

# Effective Dirac neutrino masses

with multi-component fermionic dark matter

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UNIVERSIDAD DE ANTIOQUIA

1803

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**Focus on**

**arXiv:2108.05907**

**In collaboration with**

**Nicolás Bernal**

## Dark sectors

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# Local $U(1)_\chi$

$$\mathcal{L} = -\frac{1}{4} V_{\mu\nu} V^{\mu\nu} + i\bar{\Psi}\not{D}\Psi - h\bar{\Psi}\Psi S$$

Diracness protected chiral fermion

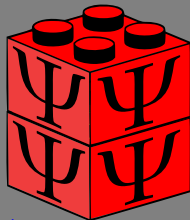
dark matter  $m_\Psi = h\langle S \rangle$

Relic abundance:

$$F_{\mu\nu} \quad V^{\mu\nu}$$

Active Symmetry:  $\mathcal{X} \rightarrow X: \Psi\bar{\Psi} \rightarrow \text{SM SM}$

Dark Symmetry:  $\mathcal{X} \rightarrow D: \Psi\bar{\Psi} \rightarrow \gamma_D \gamma_D$



$$\bar{\Psi}\Psi = \psi_1\psi_2 + \psi_1^\dagger\psi_2^\dagger \rightarrow \psi_\alpha, \quad \alpha = 1, \dots, N' \rightarrow N' > 4$$



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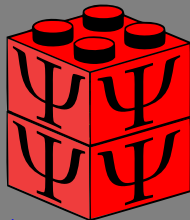
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multi-component  
dark matter

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## Standard model extended with $U(1)_{\mathcal{X}}$ gauge symmetry

Fields	$SU(2)_L$	$U(1)_Y$	$U(1)_{\mathcal{X}}$
$L^\dagger$	<b>2</b>	$+1/2$	$l$
$Q^\dagger$	<b>2</b>	$-1/6$	$q$
$d_R$	<b>1</b>	$-1/2$	$d$
$u_R$	<b>1</b>	$+2/3$	$u$
$e_R$	<b>1</b>	$-1$	$e$
$H$	<b>2</b>	$-1/2$	$h$
$\psi_\alpha$	<b>1</b>	$0$	$n_\alpha$

**Table 1:**  $l = q = d = u = e = h = 0$  for  $\mathcal{X} = D$

$$[\mathrm{SU}(3)_c]^2 \mathrm{U}(1)_X : \quad [3u + 3d] - [3 \cdot 2q] = 0$$

$$[\mathrm{SU}(2)_L]^2 \mathrm{U}(1)_X : \quad [2l + 3 \cdot 2q] = 0$$

$$[\mathrm{U}(1)_Y]^2 \mathrm{U}(1)_X : \quad \left[ (-2)^2 e + 3 \left( \frac{4}{3} \right)^2 u + 3 \left( -\frac{2}{3} \right)^2 d \right] - \left[ 2(+1)^2 l + 3 \cdot 2 \left( -\frac{1}{3} \right)^2 q \right] = 0$$



with solution

$$u = -e - \frac{2l}{3}, \quad d = e + \frac{4l}{3}, \quad q = -\frac{l}{3},$$

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For  $N$  extra quiral fields  $\psi_\alpha$  ( $\alpha = 1, \dots, N'$ ) with  $\mathcal{X}$ -charges  $n_\alpha$ :

$$[SO(1,3)]^2 U(1)_X : \quad \sum_{\alpha} n_{\alpha} + 3(e - 2l) = 0, \quad (1)$$

$$[U(1)_X]^3, \quad \sum_{\alpha} n_{\alpha}^3 + 3(e - 2l)^3 = 0 \quad (2)$$

with solution

$$u = -m + \frac{4l}{3}, \quad d = m - \frac{2l}{3}, \quad q = -\frac{l}{3}, \quad e = m - 2l,$$

which satisfy

$$\mathrm{U}(1)_Y [\mathrm{U}(1)_X]^2 : \quad [(-2)e^2 + 3\left(\frac{4}{3}\right)u^2 + 3\left(-\frac{2}{3}\right)d^2] - [2(+1)l^2 + 3 \cdot 2\left(-\frac{1}{3}\right)q^2] = 0$$

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$$[\mathrm{SO}(1,3)]^2 \mathrm{U}(1)_X : \quad \sum_{\alpha} n_{\alpha} + 3m = 0, \quad (1)$$

$$[\mathrm{U}(1)_X]^3, \quad \sum_{\alpha} n_{\alpha}^3 + 3m^3 = 0 \quad (2)$$

with solution

$$u = -m + \frac{4l}{3}, \quad d = m - \frac{2l}{3}, \quad q = -\frac{l}{3}, \quad e = m - 2l,$$

which satisfy

$$\mathrm{U}(1)_Y [\mathrm{U}(1)_X]^2 : \quad [(-2)e^2 + 3\left(\frac{4}{3}\right)u^2 + 3\left(-\frac{2}{3}\right)d^2] - [2(+1)l^2 + 3 \cdot 2\left(-\frac{1}{3}\right)q^2] = 0$$

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Any set of integers of integers  $(n_1, n_2, \dots, n_N)$  which satisfy (1) and (2) can be interpreted as  $\mathcal{X} \rightarrow D$  symmetry with  $N$  chiral fields.

If one integer,  $m$  is repeated 3 times, the set can be interpreted as  $\mathcal{X} \rightarrow X$  symmetry with  $N' = N - 3$  chiral fields.

From: arXiv:1905.13279 [PRL] Costa, *et al*

Let a vector  $\mathbf{z}$  with  $N$  non-zero integer entries such that

$$\sum_{i=1}^N z_i = 0, \quad \sum_{i=1}^N z_i^3 = 0.$$

We like to build this set of  $N$  integers from two subsets  $\ell$  and  $\mathbf{k}$  with sizes

$$\dim(\ell) = \begin{cases} \alpha = \frac{N}{2} - 1, & \text{if } N \text{ even} \\ \beta = \frac{N-3}{2}, & \text{if } N \text{ odd} \end{cases}; \quad \dim(\mathbf{k}) = \begin{cases} \alpha = \frac{N}{2} - 1, & \text{if } N \text{ even} \\ \beta + 1 = \frac{N-1}{2}, & \text{if } N \text{ odd} \end{cases}$$

- $N$  even: Consider the following two vector-like examples of  $\mathbf{z}$  such that

$$\mathbf{x} = (\ell_1, k_1, \dots, k_\alpha, -\ell_1, -k_1, \dots, -k_\alpha)$$

$$\mathbf{y} = (0, 0, \ell_1, \dots, \ell_\alpha, -\ell_1, \dots, -\ell_\alpha).$$

- $N$  odd:

$$\mathbf{x} = (0, k_1, \dots, k_{\beta+1}, -k_1, \dots, -k_{\beta+1})$$

$$\mathbf{y} = (\ell_1, \dots, \ell_\beta, k_1, 0, -\ell_1, \dots, -\ell_\beta, -k_1)$$

From any of this, we can build a final  $\mathbf{z}$  which can includes *chiral* solutions

$$\mathbf{x} \oplus \mathbf{y} \equiv \left( \sum_{i=1}^N x_i y_i^2 \right) \mathbf{x} - \left( \sum_{i=1}^N x_i^2 y_i \right) \mathbf{y}.$$

Let a vector  $\mathbf{z}$  with  $N$  non-zero integer entries such that

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$$\begin{aligned} \mathbf{x} &= (0, k_1, \dots, k_{\beta+1}, -k_1, \dots, -k_{\beta+1}) \\ \mathbf{y} &= (\ell_1, \dots, \ell_\beta, k_1, 0, -\ell_1, \dots, -\ell_\beta, -k_1) \end{aligned}$$

From any of this, we can build a final  $\mathbf{z}$  which can includes *chiral* solutions

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<https://pypi.org/project/anomalies>



# anomalies 0.1.4

<https://github.com/restrepo/anomaly/raw/main/solutions.json.gz>



`pip install anomalies`



390074 solutions: 4<N<13

Released: Nov 30, 2020

## Navigation

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Release history

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## Statistics

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★ Stars: 0

🍴 Forks: 1

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## Meta

License: BSD

Author: [restrepo](#)

## Maintainers

## Anomalies

Implement the anomaly free solution of [arXiv:1905.13729](#) [PRL]:

Obtain a numpy array  $\mathbf{z}$  of  $N$  integers which satisfy the Diophantine equations

```
>>> z.sum()
0
>>> (z**3).sum()
0
```

The input is two lists  $\mathbf{l}$  and  $\mathbf{k}$  with any  $(N-3)/2$  and  $(N-1)/2$  integers for  $N$  odd, or  $N/2-1$  and  $N/2-1$  for  $N$  even ( $N \geq 4$ ). The function is implemented below under the name: `free(l,k)`

## Install

```
$ pip install anomalies
```

## USAGE

```
>>> from anomalies import anomaly
>>> anomaly.free([-1,1],[4,-2])
array([ 3,  3,  3, -12, -12, 15])
>>> anomaly.free.gcd
3
>>> anomaly.free.simplified
array([ 1,  1,  1, -4, -4,  5])
```

$$N=6$$



$$\alpha=2$$

$$\vec{l} = (-1, 1)$$

$$\vec{k} = (4, -2)$$



September 24, 2021

Dataset

Open Access

# Set of N integers between -30 and 30 with sum and cubic sum up to zero for $4 < N < 13$

Diego Restrepo

## Anomalies

Solutions obtained with the python package: [anomalies](#) based on the method to find anomaly free solutions of the standard model extended with an Abelian Dark Symmetry with  $N$  right-handed singlet chiral fields described in [arXiv:1905.13729 \[PRL\]](#):

## Data scheme

- 'l': integer lists → input to obtain the 'solution' by using the [anomalies](#) package
- 'k': integer lists → input to obtain the 'solution' by using the [anomalies](#) package

- 'solution': list → of integers,  $Z_i$  which satisfy  $\sum_{i=1}^N Z_i = 0$  and  $\sum_{i=1}^N Z_i^3 = 0$ .

- 'n': integer → number of integers in 'solution',  $N$ .

