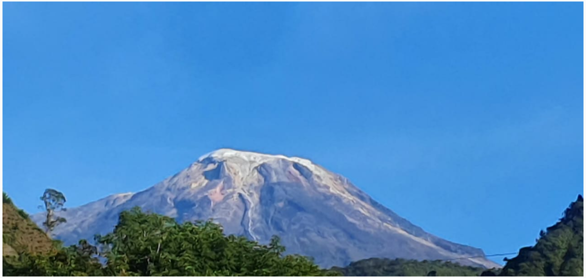
# Abelian gauge extensions with Higgs mixing

Diego Restrepo

Instituto de Física
Universidad de Antioquia
Phenomenology Group
`http://gfif.udea.edu.co`

In collaboration with
Farinaldo S. Queiroz (IIP), Clarissa Siqueira(São Paulo University)
and Carlos Yaguna (UPTC) [*In progress...*]



# Local Abelian extensions of the SM

# Anomaly cancellation of a gauge $U(1)_X$ extension

It is known that if we extend the SM by a local $U(1)_X$ and a set of $\alpha$ singlet chiral fields, $N_\alpha$, the cubic and mixed gauge anomalies can be written in terms of two $U(1)_X$ charge, e.g, $u$ and $d$, where $u$ and $d$ represent the charges of the right-handed up and down quarks, respectively

$$\sum_\alpha n_\alpha^3 = -3(2d+u)^3$$
$$\sum_\alpha n_\alpha = -3(2d+u)\,. \tag{1}$$

where $n_\alpha$ is the $U(1)_X$-charge of $N_\alpha$.

# $\alpha = 3$

If the set of $N_\alpha$ is just the set of heavy singlet chiral fields of the type-I seesaw

$$\sum_\alpha n_\alpha^3 = -3$$
$$\sum_\alpha n_\alpha = -3\,,$$

such that

$$2d + u = 1, \to u = 1 - 2d\,. \tag{2}$$

In this way, we can write the solution in terms of just one parameter: $d$

# Examples of $d$ with vector-like Dirac fermionic dark matter and mass mixing

For a vector-like fermionic dark matter candidate $\chi$, $U(1)_X$ can guarantee the stability

| Fields | $SU(2)_L$ | $U(1)_Y$ | $U(1)_X$ | $U(1)_{B-L}$ | $U(1)_R$ | $U(1)_{\not{D}}$ | $U(1)_{\not{U}}$ |
|---|---|---|---|---|---|---|---|
| $d_R$ | $\mathbf{1}$ | $-1/3$ | $d$ | $1/3$ | $1$ | $0$ | $1/2$ |
| $u_R$ | $\mathbf{1}$ | $+2/3$ | $1-2d$ | $1/3$ | $-1$ | $1$ | $0$ |
| $Q$ | $\mathbf{2}$ | $1/6$ | $1/2-d/2$ | $1/3$ | $0$ | $1/2$ | $1/6$ |
| $L$ | $\mathbf{2}$ | $-1/2$ | $-3/2+3d/2$ | $-1$ | $0$ | $-3/2$ | $-3/4$ |
| $e_R$ | $\mathbf{1}$ | $-1$ | $-2+3d$ | $-1$ | $1$ | $-2$ | $-1/2$ |
| $N_R$ | $\mathbf{1}$ | $0$ | $-1$ | $-1$ | $-1$ | $-1$ | $-1$ |
| $H$ | $\mathbf{2}$ | $1/2$ | $1/2-3d/2$ | $0$ | $-1$ | $1/2$ | $-1/4$ |
| $\Phi_s$ | $\mathbf{1}$ | $0$ | $2$ | $2$ | $2$ | $2$ | $2$ |
| $\chi_L$ | $\mathbf{1}$ | $0$ | $1/5$ | $1/5$ | $1/5$ | $1/5$ | $1/5$ |
| $\chi_R$ | $\mathbf{1}$ | $0$ | $1/5$ | $1/5$ | $1/5$ | $1/5$ | $1/5$ |

**Table 1:** $d$ will be a continuos variable reconverted to rational whenever necessary $\rightarrow$ mass mixing!

## Gauged Type-I seesaw with $\chi$ and mass mixing

$$\mathcal{L} \supset y_2^d \bar{Q} H d_R + y_2^u \bar{Q} \widetilde{H} u_R + y_2^e \bar{L} H e_R + y^D \bar{L} \widetilde{H} N_R + Y^M \overline{(N_R)^c} \Phi_s N_R + \text{h.c.}$$
$$+ i\bar{\chi}\gamma_\mu \left(\partial^\mu - i g_X \chi B_X^\mu\right) \chi - m_{\rm DM} \bar{\chi}\chi + (\mathcal{D}_\mu H)^\dagger \mathcal{D}^\mu H + (\mathcal{D}_\mu \Phi_s)^* \mathcal{D}_\mu \Phi_s - V\,, \tag{3}$$

