From: arXiv:1905.13279 [PRL] Costa, *et al*

Let a vector $\boldsymbol{z}$ with N non-zero integer entries such that

$$\sum_{i=1}^{N} z_i = 0\,, \qquad \sum_{i=1}^{N} z_i^3 = 0\,.$$

We like to build this set of N integers from two subsets $\boldsymbol{\ell}$ and $\boldsymbol{k}$ with sizes

$$\dim(\boldsymbol{\ell}) = \begin{cases} \alpha = \frac{N}{2} - 1\,, & \text{if } N \text{ even} \\ \beta = \frac{N-3}{2}\,, & \text{if } N \text{ odd} \end{cases} ; \qquad \dim(\boldsymbol{k}) = \begin{cases} \alpha = \frac{N}{2} - 1\,, & \text{if } N \text{ even} \\ \beta + 1 = \frac{N-1}{2}\,, & \text{if } N \text{ odd} \end{cases}$$

- N even: Consider the following two vector-like examples of $\boldsymbol{z}$ such that

$$\begin{aligned} \boldsymbol{x} &= (\ell_1, k_1, \ldots, k_\alpha, -\ell_1, -k_1, \ldots, -k_\alpha) \\ \boldsymbol{y} &= (0, 0, \ell_1, \ldots, \ell_\alpha, -\ell_1, \ldots, -\ell_\alpha)\ . \end{aligned}$$

- N odd:

$$\boldsymbol{x} = (0, k_1, \ldots, k_{\beta+1}, -k_1, \ldots, -k_{\beta+1})$$

$$\boldsymbol{y} = (\ell_1, \ldots, \ell_\beta, k_1, 0, -\ell_1, \ldots, -\ell_\beta, -k_1)$$

From any of this, we can build a final $\boldsymbol{z}$ which can includes *chiral* solutions

$$\boldsymbol{x} \oplus \boldsymbol{y} \equiv \left(\sum_{i=1}^{N} x_i y_i^2\right) \boldsymbol{x} - \left(\sum_{i=1}^{N} x_i^2 y_i\right) \boldsymbol{y}\,.$$