures as well as error descriptives ...

The large number of result tables presented in the supplemental material is another exemplary reporting practice worth highlighting. While the comparison of hundreds of table cells is not an easy endeavor and the general interest in these tables likely limited it protects the authors against any allegations of selective reporting and makes the assessment of replicability possible.

A similar structure could be found for the performance measures which were discussed in separate subsections separated by corresponding heading. While very readable as is, we would have however preferred the performance measures to be elaborated on as part of the method section instead of the result section.

The introduction section included the presentation and discussion of several closely related methods as well as findings from previous studies investigating the same. Due to the large amount of information surrounding highly similar methods and their implementation it took us several readings of the introduction to feel confident about having identified the version actually implemented in the study at hand. A clearer separation of the implemented methods (e.g. in a box) would have facilitated isolating the relevant implementation details.

Finally, a major factor facilitating the reproduction process was the availability of specialized SEM software in the R programming environment. As R is frequently used for simulation studies investigating SEM methodology we were able to build upon a code base that was designed for this very purpose. While such specialized software has the potential of huge time savings on the coding end and additionally is likely to minimize coding errors on the part of the replicator it consumes a significant amount of time to familiarize oneself with the exact parameters underlying the tools. The inexperienced user is at the mercy of the package documentation and the occasional peek under the hood of a given function. Having a code base from related simulation studies available would increase confidence in using such tools and avoid some trial and error while familiarizing oneself with the functionalities.

<Provide a general statement of how you experienced the replication process. Was it easy? What made it easy or difficult?>

## 4.2 Replicator degrees of freedom

<Here you can discuss the replicator degrees of freedom. What could the authors have done to make it more clear? Do you think the replicator degrees of freedom are so extensive that they could influence the results?>

## 4.3 Equivalence of results

<How would you judge the overall equivalence of results? Are the orders of magnitude comparable? Are trends in the same direction? Would you draw the same conclusions as the authors based on your replication? Were some results not comparable because of insufficient figure resolution or labeling? Did the authors omit some results which consequently cannot be compared?>

# 5 Acknowledgments

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<Acknowledge the help of anyone who assisted you in the process>

# 6 Contributions

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Authors made the following contributions according to the CRediT framework <https://casrai.org/credit/>  
Anna Lohmann:

- Data Curation
- Formal Analysis (lead)
- Investigation
- Software
- Visualization (lead)
- Writing - Original Draft Preparation
- Writing - Review & Editing

Arjan Huizing:

- Formal Analysis (supporting)
- Investigation
- Software (supporting)
- Visualization (supporting)
- Validation
- Writing - Review & Editing

## References

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- 10 Rougier, Nicolas P., Konrad Hinsén, Frédéric Alexandre, Thomas Arildsen, Lorena A. Barba, Fabien C. Y. Benureau, C. Titus Brown, et al. 2017. "Sustainable Computational Science: The ReScience Initiative." *PeerJ Computer Science* 3 (December): e142. <https://doi.org/10.7717/peerj-cs.142>.

# Appendix

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## Additional results

<insert additional results not reported in the original article or results presented in an alternative way>

### 6.1 Code organization

The code and the files associated are organized in the form of a research compendium which can be found in the following git repository <https://github.com/replisims/rhentulla-2012>

```
## .
## +-- defs.tex
## +-- figures
## |   +-- fig3.png
## |   +-- fig3_original.png
## |   +-- fig4.png
## |   +-- fig4_original.png
## |   +-- fig5.png
## |   +-- fig5_original.png
## |   +-- fig6.png
## |   +-- fig6_original.png
## |   +-- fig7.png
## |   +-- fig7_original.png
## |   +-- fig8
## |   +-- fig8_original.png
## |   +-- fig9.png
## |   +-- fig9_original.png
## |   +-- fig_3.png
## |   +-- fig_4.png
## |   +-- fig_5.png
## |   +-- fig_6.png
## |   +-- fig_8.png
## |   +-- fig_9.png
## |   +-- tabA2_A3.png
## |   +-- tabA4_A5.png
## |   +-- tabA6.png
## |   +-- tabA7.png
## |   +-- table2.png
## |   +-- tableA10.html
## |   +-- tableA2_A3.html
## |   +-- tableA4_A5.html
## |   +-- tableA6.html
## |   +-- tableA7.html
## |   +-- tableA8.html
## |   \-- tableA9.html
## +-- flowchart.PNG
## +-- Lato-Black.ttf
## +-- Lato-BlackItalic.ttf
## +-- Lato-Bold.ttf
## +-- Lato-BoldItalic.ttf
```

```
## +-- Lato-Italic.ttf
## +-- Lato-Regular.ttf
## +-- references.bib
## +-- Replication Report Rhemthulla et al 2012.Rmd
## +-- Replication Report Rhemthulla et al 2012.Rmd.bak
## +-- Replication-Report-Rhemthulla-et-al-2012.log
## +-- Replication-Report-Rhemthulla-et-al-2012.pdf
## +-- Replication-Report-Rhemthulla-et-al-2012.Rmd
## +-- Replication-Report-Rhemthulla-et-al-2012.tex
## +-- UbuntuMono-Bold.ttf
## +-- UbuntuMono-BoldItalic.ttf
## +-- UbuntuMono-Italic.ttf
## \-- UbuntuMono-Regular.ttf
```

- foldername: contains <insert description>
- filename: contains <insert description>
- ...

