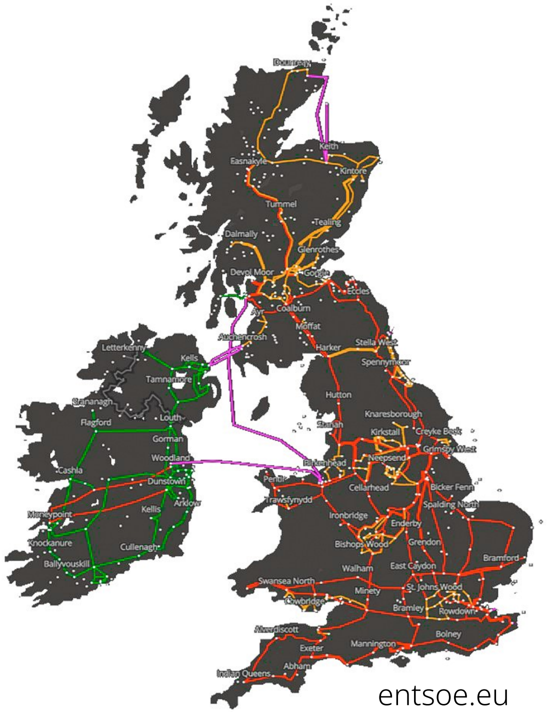
When do delays desynchronize a power grid?

Master stability conditions for inertial oscillator networks



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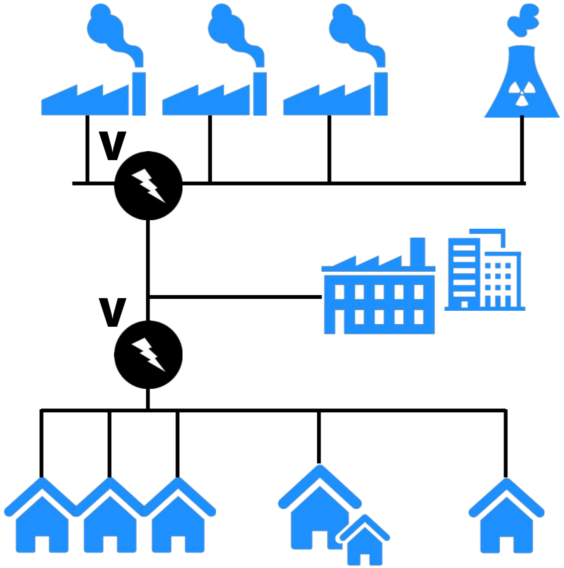
PIK