## USAGE

#Example of JSON file usage in Python with pandas (see also json module)

```
>>> import pandas as pd
>>> df=pd.read_json('solutions.json')
>>> df[:2]
```

	1	k	solution	gcd	n
0	[1, 2]	[0, -3]	[1, 5, -7, -8, 9]	1	5
1	[-2, -1]	[0, -1]	[2, 4, -7, -9, 10]	1	5

## Data:

390074 solutions with  $5 \leq N \leq 12$  integers until '132' [JSON]

17

views

4

downloads

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Indexed in

OpenAIRE

## Publication date:

September 24, 2021

## DOI:

DOI: [10.5281/zenodo.5526707](https://doi.org/10.5281/zenodo.5526707)

## Keyword(s):

[Anomaly free](#) [Diophantine equations](#) [Abelian symmetry](#)  
[Gauge Symmetry](#)

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## Versions

Version 1

Sep 24, 2021

[10.5281/zenodo.5526707](https://doi.org/10.5281/zenodo.5526707)

## Effective Dirac neutrino mass operator

$$\mathcal{L}_{\text{eff}} = h_{\nu}^{\alpha i} (\nu_{R\alpha})^{\dagger} \epsilon_{ab} L_i^a H^b \left( \frac{S^*}{\Lambda} \right)^{\delta} + \text{H.c.}, \quad \text{with } i = 1, 2, 3,$$

and  $\delta = 1, 2, \dots$  for dimension 5 (D-5) or 6 (D-6) operators, etc. Here  $h_{\nu}^{\alpha i}$  correspond to dimensionless induced couplings,  $\nu_{R\alpha}$  are at least two RHNs ( $\alpha = 1, 2, \dots$ ) with the same  $D$  or  $X$ -charge  $\nu$ ,  $L_i$  are the lepton doublets with  $X$ -charge  $-L$ ,  $H$  is the SM Higgs doublet with  $X$ -charge  $h = L - m$ ,  $S$  is the complex singlet scalar responsible for the SSB of the anomaly-free gauge symmetry with  $D$  or  $X$ -charge

$$s = -(\nu + m)/\delta,$$

## Diracness of non-zero DM and Dirac neutrinos masses from $U(1)_X$

Starting from the extended dataset with the solutions with  $N$  integers to the Diophantine equations (1) (2), we apply the following steps

- Check that the solution has two (three) repeated integers to be identified as  $\nu$  and fix  $N_\nu = 2$  ( $N_\nu = 3$ ).
- For  $\delta = 1, 2, \dots$  and all the possible combinations for  $m$  and  $\nu$  in the solution, including  $m = 0$ , find the  $s$  value compatible with the effective Dirac neutrino mass operator of  $D-4 + \delta$  according to eq. (??).
- Interpret the integers in the solution which are different from  $m$  and  $\nu$  as the  $D$ -charges for  $m = 0$  or  $X$ -charges for  $m \neq 0$  of a set of singlet chiral fermions:  $\psi_i$ ,  $i = 1, \dots, N_{\text{chiral}} - N_\nu$ . Then select the solutions for which the condition

$$|n_i + n_j| = |s| \quad (3)$$

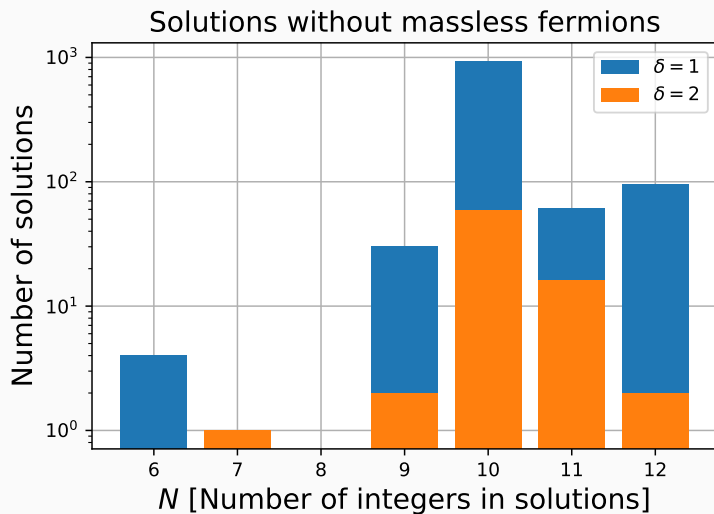
which guarantees that all the singlet chiral fermions,  $\psi_i$ , acquire masses after the spontaneous symmetry breaking of the gauge Abelian symmetry through  $\langle S \rangle$ .

## Unconditional stability

Two DM candidates with *unconditional* stability. This happens when there are two remnant symmetries such that  $\mathbb{Z}_{|s|} \cong \mathbb{Z}_p \otimes \mathbb{Z}_q$  with  $p$  and  $q$  coprimes and  $|s| = pq$ , which guarantee the stability of each lightest state under  $\mathbb{Z}_p$  and  $\mathbb{Z}_q$  respectively, without imposing any kinematical restriction. For the two DM candidates associated to the set of chiral fields  $\psi_i$  and  $\chi_j$ , we consider below the following two possibilities for  $|s|$

- $\mathbb{Z}_6 \cong \mathbb{Z}_2 \otimes \mathbb{Z}_3$ : solutions with at least a set of chiral fields with  $\psi_i \sim [\omega_6^2 \vee \omega_6^4]$  under  $\mathbb{Z}_6$ , and at least a set of chiral fields with  $\chi_i \sim \omega_6^3$  under  $\mathbb{Z}_6$ ,
- $\mathbb{Z}_{14} \cong \mathbb{Z}_2 \otimes \mathbb{Z}_7$ : solutions with at least a set of chiral fields with  $\psi_i \sim [\omega_{14}^2 \vee \omega_{14}^6 \vee \omega_{14}^8 \vee \omega_{14}^{10} \vee \omega_{14}^{12}]$  under  $\mathbb{Z}_{14}$  and at least a set of chiral fields with  $\chi_i \sim \omega_{14}^7$  under  $\mathbb{Z}_{14}$ ,

where  $\omega_{|s|} = e^{i2\pi/|s|}$ .



**Figure 1:** Distribution of solutions with  $N$  integers to the Diophantine equations (??) which allow the effective Dirac neutrino operator at  $D-4 + \delta$  for at least two right-handed neutrinos and have non-vanishing Dirac or Majorana masses for the other singlet chiral fermions in the solution.

## 48 type of representative solutions

Solution	$N$	$N_{\text{chiral}}$	$m$	$\nu$	$\delta$	$s$	$N_D$	$N_M$	$G_D$	$G_M$
(1, -2, -3, 5, 5, -6)	6	6	0	5	1	-5	2	0	1	0
(3, 3, 3, -5, -5, -7, 8)	7	4	3	-5	2	1	1	0	1	0
(1, -2, 3, 4, 6, -7, -7, -7, 9)	9	9	0	-7	1	7	3	0	1	0
(1, 1, -4, -5, 9, 9, 9, -10, -10)	9	9	0	9	1	-9	3	0	2	0
(1, 2, -6, -6, -6, 8, 9, 9, -11)	9	6	-6	9	1	-3	2	0	1	0
(1, -3, 8, 8, 8, -12, -12, -17, 19)	9	6	8	-12	2	2	2	1	1	1
(8, 8, 8, -12, -12, 15, -17, -23, 25)	9	6	8	-12	2	2	2	0	1	0
(1, -2, -2, 3, 3, -4, -4, 6, 6, -7)	10	10	0	6	1	-6	3	2	2	2
(1, -2, -2, 3, 4, -5, -5, 7, 7, -8)	10	10	0	-5	1	5	4	0	2	0
(1, -2, -2, 3, 5, -6, -6, 8, 8, -9)	10	10	0	-6	1	6	4	0	2	0
(2, 2, 3, 4, 4, -5, -6, -6, -7, 9)	10	10	0	2	1	-2	4	2	2	2
(1, 1, 5, 5, 5, -6, -6, -6, -9, 10)	10	10	0	1	1	-1	4	0	3	0
(2, 2, 4, 4, -7, -7, -9, -9, 10, 10)	10	10	0	10	2	-5	3	0	2	0
(1, 2, 2, -3, 6, 6, -8, -8, -9, 11)	10	10	0	-8	1	8	4	1	2	1
(1, -2, -3, 5, 6, -8, -9, 11, 11, -12)	10	10	0	11	1	-11	4	0	1	0
(1, 1, -3, 4, 4, -7, 8, -10, -10, 12)	10	10	0	-10	2	5	4	0	2	0
(1, 1, -2, -2, -4, 6, -10, 11, 12, -13)	10	10	0	-2	1	2	3	2	1	2
(3, 4, 4, 4, 4, -5, -8, -8, -11, 13)	10	10	0	-8	1	8	2	4	1	4
(4, 4, 5, 6, 6, -9, -10, -10, -11, 15)	10	10	0	6	1	-6	4	0	2	0
(1, -2, -4, 7, 7, -10, -12, 14, 14, -15)	10	10	0	14	1	-14	3	2	1	2
(1, 2, 2, -3, 4, -6, 12, -13, -14, 15)	10	10	0	2	1	-2	4	1	1	1
(1, 4, 4, -7, 8, 8, -9, -12, -12, 15)	10	10	0	8	1	-8	4	2	2	2
(1, 2, 2, -9, -9, 16, 16, 17, -18, -18)	10	10	0	-18	1	18	3	2	2	2
(1, -3, -6, 7, -10, 11, -16, 18, 18, -20)	10	10	0	18	2	-9	4	0	1	0

## 48 type of representative solutions

Solution	$N$	$N_{\text{chiral}}$	$m$	$\nu$	$\delta$	$s$	$N_D$	$N_M$	$G_D$	$G_M$
(1, -4, 5, -6, -6, 10, -14, 15, 20, -21)	10	10	0	-6	1	6	4	0	1	0
(2, -3, -6, 7, 12, -14, -14, 17, 20, -21)	10	10	0	-14	1	<b>14</b>	4	1	1	1
(3, 6, 6, -7, 8, 8, -14, -14, -17, 21)	10	10	0	-14	1	<b>14</b>	4	1	2	1
(8, 8, 9, 10, 10, -13, -18, -18, -27, 31)	10	10	0	-18	1	<b>18</b>	4	1	2	1
(1, 1, 1, -2, -2, -5, -5, 6, 6, 7, -8)	11	8	1	-2	1	1	3	0	2	0
(1, -2, -2, -2, -3, 4, 4, -5, 6, 7, -8)	11	8	-2	4	1	-2	3	1	1	1
(1, 1, 2, 2, 2, -4, -4, 7, -8, -9, 10)	11	8	2	-4	1	2	2	2	1	2
(2, 2, 2, -4, -4, -5, 7, -8, 9, 10, -11)	11	8	2	-4	1	2	3	0	1	0
(1, -2, -3, -3, -3, 5, 5, -7, 8, 10, -11)	11	8	-3	5	2	-1	3	0	1	0
(3, 3, 3, -4, -4, 7, 7, -8, -9, -9, 11)	11	8	3	-9	2	3	3	0	2	0
(1, 3, 5, -6, -6, -6, 8, -9, 12, 12, -14)	11	8	-6	12	1	-6	3	1	1	1
(1, -2, 6, 6, 6, -7, 8, -9, -12, -12, 15)	11	8	6	-12	1	<b>6</b>	3	0	1	0
(1, 3, 3, 6, 6, 6, -7, -10, -12, -12, 16)	11	8	6	-12	1	<b>6</b>	2	2	1	2
(1, -2, -2, -2, 3, 3, 4, 4, -5, -5, -5, 6)	12	9	-5	-2	1	<b>7</b>	3	0	2	0
(1, 1, -3, 4, 5, 5, 5, -6, -7, -7, -8, 10)	12	9	5	-7	1	2	3	2	1	2
(1, 1, 1, -2, 4, -7, -7, -7, 8, 9, 9, -10)	12	9	-7	9	1	-2	2	3	1	3
(1, 1, -3, -3, -5, -5, -5, 7, 7, 7, 9, -11)	12	9	-5	7	1	-2	3	2	2	2
(1, -3, -3, -3, 4, 6, 7, 9, -10, -10, -10, 12)	12	9	-3	-10	1	13	3	0	1	0
(1, 1, 1, 3, 3, -5, 7, 7, -11, -11, -11, 15)	12	9	1	-11	1	10	3	1	2	1
(1, 1, 1, 3, 5, 5, -5, 5, -9, -9, -13, 15)	12	9	5	-9	2	2	2	3	1	3
(1, -2, -2, 3, 6, -10, -10, -10, 13, 14, 14, -17)	12	9	-10	14	1	-4	4	2	2	2
(1, -3, 9, -11, -13, -13, -13, 15, 15, 15, 21, -23)	12	9	-13	15	1	-2	3	1	1	1

