H4.

Harry Potter

1. (12 points) For each of these partial sequences of integers, determine the next term of the sequence, and then provide a general formula or rule to generates terms of the sequence.

   (a) $3, 4, 7, 12, 19, 28, 39, 52, 67, 84, 103, \ldots$

   (b) $7, 12, 17, 22, 27, 32, 37, 42, 47, 52, 57, \ldots$

   (c) $1, 2, 2, 2, 3, 3, 3, 3, 3, 5, 5, 5, 5, 5, 5, 5, \ldots$

   (d) $3, 9, 81, 6561, 43046721 \ldots$

2. (4 points) Compute each of these double sums.

   (a) $\sum_{i=1}^{2} \sum_{j=2}^{4} (i + j/2)$

   (b) $\sum_{i=0}^{2} \sum_{j=0}^{3} (3i + 2j)$

   (c) $\sum_{i=1}^{3} \sum_{j=0}^{2} i$

   (d) $\sum_{i=0}^{2} \sum_{j=1}^{3} i^2 j$

3. (6 points) Compute each of these sums.

   (a) $\sum_{i=0}^{n} 5^{i+1} - 5^i$

   (b) $\sum_{i=0}^{2n} (-3)^i$ (hint: split series in two parts)

4. Consider the series $\sum_{k=2}^{2n+1} \frac{2}{k^2-1}$.

   (a) (4 points) Write the series as a telescoping series.

   (b) (6 points) show

   $$\sum_{k=2}^{2n+1} \frac{2}{k^2 - 1} = \frac{3}{2} - \frac{1}{2n+1} - \frac{1}{2n+2}$$

   Hint: write out at least the first six terms and the last two terms, and group them in pairs of two.

5. (8 points) Prove by induction that $\sum_{i=1}^{n} i^2 = \frac{1}{6} n(n+1)(2n+1)$.