



UNIVERSIDAD DE ANTIOQUIA
1803

Secluded Abelian extensions of the SM

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Anomaly cancellation of a gauge $U(1)_X$ extension

Any *universal* local Abelian extension of the Standard Model can be reduced to a set of integers

$$\mathbf{S} = [z_1, z_2, \dots, z_N] ,$$

which must satisfy the gravitational anomaly, $[SO(1,3)]^2 U(1)_Y$, and the cubic anomaly, $[U(1)_X]^3$ conditions:

$$\sum_{\alpha=1}^N z_{\alpha} = 0 , \qquad \sum_{\alpha=1}^N z_{\alpha}^3 = 0 , \qquad (1)$$

September 24, 2021

Dataset

Open Access

Set of N integers between -30 and 30 with sum and cubic sum up to zero for $4 < N < 13$

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Anomalies

Solutions obtained with the python package: [anomalies](#) based on the method to find anomaly free solutions of the standard model extended with an Abelian Dark Symmetry with N right-handed singlet chiral fields described in [arXiv:1905.13729](#) [PRL].

Data scheme

- 'l': integer lists → input to obtain the 'solution' by using the [anomalies](#) package
- 'k': integer lists → input to obtain the 'solution' by using the [anomalies](#) package
- 'solution': list → of integers, z_i which satisfy $\sum_{i=1}^N z_i = 0$ and $\sum_{i=1}^N z_i^3 = 0$.
- 'n': integer → number of integers in 'solution', N .

USAGE

```
#Example of JSON file usage in Python with pandas (see also json module)
>>> import pandas as pd
>>> df=pd.read_json('solutions.json.gz')
>>> df[:2]
   l      k      solution gcd n
0  [1, 2]  [0, -3]  [1, 5, -7, -8, 9]  1  5
1  [-2, -1] [0, -1]  [2, 4, -7, -9, 10]  1  5
```

Data:

2 296 615 solutions with $5 \leq N \leq 12$ integers until 'j32' [JSON]

141

views

351

downloads

[See more details...](#)

Indexed in

OpenAIRE

Publication date:

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DOI:

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Keyword(s):

[Anomaly free](#) [Diophantine equations](#) [Abelian symmetry](#)
[Gauge Symmetry](#)

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Versions

Version v2

Sep 24, 2021

[10.5281/zenodo.7380817](#)

Secluded gauge $U(1)_D$ without vector-like fermions:

$$\mathbf{S} = [\chi_1, \chi_2, \dots, \psi_1, \psi_2, \dots, \psi_{N'}]$$

- *Higgs mechanism*: Singlet scalar ϕ acquires a vev and give mass to the dark photon

$$\mathcal{L} = i\psi_a^\dagger \overline{\sigma}^\mu \left(\partial_\mu - ig_D Z_\mu^D \right) \psi_a - \frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \sum_{a < b} h_{ab} \psi_a \psi_b \phi^{(*)} + \text{h.c.} - V(\phi). \quad (2)$$

- z_α are the charges of SM-singlet left-handed chiral fermions with $N \geq 5$
 - χ_i *massless fermions* with $i = 1, \dots, N'$ with $N' \leq N$
 - ψ_a *multi-component dark matter*: massive after the spontaneous symmetry breaking of $U(1)_D$ with $a = N' + 1, \dots, N$
- *Larger parameter space*: Dark photon exclusions instead of Z'

Decrease the number of charges to be assigned to dark matter particles, ψ_i below

$$[\chi_1, \chi_2, \dots, \psi_1, \psi_2, \dots, \psi_N]$$

Secluded case:

$$[\nu, \nu, (\nu), \psi_1, \psi_2, \dots, \psi_N]$$

$$\chi_1 \rightarrow \nu_{R1}, \dots, \chi_{N_\nu} \rightarrow \nu_{RN_\nu}, \quad 2 \leq N_\nu \leq 3,$$

$$\mathcal{L}_{\text{eff}} = h_{\nu}^{ij} (\nu_{Ri})^{\dagger} \epsilon_{ab} L_j^a H^b \left(\frac{\phi^*}{\Lambda} \right)^{\delta} + \text{H.c.}, \quad \text{with } i, j = 1, 2, 3,$$

ϕ is the complex singlet scalar responsible for the SSB of the anomaly-free gauge symmetry and give mass to all ψ_a

$$\phi = -\frac{\nu}{\delta},$$

Decrease the number of charges to be assigned to dark matter particles, ψ_i below

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Decrease the number of charges to be assigned to dark matter particles, ψ_i below

$$[\chi_1, \chi_2, \dots, \psi_1, \psi_2, \dots, \psi_N]$$

Secluded case:

$$[5, 5, -3, -2, 1, -6]$$

$$\chi_1 \rightarrow \nu_{R1}, \chi_2 \rightarrow \nu_{R2}, \quad N_\nu = 2,$$

$$\mathcal{L}_{\text{eff}} = h_\nu^{aj} (\nu_{Ra})^\dagger \epsilon_{bc} L_j^b H^c \left(\frac{\phi^*}{\Lambda} \right) + \text{H.c.}, \quad \text{with } j = 1, 2, 3,$$

