UCDAVIS
Department of Animal Science
Quantitative Genetics Lab

Statistics and Data Science

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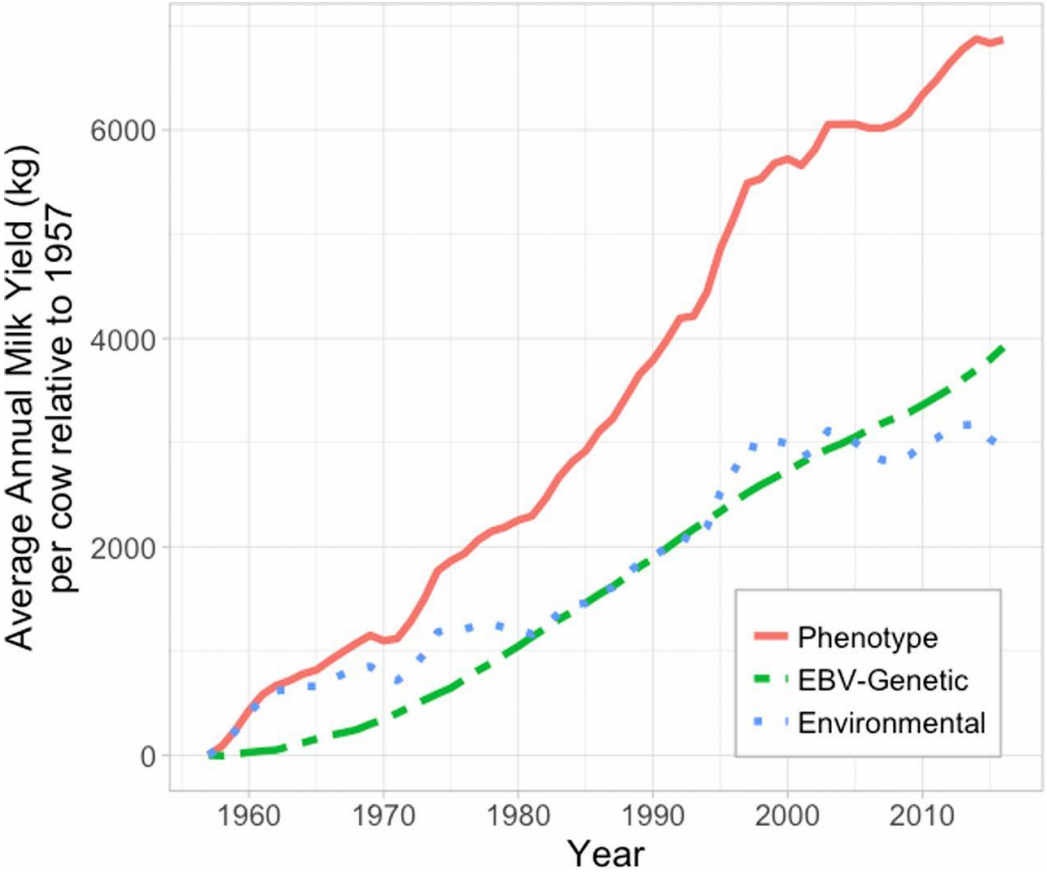
Why is Statistics important?


Average Annual Milk Yield (kg)
per cow relative to 1957
6000
4000
2000
0
1960
1970
1980
1990
2000
2010
Year
Phenotype
EBV-Genetic
Environmental

