Scotogenic seesaw and baryogenesis



with gauged Baryon number

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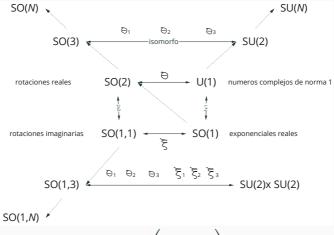
Focus on arXiv:2205.05762

In collaboration with

Andrés Rivera (UdeA), Walter Tangarife (Loyola University Chicago)

Model building

Lie groups



$$U = \exp\left(i\sum_{j} T_{j}\theta^{j}\right),\tag{1}$$

where θ^{j} are the parameters of the transformation and T_{i} are the generators.

SO(1)

Consider the 1×1

$$K = -i, (2)$$

which generates an element of dilaton group , SO(1), $R(\xi)$

$$\lambda(\xi) = e^{\xi}, \tag{3}$$

which are just the group of the real exponentials. Such a number can be transformed as

$$x \to x' = e^{\xi} x, \tag{4}$$

that corresponds to a boost by e^{ξ} . We can defin the invariant scalar product just as the division of real numbers, such that

$$x \cdot y \to x' \cdot y' \equiv \frac{x'}{y'} = \frac{e^{\xi} x}{e^{\xi} y} = \frac{x}{y} = x \cdot y. \tag{5}$$

SO(1,1)

Queremos obtener una representación 2×2 del álgebra

$$K = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix} \to K^2 = -\mathbf{1} \,, \tag{6}$$

que genera un elemento del grupo $\mathsf{SO}(1,1)$ con parámetro ξ

$$\Lambda = \exp(i\xi K) = \begin{pmatrix} \cosh \xi & \sinh \xi \\ \sinh \xi & \cosh \xi \end{pmatrix}, \qquad (7)$$

La transformación de una coordenada temporaloide y otra espacialoide (c=1)

$$\begin{pmatrix} t \\ x \end{pmatrix} = \begin{pmatrix} x^0 \\ x^1 \end{pmatrix} \to \begin{pmatrix} x'^0 \\ x'^1 \end{pmatrix} \begin{pmatrix} \cosh \xi & \sinh \xi \\ \sinh \xi & \cosh \xi \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \end{pmatrix}$$

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$$\cosh \xi = \gamma = \frac{1}{\sqrt{1 - v^2}}$$

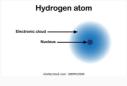
Special: parameter ξ or v is constant, e.g, inertial system invariance: *Global* conservation of E and p (still action at a distance!)

General: parameter $\xi(t, \mathbf{x})$ or $v(t, \mathbf{x})$ is constant, e.g, accelerated system invariance: **Local** conservation of E and \mathbf{p}



Noether's paradigm

U(1): From special θ to general $\theta(t, x)$



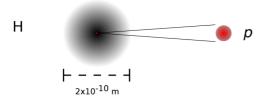
What is a particle wavicle? https://www.quantamagazine.org/what-is-a-particle-20201112/

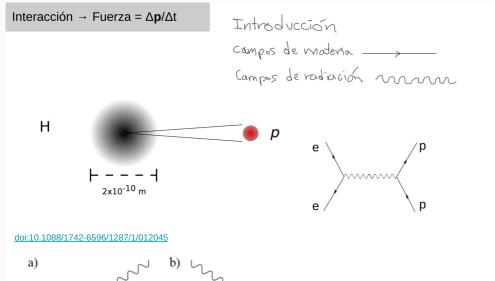
Is a "Quantum Excitation of a Field"



Is a "Irreducible Representation of a Group"

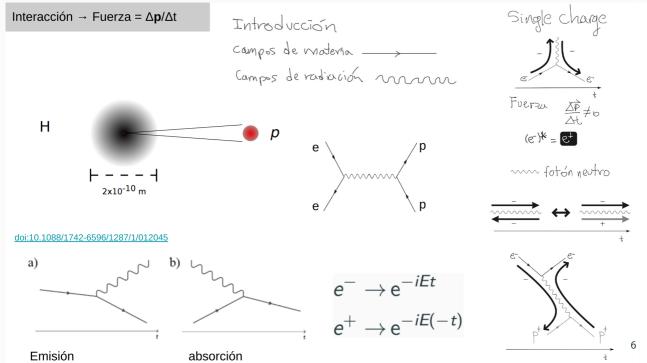






absorción

Emisión



Under a general Lorentz transformation we have.

