Controlling the stability of future power networks

How delays affect synchronization

Reyk Börner
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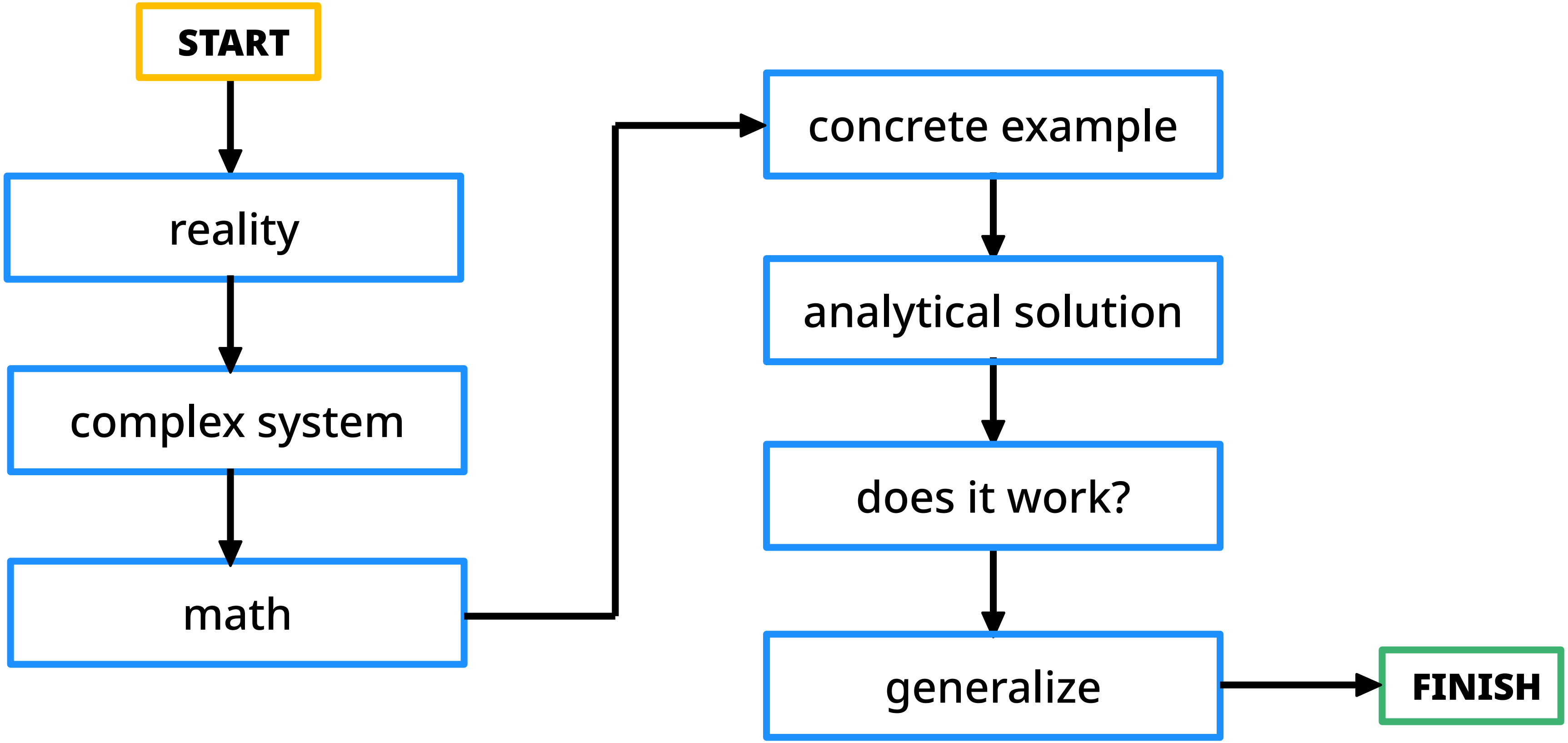
27.01.2021
When machine learning meets complex systems
Seminar @ LMU München

Freie Universität Berlin
CoNDyNet
STROMNETZE
Forschungsinitiative der Bundesregierung
PIK
Potsdam Institute for Climate Impact Research

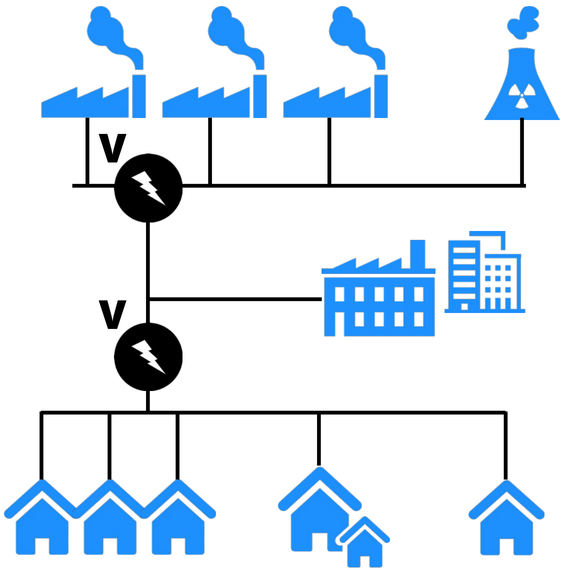

Copenhagen
Munich
NASA EOSDIS Worldview (2016)

Trajectory


START
reality
complex system
math
concrete example
analytical solution
does it work?
generalize
FINISH

