Multicomponent fermionic dark matter and dark baryogenesis

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Local Abelian extensions of the SM

Anomaly cancellation of a gauge $U(1)_x$ extension

Any *universal* local Abelian extension of the Standard Model can be reduced to a set of integers which must satisfy the gravitational anomaly, $[SO(1,3)]^2 U(1)_Y$, and the cubic anomaly, $[U(1)_X]^3$ conditions:

$$\sum_{\alpha=1}^{N} z_\alpha = 0, \qquad\qquad \sum_{\alpha=1}^{N} z_\alpha^3 = 0, \qquad (1)$$

- From a list of $N - 2$ integers, e.g., for N even

$$\boldsymbol{q} = [l_1, l_2, \cdots, l_n, k_1, k_2, \cdots, k_n], \qquad n = (N - 2)/2. \tag{2}$$

in the range $[-m, m]$, build two vector-like solutions of N integers,

$$\boldsymbol{x} = [l_1, k_1, \cdots k_n, -l_1, -k_1, \cdots - k_n,] \qquad \boldsymbol{y} = [0, 0, l_1, \cdots l_n, -l_1, \cdots - l_n] \tag{3}$$

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- Obtain a (some times) non vector-like solution with $z_{\max} = 2m$

$$\boldsymbol{z} = \boldsymbol{x} \oplus \boldsymbol{y} = \left(\sum_{i=1}^{N} x_i y_i^2 \right) \boldsymbol{x} + \left(\sum_{i=1}^{N} x_i^2 y_i \right) \boldsymbol{y}, \qquad (4)$$

2

- From a list of $N - 2$ integers, e.g., for N even

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The parameter space to be explored with $z_{\max} = 20$ ($m = 10$) has 96 153 non vector-like solutions

$$\# \text{ of } \boldsymbol{q} \text{ lists} = (2m+1)^{N-2} = \begin{cases} 9261 \to 3 & N = 5 \\ 194841 \to 38 & N = 6 \\ \vdots & \vdots \\ 1.6 \times 10^{13} \to 65910 & N = 12, \qquad \text{instead } 10^{19} \end{cases} \qquad (5)$$

2

- From a list of $N-2$ integers, e.g., for N even

$$\boldsymbol{q} = [l_1 = 2, l_2 = 3, k_1 = -1, k_2 = -3], \qquad n = 2. \tag{2}$$

in the range $[-3, 3]$, build two vector-like solutions of 6 integers,

$$\boldsymbol{x} = [2, -1, -3, -2, 1, 3,] \qquad\qquad \boldsymbol{y} = [0, 0, 2, \cdots 3, -2, \cdots -3] \tag{3}$$

- Obtain a (some times) non vector-like solution with $z_{max} = 2 \times 3 = 6$

$$\boldsymbol{z} = \boldsymbol{x} \oplus \boldsymbol{y} = \left(\sum_{i=1}^{N} x_i y_i^2\right) \boldsymbol{x} + \left(\sum_{i=1}^{N} x_i^2 y_i\right) \boldsymbol{y}, \tag{4}$$

The parameter space to be explored with $z_{max} = 20$ ($m = 10$) has $96\,153$ non vector-like solutions

$$\# \text{ of } \boldsymbol{q} \text{ lists} = (2m+1)^{N-2} = \begin{cases} 9261 \to 3 & N = 5 \to [1, -2, -3, 5, 5, -6] \\ 194841 \to 38 & N = 6 \\ \vdots & \vdots \\ 1.6 \times 10^{13} \to 65910 & N = 12, \qquad \text{instead } 10^{19} \end{cases} \tag{5}$$

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anomalies 0.2.5

`pip install anomalies` 📋

✓ Latest version

Released: Sep 6, 2022

Anomaly cancellation

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Project description

Anomalies

[Python package | passing] [Upload Python Package | passing] [DOI | 10.5281/zenodo.5526558]

Implement the anomaly free solution of [arXiv:1905.13729](#) [PRL]:

Obtain a numpy array z of N integers which satisfy the Diophantine equations

```
>>> z.sum()
0
>>> (z**3).sum()
0
```

The input is two lists l and k with any $(N-3)/2$ and $(N-1)/2$ integers for N odd, or $N/2-1$ and $N/2-1$ for N even $(N>4)$. The function is implemented below under the name: `free(l,k)`.

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Dataset Open Access

Set of N integers between -30 and 30 with sum and cubic sum up to zero for 4<N<13

🆔 Diego Restrepo

Anomalies

Solutions obtained with the python package: anomalies based on the method to find anomaly free solutions of the standard model extended with an Abelian Dark Symmetry with *N* right-handed singlet chiral fields described in arXiv:1905.13729 [PRL].

Data scheme

- 'l': integer lists → input to obtain the 'solution' by using the anomalies package
- 'k': integer lists → input to obtain the 'solution' by using hte anomalies package
- 'solution': list → of integers, z_i which satisfy $\sum_{i=1}^{N} z_i = 0$ and $\sum_{i=1}^{N} z_i^3 = 0$.
- 'n': integer → number of integers in 'solution', *N*.