ϕ is the complex singlet scalar responsible for the SSB of the anomaly-free gauge symmetry and give mass to all ψ_a

$$\phi = -\nu = -5,$$

Decrease the number of charges to be assigned to dark matter particles, ψ_i below

$$[\chi_1, \chi_2, \dots, \psi_1, \psi_2, \dots, \psi_N]$$

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ϕ is the complex singlet scalar responsible for the SSB of the anomaly-free gauge symmetry and give mass to all ψ_a

$$\phi = -\nu = -5,$$

Minimal secluded model with D-5 effective Dirac neutrino masses

$$\mathcal{L} = i\psi_i^\dagger \not{D}\psi_i - \frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \sum_{i<j} h_{ij}\psi_i\psi_j\phi^{(*)} + \text{h.c.} - V(\phi). \quad (3)$$

multi-component DM with two Dirac-fermion DM

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multi-component DM with two Dirac-fermion DM

$$\mathbf{z} = [5, 5, -3, -2, 1, -6] \rightarrow \phi = -5 \rightarrow [(5, 5), (-3, -2), (1, -6)] \quad (4)$$

Minimal secluded model with D-5 effective Dirac neutrino masses

$$\mathcal{L} = i\psi_i^\dagger \not{D}\psi_i - \frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \sum_{i<j} h_{ij}\psi_i\psi_j\phi^{(*)} + \text{h.c.} - V(\phi). \quad (3)$$

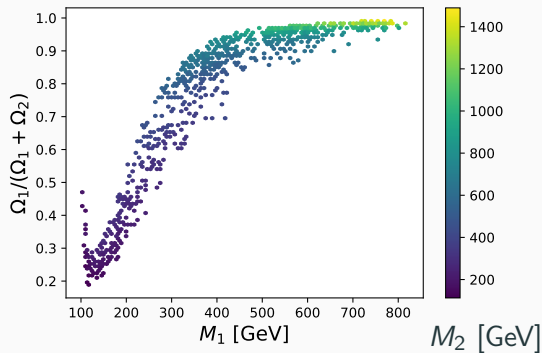
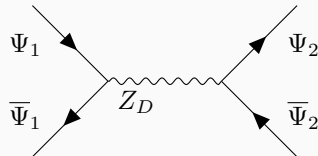
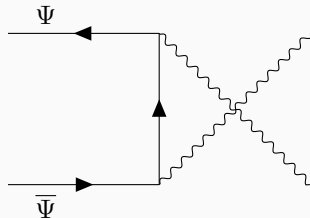
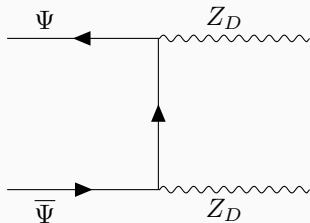
multi-component DM with two Dirac-fermion DM

$$z = [5, 5, -3, -2, 1, -6] \rightarrow \phi = -5 \rightarrow [(5, 5), (-3, -2), (1, -6)] \quad (4)$$

$$\mathcal{L} \subset h_{(-3,-2)}\psi_{-3}\psi_{-2}\phi + h_{(1,-5)}\psi_1\psi_{-6}\phi + \text{h.c.} \quad (5)$$

$U(1)_D$: two dark matter candidates without kinetic mixing: $Z_D \rightarrow \bar{\nu}\nu$

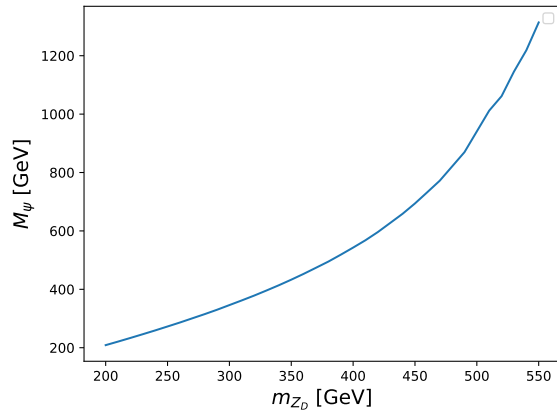
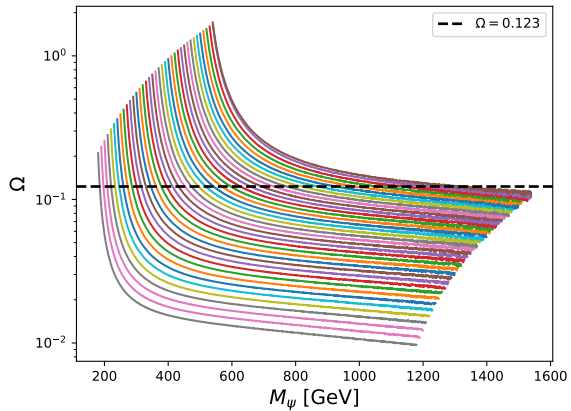
in progress...