Yesterday

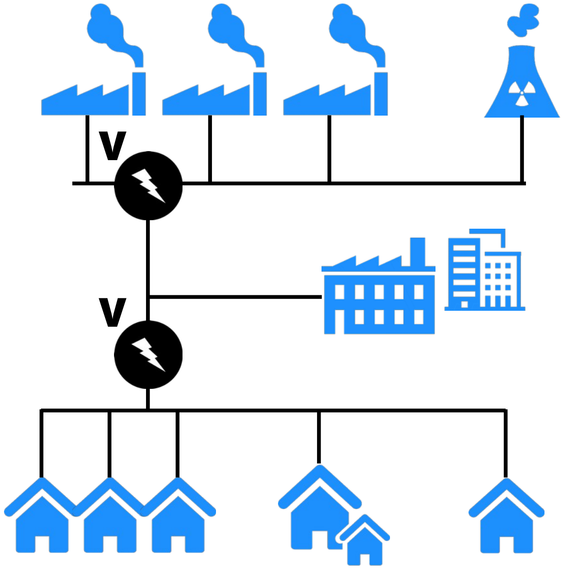
Power grid

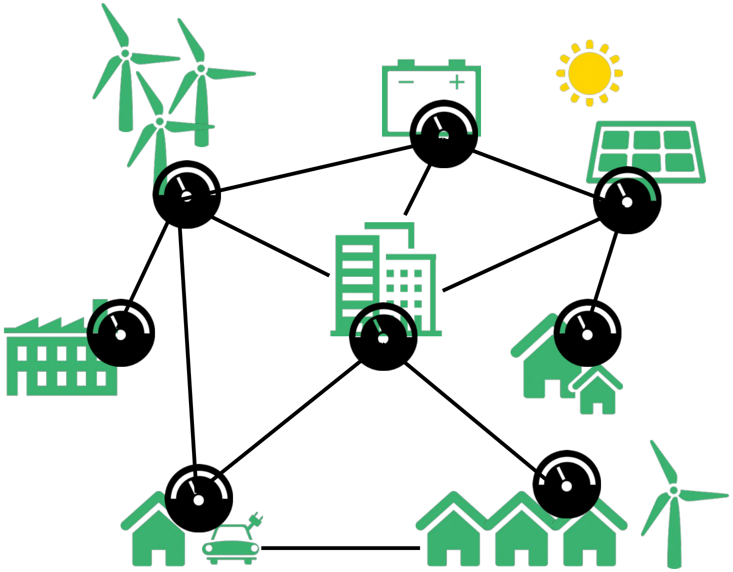


Yesterday

Power grid

Tomorrow

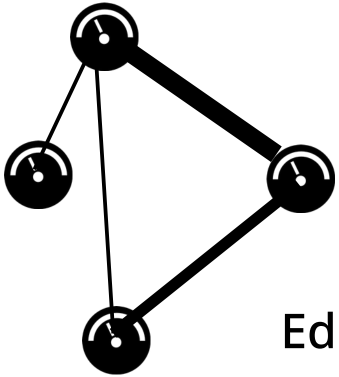




hierarchical ← **topology** → distributed

self-stabilizing ← **dynamics** → volatile

Power grids as oscillator networks



Nodes = oscillators

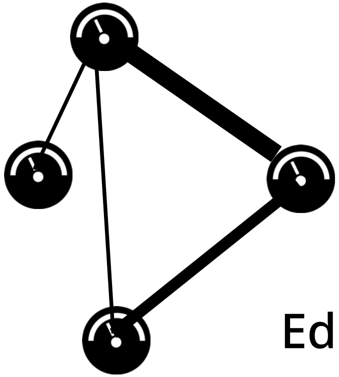
“prosumers”

inverters

Edges

transmission lines

Power grids as oscillator networks



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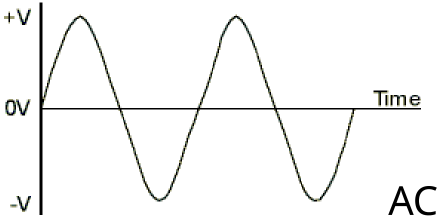
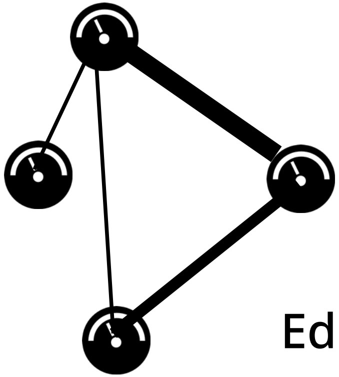


