AST121

AST121 Review

* 1 AU = ~~≡~~ 149.6 million km = 1.496×10^8 km ≐ 500 light seconds
* farthest man-made object 115 AU = 16 light hours
* farthest solar system bodies visited by man-made objects
  26 AU = 3.6 hrs

* closest star : the alpha-Centauri triple 4.4 lyrs.

$1 \text{ l.y.} \doteq 10^{13} \text{ km} \doteq 10^5 \text{ AU}$

* closest galaxy 2.5 million light-years
* Universe (observable patch) 14 billion light-years

* ~~10^{11}~~ ~100 billion (10^{11}) stars in one galaxy.
  ~100 billion (10^{11}) galaxies in observable universe

* Earth < Solar system < Milky Way galaxy < local group < Local Supercluster < Universe.

* Our local group = Milky Way + Andromeda + some dwarf galaxies (L.M, S.M, ...)

* We belong to Virgo supercluster ~100 groups of galaxies, ~~~1000~~ 10,000 galaxies

* the Copernican Principle : We don't occupy a special location in the universe

* Atom size ~ Å = 10^{-10} m
  proton size ~ fermi = 10^{-15} m
  the Planck scale : ~10^{-35} m

* Radiation Era → "Dark ages" → First stars → First galaxies.
  → HUDF (0.7 - 0.4 billions of years) → HDF (1.0)
  (Hubble Ultra deep field)

* Cosmic Calendar

Jan 1st — Big Bang
May 1st — Milky Way born
Sept. 9th — Solar system born
Sept. 14th — Earth
25th — life ?
Dec 31st. 22:30 — human (2 million yrs ago)
23:59:30 — ~~ag~~ agriculture (~10,000 yrs)

looking away in space = looking back in time

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matter = luminous matter + dark matter + dark energy

"standard model" : particles

all matter is constructed out of "fundamental particles" 'uncuttable' atoms, subatomic particles & interactions.

Mostly, up quarks + down quarks + electrons

Quarks

Charge	1st Gen	2nd Gen	3rd Gen
+2/3	up(u)	charm(c)	top(t)
−1/3	down(d)	strange(s)	bottom(b)

- Mass 3 > 2 > 1
- 1st mostly
- few heavy quarks at present.

$E = mc^2$

$1\ GeV = 10^9$ electron volt $= 1.6 \times 10^{-10}$ J.

$1\ GeV/c^2 = 1.8 \times 10^{-24}$ g ~ 1 proton mass

Most common ex:

Proton : uud $\frac{2}{3}+\frac{2}{3}-\frac{1}{3}=+1$

Neutron : udd $\frac{2}{3}+(-\frac{1}{3})+(-\frac{1}{3})=0$

- hadron = quark combinations
- baryon = three quark combinations
- meson = quark + anti-quark

Leptons

charge	1st Gen	2nd Gen	3rd Gen
-1	electron (e^-)	muno (μ^-)	tau (τ^-)
0	electron-neutrino (ν_e)	muno-neutrino (ν_μ)	tau-neutrino (ν_τ)

- first row : mass 3>2>1
- second row : only have upper limits now

Forces (interactions between particles, carried out by "force mediators")

Four BASIC Forces in Nature

	Name	Strength	Range	What?
	① Strong	1	10^{-15}m	Holds atomic nucleus together
	② Electromagnetism	1/137	infinite	Holds you together
every-day	③ Weak	10^{-5}	10^{-17}m	Breaks atoms apart
	④ Gravity	10^{-39}	infinite	Holds universe together

① Strong Force

- only between quarks. it binds quarks to form composite particles
- acts over 1 Fermi (10^{-15}m)

"Gluons" force carrier for Strong Force.

Quarks bound up since 10^{-6} second old (Quark Confinement)

* fundamental particles:

6 quarks + 6 leptons + 4 bosons (force medicators) + Higgs?

fundamental forces:

short range: strong & weak

long range: EM & gravity

③ Weak Force

- hard to describle
- neither attractive nor repulsive, but it changes ~~one~~ particals from one type to another.
- felt by both leptons & quarks
- acts over a distance ~ 1% of a proton size.

"larger generation gap"

leptons $e^- \updownarrow \nu_e$ $\quad \mu^- \updownarrow \nu_\mu$ $\quad \tau^- \updownarrow \nu_\tau$ Always

quarks $u \updownarrow d$ $\quad c \updownarrow s$ $\quad t \updownarrow b$ $\quad \leftrightarrow$ usually $\quad$ --- sometimes

* important weak force interaction — beta decay

* force = exchange of 'force carriers' (bosons)

"mirror" for anti-quarks & anti-leptons

u & $\bar{u}$: identical mass

opposite charge / baryon #

meet ⇒ "annihilate"

$E = mc^2$

Positron 正电子 is anti-electron.

* Heisenberg's uncertainty principle (1927)

$$\Delta x \Delta p \geq h$$
$$\Delta E \Delta t \geq h$$

Why are S/W forces so short-ranged?

- range is limited by the uncertainty principle.
- force carrier particle has to "get back home before it is missed".
- 'virtual particles': created & exist only during exchange
- More massive exchange particles: smaller range for the force.
- EM & gravity infinite reach: photons & gravitons are ~~part~~ massless.

Unification of forces:
GUT: 10^{15} GeV

Newton: Light was made up ~~of~~ of particles.
Thomas Young: "Two-slit Experiment" ⟹ Light is a wave (1800)

$$\boxed{E = h\nu = \frac{hc}{\lambda}}$$

"The Wave-Particle Duality"
photoelectric effect

ν → frequency

h is Planck's constant

Object has two colors:

1. reflection of light. ~~Matter~~ Material property determines.
2. thermal radiation. Temperature determines.

Blackbody Radiation

- radiates above 0K.
- thermal energy is turned into photons & the body cools — "thermal radiation".
- spectrum & intensity of this "thermal radiation" depends only on T.