USAGE

```
#Example of JSON file usage in Python with pandas (see also json module)
>>> import pandas as pd
>>> df=pd.read_json('solutions.json.gz')
>>> df[:2]
         l         k            solution  gcd  n
0    [1, 2]  [0, -3]   [1, 5, -7, -8, 9]    1  5
1  [-2, -1]  [0, -1]  [2, 4, -7, -9, 10]    1  5
```

Data:
2 296 615 solutions with $5 \leq N \leq 12$ integers until '|32|' [JSON]

141
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Publication date:
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DOI:
DOI 10.5281/zenodo.7380817

Keyword(s):
Anomaly free   Diophantine equations   Abelian symmetry
Gauge Symmetry

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Versions

Version v2   Sep 24, 2021
10.5281/zenodo.7380817

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$$\mathcal{L} = i\psi_i^\dagger \mathcal{D}\!\!\!/\,\psi_i - \frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \sum_{i<j} h_{ij}\psi_i\psi_j\phi^{(*)} + \mathsf{h.c} \tag{6}$$

$96\,153 \rightarrow 5\,196$ multi-component DM $(N = 8, 12) \rightarrow 142$ with three Dirac-fermion DM

$$\mathcal{L} = i\psi_i^\dagger \slashed{\mathcal{D}} \psi_i - \frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \sum_{i<j} h_{ij} \psi_i \psi_j \phi^{(*)} + \mathsf{h.c} \tag{6}$$

$96\,153 \to 5\,196$ multi-component DM   ($N = 8, 12) \to 142$ with three Dirac-fermion DM

$$\boldsymbol{z} = [1, -2, -2, 4, 5, -7, -7, 8] \to \phi = 9 \to [(1, 8), (-2, -7), (4, 5)] \tag{7}$$

Minimal secluded model with SM-singlet massive chiral fermions from SSB: $U(1)_D$

$$\mathcal{L} = i\psi_i^\dagger \slashed{\mathcal{D}}\psi_i - \frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \sum_{i<j} h_{ij}\psi_i\psi_j\phi^{(*)} + \text{h.c} \tag{6}$$

$96\,153 \rightarrow 5\,196$ multi-component DM $(N=8,12) \rightarrow 142$ with three Dirac-fermion DM

$$z = [1, -2, -2, 4, 5, -7, -7, 8] \rightarrow \phi = 9 \rightarrow [(1,8), (-2,-7), (4,5)] \tag{7}$$

$$\mathcal{L} \subset h_{(1,8)}\psi_1\psi_8\phi^*\phi^{(*)} + \sum_{a,b=1}^{2} h_{(-2a,-7b)}\psi_4\psi_{-7}\phi + h_{(4,5)}\psi_4\psi_5\phi^*\phi^{(*)} + \text{h.c.} \tag{8}$$

multi-flavor DM

$$\mathcal{L} = i\psi_i^\dagger \slashed{\mathcal{D}}\psi_i - \frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \sum_{i<j} h_{ij}\psi_i\psi_j\phi^{(*)} + \mathsf{h.c} \tag{9}$$

$96\,153 \to 5\,196$ multi-component DM $(N = 8, 12) \to 41$ with two Dirac-fermion DM

$$\mathcal{L} = i\psi_i^\dagger \slashed{\mathscr{D}}\psi_i - \frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \sum_{i<j} h_{ij}\psi_i\psi_j\phi^{(*)} + \text{h.c} \tag{9}$$

$96\,153 \to 5\,196$ multi-component DM $(N = 8, 12) \to 41$ with two Dirac-fermion DM

$$\boldsymbol{z} = [1, 2, 2, 4, -5, -5, -7, 8] \to \phi = 3 \to [(1, 2), (2, -5), (-5, 8), (4, -7)] \tag{10}$$

$$\mathcal{L} = i\psi_i^\dagger \mathcal{D}\!\!\!/\, \psi_i - \frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \sum_{i<j} h_{ij} \psi_i \psi_j \phi^{(*)} + \text{h.c} \tag{9}$$

$96\,153 \to 5\,196$ multi-component DM $(N = 8, 12) \to 41$ with two Dirac-fermion DM

$$\boldsymbol{z} = [1, 2, 2, 4, -5, -5, -7, 8] \to \phi = 3 \to [(1,2), (2,-5), (-5,8), (4,-7)] \tag{10}$$

$$\mathcal{L} \subset \Psi^{\mathsf{T}} \begin{array}{c} \\ 1 \\ 2 \\ 2 \\ -5 \\ -5 \\ 8 \end{array} \begin{bmatrix} 0 & h_{(1,2)} & h'_{(1,2)} & 0 & 0 & 0 \\ h_{(1,2)} & 0 & 0 & h_{(2,-5)} & h_{(2,-5)} & 0 \\ h'_{(1,2)} & 0 & 0 & 0 & 0 & 0 \\ 0 & h_{(2,-5)} & 0 & 0 & 0 & h_{(-5,8)} \\ 0 & h_{(2,-5)} & 0 & 0 & 0 & h'_{(-5,8)} \\ 0 & 0 & 0 & h_{(-5,8)} & h'_{(-5,8)} & 0 \end{bmatrix} \Psi \phi^{(*)} + h_{(4,-7)} \psi_4 \psi_{-7} \phi^*$$

