

Secluded Abelian extensions of the SM

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Anomaly cancellation of a gauge $U(1)_x$ extension

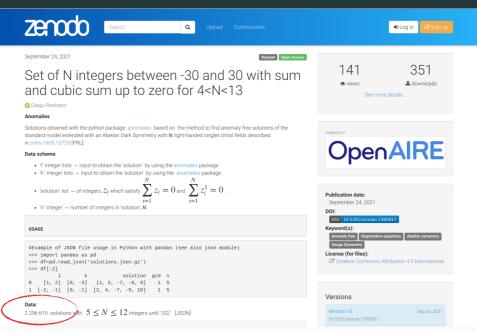
Any *universal* local Abelian extension of the Standard Model can be reduced to a set of integers

$$S = [z_1, z_2, \cdots, z_N],$$

which must satisfy the gravitational anomaly, $[SO(1,3)]^2 U(1)_Y$, and the cubic anomaly, $[U(1)_X]^3$ conditions:

$$\sum_{\alpha=1}^{N} z_{\alpha} = 0, \qquad \sum_{\alpha=1}^{N} z_{\alpha}^{3} = 0, \qquad (1)$$

https://doi.org/10.5281/zenodo.7380817



Solution	N	ν	δ	ϕ	N_D	N _M	G_D	G_M
(5, 5, -2, -3, 1, -6)	6	5	1	-5	2	0	1	0
(1, -2, 3, 4, 6, -7, -7, -7, 9)	9	-7	1	7	3	0	1	0
(1,1,-4,-5,9,9,9,-10,-10)	9	9	1	-9	3	0	2	0
(1, -2, -2, 3, 3, -4, -4, 6, 6, -7)	10	6	1	-6	3	2	2	2
(1, -2, -2, 3, 4, -5, -5, 7, 7, -8)	10	-5	1	5	4	0	2	0
(1, -2, -2, 3, 5, -6, -6, 8, 8, -9)	10	-6	1	6	4	0	2	0
(2,2,3,4,4,-5,-6,-6,-7,9)	10	2	1	-2	4	2	2	2
(1,1,5,5,5,-6,-6,-6,-9,10)	10	1	1	-1	4	0	3	0
(2, 2, 4, 4, -7, -7, -9, -9, 10, 10)	10	10	2	-5	3	0	2	0
(1, 2, 2, -3, 6, 6, -8, -8, -9, 11)	10	-8	1	8	4	1	2	1
(1, -2, -3, 5, 6, -8, -9, 11, 11, -12)	10	11	1	-11	4	0	1	0
(1, 1, -3, 4, 4, -7, 8, -10, -10, 12)	10	-10	2	5	4	0	2	0
(1,1,-2,-2,-4,6,-10,11,12,-13)	10	-2	1	2	3	2	1	2
(3, 4, 4, 4, 4, -5, -8, -8, -11, 13)	10	-8	1	8	2	4	1	4
(4,4,5,6,6,-9,-10,-10,-11,15)	10	6	1	-6	4	0	2	0
(1, -2, -4, 7, 7, -10, -12, 14, 14, -15)	10	14	1	-14	3	2	1	2
(1, 2, 2, -3, 4, -6, 12, -13, -14, 15)	10	2	1	-2	4	1	1	1
(1,4,4,-7,8,8,-9,-12,-12,15)	10	8	1	-8	4	2	2	2
(1, 2, 2, -9, -9, 16, 16, 17, -18, -18)	10	-18	1	18	3	2	2	2
(1, -3, -6, 7, -10, 11, -16, 18, 18, -20)	10	18	2	-9	4	0	1	0
(1, -4, 5, -6, -6, 10, -14, 15, 20, -21)	10	-6	1	6	4	0	1	0
(2, -3, -6, 7, 12, -14, -14, 17, 20, -21)	10	-14	1	14	4	1	1	1
(3,6,6,-7,8,8,-14,-14,-17,21)	10	-14	1	14	4	1	2	1
(8. 8. 9. 10. 1013181827. 31)	10	-18	1	18	4	1	2	1