Black Spectrum

hotter → shorter λ → more luminous

the peak of the radiation moves to higher freq. for higher T :

Wien's Law : $\boxed{\lambda_{max} = \frac{0.0029}{T}}$ m

Total energy/unit time/unit area.

Stefan-Boltzmann Law

$\boxed{E = \sigma T^4}$ J/m²s

$\sigma = 5.67 \times 10^{-8}$ W/m²K⁴

CMB is a perfect blackbody.

* the "Blackbody" spectrum is an approximation
  - Sun's b spectrum looks like a blackbody.
  - hotter star → hotter blackbody (bluer)

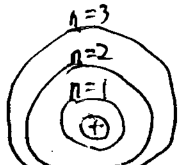
* ~~[illegible]~~ (filtering)

  O_3 absorbs UV

  H_2O, CO_2 absorbs infrared "Greenhouse effect"

Atoms/molecules absorb or emit at specific wavelengths

— Niels Bohr (1920s)



$\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

Energy of an orbit with quantum number n
- size of the orbit is quantized as

$$r = (n^2)\left(\frac{h}{2\pi}\right)^2 \frac{1}{m_e k_e e^2} = n^2 \times (0.5\,\text{Å})$$

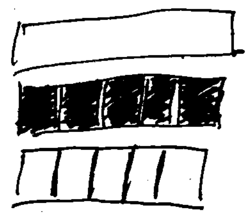
- energy $E = \frac{1}{2} m_e v^2 + \left(\frac{-k_e e^2}{r}\right) = -\frac{1}{n^2}\left(\frac{k_e^2 e^4 m_e}{2\hbar^2}\right) = -\frac{1}{n^2} \times (13.6\,\text{eV})$

- blackbody radiation from deeper, hotter layers of the star is absorbed by the cooler outer layer of gas at specific wavelengths, producing an <u>absorption spectrum</u>.

- hot gas producing an <u>emission spectrum</u>.

·3 Types of astronomical spectra.

Continuum	light?	
Emission lines	hot gas	
Absorption lines	Cold gas	



- compact fluorescent bulbs: emit only a <u>narrow range of wavelength (emission lines)</u>, and fool our brains into thinking the light is a <u>broad spectrum</u>
(Don't ~~是~~ need high T. save energy)

The (Classical) Doppler effect

blue ~~red~~ shift: • $\lambda = \lambda_0 (1 - \frac{v}{c})$

red ~~blue~~ shift • $\lambda = \lambda_0 (1 + \frac{v}{c})$

Def redshift $z = \frac{\lambda}{\lambda_0} - 1 = \frac{v}{c}$

CMB: $z \sim 1100$

Speed of light is same for everybody, always.

Time Dilation $\boxed{t' = \dfrac{t}{\sqrt{1-\frac{v^2}{c^2}}}}$

Experimental tests of special relativity I:
Muon Decay: it actually can reach the ground

Ruler Contraction?

$$\boxed{L' = \frac{L}{\gamma(v)} = \frac{L}{\sqrt{1-\frac{v^2}{c^2}}}}$$

Special Relativity: Momentum and Energy

$p = \gamma m v \rightarrow$ rest mass

$E = \gamma mc^2 = KE + mc^2 \rightarrow$ intrinsic energy

$\swarrow$ relativistic energy

$$\gamma^2 = \frac{1}{1-\beta^2}, \quad \beta = \frac{v}{c}$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$v \to c$, $E \to \infty$ unless $m = 0$

$$E^2 = p^2c^2 + m^2c^4$$

- no mass $\rightarrow$ no intrinsic energy. e.g. photon

[redacted]

- $E=pc$ this still holds even if it is massless.
- If it is massless, $p=\frac{E}{c}$, ~~it~~ photons have momentum, exert radiation pressure.

General relativity: ~~general re~~ space-time can be curved.

shortest way: "geodesics"

The Essence of General Relativity:
Mass tells space how to curve,
Space tells mass how to move.

Einstein Field Equation:

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

- light bending: space curvature same for every one, even photons.

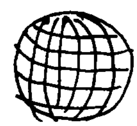
"Einstein Cross" – gravitational lens

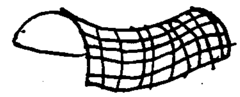
Extreme Geodesics: a blackhole (compact, no light can escape)

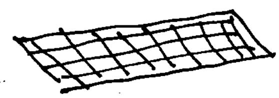
event horizon: even moving with c, all straight path heads downward. (You can't beam back video!)

! * Black-hole: don't radiate (except Hawking radiation)
* Black-body: radiate according to its T.

* Shapes of the universe
  isotropic (every direction – same)
  homogeneous (every point – same)

positive curvature 

negative —— 

zero —— 

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light redshifted by expansion, not Doppler Effect

photon emitted with λ_0 when the universe size a_0
photon stretches as it propagates toward us with speed of light

at reception, $\lambda(t)$ universe size $a(t)$

$a=a(t)$ called the scale factor of the universe.

redshift tells the SIZE when the light was emitted.
Or, the AGE of the universe.

$$1+z=\frac{\lambda(t)}{\lambda_0}=\frac{a(t)}{a_0}$$

$$z=1,\ a(t)=2a_0$$

What holds galaxies, clusters, super- together:
gravity from $\boxed{\text{Dark Matter}}$

~~The~~ Third Law: the law of Periods
The ~~square of the period of~~

$$\left(\frac{T_1}{T_2}\right)^2 \propto \left(\frac{a_1}{a_2}\right)^3$$

semimajor axis

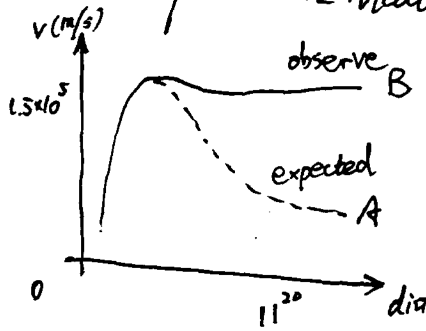
$$F = \frac{GMm}{a^2} \qquad \frac{mv^2}{a} = \frac{GMm}{a^2} \qquad v = \frac{2\pi a}{P}$$

$$F = \frac{mv^2}{a}$$

$$\boxed{P^2 = \frac{4\pi^2}{GM} a^3}$$

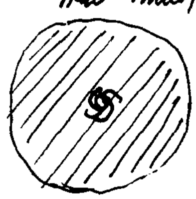
$$\left(\frac{P}{1\,yr}\right)^2 = \left(\frac{a}{1\,AU}\right)^3$$

Existence of Dark Matter : Rotation curves of galaxies



has to be "dark matter"!