with column labels $1 \quad 2 \quad 2 \quad -5 \quad -5 \quad 8$

$$(11)$$

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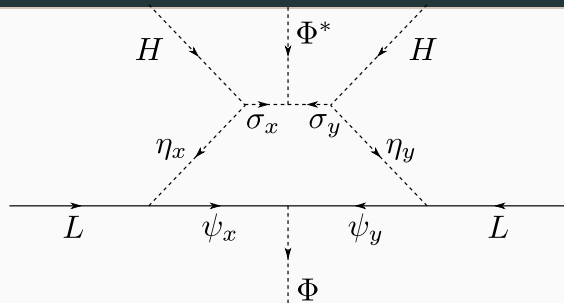
$$\frac{y}{\Lambda} LLHH$$

$$\frac{y}{\Lambda} LLHH \rightarrow \frac{y}{\Lambda} LLHH \frac{\Phi}{\Lambda} \frac{\Phi^*}{\Lambda}$$

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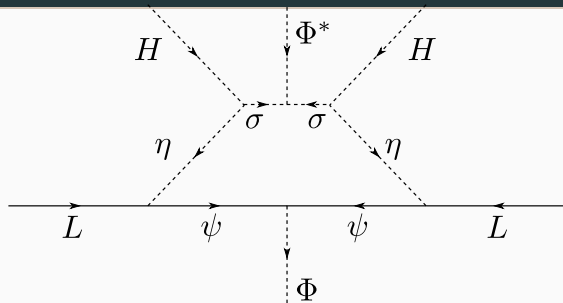
Already found by Chi-Fong Wong in arXiv:2008.08573 (subset with $N \leq 9$ and $z_{\mathsf{max}} \leq 10$)

$$\mathbf{z} = [\underbrace{1,1}_{\psi}, 2, 3, -4, -4, -5, 6] \rightarrow \phi = 2 \rightarrow [\,(1,1)_a, (2,-4), (4,-6), (4,-7)\,] \qquad (12)$$

Additional conditions to reduce multiplicity

Decrease the number of charges to be assigned to dark matter particles, ψ_i below

$$[\chi_1, \chi_2, \cdots, \psi_1, \psi_2, \cdots, \psi_{N'}]$$

Secluded case:

$$[\nu, \nu, (\nu), \psi_1, \psi_2, \cdots, \psi_{N'}]$$

$$\chi_1 \to \nu_{R1}, \cdots, \chi_{N_\nu} \to \nu_{R\,N_\nu}, \qquad 2 \leq N_\nu \leq 3,$$

$$(13)$$

$$\mathcal{L}_{\text{eff}} = h_\nu^{\alpha i} \left(\nu_{R\alpha}\right)^\dagger \epsilon_{ab} L_i^a H^b \left(\frac{\Phi^*}{\Lambda}\right)^\delta + \text{H.c.}, \qquad \text{with } i = 1, 2, 3,$$

Φ is the complex singlet scalar responsible for the SSB of the anomaly-free gauge symmetry and give mass to *all* ψ_i

$$\phi = -\frac{\nu}{\delta}, \qquad (14)$$

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Decrease the number of charges to be assigned to dark matter particles, ψ_i below

$$[\chi_1, \chi_2, \cdots, \psi_1, \psi_2, \cdots, \psi_{N'}]$$

Secluded case:

Active case:

$$[\nu, \nu, (\nu), \psi_1, \psi_2, \cdots, \psi_{N'}]$$

$$[\nu, \nu, (\nu), m, m, m, \psi_1, \psi_2, \cdots, \psi_{N'}]$$

$$\chi_1 \to \nu_{R1}, \cdots, \chi_{N_\nu} \to \nu_{R\,N_\nu}, \qquad 2 \le N_\nu \le 3, \quad X(L_i) = -L, \ X(H) = h \qquad \to m = L - h$$

$$(13)$$

$$\mathcal{L}_{\text{eff}} = h_\nu^{\alpha i} \left(\nu_{R\alpha} \right)^\dagger \epsilon_{ab} L_i^a H^b \left(\frac{\Phi^*}{\Lambda} \right)^\delta + \text{H.c.}, \qquad \text{with } i = 1, 2, 3,$$