AC Voltage

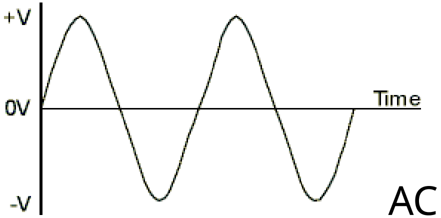
Power flow between node i and j

$$P_{i\to j} = K_{ij}\,\sin(\varphi_i - \varphi_j)$$

Power grids as oscillator networks



Nodes = oscillators
"prosumers"
inverters



AC Voltage

Edges
transmission lines

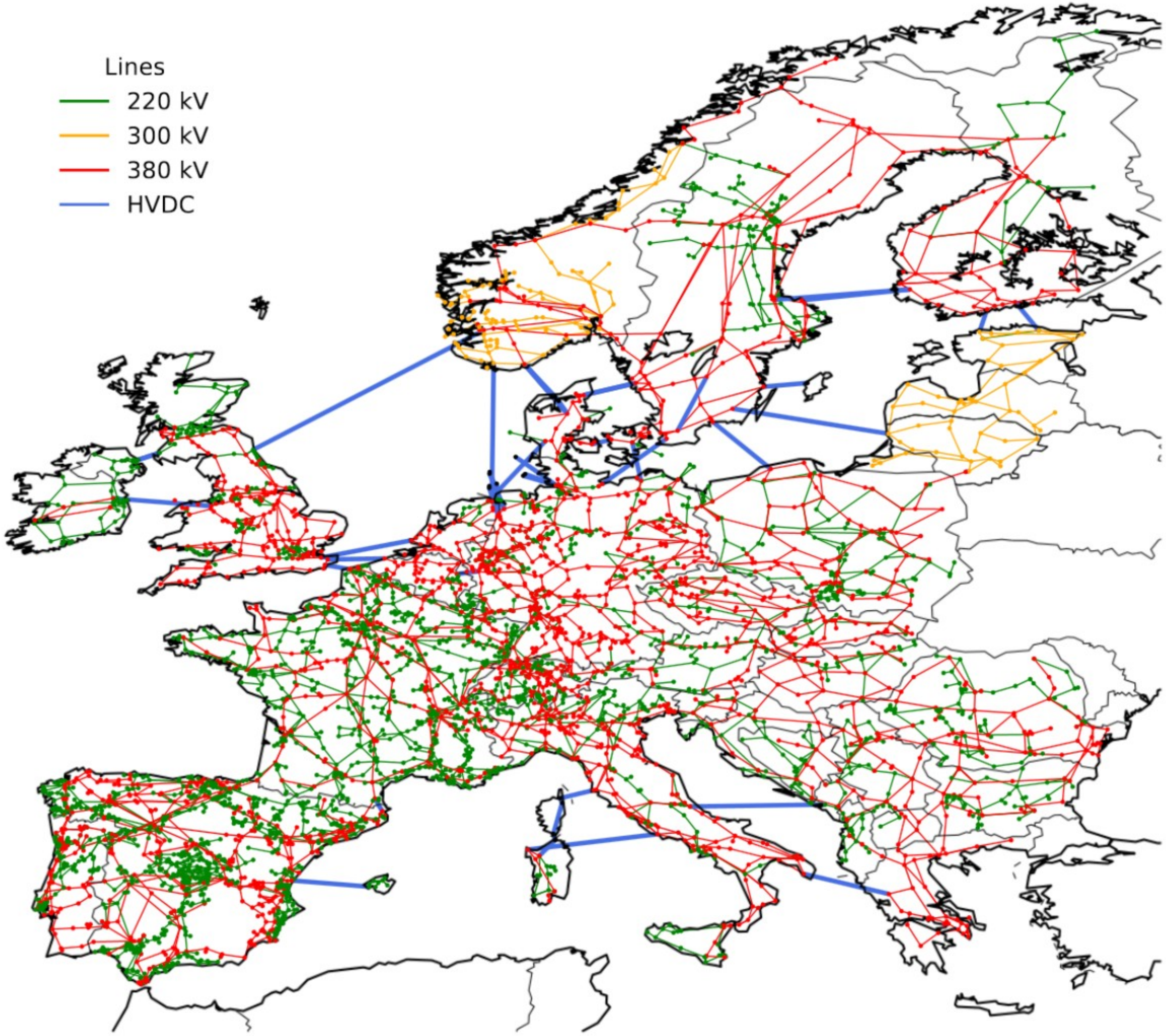
Power flow between node i and j

$$P_{i\to j} = K_{ij}\,\sin(\varphi_i - \varphi_j)$$

Weighted adjacency matrix

$$K_{ij} = \begin{cases} \ell_{ij} & i \text{ and } j \text{ connected} \\ 0 & \text{otherwise} \end{cases}$$

