

# Spacecraft Dynamics Lecture - UUM571 - Project Part 1

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- Travel to planet Mars from planet Earth.
- Wait on Mars parking orbit until return window is opened.
- **Stages:**
  - 1) Launch Platform Coordinates Selection
  - 2) Determine Appropriate Launch Date & Time
  - 3) Rocket Launch to Earth Parking Orbit (500km)
  - 4) Conduct Hyperbolic Escape Trajectory Maneuver
  - 5) Apply Hohmann Transfer to Mars
  - 6) Travel to Mars Parking Orbit with Hyperbolic Maneuver
  - 7) Wait Until Return Window is Opened

# Launch Location Selection

- Sun-synchronous Earth circular parking orbit is chosen.
- Does not have to be eligible/realistic location.
- Azimuth Angle:  $A = 279.3489$  degrees
- Orbital Inclination:  $i = 97.401$  degrees
  - average rate of change of the the right ascension:

$$\dot{\Omega} = \frac{2\pi}{365.26 * 24 * 3600}$$

- Latitude Angle:  $L = 82.499$  degrees

$$\cos(i) = \cos(\phi)\cos(A)$$

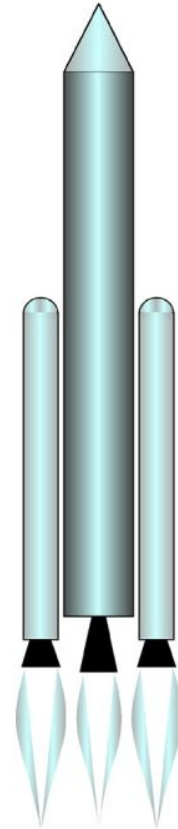


- **Design Properties:**

- Launch vehicle rocket will burnout at an altitude of **500 km**.
- Only one stage rocket is designed.

- **Parameters:**

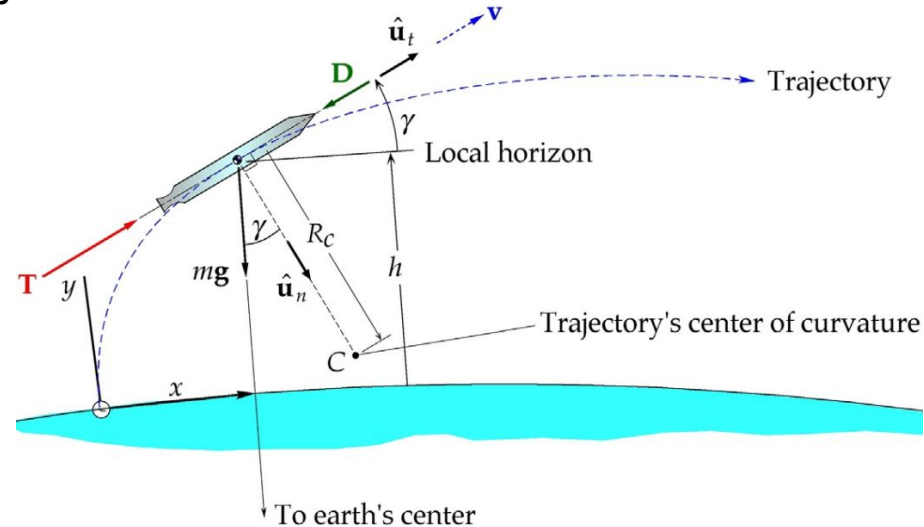
- Rocket thrust (constant):  **$T = 10^6 \text{ N}$**
- Diameter:  **$d = 3.7 \text{ m}$**
- Drag coefficient:  **$C_d = 0.5$**
- Frontal area of the launch vehicle:  **$A = 10.75 \text{ m}^2$**
- Flight path angle:  **$\gamma = 90 \text{ degrees}$**
- Earth parking orbit spacecraft mass:  **$m_{pl} = 6666 \text{ kg}$**
- Launch vehicle propellant mass:  **$m_p = 54321 \text{ kg}$**
- Rocket empty mass:  **$m_e = 12345 \text{ kg}$**
- Total Launch Vehicle tandem-stack mass:  **$m = 73332 \text{ kg}$**