$$A^{\mu}(x) \to A'^{\mu}(x) = \Lambda^{\mu}{}_{\nu}A^{\nu}(\Lambda^{-1}x).$$
 (8)

A pure underscript 4-vector is

$$\partial_{\mu} = \frac{\partial}{\partial x^{\mu}} = \left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) = (\partial_{0}, \nabla). \tag{9}$$

From

$$\frac{1}{x'^{\mu}} = \left(\Lambda^{-1}\right)^{\nu}_{\mu} \frac{1}{x^{\nu}} \,, \tag{10}$$

the tranformation properties for a $\partial_{\mu}=\partial/\partial x^{\mu}$, are

$$\partial_{\mu}^{\prime} = \left(\Lambda^{-1}\right)^{\nu}_{\mu} \partial_{\nu} \,. \tag{11}$$

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In this way, the invariant scalar product between the 4-vector field and the four-gradient is just

$$\partial_{\mu}A^{\mu} \to \partial'_{\mu}A'^{\mu} = \partial_{\mu}A^{\mu} \,. \tag{12}$$

Name		Symbol		SU(N)
scalar <i>N</i> -plet		Ψ		UΨ
scalar anti- <i>N</i> -plet		Ψ^{\dagger}		$\Psi^\dagger U^\dagger$
Name	Symbo		Lorentz	
Photon	\mathcal{A}^{μ}		$\Lambda^{\mu}_{\ \nu}$	$4^{ u}$
4-gradient	∂_{μ}		$\partial_{\nu}(I)$	$(-1)^{ u}_{\mu}$

Table 1: Scalar products: $\Psi^{\dagger}\Psi$, $\partial_{\mu}A^{\mu}$, $A^{\nu}A_{\nu}$, $\partial_{\mu}\partial^{\mu}$

Name	Symbol	Lorentz	U(1)
e _L : electron left	ξ_{α}	$S_{\alpha}{}^{\beta}\xi_{\beta}$	$e^{i\theta}\xi_{\alpha}$
$(e_L)^{\dagger}$: positron right	$(\xi_{m{lpha}})^\dagger = \xi_{\dot{m{lpha}}}^\dagger$	$\xi^{\dagger}_{\dot{eta}} ig[\mathcal{S}^{\dagger} ig]^{\dot{eta}}_{}\dot{lpha}}$	$\xi^{\dagger}_{\dot{lpha}}e^{-i heta}$
e _R : electron right	$(\eta^{lpha})^{\dagger}=\eta^{\dagger}{}^{\dot{lpha}}$	$\left[\left(S^{-1} \right)^{\dagger} \right]^{\dot{\alpha}}_{\dot{\beta}} \eta^{\dagger \dot{\beta}}$	$e^{i heta}\eta^{\dagger}\dot{lpha}$
$(e_R)^{\dagger}$: positron left	$\eta^{\color{red}lpha}$	$\eta^{\beta} [S^{-1}]_{\beta}^{\alpha}$	$e^{-i\theta}\eta^{\alpha}$

Table 2: electron components

Scalar products

- U(1) Majorana scalars: $\xi^{\alpha}\xi_{\alpha} + \xi^{\dagger}_{\dot{\alpha}}\xi^{\dagger\dot{\alpha}}$, $\eta^{\alpha}\eta_{\alpha} + \eta^{\dagger}_{\dot{\alpha}}\eta^{\dagger\dot{\alpha}}$.
- Dirac scalar: $\eta^{\alpha}\xi_{\alpha} + \xi^{\dagger}_{\dot{\alpha}}\eta^{\dagger\dot{\alpha}}$.
- Tensor under subgroup SL(2, C) but vector under SO(1,3): $S^{\dagger \dot{\alpha}}{}_{\dot{\beta}} \overline{\sigma}^{\mu \, \dot{\beta} \beta} S_{\beta}{}^{\alpha} = \Lambda^{\mu}{}_{\nu} \overline{\sigma}^{\nu \, \dot{\alpha} \alpha}$

Name	Symbol	Lorentz	U(1)
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$(e_R)^{\dagger}$: positron left	$\eta^{\color{red}lpha}$	$\eta^{\beta} [S^{-1}]_{\beta}^{\alpha}$	$e^{-i\theta}\eta^{\alpha}$

