UUM571E - Spacecraft Dynamics Project - Part 1 - Fall 2022/2023

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***Abstract*— In this project, interplanetary travel to the planet Mars from the planet Earth is designed and required calculations are conducted with appropriate launch date selection. Initially suitable date and time is chosen in order to send the spacecraft to Mars. The spacecraft is send to the parking orbit of Earth at an required altitude of 500 km with appropriate launch vehicle system. In the next stage, this spacecraft applied hyperbolic trajectory maneuver to escape Earth sphere of influence (SOI). From there, an interplanetary Hohmann transfer orbit maneuver is conducted to travel to the Mars parking orbit at required altitude of 300 km. Eventually, a minimum time required for the spacecraft returning Hohmann transfer is calculated.**

***Index Terms*— Spacecraft Dynamics, Interplanetary Orbits, Rocket Dynamics**

I. Overview

In this project, interplanetary mission to the planet Mars from planet Earth is conducted with orbital mechanics, rocket system design, and required mission duration architecture. All formulas and equations used in this project are taken from the book by Howard D. Curtis [1] and explained in Section II.

The structure of this report is constructed as follows: Section I explains overall framework of the report; Section III concludes selected Earth launch precise date and time with reasonable explanations and background of Ephemeris equations; Section IV gives rocket system selection reasons and engineering senses on the project; Section V gives details of the spacecraft departure from Earth sphere of influence (SOI) radius using hyperbolic trajectory; Section VI emphasizes interplanetary Hohmann initialization design with predefined project parameters and requirements; and in the final Section VII, general project finalization is discussed.

II. Background

In this section, all useful formulas and equations for this project are given individually with corresponding subsections. Equations in this project are taken from [1].

A. Planetary Ephemeris

The rates of changes of orbital elements of the planets could be found in Table 8.1 of reference [1]. Because it is assumed that Earth and Mars are co-planar, in this Table 8.1, orbital inclination and eccentricity of the planets are manually changed to 0 (zero) value. A Julian day conversion of UT format date is done with the following Eq. (1).

$$J_D = J_0 + \frac{UT}{24} \tag{1}$$

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where J_0 is the symbol for the Julian day number at 0 h UT as could be calculated with Eq. (2).

$$J_0 = 367y - INT\left[\frac{7\left[y + INT(\frac{m+9}{12})\right]}{4}\right] + INT\left(\frac{275m}{9}\right) + d + 1,721,013.5 \tag{2}$$

where y is a year in range of [1901, 2099], m is a month between range of [1, 12], and d is a day number in range [1, 31]. These year, month, and day information will be used in determination of Hohmann transfer orbit maneuver.

UT in Eq. (1) is determined by substituting the following equation, where h, m, and s represent hour, minute, and second respectively.

$$UT = \frac{h + (\frac{m}{60}) + (\frac{s}{3600})}{24} \tag{3}$$

The mean anomaly M is calculated as Eq. (4), where L is a mean longitude and $\hat{w} = w + \Omega$ where $\hat{w}$ longitude of perihelion, and Ω is the J2000 vernal equinox relative right ascension of the ascending node.

$$M = \frac{h + (\frac{m}{60}) + (\frac{s}{3600})}{24} \tag{4}$$

The Kepler's equation for the eccentric anomaly E is calculated as shown in Eq. (5) with iterative application of Newton's method. For more details, Algorithm 3.1 could be investigated in [1].

$$E_{i+1} = E_i - \frac{E_i - esin(E_i) - M}{1 - ecos(E_i)} \tag{5}$$

A planetary true anomaly is calculated with Eq. (6) as illustrated.

$$tan(\frac{\theta}{2}) = \sqrt{\frac{1+e}{1-e}} tan(\frac{E}{2}) \tag{6}$$

Assumptions:

- inclination planetary orbital trajectories of the planets Earth and Mars are assumed to be 0 degrees; so the Earth and Mars have the co-planar orbits.
- solar orbits of the planets Earth and Mars in heliocentric orbital plane are assumed to be circular orbits; so the eccentricity of these orbits are equal to 0.
- in part-1 of the project, solar effects and space perturbations are neglected.