$M_{Z_D} < 200$, GeV

(xBit scan)

$M_2 \gg M_1$ and $g_D = 0.1$



$$\mathcal{L} = i\psi_i^\dagger \not{D}\psi_i - \frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \sum_{i<j} h_{ij}\psi_i\psi_j\phi^{(*)} + \text{h.c.} - V(\phi). \quad (6)$$

96 153 \rightarrow 5 196 **multi-component DM** ($N = 8, 12$) \rightarrow 142 with three Dirac-fermion DM

Minimal secluded model with SM-singlet massive chiral fermions from SSB: $U(1)_D$

$$\mathcal{L} = i\psi_i^\dagger \not{D}\psi_i - \frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \sum_{i<j} h_{ij}\psi_i\psi_j\phi^{(*)} + \text{h.c.} - V(\phi). \quad (6)$$

96 153 \rightarrow 5 196 **multi-component DM** ($N = 8, 12$) \rightarrow 142 with three Dirac-fermion DM

$$\mathbf{z} = [1, -2, -2, 4, 5, -7, -7, 8] \rightarrow \phi = 9 \rightarrow [(1, 8), (-2, -7), (4, 5)] \quad (7)$$

Minimal secluded model with SM-singlet massive chiral fermions from SSB: $U(1)_D$

$$\mathcal{L} = i\psi_i^\dagger \not{D} \psi_i - \frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \sum_{i < j} h_{ij} \psi_i \psi_j \phi^{(*)} + \text{h.c.} - V(\phi). \quad (6)$$

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$$\mathbf{z} = [1, -2, -2, 4, 5, -7, -7, 8] \rightarrow \phi = 9 \rightarrow [(1, 8), (-2, -7), (4, 5)] \quad (7)$$

$$\mathcal{L} \subset h_{(1,8)} \psi_1 \psi_8 \phi^* \phi^{(*)} + \underbrace{\sum_{a,b=1}^2 h_{(-2a,-7b)} \psi_2 \psi_{-7b}}_{\text{multi-flavor DM}} + h_{(4,5)} \psi_4 \psi_5 \phi^* \phi^{(*)} + \text{h.c.} \quad (8)$$

Simplest secluded model with SM-singlet massive chiral fermions from SSB: $U(1)_D$

$$\mathcal{L} = i\psi_i^\dagger \not{D}\psi_i - \frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \sum_{i<j} h_{ij}\psi_i\psi_j\phi^{(*)} + \text{h.c} \quad (9)$$

96 153 \rightarrow 5 196 multi-component DM ($N = 8, 12$) \rightarrow 41 with two Dirac-fermion DM

Simplest secluded model with SM-singlet massive chiral fermions from SSB: $U(1)_D$

$$\mathcal{L} = i\psi_i^\dagger \not{D}\psi_i - \frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \sum_{i<j} h_{ij}\psi_i\psi_j\phi^{(*)} + \text{h.c} \quad (9)$$

96 153 \rightarrow 5 196 **multi-component DM** ($N=8,12$) \rightarrow 41 with two Dirac-fermion DM

$$\mathbf{z} = [1, 2, 2, 4, -5, -5, -7, 8] \rightarrow \phi = 3 \rightarrow [(1, 2), (2, -5), (-5, 8), (4, -7)] \quad (10)$$

Simplest secluded model with SM-singlet massive chiral fermions from SSB: $U(1)_D$

$$\mathcal{L} = i\psi_i^\dagger \not{D}\psi_i - \frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \sum_{i < j} h_{ij}\psi_i\psi_j\phi^{(*)} + \text{h.c} \quad (9)$$

96 153 \rightarrow 5 196 **multi-component DM** ($N = 8, 12$) \rightarrow 41 with two Dirac-fermion DM

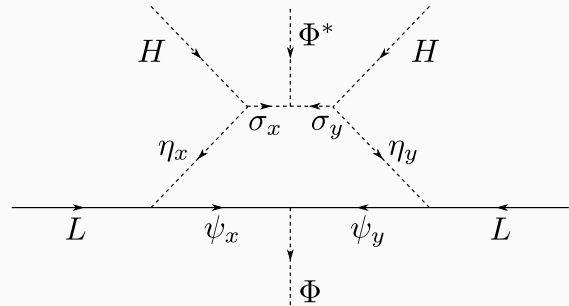
$$\mathbf{z} = [1, 2, 2, 4, -5, -5, -7, 8] \rightarrow \phi = 3 \rightarrow [(1, 2), (2, -5), (-5, 8), (4, -7)] \quad (10)$$

