Network (or spatial) autocorrelation models: From Galton's problem to Galton's opportunity

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Roadmap

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Motivation

Introduction

- In organizational research, we are often interested in questions like...
  - what factors influence employee mobility?
  - why do firms engage in acquisitions?
  - how can organizations motivate innovation?
- Researchers have looked at these questions using many methods...
  - a strength of organizational research is its methodological pluralism
- ...but the most common approach would probably be regression.

A simple model

- Imagine we are studying one of the questions above using the model...

$$y = X\beta + \epsilon \tag{1}$$

- To obtain valid inferences, we need to make some assumptions, including...
  - linearity
  - homoscedasticity
  - normality
  - **independence**

Independence assumption

- Today, I want to focus on violations of the independence assumption.
- In many organizational settings, this assumption is heroic.
- Consider the research questions/topics from earlier.
  - What factors influence employee mobility? (vacancy chains)
  - Why do firms engage in acquisitions? (opportunity)
  - How can organizations motivate innovation? (influence)
- Generally, our problems arise from a few common features of data.
  - time
  - **space**
  - groups
- In any case, we have a problem because we will underestimate our errors.

Solutions

There are many solutions...

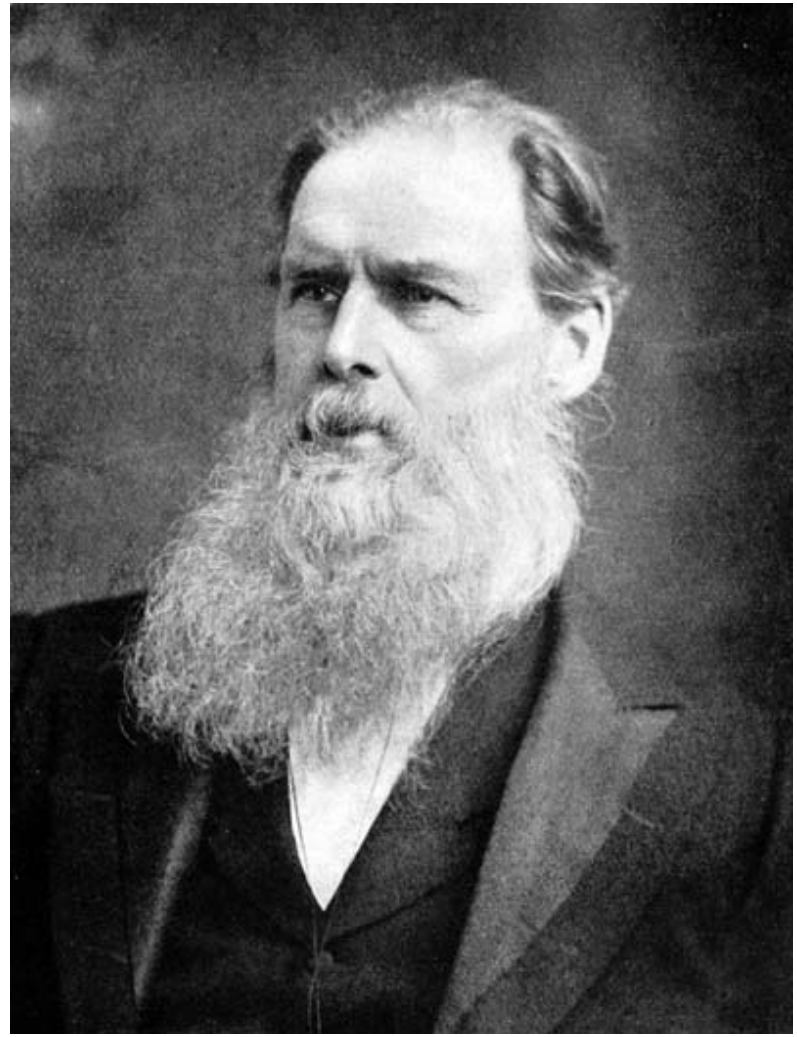
Here are a few (semi) random examples...