# Multi-component dark matter I

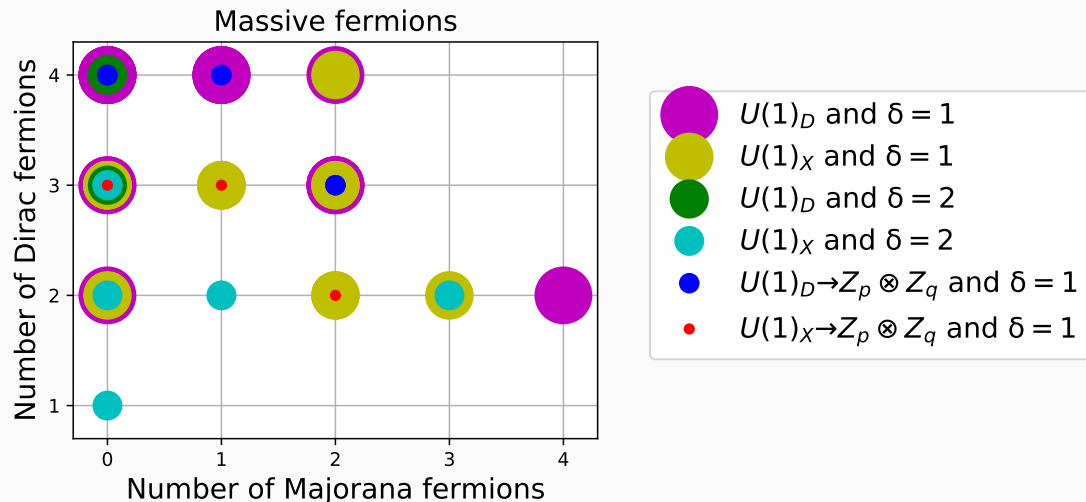
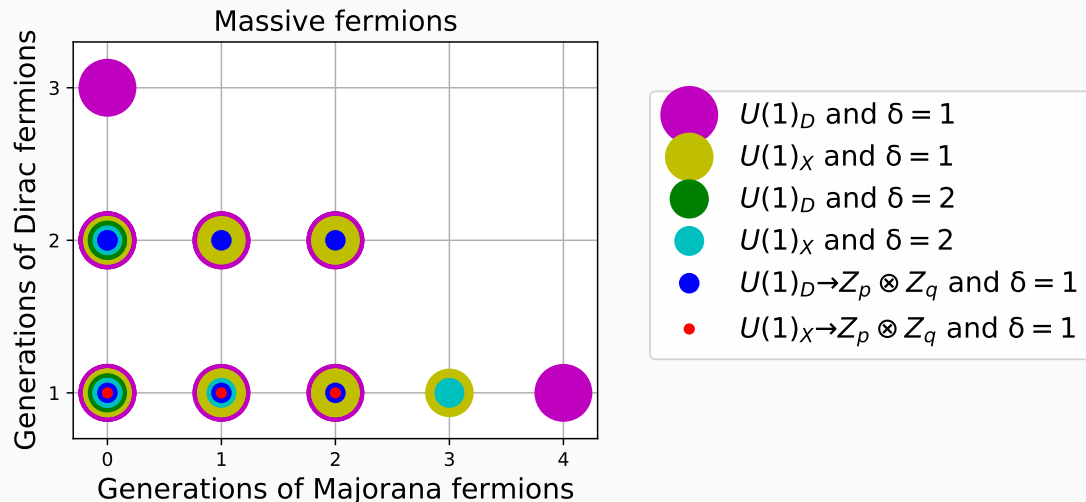


Figure 2: Number of massive Dirac and Majorana fermions in each type of the 48 types of solutions of



## Multi-component dark matter II



**Solution:**  $(3, 3, 3, -5, -5, -7, 8)$

Field	$SU(2)_L$	$U(1)_Y$	$U(1)_X$	$U(1)_{B-L}$
$L_i$	<b>2</b>	$-1/2$	$-L$	$-1$
$e_{Ri}$	<b>1</b>	$-2$	$3 - 2L$	$-1$
$\nu_{R\alpha}$	<b>1</b>	$0$	$-5$	$-5/3$
$\psi_1$	<b>1</b>	$0$	$-7$	$-7/3$
$\psi_2$	<b>1</b>	$0$	$8$	$8/3$
$H$	<b>2</b>	$1/2$	$L - 3$	$0$
$S$	<b>1</b>	$0$	$1$	$1/3$
$\sigma_1^+$	<b>1</b>	$+2$	$2L$	$2$
$\sigma_2^+$	<b>1</b>	$+2$	$-(2 - 2L)$	$4/3$

**Table 2:** Charges for last solution.  $i = 1, 2, 3$ ,  $\alpha = 1, 2, 3$ . Note that  $(\omega_n^d)^* = \omega_n^{-d} = \omega_n^{n-d}$ .

Neutrino phenomenology with J. Calle and O. Zapata: arXiv:2103.15328 [PRD]






DM Phenomenology: arXiv:1506.05107

In general, we can see that multi-component and multi-generation DM candidates are the trend for gauge Abelian extensions of the SM with massive singlet chiral fermions compatible with the effective Dirac neutrino mass operator of dimension

# One parameter $U(1)_X$ SM extension

Fields	$SU(2)_L$	$U(1)_Y$	$U(1)_X$	$U(1)_{B-L}$	$U(1)_R$	$U(1)_D$	$U(1)_G$	$U(1)^*_D$
$L$	<b>2</b>	$-1/2$	$/$	$-1$	0	$-3/2$	$-1/2$	0
$Q$	<b>2</b>	$-1/6$	$-/3$	$1/3$	0	$1/2$	$1/6$	0
$d_R$	<b>1</b>	$-1/2$	$1 + 2/3$	$1/3$	1	0	$2/3$	0
$u_R$	<b>1</b>	$+2/3$	$-1 - 4/3$	$1/3$	$-1$	1	$-1/3$	0
$e_R$	<b>1</b>	$-1$	$1 + 2/$	$-1$	1	$-2$	0	0
$H$	<b>2</b>	$1/2$	$-1 - /$	0	$-1$	$1/2$	$-1/2$	0
$\sum_\alpha n_\alpha$	<b>1</b>	0	$-3$	$-3$	$-3$	$-3$	$-3$	0
$\sum_\alpha n_\alpha^3$	<b>1</b>	0	$-3$	$-3$	$-3$	$-3$	$-3$	0







# solutions with $\sum n_\alpha = -3$ and $\sum n_\alpha^3 = -3$

$(\nu_{R1}, \nu_{R2}, \psi_{N-2}, \dots)$	Ref
$(-1, -1, -1)$	hep-ph/0611205, S. Khalil [JPG]
$(-4, -4, +5)$	 arXiv:0706.0473, Montero, V. Pleitez [PLB]
$\left(-\frac{2}{3}, -\frac{2}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$	 arXiv:1607.04029, S. Patra, W. Rodejohann, C. Yaguna [JHEP]
$\left(-\frac{8}{5}, -\frac{8}{5}, -\frac{2}{5}, -\frac{7}{5}, +2\right)$	 arXiv:1812.05523, with J. Calle, C. Yaguna, Ó. Zapata [PRD]
$\left(-1, -1, -\frac{10}{7}, -\frac{4}{7}, -\frac{2}{7}, \frac{9}{7}\right)$	 1808.03352, with N. Bernal, C. Yaguna, Ó. Zapata [PRD]
$\left(-\frac{5}{3}, -\frac{5}{3}, -\frac{7}{3}, \frac{8}{3}\right)$	  In progress...  method <sup>†</sup>

**Table 3:** Possible solutions with at least two repeated charges and until six chiral fermions.

<sup>†</sup> General  $\sum n_\alpha = 0$  solutions: see D.B Costa, *et al*, arXiv:1905.13729 [PRL]

Or... combine known solutions with  $\sum n_\alpha = 0$  and  $\sum n_\alpha^3 = 0$

$(\nu_{R1}, \nu_{R2}, \psi_{N-2}, \dots)$	Ref
$(-1, -1, -1)$	hep-ph/0611205, S. Khalil [JPG]
$(-4, -4, +5)$	 arXiv:0706.0473, Montero, V. Pleitez [PLB]
$\left(-\frac{2}{3}, -\frac{2}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$	 arXiv:1607.04029, S. Patra, W. Rodejohann, C. Yaguna [JHEP]
$\left(-\frac{8}{5}, -\frac{8}{5}, -\frac{2}{5}, -\frac{7}{5}, +2\right)$	 arXiv:1812.05523, with J. Calle, C. Yaguna, Ó. Zapata [PRD]
$\left(-1, -1, -\frac{10}{7}, -\frac{4}{7}, -\frac{2}{7}, \frac{9}{7}\right)$	 1808.03352, with N. Bernal, C. Yaguna, Ó. Zapata [PRD]
$\left(-\frac{5}{3}, -\frac{5}{3}, -\frac{7}{3}, \frac{8}{3}\right)$	  In progress...  method <sup>†</sup>

[https://en.wikipedia.org/wiki/Sums\\_of\\_three\\_cubes](https://en.wikipedia.org/wiki/Sums_of_three_cubes)

Only known integer solutions for -3 (1953)






September 2019:

$$42 = (-80538738812075974)^3 + 80435758145817515^3 + 12602123297335631^3$$

**Table 3:** Possible solutions with at least two repeated charges and until six chiral fermions.

<sup>†</sup> General  $\sum n_\alpha = 0$  solutions: see D.B Costa, *et al*, arXiv:1905.13729 [PRL]

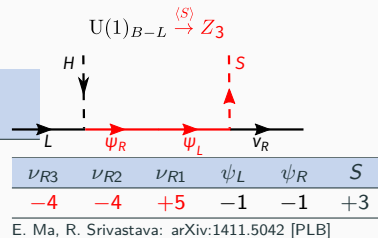
## Or... combine known solutions

$(\nu_{R1}, \nu_{R2}, \psi_{N-2}, \dots)$	Ref
$(-1, -1, -1)$	hep-ph/0611205, S. Khalil [JPG]
$(-4, -4, +5)$	 arXiv:0706.0473, Montero, V. Pleitez [PLB]
$\left(-\frac{2}{3}, -\frac{2}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$	 arXiv:1607.04029, S. Patra, W. Rodejohann, C. Yaguna [JHEP]
$\left(-\frac{8}{5}, -\frac{8}{5}, -\frac{2}{5}, -\frac{7}{5}, +2\right)$	 arXiv:1812.05523, with J. Calle, C. Yaguna, Ó. Zapata [PRD]
$\left(-1, -1, -\frac{10}{7}, -\frac{4}{7}, -\frac{2}{7}, \frac{9}{7}\right)$	 1808.03352, with N. Bernal, C. Yaguna, Ó. Zapata [PRD]
$\left(-\frac{5}{3}, -\frac{5}{3}, -\frac{7}{3}, \frac{8}{3}\right)$	  In progress...  method <sup>†</sup>

Not known solution for one-loop neutrino Majorana masses with local  $U(1)_X$ .