$$\mathcal{L} \subset \Psi^T \begin{bmatrix} & 1 & 2 & 2 & -5 & -5 & 8 \\ 1 & 0 & h_{(1,2)} & h'_{(1,2)} & 0 & 0 & 0 \\ 2 & h_{(1,2)} & 0 & 0 & h_{(2,-5)} & h_{(2,-5)} & 0 \\ 2 & h'_{(1,2)} & 0 & 0 & 0 & 0 & 0 \\ -5 & 0 & h_{(2,-5)} & 0 & 0 & 0 & h_{(-5,8)} \\ -5 & 0 & h_{(2,-5)} & 0 & 0 & 0 & h'_{(-5,8)} \\ 8 & 0 & 0 & 0 & h_{(-5,8)} & h'_{(-5,8)} & 0 \end{bmatrix} \Psi \phi^{(*)} + h_{(4,-7)}\psi_4\psi_{-7}\phi^*$$

$$\frac{y}{\Lambda} LLHH$$

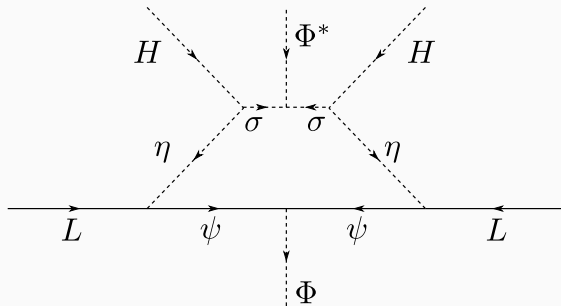
Scotogenic Majorana neutrino masses and mixings

$$\frac{y}{\Lambda} LLHH \rightarrow \frac{y}{\Lambda} LLHH \frac{\Phi}{\Lambda} \frac{\Phi^*}{\Lambda}$$



Scotogenic Majorana neutrino masses and mixings

$$\frac{y}{\Lambda} LLHH \rightarrow \frac{y}{\Lambda} LLHH \frac{\Phi}{\Lambda} \frac{\Phi^*}{\Lambda}$$



Already found by Chi-Fong Wong in arXiv:2008.08573 (subset with $N \leq 9$ and $z_{\max} \leq 10$)

$$z = [\underbrace{1, 1}_{\psi}, 2, 3, -4, -4, -5, 6] \rightarrow \phi = 2 \rightarrow [(1, 1)_a, (2, -4), (4, -6), (4, -7)] \quad (11)$$