- Dark matter haloes
- galaxy rotation curves are flat!
- galaxies are embedded in a halo of dark matter
  halo much larger than the visible (luminous) part



Dark Matter ≠ Dark Energy

- DM is matter
  - concentrated around galaxies & clusters
  - works against the expansion of the Universe
  - responsible for keeping us together
- DE is energy
  - homogeneously distributed in the Universe
  - helps expansion of the Universe
  - responsible for accelerated expansion.

We don't know them

⇓

"dark"

DM: How do we know they're there?
- gravity: what makes stuff move around
- galaxy rotation curves
- evidence from galaxy clusters

Hubble's Classification Scheme

Spiral & Elliptical Galaxies

- Spiral galaxies
  - disc, extended spiral arms
  - ongoing star formation
    - young stellar populations
    - blue colours
  - contain large discs of cold gas

- Elliptical galaxies
  - homogeneous structure, no spiral arms
  - dominated by 'random motions' of stars instead of rotation
  - little or no star formation
    - old stellar population
    - red colour
  - little or no gas (some has!)

Dwarf galaxies

- small, often irregularly shaped galaxies
  - contain at most a few million stars
  - ours has billions of stars.
- Large & Small Magellan clouds visible with naked eye from southern hemisphere.
- They're the most dark matter dominated galaxies in the Universe!

* Einstein Ring only occurs if source, lens and observer are aligned.

[redacted]

Direct evidence for dark matter : $\boxed{\text{Bullet Cluster}}$

- Two clusters 'collided' with each other.
  - galaxies : no direct hits. no prob.
  - hot gas : direct collision
- Expect most of the mass to be in the hot gas
- X-rays reveal the massive hot gas : red
- Lensing model reveals total mass : blue.
- Most of mass concentrated on stars, not on the gas.

Structure Formation
- Hierarchical formation
  - small clumps of dark matter → haloes
  - gas forms stars in these clumps → first galaxies
  - small galaxies merge into bigger ones.

<u>ALTERNATIVES</u>

MOND = MODified NewTonian Dynamics

MOND has problems explaining structure formation and cluster profiles.

e.g. ~~the~~ Bullet Cluster

not interact with EM

→ do with gravitational force

Outdated candidates:

① Massive Compact Halo Objects (MACHOs) numbers too low.

② Neutrinos : masses too low to account for <u>all dark</u> matter,

Popular candidates,

① Weakly Interacting Massive Particle (WIMP)

② SuperWIMP (have not been detected yet)

Summary: * Galaxies dominated by dark matter halo
- use rotation curves of gas disc (or stars)
- halo found for S, E & d.

* G clusters dominated by dark matter

- use velocities of galaxies and/or lensing to determine dark matter content.
- The Universe is dominated by dark matter.

Expansion of the Universe

Hubble's Law: $V = c \cdot z = H_0 \cdot d$

(Farther galaxies are moving away from us faster).

* The Universe has a beginning, a giant 'kick'
* The Universe is expanding.
  Galaxies are in for a ride.

THE cosmological principle:
viewed on a sufficiently large scale, the universe looks the same for all observers.

* Space between us and far away galaxies CAN stretch faster than speed of light. / b/c of the expansion of the universe

$H_0 = 70.8\ km/s/Mpc$

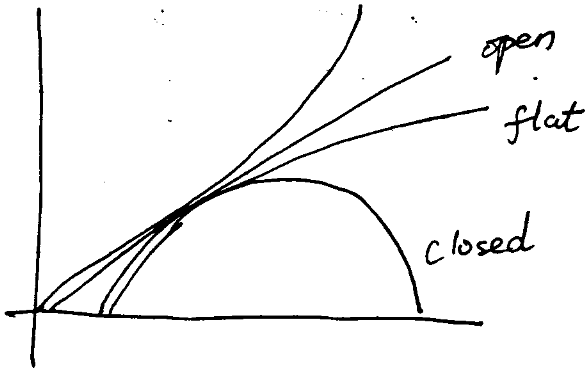
$1/H_0 = \frac{d}{V} = T_0 = 13.9$ billion years

The Hubble constant? Why? b/c same everywhere in the universe, now.

$H_0 = 0$: static $H_0 < 0$: shrinking , $H_0 > 0$: expanding

* determine H_0 using nearby galaxies for which we have independent distance measurements.

age of the universe ranges from $2/3 \frac{1}{H_0}$ (flat) to $\frac{1}{H_0}$ (empty).



Escape velo. of the U.

$E = \frac{1}{2}mv^2 - GMm/r$

v: big kick M: mass of the U

$E > 0$: open, unbound
$E = 0$: marginally bound
$E < 0$, close , bound

$$H_0 = \frac{v}{r} = \left(\frac{dr}{dt}\right)r = \left(\frac{da}{dt}\right)a$$

Friedmann's equation for the evolution of the universe

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

a is the scale factor of the universe
ρ : density
$H = \frac{da}{dt}/a$ is the ~~the~~ Hubble parameter
k is the curvature, +1, −1 or 0.

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

$E=0$, flat, $k=0$
$E>0$, open, $k=-1$
$E<0$, closed, $k=+1$

Critical Density - in a universe dominated by matter (no radiation, d E.) $\exists$ a threshold density at which the universe is flat.

$k=0$ $\qquad H^2 = \frac{8\pi G}{3}\rho + 0$

$$\boxed{\rho_{crit} = \frac{3H^2}{8\pi G}}$$

$$\boxed{\Omega_m = \frac{\rho}{\rho_{crit}} = 1 + \frac{kc^2}{a^2H^2}}$$

$\Omega_m < 1$ gravity loses. open universe. "big freeze".

