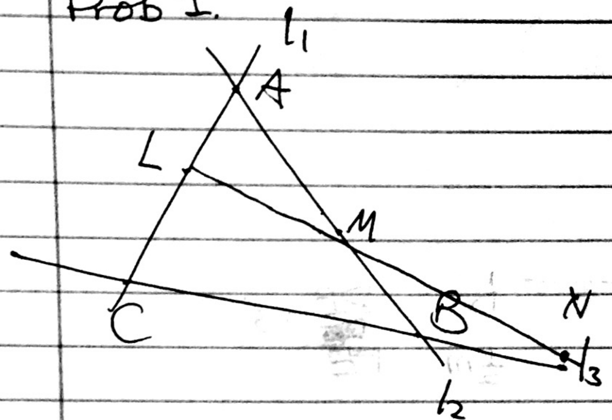
Practice Prob

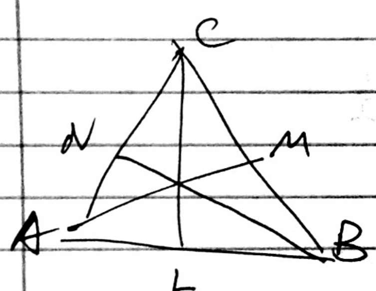
Prob 1.



Menelaus ✓

(proof by projection)

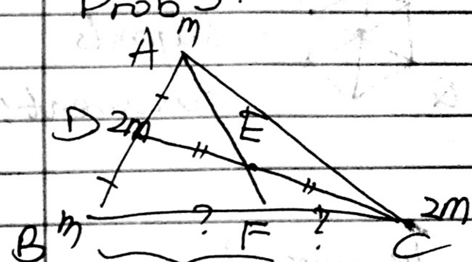
Prob. 2.



$\frac{AL}{LB} \cdot \frac{BM}{MC} \cdot \frac{CN}{NA} = 1$ pov $= 1$ ✓

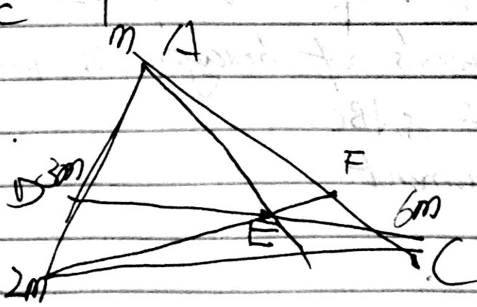
(proof by area)

Prob 3.



$\frac{BF}{FC} = \frac{2}{1}$

Prob 4.

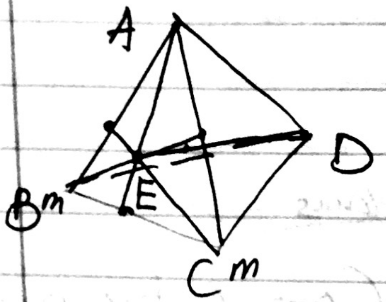


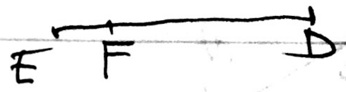
$\frac{AD}{BD} = \frac{2}{1}$

$\frac{DE}{EC} = \frac{2}{1}$

So $\frac{AF}{FC} = \frac{6m}{m} = \frac{6}{1}$

Prob. 5.

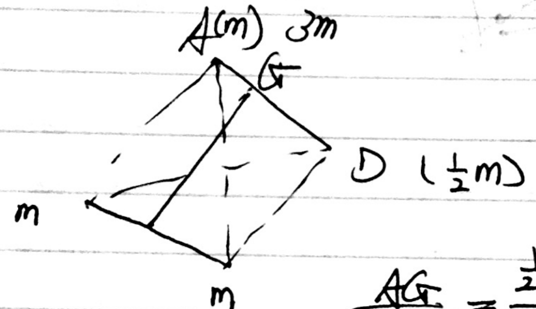




$\frac{DF}{FE} = 6$

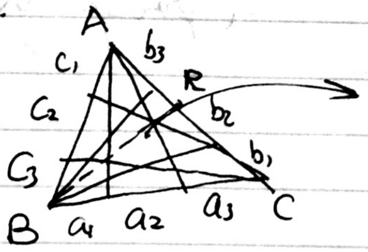
$\frac{1}{2}m$

3m

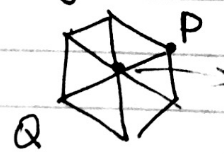


$$\frac{AG}{DG} = \frac{\frac{1}{2}m}{m} = \frac{1}{2}$$

Prob 6.