Additional conditions to reduce multiplicity

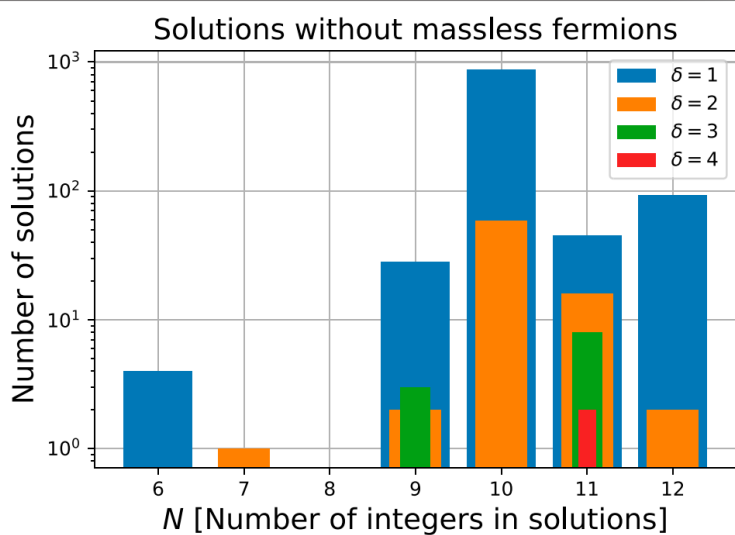
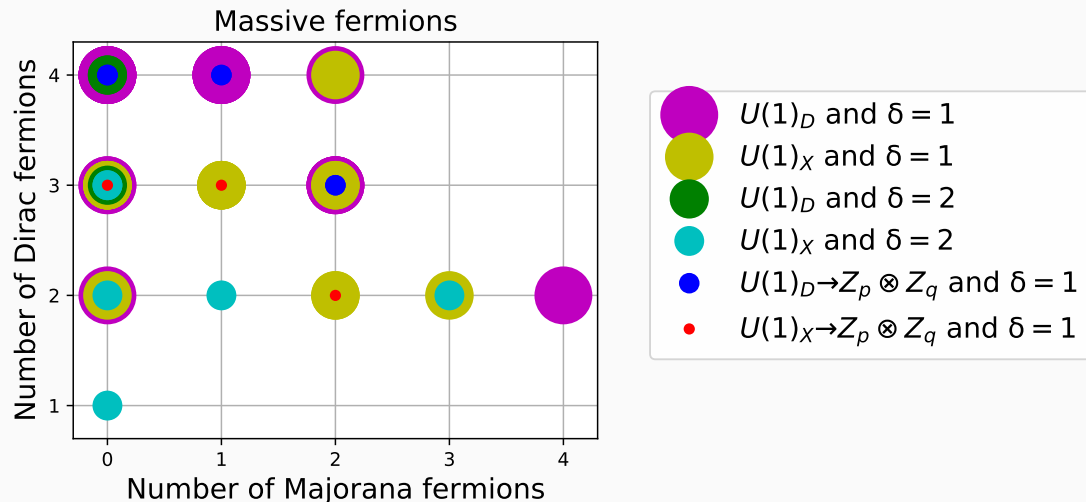
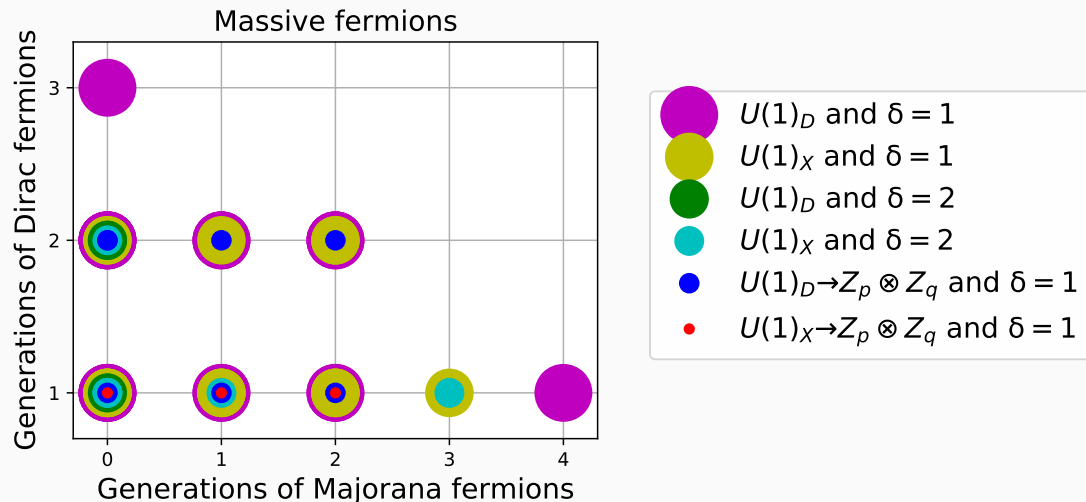


FIGURE 1 | Distribution of solutions with N integers to the Diophantine Eq. 1 which allow the effective Dirac neutrino mass operator at $d = (4 + \delta)$ for at least two right-handed neutrinos and have non-vanishing Dirac or Majorana masses for the other SM-singlet chiral fermions in the solution.

Multi-component dark matter



Multi-flavor dark matter



- Active symmetry $m = 3$

$$(-5, -5, 3, 3, 3, -7, 8)$$

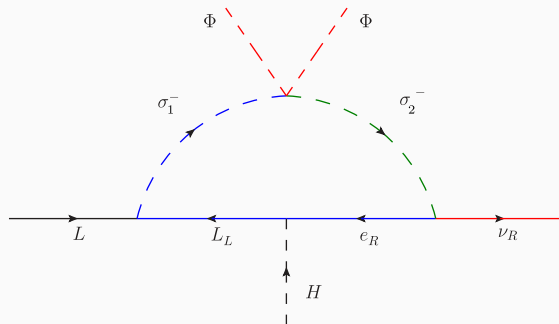
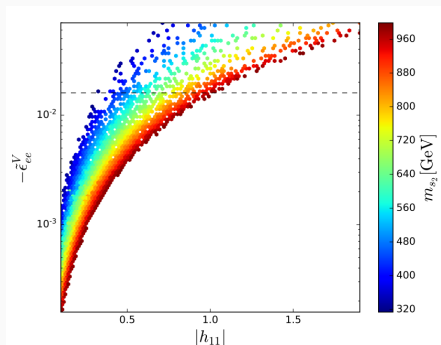
$U(1)_X$ selection with Dirac-fermionic DM

