

Effective Dirac neutrino masses

with multi-component fermionic dark matter



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1803

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Focus on

arXiv:2108.05907

In collaboration with

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Dark sectors







Local $U(1)_\mathcal{X}$

$$\mathcal{L} = -\frac{1}{4} V_{\mu\nu} V^{\mu\nu} + i\bar{\Psi}\not{D}\Psi - h\bar{\Psi}\Psi S$$

Diracness protected chiral fermion

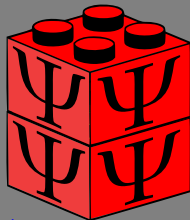
dark matter $m_\Psi = h\langle S \rangle$

Relic abundance:

$$F_{\mu\nu} \quad V^{\mu\nu}$$

Active Symmetry: $\mathcal{X} \rightarrow X: \Psi\bar{\Psi} \rightarrow \text{SM SM}$

Dark Symmetry: $\mathcal{X} \rightarrow D: \Psi\bar{\Psi} \rightarrow \gamma_D \gamma_D$



$$\bar{\Psi}\Psi = \psi_1\psi_2 + \psi_1^\dagger\psi_2^\dagger \rightarrow \psi_\alpha, \quad \alpha = 1, \dots, N' \rightarrow N' > 4$$



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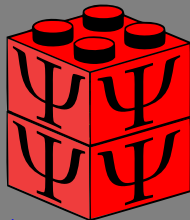
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multi-component
dark matter

$$\bar{\Psi}\Psi = \psi_1\psi_2 + \psi_1^\dagger\psi_2^\dagger \rightarrow \psi_\alpha, \quad \alpha = 1, \dots, N' \rightarrow N' > 4$$

Standard model extended with $U(1)_{\mathcal{X}}$ gauge symmetry

Fields	$SU(2)_L$	$U(1)_Y$	$U(1)_{\mathcal{X}}$
L^\dagger	2	$+1/2$	l
Q^\dagger	2	$-1/6$	q
d_R	1	$-1/2$	d
u_R	1	$+2/3$	u
e_R	1	-1	e
H	2	$-1/2$	h
ψ_α	1	0	n_α

Table 1: $l = q = d = u = e = h = 0$ for $\mathcal{X} = D$

$$[\mathrm{SU}(3)_c]^2 \mathrm{U}(1)_X : \quad [3u + 3d] - [3 \cdot 2q] = 0$$

$$[\mathrm{SU}(2)_L]^2 \mathrm{U}(1)_X : \quad [2l + 3 \cdot 2q] = 0$$

$$[\mathrm{U}(1)_Y]^2 \mathrm{U}(1)_X : \quad \left[(-2)^2 e + 3 \left(\frac{4}{3} \right)^2 u + 3 \left(-\frac{2}{3} \right)^2 d \right] - \left[2(+1)^2 l + 3 \cdot 2 \left(-\frac{1}{3} \right)^2 q \right] = 0$$

with solution

$$u = -e - \frac{2l}{3}, \quad d = e + \frac{4l}{3}, \quad q = -\frac{l}{3},$$

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which satisfy

$$U(1)_Y [U(1)_X]^2 : \quad [(-2)e^2 + 3\left(\frac{4}{3}\right)u^2 + 3\left(-\frac{2}{3}\right)d^2] - [2(+1)l^2 + 3 \cdot 2\left(-\frac{1}{3}\right)q^2] = 0$$

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For N extra quiral fields ψ_α ($\alpha = 1, \dots, N'$) with \mathcal{X} -charges n_α :

$$[SO(1,3)]^2 U(1)_X : \quad \sum_{\alpha} n_{\alpha} + 3(e - 2l) = 0, \quad (1)$$

$$[U(1)_X]^3, \quad \sum_{\alpha} n_{\alpha}^3 + 3(e - 2l)^3 = 0 \quad (2)$$

with solution

$$u = -m + \frac{4l}{3}, \quad d = m - \frac{2l}{3}, \quad q = -\frac{l}{3}, \quad e = m - 2l,$$

which satisfy

$$\mathrm{U}(1)_Y [\mathrm{U}(1)_X]^2 : \quad [(-2)e^2 + 3\left(\frac{4}{3}\right)u^2 + 3\left(-\frac{2}{3}\right)d^2] - [2(+1)l^2 + 3 \cdot 2\left(-\frac{1}{3}\right)q^2] = 0$$

For N extra quiral fields ψ_α ($\alpha = 1, \dots, N'$) with \mathcal{X} -charges n_α : $m \equiv e - 2l$

$$[\mathrm{SO}(1,3)]^2 \mathrm{U}(1)_X : \quad \sum_{\alpha} n_{\alpha} + 3m = 0, \quad (1)$$

$$[\mathrm{U}(1)_X]^3, \quad \sum_{\alpha} n_{\alpha}^3 + 3m^3 = 0 \quad (2)$$

with solution

$$u = -m + \frac{4l}{3}, \quad d = m - \frac{2l}{3}, \quad q = -\frac{l}{3}, \quad e = m - 2l,$$

which satisfy

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Any set of integers of integers (n_1, n_2, \dots, n_N) which satisfy (1) and (2) can be interpreted as $\mathcal{X} \rightarrow D$ symmetry with N chiral fields.

If one integer is repeated 3 times can be interpreted as $\mathcal{X} \rightarrow X$ symmetry with $N' = N - 3$ chiral fields.

From: arXiv:1905.13279 [PRL] Costa, *et al*

Let a vector \mathbf{z} with N non-zero integer entries such that

$$\sum_{i=1}^N z_i = 0, \quad \sum_{i=1}^N z_i^3 = 0.$$

We like to build this set of N integers from two subsets ℓ and \mathbf{k} with sizes

$$\dim(\ell) = \begin{cases} \alpha = \frac{N}{2} - 1, & \text{if } N \text{ even} \\ \beta = \frac{N-3}{2}, & \text{if } N \text{ odd} \end{cases}; \quad \dim(\mathbf{k}) = \begin{cases} \alpha = \frac{N}{2} - 1, & \text{if } N \text{ even} \\ \beta + 1 = \frac{N-1}{2}, & \text{if } N \text{ odd} \end{cases}$$

- N even: Consider the following two vector-like examples of \mathbf{z} such that

$$\mathbf{x} = (\ell_1, k_1, \dots, k_\alpha, -\ell_1, -k_1, \dots, -k_\alpha)$$

$$\mathbf{y} = (0, 0, \ell_1, \dots, \ell_\alpha, -\ell_1, \dots, -\ell_\alpha).$$

- N odd:

$$\mathbf{x} = (0, k_1, \dots, k_{\beta+1}, -k_1, \dots, -k_{\beta+1})$$

$$\mathbf{y} = (\ell_1, \dots, \ell_\beta, k_1, 0, -\ell_1, \dots, -\ell_\beta, -k_1)$$

From any of this, we can build a final \mathbf{z} which can includes *chiral* solutions

$$\mathbf{x} \oplus \mathbf{y} \equiv \left(\sum_{i=1}^N x_i y_i^2 \right) \mathbf{x} - \left(\sum_{i=1}^N x_i^2 y_i \right) \mathbf{y}.$$

Let a vector \mathbf{z} with N non-zero integer entries such that

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We like to build this set of N integers from two subsets ℓ and \mathbf{k} with sizes

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$$\begin{aligned} \mathbf{x} &= (\ell_1, k_1, \dots, k_\alpha, -\ell_1, -k_1, \dots, -k_\alpha) \\ \mathbf{y} &= (0, 0, \ell_1, \dots, \ell_\alpha, -\ell_1, \dots, -\ell_\alpha). \end{aligned}$$

- N odd:

$$\begin{aligned} \mathbf{x} &= (0, k_1, \dots, k_{\beta+1}, -k_1, \dots, -k_{\beta+1}) \\ \mathbf{y} &= (\ell_1, \dots, \ell_\beta, k_1, 0, -\ell_1, \dots, -\ell_\beta, -k_1) \end{aligned}$$

From any of this, we can build a final \mathbf{z} which can includes *chiral* solutions

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<https://pypi.org/project/anomalies>



anomalies 0.1.4

<https://github.com/restrepo/anomaly/raw/main/solutions.json.gz>



`pip install anomalies`



390074 solutions: 4<N<13

Released: Nov 30, 2020

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Meta

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Author: [restrepo](#)

Maintainers

Anomalies

Implement the anomaly free solution of [arXiv:1905.13729](#) [PRL]:

Obtain a numpy array \mathbf{z} of N integers which satisfy the Diophantine equations

```
>>> z.sum()
0
>>> (z**3).sum()
0
```

The input is two lists \mathbf{l} and \mathbf{k} with any $(N-3)/2$ and $(N-1)/2$ integers for N odd, or $N/2-1$ and $N/2-1$ for N even ($N \geq 4$). The function is implemented below under the name: `free(l,k)`

Install

```
$ pip install anomalies
```

USAGE

```
>>> from anomalies import anomaly
>>> anomaly.free([-1,1],[4,-2])
array([ 3,  3,  3, -12, -12, 15])
>>> anomaly.free.gcd
3
>>> anomaly.free.simplified
array([ 1,  1,  1, -4, -4,  5])
```

$$N=6$$



$$\alpha=2$$

$$\vec{l} = (-1, 1)$$

$$\vec{k} = (4, -2)$$

September 24, 2021

Dataset

Open Access

Set of N integers between -30 and 30 with sum and cubic sum up to zero for $4 < N < 13$

Diego Restrepo

Anomalies

Solutions obtained with the python package: [anomalies](#) based on the method to find anomaly free solutions of the standard model extended with an Abelian Dark Symmetry with N right-handed singlet chiral fields described in [arXiv:1905.13729 \[PRL\]](#):

Data scheme

- 'I': integer lists → input to obtain the 'solution' by using the [anomalies](#) package
- 'K': integer lists → input to obtain the 'solution' by using the [anomalies](#) package

- 'solution': list → of integers, Z_i which satisfy $\sum_{i=1}^N Z_i = 0$ and $\sum_{i=1}^N Z_i^3 = 0$.

- 'n': integer → number of integers in 'solution', N .

