

# Sailboat Performance Testing Techniques

By Arvel Gentry

Boeing Commercial Airplane Company

Proceedings of the Eleventh AIAA Symposium on the Aero/Hydronautics of Sailing

September 12, 1981

Seattle, Washington

## Abstract

**Testing methods, data reduction techniques, and data analysis programs used in the performance testing of racing sailboats are reviewed. Instrumentation and recording equipment are also discussed.**

## 1. Introduction

Sanding the bottom, checking and re-checking the sails with the sailmaker, and studying the local wind and tide conditions are standard practice for the serious racer. As the boats get bigger, the stakes get higher, or the desire to win increases; so does the search for other factors that might improve performance. However, a still untapped resource for many sailors is the knowledge of how his boat should perform in given sailing conditions.

Knowledge of boat performance can sometimes have an important influence on your chances of winning. One simple example is the knowledge of your boat's tacking angle in different wind and sea conditions. In light winds, the tacking angle will be quite large; on some boats, greater than 100 degrees. In smooth water and higher winds, the tacking angle may get down into the low 70's. Critical tactical decisions such as tacking to the layline, close boat crossing situations, and hitting the proper point on the finishing line will be more accurate if you know your boat's tacking angle characteristics.

Selection of sails for the next leg of a course requires either good guesswork, or a good understanding of boat performance and the apparent to true wind relationships. Many boats carry the wrong sails or fail to reef at the proper time because of a lack of knowledge of their boat's performance characteristics.

All of these problems can be solved with experience. However, proper boat testing can shorten this learning period. The history of the America's Cup races shows a continuous concern for achieving the best possible boat performance. Performance testing with a pace boat and the use of onboard electronic equipment and computers have become standard practice on America's Cup boats, for today's maxi-ocean racers, and to a lesser extent, for boats on the grand prix racing circuit.

This paper presents a basic summary of the methods used in modern sailboat testing. The information presented is based on the author's personal experience with his own boats, plus what he has learned in sailing and testing on other boats.

## 2. Personal Experiences in Sailboat Testing

Sailboat performance testing has evolved rapidly over the last ten years as a result of both intensive activity on the 12-meter boats, plus the advancement of electronic and computer technology. My own personal experiences illustrate this evolution.

My interest in boat performance started with attempts to measure the effect of heel and bow-down trim on a 14 foot dinghy under very light wind conditions. Small pieces of paper were dropped in the water and timed with a stop watch to measure changes in boatspeed with trim changes. Later, when I moved to a Cal 20 and had a knotmeter available, I collected data to determine the optimum downwind tacking angles (Reference 1).

In 1970 I found out that the 12-meter *Intrepid* was using an onboard strip recorder to record boat performance. I then set out to develop a similar device for my own boat. If the 12-meter boats made use of new technologies in order to increase their chances of winning, then why couldn't I? My objectives were simple. I did not have the experience of the people that I was racing against. I certainly didn't have the resources like the 12-meter boats did, but maybe my technical skills could help make up for my lack of sailing experience. My approach was certainly small-time by comparison to the 12's, but it is more typical of what most sailors might be able to achieve.

When I purchased *Kittiwake*, a Ranger 23, in 1972, I equipped it with full sailing instruments. The first strip recorder that I developed for use on this boat is shown in Figure 1 and was described in detail in Reference 2 (all References & Figures are at the end of the text). This device recorded boatspeed as the primary signal, and periodically switched to record a few seconds of wind speed. Wind angle and heel angle data were recorded by hand or written directly on the strip of paper as it came out of the recorder. A sample strip of output is shown in Figure 2.

In 1974, Jim Kilroy asked me to build a recorder for his new maxi-boat, *Kialoa III*, and to participate in the trials of the boat off St. Petersburg, Florida. The requirement was to develop a device that could be used on board during sea

trials that would furnish an onboard record of performance, and that would help to determine both average sailing data as well as dynamic performance during tacking. The device that I developed was more complicated than that described in Reference 2, but still used a strip recorder as the recording media. The *Kialoa III* recorder is shown in Figure 3.

After the experience on board *Kialoa III*, I built a similar recorder for my little Ranger 23, *Kittiwake*. New electronic circuits were used, but all of the basic functions of the *Kialoa III* recorder were retained. The electronic circuits were designed by a friend, Alan Sewell. This recorder is shown in use in Figures 4 and 5. It is described in more detail in a later section.

In late 1980, Jim Kilroy asked me to help out during the sea trials of his newest *Kialoa* maxi-boat (*IV*). The trials were again held out of St. Petersburg, Florida, but this time the professionals provided the computer recording equipment. The testing plan, organization, and engineering support were provided by David Pedrick. The recording and computing equipment were furnished by 12-meter performance expert, Richard McCurdy. Most of the equipment was the same as Pedrick and McCurdy had used for the 12-meter, *Clipper*, in 1980 (where it was identified as the Starship Nova system). My role on the new *Kialoa IV* was as a performance testing engineer.