European High Voltage Grid



Hörsch et al. (2018)
Energy Strategy Review 22, 207

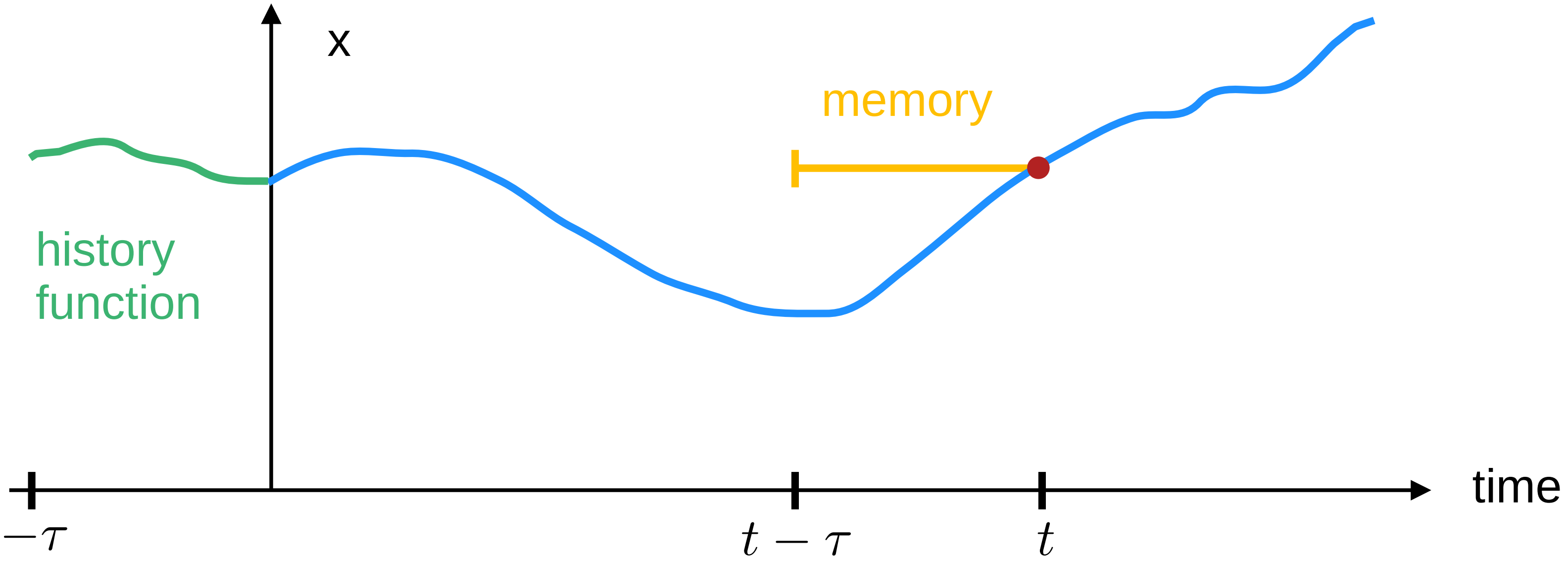
Delays & Synchronization

- Processing delay
- Transmission delay

Delay differential equations

$$\dot{x}(t) = f(x(t),\ x(t-\tau))$$

delay τ



Delay differential equations – Linear stability analysis

DDE $$\dot{x}(t) = f(x(t),\ x(t-\tau))$$

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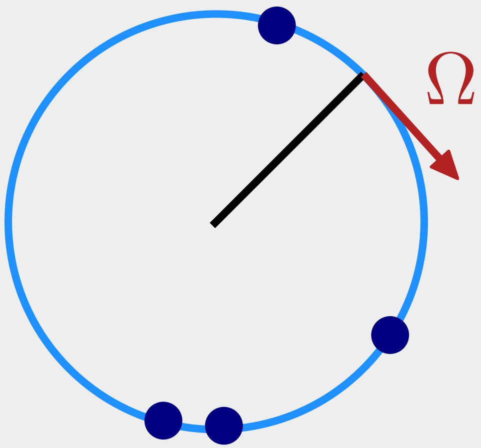
Fixed point $f(x^*) = 0$

Limit cycle

$\downarrow$ co-rotating frame

Fixed point



Delay differential equations – Linear stability analysis

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Fixed point $f(x^*) = 0$

Small deviations around fixed point $\eta(t) = x(t) - x^*$

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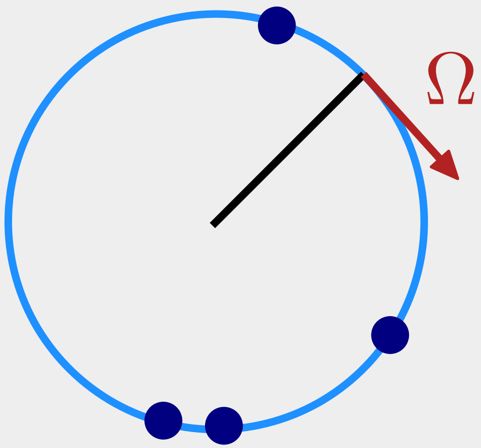
Fixed point



Delay differential equations – Linear stability analysis

DDE $\dot{x}(t) = f(x(t),\ x(t-\tau))$

Fixed point $f(x^*) = 0$

Small deviations around fixed point $\eta(t) = x(t) - x^*$

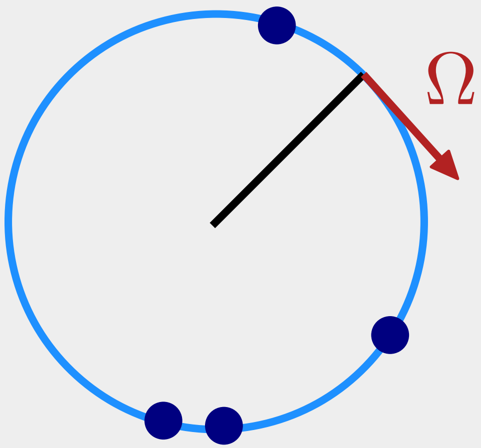
Linearize DDE

$$\dot{\eta}(t) = f(x^*) + J_0(x^*)\,\eta(t) + J_\tau(x^*)\,\eta(t-\tau) + \dots$$

Limit cycle

co-rotating frame

Fixed point



Delay differential equations – Linear stability analysis

DDE $\dot{x}(t) = f(x(t),\ x(t-\tau))$

Fixed point $f(x^*) = 0$

Small deviations around fixed point $\eta(t) = x(t) - x^*$

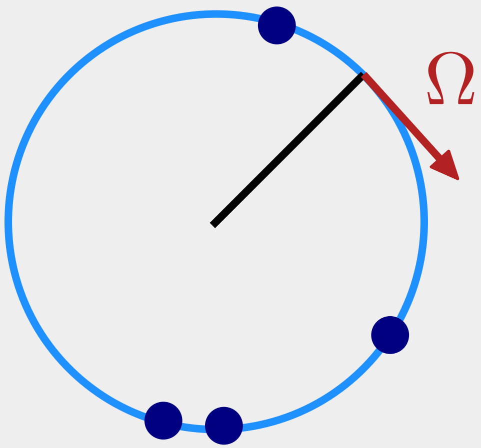
Linearize DDE

$$\dot{\eta}(t) \approx J_0(x^*)\,\eta(t) + J_\tau(x^*)\,\eta(t-\tau)$$

Limit cycle

co-rotating frame

Fixed point



Delay differential equations – Linear stability analysis

Linear DDE $$\dot{\eta}(t) \approx J_0(x^*)\,\eta(t) + J_\tau(x^*)\,\eta(t-\tau)$$

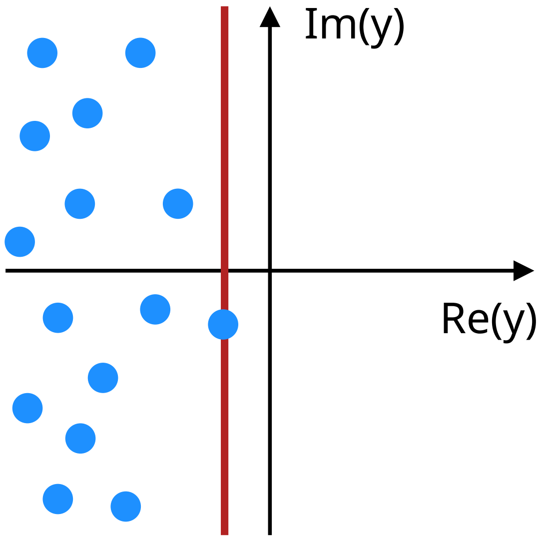
Ansatz $$\eta(t) = \eta(0)\,e^{yt}$$

Characteristic equation $$\det\left(-y\mathbb{I}_n + J_0 + e^{-y\tau} J_\tau\right) = 0$$