USAGE

#Example of JSON file usage in Python with pandas (see also json module)

```
>>> import pandas as pd
>>> df=pd.read_json('solutions.json')
>>> df[:2]
```

	1	k	solution	gcd	n
0	[1, 2]	[0, -3]	[1, 5, -7, -8, 9]	1	5
1	[-2, -1]	[0, -1]	[2, 4, -7, -9, 10]	1	5

Data:

390074 solutions with $5 \leq N \leq 12$ integers until '132' [JSON]

17

views

4

downloads

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Keyword(s):

[Anomaly free](#) [Diophantine equations](#) [Abelian symmetry](#)
[Gauge Symmetry](#)

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Versions

Version 1

Sep 24, 2021

[10.5281/zenodo.5526707](https://doi.org/10.5281/zenodo.5526707)

Effective Dirac neutrino mass operator

$$\mathcal{L}_{\text{eff}} = h_{\nu}^{\alpha i} (\nu_{R\alpha})^{\dagger} \epsilon_{ab} L_i^a H^b \left(\frac{S^*}{\Lambda} \right)^{\delta} + \text{H.c.}, \quad \text{with } i = 1, 2, 3,$$

and $\delta = 1, 2, \dots$ for dimension 5 (D-5) or 6 (D-6) operators, etc. Here $h_{\nu}^{\alpha i}$ correspond to dimensionless induced couplings, $\nu_{R\alpha}$ are at least two RHNs ($\alpha = 1, 2, \dots$) with the same D or X -charge ν , L_i are the lepton doublets with X -charge $-L$, H is the SM Higgs doublet with X -charge $h = L - m$, S is the complex singlet scalar responsible for the SSB of the anomaly-free gauge symmetry with D or X -charge

$$s = -(\nu + m)/\delta,$$

Diracness of non-zero DM and Dirac neutrinos masses from $U(1)_X$

Starting from the extended dataset with the solutions with N integers to the Diophantine equations (??), we apply the following steps

- Check that the solution has two (three) repeated integers to be identified as ν and fix $N_\nu = 2$ ($N_\nu = 3$).
- For $\delta = 1, 2, \dots$ and all the possible combinations for m and ν in the solution, including $m = 0$, find the s value compatible with the effective Dirac neutrino mass operator of $D-4 + \delta$ according to eq. (??).
- Interpret the integers in the solution which are different from m and ν as the D -charges for $m = 0$ or X -charges for $m \neq 0$ of a set of singlet chiral fermions: ψ_i , $i = 1, \dots, N_{\text{chiral}} - N_\nu$. Then select the solutions for which the condition

$$|n_i + n_j| = |s| \quad (3)$$

which guarantees that all the singlet chiral fermions, ψ_i , acquire masses after the spontaneous symmetry breaking of the gauge Abelian symmetry through $\langle S \rangle$.

Unconditional stability

Concerning the solutions with multi-component DM, we also explore the cases which feature at least two DM candidates with *unconditional* stability [?]. This happens when there are two remnant symmetries such that $\mathbb{Z}_{|s|} \cong \mathbb{Z}_p \otimes \mathbb{Z}_q$ with p and q coprimes and $|s| = pq$, which guarantee the stability of each lightest state under \mathbb{Z}_p and \mathbb{Z}_q respectively, without imposing any kinematical restriction. For the two DM candidates associated to the set of chiral fields ψ_i and χ_j , we consider below the following two possibilities for $|s|$ [?, ?]

- $\mathbb{Z}_6 \cong \mathbb{Z}_2 \otimes \mathbb{Z}_3$: solutions with at least a set of chiral fields with $\psi_i \sim [\omega_6^2 \vee \omega_6^4]$ under \mathbb{Z}_6 , and at least a set of chiral fields with $\chi_i \sim \omega_6^3$ under \mathbb{Z}_6 ,
- $\mathbb{Z}_{14} \cong \mathbb{Z}_2 \otimes \mathbb{Z}_7$: solutions with at least a set of chiral fields with $\psi_i \sim [\omega_{14}^2 \vee \omega_{14}^6 \vee \omega_{14}^8 \vee \omega_{14}^{10} \vee \omega_{14}^{12}]$ under \mathbb{Z}_{14} and at least a set of chiral fields with $\chi_i \sim \omega_{14}^7$ under \mathbb{Z}_{14} ,

where $\omega_{|s|} = e^{i2\pi/|s|}$.

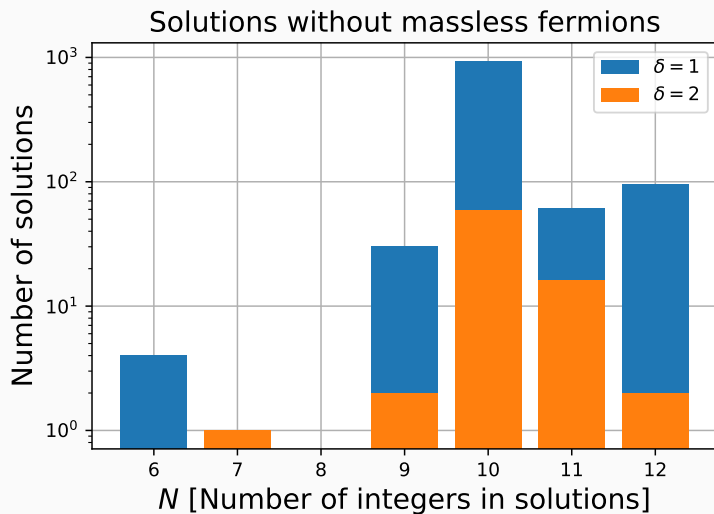


Figure 1: Distribution of solutions with N integers to the Diophantine equations (??) which allow the effective Dirac neutrino operator at $D-4 + \delta$ for at least two right-handed neutrinos and have non-vanishing Dirac or Majorana masses for the other singlet chiral fermions in the solution.

48 type of representative solutions

Solution	N	N_{chiral}	m	ν	δ	s	N_D	N_M	G_D	G_M
(1, -2, -3, 5, 5, -6)	6	6	0	5	1	-5	2	0	1	0
(3, 3, 3, -5, -5, -7, 8)	7	4	3	-5	2	1	1	0	1	0
(1, -2, 3, 4, 6, -7, -7, -7, 9)	9	9	0	-7	1	7	3	0	1	0
(1, 1, -4, -5, 9, 9, 9, -10, -10)	9	9	0	9	1	-9	3	0	2	0
(1, 2, -6, -6, -6, 8, 9, 9, -11)	9	6	-6	9	1	-3	2	0	1	0
(1, -3, 8, 8, 8, -12, -12, -17, 19)	9	6	8	-12	2	2	2	1	1	1
(8, 8, 8, -12, -12, 15, -17, -23, 25)	9	6	8	-12	2	2	2	0	1	0
(1, -2, -2, 3, 3, -4, -4, 6, 6, -7)	10	10	0	6	1	-6	3	2	2	2
(1, -2, -2, 3, 4, -5, -5, 7, 7, -8)	10	10	0	-5	1	5	4	0	2	0
(1, -2, -2, 3, 5, -6, -6, 8, 8, -9)	10	10	0	-6	1	6	4	0	2	0
(2, 2, 3, 4, 4, -5, -6, -6, -7, 9)	10	10	0	2	1	-2	4	2	2	2
(1, 1, 5, 5, 5, -6, -6, -6, -9, 10)	10	10	0	1	1	-1	4	0	3	0
(2, 2, 4, 4, -7, -7, -9, -9, 10, 10)	10	10	0	10	2	-5	3	0	2	0
(1, 2, 2, -3, 6, 6, -8, -8, -9, 11)	10	10	0	-8	1	8	4	1	2	1
(1, -2, -3, 5, 6, -8, -9, 11, 11, -12)	10	10	0	11	1	-11	4	0	1	0
(1, 1, -3, 4, 4, -7, 8, -10, -10, 12)	10	10	0	-10	2	5	4	0	2	0
(1, 1, -2, -2, -4, 6, -10, 11, 12, -13)	10	10	0	-2	1	2	3	2	1	2
(3, 4, 4, 4, 4, -5, -8, -8, -11, 13)	10	10	0	-8	1	8	2	4	1	4
(4, 4, 5, 6, 6, -9, -10, -10, -11, 15)	10	10	0	6	1	-6	4	0	2	0
(1, -2, -4, 7, 7, -10, -12, 14, 14, -15)	10	10	0	14	1	-14	3	2	1	2
(1, 2, 2, -3, 4, -6, 12, -13, -14, 15)	10	10	0	2	1	-2	4	1	1	1
(1, 4, 4, -7, 8, 8, -9, -12, -12, 15)	10	10	0	8	1	-8	4	2	2	2
(1, 2, 2, -9, -9, 16, 16, 17, -18, -18)	10	10	0	-18	1	18	3	2	2	2
(1, -3, -6, 7, -10, 11, -16, 18, 18, -20)	10	10	0	18	2	-9	4	0	1	0

48 type of representative solutions

Solution	N	N_{chiral}	m	ν	δ	s	N_D	N_M	G_D	G_M
(1, -4, 5, -6, -6, 10, -14, 15, 20, -21)	10	10	0	-6	1	6	4	0	1	0
(2, -3, -6, 7, 12, -14, -14, 17, 20, -21)	10	10	0	-14	1	14	4	1	1	1
(3, 6, 6, -7, 8, 8, -14, -14, -17, 21)	10	10	0	-14	1	14	4	1	2	1
(8, 8, 9, 10, 10, -13, -18, -18, -27, 31)	10	10	0	-18	1	18	4	1	2	1
(1, 1, 1, -2, -2, -5, -5, 6, 6, 7, -8)	11	8	1	-2	1	1	3	0	2	0
(1, -2, -2, -2, -3, 4, 4, -5, 6, 7, -8)	11	8	-2	4	1	-2	3	1	1	1
(1, 1, 2, 2, 2, -4, -4, 7, -8, -9, 10)	11	8	2	-4	1	2	2	2	1	2
(2, 2, 2, -4, -4, -5, 7, -8, 9, 10, -11)	11	8	2	-4	1	2	3	0	1	0
(1, -2, -3, -3, -3, 5, 5, -7, 8, 10, -11)	11	8	-3	5	2	-1	3	0	1	0
(3, 3, 3, -4, -4, 7, 7, -8, -9, -9, 11)	11	8	3	-9	2	3	3	0	2	0
(1, 3, 5, -6, -6, -6, 8, -9, 12, 12, -14)	11	8	-6	12	1	-6	3	1	1	1
(1, -2, 6, 6, 6, -7, 8, -9, -12, -12, 15)	11	8	6	-12	1	6	3	0	1	0
(1, 3, 3, 6, 6, 6, -7, -10, -12, -12, 16)	11	8	6	-12	1	6	2	2	1	2
(1, -2, -2, -2, 3, 3, 4, 4, -5, -5, -5, 6)	12	9	-5	-2	1	7	3	0	2	0
(1, 1, -3, 4, 5, 5, 5, -6, -7, -7, -8, 10)	12	9	5	-7	1	2	3	2	1	2
(1, 1, 1, -2, 4, -7, -7, -7, 8, 9, 9, -10)	12	9	-7	9	1	-2	2	3	1	3
(1, 1, -3, -3, -5, -5, -5, 7, 7, 7, 9, -11)	12	9	-5	7	1	-2	3	2	2	2
(1, -3, -3, -3, 4, 6, 7, 9, -10, -10, -10, 12)	12	9	-3	-10	1	13	3	0	1	0
(1, 1, 1, 3, 3, -5, 7, 7, -11, -11, -11, 15)	12	9	1	-11	1	10	3	1	2	1
(1, 1, 1, 3, 5, 5, -5, 5, -9, -9, -13, 15)	12	9	5	-9	2	2	2	3	1	3
(1, -2, -2, 3, 6, -10, -10, -10, 13, 14, 14, -17)	12	9	-10	14	1	-4	4	2	2	2
(1, -3, 9, -11, -13, -13, -13, 15, 15, 15, 21, -23)	12	9	-13	15	1	-2	3	1	1	1