Table 3: electron components

General theory: QED
$$\rightarrow D_{\mu} = i\partial_{\mu} - ieA_{\mu}$$
, $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$

$$\begin{split} \xi^{\alpha} &\to \xi'^{\alpha} = e^{i\theta(x)}\xi^{\alpha} & \eta_{\alpha} \to \eta_{\alpha}' = e^{-i\theta(x)}\eta_{\alpha} \\ D_{\mu}\xi^{\alpha} &\to (D_{\mu}\xi^{\alpha})' = e^{i\theta(x)}D_{\mu}\xi^{\alpha} & D_{\mu}\eta_{\alpha} \to (D_{\mu}\eta_{\alpha})' = e^{-i\theta(x)}D_{\mu}\eta_{\alpha} \\ \mathcal{L} &= i\xi^{\dagger}_{\dot{\alpha}} \overline{\sigma}^{\mu}{}^{\dot{\alpha}\alpha}D_{\mu}\xi_{\alpha} + i\eta^{\alpha}\sigma^{\mu}_{\alpha\dot{\alpha}}D_{\mu}\eta^{\dagger}{}^{\dot{\alpha}} - m\left(\eta^{\alpha}\xi_{\alpha} + \xi^{\dagger}_{\dot{\alpha}}\eta^{\dagger}{}^{\dot{\alpha}}\right) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \end{split}$$

Name	Symbol	Lorentz	U(1)
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$(e_L)^{\dagger}$: positron right	$(\xi_{m{lpha}})^{\dagger}=\xi_{\dot{m{lpha}}}^{\dagger}$	$\xi^{\dagger}_{\dot{eta}}ig[S^{\dagger}ig]^{\dot{eta}}_{\dot{lpha}}$	$\xi^{\dagger}_{\dot{lpha}}e^{-i heta}$
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$(e_R)^{\dagger}$: positron left	$\eta^{m{lpha}}$	$\eta^{m{eta}} ig[\mathcal{S}^{-1} ig]_{m{eta}}^{^{\prime}}}$	$e^{-i\theta}\eta^{\alpha}$

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Dirac spinor

$$\psi = \begin{pmatrix} e_L \\ e_R \end{pmatrix}$$

$$\gamma^{\mu} = \begin{pmatrix} 0 & \sigma^{\mu} \\ \overline{\sigma}^{\mu} & 0 \end{pmatrix}$$

$$\overline{\psi} = \psi^{\dagger} \gamma^{0} .$$

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$$SU(2)_L$$

17 years later... (stages of grief \rightarrow 1967)

Not mass, not charge

Field	Lorentz $SU(2)_L$
$L = \begin{pmatrix} u_L \\ e_L \end{pmatrix}$	ξ_lpha 2

Denial

$$\mathcal{L} = i(L)^{\dagger} \, \overline{\sigma}^{\mu} D_{\mu} L - \frac{1}{4} W_{\mu\nu}^{i} W_{i}^{\mu\nu}$$

Not mass, hypercharge,

Field	Lorentz	$SU(2)_L$	$U(1)_Y$
$\mathcal{L} = \begin{pmatrix} u_{L} \\ e_{L} \end{pmatrix}$	ξ_{lpha}	2	- 1/2

Denial

$$\mathcal{L} = i(L)^{\dagger} \, \overline{\sigma}^{\mu} D_{\mu} L - \frac{1}{4} W^{i}_{\mu\nu} W^{\mu\nu}_{i} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

Not mass, hypercharge, not Dirac

Field	Lorentz	$SU(2)_L$	$U(1)_Y$
$\mathcal{L} = \begin{pmatrix} u_{\mathcal{L}} \\ e_{\mathcal{L}} \end{pmatrix}$	ξ_{lpha}	2	- 1/2
$\left(e_{R} ight)^{\dagger}$	η^{lpha}	1	-1

Denial

$$\mathcal{L} = i(L)^{\dagger} \overline{\sigma}^{\mu} D_{\mu} L - \frac{1}{4} W_{\mu\nu}^{i} W_{i}^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - i(e_{R})^{\dagger} \sigma^{\mu} D_{\mu} e_{R}$$

Higgs mechanism with tachyonic mass $(\mu^2 < 0)$ and condensate

Field Lorentz
$$SU(2)_L$$
 $U(1)_Y$

$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \qquad \qquad \xi_{\alpha} \qquad \qquad \mathbf{2} \qquad -1/2$$

$$\begin{pmatrix} e_R \end{pmatrix}^{\dagger} \qquad \qquad \mathbf{1} \qquad -\mathbf{1}$$

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{bmatrix} \frac{H(x)+\nu}{\sqrt{2}} \end{bmatrix} \exp \left[i \frac{\tau^i}{2} G_i(x) \right] \qquad - \qquad \qquad \mathbf{2} \qquad \qquad 1/2$$