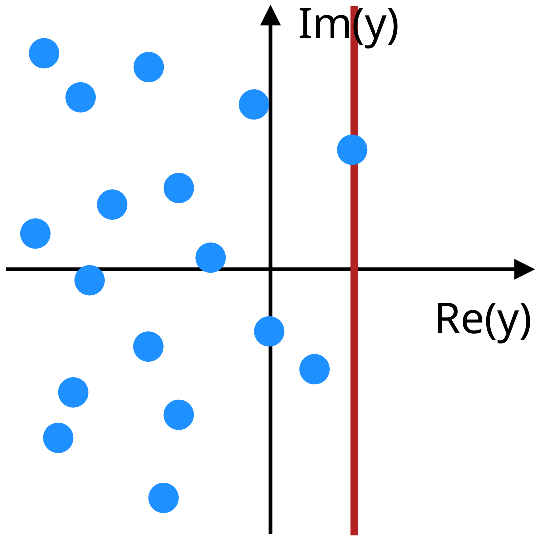
- Infinitely many roots!
- Fixed point is stable if and only if $\mathrm{Re}(y) < 0 \quad \forall\, y$ in spectrum

Delay differential equations – Characteristic spectrum

stable



instable



Application: Decentral Smart Grid Control

New Journal of Physics
The open access journal at the forefront of physics

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Decentral Smart Grid Control

Benjamin Schäfer[1], Moritz Matthiae[1], Marc Timme[1,2] and Dirk Witthaut[1,3,4]

- Match **demand** to supply
- **Grid frequency** contains all information
- Frequency fluctuation → **price adaptation**

Application: Decentral Smart Grid Control

Frequency change

Damping

Power flow

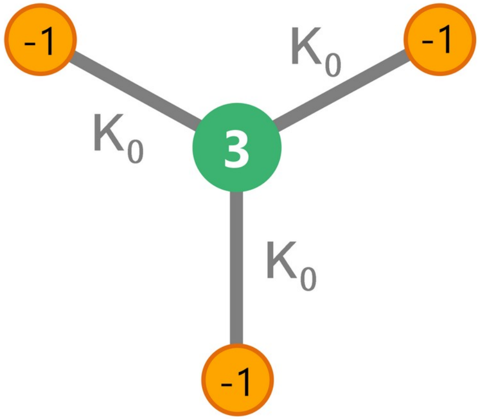
$$\ddot{\varphi}_i = P_i - \alpha\dot{\varphi}_i - \gamma\dot{\varphi}_i(t - \tau) + \sum_j K_{ij}\sin(\varphi_j - \varphi_i)$$

Power
+ production
- consumption

Price regulation

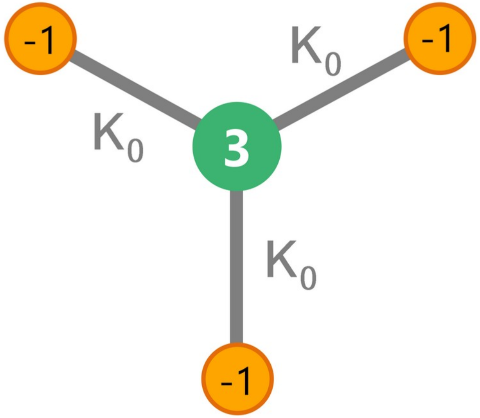
Phase	$\varphi_i(t)$
Frequency	$\omega_i(t) = \dot{\varphi}_i(t)$
	$i = 1, \ldots, N$

Application: Stability of 4-node network



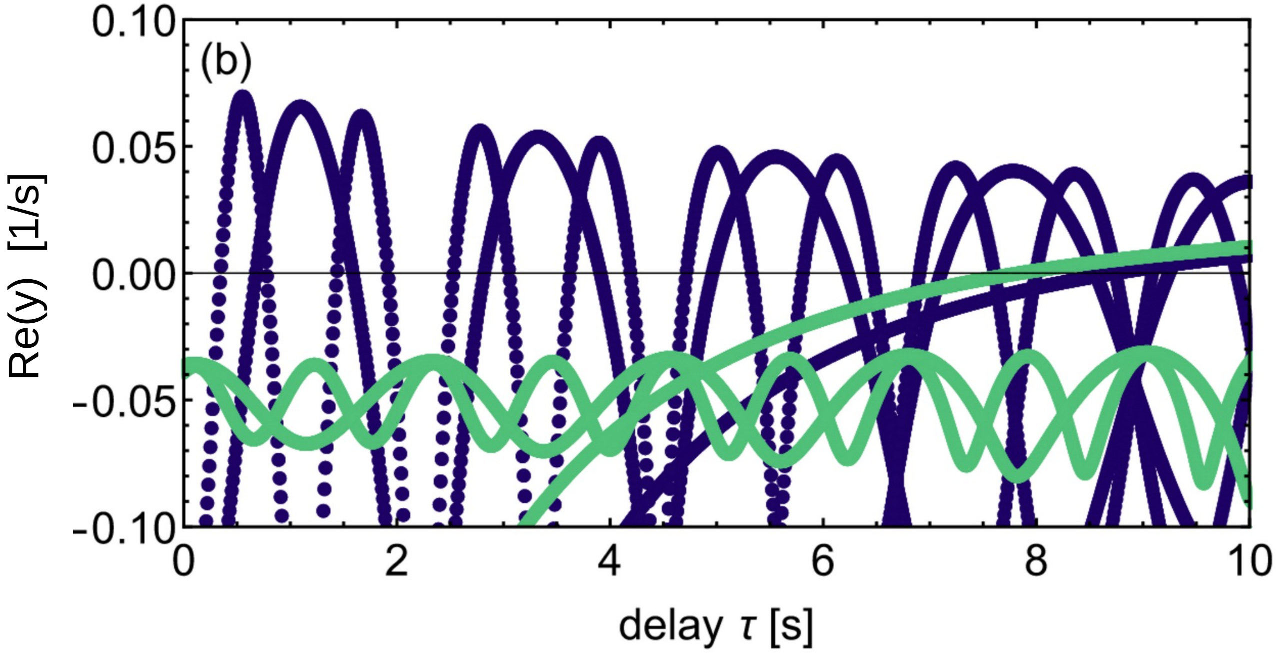
$$K = \begin{bmatrix} 0 & K_0 & K_0 & K_0 \\ K_0 & 0 & 0 & 0 \\ K_0 & 0 & 0 & 0 \\ K_0 & 0 & 0 & 0 \end{bmatrix}$$