The navigation station on *Kialoa IV* is shown in Figure 6. The CRT terminal at the left was used to control the onboard Micro-Nova computer and to enter sail trim, sea conditions, etc. The computer itself and the floppy disk system is shown in Figure 7 (McCurdy is obviously a hardware man!). Figure 8 shows the equipment used to reduce and analyze the test data. This equipment was located in a shoreside trailer and consisted of a Data General Nova Mini-Computer, several terminals, and a plotter. This shoreside computer equipment had also been used for the 12-meter, *Clipper*. The floppy disk was not used on *Clipper* since the data was sent to a shore receiver by radio. The capability of this equipment is described in more detail in Reference 3.

The Micro-Nova computer and terminal were removed from *Kialoa IV* after the sea trials. However, McCurdy has since developed the necessary interfaces between the Brookes & Gatehouse Hercules microcomputer system and an onboard Apple computer. The Apple computer reads data from the Brookes & Gatehouse data lines and the satellite navigation computer, and then performs a variety of additional performance, tactical, and navigational computations (4). The computer functions of the B & G Hercules system are turned off, and the Apple computer removed from the boat for races that do not permit this type of equipment.

## 2. Electronic Boat Instrumentation

Almost all grand prix racing boats have rather complete

sets of onboard instrumentation. Even local racing boats are sporting these expensive arrays of dials and digital readouts. The basic instruments required for normal boat testing are:

- Boatspeed
- Apparent wind speed
- Apparent wind angle (0 to 360 degrees)
- Compass
- Heel angle

Successful sailboat testing requires a thorough understanding of each of these gadgets.

### 2.1 Boatspeed

Boatspeed is usually measured by a sensor extending through the hull. Present sensors fit into three classes:

1. Paddle Wheel
2. Propeller
3. Direct Force Measurement (strain gauge)

Figure 9 shows samples of these basic types of sensors. The paddle wheel on the left is by Signet. The center propeller sensor is the part of the B&G sensor that protrudes outside of the hull. On the right is the direct force measurement sensor by Telcor.

However, the basic problem with all this equipment is the sensor location on the hull. The speed of the water past the sensor location is not necessarily the true speed of the boat. The water changes both speed and direction as it flows past the hull. Sensor position error can be quite significant, depending on the type and location of the sensor, the size of the boat, and the sailing conditions.

If the shape of the boat and internal structure permits, the best position for the sensor is usually on the centerline ahead of the keel. For the paddle wheel and strain gauge type of sensors, this will give readings that do not change from tack to tack (heeled readings may still be different from upright values). It seems obvious that boat manufacturers should provide an appropriate centerline speedo thru-hull location in with the design and construction of the boat, but this is seldom done.

Sensors that use a small propeller or spinner, however, still may read different between tacks, even when the centerline location is used. This is caused by the fact that the propeller rotates the same direction on both tacks. The effect of the prop support and the weed guard may cause the prop to spin faster on one tack than on the other.

Large boats frequently use two sensors positioned on either side of the hull in front of the keel. A gravity switch is used to automatically select the lee-side sensor. Much time and effort is required to obtain consistent readings between tacks for this type of installation. Alignment of the two sensors may have to be different in order to give consistent readings on the two tacks. This may introduce errors for the upright downwind sailing conditions. Manual switching may be required to select the most

reliable sensor signal, depending upon the sailing condition.

The boatspeed sensors should be carefully calibrated by sailing measured miles or by sailing close to a boat with well calibrated instruments. Calibrations should be performed at various heel angles on both tacks and in the upright condition. Uncorrectable errors should be recorded and the proper corrections applied to all measured boatspeed data.

Sensors that use paddle wheel or prop rotation counter circuits are usually quite stable once the instrument is properly calibrated. With time, however, wear or damage to the bearings can affect the readings. The electronic circuits, themselves, can frequently be checked at the dock by placing a 60 Hz signal near the sensor (such as a soldering iron).

A direct force measurement sensor, such as that manufactured by Telcor Instruments, is very sensitive at low speeds and is not affected by local flow angles. Calibration is accomplished just as with other sensors. However, subsequent checks of the calibration can be accomplished by simply hanging a small weight on the retracted sensor tip and checking the reading. This can even be accomplished underway.

The type of boatspeed cockpit display depends upon personal preference. An analog display can usually be averaged by eye better than the digital display. The digital display can give a more accurate instantaneous reading, but since the readings are almost always changing, average values are harder to read. Recording data manually requires some care. Either record an average reading, or record many readings and determine the average mathematically.

Some boats make use of automatic speed recording devices. The type of sensor may influence the selection and design of the recorder equipment. A digital circuit may require a D/A converter if the data is to be recorded on an analog device such as a strip recorder.

## 2.2 Wind Speed

There are several different types of wind speed sensors. Figure 10 shows three examples. The rotating cup sensor is most frequently used although it does have its problems. They are nonlinear at the low speeds, the basic calibration may be affected by heel angle, and they have bearings that wear out.