B. Constant Parameters

Table of useful parameters are given on Table I. These constant values are taken from Table A.1, Table A.2, and Table 4.3 in [1].

TABLE I

GENERAL CONSTANT PARAMETERS IN THE SOLAR SYSTEM.

Sun in Solar System		
m_{sun}	mass of the Sun	1.989 x $10^{30}kg$
r_{sun}	radius of the Sun	$696,000km$
μ_{sun}	gravitational parameter	$132,712,440,018km^3/s^2$
Planet Earth		
m_{earth}	mass of the planet Earth	5.974 x $10^{24}kg$
r_{earth}	radius of the Earth	$6378km$
μ_{earth}	gravitational parameter	$398,600km^3/s^2$
r^{earth}_{soi}	sphere of influence	$925,000km$
o_{earth}	Earth's oblateness	0.003353
$J2_{earth}$	zonal harmonics	1.08263 x 10^{-3}
Planet Mars		
m_{mars}	mass of the planet Mars	641.9 x $10^{21}kg$
r_{mars}	radius of the Mars	$3396km$
μ_{mars}	gravitational parameter	$42,828km^3/s^2$
r^{mars}_{soi}	sphere of influence	$577,000km$
o_{mars}	Mars's oblateness	0.00648
$J2_{mars}$	zonal harmonics	1.96045 x 10^{-3}

C. Orbital Elements

An orbital equation given in Eq. (7) is applicable to every type of orbits.

$$r = \frac{h^2}{\mu}\frac{1}{1+ecos(\theta)} \tag{7}$$

where r is a radius of the orbit, h is an angular momentum, e is an eccentricity of the orbit, and θ is a true anomaly.

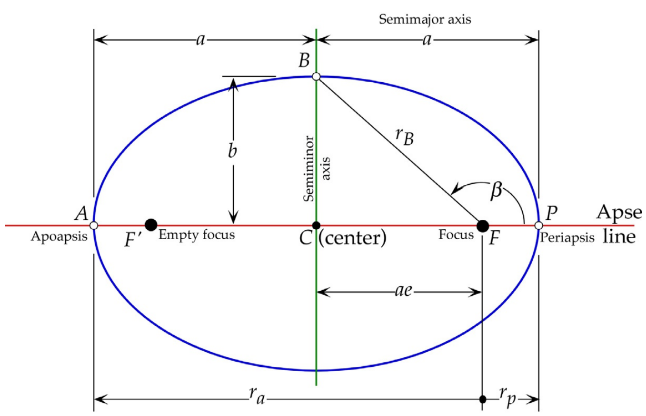

Fig. 1. Illustration of the elliptical orbit [1].

A semi-major axis (a), visualized in Fig. 1, is calculated with the following equation.

$$a = \frac{h^2}{\mu}\frac{1}{1-e^2} \tag{8}$$

where μ is a gravitational constant of the orbiting object, h is an angular momentum of the spacecraft, and e is an eccentricity. As could easily be understood from the Fig. 1, semi-major axis is equal to the radius for circular orbits.

If an orbit is a Earth circular orbit (e=0), then an angular momentum h could be calculated with fairly simple equation as in Eq. (9).

$$h = \sqrt{\mu r} \tag{9}$$

where r is an altitude from sea-level plus radius of the planet given in Table I.

An inclination i of the Earth parking sun-synchronous orbit is computed with Eq. (10).

$$\dot{\Omega} = -\left[\frac{3}{2}\frac{\sqrt{\mu}J2R^2}{(1-e^2)^2a^{7/2}}\right]cos(i) \tag{10}$$

where $\dot{\Omega}$ is an average rate of change of the the right ascension Ω, μ and R are gravitational parameter and radius of the orbiting planet, a is a semi-major axis, and e is an eccentricity of the orbit.

The spacecraft's planetary orbit inclination (i) is based on launch azimuth and latitude angles of the launch platform as illustrated in Eq. (11).

$$cos(i) = cos(\phi)cos(A) \tag{11}$$

where i is an inclination angle of the orbit, ϕ is latitude angle of the launch platform position, A is an azimuth angle of the platform. In this project, it is not required that launch position on Earth is logical or realistic.