$\Omega_m = 1$ flat

$\Omega_m > 1$ gravity wins, closed. "big crunch".

Closed universe ($\Omega_m > 1$)
* space & number of galaxies finite
* no edge, nor centre
* at some point, horizon encompasses the whole universe
* light travel geodesics (great circles).
* universe will contact, $H<0$, big cruch to a single blackhole.

Open universe ($\Omega_m < 1$)
* ——— infinite
* no edge, nor centre
* light ~~not~~ never returns to same point
* enlargement of your horizon < expansion of space
* "big freeze"

we have $\Omega_m \approx 0.3$ today

$\Omega_m = 0.30$, age = 11.2 Gyrs
this conflicts with the measured age of the universe

Theory (using Friedmann eq.)
1). space is open
2)_ universe is 11.7 Gyrs

Observation :
1). space is flat
2). 12.7 Gyrs.

* Supernova are "standard candles" : an astronomical object that has a known luminosity.

* expansion of the universe is now accelerating.

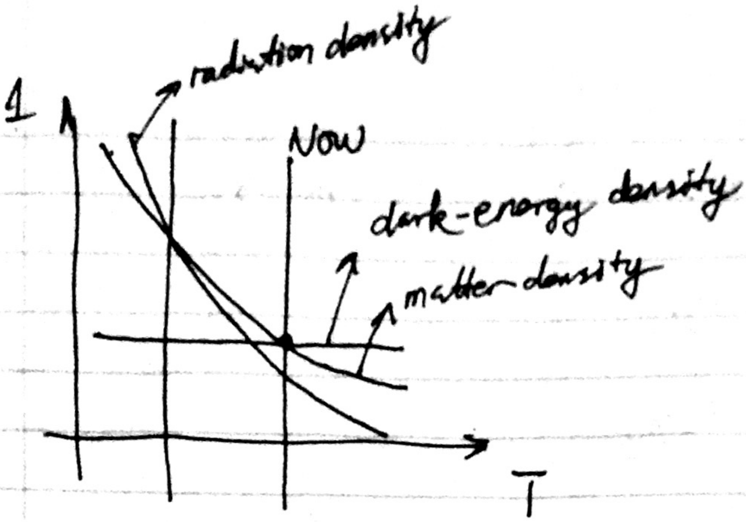
How can the universe expand faster than an empty one?

* Dark Energy (solved the conflict)

* Friedmann's equation for matter + dark energy universe

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \boxed{\frac{\Lambda c^2}{3}}$$

- repulsively (pressure)
- continuously adds energy into the U
- dominates over G at late times
- now we're at the crossroads



Now $\Omega_\Lambda = 0.7$
$\Omega_m = 0.3$

* dark energy = cosmological parameter = Λ = vaccum energy

* dark e $\Rightarrow$ BIG RIP?
every thing, even atoms!

BIG BANG
pillars of B B Cosmology
- a begining — No stars older than 13 Gyrs
  - Expanston of the universe — sth. set it into motion
  - Olber's paradox
    (Why? b/c the universe is young.
    Distant light hasn't even reached us
    yet.)
- hot begining — CMB
  - Observed He & H abundance
  - the fluctuations of matter density after a big bang explains the large scale ~~of~~ structure

* CMB : 3K (2.7K)

CMB Summary:

① 3K

② tiny fluctuations ~1 part in 10^5 cold & hot spots interspersed:

- quantum fluctuations imprinted in large ~~scale~~ scales, leading to formation of galaxies/clusters
- sound waves show up in CMB with most power at $\theta \sim 1$ deg corresponding to size of horizon at cmb.

③ cmb helps establish a "standard cos model"

- the age of the universe to be 13.73±0.12 billion years
- dark m (23%) + dark e. (72%) and inflation
- nailed down the curvature of space to within 1% of "flat" Euclidean.

* photons from the Sun embed information about the "last scattering layer".

* spot size on CMB is a "standard ruler"

 · if flat, angular size of spot = horizon (cmb)/distance

DE.

① acts like pressure, accelerate expansion

② energy density, curve the space

③ if flat at some point, then always flat,

Planck Era → GUT Era → Electroweak Era → Particle Era → Nucleosynthesis → Nuclei → Atoms → Galaxies → Big Bang!

* Two big events: ① baryon asymmetry (in particle era)

② nucleosynthesis (1 neutron for every 7 protons)

76% H, 24% He by mass

~~Summary of the BBR CMB~~

Lambda-CDM model

CDM & HDM

"Bottom-up" or "Hierarchical"

$$E = h\nu = \frac{hc}{\lambda} = mc^2$$

$$\lambda_{MAX} = \frac{0.0029}{T}$$

$$F = \sigma T^4 = \frac{L}{A}$$

$$\boxed{\begin{aligned} z &= \frac{\lambda}{\lambda_0} - 1 = \frac{v}{c} \\ 1+z &= \frac{\lambda}{\lambda_0} = \frac{a}{a_0} \end{aligned}}$$

$$v = c \cdot z = H_0 \cdot d$$

$$\rho_{crit} = \frac{3H^2}{8\pi G}$$

$$\Omega_m = \frac{\rho}{\rho_{crit}} = 1 + \frac{kc^2}{a^2H^2}$$

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$

$$\rho(t) \propto \frac{1}{a(t)^3}$$

$$1+z \propto \frac{1}{a} \propto T$$

$$\left(\frac{T_1}{T_2}\right)^2 \propto \left(\frac{a_1}{a_2}\right)^3$$

AST121 Assignment III Rui Qiu #999292509

#1

(1) Say the radius of Earth is $R = 6400$ km. the distance from ISS to the center of Earth is $r = 6400 + 380 = 6780$ km.