The wind speed sensor made by Telcor Instruments is a solid state device with no moving parts, and avoids most of these problems. The wind blowing past a thermistor tends to cool the unit. The amount of current necessary to heat the sensor back up to the balanced condition can be measured and converted to wind speed. The thermistor tip must be cleaned occasionally to remove spider webs that degrade the sensitivity.

The biggest problem with wind speed sensors is the location. The masthead is subjected to flow distortions and speed errors due to the flow created by the sails. The height of the sensor above the water must also be considered when comparing data taken on different size boats (because of the wind speed gradient with height).

## 2.3 Wind Direction

Several major problems plague boat wind direction devices. The first is that most systems on boats that I have been on are not aligned properly so that they read the same on both tacks. Careful alignment at the masthead, together with small electrical adjustments at the navigation table, should give consistent readings.

The next problem is that the wind direction sensor measures what it sees (the wind direction at its location). This may not be the correct apparent wind angle because of flow distortion due to the sails (the upwash effect), and because of heel angle. Means of correcting for these effects will be covered later.

Most wind direction sensors are integrated with the wind speed device so that the rotating cups are located under the wind vane. This means that the most practical position for the unit is on a rod extending at an angle out in front of the masthead. In this position it is subjected to strong sail upwash effects. These effects may be corrected for windward conditions, but are more difficult to account for in the running and reaching conditions. On large boats in broad reaching conditions, the removal of a staysail may significantly affect the wind direction reading.

The last problem inherent in wind direction measurements is the fact that the reading may be fluctuating quite a bit. Average readings may be difficult to obtain. Most systems have an adjustable electronic dampening control to slow down the system response so that the readings are not always jumping all over the place. This will cause problems if you are studying dynamic maneuvers such as tacks. It also means that attempts to sail by a VMG meter may lead to bad results (since the VMG computations use the apparent wind angle).

## 2.4 Compass

Little new can be said about compasses, except that they should be carefully adjusted before serious testing starts. Any errors should be noted and corrections applied to the readings before they are used in the data reduction process. Keep magnetic objects such as pliers, screw drivers, and portable radios away from the compasses during testing just as you would during a race.

If you plan on using one of the more sophisticated instrument and data recording systems, you will need a compass with an electronic readout.

## 2.5 Heel Angle

Heel angle is an important parameter that is frequently

left off sailing data sheets. However, it is required if the proper corrections are to be applied in the data reduction process. Heel angle will usually have to be recorded by hand from readings taken off of small bubble indicators. The more sophisticated systems use an electronic heel angle device. However, none of the presently available microcomputer based boat systems include a heel angle input.

My own electronic heel angle system consists of an instrumentation amplifier circuit with a weighted potentiometer furnishing the heel angle signal. This unit is on the right side of the photo in Figure 5.

## 2.6 Leeway Angle

There is presently no completely satisfactory leeway angle measuring device. Various leeway angle measurement techniques have been tried with varying success (the local flow angles measured on the boat are not the same as the true leeway angle). Sometimes, a line is towed behind the boat and a large protractor used to record the angle that the line makes with the boat centerline. However, this system is difficult to use because of the normal small angle changes in the boat heading as the wind and sea change. Careful navigation from fixed sea markers can also be used, but again, accurate results are difficult to obtain. In the data reduction procedures used in this paper we will use an empirical equation to account for leeway effects.

## 2.7 Navigation Instruments

Sophisticated modern navigation instruments may be of some help in sailboat testing and their use should be investigated if you have them on your boat. Satellite navigation systems coupled with Omega systems have been used to help detect water current variations that would affect testing.

## 2.8 Microcomputer Based Systems

The microcomputer chip is presently causing a revolution in the sailboat instrument business. Several manufacturers have systems that use microcomputer circuits. The boatspeed, wind speed and direction, and compass sensors send information to a central computer processor. The information is then sent out to the cockpit display instruments.

The new microcomputer based systems have tremendous potential. However, I find the available systems still lacking in some important areas. Most systems compute what is called speed made good to windward (VMG). Accurate computation of VMG requires that corrections be applied for both the upwash at the masthead sensor and for heel angle. None of the presently available microcomputer based systems have a heel angle input sensor. They also do not provide a means for correcting for upwash on different boats.

The Brookes & Gatehouse Hercules 190 system is shown in Figure 11. One of the best features of this system

is the multi-function display units that can be positioned about the boat where they are needed (one boat is reported to have twenty of these units). Figure 12 shows the cockpit of *Kialoa IV* with its two sets of five readouts on either side of the center hydraulic control panel. On board *Kialoa IV*, the Hercules 190 system produces data that is read by the Apple computer, and the Apple computer, in turn, puts output data back on the B&G data line for display on the multi-function units. The Hercules System 190, itself, contains 32 channels of data.

Rochester Instruments makes a microcomputer based boat instrumentation system that was used on *Freedom* in the 1980 America's Cup. A photograph of the system is shown in Figure 13. The system computes speed made good (upwind or downwind), true wind speed, and true wind direction off the bow. One nice feature is the output port for a cassette tape recorder so that the basic sailing parameters can be recorded automatically. Rochester provides a service of converting the cassette tape to printed output form.