CoNDyNet
STROMNETZE
Forschungsinitiative der Bundesregierung

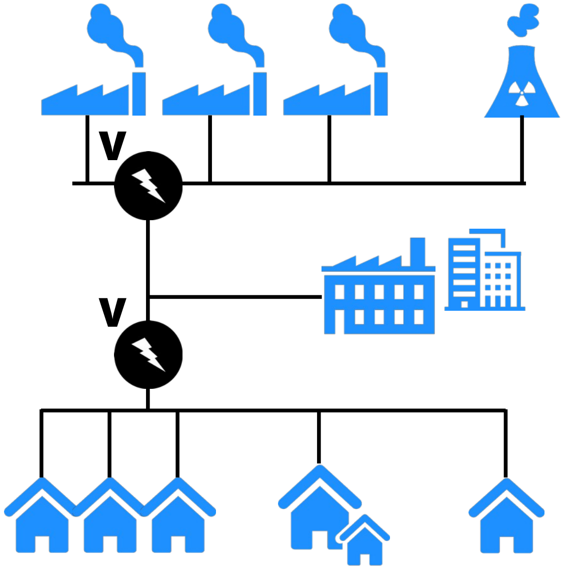
Freie Universität Berlin

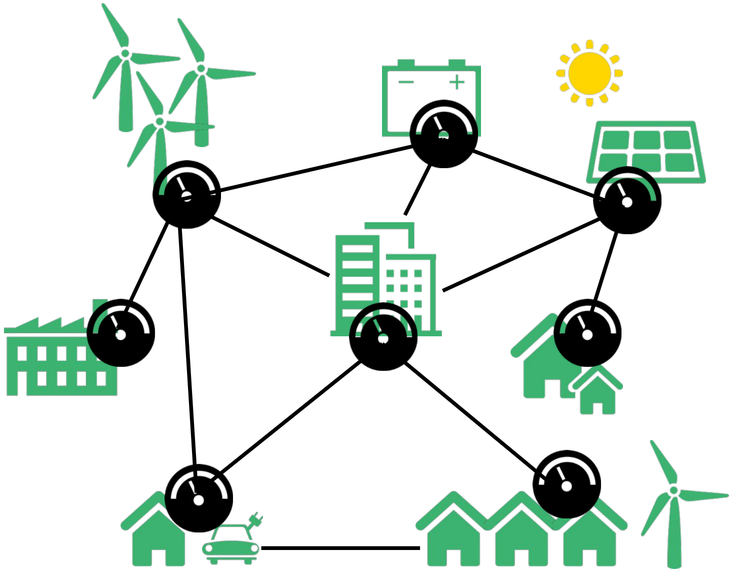
Yesterday

Power grid



Yesterday Power grid Tomorrow





hierarchical ← **topology** → distributed

self-stabilizing ← **dynamics** → volatile

Delay differential equations

DDE $$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = f(x(t),\, x(t-\tau))$$

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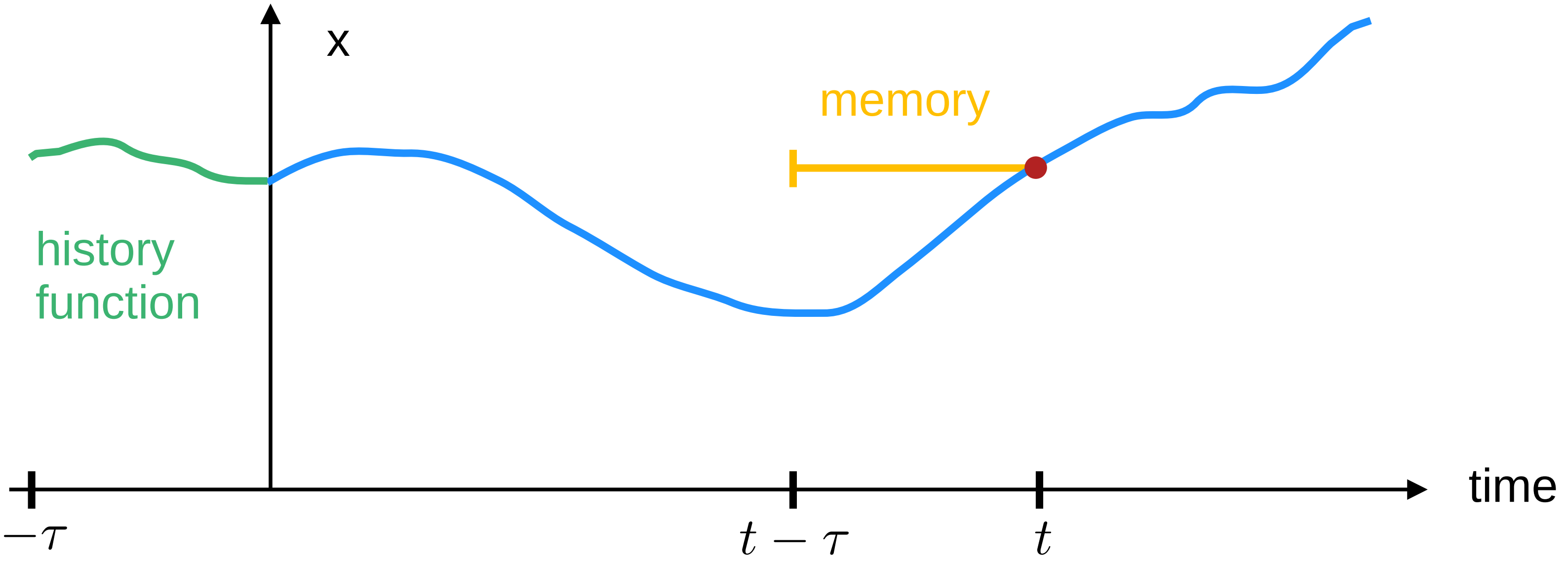