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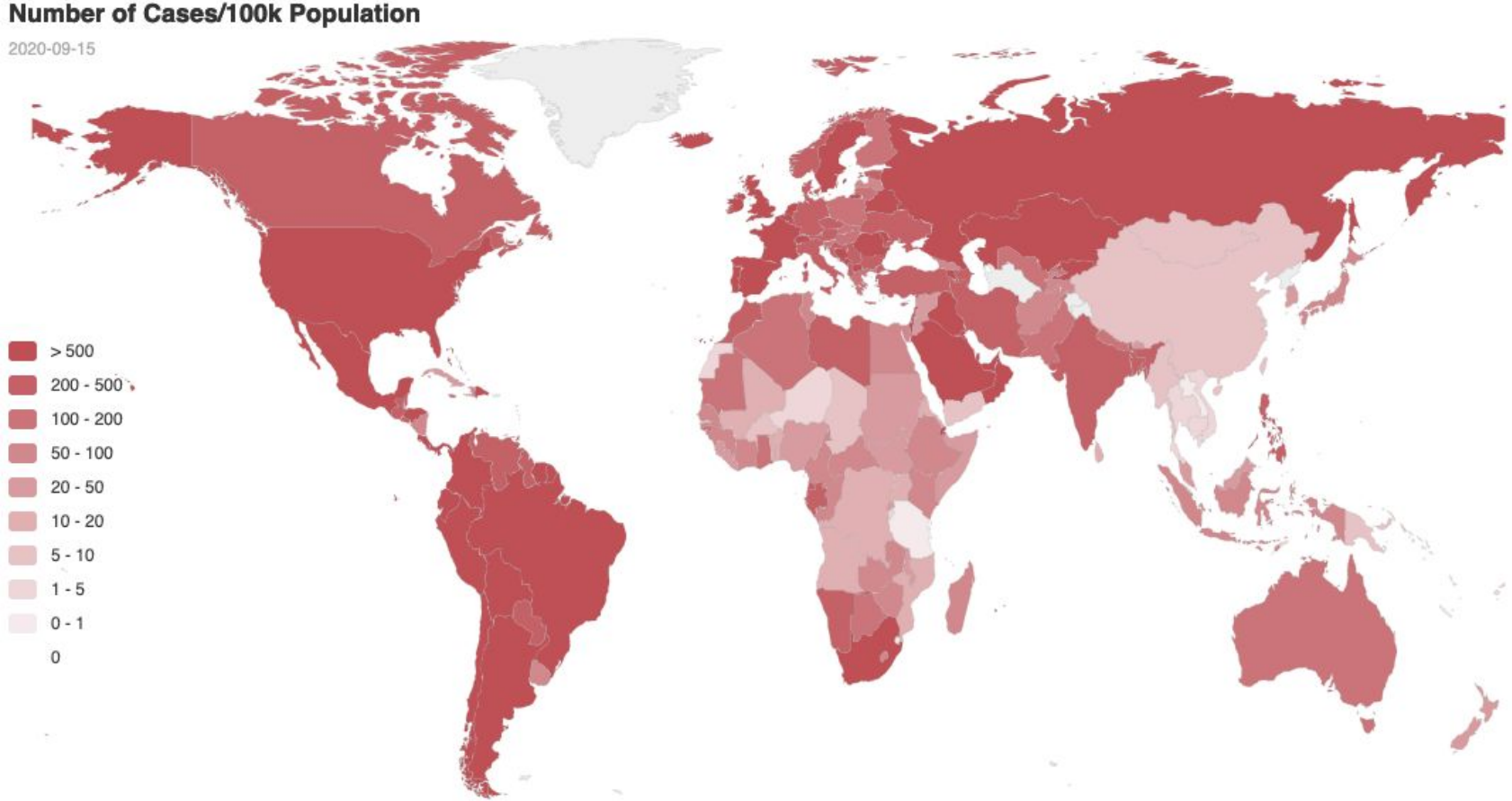
Data Science


#30 Bojangles'
Famous Chicken 'n Biscuits
749 US Locations
0:00 / 2:28


Number of Cases/100k Population
2020-09-15
> 500
200 - 500
100 - 200
50 - 100
20 - 50
10 - 20
5 - 10
1 - 5
0 - 1
0