D. Rocket Dynamics

The frontal area (A) of the selected launch vehicle is calculated with Eq. (12), where d is diameter of the rocket.

$$A = \pi\frac{d^2}{4} \tag{12}$$

One of the main property equations of the rocket system is a thrust (T) to launch vehicle weight ratio given as $T/(m_0g_0)$. The fuel flow rate $\dot{m}$ of the rocket is a design parameter given in Eq. (13).

$$\dot{m} = \frac{T}{I_{sp}g_0} \tag{13}$$

where I_{sp} is a specific impulse of the launch vehicle in seconds and g_0 is a gravitational acceleration of Earth at sea-level. Note that in this project, the thrust of the launch vehicle is assumed to be constant, but in-fact in real flight, the amount thrust varies throughout the launch.

Total propellant burning time is calculated with Eq. (14).

$$t_{burn} = \frac{m_p}{\dot{m}} \tag{14}$$

where m_p is an initial mass of the used total propellant.

In this project, the assumption of constant gravitational acceleration is **not** considered so the change of gravitational acceleration (g) with respect to the sea-level altitude is given in Eq. (15).

$$g = g_0\frac{r^2_{earth}}{(r_{earth}+z)^2} \tag{15}$$

where $r_{earth} = 6378km$ is a radius of the planet Earth given in Table I and z is a current flying altitude in km.

Although an aerodynamic drag force (D) is directed opposite of the rocket velocity vector has a magnitude given in Eq. (16).

$$D = qAC_D \tag{16}$$

where $q = \rho v^2/2$, in which ρ is a density of the atmosphere at an altitude z and v is the speed of the rocket, A is a frontal area from Eq. (12), and C_D is a design parameter constant of drag coefficient.

The velocity change (derivative) of the rocket with respect to time is calculated with Eq. (17) by considering thrust and drag effects with flight path angle γ.

$$\dot{v} = \frac{dv}{dt} = \frac{T}{m} - \frac{D}{m} - gsin(\gamma) \tag{17}$$

where thrust T is constant, drag D is computed with Eq. (16), gravitational acceleration g is calculated with Eq. (15).

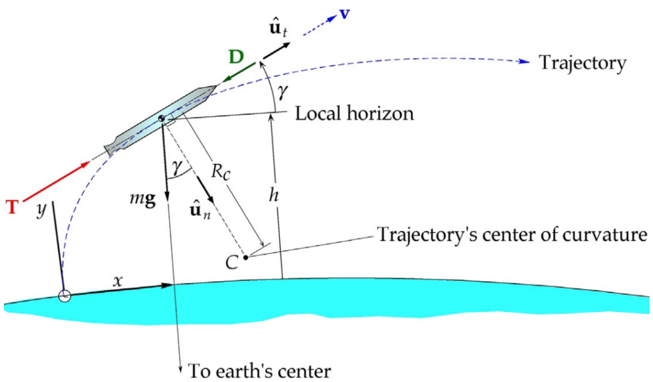

Fig. 2. Visualization of launch vehicle angles [1].

The change in flight path angle γ is calculated in Eq. (18).

$$v\dot{\gamma} = v\frac{d\gamma}{dt} = -\left[g - \frac{v^2}{r_{earth} + z}\right]cos(\gamma) \tag{18}$$

E. Hyperbolic Transfer Orbit

In order to escape an Earth SOI radius with spacecraft, the Earth centring hyperbolic trajectory should be constructed as illustrated in Fig. 3(a). A hyperbolic excess speed of the departure hyperbola shown in Fig. 3(b) and in Eq. (19).

$$v_\infty = \sqrt{\frac{\mu_{sun}}{R_1}}\left(\sqrt{\frac{2R_2}{R_1 + R_2}} - 1\right) \tag{19}$$

where μ_{sun} could be found from the Table I, R_1 and R_2 are radius of planets Earth and Mars respectively.

