STAT3032 SURVIVAL MODELS

TUTORIAL WEEK TWO

1. Using ELT 15 Ultimate mortality, find the probability that a male aged 55 will

   (a) survive to age 56 $p_{55} = 1 - q_{55} = 1 - 0.00797 = 0.99203$

   (b) die before reaching age 65 ${}_{10}q_{55} = \frac{l_{55} - l_{65}}{l_{55}} = \frac{91217 - 79293}{91217} = 0.13072$

   (c) die between ages 56 and 58 $p_{55} \cdot {}_2q_{56} = 0.99203 \times \frac{l_{56} - l_{58}}{l_{56}} = 0.01861$

   (d) die between ages 60 and 65 ${}_5p_{55} \cdot {}_5q_{60} = \frac{l_{60}}{l_{55}} \times \frac{l_{60} - l_{65}}{l_{60}} = \frac{86714}{91217} \times \frac{79293 - 86714}{86714} = 0.08136$

2. The distribution function of the future lifetime of a new-born individual in a certain district is given by:

$$F_0(t) = 1 - e^{-0.015t}$$

$x \quad x+1 \quad x+2 \quad x+3$

3. ${}_3p_x = p_x \cdot {}_2p_{x+1}$
$= {}_2p_x \cdot p_{x+2}$
$= (1 - {}_2q_x)\, p_{x+2}$

$0.912285 = (1 - 0.0398)\, p_{x+2}$

$p_{x+2} = \frac{0.912285}{1 - 0.0398} = 0.95010$

$\frac{l_{x+2}}{l_{x+1}} = 0.97$

$\frac{l_{x+3}}{l_x} = 0.912285$

$\frac{l_x - l_{x+2}}{l_x} = 0.0398$

$\frac{l_{x+3}}{l_{x+2}} = \frac{0.912285\, l_x}{0.9602\, l_x}$

$1 - \frac{l_{x+2}}{l_x} = 0.0398$

$l_{x+2} = l_x \cdot 0.9602$

What is the probability that:

(a) A new-born individual will survive to age 70? $P(T > 70) = S(70) = 1 - F(70) = e^{-0.015 \times 70} = 0.34994$

(b) A new-born individual dies not later than age 35? $P(T < 35) = 1 - e^{-0.015 \times 35} = 0.40844$

(c) A person aged 25 survives to age 50? $P(T > 50 \mid T > 25) = \frac{P(T > 50)}{P(T > 25)} = \frac{e^{-0.015 \times 50}}{e^{-0.015 \times 25}} = 0.68729$

(d) A person aged 30 dies not later than age 70? $P(T < 70 \mid T > 30) = \frac{P(30 < T < 70)}{P(T > 30)} = \frac{S(30) - S(70)}{S(30)} = 0.45119$

3. If $p_{x+1} = 0.97$, ${}_3p_x = 0.912285$, ${}_2q_x = 0.0398$, find p_{x+2}.

$= 0.950099$

4. Express in as many forms as you can, using both statistical functions (the survival function S and the distribution function F) and actuarial notation (*p*,*q*,*d* and *l* etc) the probability that a person aged 50 will die between 70 and 80.

$\mu_x = -\frac{d}{dx}\log(l_x) > 1$

5. Explain why μ_x can be greater than one.

It's instantaneous, ~~not~~ and annualised, so it's possible that at a moment, say the first few months of a year, the mortality is really high. Overall we know that even if the trend continues, μ_x should be balanced to 1 b/c ~~not~~ every body dies. but at some point, it can be greater than 1.

6. Challenge Problem

For a continuous lifetime random variable T the mean residual life function is

$$r(t) = E[(T - t) \mid T > t]$$

~~Given~~ How many years can one live given the age t.

(a) Explain in non-technical language what the mean residual life function measures.

For a person at age t, what is the expected remaining life? rest

(b) Explain why $r(t) = E[T \mid T > t] - t$.

$r(t) = E[(T - t) \mid T > t]$

$= E[T \mid T > t] - E[t \mid T > t]$

$= E[T \mid T > t] - t$

b/c t is a constant.

$f_{T|T>t}(y) = P(T=y|T>t) = \frac{P(T=y, T>t)}{P(T>t)} = \frac{f_T(y)}{1-F_T(t)} = \frac{\lambda e^{-\lambda y}}{e^{-\lambda t}}$ domain changed. $(y>t)$

$1-F_T(t) = \int_t^\infty f_T(y)$

$T \sim \exp(\lambda)$, $f_T(t) = \lambda e^{-\lambda t}$ $(t>0)$

(c) If the random variable T has an exponential distribution, show that the density of T conditional on T being greater than t is

$$f_{T|T>t}(y) = \frac{\lambda e^{-\lambda y}}{e^{-\lambda t}}.$$

$r(t) = \int_t^\infty y \cdot \frac{\lambda e^{-\lambda y}}{e^{-\lambda t}} \cdot dy = \int_t^\infty y \lambda e^{-\lambda(y-t)} dy = e^{-\lambda(y-t)}\Big|_\infty^t = 1-$

(d) Hence using an appropriate integral for conditional expectation, show that $r(t) = \frac{1}{\lambda}$.

$r(t) = \int_t^\infty y \cdot f_{T|T>t}(y) dy = e^{\lambda t} \int_0^\infty y \cdot \lambda e^{-\lambda(y-t)} dy = e^{\lambda t}[te^{-\lambda t} + \frac{1}{\lambda}e^{-\lambda t}] - t$
$= \frac{1}{\lambda}$

(e) Write down the mean of the exponential random variable with parameter λ.

(f) Explain, again using non-technical language, the significance of the fact that parts (d) and (e) yield the same result.

$r(t)$ no relation with λ
constant.
always $\frac{1}{\lambda}$.

$t=0$.
just the exponential,
E is just $\frac{1}{\lambda}$.

whatever the year t is,
alway expected to live ~~more~~ $\frac{1}{\lambda}$ more years.

"memoryless property"