Secluded gauge $U(1)_{D}$ without vector-like fermions:

$$\mathbf{S} = [\chi_1, \chi_2, \cdots, \psi_1, \psi_2, \cdots, \psi_{N'}]$$

• Higgs mechanism: Singlet scalar ϕ acquires a vev and give mass to the dark photon

$$\mathcal{L} = i\psi_{\mathsf{a}}^{\dagger} \overline{\sigma^{\mu}} \left(\partial_{\mu} - i g_{\mathsf{D}} Z_{\mu}^{\mathsf{D}} \right) \psi_{\mathsf{a}} - \frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \sum_{\mathsf{a} < \mathsf{b}} h_{\mathsf{a}\mathsf{b}} \psi_{\mathsf{a}} \psi_{\mathsf{b}} \phi^{(*)} + \text{h.c-} V(\phi) \,. \tag{2}$$

- z_{α} are the charges of SM-singlet left-handed chiral fermions with $N \geq 5$
 - χ_i massless fermions with $i=1,\cdots,N'$ with $N'\leq N$
 - ψ_a multi-component dark matter: massive after the spontaneous symmetry breaking of $U(1)_D$ with $a=N'+1,\cdots,N$
- Larger parameter space: Dark photon exclusions instead of Z'

$$\begin{split} [\chi_1,\chi_2,\cdots,\psi_1,\psi_2,\cdots,\psi_{N'}] \\ & \qquad \qquad \text{Secluded case:} \\ [\nu,\nu,(\nu),\psi_1,\psi_2,\cdots,\psi_{N'}] \\ \chi_1 \rightarrow \nu_{R1},\cdots,\chi_{N_{\nu}} \rightarrow \nu_{R\,N_{\nu}}, \qquad 2 \leq N_{\nu} \leq 3\,, \\ \mathcal{L}_{\text{eff}} = h_{\nu}^{ij} \left(\nu_{Ri}\right)^{\dagger} \epsilon_{ab} \, L_j^a \, H^b \left(\frac{\phi^*}{\Lambda}\right)^{\delta} + \text{H.c.}, \qquad \text{with } i,j=1,2,3\,, \end{split}$$

$$\phi = -\frac{\nu}{\delta} \,,$$

$$\begin{split} [\chi_1,\chi_2,\cdots,\psi_1,\psi_2,\cdots,\psi_{N'}] \\ & \qquad \qquad \text{Secluded case:} \\ [\nu,\nu,(\nu),\psi_1,\psi_2,\cdots,\psi_{N'}] \\ \chi_1 \rightarrow \nu_{R1},\cdots,\chi_{N_{\nu}} \rightarrow \nu_{R\,N_{\nu}}, \qquad 2 \leq N_{\nu} \leq 3\,, \\ \mathcal{L}_{\text{eff}} = h_{\nu}^{ij} \left(\nu_{Ri}\right)^{\dagger} \epsilon_{ab} \, L_j^a \, H^b \left(\frac{\phi^*}{\Lambda}\right)^{\delta} + \text{H.c.}, \qquad \text{with } i,j=1,2,3\,, \end{split}$$

$$\phi = -\frac{\nu}{\delta}$$
,

$$[\chi_1, \chi_2, \cdots, \psi_1, \psi_2, \cdots, \psi_{N'}]$$

Secluded case:
 $[5, 5, -3, -2, 1, -6]$

$$\chi_1 \rightarrow \nu_{R1}, \ \chi_2 \rightarrow \nu_{R2}, \qquad N_{\nu} = 2,$$

$$\mathcal{L}_{ ext{eff}} = h_{
u}^{ extstyle aj} \left(
u_{ extstyle Ra}
ight)^{\dagger} \epsilon_{ extstyle bc} \, \mathit{L}_{j}^{ extstyle b} \, \mathit{H}^{c} \left(rac{\phi^{*}}{\Lambda}
ight) + ext{H.c.}, \qquad ext{with } j = 1, 2, 3 \, ,$$