Hyperbolic trajectory maneuver is initialized when the spacecraft is in the circular orbit of the Earth having the velocity of $v_{circular}$ as given in Eq. (20).

$$v_{circular} = \sqrt{\frac{\mu_{earth}}{r_{earth} + z}} \tag{20}$$

where z is an altitude of the parking circular orbit in km.

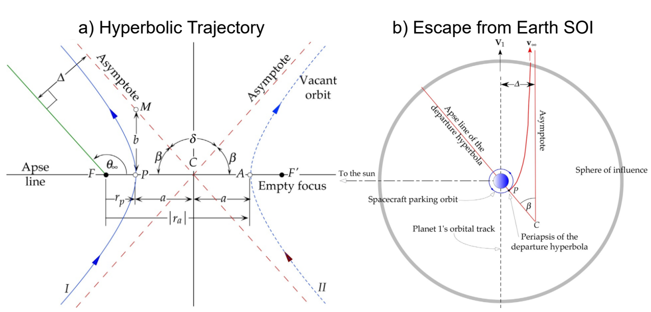

Fig. 3. Illustration of a hyperbolic trajectory to escape Earth SOI [1].

Delta-v is required to step-up to the departure hyperbolic trajectory as depicted in Eq.8.42 of the reference [1]. This required velocity difference is calculated with Eq. (21).

$$\Delta V_{hyperbola} = v_{periapsis} - v_{circular}$$

$$\Delta V_{hyperbola} = v_{circular}\left[\sqrt{2 + \left(\frac{v_\infty}{v_{circular}}\right)^2} - 1\right] \tag{21}$$

where $v_{periapsis}$ is a hyperbolic periapsis velocity of the spacecraft as defined in Eq. (22).

$$v_{periapsis} = \frac{h}{r_{periapsis}} = \sqrt{v_\infty^2 + \frac{2\mu}{r_{periapsis}}} \tag{22}$$

The location of periapsis point where $\Delta V_{hyperbola}$ will occur is determined by β angle as visualized on Fig. 3. The orientation of the apse line of the hyperbola to the planet's heliocentric velocity vector is calculated with Eq. (23).

$$\beta = cos^{-1}(\frac{1}{e}) = cos^{-1}\left(\frac{1}{1 + \frac{v_\infty^2 . r_{periapsis}}{\mu}}\right) \tag{23}$$

An eccentricity of the hyperbolic orbits as in Eq. (24).

$$e = \frac{1}{cos(\beta)} \tag{24}$$

As a results, a hyperbolic true anomaly at the infinity and at the SOI location are calculated with Eq. (25) and Eq. (26).

$$\theta_\infty = cos^{-1}(-1/e) \tag{25}$$

$$\theta_{soi} = cos^{-1}\left[\frac{\frac{h^2}{\mu_{earth}.r_{soi}^{earth}} - 1}{e}\right] \tag{26}$$

A hyperbolic eccentric anomaly F from perigee to reach SOI is calculated with the same equation Eq.3.44a in reference [1] as given in Eq. (27).

$$tan\left(\frac{F}{2}\right) = \frac{e-1}{e+1}.tan\left(\frac{\theta}{2}\right) \tag{27}$$

The hyperbolic mean anomaly M in radians is calculated with Kepler's equation as detailed in Algorithm 3.2 in of reference [1].

$$M = e.sinh(F) - F \tag{28}$$

Total duration time passed $t_{hyperbola}$ while the spacecraft travels through hyperbolic trajectory is calculated in Eq. (29).

$$M = \frac{\mu_{earth}^2}{h^3}.(e^2 - 1)^{3/2}.t_{hyperbola}$$

after rearranging this equation:

$$t_{hyperbola} = M.\frac{h^3}{\mu_{earth}^2.(e^2 - 1)^{3/2}} \tag{29}$$

Note: All of the equations used in this project are taken and referenced from the book by H. Curtis of reference [1]. These equations will be referenced later inside the sections without rewriting them. All programming and simulation codes are writing in MATLAB with corresponding book references as comments for specific lines.