Φ is the complex singlet scalar responsible for the SSB of the anomaly-free gauge symmetry and give mass to *all* ψ_i $\to [-4, -4, 1, 1, 1, 5]$

$$\phi = -\frac{(\nu + m)}{\delta}, \qquad (14)$$
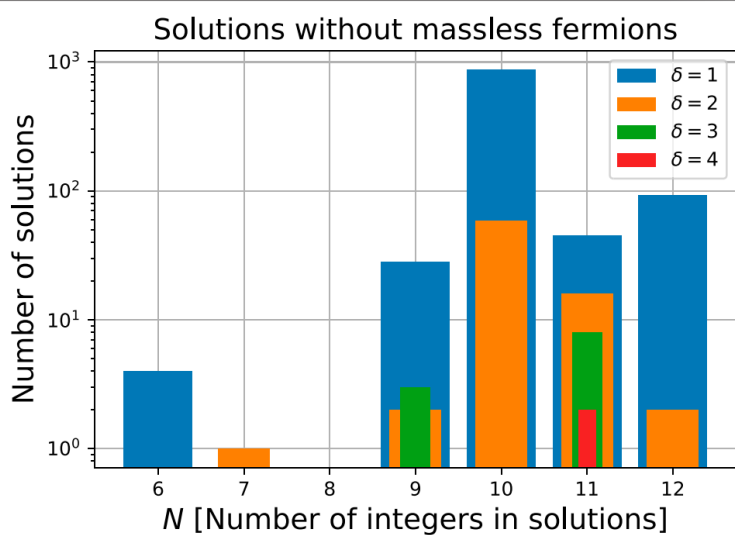
FIGURE 1 | Distribution of solutions with N integers to the Diophantine **Eq. 1** which allow the effective Dirac neutrino mass operator at $d = (4 + \delta)$ for at least two right-handed neutrinos and have non-vanishing Dirac o Majorana masses for the other SM-singlet chiral fermions in the solution.
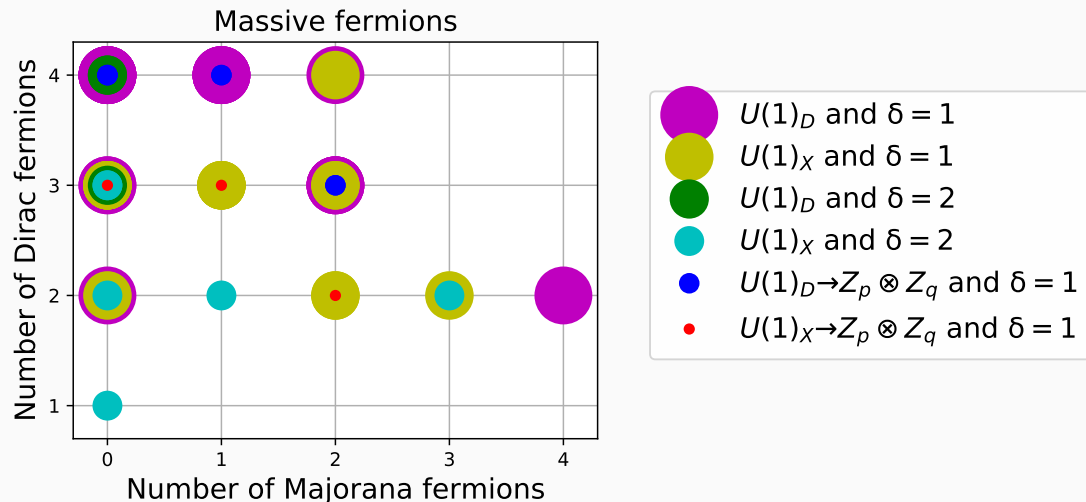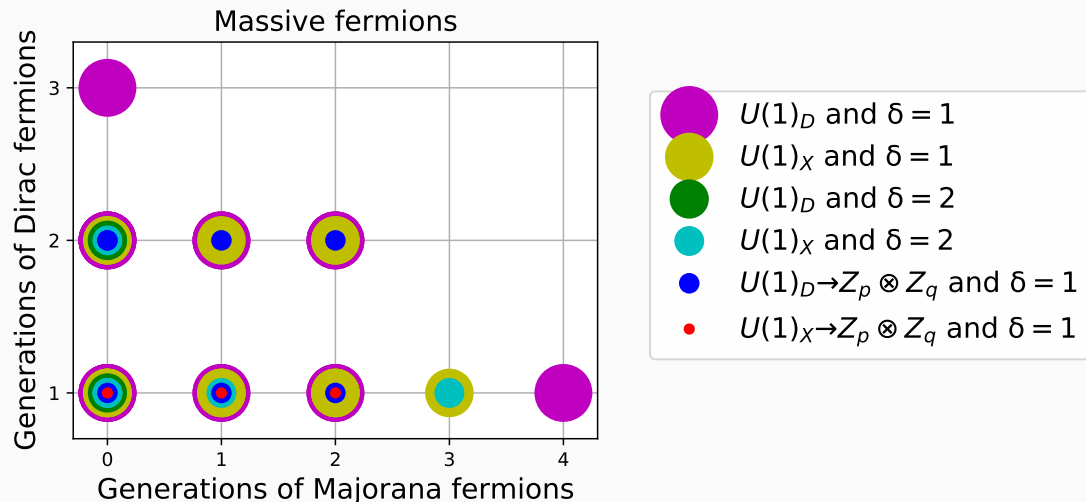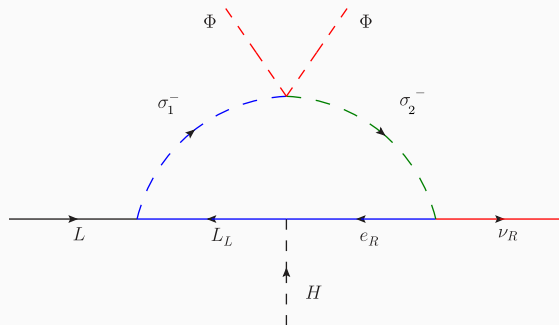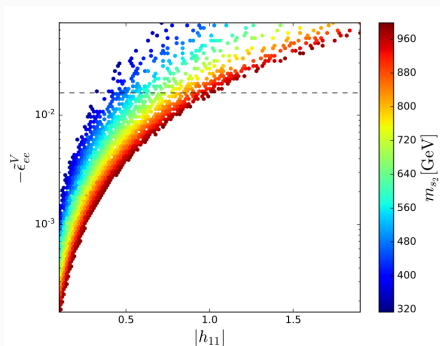
- Active symmetry $m = 3$