**Table 3:** Possible solutions with at least two repeated charges and until six chiral fermions.

<sup>†</sup> General  $\sum n_{\alpha} = 0$  solutions: see D.B Costa, *et al*, arXiv:1905.13729 [PRL]





$$m_{\text{Majorana}}^{\nu} + = \frac{h_{\nu}}{\Lambda} L \cdot H L \cdot H$$





**3 models**

$$m_{\text{Majorana}}^{\nu} + = \frac{h_{\nu}}{\Lambda} L \cdot H L \cdot H \quad (\text{three-level})$$

Type-I arXiv:1808.03352, II arXiv:1607.04029, III arXiv:1908.04308

$$\mathcal{L} = y(N_R)^\dagger L \cdot H + M_N N_R N_R + \text{h.c.}$$



$$m_{\text{Majorana}}^\nu + = \frac{h_\nu}{\Lambda} L \cdot H L \cdot H$$

Type-I  
seesaw

Type-I arXiv:1808.03352, with N. Bernal, C. Yaguna, and Ó. Zapata [PRD]

$$U(1)_{B-L} \rightarrow Z_7$$

$$\mathcal{L} = y(N_R)^\dagger L \cdot \langle H \rangle + y' \langle S \rangle N_R N_R + \text{h.c}$$



$\nu_{R3}$	$\nu_{R2}$	$S$
-1	-1	2

$$m_{\text{Majorana}}^\nu + = \frac{h_\nu}{\Lambda} L \cdot H L \cdot H \frac{S}{\Lambda}$$

Type-I arXiv:1808.03352

: Also new terms  
arise from spontaneous  
breakdown of a  
new gauge symmetry

# Local $U(1)_{B-L} \rightarrow Z_7$

$$\mathcal{L} = y(N_R)^\dagger L \cdot \langle H \rangle + y' \langle S \rangle N_R N_R + \text{h.c}$$

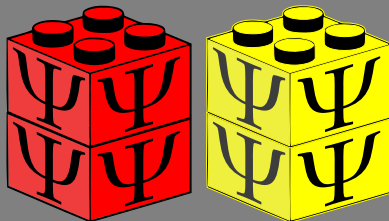


$$m_{\text{Majorana}}^\nu = \frac{h_\nu}{\Lambda} L \cdot H L \cdot H \frac{S}{\Lambda}$$

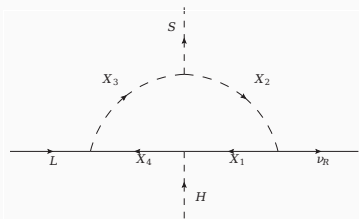
Type-I arXiv:1808.03352

: Also new terms  
arise from spontaneous  
breakdown of a  
new gauge symmetry

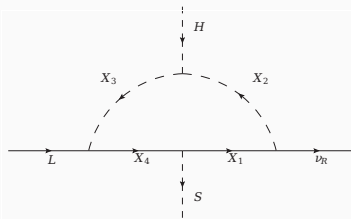
$\nu_{R3}$	$\nu_{R2}$	$\overline{\psi_{L1}}$	$\psi_{R1}$	$\psi_{R2}$	$\overline{\psi_{L2}}$	$S$	$S'$
-1	-1	$-\frac{10}{7}$	$-\frac{4}{7}$	$-\frac{2}{7}$	$\frac{9}{7}$	2	1



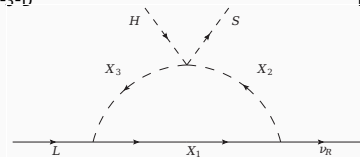
# One loop topologies $U(1)_{B-L} \oplus Z_2 \oplus Z_2$



T1-3-D

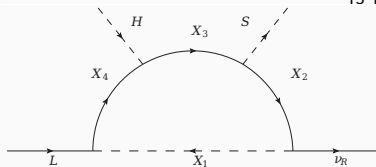


T1-3-E

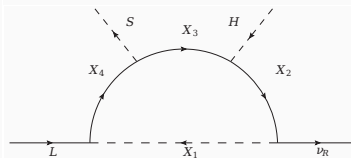


T3-1-A

Chang-Yuan Yao and Gui-Jun Ding, arXiv:1802.05231 [PRD]

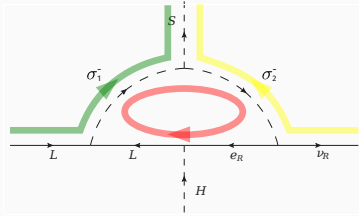


T1-2-A

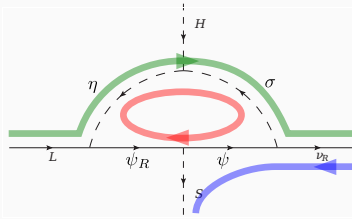


T1-2-B

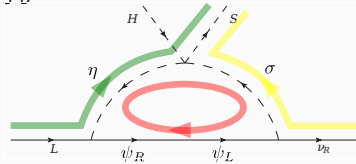
# One loop topologies $U(1)_{B-L}$ only!



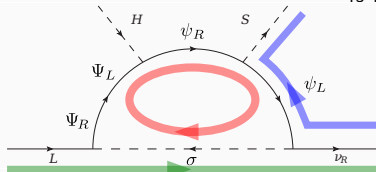
T1-3-D



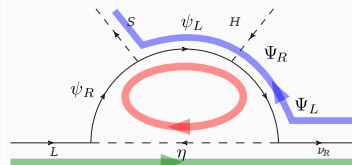
T1-3-E



T3-1-A



T1-2-A



T1-2-B

$\psi_{L,R} \rightarrow$  Singlet fermions

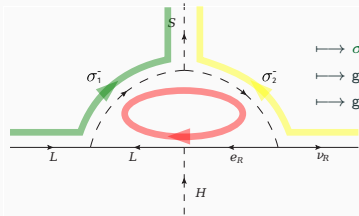
$\Psi_{L,R} \rightarrow$  Vector-like doublet fermions

$\sigma \rightarrow$  Singlet scalar

$\eta \rightarrow$  Doublet scalar

with J. Calle, C. Yaguna, and O. Zapata, arXiv:1812.05523 [PRD]

# One loop topologies $U(1)_{B-L}$ only! with J. Calle, C. Yaguna, and O. Zapata, arXiv:1812.05523 [PRD]



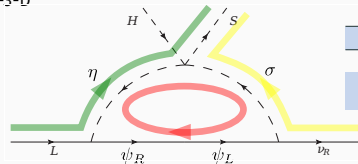
$$\mapsto \sigma_1 = -2, \quad \sigma_2 = -5,$$

$\mapsto$  generalization to two and three loops: S. Saad arXiv:1902.07259 [NPB]

$\mapsto$  generalization to  $U(1)_R$ : *et al*, S. Saad arXiv:1904.07407



T1-3-D



T3-1-A

$\psi_{L,R} \rightarrow$  Singlet fermions (vector-like)

$\sigma \rightarrow$  Singlet scalar

$\eta \rightarrow$  Doublet scalar

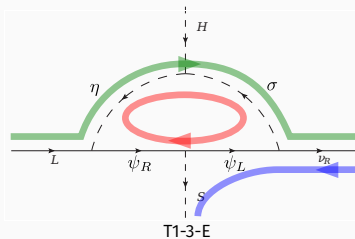
Fields: $f_i$	$(\nu_{R3})^\dagger$	$(\nu_{R2})^\dagger$	$(\nu_{R1})^\dagger$	$\psi_L$	$(\psi_R)^\dagger$	$S$
(A)	+4	+4	-5	-r	r	+3

Anomaly cancellation conditions

$$\sum_i f_i = 3$$

$$\sum_i f_i^3 = 3$$

# One loop topologies $U(1)_{B-L}$ only! with J. Calle, C. Yaguna, and O. Zapata, arXiv:1812.05523 [PRD]



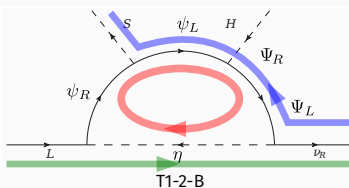
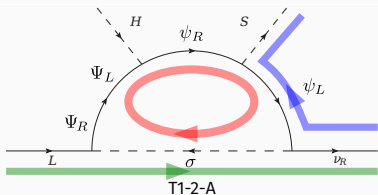
Fields: $f_i$	$(\nu_{R3})^\dagger$	$(\nu_{R2})^\dagger$	$(\nu_{R1})^\dagger$	$\psi_L$	$(\psi_R)^\dagger$	$S$
(A)	+4	+4	-5	-r	r	+3
(B)	$+\frac{8}{5}$	$+\frac{8}{5}$	$+\frac{2}{5}$	$\frac{7}{5}$	$-\frac{10}{5}$	$+\frac{3}{5}$

$\psi_{L,R} \rightarrow$  Singlet fermions (quiral)

$\Psi_{L,R} \rightarrow$  Vector-like doublet fermions

$\sigma \rightarrow$  Singlet scalar

$\eta \rightarrow$  Doublet scalar



Anomaly cancellation conditions

$$\sum_i f_i = 3$$

$$\sum_i f_i^3 = 3$$

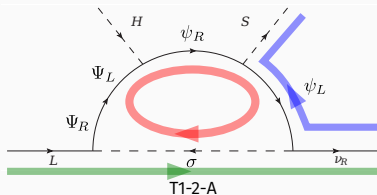


$\psi_{L,R} \rightarrow$  Singlet fermions (quiral)

$\Psi_{L,R} \rightarrow$  Vector-like doublet fermions : **10/5**

$\sigma \rightarrow$  Singlet scalar : 15/5

Fields: $f_i$	$(\nu_{R3})^\dagger$	$(\nu_{R2})^\dagger$	$(\nu_{R1})^\dagger$	$\psi_L$	$(\psi_R)^\dagger$	$S$
(A)	<b>+4</b>	<b>+4</b>	-5	<b>-r</b>	<b>r</b>	+3
(B)	$+\frac{8}{5}$	$+\frac{8}{5}$	$+\frac{2}{5}$	$\frac{7}{5}$	<b><math>-\frac{10}{5}</math></b>	<b><math>+\frac{3}{5}</math></b>