We suppose a certain ball on the surface of Earth, it has $\frac{GMm}{R^2} = mg$, where M is the mass of Earth

so $M = \frac{R^2 g}{G}$

Then for ISS, its acceleration follows:

$$\frac{GM}{r^2} = \omega^2 r$$

Since $M = \frac{R^2 g}{G}$, $T = \frac{2\pi}{\omega}$.

$$T = \frac{2\pi}{\sqrt{\frac{GM}{r^3}}} = \frac{2\pi}{\sqrt{\frac{R^2 g}{r^3}}} = \frac{2\pi}{\sqrt{\frac{(6400 \times 1000m)^2 \cdot 9.8 m/s^2}{(6780 \times 1000m)^3}}} \approx 5536.45s \approx 92.3 \text{ min}$$

Therefore it revolves the Earth every 92.3 min.

(2). Solution: Since $\frac{dr}{dt} = -0.4 \text{ km/day} \left[\frac{380 km}{r - R}\right]^6$

Move dt to one side, and dr to the other:

$$(r-6400)^6 dr = -0.4 \cdot 380^6 dt$$

Integrate two sides: fine

2

$$\int_{6400}^{6780} (r-6400)^6 dr = \int_t^0 -0.4 \cdot 380^6 dt$$

$$\frac{1}{7}(r-6400)^7 \Big|_{6400}^{6780} = -0.4 \times 380^6 t \Big|_t^0$$

$$\frac{1}{7} \cdot 380^7 - 0 = 0 + 0.4 \times 380^6 t$$

$r(t)$?

$$t = \frac{1}{7} \cdot 380^7 \cdot 2.5 \cdot 380^{-6} = 135.71 \text{ days}$$

Therefore it takes 135.71 days to crash onto ground.

(3). Solution: $T = 12h = 4.32\times10^4 s$

$$\omega = \frac{2\pi}{T}$$

$$r^3 = \frac{R^2 g}{\omega^2} = \frac{R^2 g}{\left(\frac{2\pi}{T}\right)^2} = \frac{R^2 g T^2}{4\pi^2}$$

$$H = r - R = \sqrt[3]{\frac{R^2 g T^2}{4\pi^2}} - R$$

$$= \sqrt[3]{\frac{(6.4\times10^6)^2\times 9.8\times(4.32\times10^4)^2}{4\pi^2}} - 6.4\times10^6$$

$$\doteq 20272553.41\ m$$

$$\doteq \cancel{2027\ km}\ 20272.553\ km \qquad 20272.553\ km$$

Therefore the orbital height is ~~2027km~~ from Earth surface.

~~Then for~~ $\cancel{\frac{dr}{dt} = -0.4\ km/day\left[\frac{330\ 2027\ km}{r-6400km}\right]^6}$

$$\cancel{\int_{6400}^{8427}(r-6400)^6\,dr = \int_t^0 -0.4\times2027^6\,dt}$$

$$\cancel{\frac{1}{7}(r-6400)^7\Big|_{6400}^{8427} = -0.4\times20273^6 t}\Big|_t^0$$

$$\cancel{\frac{1}{7}\times20273^7 = 0.4\times20273^6 t}$$

$$\cancel{t = 2027\times\frac{1}{7}\times\frac{5}{2}}$$

$$\cancel{\doteq 723.93\ days}$$

~~Therefore they can stay up in the sky for 723.93 days.~~

use 350

Then for $\frac{dr}{dt} = -0.4\ km/day\left[\frac{20272.553km}{r-6400km}\right]^6$

$$\int_{6400}^{26673}(r-6400)^6\,dr = \int_t^0 -0.4\times20273^6\,dt$$

$$\frac{1}{7}(r-6400)^7\Big|_{6400}^{26673} = -0.4\times20273^6\Big|_t^0$$

$$\frac{1}{7}\times20273^7 = 0.4\times20273^6 t$$

$$t = 20273\times\frac{1}{7}\times\frac{5}{2}$$

$$\doteq 7240.4\ days$$

Therefore they can stay up in the sky for 7240.4 days.

#2. Rui Qiu #999292509

(1). $[G] = \frac{[F]}{[M]^2}[L]^2 = \frac{[M][A]}{[M]^2}[L]^2 = \frac{[V]}{[T]} \cdot [M]^{-1}[L]^2 = \frac{[L]}{[T]^2}[M]^{-1}[L]^2$

$= [L]^3[M]^{-1}[T]^{-2}$

$[h] = [E]\cdot[T] = [M][V]^2[T] = [M][L]^2[T]^{-2}[T]$

$= [M][L]^2[T]^{-1}$

$[c] = [L][T]^{-1}$

(2). $[T] = [G]^\alpha [h]^\beta [c]^\gamma = \left([L]^3[M]^{-1}[T]^{-2}\right)^\alpha \cdot \left([M][L]^2[T]^{-1}\right)^\beta \cdot \left([L][T]^{-1}\right)^\gamma$

$= [L]^{3\alpha+2\beta+\gamma} \cdot [M]^{-\alpha+\beta} \cdot [T]^{-2\alpha-\beta-\gamma}$

So $\begin{cases} 3\alpha+2\beta+\gamma = 0 \\ -\alpha+\beta = 0 \\ \cancel{-\alpha+\beta}\ -2\alpha-\beta-\gamma = 1 \end{cases}$

$\Rightarrow \alpha = \frac{1}{2},\ \beta = \frac{1}{2},\ \gamma = -\frac{5}{2}$ how?

Therefore $[T] = [G]^{\frac{1}{2}}[h]^{\frac{1}{2}}[c]^{-\frac{5}{2}}$.