Multi-component dark matter I

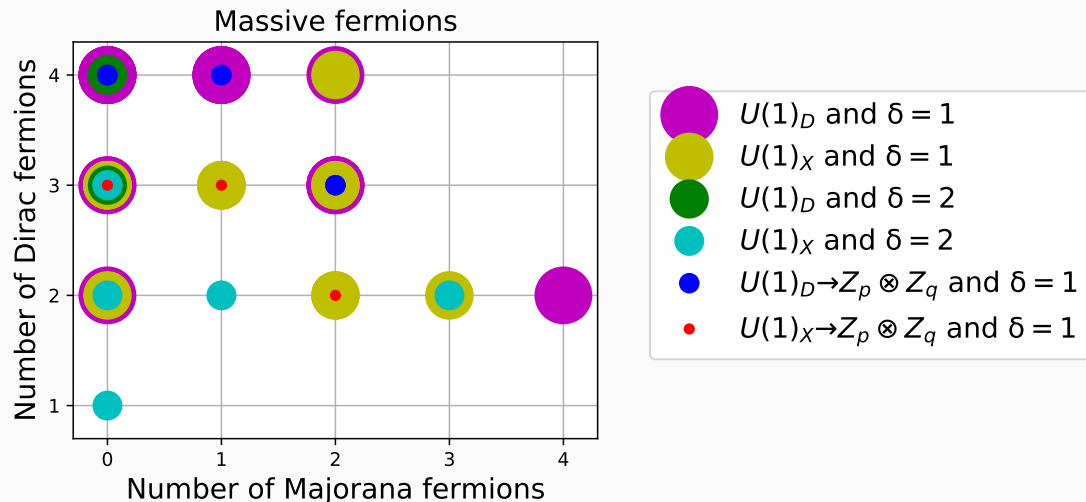
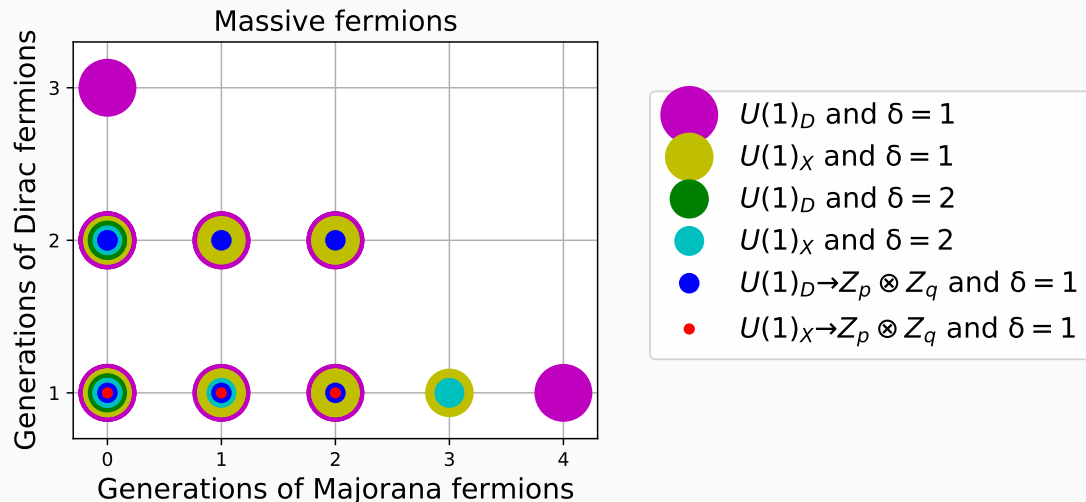


Figure 2: Number of massive Dirac and Majorana fermions in each type of the 48 types of solutions of

Multi-component dark matter II



Solution: $(3, 3, 3, -5, -5, -7, 8)$

Field	$SU(2)_L$	$U(1)_Y$	$U(1)_X$	$U(1)_{B-L}$
L_i	2	$-1/2$	$-L$	-1
e_{Ri}	1	-2	$3 - 2L$	-1
$\nu_{R\alpha}$	1	0	-5	$-5/3$
ψ_1	1	0	-7	$-7/3$
ψ_2	1	0	8	$8/3$
H	2	$1/2$	$L - 3$	0
S	1	0	1	$1/3$
σ_1^+	1	$+2$	$2L$	2
σ_2^+	1	$+2$	$-(2 - 2L)$	$4/3$

Table 2: Charges for last solution. $i = 1, 2, 3$, $\alpha = 1, 2, 3$. Note that $(\omega_n^d)^* = \omega_n^{-d} = \omega_n^{n-d}$.

Neutrino phenomenology with J. Calle and O. Zapata: arXiv:2103.15328 [PRD]

DM Phenomenology: arXiv:1506.05107

In general, we can see that multi-component and multi-generation DM candidates are the trend for gauge Abelian extensions of the SM with massive singlet chiral fermions compatible with the effective Dirac neutrino mass operator of dimension

One parameter $U(1)_X$ SM extension

Fields	$SU(2)_L$	$U(1)_Y$	$U(1)_X$	$U(1)_{B-L}$	$U(1)_R$	$U(1)_D$	$U(1)_G$	$U(1)^*_D$
L	2	$-1/2$	$/$	-1	0	$-3/2$	$-1/2$	0
Q	2	$-1/6$	$-/3$	$1/3$	0	$1/2$	$1/6$	0
d_R	1	$-1/2$	$1 + 2/3$	$1/3$	1	0	$2/3$	0
u_R	1	$+2/3$	$-1 - 4/3$	$1/3$	-1	1	$-1/3$	0
e_R	1	-1	$1 + 2/$	-1	1	-2	0	0
H	2	$1/2$	$-1 - /$	0	-1	$1/2$	$-1/2$	0
$\sum_\alpha n_\alpha$	1	0	-3	-3	-3	-3	-3	0
$\sum_\alpha n_\alpha^3$	1	0	-3	-3	-3	-3	-3	0

solutions with $\sum n_\alpha = -3$ and $\sum n_\alpha^3 = -3$












$(\nu_{R1}, \nu_{R2}, \psi_{N-2}, \dots)$	Ref
$(-1, -1, -1)$	hep-ph/0611205, S. Khalil [JPG]
$(-4, -4, +5)$	 arXiv:0706.0473, Montero, V. Pleitez [PLB]
$\left(-\frac{2}{3}, -\frac{2}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$	 arXiv:1607.04029, S. Patra, W. Rodejohann, C. Yaguna [JHEP]
$\left(-\frac{8}{5}, -\frac{8}{5}, -\frac{2}{5}, -\frac{7}{5}, +2\right)$	 arXiv:1812.05523, with J. Calle, C. Yaguna, Ó. Zapata [PRD]
$\left(-1, -1, -\frac{10}{7}, -\frac{4}{7}, -\frac{2}{7}, \frac{9}{7}\right)$	 1808.03352, with N. Bernal, C. Yaguna, Ó. Zapata [PRD]
$\left(-\frac{5}{3}, -\frac{5}{3}, -\frac{7}{3}, \frac{8}{3}\right)$	  In progress...  method [†]

Table 3: Possible solutions with at least two repeated charges and until six chiral fermions.

[†] General $\sum n_\alpha = 0$ solutions: see D.B Costa, *et al*, arXiv:1905.13729 [PRL]

Or... combine known solutions with $\sum n_\alpha = 0$ and $\sum n_\alpha^3 = 0$

$(\nu_{R1}, \nu_{R2}, \psi_{N-2}, \dots)$	Ref
$(-1, -1, -1)$	hep-ph/0611205, S. Khalil [JPG]
$(-4, -4, +5)$	 arXiv:0706.0473, Montero, V. Pleitez [PLB]
$\left(-\frac{2}{3}, -\frac{2}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$	 arXiv:1607.04029, S. Patra, W. Rodejohann, C. Yaguna [JHEP]
$\left(-\frac{8}{5}, -\frac{8}{5}, -\frac{2}{5}, -\frac{7}{5}, +2\right)$	 arXiv:1812.05523, with J. Calle, C. Yaguna, Ó. Zapata [PRD]
$\left(-1, -1, -\frac{10}{7}, -\frac{4}{7}, -\frac{2}{7}, \frac{9}{7}\right)$	 1808.03352, with N. Bernal, C. Yaguna, Ó. Zapata [PRD]
$\left(-\frac{5}{3}, -\frac{5}{3}, -\frac{7}{3}, \frac{8}{3}\right)$	  In progress...  method [†]

https://en.wikipedia.org/wiki/Sums_of_three_cubes

Only known integer solutions for -3 (1953)






September 2019:

$$42 = (-80538738812075974)^3 + 80435758145817515^3 + 12602123297335631^3$$

Table 3: Possible solutions with at least two repeated charges and until six chiral fermions.

[†] General $\sum n_\alpha = 0$ solutions: see D.B Costa, *et al*, arXiv:1905.13729 [PRL]

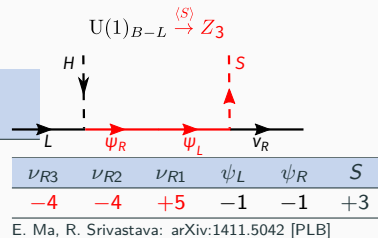
Or... combine known solutions

$(\nu_{R1}, \nu_{R2}, \psi_{N-2}, \dots)$	Ref
$(-1, -1, -1)$	hep-ph/0611205, S. Khalil [JPG]
$(-4, -4, +5)$	 arXiv:0706.0473, Montero, V. Pleitez [PLB]
$\left(-\frac{2}{3}, -\frac{2}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$	 arXiv:1607.04029, S. Patra, W. Rodejohann, C. Yaguna [JHEP]
$\left(-\frac{8}{5}, -\frac{8}{5}, -\frac{2}{5}, -\frac{7}{5}, +2\right)$	 arXiv:1812.05523, with J. Calle, C. Yaguna, Ó. Zapata [PRD]
$\left(-1, -1, -\frac{10}{7}, -\frac{4}{7}, -\frac{2}{7}, \frac{9}{7}\right)$	 1808.03352, with N. Bernal, C. Yaguna, Ó. Zapata [PRD]
$\left(-\frac{5}{3}, -\frac{5}{3}, -\frac{7}{3}, \frac{8}{3}\right)$	  In progress...  method [†]

Not known solution for one-loop neutrino Majorana masses with local $U(1)_X$.

Table 3: Possible solutions with at least two repeated charges and until six chiral fermions.

