Mr. Pumblechook

1. (5 pts) Show that if n is an integer, then $n^2 \equiv 0$ or $1 \pmod 4$

2. (5 pts) Show that a positive integer is divisible by 11 if and only if the difference of the sum of its decimal digits in even-numbered positions and the sum of its decimal digits in odd-numbered positions is divisible by 11.

3. (5 pts) Show that if a, b, and m are integers such that $m \geq 2$ and $a \equiv b \pmod m$, then $\gcd(a, m) = \gcd(b, m)$.

4. (5 pts) Use Fermat's little theorem to find $23^{1002} \mod 41$.

5. Some airline tickets have a 15-digit identification number $a_1 a_2 \ldots a_{15}$ where a_{15} is a check digit that equals $a_1 a_2 \ldots a_{14} \mod 7$.

   (a) (5 pts) Find the check digit a_{15} that follows from these initial 14 digits of an airline ticket identification number, 00193222543435.

   (b) (15 pts) Produce a function (in Python) that takes a string, $a_1 a_2 \ldots a_{15}$ and returns True if it is a valid airline ticket identification number.