(3). $t_{Planck} = (6.673\times10^{-11} m^3 kg^{-1} s^{-2})^{\frac{1}{2}} \cdot (6.625\times10^{-34} J\cdot s)^{\frac{1}{2}} \cdot (2.998\times10^{8} ms^{-1})^{-\frac{5}{2}}$

$\approx 1.351\times10^{-43}\ (m^{\frac{3}{2}} kg^{-\frac{1}{2}} s^{-1} \cdot J^{\frac{1}{2}} \cdot s^{\frac{1}{2}} \cdot m^{-\frac{5}{2}} \cdot s^{\frac{5}{2}})$

$= 1.351\times10^{-43}\ s$

(4). $[L] = [G]^\alpha[h]^\beta[c]^\gamma = [L]^{3\alpha+2\beta+\gamma} \cdot [M]^{-\alpha+\beta} \cdot [T]^{-2\alpha-\beta-\gamma}$

$\begin{cases} 3\alpha+2\beta+\gamma = 1 \\ -\alpha+\beta = 0 \\ -2\alpha-\beta-\gamma = 0 \end{cases}$

$\Rightarrow \alpha = \frac{1}{2},\ \beta = \frac{1}{2},\ \gamma = -\frac{3}{2}$

Therefore $[L] = [G]^{\frac{1}{2}}[h]^{\frac{1}{2}}[c]^{-\frac{3}{2}}$

$l_{Planck} = (6.673\times10^{-11} m^3 kg^{-1} s^{-2})^{\frac{1}{2}} \cdot (6.625\times10^{-34} J\cdot s)^{\frac{1}{2}} \cdot (2.998\times10^{8} ms^{-1})^{-\frac{3}{2}}$

$\doteq 4.050\times10^{-35}\ m$

(5). $[M]=[G]^{\alpha}[h]^{\beta}[c]^{\gamma}$

$=[L]^{3\alpha+2\beta+\gamma}\cdot[M]^{-\alpha+\beta}\cdot[T]^{-2\alpha-\beta-\gamma}$

show your work!

$$\begin{cases} 3\alpha+2\beta+\gamma=0 \\ -\alpha+\beta=1 \\ -2\alpha-\beta-\gamma=0 \end{cases}$$

$\Rightarrow \alpha=-\frac{1}{2},\ \beta=\frac{1}{2},\ \gamma=\frac{1}{2}$

Therefore. $m_{planck} = (6.673\times10^{-11}\,m^{3}\,kg^{-1}\,s^{-2})^{-\frac{1}{2}}\cdot(6.625\times10^{-34}\,J\cdot s)^{\frac{1}{2}}\cdot(2.998\times10^{8}\,ms^{-1})^{\frac{1}{2}}$

$\doteq 5.456\times10^{-8}\,kg.$

Q1: 4

Q2: 4

Total: 8

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#1. Solution: The kinetic energy is balanced by gravitational energy:

$$\frac{1}{2}mv^2 = mgr$$

where ~~the gravitational constant~~ $g = \frac{GM}{r^2}$ 2

and according to Hubble's Law $v = H_0 r$

Then

$$\frac{1}{2}mH_0^2 r^2 = \frac{GMm}{r^2} \cdot r$$

$$\frac{1}{2}H_0^2 r^3 = GM$$

$$M = \frac{H_0^2 r^3}{2G}$$

This is the total mass of the universe

Since the volumn of the universe is $V = \frac{4}{3}\pi r^3$

then the density

$$\rho_{m_0} = \frac{M}{V} = \frac{\frac{H_0^2 r^3}{2G}}{\frac{4}{3}\pi r^3} = \frac{3H_0^2}{8\pi G}$$ as desired.

Take $H_0 = 70$ km/s/Mpc

$1\,pc = 3.0857 \times 10^{16}$ m

$1\,Mpc = 3.0857 \times 10^{22}$ m

$G = 6.673 \times 10^{-11} m^3 kg^{-1} s^{-2}$

$$\rho_{m_0} = \frac{3H_0^2}{8\pi G} = \frac{3\times(70\times 1000\, m/s)^2}{8\pi \times 6.673\times 10^{-11} m^3 kg^{-1} s^{-2} \cdot (1\,Mpc)^2} = \frac{1.47\times 10^{10}\, m^2 s^{-2}}{1.597\times 10^{36}\, m^5 kg^{-1} s^{-2}}$$

$$= 9.205 \times 10^{-27}\, kg/m^3$$

In units of number of hydrogen atoms per cubic meter:

$$n_H = \frac{\rho_{m_0}}{m_H} = \frac{9.205\times 10^{-27}\, kg/m^3}{1.673\times 10^{-27}\, kg} \doteq 5.50 \text{ atoms}/m^3.$$

2

#2. Solution:

Evolution of a flat (curvature-free) universe follows

$$\rho_m(t) = \frac{3H(t)^2}{8\pi G}$$

According to the fact that total mass is conserved,

$$\rho_m(t) = \rho_{m0}\frac{a_0^3}{a(t)^3}$$

Then $$\frac{3H(t)^2}{8\pi G} = \rho_{m0}\frac{a_0^3}{a(t)^3}$$

Since $$H(t) = \frac{\dot{r}}{r(t)} = \frac{\dot{a}}{a(t)} = \frac{\frac{da}{dt}}{a(t)} = \frac{\frac{dr}{dt}}{r(t)}$$

Then $$\frac{\cancel{3H}\ 3H(t)^2}{8\pi G} = \rho_{m_0}\frac{a_0^3}{a(t)^3}$$

$$\Rightarrow \frac{\left(\frac{da}{dt}\right)^2}{a(t)^2} = \frac{\rho_{m_0} a_0^3 \cdot 8\pi G}{3a(t)^3}$$

$$\frac{da}{dt} = \sqrt{\frac{8\pi\rho_{m_0} G a_0^3}{3a(t)}} = a(t)^{-\frac{1}{2}}\sqrt{\frac{8\pi\rho_{m_0} G a_0^3}{3}}$$

more dt to RHS and $a(t)^{-\frac{1}{2}}$ to LHS.

$$\cancel{a(t)^{\frac{1}{2}}\, da} =\ a^{\frac{1}{2}}\, da = \sqrt{\frac{8\pi}{3}\rho_{m_0} G a_0^3}\, dt$$

Integrate both sides:

$$\int_{a(t=0)}^{a(t)} a^{\frac{1}{2}}\, da = \int_{t=0}^{t}\sqrt{\frac{8\pi}{3}\rho_{m_0} G a_0^3}\, dt$$

$$\cancel{\frac{2}{3}a(0)^{\frac{3}{2}} \quad \frac{2}{3}}$$

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$$\frac{2}{3}a^{\frac{3}{2}}(t) - \frac{2}{3}a^{\frac{3}{2}}(0) = \sqrt{\frac{8\pi}{3}\rho_{m_0} G a_0^3} \cdot t - 0$$

Take $a(0) = 0$ when $t = 0$.