Anomaly cancellation conditions

$$\sum_i f_i = 3$$

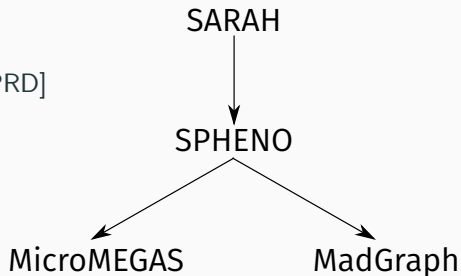
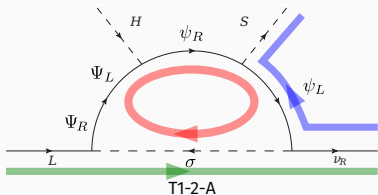
$$\sum_i f_i^3 = 3$$

# SD<sup>3</sup>M+SSDM: $\sigma_a$ ( $a = 1, 2$ )

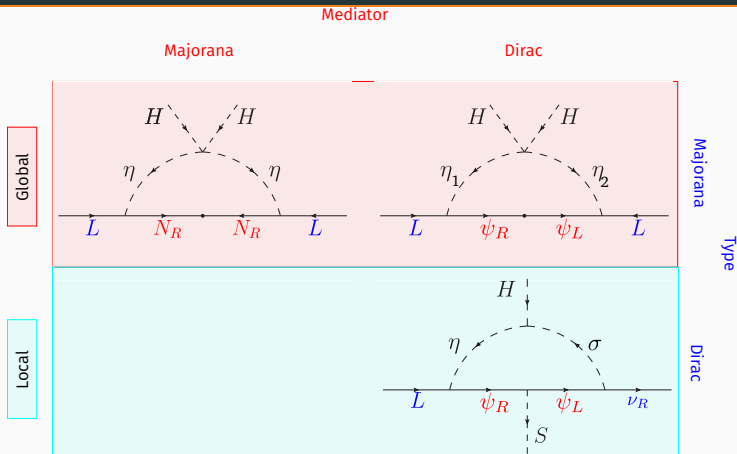
$$M_\psi = h_1 \langle S \rangle, \text{ } y_2 = 0:$$

$$\mathcal{L} = \mathcal{L}_{\text{SD}^3\text{M}} + h_3^{ia}(\widetilde{\Psi_R}) \cdot L_i \sigma_a + h_2^{\beta a} (\nu_{R\beta})^\dagger \psi_L \sigma_a^* - V(\sigma_a, S, H).$$

with A.F Rivera, W. Tangarife, arXiv:1906.09685 [PRD]



# Radiative Type-I seesaw $\rightarrow$ Local: only $U(1)_{B-L}$ ! arXiv:1812.05523, with J. Calle, C. Yaguna, Ó. Zapata [PRD]



For radiative Dirac models with only  $U(1)_X$  see also:

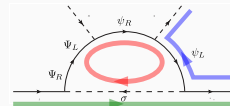
arXiv:1812.01599, 1901.06402, 1902.07259,

1903.01477, 1904.07407, 1907.08630, 1910.09537

1909.00833 1907.11557, 1909.09574

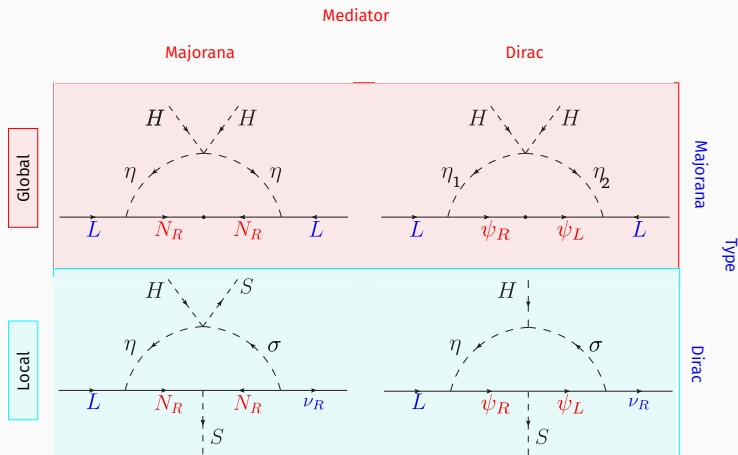
$\mathcal{O}(50)$  new models mostly with  $\sim (-4, -4, 5)$

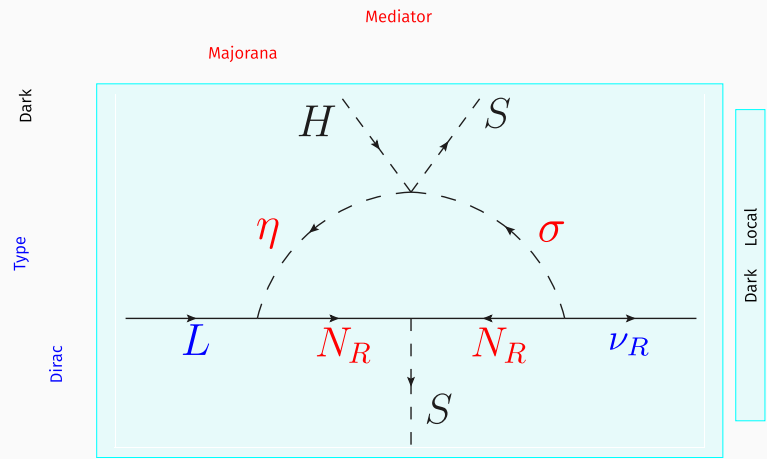
Example: **New**  $U(1)_{B-L}$



Pheno analysis with

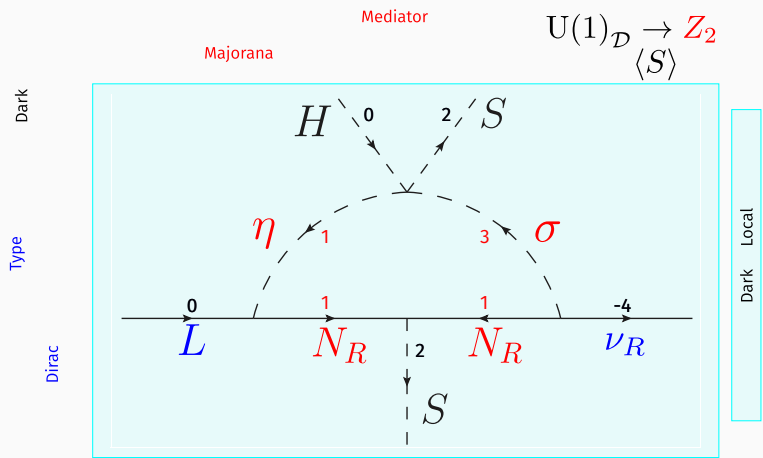
A. Rivera, W. Tangarife, arXiv:1906.09685 [PRD]





$$N = -\frac{\nu}{4}, \quad \eta = -\frac{\nu}{4}, \quad \sigma = -\frac{3\nu}{4}.$$

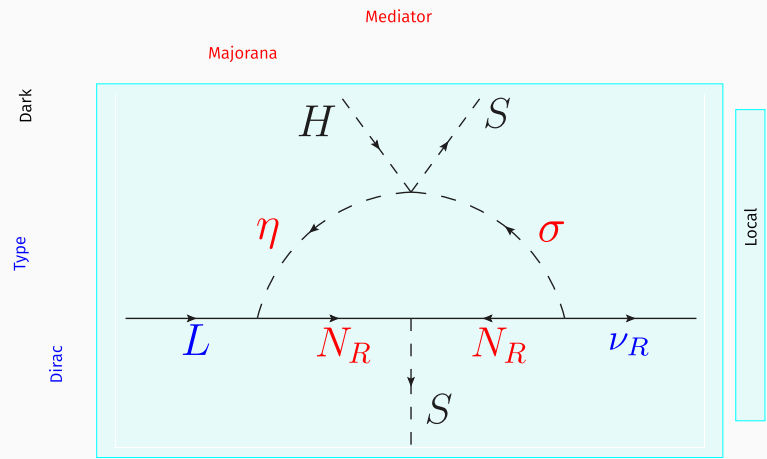
Fields	$SU(2)_L$	$U(1)_Y$	$U(1)_D$
$L$	2	$-1/2$	0
$Q$	2	$-1/6$	0
$d_R$	1	$-1/2$	0
$u_R$	1	$+2/3$	0
$e_R$	1	$-1$	0
$H$	2	$1/2$	0
$\eta$	2	$1/2$	1
$S$	1	0	2
$\sigma$	1	0	3
$\nu_{R1}$	1	0	$-4$
$\nu_{R2}$	1	0	$-4$
$\nu_{R3}$	1	0	5
$N_{R1}$	1	0	1
$N_{R2}$	1	0	1
$N_{R3}$	1	0	1
TOTAL			0 22



$$N = -\frac{\nu}{4}, \quad \eta = -\frac{\nu}{4}, \quad \sigma = -\frac{3\nu}{4}.$$

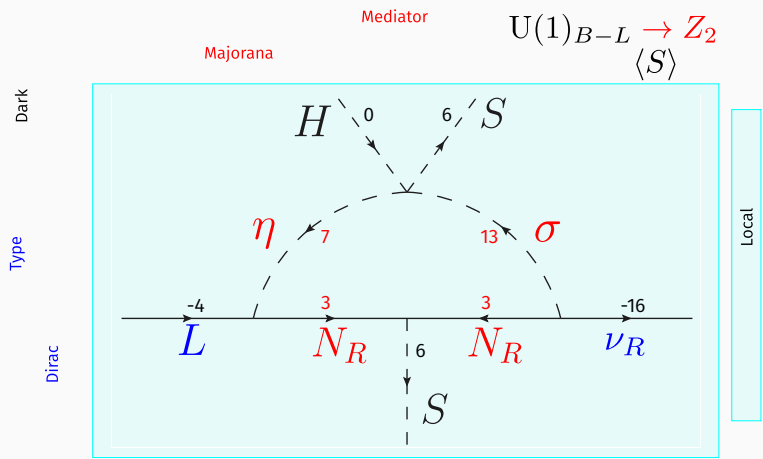
Fields	$SU(2)_L$	$U(1)_Y$	$U(1)_D$
$L$	2	$-1/2$	0
$Q$	2	$-1/6$	0
$d_R$	1	$-1/2$	0
$u_R$	1	$+2/3$	0
$e_R$	1	$-1$	0
$H$	2	$1/2$	0
$\eta$	2	$1/2$	1
$S$	1	0	2
$\sigma$	1	0	3
$\nu_{R1}$	1	0	-4
$\nu_{R2}$	1	0	-4
$\nu_{R3}$	1	0	5
$N_{R1}$	1	0	1
$N_{R2}$	1	0	1
$N_{R3}$	1	0	1
TOTAL			0 22

# Dirac Radiative Type-I seesaw with Majorana mediators with J. Calle and Ó. Zapata, arXiv:1909.09574



$$N = -\frac{\nu}{4} - \frac{1}{4}, \quad \eta = -\frac{\nu}{4} - \frac{1}{4} - l, \quad \sigma = -\frac{3\nu}{4} + \frac{1}{4}.$$