Signet also produces a microcomputer based boat system. This system computes speed made good (VMG), true wind speed and direction, and has a start timer.

The present boat microcomputer systems have only limited capacity for the more complex computations. In my opinion, several practical implementation problems have not been solved. As stated previously, none of the systems have a heel angle input, and none provide a means for correcting for upwash. This makes their VMG and true wind results suspect.

A display of relative boat performance would be helpful (as compared to stored polars). The Hercules 190 system uses a built-in set of data that represents general boat performance characteristics (using your input IOR rating). The boat performance is compared with information stored in the computer, and a performance percentage number displayed. Data is provided for either windward or reaching conditions.

Ideally, the user should be able to determine his own boat's performance, and to load it into an EPROM for use by the boat microcomputer system. Another possibility would be to have the data prepared by a home computer (or by a service provided by the instrument manufacturer), and then loaded into the boat microcomputer through a cassette tape.

An onboard computer that is separate from the boat instruments, such as the Apple computer on *Kialoa IV*, provides a powerful system to assist and supplement the normal boat microcomputer instrument.

However, the use of something like the Apple computer requires a number of difficult interfaces with the boat's instrumentation, plus some sophisticated software (4). And last, it would require a boat owner (plus probably a navigator) who could understand and make maximum

use of such a system.

## 2.9 Automatic Data Recorders

Performance data can always be recorded by hand onto data forms. However, this means of gathering data depends upon the judgment and diligence of the person writing down the numbers. Automatic recorders are a more reliable means of recording the number data but have their own problems which must be solved. The primary one is the recording of data that is not available by electronic means.

Data is useless if you do not know what was happening on the boat when the data was taken. This should include such data as the sail configuration, all the sail trim parameters (genoa car location, outhaul, halyard tension, etc.), backstay pressure, running backstay pressures, babystay pressures, helmsman, sea conditions, etc.

### 2.9.1 The Strip Recorder

The strip recorder provides one means of recording both the electronically generated data and the other information mentioned above. Notes can be made right on the strip of paper as it comes out of the machine. The recorder developed by the author for use on *Kialoa III*, and on his own boat, *Kittiwake*, was a rather sophisticated instrument for its time.

This recorder as it was used on *Kittiwake* is shown in Figures 4 and 5. The recorder was kept below during races but used in the cockpit during other testing periods. During short races, an audio cassette recorder was started with the strip recorder and recorded all of sail trim, tacking, and tactical information. A typical record from this recorder is shown in Figure 14.

On the *Kialoa III* and *Kittiwake* recorders, a total of six data signals could be input to the control unit. Only two signals could be recorded at a time, but a combination of automatic and manual switching permitted the effective recording of six parameters on a single strip of recorder paper. Boatspeed (VS) was the primary signal on the lower channel. The two secondary parameters on the lower channel were the apparent wind speed and a spare channel (used for the Brookes & Gatehouse Horatio computer output on *Kialoa III*).

Any signal could be recorded full-time, or the primary signal and one of the selected secondary signals could be automatically alternated. The upper channel had two primary signals that were selected by a switch on the control unit. These were the apparent wind angle scaled from 0 to 180 degrees and the closehauled wind angle. The signal from an electronic heel angle indicator was the single secondary signal for the upper channel. This recorder was used on board *Kittiwake* for all of its races and practice sessions for over two years.

The disadvantages of the strip recorder are that it needs someone to write all of the notes on the recorder paper,

and that the output is not immediately available for use within modern computers.

### 2.9.2 Electronic Recording

The electronic recording and processing of sailboat performance was used extensively in the 1980 America's Cup season. David Pedrick and Richard McCurdy developed a rather sophisticated system for *Clipper* in an attempt to shorten the learning and boat tuning time (3).

Much of the equipment from *Clipper* was used during the sea trials of Jim Kilroy's new *Kialoa IV* in 1981. For *Kialoa IV*, McCurdy had to develop interfaces, incorporate the onboard floppy disk system, and develop new shoreside computer software. As a result, this sophisticated equipment was not ready for the first part of the sea trials. During this period it was necessary to record data by hand and to do all of the data reduction on an HP-41C programmable calculator (that was my job).

During the *Kialoa IV* sea trials, performance polars were updated daily as new data was gathered. The performance testing on *Kialoa IV* was probably the most complete and sophisticated yet applied on a racing yacht (including the 12-meters).

## 3. Testing Techniques

The most important time for serious testing is right after the boat is completed and before the first race. Most owners want to get the most out of their boat as soon as possible, and careful testing can aid significantly in accomplishing this. However, the performance testing must not interfere with other equally important aspects such as crew training, general boat familiarization, sail inventory checks, and rig tuning.

Initial sea trial testing provides the first indications as to how the boat will perform under various conditions. Data gathered during this period should be considered as being preliminary since significant improvements in performance will usually be obtained during actual racing conditions. These initial tests, however, will usually provide a chance to obtain general trends that will be useful in correlating the data obtained during racing conditions.

