August 8th

REVIEW SESSION

Chapter 1–2 : 35
3 : 15
§4.1–4.2 : 17
§4.3–5.8 : 53

8 Questions in total

Chapter 3

§3.1 The IFT

Eg. $x^2+y^2+z^2=1$ in $\mathbb{R}^3$ near a point

Find $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ conditions: when we can find the derivative

§3.2 3 presentations of curves ($\mathbb{R}^2$)

C^1 (iii) $\vec{f}(t): \mathbb{R} \to \mathbb{R}^2$, $\vec{f}(t) = (\phi(t), \psi(t))$

§3.3 3 presentations of curves ($\mathbb{R}^3$)

(iii) $\vec{f}(t): \mathbb{R} \to \mathbb{R}^3$, $\vec{f}(t) = (\phi(t), \psi(t), \varphi(t))$

3 presentations of surfaces

know how to convert one to another & what is the condition (IFT)

§3.4 transformation & coordinates

The purpose of this section is building tools for the change of variable formulas $\underbrace{\text{in the integration}}_{\S 4.4}$

Eg 2012 August question 5

(a) give the three representations of the smooth curves in R^3 in the order they appeared in the text book

(b) choose one of the representation (II) or(iii) and state a regularity condition that guarantees this representation can be translated to representation (I) (IFT)

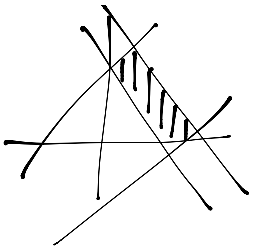
2012 August question 6

(a) Present the change of variable formula for double integrals for a linear transformation $\vec{G}: \mathbb{R}^2 \to \mathbb{R}^2$ with $(x, y) = \vec{G}(u, v)$ a measurable region S and integrable function $f(x,y)$ on S

Thm 4.37

$$\iint_S f(\vec{x})\,dx\,dy = |\det A| \iint_{G^{-1}(S)} f(A\vec{u})\,du\,dv \quad , \vec{G}(u) = A\vec{u}$$

b) Apply your formula from part (a) to integrate $\iint_S (2x+y)^2 e^{x-y}\,dA$ where the region S is the region bdd by $\underbrace{2x+y}_{u}=1$, $2x+y=4$, $\overbrace{x-y}^{v}=-1$ & $x-y=1$



$u = 2x + y$
$v = x - y$

$$\frac{\partial(u,v)}{\partial(x,y)} = \det\begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix} = -2 - 1 = -3$$

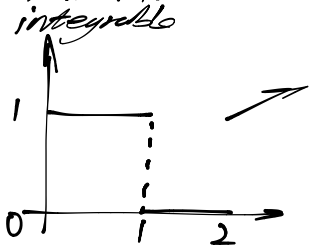
$$\iint_S (2x+y)^2 e^{x-y} d\vec{x} = \int_1^4 \int_{-1}^1 u^2 e^v \, dv \, du = \cdots$$

Chapter 4

§ 4.1 Integration on the line

- Def the Riemann integration → partitions → refinements → lower & upper sums → Riemann Sums
- Riemann integrability
- lemma 4.5 ε-characterization of integrability

$\Rightarrow$ thm 4.10 bdd & monotone on $[a,b] \Rightarrow$ integrable
$\Rightarrow$ Thm 4.11 continuous on $[a,b] \Rightarrow$ integrable
$\Rightarrow$ Thm 4.12 Continuous at all except finitely many points on $[a,b] \Rightarrow$ integrable



construct a partition to show f is integrable.

- The FTC a). $F(x) = \int_a^x f(t)\,dt \Rightarrow F'(x) = f(t)$

b). $F'(x) = f \Rightarrow \int_a^b f(t)\,dt = F(b) - F(a)$

§ 4.2 Higher Dim

Def the Riemann integration

- characteristic func
- The mean value thm of integrals
- f, g continuous, $g \geq 0 \Rightarrow \iint_S f(\vec{x}) g(\vec{x})\, d\vec{x} = f(\vec{a}) \iint_S g(\vec{x})\, d\vec{x}$ for some $\vec{a} \in S$

§ 4.3 Compute the Higher Dim integration

Fubini → double integration → iterated integration
→ integrate y first or x first

Condition: when they're equal

Draw the graph before compute the integral

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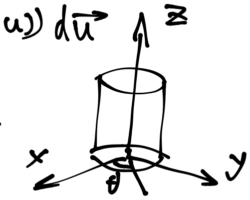
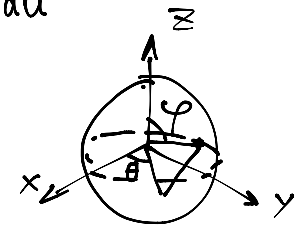
真迹