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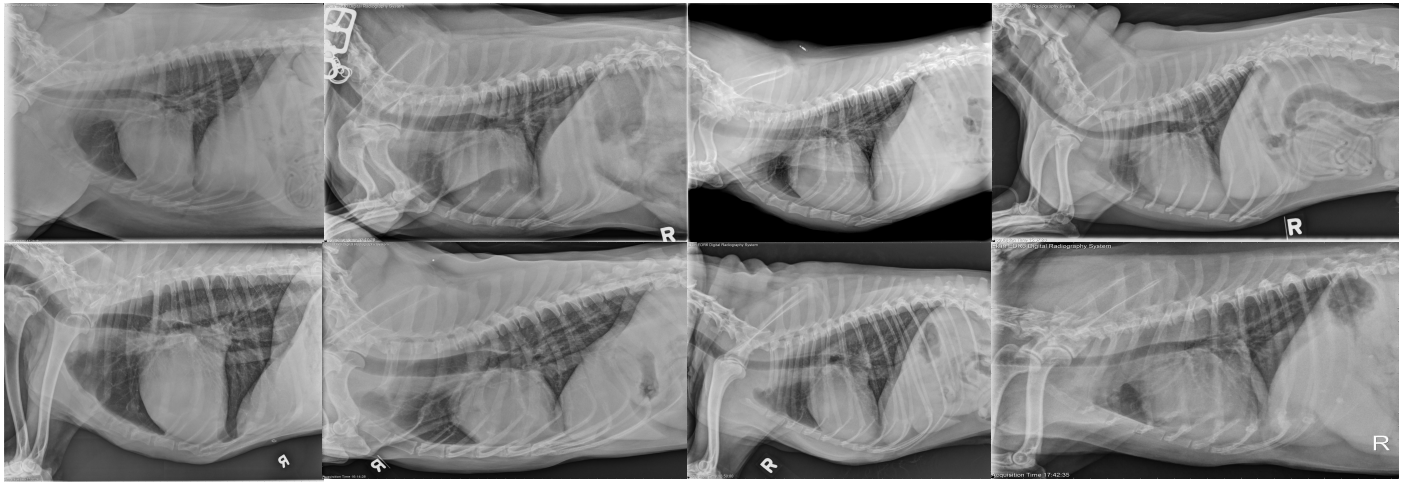
Pilot study: Application of artificial intelligence for detecting left atrial enlargement on canine thoracic radiographs

Shen Li, Zigui Wang, Lance C. Visser, Erik R. Wisner, Hao Cheng

First published: 11 August 2020 | https://doi.org/10.1111/vru.12901



Quiz 1: What is p-value?

The p-value is the probability of obtaining a value of the test statistic as or more extreme than the observed value of the test statistic when the null hypothesis is true.

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Creative Writing Study

- Experimental units: writers
- Treatment: type of questionnaire given at the beginning of the study (questions emphasize either intrinsic or extrinsic rewards)
- Random assignment: 24 intrinsic, 23 extrinsic
- Response: average of 12 evaluations of creativity on a 40 point scale

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Observed data

intrinsic:	12.0	12.0	12.9	13.6	16.6	17.2
	17.5	18.2	19.1	19.3	19.8	20.3
	20.5	20.6	21.3	21.6	22.1	22.2
	22.6	23.1	24.0	24.3	26.7	29.7
extrinsic:	5.0	5.4	6.1	10.9	11.8	12.0
	12.3	14.8	15.0	16.8	17.2	17.2
	17.4	17.5	18.5	18.7	18.7	19.2
	19.5	20.7	21.2	22.1	24.0	

null hypothesis:

Treatments have the same effect.

model-based

(relies on the specification of a model)

model-based

- two independent random samples
- homogeneous population variances
- normality

Randomization

(design-based)

Quiz 2: What is type I (II) error?