Accurate performance polars require hundreds of data points. If possible, the data gathering process should continue throughout the racing life of the boat. This will provide an excellent baseline for comparison if modifications are subsequently made to the boat.

It is important that the maximum amount of data be gathered for the boat sailing in smooth water conditions. This usually gives the maximum performance characteristics for the boat. If you know what the boat should be able to do under ideal smooth water conditions, you are better able to judge how the boat should be sailed as the sea conditions deteriorate. Rough sea conditions degrade the performance of the boat. Eventually you will want to

prepare two or more boat performance polars for different sea conditions.

### 3.1 Data Gathering

Since most boats will not have the sophisticated equipment of a *Clipper* 12-meter or a *Kialoa IV* maxi-boat, the rest of this discussion will assume that the data are to be gathered by the hand recording method.

Data to be used for constructing speed polars should normally be recorded when the boat is settled down and at maximum speed for the sailing conditions. However, you at times may find it useful to record data at other odd conditions as the situations arise. For example, odd relative bits of data may help in correlating VMG and performance numbers computed by the onboard instrument system with subsequent values computed by the data reduction program that will include upwash and heel angle.

One of the major problems in the data correlation process is the determination of the true wind strength and direction. As stated previously, heel angle and upwash at the masthead complicate this process. During the testing, it is usually helpful to periodically bring the boat to a complete stop and head to the wind. Record the wind speed and compass direction, and use this data as a check against the true wind values calculated during the data reduction process.

#### 3.1.1 Data Recording Form

Everyone has his own preference for the format and arrangement of the data recording form. One of the ones that I have used is shown in Figure 15.

The data point sequence number is recorded in the "Point No." column. This number may be useful in the data correlation process. The clock time is input in the next column. The P/S column is used to indicate the tack (port or starboard). Next comes the basic boat performance parameters:

VS	Boatspeed
VA	Apparent wind speed
$\beta_A$	Apparent wind angle
$\phi$	Heel Angle
CH	Compass heading

The remaining data columns will be filled out during the data reduction process. These parameters are:

VT	True wind speed
$\gamma$	Polar wind angle (/includes leeway)
Tack/Jibe	Tacking angle for windward work or jibing angle for running
VMG	Speed made good to windward
$\gamma - \lambda$	True wind angle without leeway
WD	Direction true wind is blowing from
$\epsilon$	Upwash correction
$\lambda$	Leeway angle

The area at the right is used for general notes as to sea conditions, sail trim, rigging pressures, comments on the estimated value of the data point, etc..

It is usually a good idea to record the data numbers close to the bottom line on each row. This will leave the top part of the row for subsequent data corrections (i.e., for calibration errors, computations using different upwash constants, etc.). A typical set of data is shown in Figure 16.

#### 3.1.2 Windward Testing

The highest priority should be placed on windward testing. The general procedure, at first, will be to let the helmsman sail the boat at what he thinks is the best windward point of sail. Give the helmsman time to get used to the boat and to get what he thinks is the best performance (the best seat-of-the-pants condition). Record a series of data points over several minutes (I find that one complete set of data taken every minute to be a good procedure).

Then tack the boat and repeat the data on the new tack. This is very important since the compass tacking angle data is necessary in the correlation process to determine the proper empirical upwash constants for the particular boat. It is difficult to obtain useful windward data correlations without a sufficient amount of tacking data.

Next, have the helmsman intentionally pinch the boat a degree or two closer to the wind than he normally thinks is best. Again, gather data on both tacks. Have the helmsman sail at conditions that are two degrees and four degrees wider than he thinks is best. Be sure that the sails are trimmed as good as possible for each data point.

Later, you will calculate the VMG for each of these conditions and plot the results as a function of apparent wind angle. This will tell you what the proper apparent wind angle is for the given wind and sea conditions. The data to be used in constructing the windward performance curves will be picked off of these plots at the maximum VMG speed. The computed true wind directions should be approximately the same (providing the wind is not shifting). This can be checked by noting the compass and apparent wind readings.

#### 3.1.3 Reaching Conditions

Testing for reaching conditions has its own particular problems. Upwash effects may be smaller than the windward conditions but are more difficult to determine. You should frequently bring the boat to a stop heading into the wind, so that you will have some way of determining the upwash correlation constants.

Sail selection is very important for the reaching conditions. The test apparent wind angles should cover the range for each sail configuration. Additional testing beyond what you normally think is the limit angle for a sail configuration will help identify the precise sail change point for best performance.

### 3.1.4 Running Conditions

The primary purpose of testing in the running condition is to determine the optimum jibing angles. As always, sail trim is very important. Be sure that you also have all of the proper sails up (staysails or blooper). First sail dead downwind according to the masthead wind direction indicator. Record data and then jibe over and repeat the dead downwind condition again. You may find a difference in compass readings. This indicates that you have upwash at the wind direction indicator just as with the other sailing conditions.

Next, sail the boat 10, 20, 30, and 40 degrees off of the dead downwind condition. If possible repeat the data on each jibe before going to the next apparent wind angle (a good time for crew training!). Under high wind conditions, be sure to record data both at the peak surfing speeds and at the lower speed lulls between surfing spurts. Mark the data somehow so that you will remember to use only the average of the two readings.