Fields	$SU(2)_L$	$U(1)_Y$	$U(1)_X$
$L$	2	$-1/2$	$l$
$Q$	2	$-1/6$	$-l/3$
$d_R$	1	$-1/2$	$1 + 2l/3$
$u_R$	1	$+2/3$	$-1 - 4l/3$
$e_R$	1	$-1$	$1 + 2l$
$H$	2	$1/2$	$-1 - l$
$\eta$	2	$1/2$	$3/4 - l$
$S$	1	0	$3/2$
$\sigma$	1	0	$13/4$
$\nu_{R1}$	1	0	$-4$
$\nu_{R2}$	1	0	$-4$
$\nu_{R3}$	1	0	5
$N_{R1}$	1	0	$3/4$
$N_{R2}$	1	0	$3/4$
$N_{R3}$	1	0	$3/4$
$\xi_{L\alpha}$	1	0	$3/4$ <sup>22</sup>



$$N = -\frac{\nu}{4} - \frac{1}{4}, \quad \eta = -\frac{\nu}{4} - \frac{1}{4} + 1, \quad \sigma = -\frac{3\nu}{4} + \frac{1}{4}.$$

Fields	$SU(2)_L$	$U(1)_Y$	$U(1)_{B-L}$
$L$	2	$-1/2$	$-1$
$Q$	2	$-1/6$	$1/3$
$d_R$	1	$-1/2$	$1/3$
$u_R$	1	$+2/3$	$1/3$
$e_R$	1	$-1$	$-1$
$H$	2	$1/2$	0
$\eta$	2	$1/2$	$7/4$
$S$	1	0	$3/2$
$\sigma$	1	0	$13/4$
$\nu_{R1}$	1	0	$-4$
$\nu_{R2}$	1	0	$-4$
$\nu_{R3}$	1	0	5
$N_{R1}$	1	0	$3/4$
$N_{R2}$	1	0	$3/4$
$N_{R3}$	1	0	$3/4$
$\xi_{L\alpha}$	1	0	$3/4$ 22



$$\begin{aligned} \mathcal{L} \supset & - g' Z'_\mu \sum_F q_F \bar{F} \gamma^\mu F + \sum_\phi |(\partial_\mu + i g' q_\phi Z'_\mu) \phi|^2 \\ & - [h_{i\alpha} \bar{L}_i \tilde{\eta} N_{R\alpha} + y_{j\alpha} \bar{\nu}_{Rj} \sigma^* N_{R\alpha}^c + k_\alpha \overline{N_{R\alpha}^c} N_{R\alpha} S^* + \text{h.c.}] - \mathcal{V}(H, S, \eta, \sigma). \end{aligned}$$

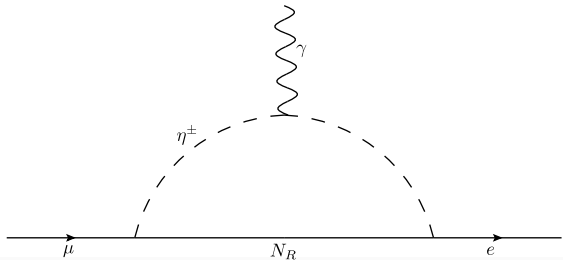
$F(\phi)$  denote the new fermions (scalars)

$$\begin{aligned} \mathcal{V}(H, S, \eta, \sigma) = & V(H) + V(S) + V(\eta) + V(\sigma) \\ & + \lambda_{HS} (H^\dagger H) (S^* S) + \lambda_2 (H^\dagger H) (\sigma^* \sigma) + \lambda_3 (H^\dagger H) (\eta^\dagger \eta) \\ & + \lambda_4 (S^* S) (\sigma^* \sigma) + \lambda_5 (S^* S) (\eta^\dagger \eta) + \lambda_6 (\eta^\dagger \eta) (\sigma^* \sigma) + \lambda_7 (\eta^\dagger H) (H^\dagger \eta) \\ & + \lambda_8 (\eta^\dagger H S^* \sigma + \text{h.c.}), \end{aligned}$$

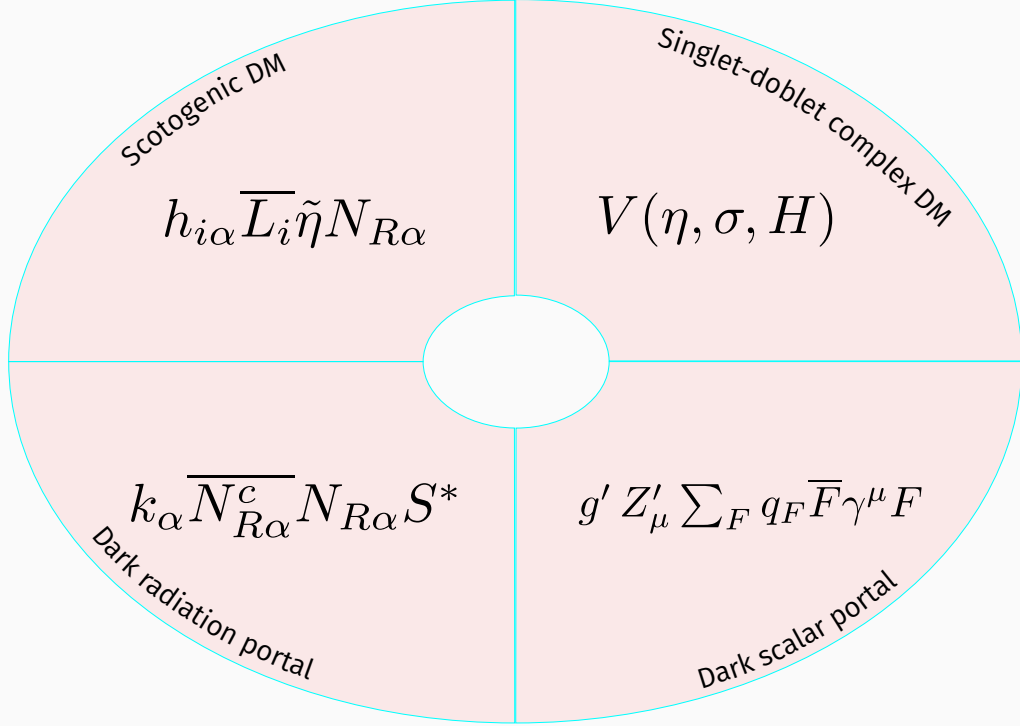
$$(\mathcal{M}_\nu)_{ij} = \frac{1}{32\pi^2} \frac{\lambda_8 v_S^2 v_H}{m_{\eta_R^0}^2 - m_{\sigma_R^0}^2} \sum_{\alpha=1}^3 h_{i\alpha} k_\alpha y_{j\alpha}^* \left[ F\left(\frac{m_{\eta_R^0}^2}{M_{N_\alpha}^2}\right) - F\left(\frac{m_{\sigma_R^0}^2}{M_{N_\alpha}^2}\right) \right] + (R \rightarrow I),$$

where  $F(x) = x \log x / (x - 1)$ .

$\mu \rightarrow e \gamma$



$$\left| \sum_{\alpha} h_{2\alpha} h_{1\alpha}^* \right| \lesssim 0.02 \left( \frac{m_\chi}{2 \text{ TeV}} \right)^2.$$



Scotogenic DM

$$h_{i\alpha} \overline{L}_i \tilde{\eta} N_{R\alpha}$$

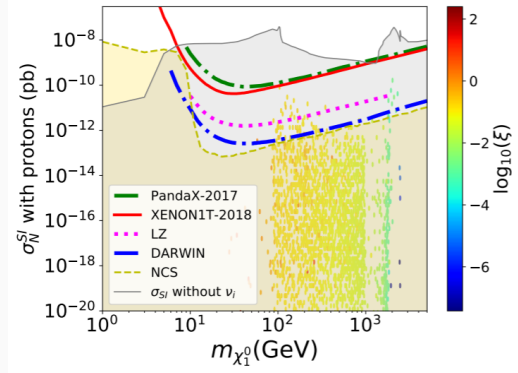
A. Ibarra, C. Yaguna, Ó. Zapata,  
arXiv:1601.01163 [PRD]

Scotogenic DM

$$h_{i\alpha} \overline{L}_i \tilde{\eta} N_{R\alpha}$$

$$N_{R2} \rightarrow \Sigma$$

with A. Rivera, arXiv:1907.11938



$$(\chi_1^0 \ \chi_2^0)^T = R(\textcolor{red}{N}_R \ \textcolor{blue}{\Sigma})^T$$

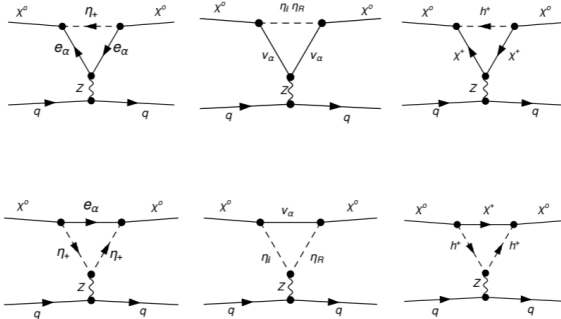
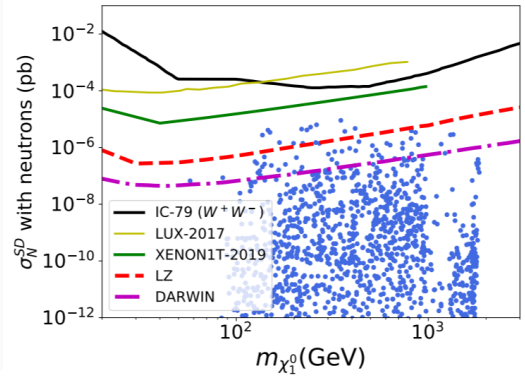
$$\xi = \frac{|M_{\Sigma} - m_{\chi_1^0}|}{m_{\chi_1^0}}$$