[†] General $\sum n_{\alpha} = 0$ solutions: see D.B Costa, *et al*, arXiv:1905.13729 [PRL]





$$m_{\text{Majorana}}^{\nu} + = \frac{h_{\nu}}{\Lambda} L \cdot H L \cdot H$$



3 models

$$m_{\text{Majorana}}^{\nu+} = \frac{h_{\nu}}{\Lambda} L \cdot H L \cdot H \quad (\text{three-level})$$

Type-I arXiv:1808.03352, II arXiv:1607.04029, III arXiv:1908.04308

$$\mathcal{L} = y(N_R)^\dagger L \cdot H + M_N N_R N_R + \text{h.c.}$$



$$m_{\text{Majorana}}^\nu + = \frac{h_\nu}{\Lambda} L \cdot H L \cdot H$$

Type-I
seesaw

Type-I arXiv:1808.03352, with N. Bernal, C. Yaguna, and Ó. Zapata [PRD]

$$U(1)_{B-L} \rightarrow Z_7$$

$$\mathcal{L} = y(N_R)^\dagger L \cdot \langle H \rangle + y' \langle S \rangle N_R N_R + \text{h.c}$$



$$m_{\text{Majorana}}^\nu = \frac{h_\nu}{\Lambda} L \cdot H L \cdot H \frac{S}{\Lambda}$$

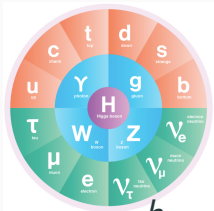
ν_{R3}	ν_{R2}	S
-1	-1	2

Type-I arXiv:1808.03352

: Also new terms
arise from spontaneous
breakdown of a
new gauge symmetry

Local $U(1)_{B-L} \rightarrow Z_7$

$$\mathcal{L} = y(N_R)^\dagger L \cdot \langle H \rangle + y' \langle S \rangle N_R N_R + \text{h.c}$$

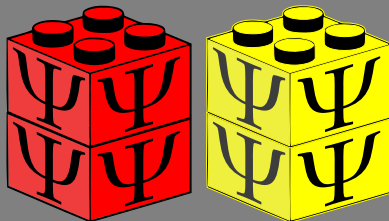


$$m_{\text{Majorana}}^\nu = \frac{h_\nu}{\Lambda} L \cdot H L \cdot H \frac{S}{\Lambda}$$

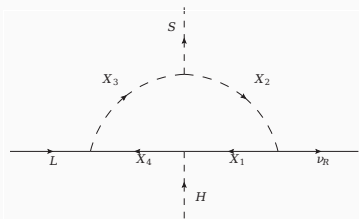
Type-I arXiv:1808.03352

: Also new terms
arise from spontaneous
breakdown of a
new gauge symmetry

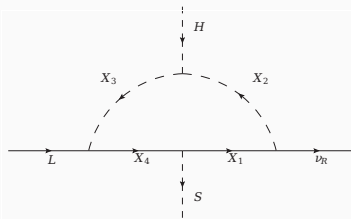
ν_{R3}	ν_{R2}	$\overline{\psi_{L1}}$	ψ_{R1}	ψ_{R2}	$\overline{\psi_{L2}}$	S	S'
-1	-1	$-\frac{10}{7}$	$-\frac{4}{7}$	$-\frac{2}{7}$	$\frac{9}{7}$	2	1



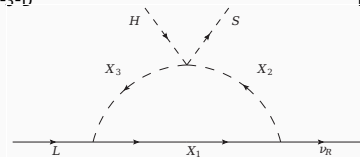
One loop topologies $U(1)_{B-L} \oplus Z_2 \oplus Z_2$



T1-3-D

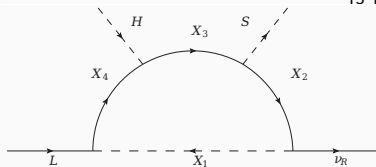


T1-3-E

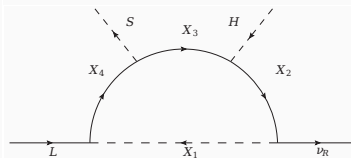


T3-1-A

Chang-Yuan Yao and Gui-Jun Ding, arXiv:1802.05231 [PRD]

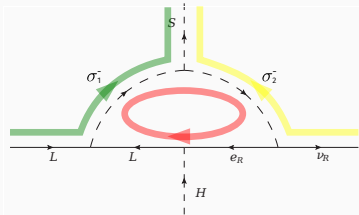


T1-2-A

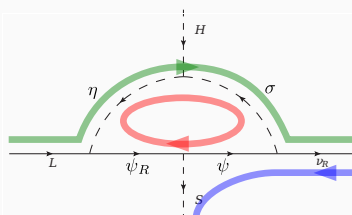


T1-2-B

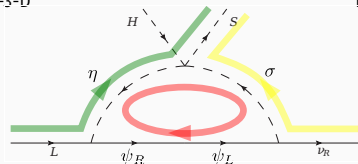
One loop topologies $U(1)_{B-L}$ only!



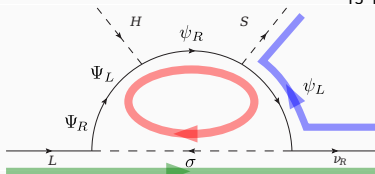
T1-3-D



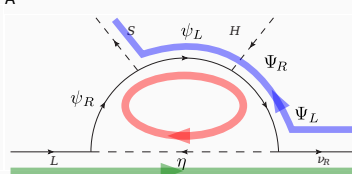
T1-3-E



T3-1-A



T1-2-A



T1-2-B

$\psi_{L,R} \rightarrow$ Singlet fermions

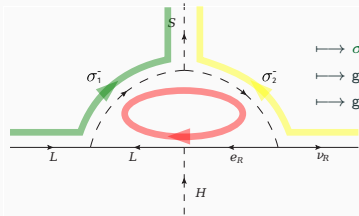
$\Psi_{L,R} \rightarrow$ Vector-like doublet fermions

$\sigma \rightarrow$ Singlet scalar

$\eta \rightarrow$ Doublet scalar

with J. Calle, C. Yaguna, and O. Zapata, arXiv:1812.05523 [PRD]

One loop topologies $U(1)_{B-L}$ only! with J. Calle, C. Yaguna, and O. Zapata, arXiv:1812.05523 [PRD]



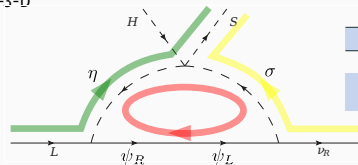
$$\mapsto \sigma_1 = -2, \quad \sigma_2 = -5,$$

\mapsto generalization to two and three loops: S. Saad arXiv:1902.07259 [NPB]

\mapsto generalization to $U(1)_R$: *et al*, S. Saad arXiv:1904.07407



T1-3-D



T3-1-A

$\psi_{L,R} \rightarrow$ Singlet fermions (vector-like)

$\sigma \rightarrow$ Singlet scalar

$\eta \rightarrow$ Doublet scalar

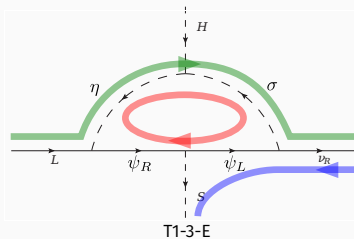
Fields: f_i	$(\nu_{R3})^\dagger$	$(\nu_{R2})^\dagger$	$(\nu_{R1})^\dagger$	ψ_L	$(\psi_R)^\dagger$	S
(A)	+4	+4	-5	-r	r	+3

Anomaly cancellation conditions

$$\sum_i f_i = 3$$

$$\sum_i f_i^3 = 3$$

One loop topologies $U(1)_{B-L}$ only! with J. Calle, C. Yaguna, and O. Zapata, arXiv:1812.05523 [PRD]



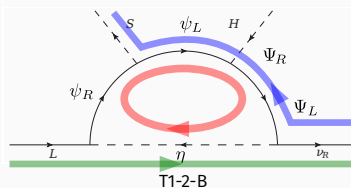
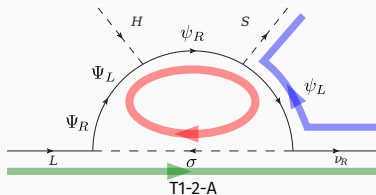
Fields: f_i	$(\nu_{R3})^\dagger$	$(\nu_{R2})^\dagger$	$(\nu_{R1})^\dagger$	ψ_L	$(\psi_R)^\dagger$	S
(A)	+4	+4	-5	-r	r	+3
(B)	$+\frac{8}{5}$	$+\frac{8}{5}$	$+\frac{2}{5}$	$\frac{7}{5}$	$-\frac{10}{5}$	$+\frac{3}{5}$

$\psi_{L,R} \rightarrow$ Singlet fermions (quiral)

$\Psi_{L,R} \rightarrow$ Vector-like doublet fermions

$\sigma \rightarrow$ Singlet scalar

$\eta \rightarrow$ Doublet scalar



Anomaly cancellation conditions

$$\sum_i f_i = 3$$

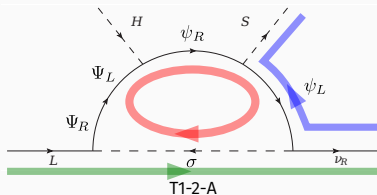
$$\sum_i f_i^3 = 3$$

$\psi_{L,R} \rightarrow$ Singlet fermions (quiral)

$\Psi_{L,R} \rightarrow$ Vector-like doublet fermions : **10/5**

$\sigma \rightarrow$ Singlet scalar : 15/5

Fields: f_i	$(\nu_{R3})^\dagger$	$(\nu_{R2})^\dagger$	$(\nu_{R1})^\dagger$	ψ_L	$(\psi_R)^\dagger$	S
(A)	+4	+4	-5	-r	r	+3
(B)	$+\frac{8}{5}$	$+\frac{8}{5}$	$+\frac{2}{5}$	$\frac{7}{5}$	$-\frac{10}{5}$	$+\frac{3}{5}$



Anomaly cancellation conditions

$$\sum_i f_i = 3$$

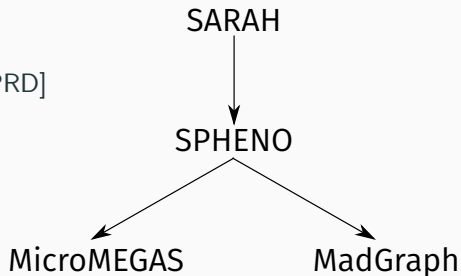
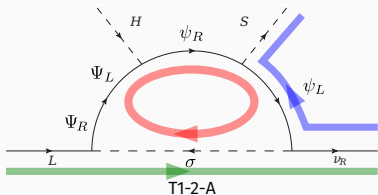
$$\sum_i f_i^3 = 3$$