$$\phi = -\nu = -5,$$

$$\begin{split} [\chi_1,\chi_2,\cdots,\psi_1,\psi_2,\cdots,\psi_{N'}] \\ &\quad \text{Secluded case:} \\ [5,5,-3,-2,1,-6] \\ \chi_1 \rightarrow \nu_{R1},\ \chi_2 \rightarrow \nu_{R2}, \qquad \textit{N}_{\nu} = 2\,, \\ \mathcal{L}_{\text{eff}} = \textit{h}_{\nu}^{aj} \left(\nu_{Ra}\right)^{\dagger} \epsilon_{\textit{bc}} \textit{L}_{\textit{j}}^{\textit{b}} \textit{H}^{\textit{c}} \left(\frac{\phi^*}{\Lambda}\right) + \text{H.c.}, \qquad \text{with } \textit{j} = 1,2,3\,, \end{split}$$

$$\phi = -\nu = -5,$$

Minimal secluded model with D-5 effective Dirac neutrino masses

$$\mathcal{L} = i\psi_i^{\dagger} \mathcal{D} \psi_i - \frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \sum_{i < j} h_{ij} \psi_i \psi_j \phi^{(*)} + \text{h.c-} V(\phi).$$
 (3)

multi-component DM with two Dirac-fermion DM

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 (3)

multi-component DM with two Dirac-fermion DM

$$z = [5, 5, -3, -2, 1, -6] \rightarrow \phi = -5 \rightarrow [(5, 5), (-3, -2), (1, -6)]$$
 (4)

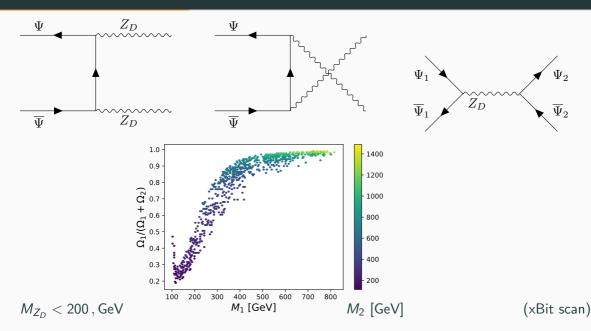
Minimal secluded model with D-5 effective Dirac neutrino masses

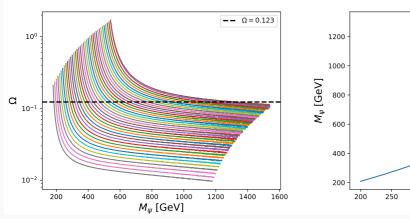
$$\mathcal{L} = i\psi_i^{\dagger} \mathcal{D} \psi_i - \frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \sum_{i < j} h_{ij} \psi_i \psi_j \phi^{(*)} + \text{h.c-} V(\phi).$$
 (3)

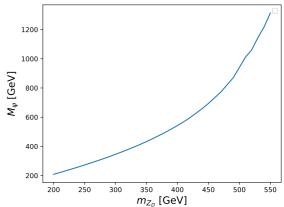
multi-component DM with two Dirac-fermion DM

$$z = [5, 5, -3, -2, 1, -6] \rightarrow \phi = -5 \rightarrow [(5, 5), (-3, -2), (1, -6)]$$
 (4)

$$\mathcal{L} \subset h_{(-3,-2)}\psi_{-3}\psi_{-2}\phi + h_{(1,-5)}\psi_{1}\psi_{-6}\phi + \text{h.c.}$$
 (5)







Minimal secluded model with SM-singlet massive chiral fermions from SSB: $U(1)_D$

$$\mathcal{L} = i\psi_i^{\dagger} \mathcal{D} \psi_i - \frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \sum_{i < j} h_{ij} \psi_i \psi_j \phi^{(*)} + \text{h.c-} V(\phi).$$
 (6)

 $96\,153 \rightarrow 5\,196$

multi-component DM (N=8,12)
ightarrow 142 with three Dirac-fermion DM

Minimal secluded model with SM-singlet massive chiral fermions from SSB: $U(1)_D$

$$\mathcal{L} = i\psi_i^{\dagger} \mathcal{D} \psi_i - \frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \sum_{i < j} h_{ij} \psi_i \psi_j \phi^{(*)} + \text{h.c-} V(\phi).$$
 (6)

 $96\,153
ightarrow 5\,196$ multi-component DM (N=8,12) ightarrow 142 with three Dirac-fermion DM

$$z = [1, -2, -2, 4, 5, -7, -7, 8] \rightarrow \phi = 9 \rightarrow [(1, 8), (-2, -7), (4, 5)]$$
 (7)