Delay differential equations – Linear stability analysis

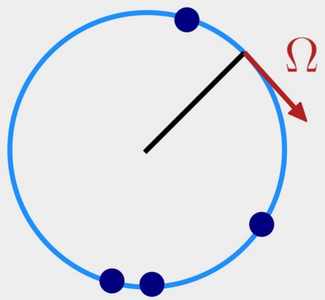
DDE $$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = f(x(t), x(t-\tau))$$

Fixed point $$f(x^*) = 0$$

Limit cycle

$\downarrow$ co-rotating frame

Fixed point



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Linearize near sync state

$$\eta(t) = x(t) - x^*$$

$$\dot{\eta}(t) \approx J_0(x^*)\,\eta(t) + J_\tau(x^*)\,\eta(t-\tau)$$

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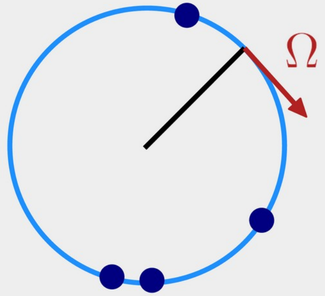

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Characteristic equation

$$\eta(t) = \eta(0)\, e^{yt}$$

$$\det\left(-y\mathbb{I}_n + J_0 + e^{-y\tau} J_\tau\right) = 0$$

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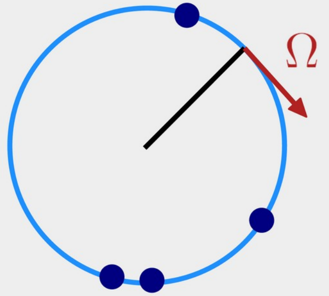

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- **Infinite spectrum!**
- Linearly stable if and only if

$$\mathrm{Re}(y) < 0 \quad \forall\, y$$

Power grid model: Decentral Smart Grid Control


New Journal of Physics
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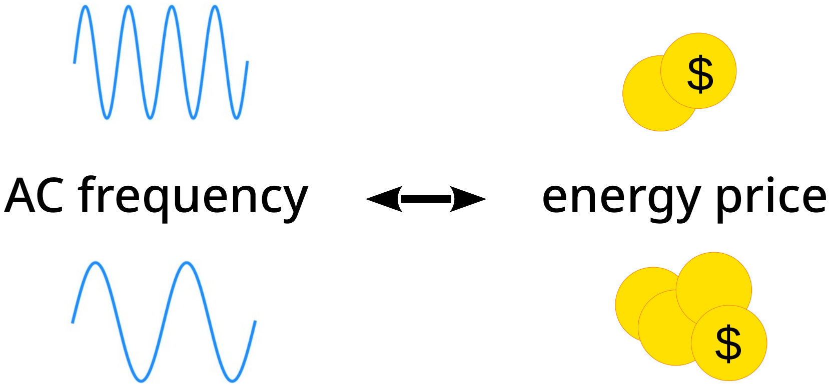
Decentral Smart Grid Control
Benjamin Schäfer[1], Moritz Matthiae[1], Marc Timme[1,2] and Dirk Witthaut[1,3,4]



Schäfer et al. (2015) New J. Phys. 17 015002

Power grid model: Decentral Smart Grid Control

$$\ddot{\varphi}_i = P_i - \alpha\dot{\varphi}_i - \gamma\dot{\varphi}_i(t-\tau) + \sum_j K_{ij}\sin(\varphi_j - \varphi_i)$$

Phase	$\varphi_i(t)$
Frequency	$\omega_i(t) = \dot{\varphi}_i(t)$
	$i = 1, \ldots, N$

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Power
+ production
- consumption

Phase $\varphi_i(t)$

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Price regulation

Phase	$\varphi_i(t)$
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Power grid model: Decentral Smart Grid Control

Frequency change

Damping

Power flow

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Delay master stability approach

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Linearize around synchronous state

$\varphi \rightarrow \eta = \varphi^* - \varphi$

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Graph Laplacian

$\mathcal{L} \equiv -\mathcal{K}_{ij} + \delta_{ij}\sum_j \mathcal{K}_{ij}$

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Linear transformation

$\eta \to \xi = T\eta$

$T\mathcal{L}T^{-1} = diag(\lambda_1, \ldots, \lambda_N)$

$$\ddot{\xi}_k = -\alpha\dot{\xi}_k - \gamma\dot{\xi}_k^\tau - \lambda_k\xi_k$$

N equations!