## 4. Data Reduction

The basic purpose of the data reduction process is to convert the data from apparent wind conditions to true wind conditions. Some of the data, such as the windward performance, will eventually be plotted again in terms of apparent wind conditions for use during races. However, even the windward data must at first be converted to true wind conditions in order to establish the best VMG conditions. The basic relationships used in the data reduction process are discussed below, followed by a description of the computer program itself.

### 4.1 Upwash Correction

The upwash correction is probably the least understood by the average sailor. The important thing, however, is to realize that no single upwash correction equation is going to work for all boats. My approach has been to select a form for the correction equation that has the necessary empirical constants to allow me to match the characteristics of a given boat. Although I have used several different forms for this equation, it is instructive to follow the correction as I first derived it in the early 1970's.

First let's look at a very simplified picture and aerodynamic representation of the lift created by the sails. The drawing in Figure 17 is a representation of the vortex systems that can be used to approximate the effect of the sails on the surrounding flow. This is the bound vortex system that represents the lift of the sails and the trailing tip vortex. Detailed aerodynamic analysis also requires an image system below the water, but I won't go into these details here.

From this idealized representation of the sails, we can readily see that the flow direction and speed at the masthead measuring unit are influenced by the vortex system used to represent the sail lift. By looking at this simplified

vortex representation of the sail (which is a quite conventional approach in aerodynamic theory), we can get some idea of how conditions at the measuring unit are affected by the flow field caused by the sails.

For the masthead unit position shown in Figure 17, the bound vortex and the tip trailing vortex are additive in producing an upwash flow field. Without going into the details of the aerodynamic theory, it is sufficient to just state that the upwash velocity at any point ( $w$ ) is directly proportional to the sail lift coefficient.

$$\frac{w}{V_\infty} = A C_L$$

where  $w$  is the upwash velocity  
 $V_\infty$  is the free stream velocity  
 $A$  is a correlation constant that is dependent upon the position of the wind measuring device  
 $C_L$  is the sail lift coefficient

For windward work,  $C_L$  is a maximum at low apparent wind velocities (where the sails are quite full and the boat usually footed off) and is at a minimum at the high apparent wind speeds (where the sails are trimmed quite flat and the boat pinched to keep it upright).

To arrive at a basic form for the upwash correction equation, I used the old classical "Gimcrack" test data. This data is shown in Figure 18 along with the equation for  $C_L$ .

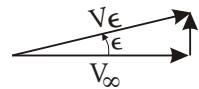
$$C_L = C_1 \cos(C_2 * VA) + C_3$$

The constants required to match the Gimcrack data were as follows:

$$\begin{aligned} C_1 &= 0.65 \\ C_2 &= 6.0 \\ C_3 &= 1.05 \end{aligned}$$

The actual value of the  $C_L$  is not too important. We are only interested in getting a rough idea as to how it changes with apparent wind speed. It is important, however, to remember that these constants will not match your particular boat. You must arrive at the proper constants through repeated trial calculations with different assumed values. The objective is to find the right set of constants that will give the correct tacking angle as measured on the compass for the various apparent wind conditions.

The relationships used to convert the upwash velocity,  $w$ , to an upwash angle are as follows:



$$\begin{aligned} \epsilon &= \tan^{-1}(w/V_\infty) \\ (w/V_\infty) &= A C_L \\ \text{Upwash angle} &= \epsilon = \tan^{-1}(A C_L) \end{aligned}$$

The above relationships are quite flexible, providing you are only working with windward data. However, we really need a single equation that can also be applied to the reaching and running conditions. The general equation

that I have used for these purposes is as follows:

$$\text{Upwash} = \epsilon = S1 * \cos(S0 * VA) * \cos(\beta_A)$$

(when  $VA > S2$  use  $VA = S2$ )

Again, the constants,  $S0$ ,  $S1$ , and  $S2$  must be determined empirically from the test data. For the larger boats, the following ranges have been used (depending upon the boat).

$$\begin{aligned} S0 &= 2 \text{ to } 3 \\ S1 &= 14 \text{ to } 16 \\ S2 &= 25 \text{ to } 30 \end{aligned}$$

The plot shown in Figure 19 gives some idea as to how the upwash changes with different values for the  $S1$  constant. For small boats, the  $S1$  parameter may be small (or even zero for no upwash correction).

## 4.2 Heel Angle Corrections

We know that the apparent wind angle would be correct if the boat were in the upright condition. At a 90 degree heel angle, the wind angle indicator would be useless in measuring the apparent wind angle (the reading would approach zero). These two extreme conditions tell us that we will have to have some correction equation that must be applied to the measured values.

Complex diagrams and geometry can be used to arrive at the heel angle correction equation. However, a much simpler way is to use matrix notation and conventional rotation relationships. To start this analysis, the boat is assumed to be pointed directly into the apparent wind. The boat is then yawed about the vertical Z-axis to the correct boat apparent wind angle ( $\beta$ ) and then rolled about the hull centerline X-axis by the heel angle ( $\phi$ ).