§4.4 change of the variable

$$\int \cdots \int_S f(\vec{x})\, d^n\vec{x} = \int \cdots \int_T f(\vec{G}(\vec{u}))\,|\det D\vec{G}(\vec{u})|\, d\vec{u}$$

when $\vec{G} = A\vec{u}$ $= |\det A| \int \cdots \int_T f(\vec{G}(u))\, d\vec{u}$





- cylinderical coordinates coordinates

$G_{cyl}(r, \theta, z) = (r\cos\theta, r\sin\theta, z)$

$\det D\vec{G} = r$

- Spherical coordinates $\quad G_{sph}(r\sin\varphi\cos\theta, r\sin\varphi\sin\theta, r\cos\varphi)$

$\det D\vec{G} = r^2 \sin\varphi$

§4.5

Thm 4.46 $\quad \lim_{x \to x_0} \iint_S f(\vec{x}, \vec{y})\, d\vec{y} = \iint_S \lim_{x \to x_0} f(\vec{x}, \vec{y})\, d\vec{y}$

Thm 4.47 $\quad \frac{\partial}{\partial x_j} \iint_S f(\vec{x}, \vec{y})\, d\vec{y} = \iint_S \frac{\partial}{\partial x_j} f(\vec{x}, \vec{y})\, dy$

$$F(x) = \int_{\varphi(x)}^{\psi(x)} f(x, y)\, dy$$

Find $F'(x)$ recall FTC $F(x) = \int_a^x f(t)\, dt \Rightarrow F'(x) = f(x)$

let $u = \varphi(x)$, $v = \psi(x)$

$F(x) = \int_v^u f(x, y)\, dy = H(u, v, x)$

$$F'(x) = \frac{\partial H}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial H}{\partial v}\frac{\partial v}{\partial x} + \frac{\partial H}{\partial x}$$

$\frac{\partial H}{\partial u} = \frac{\partial}{\partial u}\left(-\int_v^u f(x, y)\, dy\right)$, $\quad \frac{\partial H}{\partial v} = \frac{\partial}{\partial v}\left(\int_v^u f(x, y)\, dy\right) = f(x, v)$

$\frac{\partial H}{\partial x} = \frac{\partial}{\partial x}\left(\int_v^u f(x, y)\, dy\right) = \int_u^v \frac{\partial}{\partial x} f(x, y)\, dy$

$$F'(x) = -f(x, \varphi(x))\varphi'(x) + f(x, \psi(x))\psi'(x) + \int_{\varphi(x)}^{\psi(x)} \frac{\partial f}{\partial x}(x, y)\, dy$$

Integration : change variable

$$\int_0^1 \frac{ax(a^2 - x^2)}{(x^2 + a^2)^3}\, dx, \quad x^2 + a^2 = u$$

$du = 2x\,dx$ $\quad = \int_{a^2}^{a^2+1} \frac{a(2a^2 - u)}{u^3} \cdot \frac{du}{2} = \int_{a^2}^{a^2+1} \frac{a^3}{u^3} - \frac{a}{2u^2})\, du = -\frac{a^3}{2u^2}\Big|_{a^2}^{a^2+1}$

when $x = 0$, $u = a^2$

$x = 1$, $u = 1 + a^2$

$$+ \frac{a}{2u}\Big|_{a^2}^{a^2+1}$$

$$= -\frac{a^3}{2(a^2+1)^2} + \frac{1}{2a} + \frac{a}{2(a^2+1)}$$

$$-\frac{a}{2a}$$

$$= \frac{-a^3+a(a^2+1)}{2(a^2+1)^2} = \frac{a}{2(a^2+1)^2}$$

- Integration by part

$$\int u(x)v'(x)\,dx = u(x)v(x) - \int u'(x)v(x)\,dx$$

Eg. Last problem in test 2

Chapter 5

§5.1 Find length of a line $d\vec{x}$, ds

- Line integration $\int_C F\,ds = \int_C \vec{F}\cdot d\vec{x}$

§5.2 Green's Thm $\int_{\partial S} \vec{F}\cdot ds = \iint_S \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) dA$

$$\int_{\partial S} \vec{F}\cdot\vec{n}\,dS = \iint_S \left(\frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y}\right) dA$$

§5.3 Surface integration

- Find the area of a surface

$$dA = \left|\frac{\partial\vec{G}}{\partial u} \times \frac{\partial\vec{G}}{\partial v}\right| du\,dv$$