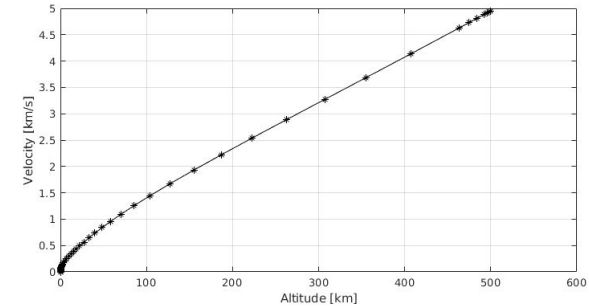
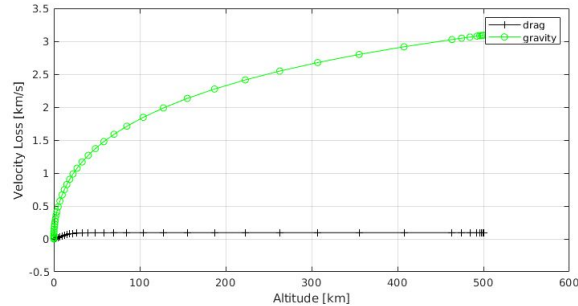
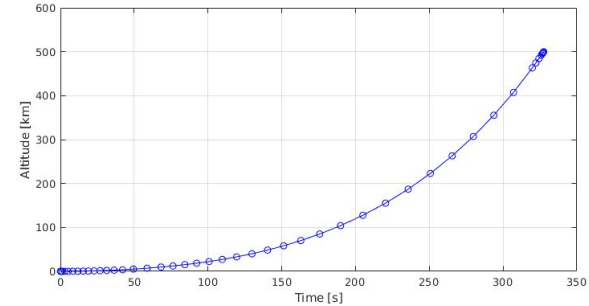
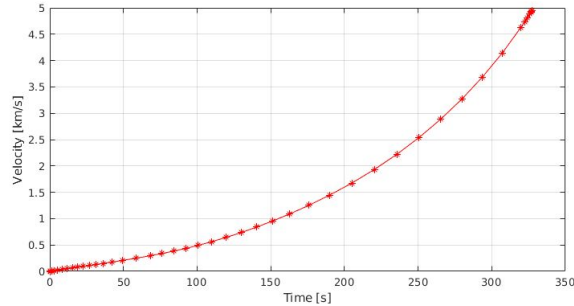
- Changing gravitational acceleration w.r.t. altitude
- Fuel flow rate is considered constant:  $\dot{m}_d = 165.8728 \text{ kg/s}$
- Experimentally selected specific impulse:  $I = 614.548 \text{ s}$
- Assumed that flight path angle is constant
- Total fuel burning time until reached to Earth parking orbit:  $t = 327.48588 \text{ s}$

$$t_{burn} = \frac{m_p}{\dot{m}}$$

- **5 minutes and 28 seconds**
- Spacecraft speed after burnout:  $v = 4.95288 \text{ km/s}$



# Rocket Performance



- Velocity at Earth circular parking orbit:  $\mathbf{v = 7.61258 \text{ km/s}}$
- Spacecraft velocity after rocket burnout:  $\mathbf{v = 4.95288 \text{ km/s}}$
- $\mathbf{\Delta V = 2.6597 \text{ km/s}}$

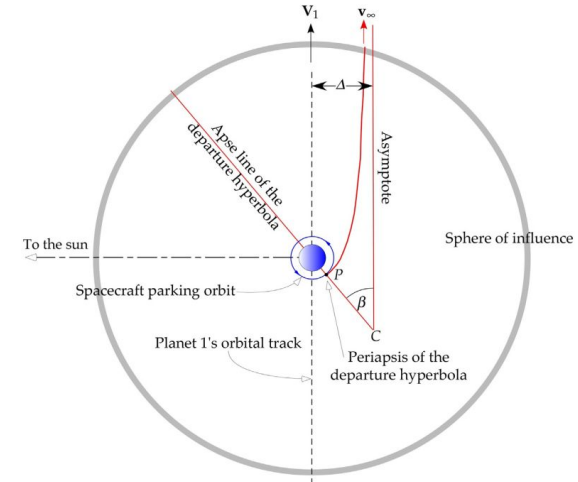
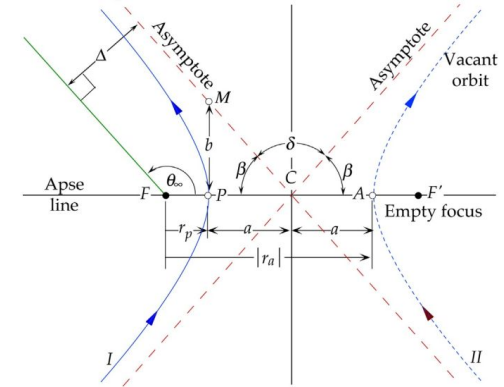
# Escape Earth SOI