Application: Stability of 4-node network



$$K = \begin{bmatrix} 0 & K_0 & K_0 & K_0 \\ K_0 & 0 & 0 & 0 \\ K_0 & 0 & 0 & 0 \\ K_0 & 0 & 0 & 0 \end{bmatrix}$$

Characteristic roots as function of delay



Schäfer et al. (2016) Eur. Phys. J. Special Topics 225, 569-582

Analytical approach

$$\ddot{\varphi}_i = P_i - \alpha\dot{\varphi}_i - \gamma\dot{\varphi}_i(t-\tau) + \sum_j K_{ij}\sin(\varphi_j - \varphi_i)$$

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Linearize around synchronous state

$\varphi \rightarrow \eta = \varphi^* - \varphi$

$$\ddot{\eta}_i = -\alpha\dot{\eta}_i - \gamma\dot{\eta}_i^\tau + \sum_{j=1}^{N} K_{ij}\cos(\varphi_j^* - \varphi_i^*)\big(\eta_i - \eta_j\big)$$

Analytical approach

$$\ddot{\varphi}_i = P_i - \alpha\dot{\varphi}_i - \gamma\dot{\varphi}_i(t-\tau) + \sum_j K_{ij}\sin(\varphi_j - \varphi_i)$$

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Graph Laplacian

$\mathcal{L} \equiv -\mathcal{K}_{ij} + \delta_{ij}\sum_j \mathcal{K}_{ij}$

$$\ddot{\eta}_i = -\alpha\dot{\eta}_i - \gamma\dot{\eta}_i^{\tau} + \sum_{j=1}^{N} \mathcal{L}_{ij}\eta_j$$

Analytical approach

$$\ddot{\varphi}_i = P_i - \alpha\dot{\varphi}_i - \gamma\dot{\varphi}_i(t-\tau) + \sum_j K_{ij}\sin(\varphi_j - \varphi_i)$$

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Linear transformation

$\eta \rightarrow \xi = T\eta$

$T\mathcal{L}T^{-1} = diag(\lambda_1, \ldots, \lambda_N)$

$$\ddot{\xi}_k = -\alpha\dot{\xi}_k - \gamma\dot{\xi}_k^\tau - \lambda_k\xi_k$$

N equations!

Analytical approach – Success!

Stability condition

$$\boxed{\gamma\tau < \frac{1}{y_k^*}\sqrt{({y_k^*}^2 - \lambda_k\tau^2)^2 + (\alpha\tau y_k^*)^2}}$$

Analytical approach – Success!

Stability condition

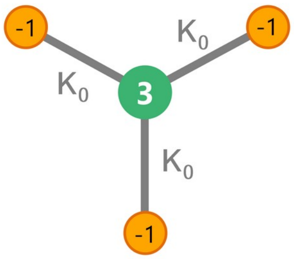
$$\boxed{\gamma\tau < \frac{1}{y_k^*}\sqrt{({y_k^*}^2 - \lambda_k\tau^2)^2 + (\alpha\tau y_k^*)^2}}$$