The yaw rotation about the vertical Z-axis is given by the matrix:

$$[\beta] = \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The roll to the required heel angle is given by:

$$[\phi] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}$$

The complete rotation matrix is obtained as follows:

$$[A] = [\beta][\phi] = \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\cos\phi \sin\beta & \cos\phi \cos\beta & \sin\phi \\ \sin\phi \sin\beta & -\sin\phi \cos\beta & \cos\phi \end{bmatrix}$$

The free stream apparent wind vector,  $V_\infty$ , may be expressed in terms of its components in the boat coordinate system ( $V_\infty x$  along the hull centerline,  $V_\infty y$  out to the side, and  $V_\infty z$  up the mast).

$$\begin{bmatrix} V_\infty x \\ V_\infty y \\ V_\infty z \end{bmatrix} = [A] \begin{bmatrix} -V_\infty \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} V_\infty \cos\beta \\ -V_\infty \cos\phi \sin\beta \\ V_\infty \sin\phi \sin\beta \end{bmatrix}$$

The final components of the apparent wind vector in terms of the boat coordinate system are, therefore, given by:

$$\begin{aligned} V_\infty x &= -V_\infty \cos\beta \\ V_\infty y &= V_\infty \cos\phi \sin\beta \\ V_\infty z &= -V_\infty \sin\phi \sin\beta \end{aligned}$$

The sign conventions used in the above equations assume a + yaw to the left and a + roll to heel the boat.

From the above we see that the wind vane at the masthead is exposed to a wind vector,  $V_\infty$ , that is composed of three components:

$V_\infty x$  is the component parallel to the boat centerline and is negative when the wind is forward of the beam.

$V_\infty y$  is the component from the side.

$V_\infty z$  is the component along the mast axis and is negative when the boat is heeled.

The masthead wind vane only rotates about the mast line (Z-axis) and, therefore, does not respond to the  $V_\infty z$  component of the velocity. The angle of the wind measured by the wind vane is therefore given by:

$$\begin{aligned} \tan\theta &= \cos\phi \tan\beta \\ \beta &= \arctan(\tan\theta / \cos\phi) \end{aligned}$$

where  $\beta$  is the angle of the boat centerline to the apparent wind

$\phi$  is the heel angle

$\theta$  is the apparent wind angle measured by the masthead wind vane

The wind speed measured by the rotating cups also may be in error because of the heel angle. However, the rotating cups themselves may have a fundamental calibration error when the flow is not perpendicular to the rotation shaft. It, therefore, becomes somewhat arbitrary in applying a geometric heel angle correction to the wind speed. The correction equation used by David Pedrick and Richard McCurdy (3) is as follows.

$$VA_{\text{corrected}} = VA_{\text{indicated}} \frac{\cos\beta_{\text{uncorrected}}}{\cos\beta_{\text{corrected}}}$$

I have used this same equation in my own data reduction programs so that my answers can be compared directly with Pedrick and McCurdy's results.

## 4.3 Leeway Correction

Since the leeway angle is not measured directly during the testing, we will make use of an empirical relationship as part of the data reduction process. The basic relationship used was derived by David Pedrick (3).

$$\lambda = K \frac{\phi}{VS^2}$$

where  $\lambda$  is the leeway angle in degrees  
 $K$  is the leeway correlation constant  
 $\phi$  is the heel angle in degrees  
 $VS$  is the boatspeed in knots

Values for the constant  $K$  vary between 9 and about 16, depending on the windward efficiency of the boat. This constant will have to be empirically adjusted as experience is gained with a given boat.

#### 4.4 Data Reduction Computer Program

Hand reduction of the data is difficult and time-consuming and not worth the effort since excellent programmable calculators are available. The necessary equations can be programmed on an HP-65 or an HP-41C. With these little handheld computers, the data reduction can be performed on deck during the actual boat testing runs as I did on *Kialoa IV*.

Subsequent analysis and plotting of the data can be accomplished ashore. Mini-computers, such as the Data General Nova Mini-computer, provide the much needed capacity for the storage and analysis of hundreds of data points and for the computer generation of speed polar plots. Home microcomputers, such as the Apple computer, also do an excellent job.

Whether done on an Apple computer or an HP-41C, the basic data reduction process is the same. The basic wind triangle relationships are shown in Figure 20. The following data reduction steps are required.

- (1) Set upwash and leeway correction constants.
- (2) Correct apparent wind angle for upwash.
- (3) Correct wind angle for heel angle.
- (4) Correct apparent wind speed for heel angle.
- (5) Solve the wind triangle to get true wind speed and angle.
- (6) Calculate tacking or jibing angle.
- (7) Calculate VMG.
- (8) Calculate leeway angle.
- (9) Display wind angle. Change sign (CHS) if on port.
- (10) Calculate wind direction.