- Testing
  - Durbin–Watson statistic (compare errors across adjacent lags)
  - Mantel test (compare two distance matrices)
  - Moran's *I* (measure dispersion on a variable)
  - Moulton factor (compare error variance with/without cluster structure)
- Adjusting
  - Clustered standard errors
  - Cochran-Orcutt (model residuals and transform model)
  - Block bootstrap (resample using cluster structure)
  - Use group level data with weights
  - Quadratic assignment procedure (resampling)
- Modeling
  - exponential random graph models
  - **network autocorrelation model**

History lesson

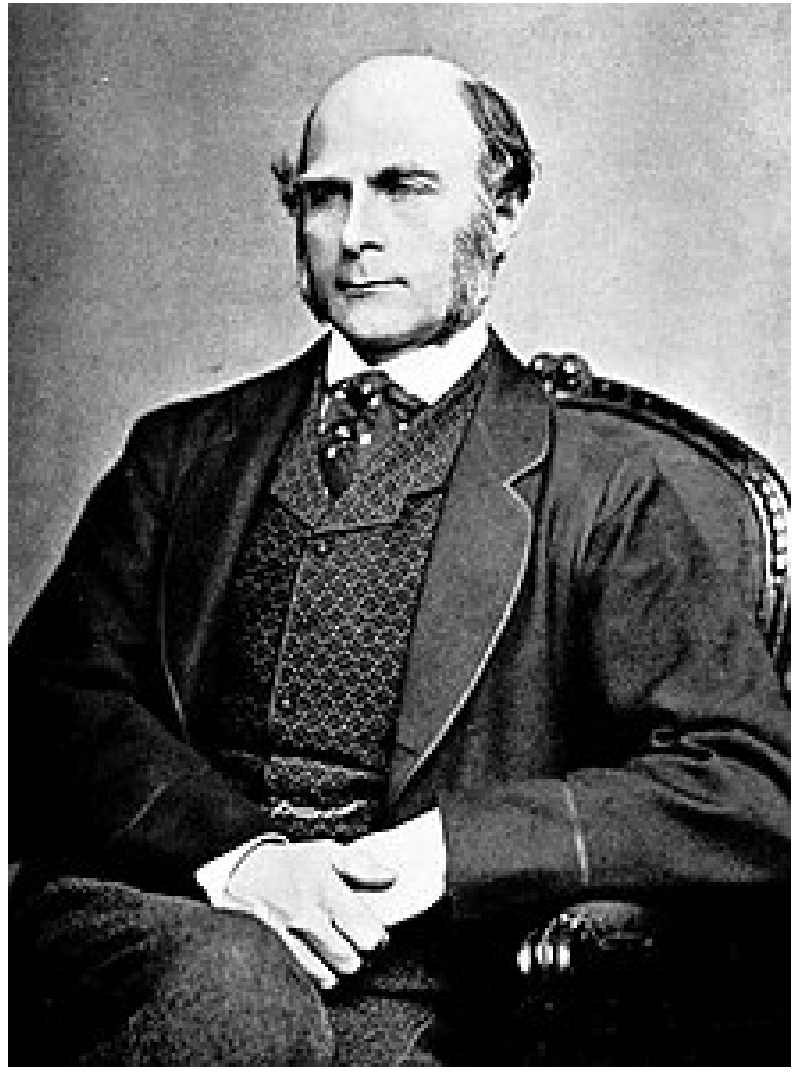
There's some interesting history here...

- It's 1899. At the Royal Anthropological Institute.
- "Big" data revolution 1.0.
- Edward B. Tylor is presenting a paper, "On a method of investigating the development of institutions applied to the laws of marriage and descent."
- Tylor's goal is to introduce a new method for the study of cultural evolution.
- The basic idea is to demonstrate the co-occurrence of institutions across cultures.
- Things are going well, until...



This guy raises his hand...

- Enter Francis Galton.
- Per Wikipedia, a "statistician, progressive, polymath, sociologist, psychologist, anthropologist, eugenicist, tropical explorer, geographer, inventor, meteorologist, proto-geneticist, and psychometrician." (Feeling exhausted?)
- Galton points out that we need to consider whether the institutions of the cultures studied are independent.
- "It might be, that some of the tribes had derived them from a common source, so that they were duplicate copies of the same original."
- He asks to see a map of the geographical distribution of the cultures and their institutions.



Fast forward...

- Galton's problem major issue in cultural anthropology.
- If cultures are interdependent, how can we conduct cross cultural research?
- Network autocorrelation has emerged as a powerful modeling framework.
- These models are related to spatial autocorrelation models in geography.
- But overall, they are not particularly well known in organizational research.

Network autocorrelation

Network autocorrelation model

Background

- There are several versions of the model.
- The simplest and most common is known as the "effects" model.

Effects model

- Hypothesizes i's values on y depend on y's of observations to which i is linked.
- We can think of y as a function of **interaction** and **local** effects.

$$y = \rho W y + X\beta + \epsilon \tag{2}$$

Parameters

- W is an adjacency matrix that captures the interdependence among observations.
  - Typically set diagonals to 0.
- We will come back to W in a bit.
- ρ is analogous to a correlation coefficient capturing the strength of interdependence.
- When $\rho = 0$, we have a standard regression.

