Lecture 1
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Q1 12% + Q2 12% + midterm 30% + final 46%
(1-side formula sheet) (2-side)

Classes of Stochastic Processes

$\{X(t), t \in T\}$ — index set

$\downarrow$ time argument

Classification
- discrete time: $t \in \{1, 2, \cdots\}$, $\{0, 1, \cdots\}$ finite or countable

  some time series
  Markov chains with discrete time
- continuous. $t \in [0, \infty)$
  $t \in [0, 1]$
  $t \in [0, T]$

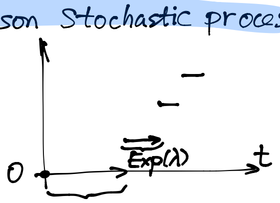
NOT the only classification way.

We look at typical trajectories (paths)

$X(t)$: $X(t, \omega)$ if you fix ω, can get function of time variable.
r.v.

Poisson Stochastic process

Purely discontinuous



$\lambda e^{-\lambda x}$ Exp(rate=λ)

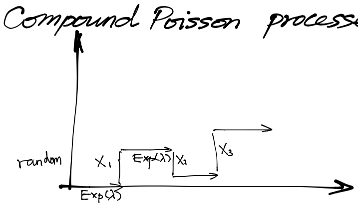
Exp(β)

$\frac{1}{\beta} e^{-x/\beta}$

The # of arrivals is Poiss(λt)
so it's called poisson stochastic process.

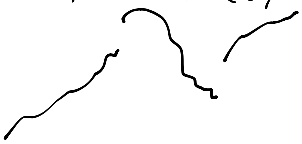
A completely opposition example: Brownian motion

Compound Poisson processes



$X_1, X_2, X_3, \cdots$ iid r.v's
"2 randomness together"

diffusion processes (Markov)
Gaussian Processes (separate)



Such processes with stationary & independent increments which start from 0.

meaning: $X(b)-X(a)$, $X(d)-X(c)$ are independent ($a \sim b$, $c \sim d$)

Lévy stochastic process:

Poisson & Normal distribution

Raikov's thm
Cramèr's thm

Poisson law of small #'s
$B(n,p) \approx Poiss(\lambda = np)$ if $n \cdot p \leq 7$

i.e. $P(B(n,p)=x) \approx e^{-\lambda} \frac{\lambda^x}{x!}$ (How? $\binom{n}{x} p^x (1-p)^{n-x}$ Stirlin approx to prove)

$\lambda = np$ if $n \cdot p \leq 7$

80 (p. 90)

$$\lim_{n\to\infty} \underbrace{\left\{e^{-n} \sum_{k=0}^{n} \frac{n^k}{k!}\right\}} = \frac{1}{2}$$ deMoivre–Laplace thm

$Poiss(\lambda_1+\lambda_2) = Poiss(\lambda_1) + Poiss(\lambda_2)$

$$P\{0 \leq \underset{n\,Poiss(1)}{Poiss(n)} \leq n\}$$

$$= P\left\{ \frac{\overbrace{X_1+\cdots+X_n}^{Poiss(n)} - n}{\sqrt{n}} \leq 0 \right\}$$

$$\overset{CLT}{=} P\{Z \leq 0\} = \frac{1}{2}$$

Ch3
(pp 93–163)
§3.2 & §3.3
Conditional distributions
discrete case: $P(x,y) = P(X=x, Y=y)$ $P_X(x) = \sum$ ...

joint pdf $f(x,y) = \cdots$

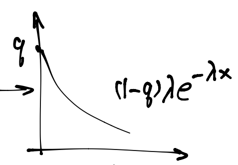
§3.4 (pp100–114)
Computing Expectations by conditioning

p100 double expectation thm

$E(X) = E(\underbrace{E(X|Y)}_{\text{r.v. which is a NON random function of r.v. } Y})$

w.r.t. probability law of r.v.

ZME zero-modified exponential distribution



descrete pointmass $0<q<1$

$P(X=0) = q \in (0,1)$

Ward identity

$\boxed{E(S) = EN \cdot EX_i}$

compound r.v.

r.v. $S = \sum_{i=1}^{N} X_i$ $\qquad X_1, X_2, \cdots, X_n \sim$ iid r.v.'s

N — non-negative integer counting r.v.

Proposition 3.1 (p112)

Conditional / Total > Variance formula

$Var(X) = Var(E(X|Y)) + E(Var(X|Y))$

compound random sum

r.v. $S = \sum_{i=1}^{N} x_i$, x_i : $E = \mu$, $var = \sigma^2$

$Var\, S = \sigma^2 E(N) + \mu^2 Var N = \lambda E(X_i^2)$

What if $N \sim Poiss(\lambda)$

Compound Poisson (λ)

$Var\, S = \lambda(\mu^2 + \sigma^2)$

$Var(x) = E(X^2) - (EX)^2$

additional assumption:

$X_i \geq 1$

integer valued

$E(X_i^2) = \sum_{x=1}^{\infty} x^2 p(x)$

$E(X_i) = \sum_{x=1}^{\infty} x p(x)$

$ES = \lambda EX$

$Var S = \lambda E(X^2)$

$Var S > ES$ overdispersion

(usually $Var S < ES$)

HW: Read §2.9, Ch3

Ch.2 #46, 76, 85, 87

Ch.3 #9, 10, 16