F. Hohmann Transfer

For interplanetary trajectories the most energy efficient method Hohmann transfer maneuver is applied by constructing a semi-elliptic transfer trajectory as depicted in Fig. 4.

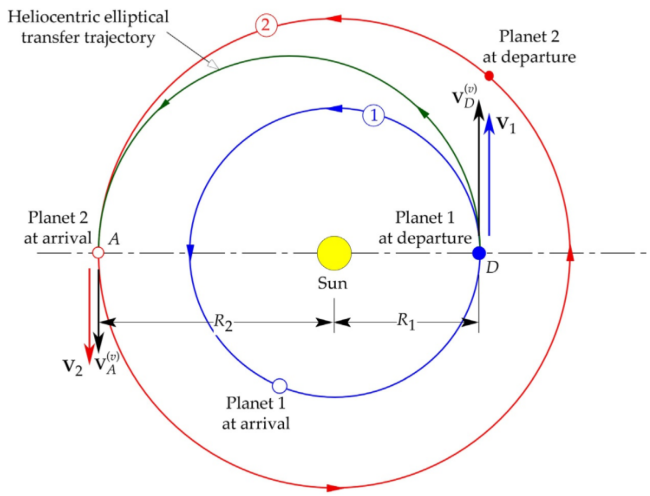

Fig. 4. Visualization of Hohmann transfer maneuver [1].

The semimajor (a) axis of Hohmann transfer ellipse could be calculated with Eq. (30).

$$a = \frac{1}{2} . \frac{1}{r_{earth} + r_{mars}} \tag{30}$$

where radius of Earth and Mars are given in Table I.

The spacecraft speed at SOI distance from the planet is calculated in Eq. (31) and would be in the direction of the planet's heliocentric motion.

$$v_{soi} = \sqrt{\frac{\mu_{sun}}{R}} \tag{31}$$

where μ_{sun} is constant and given in Table I, whereas R is a planet's heliocentric orbit radius; for Earth $R_E = 149.6$ x $10^6 km$ and for Mars $R_M = 227.9$ x $10^6 km$ [1].

The spacecraft's heliocentric velocity $v_{departure}$ at the Earth departure point is computed with Eq. (32) as the same as in Eq.8.2 in reference [1].

$$v_{departure} = \sqrt{\mu_{sun} . \left[\frac{2}{R_E} - \frac{1}{a} \right]} \tag{32}$$

where R_E is an Earth heliocentric radius and a is a semimajor axis. In Fig. 4, R_1 refer to the heliocentric circular radius of Earth R_E.

The required delta-v to transfer to Hohmann transfer orbit, as given in Eq.8.3 in reference [1], is simply calculated as follows:

$$\Delta V_{hyperbola} = v_{departure} - v_{soi} \tag{33}$$

A total time required for the Hohmann transfer period is calculated with Eq. (34).

$$t_{hohmann} = \frac{\pi}{\sqrt{\mu}} . \left(\frac{R_E + R_M}{2} \right)^{(3/2)} \tag{34}$$

III. Launch Date and Time Selection

As detailed in Section II-A, the launch day and time selection depends on the launch window from the planet Earth to planet Mars. It is required that the Hohmann transfer from Earth to Mars should occur when planet Earth has true anomaly θ angle of approximately 44.57 degrees between planet Mars.

Initially launch vehicle (rocket) system design is conducted and applicable rocket is selected that will be used to place the spacecraft into the parking orbit of Earth at $500km$. As we know the rocket system that is used t-in this project, total duration of launch vehicle flight until reaching desired parking orbit altitude after burnout is calculated with Eq. (14) in Section II-D.

After reaching the desired and predefined altitude of Earth parking orbit, parking duration and total duration of hyperbolic trajectory is calculated using equation Eq. (29). As soon as the hyperbolic escape trajectory is conducted, the spacecraft will start a Hohmann transfer at approximately 44.56 degrees of true anomaly between Mars.