So $\frac{2}{3}a^{\frac{3}{2}}(t) = \sqrt{\frac{8\pi}{3}\rho_{m_0} G a_0^3} \cdot t$

$$a^{\frac{3}{2}}(t) = \frac{3}{2}\sqrt{\frac{8\pi}{3}\rho_{m_0} G a_0^3} \cdot t$$

$$a(t) = \left(\frac{3}{2}\sqrt{\frac{8\pi}{3}\rho_{m_0} G a_0^3}\ t\right)^{\frac{2}{3}}$$

Let $\left(\frac{3}{2}\sqrt{\frac{8\pi}{3}\rho_{m_0} G a_0^3}\right)^{\frac{2}{3}} = C$ which is a constant

$$a(t) = C \cdot t^{\frac{2}{3}}$$

Then by the definition of $H(t)$,

$$H(t) = \frac{\frac{da}{dt}}{a} = \frac{\frac{2}{3}C t^{-\frac{1}{3}}}{C \cdot t^{\frac{2}{3}}} = \frac{2}{3}t^{-1}.$$

Therefore $t = \frac{2}{3H(t)}$

Take $H_0 = 70$ km/s/Mpc

$$t = \frac{2}{3 \times 70 \text{km/s/Mpc}} = \frac{2 \times 3.0857 \times 10^{22} \text{m}}{3 \times 70 \times 10^3 \text{m/s}} \doteq 2.939 \times 10^{17} \text{s}$$

$\doteq 9.319 \times 10^9$ yrs

$= 9.319$ Gyrs.

\#3. Solution: In Question \#2. we know that $a(t) = C \cdot t^{\frac{2}{3}}$ by calculation

Therefore

$$\frac{a_0}{a_t} = \frac{C t_0^{\frac{2}{3}}}{C t^{\frac{2}{3}}} = \left(\frac{t_0}{t}\right)^{\frac{2}{3}} = 1 + z$$

$$\frac{t_0}{t} = (1+z)^{\frac{3}{2}} = 11^{\frac{3}{2}}$$

Then $t = \frac{t_0}{11^{\frac{3}{2}}} = \frac{9.319 \text{Gyrs}}{11^{\frac{3}{2}}} \doteq 0.255$ Gyrs

2

#4. Solution: We have these equations:

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

$$E = -\frac{kmc^2 r_0^2}{2a_0^2}$$

$$v = Hr$$

So $\frac{1}{2}mv^2 = \frac{GMm}{r} - \frac{kmc^2r_0^2}{2a_0^2}$ ✓

$$\frac{1}{2}H^2r^2 = \frac{G}{r}\rho_m \frac{4}{3}\pi r^3 - \frac{kc^2r_0^2}{2a_0^2}$$

$$= \frac{4\pi G}{3}r^2\rho_m - \frac{1}{2}kc^2\left(\frac{r_0^2}{a_0^2}\right)$$

1.5

Since $\frac{r}{r_0} = \frac{a(t)}{a_0}$ ✓

Then $\frac{r_0^2}{a_0^2 \cdot r^2} = \frac{1}{a(t)^2}$

So $H^2 = \frac{8\pi G}{3}\rho_m - k^2c^2\left(\frac{r_0^2}{a_0^2 r^2}\right)$

$= \frac{8\pi G}{3}\rho_m - k\frac{c^2}{a^2}$ as desired.

In an empty universe, $k=-1$, $\rho_m=0$.

$H^2 = -k\frac{c^2}{a^2} = \frac{c^2}{a^2}$ show this!

As $a=ct$, $H=\sqrt{\frac{c^2}{a^2}} = \frac{c}{a} = \frac{c}{ct} = t^{-1}$

Thus $t = H(t)^{-1}$.

Using $H_0 = 70$ km/s/Mpc, ✓

$$t = \frac{1}{70\times10^3 \text{ m/s}/3.0857\times10^{22}\text{ m}} \doteq 4.408\times10^{17}\text{ s} \doteq 1.398\times10^{10}\text{ yrs}$$

$= 13.98$ Gyrs

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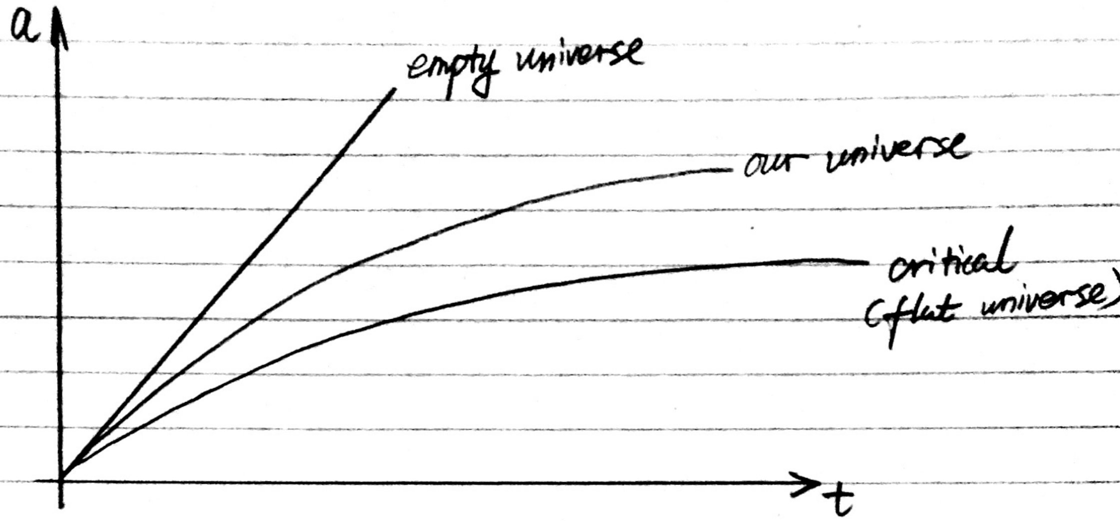
#5. Solution:



Thus our universe is open, which means it will forever expand, but with slowing down expanding rate.