Estimation

- The model is typically estimated via maximum likelihood.
- Standard errors and R^2 using OLS are incorrect; need to be adjusted.

Network autocorrelation model

Disturbances model

- Hypothesizes that autocorrelation affects the disturbances term.

$$y = X\beta + \epsilon \tag{3}$$

$$\epsilon = \rho W\epsilon + u \tag{4}$$

Parameters

- W is an adjacency matrix that captures the interdependence among observations.
- ρ is analogous to a correlation coefficient capturing the strength of interdependence.

Which model should you use?

- It all comes down to your theory.
- In the effects model, i's y is responsive to other's y.
- In the disturbances model, i's y is responsive to other's deviations from expected y.

Other variations on the network autocorrelation model

Multiple regimes model

- We can have multiple weight matrices.
- Possible for both the effects and disturbances models.

$$y = \rho_1 W_1 y + \rho_2 W_2 y + X\beta + \epsilon \tag{5}$$

$$y = X\beta + \epsilon, \epsilon = \rho_1 W_1 \epsilon + \rho_2 W_2 \epsilon + u \tag{6}$$

Joint effects and disturbances model

- We can have a joint model with effects and disturbances.
- Possible for $W_1 = W2$ if we wish.

$$y = \rho_1 W_1 y + X\beta + \epsilon \tag{7}$$

$$\epsilon = \rho_2 W_2 \epsilon + u \tag{8}$$

Two views of the network autocorrelation model. . .

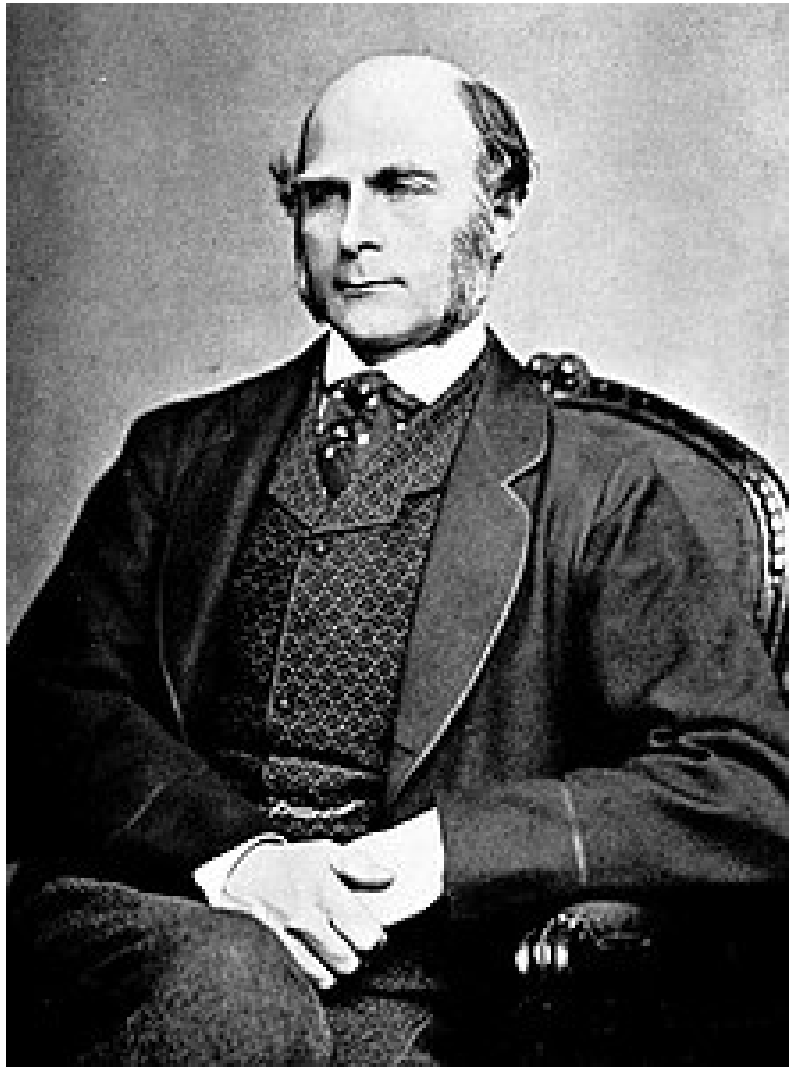
We can think about the network autocorrelation model in two ways. . .