Delay master stability approach

$$\ddot{\varphi}_i = P_i - \alpha\dot{\varphi}_i - \gamma\dot{\varphi}_i(t-\tau) + \sum_j K_{ij}\sin(\varphi_j - \varphi_i)$$

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N equations!

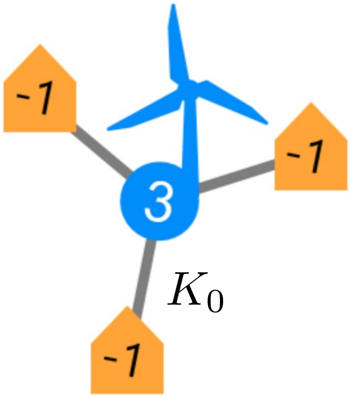
- Critical characteristic roots y_k^*
- **Delay master stability function**

$$\sigma_k(\tau, \lambda_k, y_k^*, \alpha, \gamma)$$

Stable if and only if

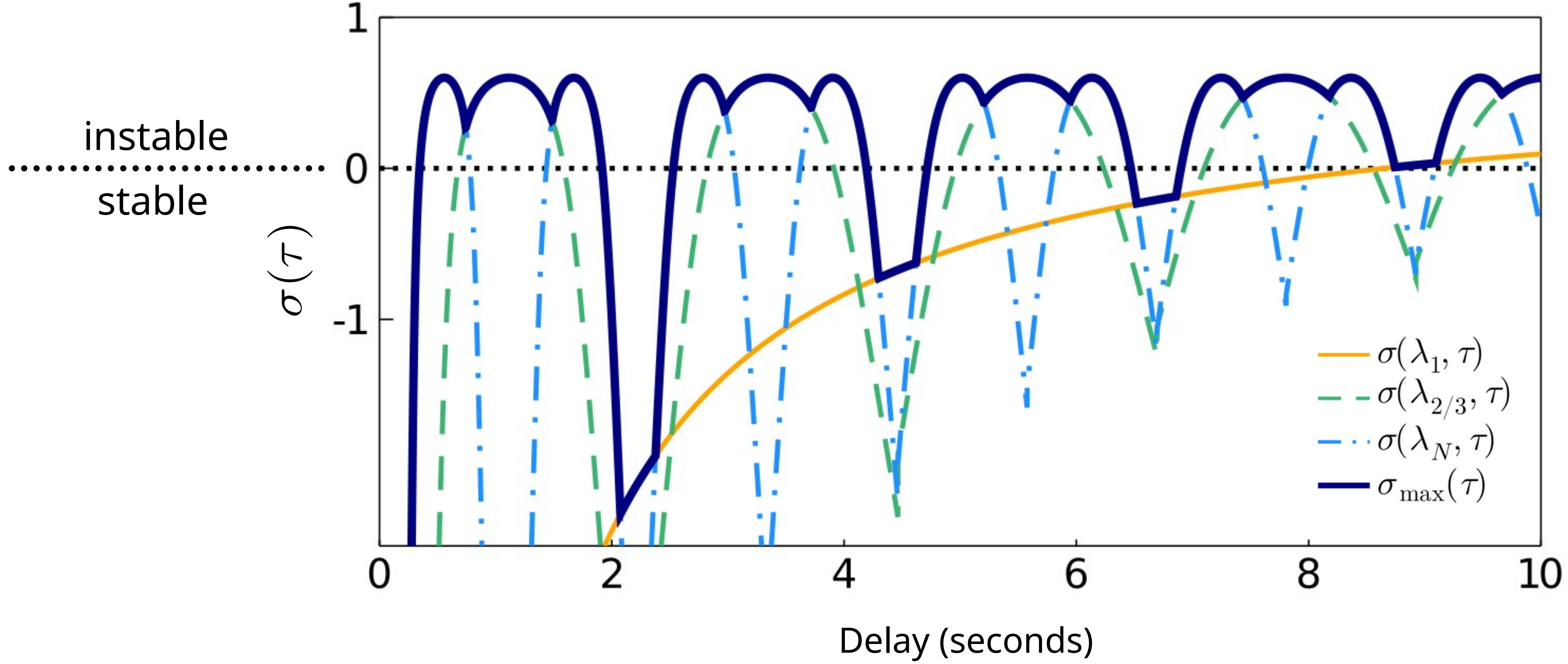
$$\sigma_k < 0 \quad \forall k = 1, \ldots, N$$