Minimal secluded model with SM-singlet massive chiral fermions from SSB: $U(1)_D$

$$\mathcal{L} = i\psi_i^{\dagger} \mathcal{D} \psi_i - \frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \sum_{i < j} h_{ij} \psi_i \psi_j \phi^{(*)} + \text{h.c-} V(\phi).$$
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$$z = [1, -2, -2, 4, 5, -7, -7, 8] \rightarrow \phi = 9 \rightarrow [(1, 8), (-2, -7), (4, 5)]$$
 (7)

$$\mathcal{L} \subset h_{(1,8)} \psi_1 \psi_8 \phi^* \phi^{(*)} + \sum_{a,b=1}^2 h_{(-2a,-7b)} \psi_2 \psi_{-7} \phi + h_{(4,5)} \psi_4 \psi_5 \phi^* \phi^{(*)} + \text{h.c.}$$
(8)

Simplest secluded model with SM-singlet massive chiral fermions from SSB: $U(1)_{D}$

$$\mathcal{L} = i\psi_i^{\dagger} \mathcal{D} \psi_i - \frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \sum_{i < j} h_{ij} \psi_i \psi_j \phi^{(*)} + \text{h.c}$$
 (9)

 $96\,153
ightarrow 5\,196$ multi-component DM (N=8,12) ightarrow 41 with two Dirac-fermion DM

Simplest secluded model with SM-singlet massive chiral fermions from SSB: $U(1)_D$

$$\mathcal{L} = i\psi_i^{\dagger} \mathcal{D} \psi_i - \frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \sum_{i < j} h_{ij} \psi_i \psi_j \phi^{(*)} + \text{h.c}$$
 (9)

 $96\,153
ightarrow 5\,196$ multi-component DM (N=8,12)
ightarrow 41 with two Dirac-fermion DM

$$z = [1, 2, 2, 4, -5, -5, -7, 8] \rightarrow \phi = 3 \rightarrow [(1, 2), (2, -5), (-5, 8), (4, -7)]$$
 (10)

Simplest secluded model with SM-singlet massive chiral fermions from SSB: $\mathrm{U}(1)_{D}$

$$\mathcal{L} = i\psi_i^{\dagger} \mathcal{D} \psi_i - \frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \sum_{i < j} h_{ij} \psi_i \psi_j \phi^{(*)} + \text{h.c}$$
 (9)

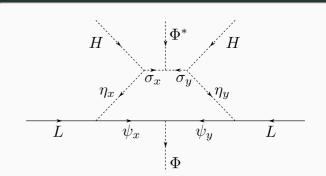
 $96\,153
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Majorana neutrino masses and mixings

$$\frac{y}{\Lambda}$$
LLHH

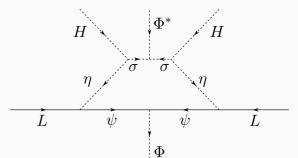
Scotogenic Majorana neutrino masses and mixings

$$\frac{y}{\Lambda}LLHH \to \frac{y}{\Lambda}LLHH\frac{\Phi}{\Lambda}\frac{\Phi^*}{\Lambda}$$



Scotogenic Majorana neutrino masses and mixings





Already found by Chi-Fong Wong in arXiv:2008.08573 (subset with $N \leq 9$ and $z_{\text{max}} \leq 10$)

$$z = [1, 1, 2, 3, -4, -4, -5, 6] \rightarrow \phi = 2 \rightarrow [(1, 1)_a, (2, -4), (4, -6), (4, -7)]$$
 (11)

Additional conditions to reduce

multiplicity

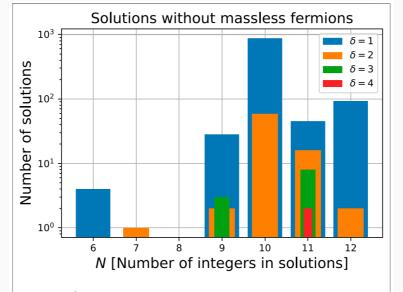
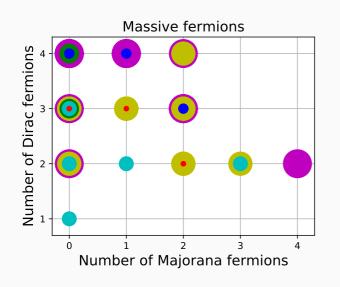
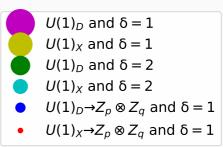