SD³M+SSDM: σ_a ($a = 1, 2$)

$$M_\psi = h_1 \langle S \rangle, \text{ } y_2 = 0:$$

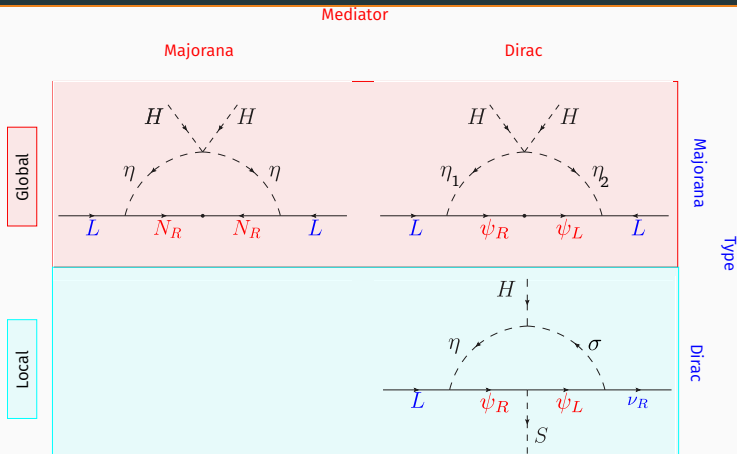
$$\mathcal{L} = \mathcal{L}_{\text{SD}^3\text{M}} + h_3^{ia}(\widetilde{\Psi_R}) \cdot L_i \sigma_a + h_2^{\beta a} (\nu_{R\beta})^\dagger \psi_L \sigma_a^* - V(\sigma_a, S, H).$$

with A.F Rivera, W. Tangarife, arXiv:1906.09685 [PRD]



Radiative Type-I seesaw \rightarrow Local: only $U(1)_{B-L}$!

arXiv:1812.05523, with J. Calle, C. Yaguna, Ó. Zapata [PRD]



For radiative Dirac models with only $U(1)_X$ see also:

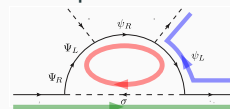
arXiv:1812.01599, 1901.06402, 1902.07259,

1903.01477, 1904.07407, 1907.08630, 1910.09537

1909.00833 1907.11557, 1909.09574

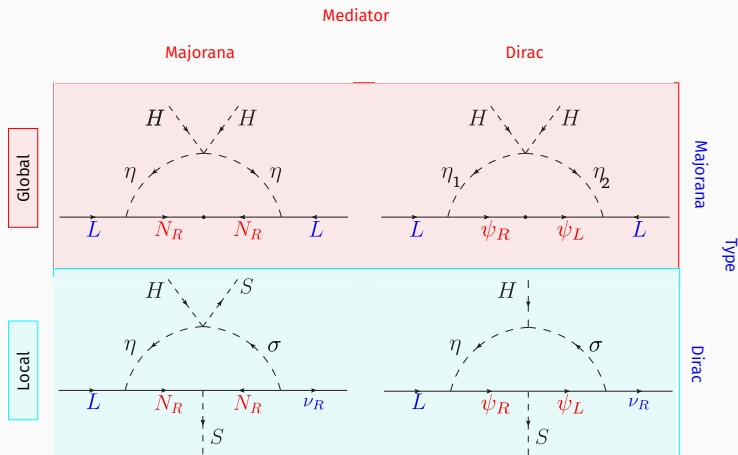
$\mathcal{O}(50)$ new models mostly with $\sim (-4, -4, 5)$

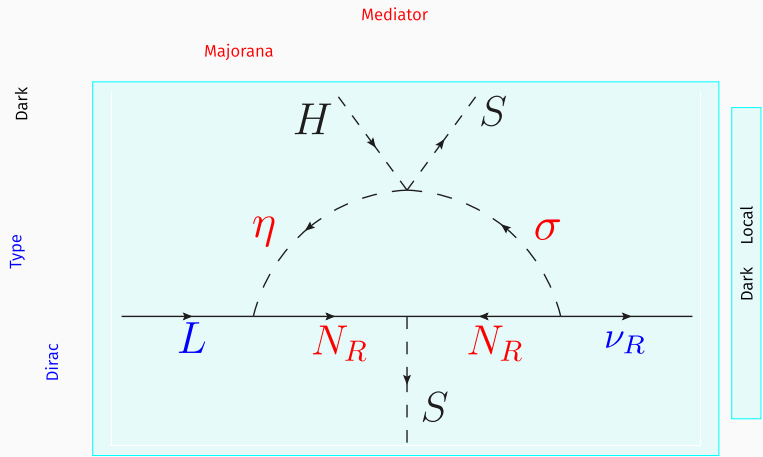
Example:  $U(1)_{B-L}$



Pheno analysis with

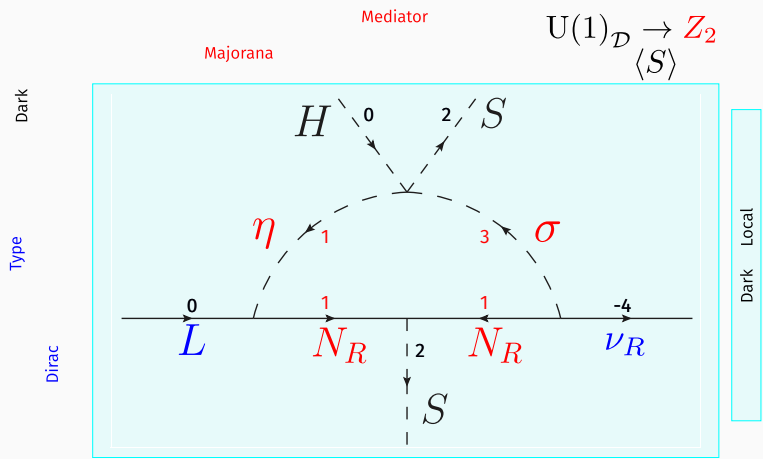
A. Rivera, W. Tangarife, arXiv:1906.09685 [PRD]





$$N = -\frac{\nu}{4}, \quad \eta = -\frac{\nu}{4}, \quad \sigma = -\frac{3\nu}{4}.$$

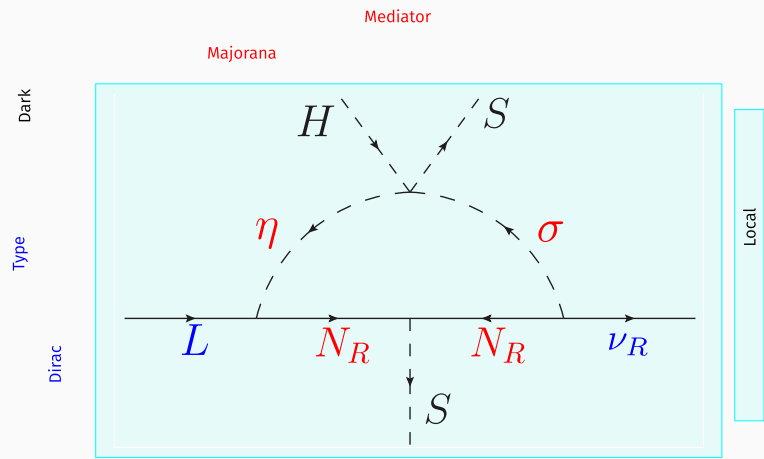
Fields	$SU(2)_L$	$U(1)_Y$	$U(1)_D$
L	2	$-1/2$	0
Q	2	$-1/6$	0
d_R	1	$-1/2$	0
u_R	1	$+2/3$	0
e_R	1	-1	0
H	2	$1/2$	0
η	2	$1/2$	1
S	1	0	2
σ	1	0	3
ν_{R1}	1	0	-4
ν_{R2}	1	0	-4
ν_{R3}	1	0	5
N_{R1}	1	0	1
N_{R2}	1	0	1
N_{R3}	1	0	1
TOTAL			0 22



$$N = -\frac{\nu}{4}, \quad \eta = -\frac{\nu}{4}, \quad \sigma = -\frac{3\nu}{4}.$$

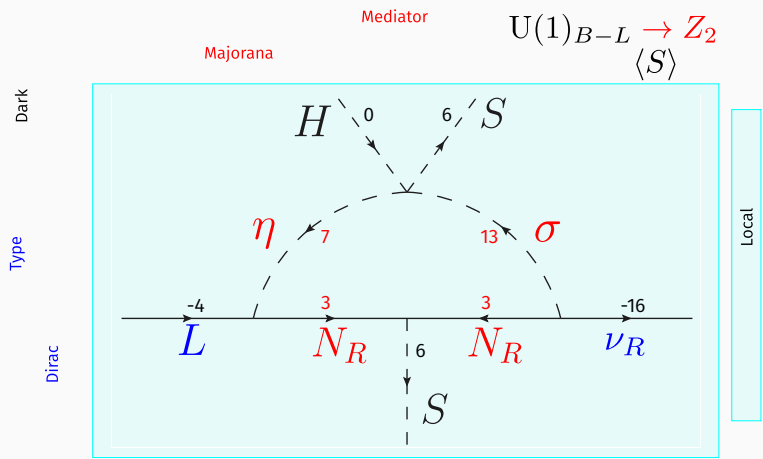
Fields	$SU(2)_L$	$U(1)_Y$	$U(1)_D$
L	2	$-1/2$	0
Q	2	$-1/6$	0
d_R	1	$-1/2$	0
u_R	1	$+2/3$	0
e_R	1	-1	0
H	2	$1/2$	0
η	2	$1/2$	1
S	1	0	2
σ	1	0	3
ν_{R1}	1	0	-4
ν_{R2}	1	0	-4
ν_{R3}	1	0	5
N_{R1}	1	0	1
N_{R2}	1	0	1
N_{R3}	1	0	1
TOTAL			0 22

Dirac Radiative Type-I seesaw with Majorana mediators with J. Calle and Ó. Zapata, arXiv:1909.09574



$$N = -\frac{\nu}{4} - \frac{1}{4}, \quad \eta = -\frac{\nu}{4} - \frac{1}{4} - l, \quad \sigma = -\frac{3\nu}{4} + \frac{1}{4}.$$

Fields	$SU(2)_L$	$U(1)_Y$	$U(1)_X$
L	2	$-1/2$	l
Q	2	$-1/6$	$-l/3$
d_R	1	$-1/2$	$1 + 2l/3$
u_R	1	$+2/3$	$-1 - 4l/3$
e_R	1	-1	$1 + 2l$
H	2	$1/2$	$-1 - l$
η	2	$1/2$	$3/4 - l$
S	1	0	$3/2$
σ	1	0	$13/4$
ν_{R1}	1	0	-4
ν_{R2}	1	0	-4
ν_{R3}	1	0	5
N_{R1}	1	0	$3/4$
N_{R2}	1	0	$3/4$
N_{R3}	1	0	$3/4$
$\xi_{L\alpha}$	1	0	$3/4^{22}$



$$N = -\frac{\nu}{4} - \frac{1}{4}, \quad \eta = -\frac{\nu}{4} - \frac{1}{4} + 1, \quad \sigma = -\frac{3\nu}{4} + \frac{1}{4}.$$