By making a trial and errors, because of knowing duration $t_{hyperbola}$ and t_{burn}, an initial rocket launch date and time are determined to be as follows:

$\boxed{year : 2023}$ $\boxed{month : May}$ $\boxed{day : 7}$

$\boxed{hour : 11a.m.}$ $\boxed{minute : 11}$ $\boxed{second : 11}$

IV. Rocket Launch

For simplicity, a sun-synchronous Earth circular orbit at $500km$ altitude is selected where the burnout of the rocket will occur at initial stage of the launch. To place the spacecraft into sun-synchronous parking orbit, latitude ϕ and azimuth A angles of the launch platform are determined. As this project is a concept project, it is not crucial whether the selected launch location actually feasible for rocket launch with all maintenance. So, an azimuth angle is selected randomly as follows:

$$\boxed{A = 279.3489deg}$$

We know that with Eq. (10), an inclination of the Earth parking orbit as a sun-synchronous orbit could be calculated with Eq.4.52 in reference [1], where average rate of change of the the right ascension $\dot{\Omega}$ is obtained as follows:

$$\dot{\Omega} = \frac{2\pi}{365.26 * 24 * 3600}$$

because selected orbit is a sun-synchronous Earth circular orbit. After substituting constants into Eq. (10), Earth parking orbit's inclination angle is determined as:

$$\boxed{i = 97.401deg}$$

which is closer to polar orbits as expected. Eventually, the latitude angle of the launch platform could be calculated with Eq. (11). The selected latitude of the launch platform is determined as:

$$\boxed{\phi = 82.499deg}$$

In this project, the selection of the specific impulse I_{sp}, diameter d, rocket constant thrust T, propellant mass ratio, and drag coefficient C_D of the launch vehicle is not restricted with realistic rockets; so these parameters are the design choices for our project. To be easily interpretable, applied rocket thrust $\boxed{T = 10^6 N}$, diameter of the launch vehicle $\boxed{d = 3.7m}$, and drag coefficient is chosen as $\boxed{C_D = 0.5}$, which are similar parameters of the Falcon 9 rockets. By applying Eq. (12), the frontal area of the launch vehicle is determined as:

$$\boxed{A = 10.7521m^2}$$

The flight path angle γ is chosen to be constant value as $\boxed{\gamma = 90deg}$. Runga-Kutta 4(5) algorithm is implemented to integrate equations Eq. (13, 15, 16, 17) at each step after launching. Fig. 5 shows the profile of launching of rocket to 500km parking orbit with determined configuration parameters.

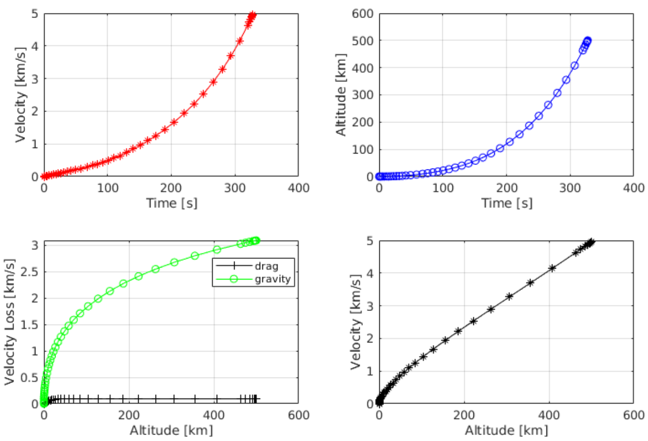

Fig. 5. Launch spacecraft to 500 km orbit.

Total mass of the spacecraft until reaching Earth parking orbit is selected to be $\boxed{m_{PL} = 6666kg}$, the propellant mass is chosen to be interpretative randomly selected as $\boxed{m_P = 54321kg}$, and the empty mass of the rocket is finally chosen to be selected as $\boxed{m_E = 12345kg}$. These parameters are all design choices and predefined without any considerations. A tandem-stacked launch vehicle before take-off is:

$$\boxed{m_0 = m_P L + m_P + m_E = 73332kg}$$

By using Eq. (13), as experimentally selected specific impulse $\boxed{I_{sp} = 614.548s}$ and rocket thrust T are assumed to be constant for launch vehicle, propellant mass flow rate $\dot{m}$ is calculated as follows:

$$\boxed{\dot{m} = 165.8728kg/s}$$

Total duration of flight of the launch vehicle with spacecraft until burnout is calculated with Eq. (14) and result is as follows:

$$\boxed{t_{burn} = 327.48588s}$$

which resembles **5 minutes** and **28 seconds** of rocket flight.

