Lecture 2

2 Questions in Quiz, All from HW

Quiz 1 Jan 22nd, 2015 6:10–6:35 pm
Quiz 2 March 19th, 2015 6:10–6:35 pm
Midterm Feb 5th, 2015 6:10–7:10 pm SS1085/7

} calculators

1-sided formula sheet

Monday, Feb 2nd, 2015. 2–3pm @ Fields Institute Stewart Library Review Seminar
answer questions @ 3pm, 2nd floor

P85, #46
Assume Independence of r.v.'s X & Y.

P165, #16
classical Stuff on the bivariate normal d'n. (needed in chapter 10)

P169 #37 (A particular Poisson mixture)
Intuition versus theorem

for	Poisson (mean)	
A	λ_1	2.6
B	λ_2	3
C	λ_3	3.4

r.v. is # of errors
$E(X)$, $Var(X)$

equally likely to make mistakes

Intuition: $E(X)=\frac{2.6+3+3.4}{3}=3$ (correct but not proper)

Thm: r.v. $Y=\begin{cases} A(1) & 1/3 \\ B(2) & 1/3 \\ C(3) & 1/3 \end{cases}$

Law of Y.

$E(X)=E(E(X|Y))$

$=\sum_{y=1}^{3} \underbrace{P(Y=y)}_{1/3} \underbrace{E(X|Y=y)}_{E\,Poiss(Y=y)} = \frac{1}{3}(2.6+3+3.4)=3$

r.v. $U=X^2$, $EU=E(E(U|Y)) = \frac{1}{3}\{(\lambda_1+\lambda_1^2)+(\lambda_2+\lambda_2^2)+(\lambda_3+\lambda_3^2)\}=12.1067$

$E(U|Y=y)=E(Poiss^2(\lambda_y))$

$Var(X)=12.1067-9=3.1067$

$E(X^2)=Var+E^2=\lambda_y+\lambda_y^2$

P171, #44
double expectation thm
$EY=E(E(Y|N))$ ↪ # of customers
↓
total amount earned on a given day.

Wald identity
$EY=EN\cdot EX_i=10\cdot 50=500$

for a compound Poisson r.v. $S=\sum_{i=1}^{N} X_i$

$N\sim Poiss(\lambda)$

$Var\,S=\lambda E(X_i^2)$

$E=\frac{a+b}{2}$, $Var=\frac{(b-a)^2}{12}=\frac{100^2}{12}$

X_i's iid r.v.'s with common $U(0,100)$ d'n.
N does not depend on X_i's

$\mathrm{Var}\,S = \underset{\lambda}{10}(50^2 + \frac{100^2}{12}) = 33{,}333$

#53 on P172

$X \sim Poiss(\lambda)$ r.v. $\lambda \sim Exp(1)$

$P(X=n) = (\frac{1}{2})^{n+1},\ n \geq 0$

marginals : negative binomial

Yule stochastic process (pure birth process)

Ex 63 p360

Ex 68 p367–368

total probability formula

$$P\{X=n\} = \int_0^\infty P\{X=n \mid \lambda\} \underset{\text{pdf of } Exp(1)}{e^{-\lambda}} d\lambda = \int_0^\infty e^{-\lambda}\frac{\lambda^n}{n!} e^{-\lambda} d\lambda$$

$$= \frac{1}{n!}\int_0^\infty \lambda^n e^{-2\lambda} d\lambda$$ change of variables $t = 2\lambda$

$$\lambda^n d\lambda = (\tfrac{1}{2})^{n+1} t^n dt$$

$$= \frac{1}{n!}(\tfrac{1}{2})^{n+1}\int_0^\infty t^{n+1-1} e^{-t} dt$$

$\Gamma(n+1)$ $\qquad \Gamma(\alpha) \overset{def}{=} \int_0^\infty x^{\alpha-1} e^{-x} dx$

$$P(X=n) = \frac{\Gamma(n+1)}{n!}(\tfrac{1}{2})^{n+1}$$

(C.R.) Rao damage process

consider compound Poisson r.v.

$$U = \sum_{i=1}^N B_i$$

N indpt of $\{B_n, n \geq 1\}$ – iid Bernoulli r.v.'s $\quad B_n = \begin{cases} 1 & ; p \\ 0 & ; q = 1-p \end{cases}$

$EU = p\lambda$

$U \sim Poiss(p\lambda)$

formula for mgf of a compound d'n

$m_U(t) = m_N(\ln m_B(t))$ Ch2.

$m_N(v) = e^{\lambda(e^v - 1)}$ $\qquad m_B(t) = 1 - p + pe^t = 1 + p(e^t - 1)$

$m_U(t) = e^{\lambda(\exp\{\ln(1+p(e^t-1))\}-1)} = \exp\{\lambda(\cancel{1} + p(e^t-1) - \cancel{1})\} = e^{p\lambda(e^t-1)} \Rightarrow$ method of mgf's

r.v. $U \sim Poiss(p\lambda)$

two parameter class of non-negative integer valued r.v.

$Y_{\gamma,r}$ $\quad \gamma \in (0,1)$

$\max(-1, -\frac{1-\gamma}{\gamma}) < r < 1$

Def given (γ, r). $P(Y_{\gamma,r}=0)=\gamma$

$k \geq 1$

$P\{Y_{\gamma,r}=k\}=\gamma(1-\gamma)(1-r)(1-\gamma+\gamma r)^{k-1} = \gamma(1-\gamma)^k$ special case $(\gamma=r)=0$

$D_p = B(\underset{n}{1}, p)$

$\sum_{i=1}^{D_p} Y_{\gamma,0}$

geometric variable

HW: read CH2–3

ex of ch3: 67, 73, 77, 92, 96, 97, 98, 99