H5.

Ronald Weasley

1. (10 points) Arrange the functions

$$3^n, 2^n, n2^n, n^{30}, (\log n)^3, \sqrt{n}\log^2 n, n\log n, \sqrt{n!}, n^{29}+n^{28}, n^{\sqrt{n}}$$

   into increasing order of growth rates.

2. (10 points) To solve a particular problem you have access to two algorithms. The execution time of the first algorithm can be given as a function of the input size n as $f(n) = n^{1.5}\log^2 n$. The execution time of the second algorithm is similarly: $g(n) = n^2$. Which algorithm is faster asymptotically? Is this algorithm faster for small n? Find the minimum problem size n needed so that the fastest asymptotic algorithm becomes faster than the other one. Hint: limit your search in powers of 2. You may use calculators to help you but the answer must self contained.

3. (10 points) What is the largest problem size n that we can solve in no more than **one hour** using an algorithm that requires $f(n)$ operations, where each operation takes 10^{-9} seconds (this is close to a today's computer), with the following $f(n)$?

   (a) $\log_2 n$
   (b) $\log_2^5 n$
   (c) $4n$
   (d) $2n\log_2 n$
   (e) $\frac{n}{2}\log_2^2 n$
   (f) n^2
   (g) $(n/2)^3$
   (h) 2^n
   (i) $n!$
   (j) n^n

4. (10 points) Use pseudocode to describe an algorithm that determines whether a given function from a finite set to another finite set is one-to-one.

   Hint: You may assume that the domain is $A = \{a_1, \ldots, a_m\}$ and the co-domain is $B = \{b_1, \ldots, b_n\}$. The function $f : A \to B$ is given as a set of pairs $\{(a_i, f(a_i)) | \forall a_i \in A\}$.