		True condition			
	Total population	Condition positive	Condition negative	Prevalence $= \frac{\Sigma \text{ Condition positive}}{\Sigma \text{ Total population}}$	Accuracy (ACC) $= \frac{\Sigma \text{ True positive} + \Sigma \text{ True negative}}{\Sigma \text{ Total population}}$
Predicted condition	Predicted condition positive	**True positive**	**False positive**, Type I error	Positive predictive value (PPV), Precision $= \frac{\Sigma \text{ True positive}}{\Sigma \text{ Predicted condition positive}}$	False discovery rate (FDR) $= \frac{\Sigma \text{ False positive}}{\Sigma \text{ Predicted condition positive}}$
	Predicted condition negative	**False negative**, Type II error	**True negative**	False omission rate (FOR) $= \frac{\Sigma \text{ False negative}}{\Sigma \text{ Predicted condition negative}}$	Negative predictive value (NPV) $= \frac{\Sigma \text{ True negative}}{\Sigma \text{ Predicted condition negative}}$
		True positive rate (TPR), Recall, Sensitivity, probability of detection, Power $= \frac{\Sigma \text{ True positive}}{\Sigma \text{ Condition positive}}$	False positive rate (FPR), Fall-out, probability of false alarm $= \frac{\Sigma \text{ False positive}}{\Sigma \text{ Condition negative}}$	Positive likelihood ratio (LR+) $= \frac{\text{TPR}}{\text{FPR}}$	Diagnostic odds ratio (DOR) $= \frac{\text{LR+}}{\text{LR−}}$
		False negative rate (FNR), Miss rate $= \frac{\Sigma \text{ False negative}}{\Sigma \text{ Condition positive}}$	Specificity (SPC), Selectivity, True negative rate (TNR) $= \frac{\Sigma \text{ True negative}}{\Sigma \text{ Condition negative}}$	Negative likelihood ratio (LR−) $= \frac{\text{FNR}}{\text{TNR}}$	F_1 score $= 2 \cdot \frac{\text{Precision} \cdot \text{Recall}}{\text{Precision} + \text{Recall}}$

https://en.wikipedia.org/wiki/Sensitivity_and_specificity

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Common Statistical Tests

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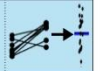
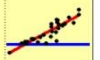
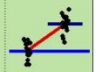
Common statistical tests are linear models

Last updated: 28 June, 2019.Also check out the Python version!

See worked examples and more details at the accompanying notebook: https://lindeloev.github.io/tests-as-linear

	Common name	Built-in function in R	Equivalent linear model in R	Exact?	The linear model in words	Icon
Simple regression: lm(y ~ 1 + x)	**y is independent of x** P: One-sample t-test N: Wilcoxon signed-rank	t.test(y) wilcox.test(y)	lm(y ~ 1) lm(signed_rank(y) ~ 1)	✓ for N >14	One number (intercept, i.e., the mean) predicts **y**. - (Same, but it predicts the *signed rank* of **y**.)	
	P: Paired-sample t-test N: Wilcoxon matched pairs	t.test(y_1, y_2, paired=TRUE) wilcox.test(y_1, y_2, paired=TRUE)	lm(y_2 - y_1 ~ 1) lm(signed_rank(y_2 - y_1) ~ 1)	✓ for N >14	One intercept predicts the pairwise $\mathbf{y_2}$-$\mathbf{y_1}$ differences. - (Same, but it predicts the *signed rank* of $\mathbf{y_2}$-$\mathbf{y_1}$.)	
	y ~ continuous x P: Pearson correlation N: Spearman correlation	cor.test(x, y, method='Pearson') cor.test(x, y, method='Spearman')	lm(y ~ 1 + x) lm(rank(y) ~ 1 + rank(x))	✓ for N >10	One intercept plus **x** multiplied by a number (slope) predicts **y**. - (Same, but with *ranked* **x** and **y**)	
	y ~ discrete x P: Two-sample t-test P: Welch's t-test N: Mann-Whitney U	t.test(y_1, y_2, var.equal=TRUE) t.test(y_1, y_2, var.equal=FALSE) wilcox.test(y_1, y_2)	lm(y ~ 1 + G_2)[A] gls(y ~ 1 + G_2, weights=…[B])[A] lm(signed_rank(y) ~ 1 + G_2)[A]	✓ ✓ for N >11	An intercept for **group 1** (plus a difference if **group 2**) predicts **y**. - (Same, but with one variance *per group* instead of one common.) - (Same, but it predicts the *signed rank* of **y**.)	
Multiple regression: lm(y ~ 1 + x_1 + x_2 +…)	P: One-way ANOVA N: Kruskal-Wallis	aov(y ~ group) kruskal.test(y ~ group)	lm(y ~ 1 + G_2 + G_3 +…+ G_N)[A] lm(rank(y) ~ 1 + G_2 + G_3 +…+ G_N)[A]	✓ for N >11	An intercept for **group 1** (plus a difference if group ≠ 1) predicts **y**. - (Same, but it predicts the *rank* of **y**.)	
	P: One-way ANCOVA	aov(y ~ group + x)	lm(y ~ 1 + G_2 + G_3 +…+ G_N + x)[A]	✓	- (Same, but plus a slope on **x**.) *Note: this is discrete AND continuous. ANCOVAs are ANOVAs with a continuous x.*	
	P: Two-way ANOVA	aov(y ~ group * sex)	lm(y ~ 1 + G_2 + G_3 +…+ G_N + S_2 + S_3 +…+ S_K + G_2*S_2 + G_3*S_3 + … + G_N*S_K)	✓	Interaction term: changing **sex** changes the **y ~ group** parameters. *Note: $\mathbf{G_{2\,to\,N}}$ is an indicator (0 or 1) for each non-intercept levels of the **group** variable. Similarly for $\mathbf{S_{2\,to\,K}}$ for sex. The first line (with G_i) is main effect of group, the second (with S_i) for sex and the third is the **group × sex** interaction. For two levels (e.g. male/female), line 2 would just be "S_2" and line 3 would be S_2 multiplied with each G_i.*	*[Coming]*
	Counts ~ discrete x N: Chi-square test	chisq.test(groupXsex_table)	**Equivalent log-linear model** glm(y ~ 1 + G_2 + G_3 + … + G_N + S_2 + S_3+ … + S_K + G_2*S_2 + G_3*S_3 +…+ G_N*S_K, family=…)[A]	✓	Interaction term: (Same as Two-way ANOVA.) *Note: Run glm using the following arguments: `glm(model, family=poisson())`* *As linear-model, the Chi-square test is log(y_i) = log(N) + log(α_i) + log(β_i) + log($\alpha_i\beta_i$) where α_i and β_i are proportions. See more info in the accompanying notebook.*	*Same as Two-way ANOVA*
	N: Goodness of fit	chisq.test(y)	glm(y ~ 1 + G_2 + G_3 +…+ G_N, family=…)[A]	✓	(Same as One-way ANOVA and see Chi-Square note.)	*1W-ANOVA*