- Active symmetry $m = 3$
- Effective neutrino mass $\delta = 2 \rightarrow \nu = -5$:

$$(-5, -5, 3, 3, 3, -7, 8)$$

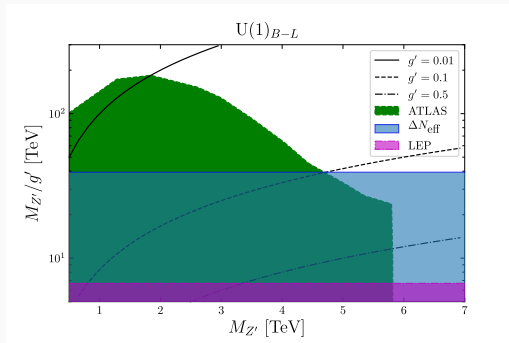
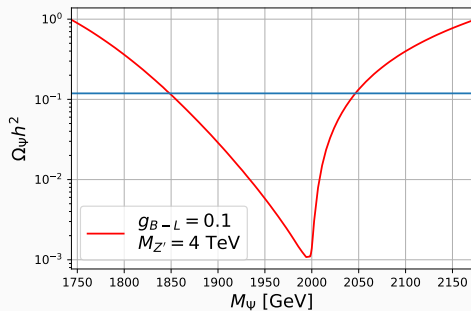
- Active symmetry $m = 3$
- Effective neutrino mass $\delta = 2 \rightarrow \nu = -5$:
- Active symmetry: $m = 3 \rightarrow \phi = -(\nu + m)/\delta = 1$

$$(-5, -5, 3, 3, 3, -7, 8)$$



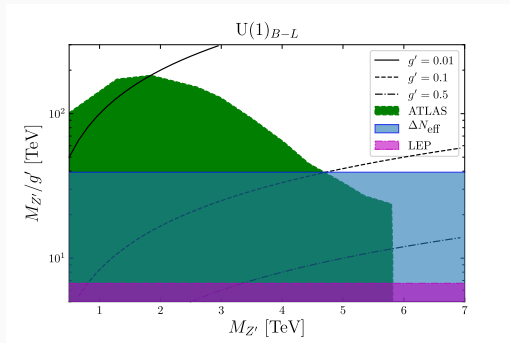
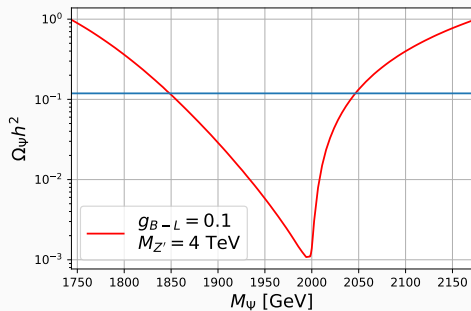
- Active symmetry $m = 3$
- Effective neutrino mass $\delta = 2 \rightarrow \nu = -5$:
- Active symmetry: $m = 3 \rightarrow \phi = -(\nu + m)/\delta = 1$
- Dirac-fermionic DM: $(\psi_L)^\dagger \psi_R'' \Phi^* \rightarrow z_6 = -7, z_7 = 8$

$$(-5, -5, 3, 3, 3, -7, 8)$$



- Active symmetry $m = 3$
- Effective neutrino mass $\delta = 2 \rightarrow \nu = -5$:
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$$(-5, -5, 3, 3, 3, -7, 8)$$



Beyond SM-fermion singlets

Standard model extended with $U(1)_{\mathcal{X}=\textcolor{blue}{X} \text{ or } \textcolor{red}{D}}$ gauge symmetry

Fields	$SU(2)_L$	$U(1)_Y$	$U(1)_{\mathcal{X}=\textcolor{red}{D} \text{ or } \textcolor{blue}{X}}$
Q_i^\dagger	2	$-1/6$	$\textcolor{red}{Q}$
d_{Ri}	1	$-1/2$	$\textcolor{red}{d}$
u_{Ri}	1	$+2/3$	$\textcolor{red}{u}$
L_i^\dagger	2	$+1/2$	$\textcolor{blue}{L}$
e_{Ri}	1	-1	$\textcolor{blue}{e}$
H	2	$1/2$	h
χ_α	1	0	z_α

Φ	1	0	ϕ
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Table 1: LHC: hadronic production and dileptonic decay

$$i = 1, 2, 3, \alpha = 1, 2, \dots, N'$$

Standard model extended with $U(1)_{\mathcal{X}=L \text{ or } B}$ gauge symmetry

Fields	$SU(2)_L$	$U(1)_Y$	$U(1)_{\mathcal{X}=B \text{ or } L}$
Q_i^\dagger	2	$-1/6$	Q
d_{Ri}	1	$-1/2$	d
u_{Ri}	1	$+2/3$	u
L_i^\dagger	2	$+1/2$	L
e_{Ri}	1	-1	e
H	2	$1/2$	$h = 0$
χ_α	1	0	z_α
$(L'_L)^\dagger$	2	$1/2$	$-\mathcal{X}'$
L''_R	2	$-1/2$	\mathcal{X}''
e'_R	1	-1	\mathcal{X}'
$(e''_L)^\dagger$	1	1	$-\mathcal{X}''$
Φ	1	0	ϕ
S	1	0	s

Table 1: minimal set of new fermion content: $L = e = 0$ for $\mathcal{X} = B$. Or $Q = u = d = 0$ for $\mathcal{X} = L$.
 $i = 1, 2, 3, \alpha = 1, 2, \dots, N'$