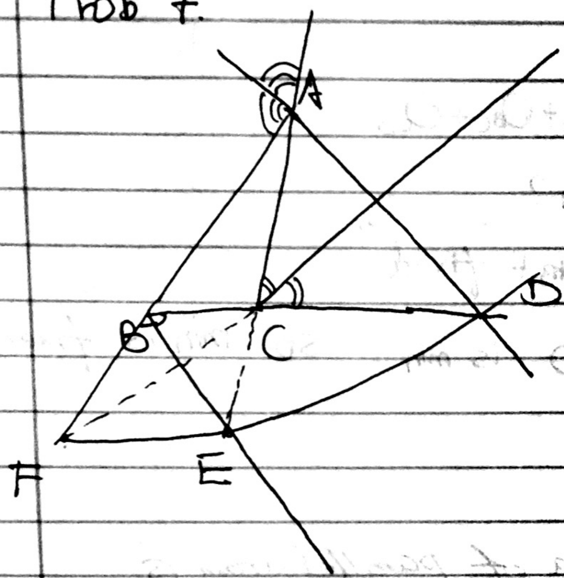
for 6-gon



diagonals concurrent!

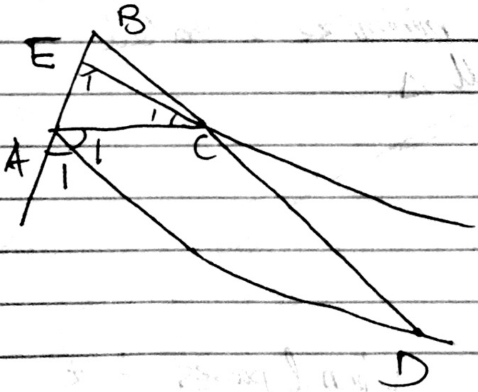
AR = RC by ceva.
AR passes P
similarly, AR = RC by ceva, AR passes Q.
So PQ is a part of median of AC.
so the 3 diagonals of hexagon is parts of 3 medians of $\triangle ABC$.
Hence concurrent.

Prob 7.



DEF collinear.

外角平分线定理



construct $CE \parallel AD$

$\frac{BD}{CD} = \frac{BA}{EA}$

$\angle AEC = \angle 1$

$\angle CAD = \angle 1$

So $\angle AEC = \angle ACE$

So $AE = AC$

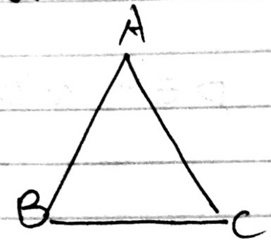
$\frac{BD}{CD} = \frac{BA}{AC}$

done.

$$1 = \left\{ \begin{array}{l} \frac{BD}{CD} = \frac{AB}{AC} \\ \frac{CE}{EA} = \frac{BC}{BA} \\ \frac{AF}{FD} = \frac{CA}{CB} \end{array} \right\} = 1$$

By Menelaus, collinear.

Prob 8.



$4O_{AB} + O_{BC} + O_{AO}$

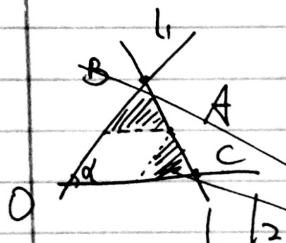
min ?

$3O_{AB}$ + "fixed"

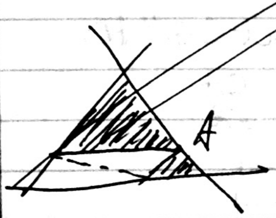
$\downarrow$

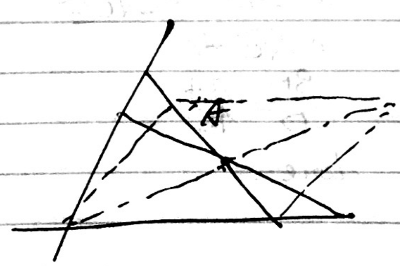
O is min   so min is "fixed"

Prob 9.