List of common parametric (P) non-parametric (N) tests and equivalent linear models. The notation y ~ 1 + x is R shorthand for y = 1·b + a·x which most of us learned in school. Models in similar colors are highly similar, but really, notice how similar they *all* are across colors! For non-parametric models, the linear models are reasonable approximations for non-small sample sizes (see "Exact" column and click links to see simulations). Other less accurate approximations exist, e.g., Wilcoxon for the sign test and Goodness-of-fit for the binomial test. The signed rank function is `signed_rank = function(x) sign(x) * rank(abs(x))`. The variables G_i and S_i are "dummy coded" indicator variables (either 0 or 1) exploiting the fact that when Δx = 1 between categories the difference equals the slope. Subscripts (e.g., G_2 or y_1) indicate different columns in data. lm requires long-format data for all non-continuous models. All of this is exposed in greater detail and worked examples at https://lindeloev.github.io/tests-as-linear.

[A] See the note to the two-way ANOVA for explanation of the notation.
[B] Same model, but with one variance per group: `gls(value ~ 1 + G2, weights = varIdent(form = ~1|group), method="ML")`.

Jonas Kristoffer Lindeløv
https://lindeloev.net

https://lindeloev.github.io/tests-as-linear/#51_independent_t-test_and_mann-whitney_u

Some Matrix Algebra

		movie		
		1	2	3
customer	1	4	1	?
	2	?	3	5
	3	?	?	3
	4	3	1	?

Can we guess ratings for customer/movie combinations not in the dataset?

y_{ij} = customer i's rating of movie j

Which movie is best?

$$y_{ij} = \mu + c_i + m_j + \epsilon_{ij}$$

		movie		
		1	2	3
customer	1	4	1	?
	2	?	3	5
	3	?	?	3
	4	3	1	?

Can we guess ratings for customer/movie combinations not in the dataset?

y_{ij} = customer i's rating of movie j

Which movie is best?

$$y_{ij} = \mu_{ij} + \epsilon_{ij}$$