Fields	$SU(2)_L$	$U(1)_Y$	$U(1)_{B-L}$
L	2	$-1/2$	-1
Q	2	$-1/6$	$1/3$
d_R	1	$-1/2$	$1/3$
u_R	1	$+2/3$	$1/3$
e_R	1	-1	-1
H	2	$1/2$	0
η	2	$1/2$	$7/4$
S	1	0	$3/2$
σ	1	0	$13/4$
ν_{R1}	1	0	-4
ν_{R2}	1	0	-4
ν_{R3}	1	0	5
N_{R1}	1	0	$3/4$
N_{R2}	1	0	$3/4$
N_{R3}	1	0	$3/4$
$\xi_{L\alpha}$	1	0	$3/4$ 22

$$\begin{aligned} \mathcal{L} \supset & - g' Z'_\mu \sum_F q_F \bar{F} \gamma^\mu F + \sum_\phi |(\partial_\mu + i g' q_\phi Z'_\mu) \phi|^2 \\ & - [h_{i\alpha} \bar{L}_i \tilde{\eta} N_{R\alpha} + y_{j\alpha} \bar{\nu}_{Rj} \sigma^* N_{R\alpha}^c + k_\alpha \overline{N_{R\alpha}^c} N_{R\alpha} S^* + \text{h.c.}] - \mathcal{V}(H, S, \eta, \sigma). \end{aligned}$$

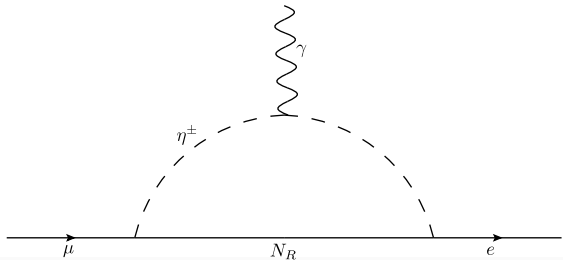
$F(\phi)$ denote the new fermions (scalars)

$$\begin{aligned} \mathcal{V}(H, S, \eta, \sigma) = & V(H) + V(S) + V(\eta) + V(\sigma) \\ & + \lambda_{HS} (H^\dagger H) (S^* S) + \lambda_2 (H^\dagger H) (\sigma^* \sigma) + \lambda_3 (H^\dagger H) (\eta^\dagger \eta) \\ & + \lambda_4 (S^* S) (\sigma^* \sigma) + \lambda_5 (S^* S) (\eta^\dagger \eta) + \lambda_6 (\eta^\dagger \eta) (\sigma^* \sigma) + \lambda_7 (\eta^\dagger H) (H^\dagger \eta) \\ & + \lambda_8 (\eta^\dagger H S^* \sigma + \text{h.c.}), \end{aligned}$$

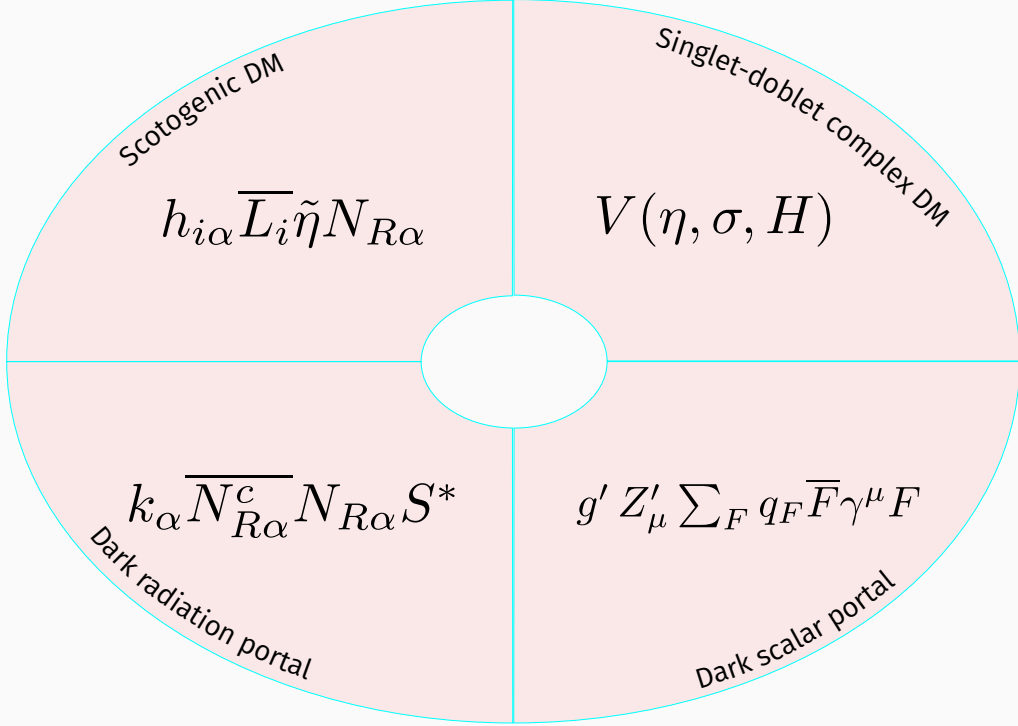
$$(\mathcal{M}_\nu)_{ij} = \frac{1}{32\pi^2} \frac{\lambda_8 v_S^2 v_H}{m_{\eta_R^0}^2 - m_{\sigma_R^0}^2} \sum_{\alpha=1}^3 h_{i\alpha} k_\alpha y_{j\alpha}^* \left[F\left(\frac{m_{\eta_R^0}^2}{M_{N_\alpha}^2}\right) - F\left(\frac{m_{\sigma_R^0}^2}{M_{N_\alpha}^2}\right) \right] + (R \rightarrow I),$$

where $F(x) = x \log x / (x - 1)$.

$\mu \rightarrow e \gamma$



$$\left| \sum_{\alpha} h_{2\alpha} h_{1\alpha}^* \right| \lesssim 0.02 \left(\frac{m_\chi}{2 \text{ TeV}} \right)^2.$$



Scotogenic DM

$$h_{i\alpha} \overline{L}_i \tilde{\eta} N_{R\alpha}$$

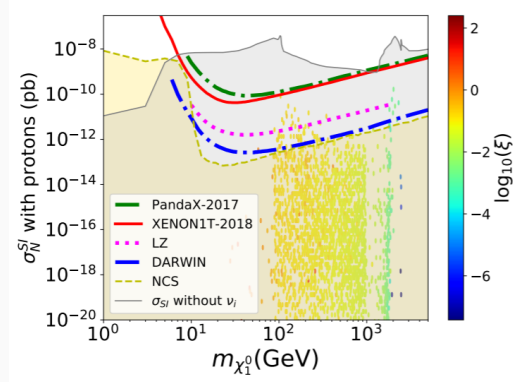
A. Ibarra, C. Yaguna, Ó. Zapata,
arXiv:1601.01163 [PRD]

Scotogenic DM

$$h_{i\alpha} \overline{L}_i \tilde{\eta} N_{R\alpha}$$

$$N_{R2} \rightarrow \Sigma$$

with A. Rivera, arXiv:1907.11938



$$(\chi_1^0 \ \chi_2^0)^T = R(\textcolor{red}{N}_R \ \textcolor{blue}{\Sigma})^T$$

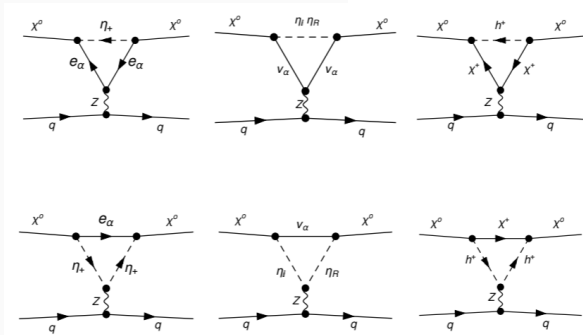
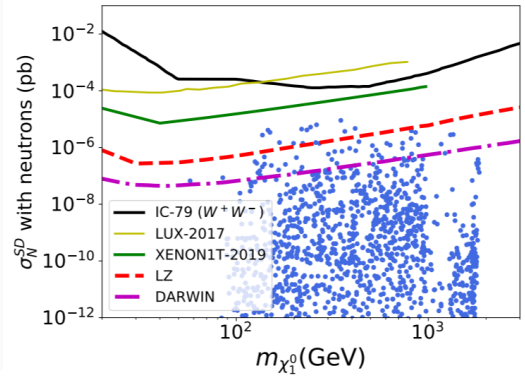
$$\xi = \frac{|M_{\Sigma} - m_{\chi_1^0}|}{m_{\chi_1^0}}$$

Scotogenic DM

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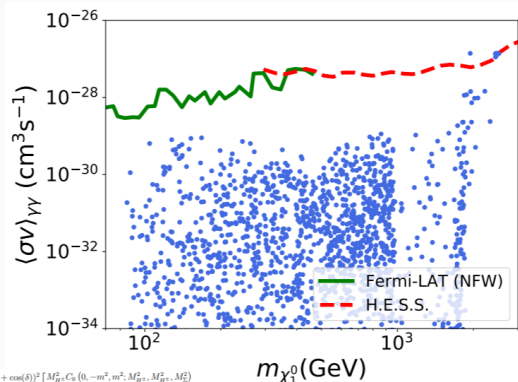
Scotogenic DM