- Specific impulse of spacecraft to make an interplanetary delta-v maneuver:  **$I = 321$  s.**
- Hyperbolic excess speed of the Earth departure hyperbola:  **$V_{\infty} = 2.94346$  km/s**

$$\Delta V_{hyperbola} = v_{periapsis} - v_{circular}$$

$$\Delta V_{hyperbola} = v_{circular} \left[ \sqrt{2 + \left( \frac{v_{\infty}}{v_{circular}} \right)^2} - 1 \right]$$

- **Delta V = 3.54840 km/s**
- Perigee of a departure hyperbola with respect to the Earth velocity vector:
  - **beta = 29.54795 degrees**
- Using an orbit equation for reaching Earth SOI radius:
  - **theta = 148.87471 degrees**
- Total duration of the spacecraft while in hyperbolic trajectory:  **$t = 273086.80638$  s**
- **3 days, 3 hours, 51 minutes, and 27 seconds**



- After adding additional **3 days, 3 hours, 56 minutes, and 55 seconds** to the launch date and time, the Hohmann transfer maneuver will be initialized:

- Year : 2023**
- Month : May**
- Day : 10**
- Hour : 3 p.m.**
- Minute : 8**
- Second : 6**

$$J_0 = 367y - INT \left[ \frac{7 \left[ y + INT \left( \frac{m+9}{12} \right) \right]}{4} \right] + INT \left( \frac{275m}{9} \right) + d + 1,721,013.5$$

- Julian day number:

- Using Kepler's equation to calculate eccentric anomaly E.

- Mean anomaly:**

$$M = \frac{h + \left( \frac{m}{60} \right) + \left( \frac{s}{3600} \right)}{24}$$

- Planetary True Anomaly:**

$$\tan\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+e}{1-e}} \tan\left(\frac{E}{2}\right)$$

**Table 8.1 Planetary orbital elements and their centennial rates**

	$a$ (AU) $\dot{a}$ (AU/Cy)	$e$ $\dot{e}$ (1/Cy)	$i$ (°) $\dot{i}$ (°/Cy)	$\Omega$ (°) $\dot{\Omega}$ (°/Cy)	$\varpi$ (°) $\dot{\varpi}$ (°/Cy)	$L$ (°) $\dot{L}$ (°/Cy)
Mercury	0.38709927 0.00000037	0.20563593 0.00001906	7.00497902 -0.00594749	48.33076593 -0.12534081	77.45779628 0.16047689	252.25032350 149.472.67411175
Venus	0.72333566 0.00000390	0.00677672 -0.00004107	3.39467605 -0.00078890	76.67984255 -0.27769418	131.60246718 0.00268329	181.97909950 58,517.81538729
Earth	1.00000261 0.00000562	0.01671123 -0.00004392	-0.00001531 -0.01294668	0.0 0.0	102.93768193 0.32327364	100.46457166 35,999.37244981
Mars	1.52371034 0.0001847	0.09339410 0.00007882	1.84969142 -0.00813131	49.55953891 -0.29257343	-23.94362959 0.44441088	-4.55343205 19,140.30268499
Jupiter	5.20288700 -0.00011607	0.04838624 -0.00013253	1.30439695 -0.00183714	100.47390909 0.20469106	14.72847983 0.21252668	34.39644501 3034.74612775
Saturn	9.53667594 -0.00125060	0.05386179 -0.00050991	2.48599187 0.00193609	113.66242448 -0.28867794	92.59887831 -0.41897216	49.95424423 1222.49362201
Uranus	19.18916464 -0.00196176	0.04725744 -0.00004397	0.77263783 -0.00242939	74.01692503 0.04240589	170.95427630 0.40805281	313.23810451 428.48202785
Neptune	30.06992276 0.00026291	0.00859048 0.00005105	1.77004347 0.00035372	131.78422574 -0.00508664	44.96476227 -0.32241464	-55.12002969 218.45945325
(Pluto)	39.48211675 -0.00031596	0.24882730 0.00005170	17.14001206 0.00004818	110.30393684 -0.01183482	224.06891629 -0.04062942	238.92903833 145.20780515