Note: the specific impulse is chosen with trial and errors that showed the burnout altitude at 500km. Additional first Delta-V $\boxed{\Delta V_{circular} = 2.6597km/s}$ is applied to the spacecraft for aligning with circular orbit velocity as final burnout velocity is $v_{burnout} = 4.95288km/s$ using Eq. (17).

V. Escape Earth SOI

After reaching to the 500km parking orbit of Earth, the spacecraft is directly towarded to hyperbolic trajectory maneuver to escape Earth SOI. The specific impulse of the spacecraft to make an interplanetary delta-v maneuvers is selected as $\boxed{I_{sp}^s = 321s}$. Because the spacecraft is near polar orbit, in other words, in a sun-synchronous Earth orbit, we assume that hyperbolic trajectory maneuver could be applied at any moment while spacecraft is orbiting Earth.

The first delta-v (Δv) after the launch is conducted to escape an Earth SOI with hyperbolic trajectory. By using Eq. (19), the hyperbolic excess speed of the Earth departure hyperbola is calculated as:

$$\boxed{v_\infty = 2.94346km/s}$$

and the speed of the spacecraft in a circular parking orbit is calculated with Eq. (20):

$$\boxed{v_{circular} = 7.61268km/s}$$

By using a hyperbolic excess speed and a parking orbit speed above, the delta-v required to step up to the departure hyperbola could be computed using Eq. (21):

$$\boxed{\Delta V_{escape} = 3.54840km/s}$$

The perigee of a departure hyperbola with respect to the Earth velocity vector is calculated using Eq. (23):

$$\boxed{\beta = 29.54795deg}$$

It is determined that the ratio of required propellant mass to the spacecraft mass is as follows:

$$m_{ratio} = 1 - e^{\frac{-\Delta V_{escape}}{I_{sp}^s \cdot g_0}} = 0.67594 \tag{35}$$

meaning that the mass of the propellant used for making ΔV_{escape} is %67.6 of the total mass of the spacecraft.

An eccentricity of the hyperbolic trajectory is defined using Eq. (24):

$$\boxed{e_{hyperbola} = 1.14950}$$

which is ¿1, meaning that the calculation is correct.

The spacecraft forwarding final location in the hyperbolic trajectory is the Earth SOI distance, which is given in Table I. The true anomaly of the asymptote of the hyperbola is calculated using Eq. (25):

$$\boxed{\theta_\infty = 150.45205deg}$$

while by using an orbit equation for reaching Earth SOI radius is calculated as:

$$\boxed{\theta_{soi} = 148.87471deg}$$

By using above obtained results and Eq. (27) for hyperbolic eccentric anomaly from perigee to reach Earth SOI, and Eq. (4) mean anomaly is calculated with Kepler's equation for hyperbola, respectively:

$$\boxed{F = 206.41524deg \quad M = 17.47185rad}$$

With these calculated information, a total duration of the spacecraft while in hyperbolic trajectory is determined with Eq. (29) as:

$$\boxed{t_{hyperbola} = 273086.80638s}$$

which could be interpreted as **3 days**, **3 hours**, **51 minutes**, and **27 seconds**.

VI. Travel to Mars

Note: as the spacecraft directly applies delta-v for escapee Earth SOI after the burnout of the launch vehicle, the total duration of flight before starting a Hohmann transfer maneuver is $t_{burn} + t_{hyperbola}$.

After adding **additional** 3 days, 3 hours, 56 minutes, and 55 seconds to the launch date and time, the Hohmann transfer maneuver will be initialized:

$$\boxed{year : 2023} \quad \boxed{month : May} \quad \boxed{day : 10}$$

$$\boxed{hour : 03p.m.} \quad \boxed{minute : 08} \quad \boxed{second : 06}$$

At this date and time, a true anomaly between Earth and Mars is approximately $\theta = 44.32deg$.