## Reproducibility Information

This report was last updated on 2022-05-24 20:34:23. The simulation replication was conducted using the following computational environment and dependencies:

```
## - Session info -----
## setting value
## version R version 4.1.3 (2022-03-10)
## os Windows 10 x64 (build 19043)
## system x86_64, mingw32
## ui RTerm
## language (EN)
## collate English_United States.1252
## ctype English_United States.1252
## tz Europe/Berlin
## date 2022-05-24
## pandoc 2.17.1.1 @ C:/Program Files/RStudio/bin/quarto/bin/ (via rmarkdown)
##
## - Packages -----
## package * version date (UTC) lib source
## assertthat 0.2.1 2019-03-21 [1] CRAN (R 4.1.2)
## cachem 1.0.6 2021-08-19 [1] CRAN (R 4.1.2)
## callr 3.7.0 2021-04-20 [1] CRAN (R 4.1.2)
## cli 3.1.0 2021-10-27 [1] CRAN (R 4.1.2)
## crayon 1.5.1 2022-03-26 [1] CRAN (R 4.1.3)
## DBI 1.1.2 2021-12-20 [1] CRAN (R 4.1.2)
## desc 1.4.1 2022-03-06 [1] CRAN (R 4.1.3)
## devtools 2.4.3 2021-11-30 [1] CRAN (R 4.1.2)
## digest 0.6.29 2021-12-01 [1] CRAN (R 4.1.2)
## dplyr * 1.0.8 2022-02-08 [1] CRAN (R 4.1.2)
## ellipsis 0.3.2 2021-04-29 [1] CRAN (R 4.1.2)
## evaluate 0.15 2022-02-18 [1] CRAN (R 4.1.3)
## fansi 1.0.3 2022-03-24 [1] CRAN (R 4.1.3)
## fastmap 1.1.0 2021-01-25 [1] CRAN (R 4.1.2)
## fs 1.5.2 2021-12-08 [1] CRAN (R 4.1.2)
```



```
## generics      0.1.2      2022-01-31 [1] CRAN (R 4.1.2)
## glue          1.6.2      2022-02-24 [1] CRAN (R 4.1.2)
## htmltools     0.5.2      2021-08-25 [1] CRAN (R 4.1.2)
## knitr          * 1.38      2022-03-25 [1] CRAN (R 4.1.3)
## lifecycle     1.0.1      2021-09-24 [1] CRAN (R 4.1.2)
## magrittr      2.0.2      2022-01-26 [1] CRAN (R 4.1.2)
## memoise       2.0.1      2021-11-26 [1] CRAN (R 4.1.2)
## pillar        1.7.0      2022-02-01 [1] CRAN (R 4.1.2)
## pkgbuild       1.3.1      2021-12-20 [1] CRAN (R 4.1.2)
## pkgconfig     2.0.3      2019-09-22 [1] CRAN (R 4.1.2)
## pkgload       1.2.4      2021-11-30 [1] CRAN (R 4.1.2)
## prettyunits   1.1.1      2020-01-24 [1] CRAN (R 4.1.2)
## processx      3.5.2      2021-04-30 [1] CRAN (R 4.1.2)
## ps            1.6.0      2021-02-28 [1] CRAN (R 4.1.2)
## purrr         0.3.4      2020-04-17 [1] CRAN (R 4.1.2)
## R6            2.5.1      2021-08-19 [1] CRAN (R 4.1.2)
## remotes       2.4.2      2021-11-30 [1] CRAN (R 4.1.2)
## ReplisimReport 0.0.0.9000 2022-02-03 [1] Github (replisims/ReplisimReport@5f14003)
## rlang         1.0.1      2022-02-03 [1] CRAN (R 4.1.2)
## rmarkdown     2.13       2022-03-10 [1] CRAN (R 4.1.3)
## rprojroot     2.0.2      2020-11-15 [1] CRAN (R 4.1.2)
## rstudioapi    0.13       2020-11-12 [1] CRAN (R 4.1.2)
## sessioninfo   1.2.2      2021-12-06 [1] CRAN (R 4.1.2)
## stringi       1.7.6      2021-11-29 [1] CRAN (R 4.1.2)
## stringr       1.4.0      2019-02-10 [1] CRAN (R 4.1.2)
## testthat      3.1.1      2021-12-03 [1] CRAN (R 4.1.2)
## tibble        3.1.6      2021-11-07 [1] CRAN (R 4.1.2)
## tidyselect    1.1.2      2022-02-21 [1] CRAN (R 4.1.3)
## usethis       2.1.5      2021-12-09 [1] CRAN (R 4.1.2)
## utf8          1.2.2      2021-07-24 [1] CRAN (R 4.1.2)
## vctrs         0.3.8      2021-04-29 [1] CRAN (R 4.1.3)
## withr         2.5.0      2022-03-03 [1] CRAN (R 4.1.3)
## xfun          0.30       2022-03-02 [1] CRAN (R 4.1.3)
## xtable        * 1.8-4      2019-04-21 [1] CRAN (R 4.1.2)
## yaml          2.3.5      2022-02-21 [1] CRAN (R 4.1.2)
##
## [1] C:/Users/alohmann/Documents/R/win-library/4.1
## [2] C:/Program Files/R/R-4.1.3/library
##
## -----
```

The current Git commit details are:

```
## Local:   test C:/Users/alohmann/Dropbox (Personal)/anna/projects_new/replisims/replications/rhemtulla-2012
## Remote:  test @ origin (https://github.com/replisims/rhemtulla-2012.git)
## Head:    [2bd9077] 2022-05-23: Add table 2 to report
```