Anomaly cancellation: $\mathcal{X} = L$ or B : beyond SM-singlet fermions

The anomaly-cancellation conditions on $[SU(3)_c]^2 U(1)_X$, $[SU(2)_L]^2 U(1)_X$, $[U(1)_Y]^2 U(1)_X$, allow us to express three of the X -charges in terms of the others

$$u = -e - \frac{2}{3}L - \frac{1}{9}(x' - x''), \quad d = e + \frac{4}{3}L - \frac{1}{9}(x' - x''), \quad Q = -\frac{1}{3}L + \frac{1}{9}(x' - x''), \quad (12)$$

while the $[U(1)_X]^2 U(1)_Y$ anomaly condition reduces to

$$(e + L)(x' - x'') = 0. \quad (13)$$

- Previously: $x' = x''$
- We choose instead ($h = 0$):

$$e = -L, \quad (14)$$

so that (L is still a free parameter)

$$Q = -u = -d = -\frac{1}{3}L + \frac{1}{9}(x' - x''). \quad (15)$$

Anomaly cancellation: $\mathcal{X} = L$ or B

The gravitational anomaly, $[\mathrm{SO}(1,3)]^2 \mathrm{U}(1)_Y$, and the cubic anomaly, $[\mathrm{U}(1)_X]^3$, can be written as the following system of Diophantine equations, respectively,

$$\sum_{\alpha=1}^N z_{\alpha} = 0, \quad \sum_{\alpha=1}^N z_{\alpha}^3 = 0, \quad (16)$$

where

$$\begin{aligned} z_1 &= -x', & z_2 &= x'', \\ z_{2+i} &= L, \quad i = 1, 2, 3 \end{aligned} \quad (17)$$

\rightarrow

$$9Q = -\sum_{\alpha=1}^5 z_{\alpha} = -x' + x'' + L + L + L, \quad (18)$$

$L = 0 \rightarrow \mathrm{U}(1)_B$ but $Q = 0 \not\rightarrow \mathrm{U}(1)_L$

- $L = 0$

$$(5, 5, -3, -2, 1, -6)$$

$U(1)_B$ selection: Neutrinos, dark matter and baryogenesis

- $L = 0$
- Effective Dirac neutrino masses: $\phi = -\nu = -5$

$$(5, 5, -3, -2, 1, -6)$$

- $L = 0$
- Effective Dirac neutrino masses: $\phi = -\nu = -5$
- Electroweak-scale vector-like fermions:

$$(L'_L)^\dagger L''_R \Phi^* \rightarrow x' = -1, x'' = 6$$

$$(5, 5, -3, -2, 1, -6)$$

- $L = 0$
- Effective Dirac neutrino masses: $\phi = -\nu = -5$
- Electroweak-scale vector-like fermions:
 $(L'_L)^\dagger L''_R \Phi^* \rightarrow x' = -1, x'' = 6$
- Dirac-fermionic DM: $(\chi_L)^\dagger \chi''_R \Phi^* \rightarrow z_3 = -3, z_4 = -2$

$$(5, 5, -3, -2, 1, -6)$$

$U(1)_B$ selection: Neutrinos, dark matter and baryogenesis

- $L = 0$

- Effective Dirac neutrino masses: $\phi = -\nu = -5$

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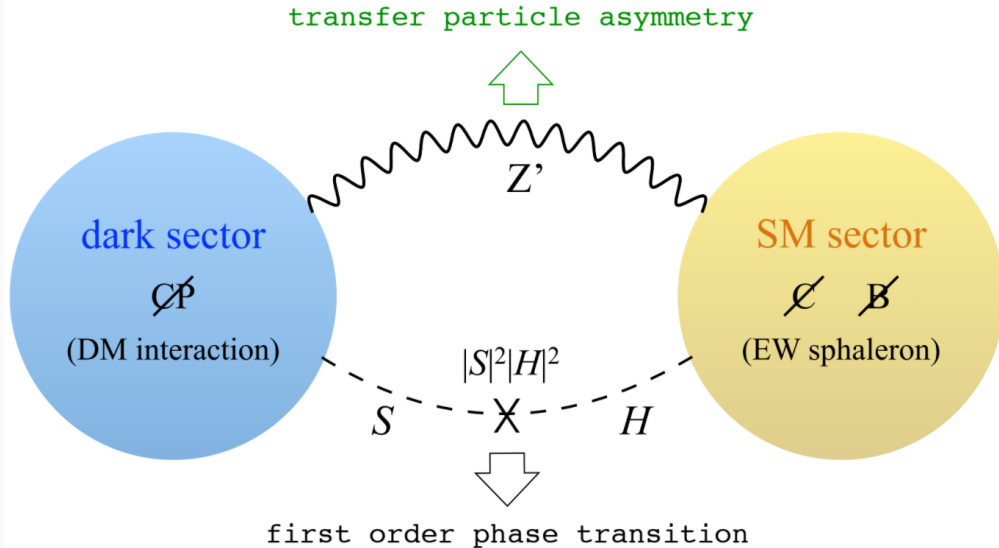
$$(L'_L)^\dagger L''_R \Phi^* \rightarrow x' = -1, x'' = 6$$

$$(5, 5, -3, -2, 1, -6)$$

- Dirac-fermionic DM: $(\chi_L)^\dagger \chi''_R \Phi^* \rightarrow z_3 = -3, z_4 = -2$