The spacecraft speed at the point of Earth departure is calculated with Eq. (32):

$$\boxed{v_{departure} = 32.72794km/s}$$

The semimajor axis of the Hohmann transfer orbit is $a = 1.8875$ x $10^8 km$. The total elapsed time during Hohmann transfer maneuver to Mars is calculated with Eq. (34):

$$\boxed{t_{hohmann} = 22362713.30644s}$$

which could be interpreted as **258 days**, **19 hours**, **51 minutes**, and **53 seconds**.

The spacecraft will enter to the Mars SOI in 2024 year, January month, 24th day, hour 10 a.m., 59 minutes, and 59 seconds. The speed of the spacecraft after Hohmann transfer at Mars SOI location is calculated as similar to Eq. (32):

$$v_{arrival} = \sqrt{\mu_{sun}\left(\frac{2}{R_M} - \frac{1}{a}\right)} = \boxed{21.48355km/s} \tag{36}$$

The speed at Mars capture orbit (at Mars SOI radius) with respect to heliocentric trajectory is calculate as:

$$v_{mars} = \sqrt{\frac{\mu_{sun}}{R_M}} = \boxed{24.13146km/s} \tag{37}$$

and the hyperbolic excess speed at Mars arrival hyperbola is

$$v_\infty^{mars} = v_{arrival} - v_{mars} = \boxed{-2.64792km/s}$$

The spacecraft velocity relative to Mars at periapse of approach hyperbola could be computed as:

$$v_{perigee}^{mars} = \sqrt{(v_\infty^{mars})^2 + \left(\frac{2\mu_{mars}}{r_{mars} + z_{mars}}\right)} \tag{38}$$

where z_{mars} is an altitude of the Mars parking orbit, which is predefined to be 300km.

Finally, delta-V required to stay on the given circular parking orbit of the planet Mars is calculated as:

$$\Delta V_{arrive} = v_{circular}^{mars} - v_{perigee}^{mars} = \boxed{-2.09018km/s} \tag{39}$$

where $v_{circular}^{mars} = 3.40407km/s$ is a Mars parking orbit velocity calculated using Eq. (20) and minus sign in the delta-v resembles that the spacecraft should decrease its speed to arrive at a circular parking orbit when reached to Mars SOI location.

The waiting duration on the parking orbit of Mars is determined by observing planetary ephemeris for appropriate window of Hohmann transfer.

An Earth planet mean angular velocity (circular orbit assumption) is calculated as follows:

$$n_E = \frac{V_{earth}}{R_E} = \frac{\sqrt{\frac{\mu_{sun}}{R_E}}}{R_E} \tag{40}$$

similarly n_M, Mars planet mean angular velocity is also calculated by using Table I.

An initial phase angle between Earth and Mars is conducted with Eq.8.12 in [1] as:

$$\phi_0 = \pi - (n_M * t_{hohmann}) = \boxed{44.32deg} \tag{41}$$

A final phase angle between Earth and Mars when arrival to Mars is calculated with Eq.8.13 in [1] as:

$$\phi_f = \phi_0 + (n_M - n_E) * t_{hohmann} = \boxed{-75.09712deg} \tag{42}$$

Waiting duration is computed with Eq.8.15 in [1] as:

$$t_{wait} = \frac{-2.\phi_f}{n_M - n_E} \tag{43}$$

Total wait time until next Hohmann transfer back to Earth is:

$$\boxed{t_{wait} = 39286373.02635s}$$

in other words, **454 days**, **16 hours**, **52 minutes**, and **53 seconds**.

VII. Conclusion

In conclusion, all of the calculations and simulations of a part-1 of this project is completed. All simulations are calculated with custom scripts in MATLAB. Each script line has a comment for better readability and in these comments useful equation numbers in reference [1] are given.

References

[1] H. Curtis, *Orbital mechanics for engineering students.* Butterworth-Heinemann, 2013.