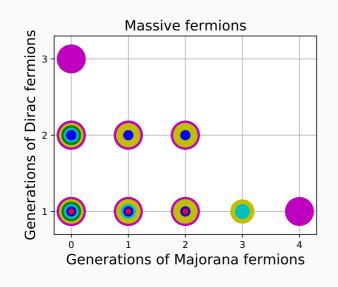
FIGURE 1 Distribution of solutions with N integers to the Diophantine **Eq. 1** which allow the effective Dirac neutrino mass operator at $d=(4+\delta)$ for at least two right-handed neutrinos and have non-vanishing Dirac o Majorana masses for the other SM-singlet chiral fermions in the solution.

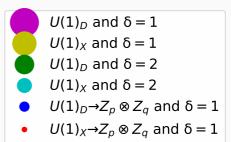
Multi-component dark matter





Multi-flavor dark matter





$U(1)_X$ selection with Dirac-fermionic DM

• Active symmetry m = 3

$$(-5, -5, 3, 3, 3, -7, 8)$$

$\overline{U(1)_X}$ selection with Dirac-fermionic DM

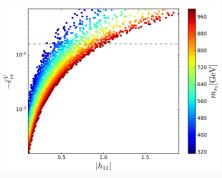
- Active symmetry m = 3
- Effective neutrino mass $\delta = 2 \rightarrow \nu = -5$:

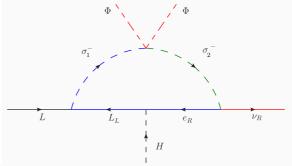
$$(-5, -5, 3, 3, 3, -7, 8)$$

$U(1)_X$ selection with Dirac-fermionic DM

- Active symmetry m = 3
- Effective neutrino mass $\delta = 2 \rightarrow \nu = -5$:
- Active symmetry: $m = 3 \rightarrow \phi = -(\nu + m)/\delta = 1$

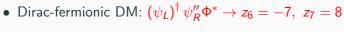
(-5, -5, 3, 3, 3, -7, 8)

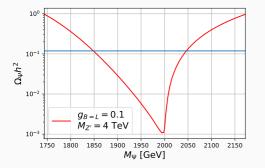


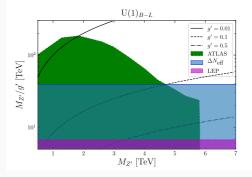


(-5, -5, 3, 3, 3, -7, 8)

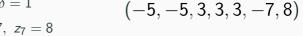
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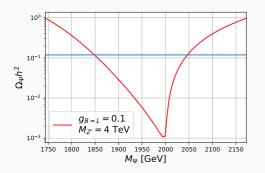


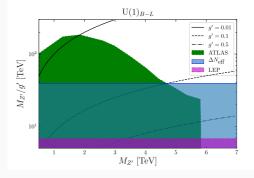




- Active symmetry m = 3
- Effective neutrino mass $\delta = 2 \rightarrow \nu = -5$:
- Active symmetry: $m = 3 \rightarrow \phi = -(\nu + m)/\delta = 1$
- Dirac-fermionic DM: $(\psi_L)^{\dagger} \psi_R'' \Phi^* \rightarrow z_6 = -7, \ z_7 = 8$







Beyond SM-fermion singlets

Standard model extended with $U(1)_{\mathcal{X}=X \text{ or } \mathbf{D}}$ gauge symmetry

Fields	$SU(2)_L$	$U(1)_Y$	$U(1)_{\mathcal{X}=\mathbf{D} \text{ or } X}$
Q_i^{\dagger}	2	-1/6	Q
d_{Ri}	1	-1/2	d
u_{Ri}	1	+2/3	и
L_i^{\dagger}	2	+1/2	L
e_{Ri}	1	-1	e
Н	2	1/2	h
χ_{α}	1	0	Z_{α}

Φ	1	0	ϕ

Table 2: LHC: hadronic production and dileptonic decay

$$i = 1, 2, 3, \ \alpha = 1, 2, \dots, N'$$

Standard model extended with $U(1)_{\mathcal{X}=L \text{ or } \mathbf{B}}$ gauge symmetry