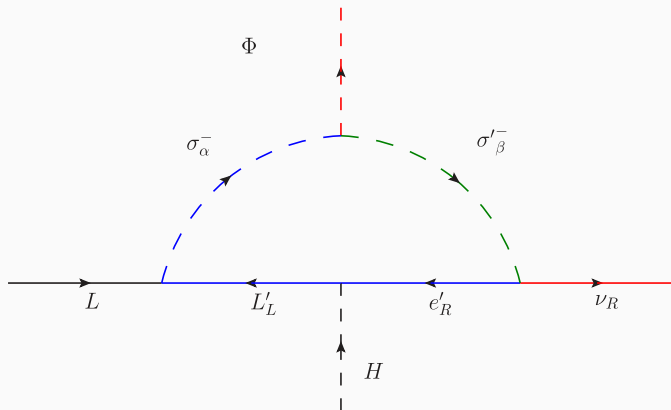
959 solutions

Dark sector baryogenesis



Gauge Baryon number scotogenic realization: arXiv:2205.05762 [PRD]

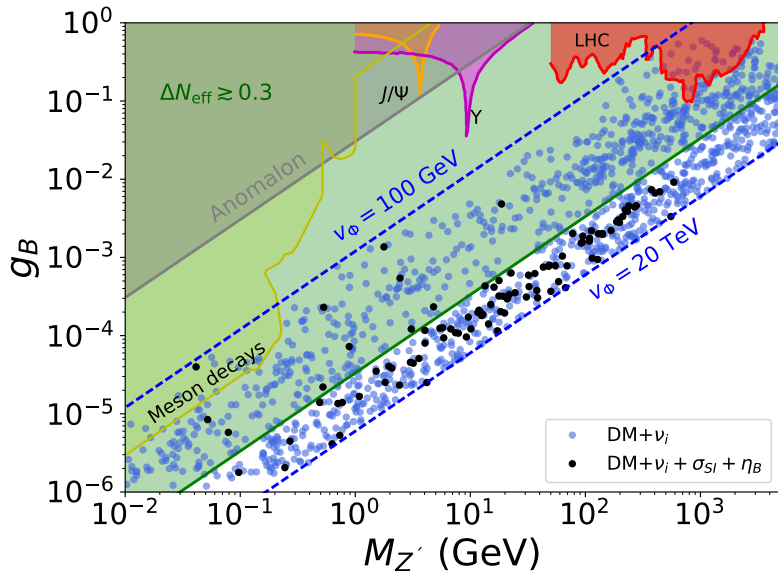
with Andrés Rivera (UdeA) and Walter Tangarife (Loyola U.)



Field	$SU(2)_L$	$U(1)_Y$	$U(1)_B$
u_{Ri}	1	2/3	$u = 1/3$
d_{Ri}	1	-1/3	$d = 1/3$
$(Q_i)^\dagger$	2	-1/6	$Q = -1/3$
$(L_i)^\dagger$	2	1/2	$L = 0$
e_R	1	-1	$e = 0$
$(L'_L)^\dagger$	2	1/2	$-x' = -3/5$
e'_R	1	-1	$x' = 3/5$
L''_R	2	-1/2	$x'' = 18/5$
$(e'_L)^\dagger$	1	1	$-x'' = -18/5$
$\nu_{R,1}$	1	0	-3
$\nu_{R,2}$	1	0	-3
χ_R	1	0	6/5
$(\chi_L)^\dagger$	1	0	9/5
H	2	1/2	0
S	1	0	3
Φ	1	0	3
σ_α^-	1	1	3/5
σ'^-_α	1	-1	-12/5

- SARAH→SPheno→MicroMegas
- η_B calculation code
- Python notebook with the scan

Black points: Dirac neutrinos with proper DM and baryon assymetry



A methodology was designed to find all the *universal* gauge Abelian extensions of the standard model:

All of the extensions can be reformulated as the solution of

$$\sum_{\alpha=1}^N z_{\alpha} = 0, \quad \sum_{\alpha=1}^N z_{\alpha}^3 = 0,$$

which we thoroughly scan in an efficient way until $N = 12$ and $|z_{\max}| = 20$

Once the physical conditions are established, the full set of self-consistent models are found from a simple data-analysis procedure, providing enough freedom to solve several phenomenological problems.