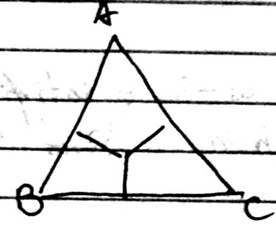
The area of parallelogram is fixed by A & α.
then if we want $\min(S_\Delta)$ is to minimize 2 shaded small Δ.





when l passes A &
A bisects l.

Prob 10



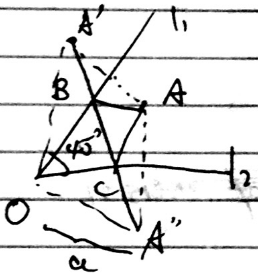
smallest $O_{AB}+2O_{BC}+3O_{CA}$

$= \text{fixed} + O_{BC} + 2O_{CA}$

$\geq \text{fixed} + 0 + 0 = \text{fixed}.$

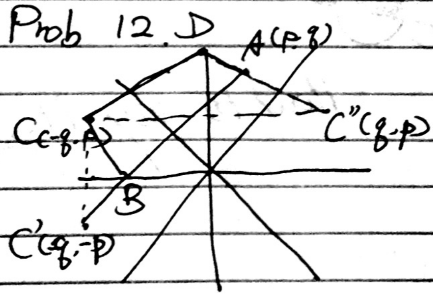
O is at pt C.

Prob 11.



$\sqrt{2}\,a$

Prob 12.



$|AB+BC|\ \min \Rightarrow |AC| = \sqrt{(p+q)^2+(q-p)^2}$

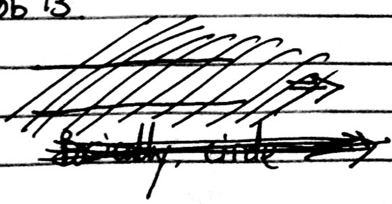
$= \sqrt{2}\,(p+q)$

$|CD-DA|\ \max \Rightarrow \ldots$

skip calculation.

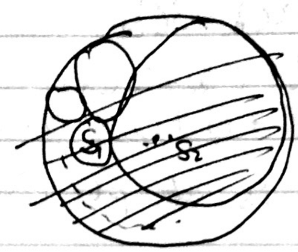
Prob 13.

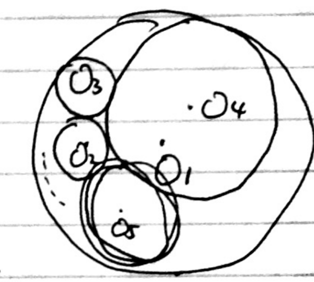
分类讨论

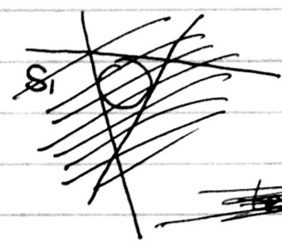


P14.

if a chain $S_3 - S_{2005}$ satisfies
then any chain $S_3' - S_{2005}'$ satisfies.

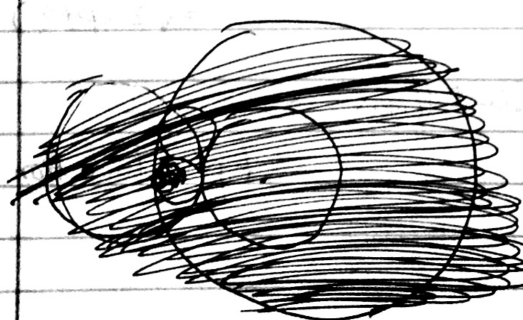






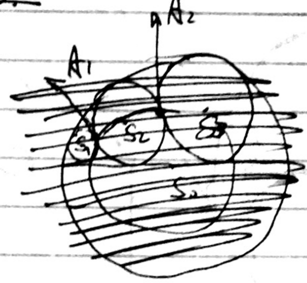
~~let O_1 be center of inversion~~

How to say it's true



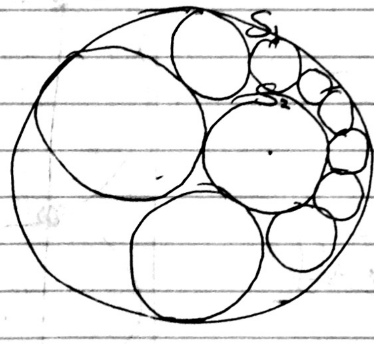
15. Prob. (same as Prob 13)