Galton's problem

- Network autocorrelation is a nuisance to be controlled or adjusted for.

Galton's opportunity

- Network autocorrelation is indicative of an interesting phenomenon in our research context that we should model and study.



Practical considerations

Constructing the weight matrix

- It's important to think carefully about how to construct the weight matrix.
- Use a different matrix, get a different set of results.
- Most critically, the choice should be based on your theory.
- Must-read paper on this point is Leenders (2002).

Example 1—Geography

- inverse geographic distances
- indicators for common membership (e.g., city, state, country)

Example 2—Social influence

- cohesion (i.e., direct connections)
  - helps get at communication
- structural equivalence (i.e., similarity of connection)
  - helps get at comparison

Normalizing the weight matrix

- Typically, you will want to normalize the weight matrix.
- Row versus column matrix doesn't really matter when you have undirected edges.
- But it makes a substantive difference with directed edges.
- **Row normalization**
  - Weight proportional to outdegree
  - Total amount of accepted influence equal for all actors
  - Same weight attached to each outgoing tie
  - Deals with accepted/received influence
- **Column normalization**
  - Weight proportional to indegree
  - Total amount of exerted influence equal for all actors
  - Same weight attached to each incoming tie
  - Deals with exerted/executed influence

Comparing different weight matrices

- What do you do when you can't choose between several weight matrices?
- There are several statistical tests available for differentiating.
- Not going to go into the details today.
- But for a great introduction, see Leenders (2002).

The problem of network density

- Research has identified some systematic bias in the network autocorrelation model.
- The bias increases with increases in network density (Mizruchi and Neuman, 2010).
- We tend to see negatively biased values of ρ.
- But their is good news
  - The bias tends to kick in at high densities relative to what we typically observe.
  - The bias is in a direction that likely works against our hypotheses.

"Given the fact that the estimates of ρ tend to be biased downward, any findings of significantly positive ρs in substantive analyses should be taken as especially strong, particularly those based on high-density networks."—Mizruchi and Neuman (2008).