$$(-5, -5, 3, 3, 3, -7, 8)$$

$U(1)_x$ selection with Dirac-fermionic DM

- Active symmetry $m = 3$
- Effective neutrino mass $\delta = 2 \rightarrow \nu = -5$:
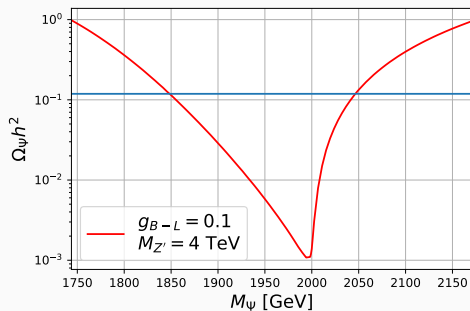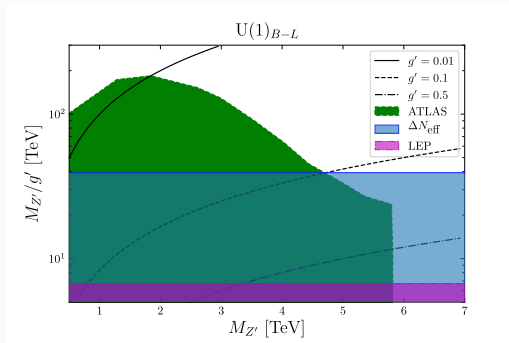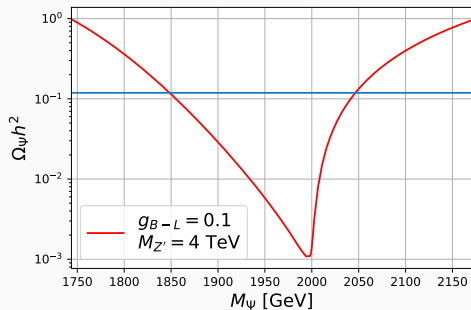
$$(-5, -5, 3, 3, 3, -7, 8)$$

- Active symmetry $m = 3$
- Effective neutrino mass $\delta = 2 \rightarrow \nu = -5$:
- Active symmetry: $m = 3 \rightarrow \phi = -(\nu + m)/\delta = 1$
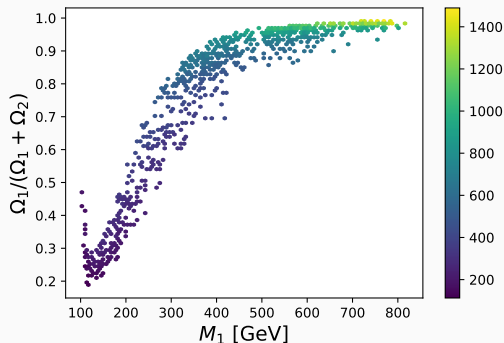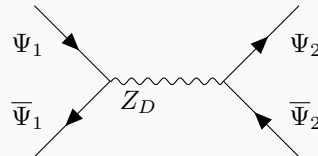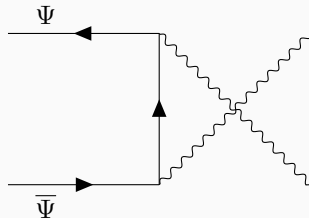
$$(-5, -5, 3, 3, 3, -7, 8)$$