~~16. Prob:~~



Prob 14. Proof of Steiner Chain:

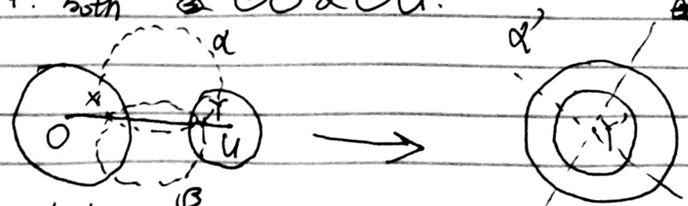
S_1 be larger circle, S_2 disjoint, smaller, inside S_1.



Need lemma to prove this

Lemma: Given 2 non-intersecting circles S_1 & S_2 (non-concentric) there is an inversion s.t. transforms them into concentric circles.

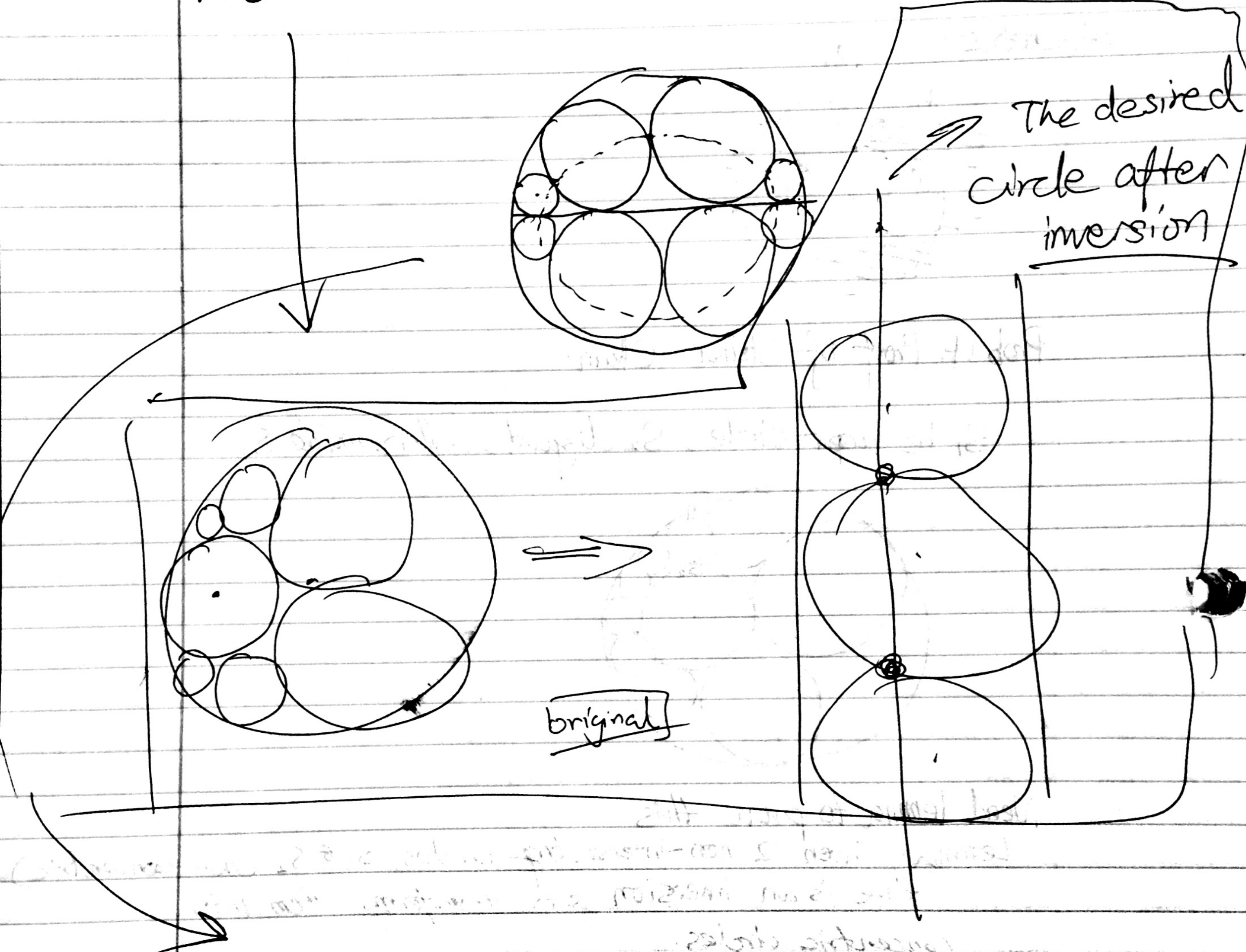
By another lemma (given 2 circles, & a pt P not on either circle, can draw a circle through P & $\perp$ both circles) we can find such circles at X & Y refered to ~~lemma~~ another lemma (2 non-intersecting circles with center O & U, $O \neq U$, can find X & Y that are inverses to each other w.r.t. both ~~&~~ ⊙O & ⊙U.