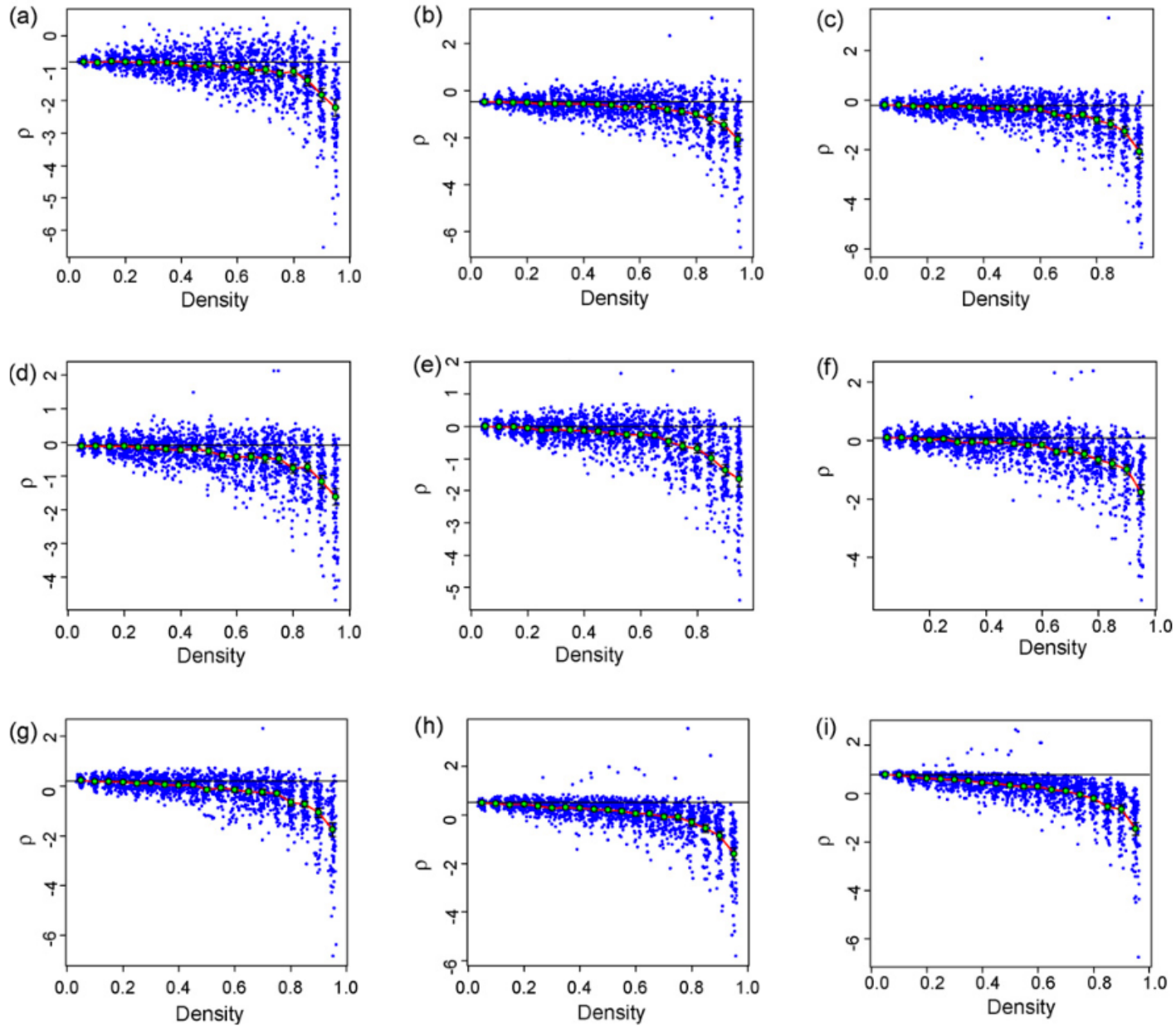


Fig. 1. Scatterplot of the estimated network effect (ρ) against the actual density of W for each of 100 replications at 19 different target values (.05–.95) of the density of W at 9 different population values of ρ. The model includes 3 X variables and a constant drawn from a standard normal distribution, a random row-normalized W of size 50, and error terms drawn from a standard normal distribution. (a) $\rho = -0.8$; (b) $\rho = -0.5$; (c) $\rho = -0.2$; (d) $\rho = -0.1$; (e) $\rho = 0$; (f) $\rho = +0.1$; (g) $\rho = +0.2$; (h) $\rho = +0.5$; (i) $\rho = +0.8$.

Software

Software

The good news is, there are many options for estimating network autocorrelation models.

R

- `sna` package (see the `lnam` function)
- `tnam` package (temporal network autocorrelation models)

Stata

- Full suite of built-in commands.
- `spregress`—spatial autoregressive model
- `spivregress`—Spatial autoregressive models with endogenous covariates
- `spxtregress`—Spatial autoregressive models for panel data

Further reading

Further reading

Methodological papers

- Dow, Malcolm M. "Galton's problem as multiple network autocorrelation effects: cultural trait transmission and ecological constraint." **Cross-Cultural Research** 41, no. 4 (2007): 336-363.
- Dow, Malcolm M., Michael L. Burton, Douglas R. White, and Karl P. Reitz. "Galton's problem as network autocorrelation." **American Ethnologist** 11, no. 4 (1984): 754-770.
- Leenders, Roger Th AJ. "Modeling social influence through network autocorrelation: constructing the weight matrix." **Social Networks** 24, no. 1 (2002): 21-47.
- Mizruchi, Mark S., and Eric J. Neuman. "The effect of density on the level of bias in the network autocorrelation model." **Social Networks** 30, no. 3 (2008): 190-200.

Substantive papers

- Burt, Ronald S. "Social contagion and innovation: Cohesion versus structural equivalence." **American Journal of Sociology** 92, no. 6 (1987): 1287-1335.
- DellaPosta, Daniel, Yongren Shi, and Michael Macy. "Why do liberals drink lattes?" **American Journal of Sociology** 120, no. 5 (2015): 1473-1511.
- Mizruchi, Mark S., Linda Brewster Stearns, and Christoper C. Marquis. "The conditional nature of embeddedness: A study of borrowing by large US firms, 1973–1994." **American Sociological Review** 71, no. 2 (2006): 310-333.
- Papachristos, Andrew V. "Murder by structure: Dominance relations and the social structure of gang homicide." **American Journal of Sociology** 115, no. 1 (2009): 74-128.

Wrap up

Wrap up

Recap (+ a little soap box)

- Nonindependence is a major issue in almost all areas of organizational research.
  - People like to beat up on social network research for violating independence.
  - But, I would argue that most areas are equally subject to this challenge.
  - The problem is often just not acknowledged.
- Network techniques are a great technique for addressing nonindependence.
- More than seeing nonindependence as a nuisance, these offer substantive insight.

Appendix