Application: Stability of 4-node network



$$K = \begin{bmatrix} 0 & K_0 & K_0 & K_0 \\ K_0 & 0 & 0 & 0 \\ K_0 & 0 & 0 & 0 \\ K_0 & 0 & 0 & 0 \end{bmatrix}$$

Application: Stability of 4-node network



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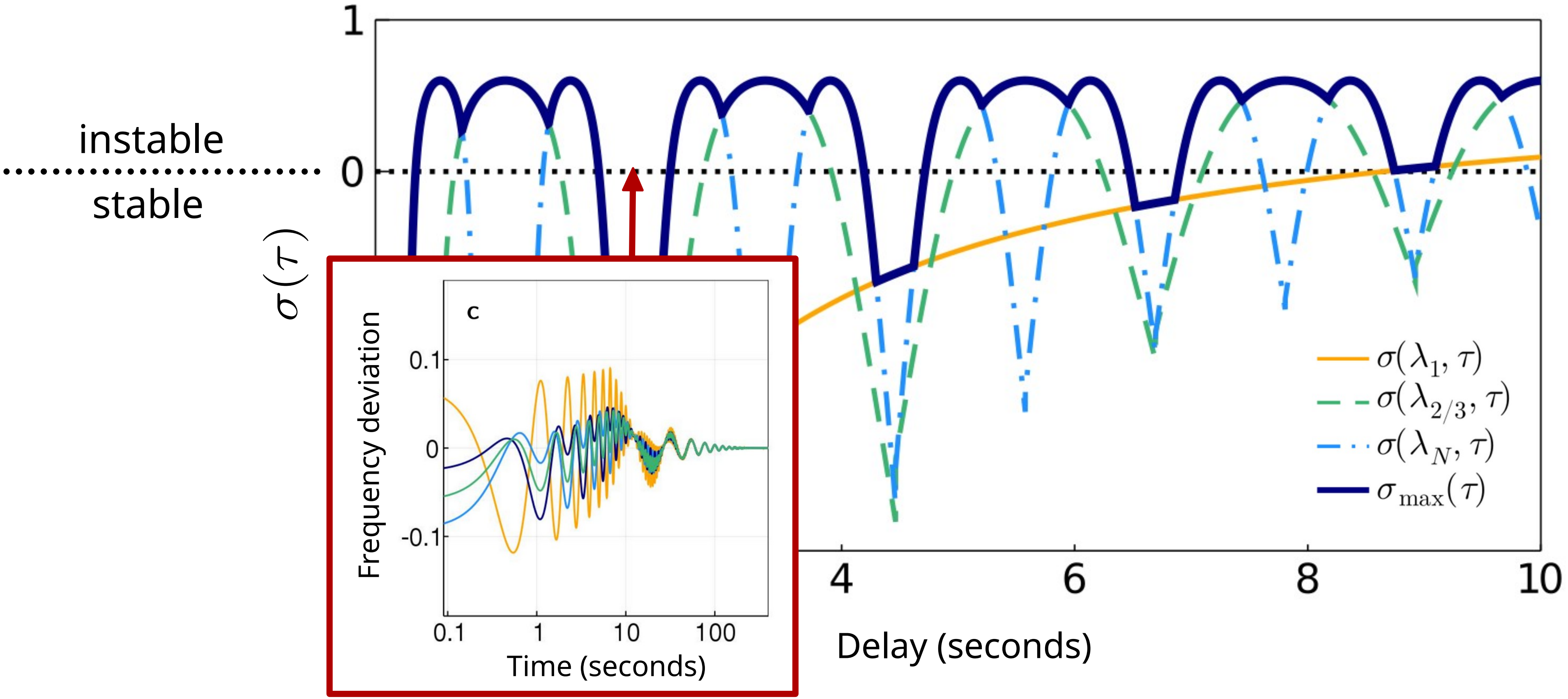