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$$N_{R2} \rightarrow \Sigma$$

with A. Rivera, arXiv:1907.11938

$$\sigma v (\chi_1^0 \chi_1^0 \rightarrow \gamma\gamma) = \frac{|\mathcal{B}|^2}{32\pi m_{\chi_1^0}^2}$$



$$\mathcal{B} = \frac{\sqrt{2}\alpha m^2 \sin^2(\alpha) Y_{\tilde{G}}^2 (\sin(\delta) + \cos(\delta))^2}{\pi} \left[\frac{M_{H^\pm}^2 C_0(0, -m^2, m^2; M_{H^\pm}^2, M_{H^\pm}^2, M_{\tilde{G}}^2)}{M_{H^\pm}^2 - M_{\tilde{G}}^2} \right. \\ - \frac{M_{\tilde{G}}(-2m M_{H^\pm}^2 - M_{\tilde{G}} M_{H^\pm}^2 + m^2 M_{\tilde{G}} + 2m M_{\tilde{G}}^2 + M_{\tilde{G}}^2) C_0(0, -m^2, m^2; M_{\tilde{G}}^2, M_{\tilde{G}}^2, M_{H^\pm}^2)}{(M_{H^\pm}^2 - M_{\tilde{G}}^2)(M_{H^\pm}^2 + m^2 - M_{\tilde{G}}^2)} \\ + \frac{2M_{\tilde{G}}(m + M_{\tilde{G}}) C_0(0, 0, 4m^2; M_{\tilde{G}}^2, M_{\tilde{G}}^2, M_{\tilde{G}}^2)}{-M_{H^\pm}^2 - m^2 + M_{\tilde{G}}^2} \left. \right] \\ + \frac{\alpha m^2 \sin(\alpha) \cos(\alpha) Y_{\tilde{G}}^2}{\pi} \left[- \frac{m_{\tilde{G}}^2 C_0(0, -m^2, m^2; m_{\tilde{G}}^2, m_{\tilde{G}}^2, m_{\tilde{G}}^2)}{m_{\tilde{G}}^2 - m_{\tilde{G}}^2} \right. \\ + \frac{m_{\tilde{G}}^2(m_{\tilde{G}}^2 + m^2 - m_{\tilde{G}}^2) C_0(0, -m^2, m^2; m_{\tilde{G}}^2, m_{\tilde{G}}^2, m_{\tilde{G}}^2)}{(m_{\tilde{G}}^2 - m_{\tilde{G}}^2)(-m_{\tilde{G}}^2 + m^2 + m_{\tilde{G}}^2)} + \frac{2m_{\tilde{G}}^2 C_0(0, 0, 4m^2; m_{\tilde{G}}^2, m_{\tilde{G}}^2, m_{\tilde{G}}^2)}{-m_{\tilde{G}}^2 + m^2 + m_{\tilde{G}}^2} \left. \right] \\ + \frac{\alpha m^2 \cos^2(\alpha) Y_{\tilde{G}}^2}{2\sqrt{2}\pi} \left[\frac{m_{\tilde{G}}^2 C_0(0, -m^2, m^2; m_{\tilde{G}}^2, m_{\tilde{G}}^2, m_{\tilde{G}}^2)}{m_{\tilde{G}}^2 - m_{\tilde{G}}^2} \right. \\ - \frac{m_{\tilde{G}}^2(m_{\tilde{G}}^2 + m^2 - m_{\tilde{G}}^2) C_0(0, -m^2, m^2; m_{\tilde{G}}^2, m_{\tilde{G}}^2, m_{\tilde{G}}^2)}{(m_{\tilde{G}}^2 - m_{\tilde{G}}^2)(-m_{\tilde{G}}^2 + m^2 + m_{\tilde{G}}^2)} - \frac{2m_{\tilde{G}}^2 C_0(0, 0, 4m^2; m_{\tilde{G}}^2, m_{\tilde{G}}^2, m_{\tilde{G}}^2)}{-m_{\tilde{G}}^2 + m^2 + m_{\tilde{G}}^2} \left. \right] \\ + \frac{\sqrt{2}\alpha m^2 \sin^2(\alpha) Y_{\tilde{G}}^2}{2\pi} \left[\frac{m_{\tilde{G}}^2 C_0(0, -m^2, m^2; m_{\tilde{G}}^2, m_{\tilde{G}}^2, m_{\tilde{G}}^2)}{m_{\tilde{G}}^2 - m_{\tilde{G}}^2} \right. \\ - \frac{m_{\tilde{G}}^2(m_{\tilde{G}}^2 + m^2 - m_{\tilde{G}}^2) C_0(0, -m^2, m^2; m_{\tilde{G}}^2, m_{\tilde{G}}^2, m_{\tilde{G}}^2)}{(m_{\tilde{G}}^2 - m_{\tilde{G}}^2)(-m_{\tilde{G}}^2 + m^2 + m_{\tilde{G}}^2)} - \frac{2m_{\tilde{G}}^2 C_0(0, 0, 4m^2; m_{\tilde{G}}^2, m_{\tilde{G}}^2, m_{\tilde{G}}^2)}{-m_{\tilde{G}}^2 + m^2 + m_{\tilde{G}}^2} \left. \right] \\ - \frac{8\sqrt{2}\alpha m^2 \cos^2(\alpha) M_{\tilde{G}}^2}{\pi (M_{\tilde{G}}^2 - M_{\tilde{G}}^2)(4v_{\tilde{G}}^2 + v_{\tilde{G}}^2)(m^2 - M_{\tilde{G}}^2 + M_{\tilde{G}}^2)(m^2 + M_{\tilde{G}}^2 - M_{\tilde{G}}^2)} \\ \left[4(m^2 - M_{\tilde{G}}^2)(M_{\tilde{G}}^2 - M_{\tilde{G}}^2)(m^2 - M_{\tilde{G}}^2 + M_{\tilde{G}}^2) C_0(0, 0, 4m^2; M_{\tilde{G}}^2, M_{\tilde{G}}^2, M_{\tilde{G}}^2) \right. \\ + 2M_{\tilde{G}}(2m - M_{\tilde{G}})(M_{\tilde{G}}^2 - M_{\tilde{G}}^2)(m^2 + M_{\tilde{G}}^2 - M_{\tilde{G}}^2) C_0(0, 0, 4m^2; M_{\tilde{G}}^2, M_{\tilde{G}}^2, M_{\tilde{G}}^2) \\ - (m^2 - M_{\tilde{G}}^2 + M_{\tilde{G}}^2)(-M_{\tilde{G}}^2(m^2 + M_{\tilde{G}}^2) - 4m M_{\tilde{G}}(m^2 + M_{\tilde{G}}^2 - M_{\tilde{G}}^2) + 4M_{\tilde{G}}^2 + M_{\tilde{G}}^2) \\ C_0(0, -m^2, m^2; M_{\tilde{G}}^2, M_{\tilde{G}}^2, M_{\tilde{G}}^2) - M_{\tilde{G}}(m^2 + M_{\tilde{G}}^2 - M_{\tilde{G}}^2)(4m^3 - 3m^2 M_{\tilde{G}} + M_{\tilde{G}}^2 - M_{\tilde{G}} M_{\tilde{G}}^2) \\ \left. C_0(0, -m^2, m^2; M_{\tilde{G}}^2, M_{\tilde{G}}^2, M_{\tilde{G}}^2) \right].$$

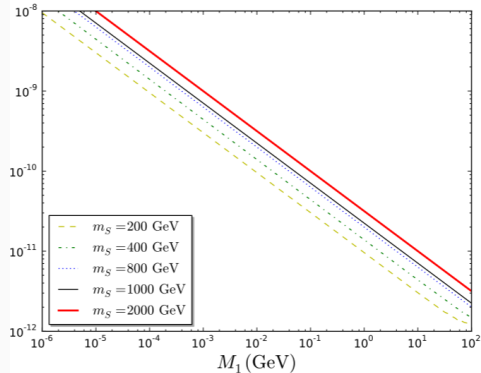
Scotogenic DM

FIMP Scenario

$$h_{i\alpha} \overline{L}_i \tilde{\eta} N_{R\alpha}$$

F. Molinaro, C. Yaguna, Ó. Zapata,
arXiv:1405.1259 [JCAP]

$$h_1 \sim h_{1\alpha}$$

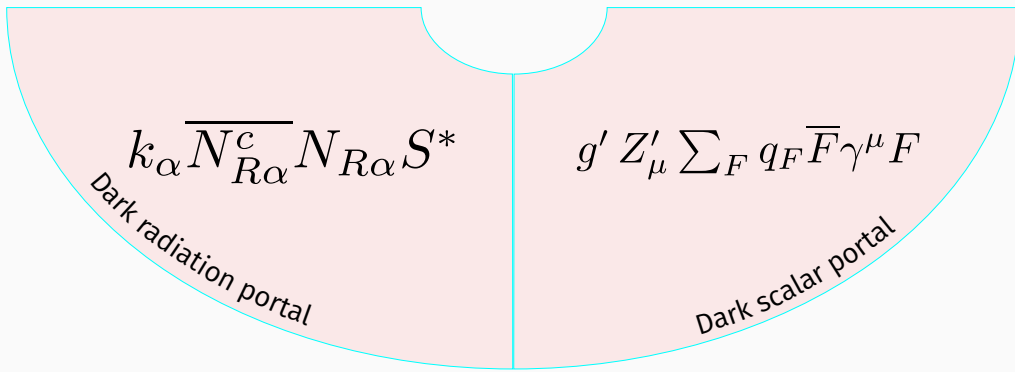


$$l(\eta^+) = 3 \times 10^5 \text{cm} \left(\frac{M_1}{1 \text{GeV}} \right) \left(\frac{1 \text{TeV}}{m_{\eta^+}} \right)^2$$

$$\lesssim 3 \text{ meters} \left(\frac{1 \text{TeV}}{m_{\eta^+}} \right)^2 \quad \text{for} \quad M_1 \lesssim 1 \text{MeV}$$

$$N_R N_R \rightarrow \nu_R \nu_R$$

$$\Delta N_{\text{eff}} \sim 0.2$$



(One-loop) Dirac neutrino masses

Small Dirac neutrino masses

To explain the **smallness** of Dirac neutrino masses choose $U(1)_X$ which:

- Forbids tree-level mass (TL) term ($Y(H) = +1/2$)

$$\begin{aligned}\mathcal{L}_{\text{T.L}} &= h_D \epsilon_{ab} (\nu_R)^\dagger L^a H^b + \text{h.c.} \\ &= h_D (\nu_R)^\dagger L \cdot H + \text{h.c.}\end{aligned}$$

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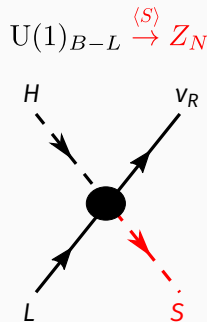
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- Realizes of the 5-dimension operator which conserves lepton number in $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$:

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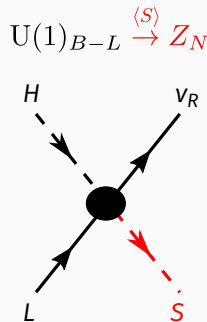
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- Enhancement to the *effective number of degrees of freedom in the early Universe* $\Delta N_{\text{eff}} = N_{\text{eff}} - N_{\text{eff}}^{\text{SM}}$ (see arXiv:1211.0186)