- Active symmetry $m = 3$
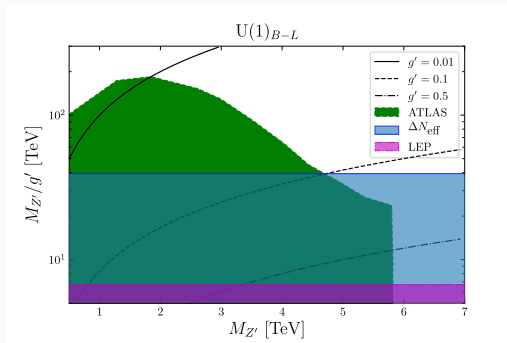
- Effective neutrino mass $\delta = 2 \rightarrow \nu = -5$:

- Active symmetry: $m = 3 \rightarrow \phi = -(\nu + m)/\delta = 1$

- Dirac-fermionic DM: $(\psi_L)^\dagger \psi_R'' \Phi^* \rightarrow z_6 = -7,\ z_7 = 8$
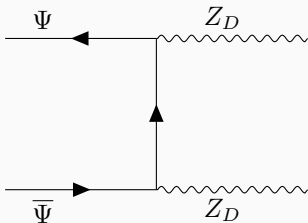
$$(-5, -5, 3, 3, 3, -7, 8)$$

- Active symmetry $m = 3$

- Effective neutrino mass $\delta = 2 \to \nu = -5$:

- Active symmetry: $m = 3 \to \phi = -(\nu + m)/\delta = 1$

- Dirac-fermionic DM: $(\psi_L)^\dagger \psi_R'' \Phi^* \to z_6 = -7,\; z_7 = 8$

$$(-5, -5, 3, 3, 3, -7, 8)$$

Beyond SM-fermion singlets

Standard model extended with $U(1)_{\mathcal{X}=X \text{ or } D}$ gauge symmetry

Fields	$SU(2)_L$	$U(1)_Y$	$U(1)_{\mathcal{X}=D \text{ or } X}$
$Q_i^\dagger$	**2**	$-1/6$	Q
d_{Ri}	**1**	$-1/2$	d
u_{Ri}	**1**	$+2/3$	u
$L_i^\dagger$	**2**	$+1/2$	L
e_{Ri}	**1**	-1	e
H	**2**	$1/2$	h
χ_α	**1**	0	z_α
Φ	**1**	0	ϕ

Table 1: LHC: hadronic production and dileptonic decay

$$i = 1, 2, 3, \ \alpha = 1, 2, \ldots, N'$$

Standard model extended with $U(1)_{\mathcal{X}=L \text{ or } B}$ gauge symmetry

Fields	$SU(2)_L$	$U(1)_Y$	$U(1)_{\mathcal{X}=B \text{ or } L}$
$Q_i^\dagger$	**2**	$-1/6$	Q
d_{Ri}	**1**	$-1/2$	d
u_{Ri}	**1**	$+2/3$	u
$L_i^\dagger$	**2**	$+1/2$	L
e_{Ri}	**1**	-1	e
H	**2**	$1/2$	$h = 0$
χ_α	**1**	0	z_α
$(L'_L)^\dagger$	**2**	$1/2$	$-x'$
L''_R	**2**	$-1/2$	x''
e'_R	**1**	-1	x'
$(e''_L)^\dagger$	**1**	1	$-x''$
Φ	**1**	0	ϕ
S	**1**	0	s

Table 1: minimal set of new fermion content: $L = e = 0$ for $\mathcal{X} = B$. Or $Q = u = d = 0$ for $\mathcal{X} = L$.
$i = 1, 2, 3$, $\alpha = 1, 2, \ldots, N'$

The anomaly-cancellation conditions on $[SU(3)_c]^2\, U(1)_X$, $[SU(2)_L]^2\, U(1)_X$, $[U(1)_Y]^2\, U(1)_X$, allow us to express three of the X-charges in terms of the others

$$u = -e - \frac{2}{3}L - \frac{1}{9}\left(x' - x''\right), \quad d = e + \frac{4}{3}L - \frac{1}{9}\left(x' - x''\right), \quad Q = -\frac{1}{3}L + \frac{1}{9}\left(x' - x''\right),$$
(15)

while the $[U(1)_X]^2\, U(1)_Y$ anomaly condition reduces to

$$(e + L)(x' - x'') = 0.$$
(16)

- Previously: $x' = x''$
- We choose instead ($h = 0$):

$$e = -L,$$
(17)

so that (L is still a free parameter)

$$Q = -u = -d = -\frac{1}{3}L + \frac{1}{9}\left(x' - x''\right).$$
(18)

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The gravitational anomaly, $[SO(1,3)]^2\, U(1)_Y$, and the cubic anomaly, $[U(1)_X]^3$, can be written as the following system of Diophantine equations, respectively,

$$\sum_{\alpha=1}^{N} z_\alpha = 0\,, \qquad\qquad \sum_{\alpha=1}^{N} z_\alpha^3 = 0\,, \qquad\qquad (19)$$

where

$$z_1 = -x'\,, \qquad\qquad\qquad\qquad\qquad\qquad z_2 = x''\,,$$

$$z_{2+i} = L\,, \quad i = 1, 2, 3 \qquad\qquad\qquad (20)$$

$\rightarrow$

$$9Q = -\sum_{\alpha=1}^{5} z_\alpha = -x' + x'' + L + L + L\,, \qquad\qquad (21)$$