Contempt

$$\mathcal{L} = i(L)^{\dagger} \overline{\sigma}^{\mu} D_{\mu} L - \frac{1}{4} W_{\mu\nu}^{i} W_{i}^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - i(e_{R})^{\dagger} \sigma^{\mu} D_{\mu} e_{R} + (e_{R})^{\dagger} \Phi^{\dagger} L - (D^{\mu} \Phi)^{\dagger} D_{\mu} \Phi - \mu^{2} \Phi^{\dagger} \Phi - \lambda \left(\Phi^{\dagger} \Phi\right)^{2}$$

Higgs mechanism with tachyonic mass $(\mu^2 < 0)$ and condensate

Field Lorentz
$$SU(2)_L$$
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$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \qquad \xi_{\alpha} \qquad \mathbf{2} \qquad -1/2$$

$$\begin{pmatrix} e_R \end{pmatrix}^{\dagger} \qquad \boldsymbol{\eta}^{\alpha} \qquad \mathbf{1} \qquad -1$$

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{bmatrix} \frac{H(x)+\nu}{\sqrt{2}} \end{bmatrix} \exp\left[i\frac{\tau^i}{2}G_i(x)\right] \qquad - \qquad \mathbf{2} \qquad 1/2$$

Contempt

$$\begin{split} \mathcal{L} &= i(L)^{\dagger} \, \overline{\sigma}^{\mu} D_{\mu} L - \tfrac{1}{4} W^{i}_{\mu\nu} \, W^{\mu\nu}_{i} - \tfrac{1}{4} B_{\mu\nu} B^{\mu\nu} - i \big(e_{R} \big)^{\dagger} \, \sigma^{\mu} D_{\mu} e_{R} + \big(e_{R} \big)^{\dagger} \, \Phi^{\dagger} L - \big(D^{\mu} \Phi \big)^{\dagger} \, D_{\mu} \Phi - \mu^{2} \Phi^{\dagger} \Phi - \lambda \left(\Phi^{\dagger} \Phi \right)^{2} \\ & \Phi \rightarrow \Phi' = \exp \left[i \tfrac{\tau^{i}}{2} \theta_{i}(x) \right] \Phi = \tfrac{1}{\sqrt{2}} [H(x) + v] \\ \mathcal{L} &= i \overline{\psi} \gamma^{\mu} \partial \psi - m_{e} \overline{\psi} \psi - i \big(\nu_{L} \big)^{\dagger} \, \overline{\sigma}^{\mu} \partial_{\mu} \nu_{L} - \big(D^{\mu} \Phi \big)^{\dagger} \, D_{\mu} \Phi - \mu^{2} \Phi^{\dagger} \Phi - \lambda \left(\Phi^{\dagger} \Phi \right)^{2} - \mathcal{L}_{gauge} + Z \nu \nu + m_{e} \overline{\psi} \psi H \end{split}$$

Z and W phenomenology and discovery

Field Lorentz
$$SU(2)_L$$
 $U(1)_Y$

$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \qquad \qquad \xi_{\alpha} \qquad \qquad \mathbf{2} \qquad -1/2$$

$$\begin{pmatrix} e_R \end{pmatrix}^{\dagger} \qquad \qquad \boldsymbol{\eta}^{\alpha} \qquad \qquad \mathbf{1} \qquad -1$$

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{bmatrix} \frac{H(x)+v}{\sqrt{2}} \end{bmatrix} \exp \left[i \frac{\tau^i}{2} G_i(x) \right] \qquad - \qquad \qquad \mathbf{2} \qquad \qquad 1/2$$

Bargaining

$$\mathcal{L} = i(L)^{\dagger} \, \overline{\sigma}^{\mu} D_{\mu} L - \frac{1}{4} W_{\mu\nu}^{i} W_{i}^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - i(e_{R})^{\dagger} \, \sigma^{\mu} D_{\mu} e_{R} + (e_{R})^{\dagger} \, \Phi^{\dagger} L - (D^{\mu} \Phi)^{\dagger} \, D_{\mu} \Phi - \mu^{2} \Phi^{\dagger} \Phi - \lambda \left(\Phi^{\dagger} \Phi\right)^{2}$$

$$\Phi \rightarrow \Phi' = \exp \left[i \frac{\tau^{i}}{2} \theta_{i}(x) \right] \Phi = \frac{1}{\sqrt{2}} [H(x) + v]$$

$$\mathcal{L} = i \overline{\psi} \gamma^{\mu} \partial \psi - m_{e} \overline{\psi} \psi - i(\nu_{L})^{\dagger} \, \overline{\sigma}^{\mu} \partial_{\mu} \nu_{L} - (D^{\mu} \Phi)^{\dagger} \, D_{\mu} \Phi - \mu^{2} \Phi^{\dagger} \Phi - \lambda \left(\Phi^{\dagger} \Phi\right)^{2} - \mathcal{L}_{gauge} + Z \nu \nu + m_{e} \overline{\psi} \psi H$$