Scotogenic DM

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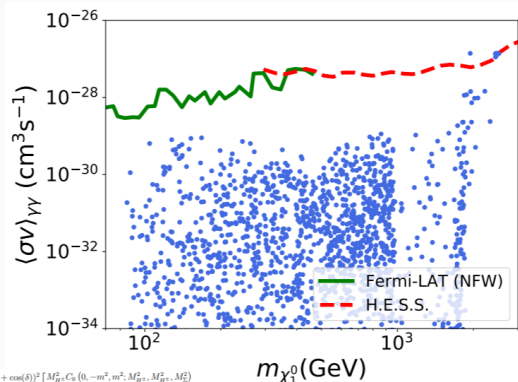
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with A. Rivera, arXiv:1907.11938

$$\sigma v (\chi_1^0 \chi_1^0 \rightarrow \gamma\gamma) = \frac{|\mathcal{B}|^2}{32\pi m_{\chi_1^0}^2}$$



$$\mathcal{B} = \frac{\sqrt{2}\alpha m^2 \sin^2(\alpha) Y_{\tilde{G}}^2 (\sin(\delta) + \cos(\delta))^2}{\pi} \left[ \frac{M_{H^\pm}^2 C_0 (0, -m^2, m^2; M_{H^\pm}^2, M_{H^\pm}^2, M_{\tilde{G}}^2)}{M_{H^\pm}^2 - M_{\tilde{G}}^2} \right. \\ - \frac{M_{\tilde{G}} (-2m M_{H^\pm}^2 - M_{\tilde{G}} M_{H^\pm}^2 + m^2 M_{\tilde{G}} + 2m M_{\tilde{G}}^2 + M_{\tilde{G}}^2) C_0 (0, -m^2, m^2; M_{\tilde{G}}^2, M_{\tilde{G}}^2, M_{H^\pm}^2)}{(M_{H^\pm}^2 - M_{\tilde{G}}^2) (M_{H^\pm}^2 + m^2 - M_{\tilde{G}}^2)} \\ + \frac{2M_{\tilde{G}} (m + M_{\tilde{G}}) C_0 (0, 0, 4m^2; M_{\tilde{G}}^2, M_{\tilde{G}}^2, M_{\tilde{G}}^2)}{-M_{H^\pm}^2 - m^2 + M_{\tilde{G}}^2} \left. \right] \\ + \frac{\alpha m^2 \sin(\alpha) \cos(\alpha) Y_{\tilde{G}}^2}{\pi} \left[ - \frac{m_{\tilde{G}}^2 C_0 (0, -m^2, m^2; m_{\tilde{G}}^2, m_{\tilde{G}}^2, m_{\tilde{G}}^2)}{m_{\tilde{G}}^2 - m_{\tilde{G}}^2} \right. \\ + \frac{m_{\tilde{G}}^2 (m_{\tilde{G}}^2 + m^2 - m_{\tilde{G}}^2) C_0 (0, -m^2, m^2; m_{\tilde{G}}^2, m_{\tilde{G}}^2, m_{\tilde{G}}^2)}{(m_{\tilde{G}}^2 - m_{\tilde{G}}^2) (-m_{\tilde{G}}^2 + m^2 + m_{\tilde{G}}^2)} + \frac{2m_{\tilde{G}}^2 C_0 (0, 0, 4m^2; m_{\tilde{G}}^2, m_{\tilde{G}}^2, m_{\tilde{G}}^2)}{-m_{\tilde{G}}^2 + m^2 + m_{\tilde{G}}^2} \left. \right] \\ + \frac{\alpha m^2 \cos^2(\alpha) Y_{\tilde{G}}^2}{2\sqrt{2}\pi} \left[ \frac{m_{\tilde{G}}^2 C_0 (0, -m^2, m^2; m_{\tilde{G}}^2, m_{\tilde{G}}^2, m_{\tilde{G}}^2)}{m_{\tilde{G}}^2 - m_{\tilde{G}}^2} \right. \\ - \frac{m_{\tilde{G}}^2 (m_{\tilde{G}}^2 + m^2 - m_{\tilde{G}}^2) C_0 (0, -m^2, m^2; m_{\tilde{G}}^2, m_{\tilde{G}}^2, m_{\tilde{G}}^2)}{(m_{\tilde{G}}^2 - m_{\tilde{G}}^2) (-m_{\tilde{G}}^2 + m^2 + m_{\tilde{G}}^2)} - \frac{2m_{\tilde{G}}^2 C_0 (0, 0, 4m^2; m_{\tilde{G}}^2, m_{\tilde{G}}^2, m_{\tilde{G}}^2)}{-m_{\tilde{G}}^2 + m^2 + m_{\tilde{G}}^2} \left. \right] \\ + \frac{\sqrt{2}\alpha m^2 \sin^2(\alpha) Y_{\tilde{G}}^2}{2\pi} \left[ \frac{m_{\tilde{G}}^2 C_0 (0, -m^2, m^2; m_{\tilde{G}}^2, m_{\tilde{G}}^2, m_{\tilde{G}}^2)}{m_{\tilde{G}}^2 - m_{\tilde{G}}^2} \right. \\ - \frac{m_{\tilde{G}}^2 (m_{\tilde{G}}^2 + m^2 - m_{\tilde{G}}^2) C_0 (0, -m^2, m^2; m_{\tilde{G}}^2, m_{\tilde{G}}^2, m_{\tilde{G}}^2)}{(m_{\tilde{G}}^2 - m_{\tilde{G}}^2) (-m_{\tilde{G}}^2 + m^2 + m_{\tilde{G}}^2)} - \frac{2m_{\tilde{G}}^2 C_0 (0, 0, 4m^2; m_{\tilde{G}}^2, m_{\tilde{G}}^2, m_{\tilde{G}}^2)}{-m_{\tilde{G}}^2 + m^2 + m_{\tilde{G}}^2} \left. \right] \\ - \frac{8\sqrt{2}\alpha m^2 \cos^2(\alpha) M_{\tilde{G}}^2}{\pi (M_{\tilde{G}}^2 - M_{\tilde{G}}^2) (4v_{\tilde{G}}^2 + v_{\tilde{G}}^2) (m^2 - M_{\tilde{G}}^2 + M_{\tilde{G}}^2) (m^2 + M_{\tilde{G}}^2 - M_{\tilde{G}}^2)} \\ \left[ 4(m^2 - M_{\tilde{G}}^2) (M_{\tilde{G}}^2 - M_{\tilde{G}}^2) (m^2 - M_{\tilde{G}}^2 + M_{\tilde{G}}^2) C_0 (0, 0, 4m^2; M_{\tilde{G}}^2, M_{\tilde{G}}^2, M_{\tilde{G}}^2) \right. \\ + 2M_{\tilde{G}} (2m - M_{\tilde{G}}) (M_{\tilde{G}}^2 - M_{\tilde{G}}^2) (m^2 + M_{\tilde{G}}^2 - M_{\tilde{G}}^2) C_0 (0, 0, 4m^2; M_{\tilde{G}}^2, M_{\tilde{G}}^2, M_{\tilde{G}}^2) \\ - (m^2 - M_{\tilde{G}}^2 + M_{\tilde{G}}^2) (-M_{\tilde{G}}^2 (m^2 + M_{\tilde{G}}^2) - 4m M_{\tilde{G}} (m^2 + M_{\tilde{G}}^2 - M_{\tilde{G}}^2) + 4M_{\tilde{G}}^2 + M_{\tilde{G}}^2) \\ C_0 (0, -m^2, m^2; M_{\tilde{G}}^2, M_{\tilde{G}}^2, M_{\tilde{G}}^2) - M_{\tilde{G}} (m^2 + M_{\tilde{G}}^2 - M_{\tilde{G}}^2) (4m^3 - 3m^2 M_{\tilde{G}} + M_{\tilde{G}}^2 - M_{\tilde{G}} M_{\tilde{G}}^2) \\ \left. C_0 (0, -m^2, m^2; M_{\tilde{G}}^2, M_{\tilde{G}}^2, M_{\tilde{G}}^2) \right].$$

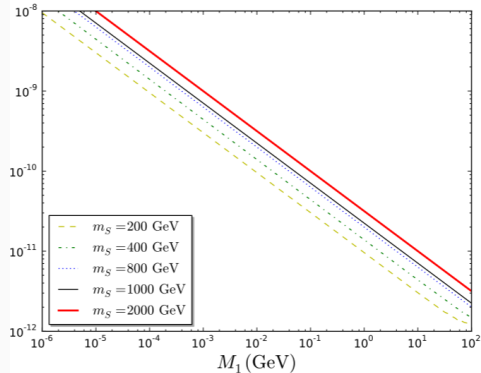
Scotogenic DM

**FIMP Scenario**

$$h_{i\alpha} \overline{L}_i \tilde{\eta} N_{R\alpha}$$

F. Molinaro, C. Yaguna, Ó. Zapata,  
arXiv:1405.1259 [JCAP]

$$h_1 \sim h_{1\alpha}$$



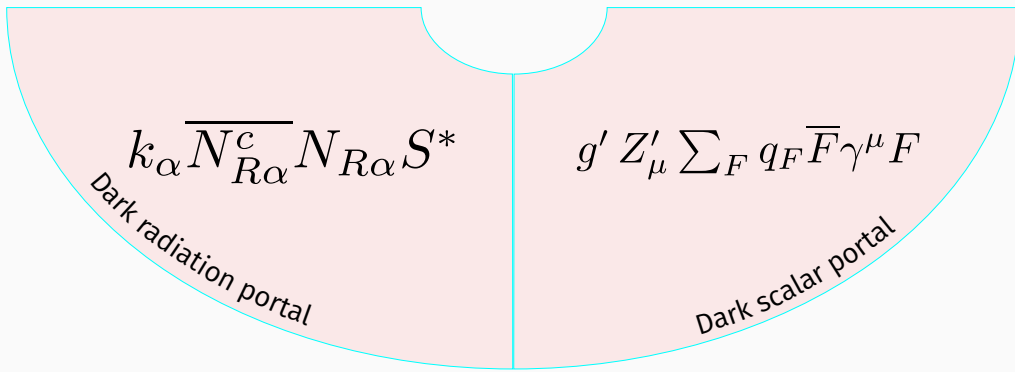
$$l(\eta^+) = 3 \times 10^5 \text{ cm} \left( \frac{M_1}{1 \text{ GeV}} \right) \left( \frac{1 \text{ TeV}}{m_{\eta^+}} \right)^2$$

$$\lesssim 3 \text{ meters} \left( \frac{1 \text{ TeV}}{m_{\eta^+}} \right)^2 \quad \text{for} \quad M_1 \lesssim 1 \text{ MeV}$$



$$N_R N_R \rightarrow \nu_R \nu_R$$

$$\Delta N_{\text{eff}} \sim 0.2$$



## **(One-loop) Dirac neutrino masses**

---

## Small Dirac neutrino masses

To explain the **smallness** of Dirac neutrino masses choose  $U(1)_X$  which:

- Forbids tree-level mass (TL) term (  $Y(H) = +1/2$  )

$$\begin{aligned}\mathcal{L}_{\text{T.L}} &= h_D \epsilon_{ab} (\nu_R)^\dagger L^a H^b + \text{h.c.} \\ &= h_D (\nu_R)^\dagger L \cdot H + \text{h.c.}\end{aligned}$$

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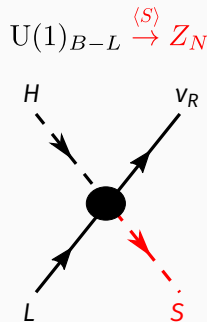
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- Realizes of the 5-dimension operator which conserves lepton number in  $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ :

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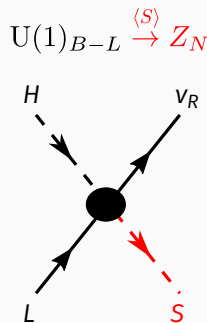
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- Enhancement to the *effective number of degrees of freedom in the early Universe*  $\Delta N_{\text{eff}} = N_{\text{eff}} - N_{\text{eff}}^{\text{SM}}$  (see arXiv:1211.0186)