$L = 0 \rightarrow U(1)_B$ but $Q = 0 \nrightarrow U(1)_L$

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- $L = 0$

$$(5, 5, -3, -2, 1, -6)$$

- $L = 0$
- Effective Dirac neutrino masses: $\phi = -\nu = -5$

$$(5, 5, -3, -2, 1, -6)$$

- $L = 0$
- Effective Dirac neutrino masses: $\phi = -\nu = -5$
- Electroweak-scale vector-like fermions:
  $(L'_L)^\dagger L''_R \Phi^* \rightarrow x' = -1,\ x'' = 6$

$$(5, 5, -3, -2, 1, -6)$$

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  $(L'_L)^\dagger L''_R \Phi^* \to x' = -1,\ x'' = 6$
- Dirac-fermionic DM: $(\chi_L)^\dagger \chi''_R \Phi^* \to z_3 = -3,\ z_4 = -2$

$$(5, 5, -3, -2, 1, -6)$$

- $L = 0$
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  $(L'_L)^\dagger L''_R \Phi^* \to x' = -1,\ x'' = 6$
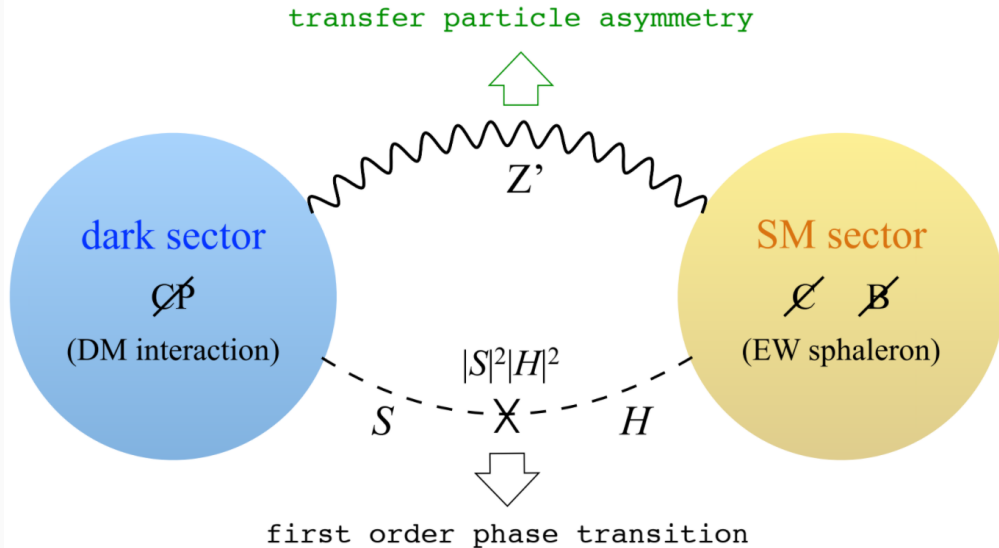- Dirac-fermionic DM: $(\chi_L)^\dagger \chi''_R \Phi^* \to z_3 = -3,\ z_4 = -2$

959 solutions

$$(5, 5, -3, -2, 1, -6)$$

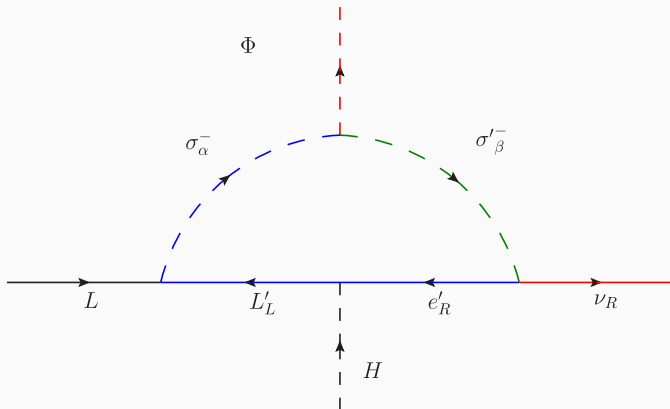transfer particle asymmetry

Z'

dark sector
$C\!\!\!/P\!\!\!/$
(DM interaction)

SM sector
$C\!\!\!/$ $B\!\!\!/$
(EW sphaleron)