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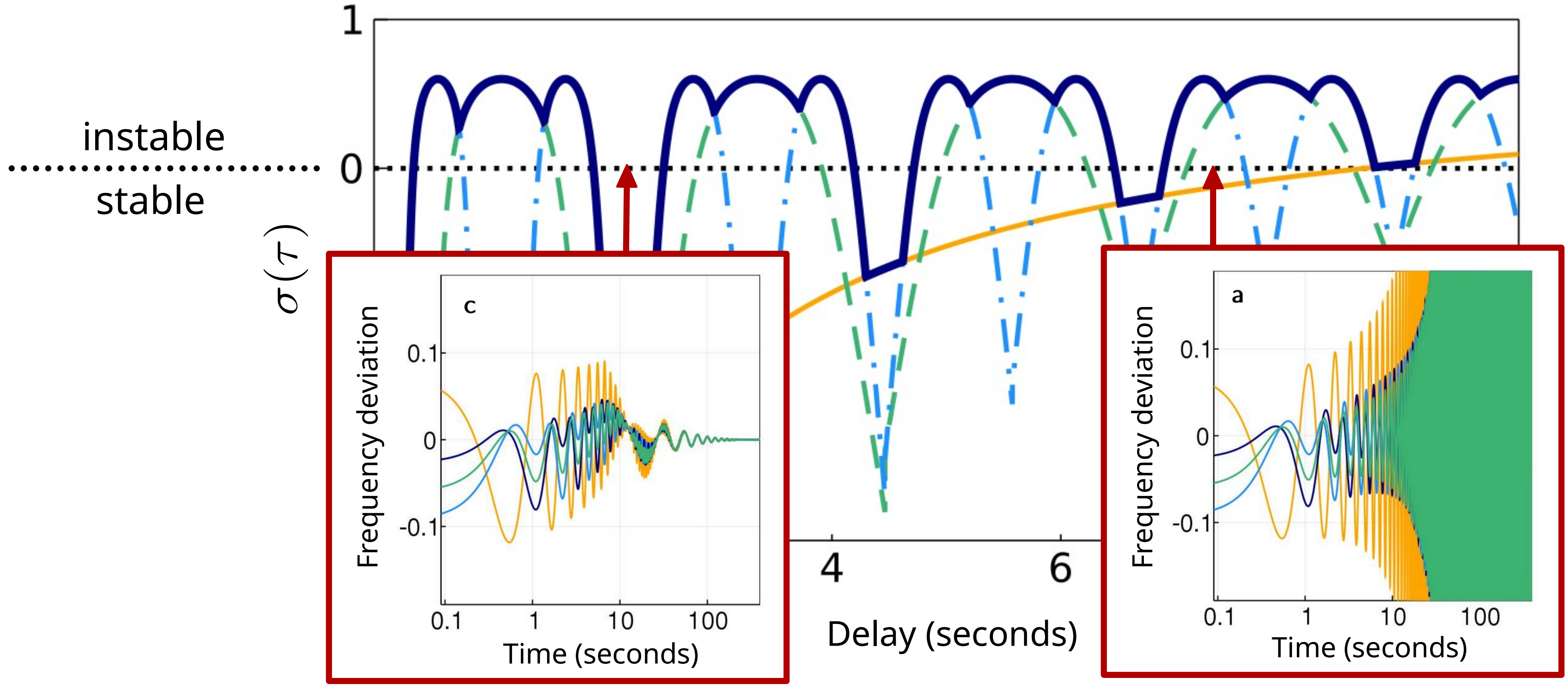

Conclusion

- **necessary and sufficient** delay master stability conditions
- any **weighted, undirected graph**
- **inertial oscillators** with nonlinear diffusive coupling

References

- Börner et al. (2020) Phys. Rev. Research 2, 023409
- more on github.com/reykboerner/delay-networks