See E. Ma, Rahul Srivastava: arXiv:1411.5042 [PLB] for tree-level realization

From 1210.6350 and 1805.02025: $\Delta N_{\text{eff}} = 3 (T_{\nu_R} / T_{\nu_L})^4$

$$\Gamma_{\nu_R}(T) = n_{\nu_R}(T) \sum_f \langle \sigma_f(\nu_R \bar{\nu}_R \rightarrow \bar{f} f) v \rangle$$

$$= \sum_f \frac{g_{\nu_R}^2}{n_{\nu_R}} \int \frac{d^3 p}{(2\pi)^3} \frac{d^3 q}{(2\pi)^3} f_{\nu_R}(p) f_{\nu_R}(q) \sigma_f(s) (1 - \cos \theta),$$

$$s = 2pq(1 - \cos \theta),$$

$$f_{\nu_R}(k) = 1/(e^{k/T} + 1)$$

$$n_{\nu_R}(T) = g_{\nu_R} \int \frac{d^3 k}{(2\pi)^3} f_{\nu_R}(k),$$

$$\text{with } g_{\nu_R} = 2$$

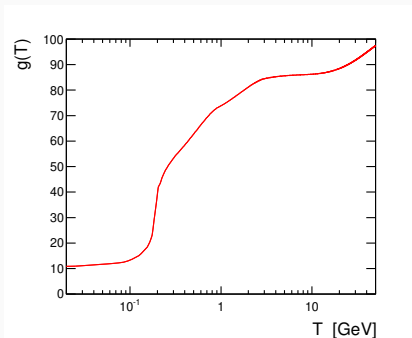
$$\sigma_f(s) \simeq \frac{N_C^f (Q_{BL}^f)^2 Q^2 s}{12\pi} \left(\frac{g'}{M_{Z'}} \right)^4, \quad \text{In the limit } M_{Z'}^2 \gg s.$$

with three right-handed neutrinos, the Hubble parameter is

$$H(T) = \sqrt{\frac{4\pi^3 G_N [g(T) + 21/4]}{45}} T^2.$$

The right-handed neutrinos decouple when

$$\Gamma_{\nu_R}(T_{\text{dec}}^{\nu_R}) = H(T_{\text{dec}}^{\nu_R}).$$



A. Solaguren-Beascoa, M. C. Gonzalez-Garcia: arXiv:1210.6350 [PLB]

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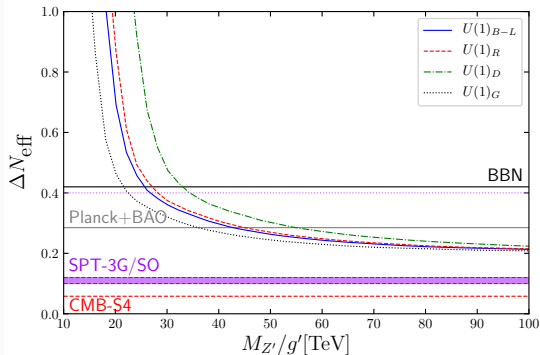
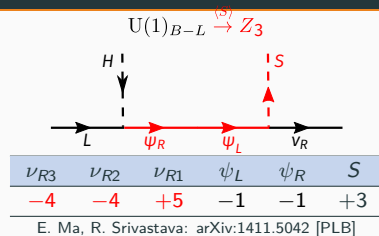
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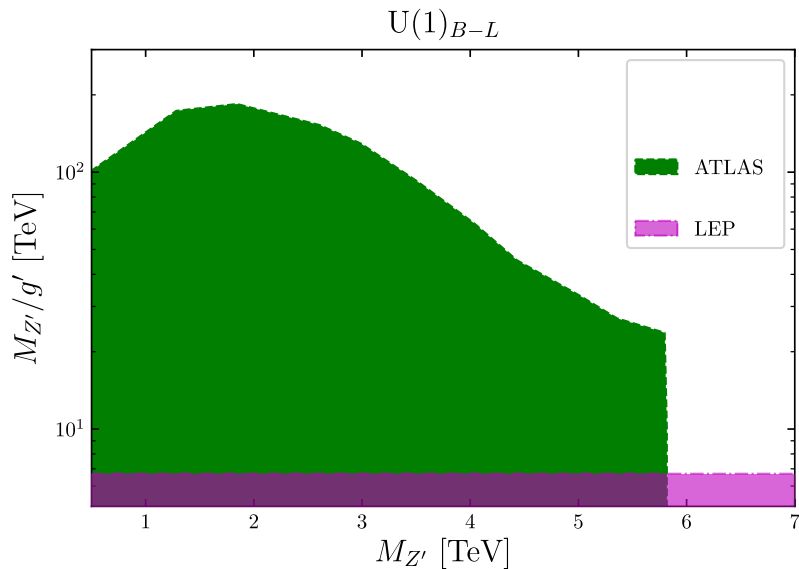
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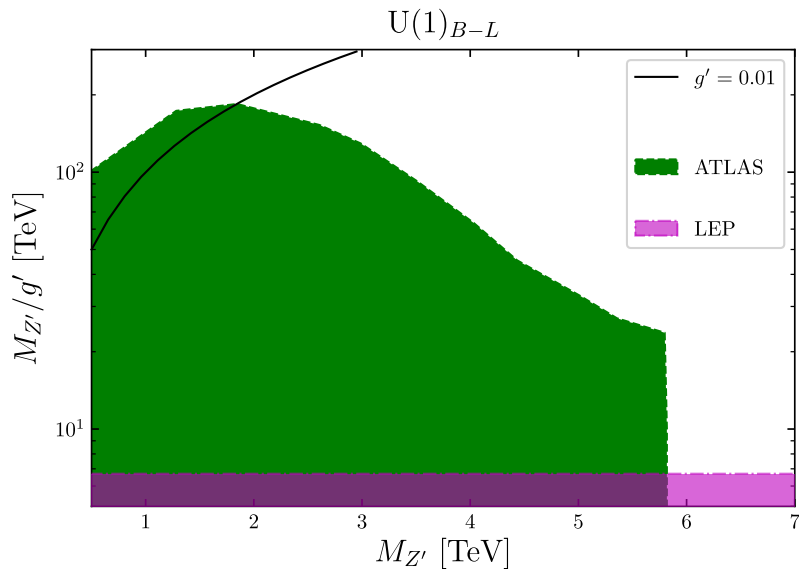
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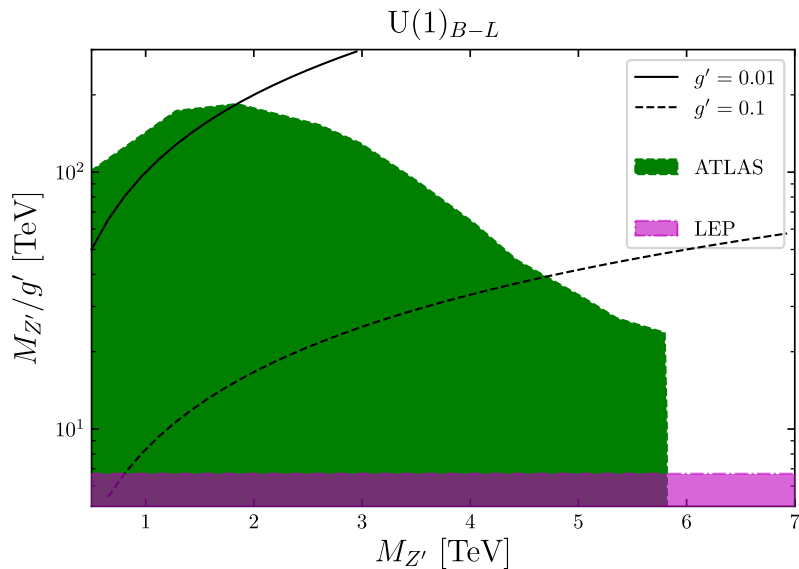
Same constraints as before



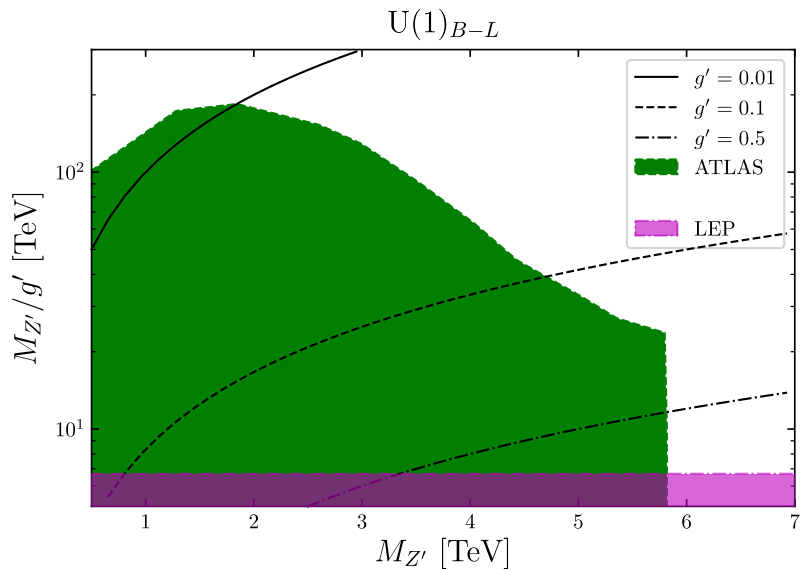
Same constraints as before



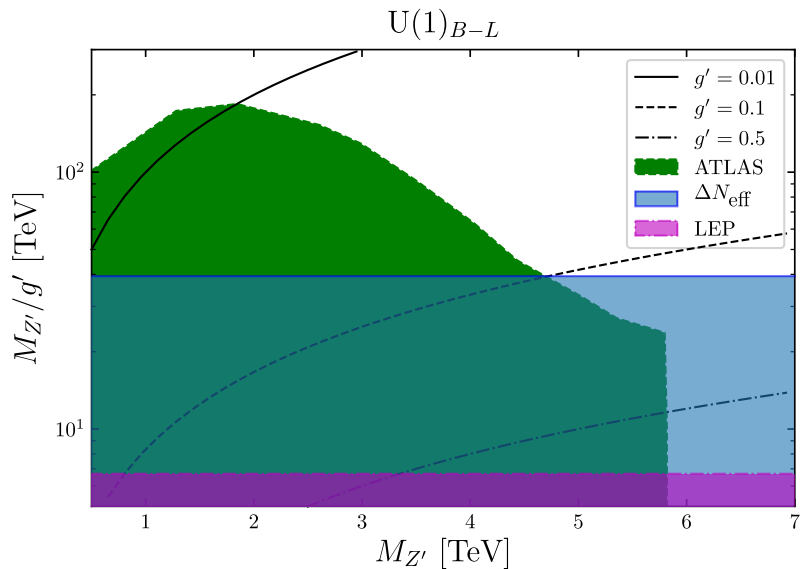
Same constraints as before



Same constraints as before



Same constraints as before



It makes sense to focus our attention on models that can account for neutrino masses and dark matter (DM) **without adhoc symmetries**

One-loop Dirac neutrino masses

A single $U(1)_X$ gauge symmetry to explain both the smallness of Dirac neutrino masses and the stability of Dirac fermion dark matter

- Spontaneously broken $U(1)_X$ generates a radiative Dirac neutrino masses
- A remnant symmetry makes the lightest field circulating the loop stable and good dark matter candidate.
- For T1-2-A: Either Singlet Doublet Dirac Dark Matter or Singlet Scalar Dark Matter with extra scalar and vector portal
- Dark symmetry for Majorana mediators

Thanks!