and the scalar potential is,

$$V = \mu^2 H^\dagger H + \frac{\lambda}{2}\left(H^\dagger H\right)^2 + m_s^2 \Phi_s^\dagger \Phi_s + \frac{\lambda_s}{2}\left(\Phi_s^\dagger \Phi_s\right)^2 + \lambda_{sH} H^\dagger H \Phi_s^\dagger \Phi_s \tag{4}$$

where

$$H = \begin{pmatrix} G^+ \\ (v + H^0 + i\,G^0)/\sqrt{2} \end{pmatrix} \tag{5}$$

and,

$$\Phi_s = \left(S + v_s + iG'^0\right)/\sqrt{2}. \tag{6}$$

for any $\psi$

$$\mathcal{L} \supset -\frac{g_X}{4}\left[(X_R + X_L)\psi\gamma_\mu\psi + (X_R - X_L)\psi\gamma_\mu\gamma_5\psi\right] B_X^\mu, \tag{7}$$

# Neutrino Masses

Neutrino masses are generated via the last two terms of Eq.(3). They induce the presence of Dirac and Majorana mass terms, which consequently lead to the mass matrix,

$$\begin{pmatrix} \nu & N_R \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu \\ N_R \end{pmatrix} \tag{8}$$

where $m_D = y^D v_2/(2\sqrt{2})$ and $M_R = y^M v_s/(s\sqrt{2})$. From the diagonalization procedure, we conclude that,

$$m_\nu = m_D^T m_D / M_R, \ m_N = M_R, \tag{9}$$

for $M_R \gg m_D$.

Therefore, it is clear that our model naturally account for neutrino masses via the type-I seesaw mechanism, and usual can reproduce neutrino oscillation data the main annihilation channels that set the dark matter annihilation rate are indeed those shown in Fig.1. The approximate expressions for the annihilation cross-sections are given by,

$$\sigma v\left(\chi\bar{\chi} \to \bar{f}f\right) \simeq \frac{n_c(1-m_f^2/m_\chi^2)^{1/2}}{2\pi M_{Z'}^4(4m_\chi^2 - M_{Z'}^2)^2}\left[2g_\chi^2 g_{fa}^2 M_{Z'}^4(m_\chi^2 - m_f^2)\right.$$
$$\left. + \; g_\chi^2 g_{fv}^2 M_{Z'}^4(2m_\chi^2 + m_f^2)\right], \qquad (10)$$

where $g_\chi = g_X \chi^2$

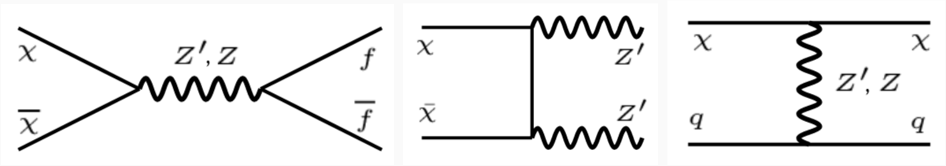



**Figure 1:** Feynman diagrams that govern the dark matter phenomenology. The first dictates the relic density and the second the scattering rate, including diagrams due to the presence $Z - Z'$ mixing. See the text.

# Isospin violation

# Direct detection

The spin-independent dark matter-nucleus scattering cross-section induced by a vector current mediated by a $Z'$ of mass $M_{Z'}$

$$\mathcal{L} = f_\chi \overline{\chi}\gamma^\mu \chi \, Z'_\mu + \sum_{q=u,d} f_q \overline{q}\gamma^\mu q \, Z'_\mu \, . \tag{11}$$

where $f_{u,d}$ and $f_\chi$ are the corresponding coefficients of the vector currents which gives to

$$\mathcal{L}^q_{\text{eff}} = \sum_{q=u,d} \frac{f_\chi f_q}{M^2_{Z'}} \overline{\chi}\gamma^\mu \chi \overline{q}\gamma_\mu \, q \, . \tag{12}$$

The nucleon Abelian charge is obtained by summing over the valence quarks of the nucleons

$$\mathcal{L}_{\text{eff}} = \sum_{N=p,n} \frac{f_\chi f_N}{M^2_{Z'}} \overline{\chi}\gamma^\mu \chi \overline{N}\gamma_\mu \, N \, . \tag{13}$$

where (no contribution of sea quarks or gluons to the effective couplings)

$$\begin{aligned} f_n &= f_u + 2 f_d, \\ f_p &= 2 f_u + f_d \, , \end{aligned} \tag{14}$$