Fields	$SU(2)_L$	$U(1)_Y$	$U(1)_{\mathcal{X}=\mathbf{B}}$ or \mathbf{L}
Q_i^{\dagger}	2	-1/6	Q
d_{Ri}	1	-1/2	d
u_{Ri}	1	+2/3	и
L_i^{\dagger}	2	+1/2	L
e_{Ri}	1	-1	e
Н	2	1/2	h = 0
χ_{α}	1	0	z_{α}
$(L'_L)^{\dagger}$	2	1/2	-x'
$L_R^{\prime\prime}$	2	-1/2	x''
e_R'	1	-1	x'
$(e_L^{\prime\prime})^\dagger$	1	1	-x"
Ф	1	0	φ
S	1	0	S

Table 2: minimal set of new fermion content: L = e = 0 for $\mathcal{X} = B$. Or Q = u = d = 0 for $\mathcal{X} = L$. $i = 1, 2, 3, \alpha = 1, 2, \dots, N'$

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Anomaly cancellation: $\mathcal{X} = L$ or **B**: beyond SM-singlet fermions

The anomaly-cancellation conditions on $[SU(3)_c]^2 U(1)_X$, $[SU(2)_L]^2 U(1)_X$, $[U(1)_Y]^2 U(1)_X$, allow us to express three of the X-charges in terms of the others

$$\mathbf{u} = -\mathbf{e} - \frac{2}{3}\mathbf{L} - \frac{1}{9}(x' - x'') , \quad \mathbf{d} = \mathbf{e} + \frac{4}{3}\mathbf{L} - \frac{1}{9}(x' - x'') , \quad \mathbf{Q} = -\frac{1}{3}\mathbf{L} + \frac{1}{9}(x' - x'') ,$$
(12)

while the $[U(1)_X]^2 U(1)_Y$ anomaly condition reduces to

$$(e+L)(x'-x'')=0.$$
 (13)

- Previously: x' = x''
- We choose instead (h = 0):

$$e = -L, (14)$$

so that (L is still a free parameter)

$$Q = -u = -d = -\frac{1}{3}L + \frac{1}{9}(x' - x'').$$
 (15)

Anomaly cancellation: $\mathcal{X} = L$ or B

The gravitational anomaly, $[SO(1,3)]^2 U(1)_Y$, and the cubic anomaly, $[U(1)_X]^3$, can be written as the following system of Diophantine equations, respectively,

$$\sum_{\alpha=1}^{N} z_{\alpha} = 0, \qquad \sum_{\alpha=1}^{N} z_{\alpha}^{3} = 0, \qquad (16)$$

where

$$z_1 = -x',$$
 $z_2 = x'',$ $z_{2+i} = L, \quad i = 1, 2, 3$ (17)

 \rightarrow

$$9Q = -\sum_{\alpha=1}^{5} z_{\alpha} = -x' + x'' + L + L + L, \qquad (18)$$

$$L = 0 \rightarrow U(1)_B$$
 but $Q = 0 \rightarrow U(1)_L$

$U(1)_{\it B}$ selection: Neutrinos, dark matter and baryogenesis

•
$$L = 0$$

$$(5,5,-3,-2,1,-6)$$

$U(1)_{B}$ selection: Neutrinos, dark matter and baryogenesis

- L = 0
- Effective Dirac neutrino masses: $\phi = -\nu = -5$

$$(5, 5, -3, -2, 1, -6)$$

$U(1)_B$ selection: Neutrinos, dark matter and baryogenesis

- L = 0
- Effective Dirac neutrino masses: $\phi = -\nu = -5$
- Electroweak-scale vector-like fermions:

$$(L'_L)^{\dagger} L''_R \Phi^* \to x' = -1, \ x'' = 6$$

$$(5,5,-3,-2,1,-6)$$

$U(1)_{B}$ selection: Neutrinos, dark matter and baryogenesis

- L = 0
- Effective Dirac neutrino masses: $\phi = -\nu = -5$
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$$(L'_L)^{\dagger} L''_R \Phi^* \to x' = -1, \ x'' = 6$$