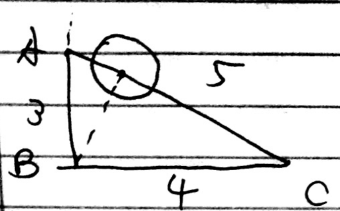
then it's simple.

non-concentiric ⇒ concentric (n steps) / . . .

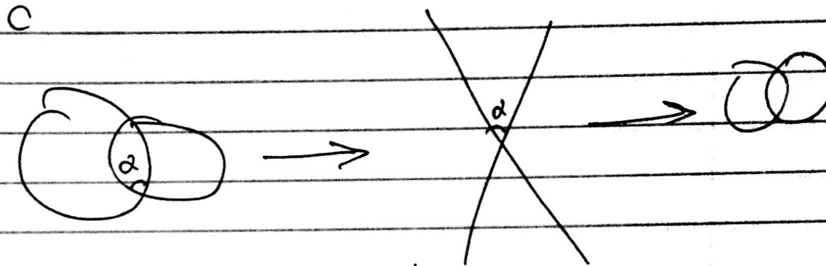
Prob. 16.



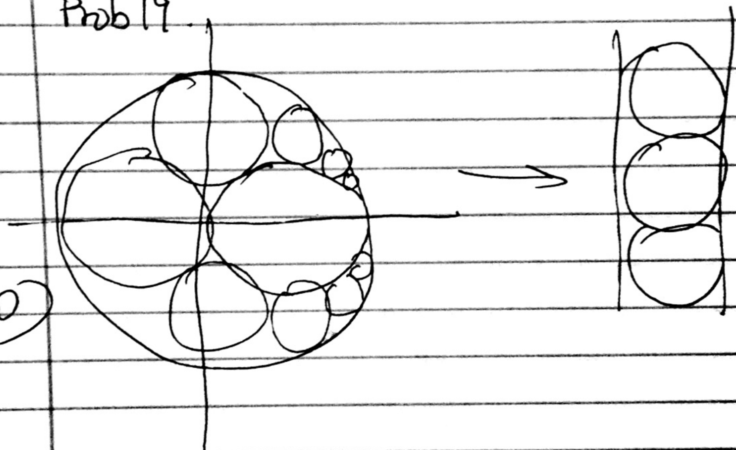
Prob 17



Prob 18



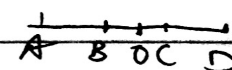
Prob 19



(-2, 0)

P20.

shift to



so $f(A) = -D$

$f(B) = -C$

P21. f_0 vertices

at least 3 edges meet at 1 vertices.

so $f_1 \geq 3f_0/2$

$2f_1 \geq 3f_0$ $\qquad f_0 \leq \frac{2}{3} f_1$

$f_0 - f_1 + f_2 = 2$

$f_0 = 2 + f_1 - f_2 \leq \frac{2}{3} f_1 \Rightarrow 6 + 3f_1 - f_2 < 2f_1$

$f_2 - f_1 > 6$