# Isospin conservation

For $U(1)_{B-L}$ $f_n = f_p$ and we can use directly the direct detection proton cross section given by micrOMEGAS and the reported by the direct detection experiments:

$$\sigma_p = \frac{f_\chi^{\ 2} \mu_p^2}{4\pi M_{Z'}^4} f_p^{\ 2} \,, \tag{15}$$

**micrOMEGAS output**

```
CDM-nucleon cross sections[pb]:
 proton  SI 4.440E-10  SD 0.000E+00
 neutron SI 4.440E-10  SD 0.000E+00
```

$$\frac{f_n}{f_p} = \frac{Q + u + 2(Q + d)}{2(Q + u) + Q + d} = 1 \,.$$

Note that the $X$-charge of the SM Higgs: $H = 0$. Therefore $B_X^\mu = Z'^\mu$.

# Isospin violation

In general, $f_n \neq f_p$. The dark matter-nucleon scattering cross-section is

$$\sigma_N^Z = \sigma_p \frac{\sum_i \eta_i \mu_{A_i}^2 \left[Z + (A_i - Z) f_n / f_p\right]^2}{\sum_i \eta_i \mu_{A_i}^2 A_i^2} \tag{16}$$

where

$$\mu_{A_i} = \frac{M_{A_i} m_\chi}{M_{A_i} + m_\chi}\,, \tag{17}$$

where $\eta_i$ the natural abundance of each isotope $i$, with mass $M_{A_i}$.

# Xenon detectors

We have $A = 131$ and $Z = 54$, and

$$\sigma_N^Z = \frac{\sigma_p}{A^2}\left[Z + (A - Z)\frac{f_n}{f_p}\right]^2 , \tag{18}$$

Therefore, the Xenonphobic condition, $\sigma_N^Z = 0$, imply

$$f_n/f_p = -\,54/77 \approx -0.7\,.$$

For the numerical calculation we use the micrOMEGAS amplitudes $\mathcal{A}_p$, $\mathcal{A}_n$:

**micrOMEGAS output**

```
    CDM-nucleon micrOMEGAs amplitudes:
proton:  SI  -1.009E-09  SD  0.000E+00
neutron: SI  -1.009E-09  SD  0.000E+00
```

and

$$\frac{f_n}{f_p} = \frac{\mathcal{A}_n}{\mathcal{A}_p} \tag{19}$$

# Mass mixing I

Whenever $f_n \neq f_p$ the $X$-charge of the SM Higgs: $H \neq 0$. Therefore $B_X^{\mu} \neq Z'^{\mu}$ and

$$\frac{f_n}{f_p} \neq \frac{Q + u + 2(Q + d)}{2(Q + u) + Q + d}\,.$$

# Mass mixing II

$$\mathcal{L} \supset (\mathcal{D}_\mu H)^\dagger \mathcal{D}^\mu H + (\mathcal{D}_\mu \Phi_s)^* \mathcal{D}_\mu \Phi_s \tag{20}$$

We focus in the diagonal part for the first part

$$\begin{aligned}\mathcal{L} \supset & \frac{1}{2}\left(-\tfrac{1}{2} g_2 W_3^\mu - g_1 Y_\Phi B^\mu - g_X H B_X^\mu\right)^2 (H^0 + v)^2 + g_X s B_X^\mu (S + v_s)^2 \,. \\ \supset & \delta M^2 W_{3\mu} B_X^\mu + \cdots \end{aligned} \tag{21}$$

where

$$\delta M^2 = \frac{1}{2} g_2 g_X H v^2 \,. \tag{22}$$

with leads to the mass eigenstates [`Babu:1997st`]

# Mass mixing III

$$\begin{pmatrix} A_\mu \\ Z_\mu \\ Z'_\mu \end{pmatrix} \approx \begin{pmatrix} \cos\theta_W & \sin\theta_W & 0 \\ -\sin\theta_W & \cos\theta_W & \epsilon \\ \epsilon\sin\theta_W & -\epsilon\cos\theta_W & 1 \end{pmatrix} \begin{pmatrix} B_Y^\mu \\ W^{3\mu} \\ B_X^\mu \end{pmatrix}$$

$$\epsilon \approx \frac{-\delta M^2}{M_{Z'}^2 - M_Z^2}, \tag{23}$$

by using eq. (22)

$$\frac{\epsilon}{g_X} = -\frac{g_2 v^2}{M_{Z'}^2 - M_Z^2}(H/2) \tag{24}$$

$$= -\frac{2\cos\theta_W}{\left(\sqrt{2}G_F\right)^{1/2}} \frac{M_Z}{M_{Z'}^2 - M_Z^2}(H/2). \tag{25}$$