• Dirac-fermionic DM: $(\chi_L)^{\dagger} \chi_R'' \Phi^* \rightarrow z_3 = -3, z_4 = -2$

$$(5, 5, -3, -2, 1, -6)$$

$U(1)_B$ selection: Neutrinos, dark matter and baryogenesis

- L = 0
- Effective Dirac neutrino masses: $\phi = -\nu = -5$
- Electroweak-scale vector-like fermions:

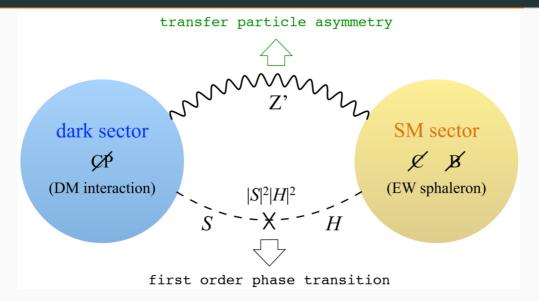
$$(L'_L)^{\dagger} L''_R \Phi^* \to x' = -1, \ x'' = 6$$

• Dirac-fermionic DM: $(\chi_L)^{\dagger} \chi_R'' \Phi^* \rightarrow z_3 = -3, \ z_4 = -2$

959 solutions

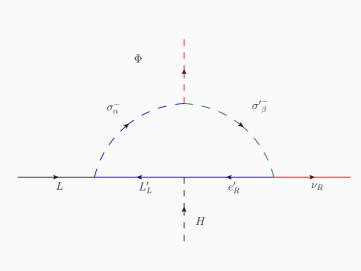
$$(5,5,-3,-2,1,-6)$$

Dark sector baryogenesis



Gauge Baryon number scotogenic realization: arXiv:2205.05762 [PRD]

with Andrés Rivera (UdeA) and Walter Tangarife (Loyola U.)

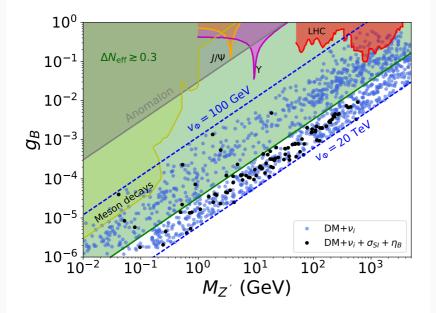


0.)			
Field	$SU(2)_L$	$U(1)_Y$	$U(1)_B$
u_{Ri}	1	2/3	u = 1/3
d_{Ri}	1	-1/3	d = 1/3
$(Q_i)^{\dagger}$	2	-1/6	Q = -1/3
$(L_i)^{\dagger}$	2	1/2	L=0
e_R	1	-1	e = 0
$(L'_L)^{\dagger}$	2	1/2	-x' = -3/5
e'_R	1	-1	x' = 3/5
$L_R^{\prime\prime}$	2	-1/2	x'' = 18/5
$\left(e_L^{\prime\prime} ight)^\dagger$	1	1	-x'' = -18/5
$ u_{R,1}$	1	0	-3
$ u_{R,2}$	1	0	-3
χ_R	1	0	6/5
$(\chi_L)^{\dagger}$	1	0	9/5
Н	2	1/2	0
S	1	0	3
Φ	1	0	3
σ_{lpha}^-	1	1	3/5
σ'_{α}^{-}	1	-1	-12/5

arXiv:2205.05762 [PRD] https://github.com/anferivera/DarkBariogenesis

- SARAH \rightarrow SPheno \rightarrow MicroMegas
- η_B calculation code
- Python notebook with the scan

Black points: Dirac neutrinos with proper DM and baryon assymetry



Conclusions

A methodology was designed to find all the *universal* gauge Abelian extensions of the standard model:

All of the extensions can be reformulated as the solution of

$$\sum_{\alpha=1}^{N} z_{\alpha} = 0, \qquad \sum_{\alpha=1}^{N} z_{\alpha}^{3} = 0,$$

which we thoroughly scan in an efficient way until N=12 and $|z_{\rm max}|=20$

Once the physical conditions are stablished, the full set of self-consistent models are found from a simple data-analysis procedure, providing enough freedom to solve several phenomenological problems.