Q	1	2	3	4	5
	2	2	2	1.5	1

8.5

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Assignment IX

Q1.

Answer: From staring at the Cosmetc Microwave Background, qualitatively speaking, we can learn that ① the universe consists of different components: energy, matters, void, etc. (shown as cold and hot spots in CMB); ② the universe is quite flat.

Q2.

(a).

$$dF = \frac{L}{4\pi r^2} dN = \frac{L}{4\pi r^2} \cdot n 4\pi r^2 dr = L \cdot n dr$$

$$\int_b^a dF = \int_0^\infty L \cdot n dr = L \cdot n r\Big|_{r=\infty} - L \cdot n r\Big|_{r=0}$$

$$= L \cdot n r\Big|_{r=\infty}$$

so the total flux (dF) diverses.

(b). The solid angle: $\Omega = \frac{\pi R^2}{r^2}$

By definition, $\Omega = \frac{S}{r^2}$

So $S = \pi R^2$ (a tiny curve surface part of the huge sphere)

Number of stars $dN = \frac{S_r}{S} = \frac{4\pi r^2}{\pi R^2} = \frac{4r^2}{R^2}$

$$dF = \frac{L}{4\pi r^2} \cdot dN = \frac{L}{4\pi r^2} \cdot \frac{4r^2}{R^2} = \frac{L}{\pi R^2}$$

Hence the total energy received is independent of r.

(c). $1° = \frac{\pi}{180}$ rad

$1 \text{ square deg} = 1° \cdot 1° = \frac{\pi^2}{180^2}$ sr

Say $D = 1$ AU , $F = \frac{L}{4\pi D^2} n$

$$n = \frac{4\pi \text{ sr}}{(0.2dg)^2} = \frac{4\pi \text{ sr}}{0.2 \times \frac{\pi^2}{180^2} \text{ sr}} \doteq 2.063 \times 10^5$$

So $F = \frac{L}{4\pi D^2} \cdot 2.063 \times 10^5$

Hence the Earth will receive 2.063×10^5 times higher flux of radiation.

(d). $F = \frac{L}{A} = \sigma T^4$ with $T = 270K$.

$$F' = 2.063 \times 10^5 \times F$$
$$= 2.063 \times 10^5 \sigma T^4$$
$$= \sigma (T')^4$$

$$T' = \sqrt[4]{2.063 \times 10^5} \cdot T$$
$$\doteq 21.312 \times 270K$$
$$\doteq 5754 K.$$

The Earth would radiate in 5754K temperature.

e). ?

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Q3.

(a). size of hotspot $\Rightarrow a = c \cdot \times 300000$ yrs $= 3\times10^5$ ly

since the universe is expanding with $\Omega = \Omega_m = 1$.

$a \propto t^{2/3}$

1.5

$\Rightarrow a = c' t^{2/3}$

$a_0 = c' t_0^{2/3} \Rightarrow c' = \dfrac{a_0}{t_0^{2/3}}$

So $a = \dfrac{a_0}{t_0^{2/3}} \cdot t^{2/3}$

then $a_0 = \cancel{\left(\frac{t}{t_0}\right)^{2/3}} \cdot a$

Careful! We are not solving for a!

$= \left(\dfrac{13.7\times10^9}{3\times10^5}\right)^{2/3} \times 3\times10^5$ ly

use $\dfrac{r}{r_0} = \dfrac{a}{a_0}$

$\approx 3.83\times10^8$ ly

The size of hot spot is now 3.83×10^8 ly.

(b). Note that

2

today's critical density $\rho_{cr} = 10^{-29}\ g/cm^3$

matter density today $0.3\rho_{cr} = 3\times10^{-30}\ g/cm^3$

volumn increase ratio:

$$\left(\frac{a_0}{a}\right)^3 = \left(\frac{t_0}{t}\right)^2 = \left(\frac{13.7\times10^9}{3\times10^5}\right)^2 \simeq 2.09\times10^9$$

<u>Since the hotspot does not expand so $\rho_{hotspot}$ is constant.</u>

$\rho_{0m} = 0.3\rho_{cr} = 3\times10^{-30}\ g/cm^3 = \dfrac{m}{V_0}$

$m = 3\times10^{-30} V_0 = 2.09\times10^9 \times 3\times10^{-30} \cdot V = 6.27\times10^{-21} V$

$\rho = \dfrac{m}{V} = 6.27\times10^{-21}\ g/cm^3 > 3\times10^{-30}\ g/cm^3$

Hence we conclude that the hotspot density today is much higher than the average density.

(To be specific it's $\frac{6.27\times10^{-21}}{3\times10^{-30}} = 2.09\times10^{9}$ times higher)

(c). $V = \frac{m}{\rho_{m_0}} = \frac{10^{42} kg}{3\times10^{-30} g/cm^3} = 3.33\times10^{74} cm^3$

1

① To simplify, we treat it as a cube.

so $a = \sqrt[3]{V} = \sqrt[3]{3.33\times10^{74} cm^3} \doteq 7.33\times10^{6}\ ly$

since distance from Milky Way to Andromeda is 2.5×10^{6} ly.

Hence it is smaller than the radius of "Milky Way".

② Or say one galaxy is a sphere

so $r = \sqrt[3]{\frac{3}{4}\cdot\frac{1}{\pi}V} \doteq 4.30\times10^{24}$ cm
$= 4.30\times10^{22}$ m
$\doteq 4.54\times10^{6}$ ly

then the distance $d = 2r \approx 9.08$ ly

Q1	Q2	Q3
1	3.5	4.5