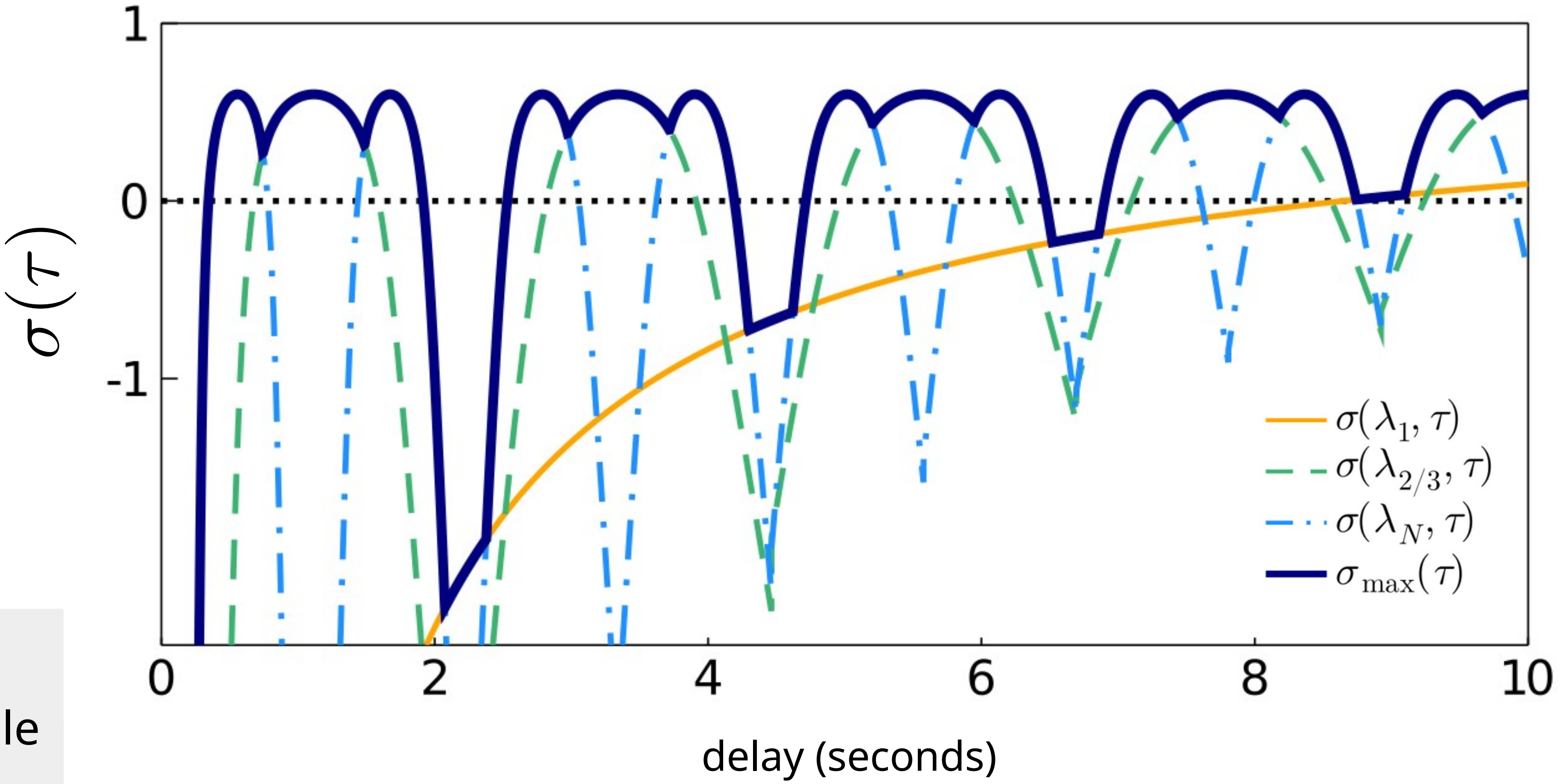
Delay master stability function (dMSF)

$$\boxed{\sigma(\lambda,\tau) = 1 - \frac{1}{\gamma\tau y^*(\lambda,\tau)}\sqrt{\left(y^*(\lambda,\tau)^2 - \lambda\tau^2\right)^2 + \left(\alpha\tau y^*(\lambda,\tau)\right)^2}}$$

- Stable if and only if $\sigma(\lambda_k,\tau) < 0 \quad \forall\ \lambda_k$

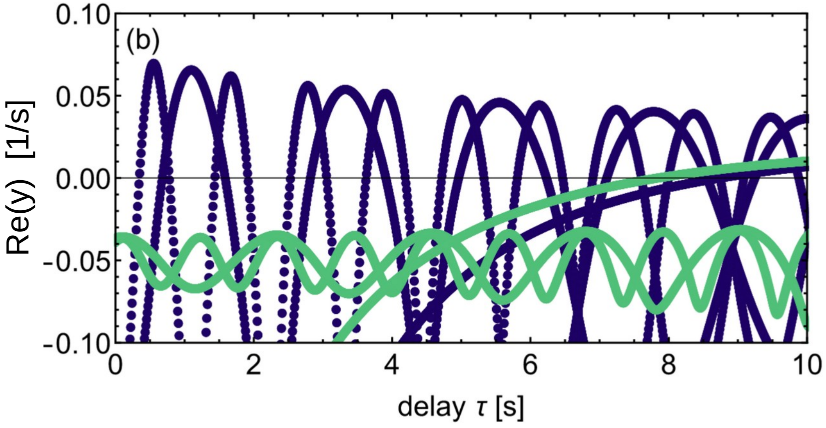
Analytical approach – Delay master stability function

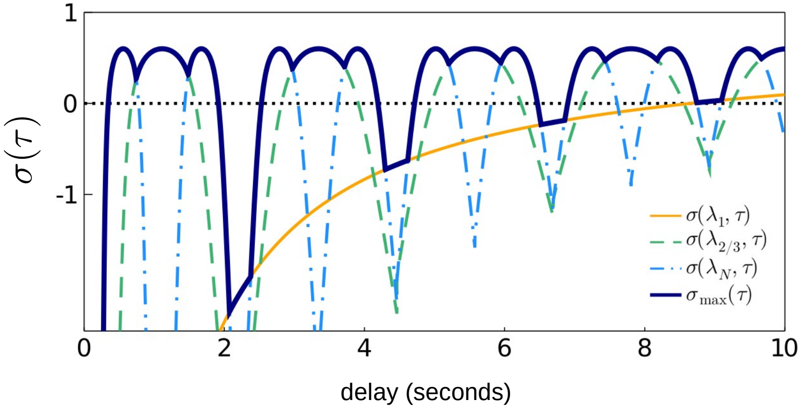




$\sigma < 0$ stable
$\sigma > 0$ instable

Analytical vs. numerical





Numerical

- Compute large number of roots (e.g. 10^7)
- Possibly not all critical roots found

Analytical

- Compute max. 2N roots
- Necessary and sufficient stability conditions

Example 2: Kuramoto model with delayed coupling

Second-order Kuramoto model

$$\boxed{\ddot{\varphi}_i = -\alpha\dot{\varphi}_i + \beta\left(P_i^d - \sum_{j=1}^{N} K_{ij}\,\sin\left(\varphi_i^\tau - \varphi_j^\tau\right)\right)}$$