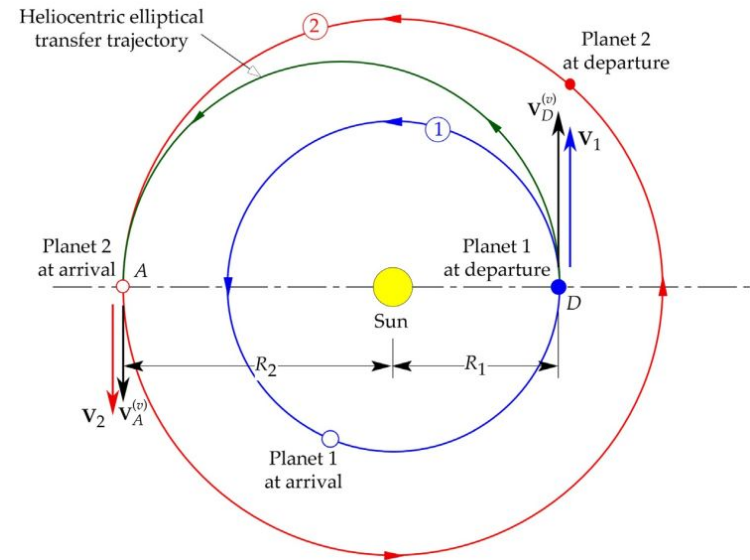
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## Assumptions:

- **inclination** of planetary orbital trajectories of the planets Earth and Mars are assumed to be 0 degrees; so the Earth and Mars have the co-planar orbits.
- solar orbits of the planets Earth and Mars in heliocentric orbital plane are assumed to be **circular orbits**; so the eccentricity of these orbits are equal to 0.
- Solar effects and space **perturbations are neglected**.
- True anomaly difference between planets is **44.32 degrees**.
- The duration of Hohmann transfer:  
 **$t = 22362713.30644$  s**
- **258 days, 19 hours, 51 minutes, and 53 seconds**
- Spacecraft's heliocentric velocity:  **$V = 32.72794$  km/s**

$$v_{departure} = \sqrt{\mu_{sun} \cdot \left[ \frac{2}{R_E} - \frac{1}{a} \right]}$$



- The spacecraft **will enter** to the Mars SOI in 2024 year, January, 24th, 10 a.m., 59 min, and 59 sec.
- The speed of the spacecraft after Hohmann transfer **at Mars SOI** location:

$$v_{arrival} = \sqrt{\mu_{sun} \left( \frac{2}{R_M} - \frac{1}{a} \right)} = \boxed{21.48355 km/s}$$

- The speed at Mars **capture orbit** (at Mars SOI radius):

$$v_{mars} = \sqrt{\frac{\mu_{sun}}{R_M}} = \boxed{24.13146 km/s}$$

- The spacecraft **velocity relative to Mars** at periapse of approach hyperbola:

$$v_{perigee}^{mars} = \sqrt{(v_{\infty}^{mars})^2 + \left( \frac{2\mu_{mars}}{r_{mars} + z_{mars}} \right)}$$

- **Delta-V** for stay in mars circular orbit:

$$\Delta V_{arrive} = v_{circular}^{mars} - v_{perigee}^{mars} = \boxed{-2.09018 km/s}$$

# Wait for Next Window

- The spacecraft **will enter** to the Mars SOI in 2024 year, January, 24th, 10 a.m., 59 min, and 59 sec.
- Earth and Mars planets **mean angular velocity** (circular orbit assumption):

$$n_E = \frac{V_{earth}}{R_E} = \frac{\sqrt{\frac{\mu_{sun}}{R_E}}}{R_E}$$

- **Initial phase angle** between Earth and Mars:

$$\phi_0 = \pi - (n_M * t_{hohmann}) = \boxed{44.32deg}$$

- **Final phase angle** between Earth and Mars:

$$\phi_f = \phi_0 + (n_M - n_E) * t_{hohmann} = \boxed{-75.09712deg}$$

- **Waiting time:**

$$\boxed{t_{wait} = 39286373.02635s}$$

in other words, **454 days, 16 hours, 52 minutes, and 53 seconds.**

- Codes and simulation are conducted in MATLAB.
- Detailed report is submitted to the Ninova system with all codes and simulations, including LaTeX file.
- Only one reference is used throughout the study:

“H. Curtis, Orbital mechanics for engineering students. ButterworthHeinemann, 2013”
- Report is 6 pages in IEEE format written in LaTeX.
- To-Do:
  - Detailed readme to be added to the simulation codes.
  - Simulation graphics will be added in the next part of the project.
  - Align with others to make sure that ephemeris and delta-v calculations are correct.