Find the area of the surface of sphere in $\mathbb{R}^3$

- area ellipse in $\mathbb{R}^2$
- If $\vec{G} = (x, y, \varphi(x,y))$, $dA = \sqrt{1+(\partial_x\varphi)^2+(\partial_y\varphi)^2}$

Surface integral $\iint_S f\,dA = \iint_W f(\vec{G}(u,v))\left|\frac{\partial\vec{G}}{\partial u} \times \frac{\partial\vec{G}}{\partial v}\right| du\,dv$

$$\iint_S \vec{F}\cdot\vec{n}\,dA = \iint_W \vec{F}(\vec{G}(u,v))\cdot\frac{\partial\vec{G}}{\partial u} \times \frac{\partial\vec{G}}{\partial v}\,du\,dv$$

§5.4 Vector Derivatives

$\nabla = (\partial_1, \cdots, \partial_n)$

$\operatorname{div}\vec{F} = \nabla\cdot\vec{F}$

$\operatorname{curl}\vec{F} = \nabla\times\vec{F}$

§5.5 Divergence Thm $\iint_{\partial R} \vec{F}\cdot\vec{n}\,dA = \iint_R \operatorname{div}\vec{F}\,dV$

$\Rightarrow$ Green's Formulas

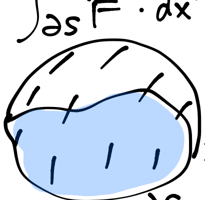
$$\iint_{\partial R} f\nabla g\cdot\vec{n}\,dA = \iint_R \nabla f\cdot\nabla g + f\nabla^2 g\,dV$$

$$\iint_{\partial R} f\nabla g - g\nabla f\cdot\vec{n}\,dA = \iint_R f\nabla^2 g - g\nabla^2 f\,dV$$

§5.7 . Stoke's Thm

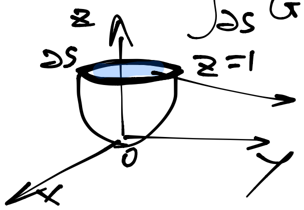
$$\int_{\partial S} \vec{F}\cdot d\vec{x} = \iint_S (\text{curl } \vec{F})\, \vec{n}\, dA$$

2010 Apr. Q4(b)



Let surface S be the portion of z=x^2+y^2 inside the. Cylinder x^2+y^2=1 with the downward orientation.

Calculate $\int_{\partial S} \vec{G}(\vec{x})\, d\vec{x}$ where $\vec{G}(x,y,z) = (\cos z,\ zx + \log(y^2+1),\ yz^2)$



Use the disk $D = \{(x,y,z),\ x^2+y^2 \le 1,\ z=1\}$

$\partial S = \partial D$

$\vec{n} = (0,0,-1)$

$$\text{curl } \vec{G} = \det\begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial x & \partial y & \partial z \\ \cos z & zx+\log(y^2+1) & yz^2 \end{pmatrix}$$

$$= (z^2 - x)\vec{i} + (-\sin z)\vec{j} - z\vec{k}$$

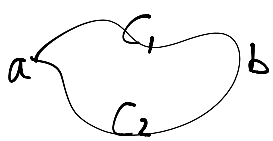
$$\int_{\partial S} \vec{G}(\vec{x})\cdot d\vec{x} \overset{\text{Stoke's}}{=\!=\!=} \iint_D \text{curl } \vec{F} \cdot \vec{n}\, dA = \iint_D z\, dA = \pi$$

§5.8

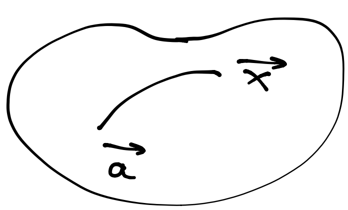
$\nabla f = \vec{G} \longrightarrow$ conservative : Def:(a) $\int_{C_1} \vec{G}\cdot d\vec{x} = \int_{C_2} \vec{G}\cdot d\vec{x}$ C_1, C_2 have same start & ending pt

$\Updownarrow$

(b) $\int_C \vec{G}\cdot d\vec{x} = 0$ when C is a closed curve



• $\exists f$, s.t. $\nabla f = \vec{G} \iff \vec{G}$ is conservative



R is convex & open
curl $\vec{G} = 0$
$\Rightarrow$ conservative

· Given G, solve the equations to find f : General sol'n $f_0 + C$

curl $\vec{F} = \vec{G}$

· convex, open R

when $\text{div } \vec{G} = 0 \Rightarrow \exists \vec{F}$, s.t. curl $\vec{F} = \vec{G}$

how to compute $\vec{F}$ for given $\vec{G}$. Let $F_3 = 0$. General Sol'n $F_0 + \nabla f$