$|S|^2|H|^2$
S H

first order phase transition

from: [1] Carena, *et al*, arXiv:1908.04818 [PRD] (see also: arXiv:1811.09719 [PRL])

with Andrés Rivera (UdeA) and Walter Tangarife (Loyola U.)
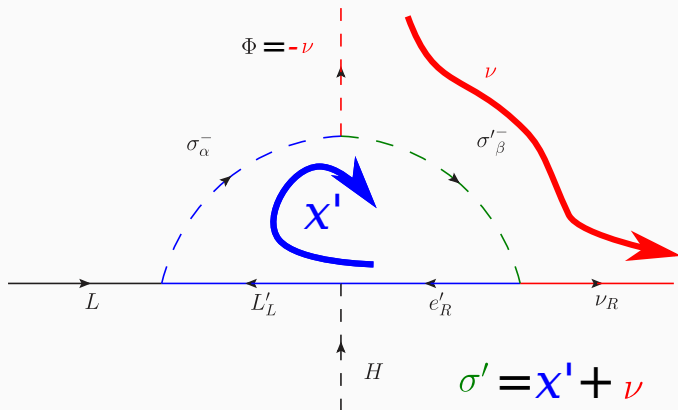
with Andrés Rivera (UdeA) and Walter Tangarife (Loyola U.)

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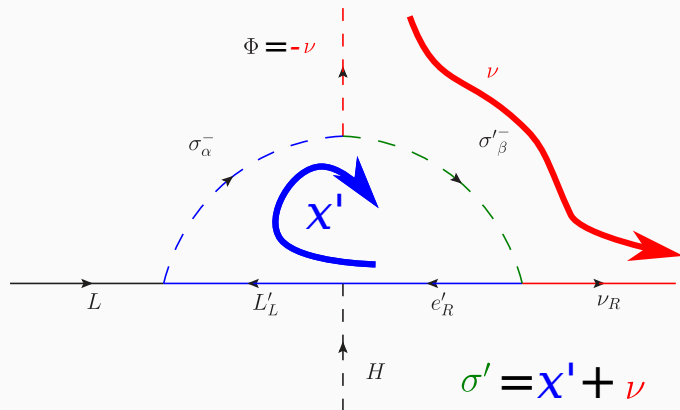
Field	$SU(2)_L$	$U(1)_Y$	$U(1)_B$
u_{Ri}	**1**	2/3	$u = 1/3$
d_{Ri}	**1**	$-1/3$	$d = 1/3$
$(Q_i)^\dagger$	**2**	$-1/6$	$Q = -1/3$
$(L_i)^\dagger$	**2**	1/2	$L = 0$
e_R	**1**	-1	$e = 0$
$(L'_L)^\dagger$	**2**	1/2	$-x' = -3/5$
e'_R	**1**	-1	$x' = 3/5$
L''_R	**2**	$-1/2$	$x'' = 18/5$
$(e''_L)^\dagger$	**1**	1	$-x'' = -18/5$
$\nu_{R,1}$	**1**	**0**	-3
$\nu_{R,2}$	**1**	**0**	-3
χ_R	**1**	0	6/5
$(\chi_L)^\dagger$	**1**	0	9/5
H	**2**	1/2	0
S	**1**	0	3
Φ	**1**	**0**	**3**
σ_α^-	**1**	**1**	**3/5**
σ'^-_α	**1**	-1	$-12/5$

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- SARAH→SPheno→MicroMegas
- η_B calculation code
- Python notebook with the scan

Conclusions
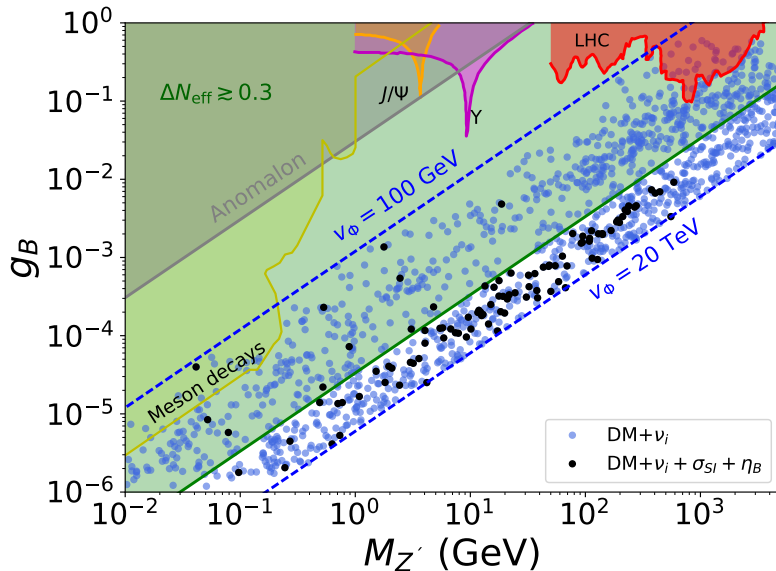
A methodology to find all the *universal* gauge Abelian extensions of the standard model is designed.

All of the extensions can be reformulated as the solution of

$$\sum_{\alpha=1}^{N} z_\alpha = 0 \, , \qquad\qquad \sum_{\alpha=1}^{N} z_\alpha^3 = 0 \, ,$$

which we fully scan until $N = 12$ and $|z_{\max}| = 20$

Once the physical conditions are stablished, the full set of self-consistent models are found from a simple data-analysis procedure. With enough freedom to solve several phenomenological problems.