PHL245 Test 2 Review

Unit 4.

- valid arguments: truth ~~value~~ of conclusion follows the truth of premises

PQ ?

- semantic value of a sentence is its truth-value.

Truth-value assignment (TVA)

- Tautologies, contradictions & contingent sentence

Tautology	true by definition	true for every TVA.
Contradiction	cannot be true	false for every TVA
~~Contigent~~ sentence Contingent ~~sentence~~	can be true or false	true on some TVA false on some TVA

- Equivalency, Consistency and inconsistency
  - sentence ϕ & ψ are logically equivalent iff no truth-value ~~ass-~~ ~~∃~~ assignment on which ϕ & ψ different truth-values.
  - consistent, at least one TVA s.t. ϕ & ψ have the set true.
  - inconsistent, no ~~true~~ TVA s.t. ϕ & ψ ~~have~~ are true.

- Validity:

valid iff ~~all~~ true when all premises true.

- all theorems are tautologies b/c they are true on every TVAs.

- A set of sentences tautologically implies a sentence ϕ iff there is no truth-value assignment for which all the sentences in the set are true and ϕ is false. (No TVA on all $\{\psi, \chi \dots\}$ are T and ϕ is F)
- No TVA on which all premises T and conclusion F.

$\exists x(Hx \wedge \forall y(Ey \to N(x,y)))$

- A sentence ϕ is a tautology iff $\{\sim\phi\}$ is inconsistent.
- A sentence ϕ is a contradiction iff $\{\phi\}$ is inconsistent
- ϕ & ψ are equivalent iff $\{\sim\phi \leftrightarrow \psi\}$ is inconsistent
- ϕ is contingent iff both $\{\phi\}$ & $\{\sim\phi\}$ are consistent

- A set of sentences tautologically implies a sentence ϕ iff the set formed by the original set of sentences together with $\sim\phi$ forms is inconsistent.
- An ~~arg~~ argument is valid iff the set of sentences consisting of the premises & the negation of the conclusion is inconsistent.

Unit 5.

Canonical form of the universal quantified sentence:

$\forall x\,(Fx \rightarrow Gx)$

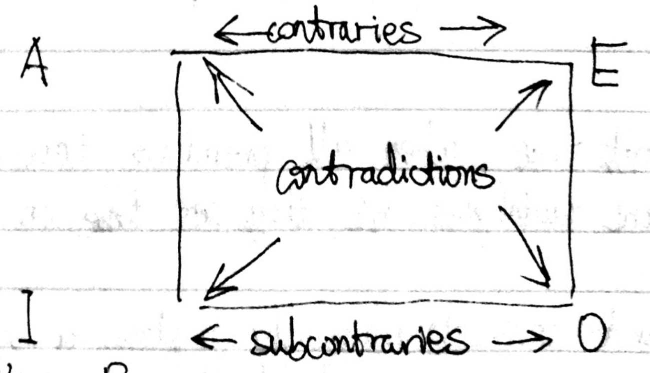
Canonical form ——— existential quantified sentence:

$\exists x\,(Fx \land Gx)$

Square of oposition:

Universal affirmative
All A's are B
$\forall x\,(Fx \rightarrow Gx)$

Universal negative
No A's are B
$\forall x\,(Fx \rightarrow \sim Gx)$



- contradiction (if one true, the other is false)
- contraries (can both ~~true~~, false, cannot both true)
- subcontraries (can both true, not cannot both false)

Some A's are B
Particular affirmative
$\exists x\,(Fx \land Gx)$

Some A's are not B
Particular negative
$\exists x\,(Fx \land \sim Gx)$

Quantifier Negation

$$\forall x\,\phi \leftrightarrow \sim\exists x \sim\phi$$
$$\exists x\,\phi \leftrightarrow \sim\forall x \sim\phi$$
$$\forall x \sim\phi \leftrightarrow \sim\exists x\,\phi$$
$$\exists x \sim\phi \leftrightarrow \sim\forall x\,\phi$$

Exportation: $\forall x(Fx \to (Gx \to Hx))$

$\forall x((Fx \land Gx) \to Hx)$

General Rule: Confinement for conditionals when the antecedent is quantified

$$\exists x Fx \to \phi \leftrightarrow \forall x(Fx \to \phi)$$
$$\forall x Fx \to \phi \leftrightarrow \exists x(Fx \to \phi)$$

General Rule: ___ consequent ___

$$\phi \to \exists x Fx \leftrightarrow \exists x(\phi \to Fx)$$
$$\phi \to \forall x Fx \leftrightarrow \forall x(\phi \to Fx)$$