The non-zero squared gauge boson masses are

$$M_Z^2 = \frac{g_2^2}{4\cos^2\theta_W} v^2 \left[1 + O\left(\epsilon^2\right)\right]$$

$$M_{Z'}^2 = \frac{g_X^2}{4}\left(H^2 v^2 + s^2 v_s^2\right)\left[1 + O\left(\epsilon^2\right)\right]$$

$$= \frac{g_X^2}{2}\left(H^2 \frac{v^2}{2} + s^2 \frac{v_s^2}{2}\right)\left[1 + O\left(\epsilon^2\right)\right]$$

# Direct detection with mass mixing I

$$\mathcal{L}_{\text{eff}}^{q} \supset \frac{g_X^2 \chi}{2 M_{Z'}^2} \sum_{q=u,d} \left[ Q + q - x_q \right] \overline{\chi} \gamma^{\mu} \chi \bar{q} \gamma_{\mu} q \,, \tag{26}$$

where

$$f_q = \frac{g_X}{2} (Q + q - x_q). \qquad f_\chi = g_X \chi \,. \tag{27}$$

and

$$x_q = K \frac{e v_q}{2 \cos\theta_W \sin\theta_W} H \,, \tag{28}$$

with

$$K = \frac{2 \cos\theta_W}{\left( \sqrt{2} G_F M_Z^2 \right)^{1/2}} \tag{29}$$

| | $u$ | $d$ | $\nu_e$ | $e$ |
|---|---|---|---|---|
| $2 v_f$ | $1 - \frac{8}{3} \sin^2\theta_W$ | $-1 + \frac{4}{3} \sin^2\theta_W$ | $1$ | $-1 + 4 \sin^2\theta_W$ |

**Table 2:** Neutral current couplings

# Direct detection with mass mixing II

$$\frac{f_n}{f_p} = \frac{f_u + 2f_d}{2f_u + f_d} = \frac{5 - 3d - 2(x_u - 2x_d)}{7 - 9d - 2(2x_u + x_d)}$$
$$= \frac{5 - 3d - Ke(2v_d + v_u)2H/(2\cos\theta_W \sin\theta_W)}{7 - 9d - Ke(2v_u + v_d)2H/(2\cos\theta_W \sin\theta_W)} \, .$$
$$= \frac{5 - 3d - Ke(2v_d + v_u)(1 - 3d)/(2\cos\theta_W \sin\theta_W)}{7 - 9d - Ke(2v_u + v_d)(1 - 3d)/(2\cos\theta_W \sin\theta_W)} = y \, . \tag{30}$$

We have explored all possible gauge Abelian symmetry from the canonical $B - L$ one without mass mixing at $d = 1/3$ until $d = 3/2$. For each one we have obtained the dark matter-nucleon scattering cross-section (18) numerically with micrOMEGAS, by using the reported $\sigma_p$ and the ratio of neutron proton amplitudes to obtain $f_n/f_p = \mathcal{A}_n/\mathcal{A}_p$.

# Xenonphobic condition I

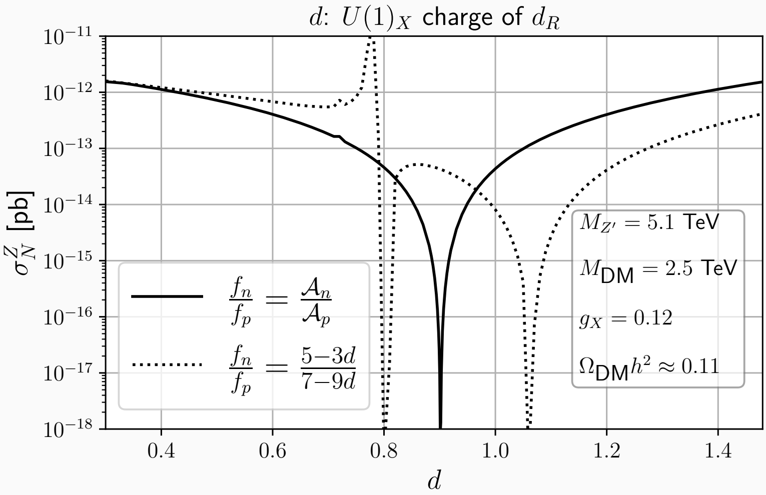



**Figure 2:** Xenonphobic solutions neglecting mass mixing: dashed; and with mass mixing: solid black

# Xenonphobic condition I

- The several solutions are displayed in Fig. 2 along the solid black line, and we can see that for $d = 9/10$ we can have a suppression of at least five orders of magnitude which depends only in SM parameters.
- In Fig. 2 we have also displayed the approximation without mass mixing for $f_n/fp$, along the dashed line. We can see that this approximation is only good in the limit of $H \to 0$ close to the canonical $B - L$ value for $d = 1/3$.

# Conclusion

The mass mixing must be taking into account to calculate the dark matter-nucleon scattering cross-section