Hierarchy problem

Field Lorentz
$$SU(2)_L$$
 $U(1)_Y$

$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \qquad \qquad \xi_{\alpha} \qquad \qquad 2 \qquad -1/2$$

$$\begin{pmatrix} e_R \end{pmatrix}^{\dagger} \qquad \qquad \eta^{\alpha} \qquad \qquad 1 \qquad -1$$

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{bmatrix} \frac{H(x)+v}{\sqrt{2}} \end{bmatrix} \exp\left[i\frac{\tau^i}{2}G_i(x)\right] \qquad - \qquad \qquad 2 \qquad \qquad 1/2$$

Depression

$$\mathcal{L} = i(L)^{\dagger} \, \overline{\sigma}^{\mu} D_{\mu} L - \frac{1}{4} W_{\mu\nu}^{i} W_{i}^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - i(e_{R})^{\dagger} \, \sigma^{\mu} D_{\mu} e_{R} + (e_{R})^{\dagger} \, \Phi^{\dagger} L - (D^{\mu} \Phi)^{\dagger} D_{\mu} \Phi - \mu^{2} \Phi^{\dagger} \Phi - \lambda \left(\Phi^{\dagger} \Phi\right)^{2}$$

$$\Phi \rightarrow \Phi' = \exp \left[i \frac{\tau^{i}}{2} \theta_{i}(x) \right] \Phi = \frac{1}{\sqrt{2}} [H(x) + \nu]$$

$$\mathcal{L} = i \overline{\psi} \gamma^{\mu} \partial \psi - m_{e} \overline{\psi} \psi - i(\nu_{L})^{\dagger} \, \overline{\sigma}^{\mu} \partial_{\mu} \nu_{L} - (D^{\mu} \Phi)^{\dagger} D_{\mu} \Phi - \mu^{2} \Phi^{\dagger} \Phi - \lambda \left(\Phi^{\dagger} \Phi\right)^{2} - \mathcal{L}_{gauge} + Z \nu \nu + m_{e} \overline{\psi} \psi H$$

Higgs discovery!

Field Lorentz
$$SU(2)_L$$
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$$\begin{pmatrix} e_R \end{pmatrix}^{\dagger} \qquad \qquad \boldsymbol{\eta}^{\alpha} \qquad \qquad \mathbf{1} \qquad -1$$

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{bmatrix} \frac{H(x)+v}{\sqrt{2}} \end{bmatrix} \exp \left[i\frac{\tau^i}{2}G_i(x)\right] \qquad - \qquad \qquad \mathbf{2} \qquad \qquad 1/2$$

Acceptance

$$\begin{split} \mathcal{L} &= i(L)^{\dagger} \, \overline{\sigma}^{\mu} D_{\mu} L - \tfrac{1}{4} W^{i}_{\mu\nu} \, W^{\mu\nu}_{i} - \tfrac{1}{4} B_{\mu\nu} B^{\mu\nu} - i \big(e_{R} \big)^{\dagger} \, \sigma^{\mu} D_{\mu} e_{R} + \big(e_{R} \big)^{\dagger} \, \Phi^{\dagger} L - \big(D^{\mu} \Phi \big)^{\dagger} \, D_{\mu} \Phi - \mu^{2} \Phi^{\dagger} \Phi - \lambda \, \big(\Phi^{\dagger} \Phi \big)^{2} \\ & \Phi \rightarrow \Phi' = \exp \left[i \tfrac{\tau^{i}}{2} \theta_{i}(x) \right] \Phi = \tfrac{1}{\sqrt{2}} [H(x) + v] \\ \mathcal{L} &= i \overline{\psi} \gamma^{\mu} \partial \psi - m_{e} \overline{\psi} \psi - i \big(\nu_{L} \big)^{\dagger} \, \overline{\sigma}^{\mu} \partial_{\mu} \nu_{L} - \big(D^{\mu} \Phi \big)^{\dagger} \, D_{\mu} \Phi - \mu^{2} \Phi^{\dagger} \Phi - \lambda \, \big(\Phi^{\dagger} \Phi \big)^{2} - \mathcal{L}_{gauge} + Z \nu \nu + m_{e} \overline{\psi} \psi H \end{split}$$