General Rule: ___ biconditionals

$$(\forall x Fx \leftrightarrow \phi) \leftrightarrow (\exists x(Fx \to \phi) \land \forall x(\phi \to Fx))$$
$$(\exists x Fx \leftrightarrow \phi) \leftrightarrow (\forall x(Fx \to \phi) \land \exists x(\phi \to Fx))$$

GR: ___ conjunctions & disjunctions

$$(\forall x Fx \land \phi) \leftrightarrow \forall x(Fx \land \phi) \qquad (\exists x Fx \land \phi) \leftrightarrow \exists x(Fx \land \phi)$$
$$(\forall x Fx \lor \phi) \leftrightarrow \forall x(Fx \lor \phi) \qquad (\exists x Fx \lor \phi) \leftrightarrow \exists x(Fx \lor \phi)$$

Properties of Binary Predicates or Relations

Symmetry: $\forall x \forall y\, (F(xy) \leftrightarrow F(yx))$

Asymmetry: $\forall x \forall y\, (F(xy) \leftrightarrow \sim F(yx))$

Transitivity: $\forall x \forall y \forall z\, [(F(xy) \wedge F(yz)) \rightarrow F(xz)]$

Intransitivity: $\forall x \forall y \forall z\, [(F(xy) \wedge F(yz)) \rightarrow \sim F(xz)]$

Reflexivity: $\forall x\, F(xx)$

Irreflexivity: $\forall x \sim F(xx)$

Identity: $a = b$

Uniqueness:

canonical form of the unique individual sentence

$\exists x \forall y (Fy \leftrightarrow x = y)$

There is exactly one individual with property F.

$\exists x \forall y (Fy \leftrightarrow x = y) \wedge Gx)$

The one individual that has property F also has property G

Midterm 2 Review Practice

~~Show~~ $\sim\exists z Mz \to \sim\forall y L(yy)$	show conc
$\sim\exists z Mz$	ass cd
~~Show~~ $\sim\forall y L(yy)$	show cons
$\forall y L(yy)$	ass id
$\forall y(H(iyy) \to \forall z \sim L(iz))$	pr1 ei
~~Show~~ $\forall x\forall y(Gx \to \exists z K(zy))$	show ant pr2
~~Show~~ $\forall y(Gx \to \exists z K(zy))$	show inst
$Gx \to \exists z K(zy)$	show inst
Gx	ass cd
$\exists z(Gz \wedge \sim Mz) \to Kyy$	pr3 ui
$\forall z \sim Mz$	2 qn
$\sim Mx$	11 ui
$Gx \wedge \sim Mx$	9 12 adj
$\exists z(Gx \wedge Mx)$	13 eg
Kyy	10 14 mp
$\exists z K(zy)$	15 eg
	16 cd
	8 ud
	7 ud
$\forall x\exists y\forall z H(xyz)$	pr2 6 mp
$\exists y\forall z H(iyz)$	20 ui
$\forall z H(ikz)$	21 ei
$H(ikk) \to \forall z \sim L(iz)$	5 ui
$H(ikk)$	22 ui
$\forall z \sim L(iz)$	23 24 mp
$L(ii)$	4 ui
$\sim L(ii)$	25 ui
	26 27 id
	3 cd

(c).

1	Show $\sim\forall x\exists y A(xy) \rightarrow \sim\forall x(A(xa) \vee \sim B(xx))$	show conc
2	$\sim\forall x\exists y A(xy)$	ass cd
3	Show $\sim\forall x(A(xa) \vee \sim B(xx))$	show conc
4	$\forall x(A(xa) \vee \sim B(xx))$	ass id
5	$\exists x \sim\exists y A(xy)$	2 qn
6	$\sim\exists y A(iy)$	5 ei
7	$\forall y \sim A(iy)$	6 qn
8	$A(ia) \vee \sim B(ii)$	4 ui
9	$\sim A(ia)$	7 ui
10	$\sim B(ii)$	8 9 mtp
11	$\forall y F(d(k) y d(y))$	pr 1 ei
12	$F(d(k) d(k) d(d(k)))$	11 ui
13	$\exists x F(x x d(x))$	12 eg
14	$\forall w \forall z \sim(A(wz) \leftrightarrow B(wz))$	13 pr2 mp
15	$\forall z \sim(A(iz) \leftrightarrow B(iz))$	14 ui
16	$\sim(A(ii) \leftrightarrow B(ii))$	15 ui
17	$A(ii) \leftrightarrow \sim B(ii)$	16 nb
18	$\sim B(ii) \rightarrow A(ii)$	17 bc
19	$A(ii)$	10 18 mp
20	$\sim A(ii)$	7 ui
21		19 20 id
22		3 cd
23		
24		