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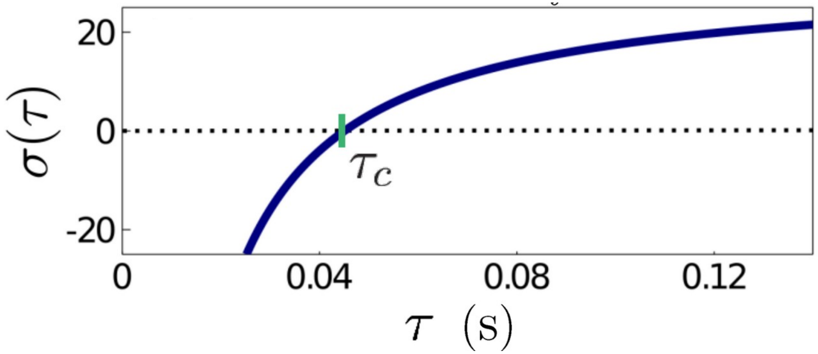
Stability condition

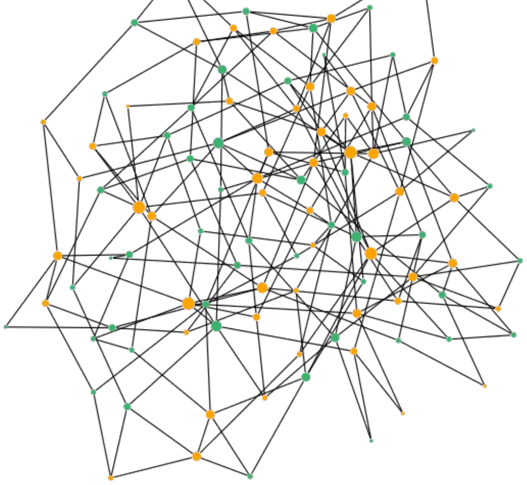
$$\lambda_N < \frac{1}{\beta}\sqrt{\left(\frac{y_1}{\tau}\right)^4 + \alpha^2\left(\frac{y_1}{\tau}\right)^2}$$

$$y_1 = \alpha\tau\cot y_1$$

Delay master stability function

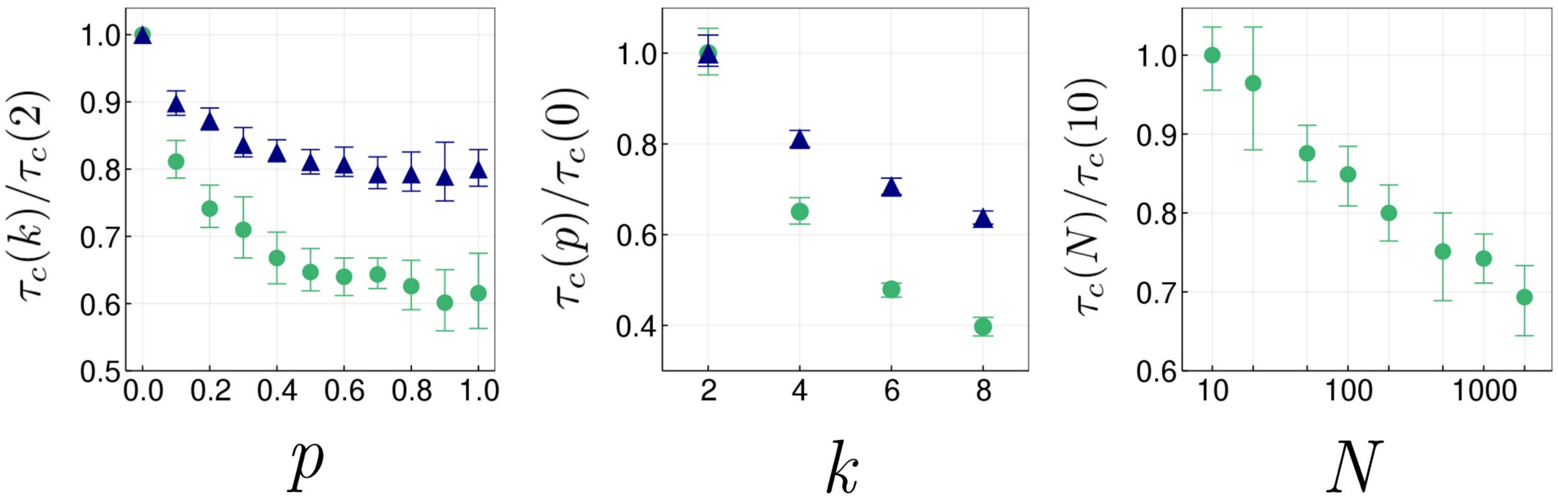


Critical delay in small world networks



fixed values
p = 0.5, k = 4, N = 100

● Kuramoto-like model
▲ Smart grid control model



- **Energy transition:** challenge for design and control of stable power grids

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- Complex oscillator networks: powerful framework to study dynamics of real system

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- **Complex oscillator networks:** powerful framework to study dynamics of real system
- **Delay master stability approach:** necessary and sufficient stability conditions

- Energy transition: challenge for design and control of stable power grids
- Complex oscillator networks: powerful framework to study dynamics of real system
- Delay master stability approach: necessary and sufficient stability conditions
- Old showers: have patience

Thank you!

References

- Schäfer et al. (2015) New J. Physics 17, 015002
- Schäfer et al. (2016) Eur. Phys. J. Special Topics 225, 569-582
- Börner et al. (2020) Phys. Rev. Research 2, 023409
- more on github.com/reykboerner/delay-networks