See E. Ma, Rahul Srivastava: arXiv:1411.5042 [PLB] for tree-level realization

From 1210.6350 and 1805.02025:  $\Delta N_{\text{eff}} = 3 (T_{\nu_R} / T_{\nu_L})^4$

$$\Gamma_{\nu_R}(T) = n_{\nu_R}(T) \sum_f \langle \sigma_f(\nu_R \bar{\nu}_R \rightarrow \bar{f} f) v \rangle$$

$$= \sum_f \frac{g_{\nu_R}^2}{n_{\nu_R}} \int \frac{d^3 p}{(2\pi)^3} \frac{d^3 q}{(2\pi)^3} f_{\nu_R}(p) f_{\nu_R}(q) \sigma_f(s) (1 - \cos \theta),$$

$$s = 2pq(1 - \cos \theta),$$

$$f_{\nu_R}(k) = 1/(e^{k/T} + 1)$$

$$n_{\nu_R}(T) = g_{\nu_R} \int \frac{d^3 k}{(2\pi)^3} f_{\nu_R}(k),$$

$$\text{with } g_{\nu_R} = 2$$

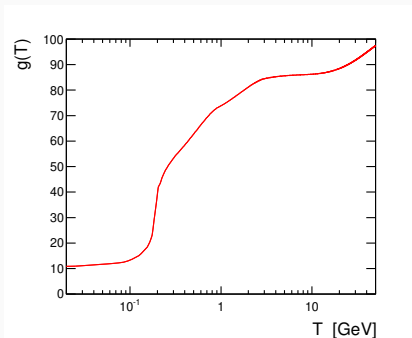
$$\sigma_f(s) \simeq \frac{N_C^f (Q_{BL}^f)^2 Q^2 s}{12\pi} \left( \frac{g'}{M_{Z'}} \right)^4, \quad \text{In the limit } M_{Z'}^2 \gg s.$$

with three right-handed neutrinos, the Hubble parameter is

$$H(T) = \sqrt{\frac{4\pi^3 G_N [g(T) + 21/4]}{45}} T^2.$$

The right-handed neutrinos decouple when

$$\Gamma_{\nu_R}(T_{\text{dec}}^{\nu_R}) = H(T_{\text{dec}}^{\nu_R}).$$



A. Solaguren-Beascoa, M. C. Gonzalez-Garcia: arXiv:1210.6350 [PLB]

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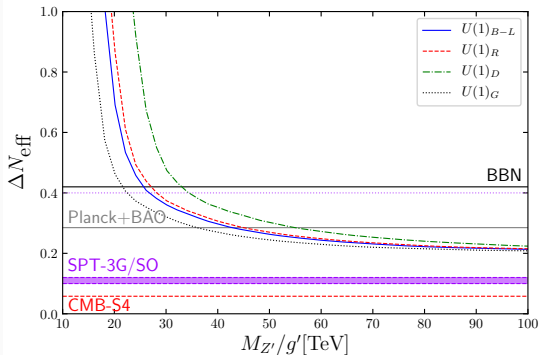
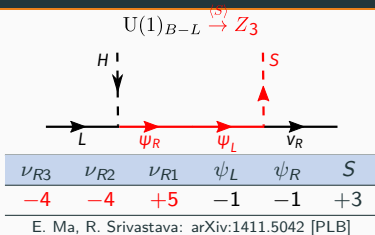
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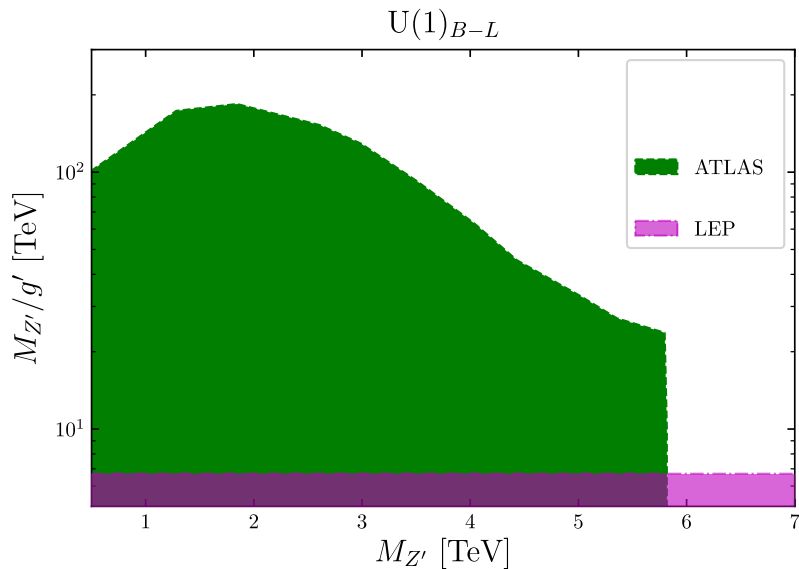
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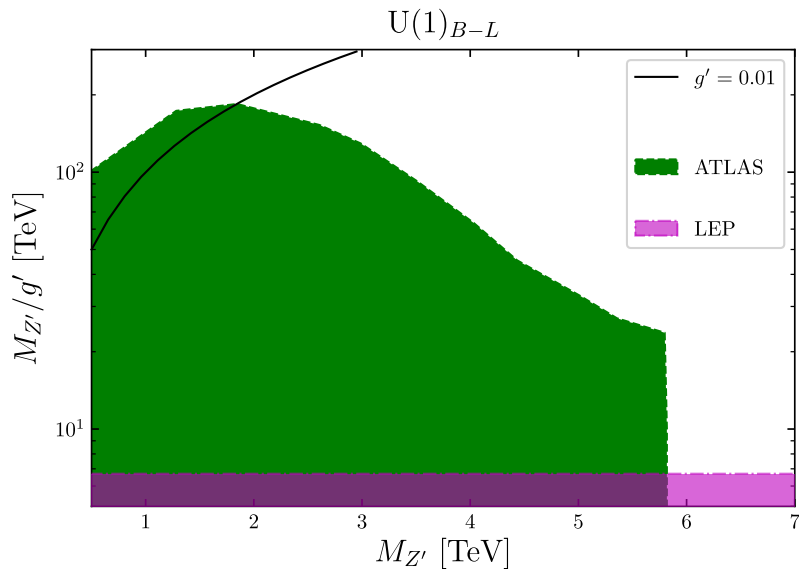




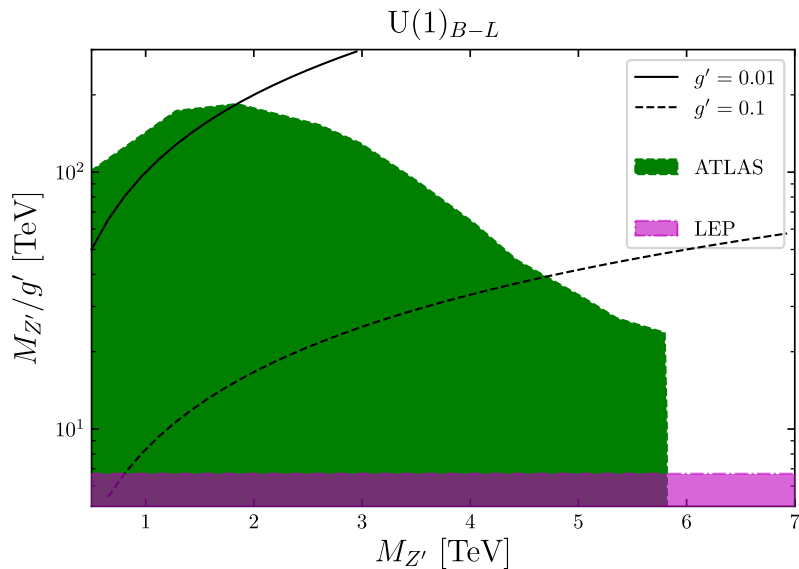
## Same constraints as before



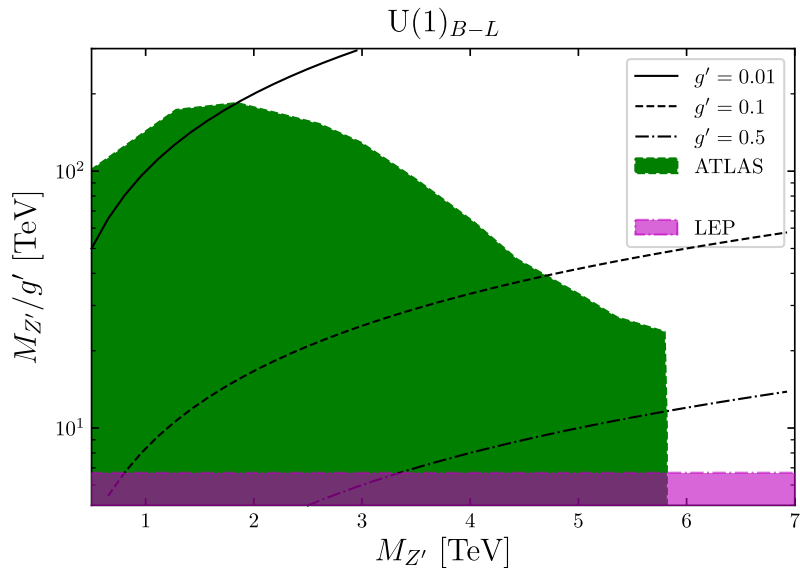
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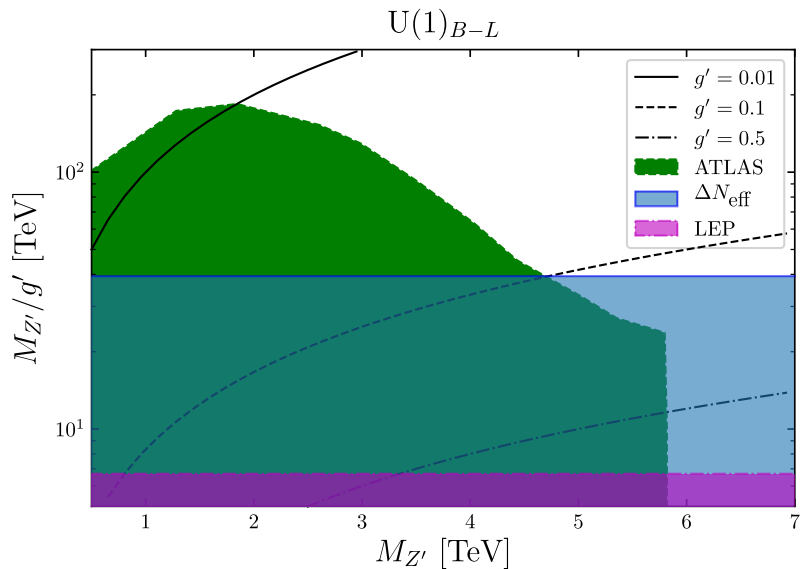
## Same constraints as before



## Same constraints as before



## Same constraints as before



It makes sense to focus our attention on models that can account for neutrino masses and dark matter (DM) **without adhoc symmetries**

## One-loop Dirac neutrino masses

A single  $U(1)_X$  gauge symmetry to explain both the smallness of Dirac neutrino masses and the stability of Dirac fermion dark matter

- Spontaneously broken  $U(1)_X$  generates a radiative Dirac neutrino masses
- A remnant symmetry makes the lightest field circulating the loop stable and good dark matter candidate.
- For T1-2-A: Either Singlet Doublet Dirac Dark Matter or Singlet Scalar Dark Matter with extra scalar and vector portal
- Dark symmetry for Majorana mediators

**Thanks!**