An HP-41C program for this basic data reduction process is given in Figure 21. The name of the boat should be stored in location 02. Estimates for the upwash constants for your boat must be stored at locations 96, 98, and 100 before running the program. Start with an SI value of 0.0 (no upwash). Set S0 = 3, and S2 = 30. If you have significant upwash at the masthead, the calculated tacking angle will be larger than the value read on the compass between tacks. With a trial and error process, and the use of plots similar to Figure 19, repeat the calculations until the calculated tacking angle is close to the measured value. As you gather data at different apparent wind speeds, you will have to adjust the S0 and S2 constants and then again search for the proper S1 constant. The leeway constant is

stored at location 104 in the program. Start with a value of about 10 and adjust it as you gain experience with the boat.

The program prompts for each of the input parameters. The output values are also identified. A sample output from the program is shown in the lower right corner of Figure 21.

### 5. Data Analysis

The purpose of this whole exercise is to be able to prepare plots of the boat's performance under different sailing conditions. These plots take two different forms.

#### 5.1 Windward Performance

The optimum windward conditions are determined by plotting the VMG data against the indicated apparent wind angle. A sample set of data at two different wind speeds is given in Figure 22.

The optimum VMG correlations are then used to create a complete picture of the windward performance of the boat. This data will be for the optimum windward conditions in smooth water and will be plotted as a function of indicated apparent wind speed. A typical set of plots is shown in Figures 23 and 24. Figure 23 gives the optimum windward boat speed. Figure 24 shows the tacking angle, heel angle, and indicated apparent wind angle.

As experience with the boat increases, you will be able to prepare similar plots for different sea states.

#### 5.2 Boatspeed Polar Diagram

The next major task is to prepare the complete speed polar for the boat. This will require the use of the windward performance data discussed above, plus the reaching and running data (and a lot of guess work).

The usual form of the speed polar is shown in Figure 25. The angle parameter is the true wind angle plus the leeway angle. The heart-shaped boat speed lines are prepared at constant true wind speeds. This plot should really have several discontinuities in slope at the points where the sails are changed. However, the available test data is usually so scarce and contains so much scatter that it is the usual practice to just draw smooth polar curves.

I find that a Cartesian form of the speed polar is easier to work with on the boat than the polar coordinate plot. A sample plot is shown in Figure 26. This plot contains a combination of true wind data and apparent wind data.

The solid lines running from left to right are the boatspeed curves at constant true wind speed. The long dashed lines, running roughly from the top of the plot to the bottom, are lines of constant indicated apparent wind speed. The very short dashed lines running from the left up toward the right are lines of constant apparent wind speed. The optimum windward sailing conditions are shown at the left side of the plot. Optimum downwind

tacking conditions are indicated by the VMG<sub>MAX</sub> line at the right side of the plot.

The generation of this plot is quite an art. The basic problem is that you never have enough data, and what data you do have seems to always have too much scatter. I find it helpful to draw a complete polar plot before even looking at the data obtained on the boat. This purely "guessed" polar can then be used both as a means of analyzing the test data and as a basis for the subsequent polars based on the experimental data. As more test data is obtained, the polar can be shifted and revised so as to better match the test results. Owners of MHS-rated boats can now get a computer generated polar diagram along with their rating. However, this curve should only be considered as the starting point for your own diagram.

## 6. Use of Performance Data

Good boat performance data curves have their greatest use as a yardstick for measuring boat performance during races. Current boat performance can always be compared with the windward plots or the polar data and adjustments sought that will bring the performance up to or greater than the plots. The data can also be used as basic input information for detailed tactical situations (such as selecting the optimum current crossings). As both a tactical and navigational aid, the data can help predict where you will be at a later point in a race and allow studies of possible tactical decisions under different future true wind conditions.

## 7. Conclusions

Knowledge of your boat's detailed performance characteristics can have a significant effect on your chances of winning. Experience in the 12-meter America's Cup boats and maxi-ocean racers clearly illustrate this. However, even the local sailor can improve his chances if he learns more about his boat. The technology required to assess boat performance is available to the average serious sailor. The required equipment includes a complete set of sailing instruments, a programmable calculator such as the HP-41C, and a technical understanding of the testing and data analysis process.

## 8. References

- (1) Arvel E. Gentry, Optimum Downwind Tacking, SEA Magazine, February 1970.
- (2) Arvel E. Gentry, Are You at Optimum Trim?, SAIL Magazine, March 1974.
- (3) David R. Pedrick and Richard S. McCurdy, Yacht Performance Analysis with Computers, Chesapeake Sailing Yacht Symposium, January 1981.
- (4) Joanne Fishman, Kilroy Is Here, MOTOR BOATING and SAILING Magazine, August 1981.

## Biography

Arvel Gentry is presently a research supervisor in the Aerodynamics Research Department at the Boeing Commercial Airplane Company. He has raced his own boats very successfully (primarily in Southern California), and has extensive crewing experience on larger ocean racing yachts. He has authored numerous magazine articles on sailing aerodynamics and sailboat performance. He has conducted research efforts in support of America's Cup projects, and designed the mast section shapes used on *Courageous* and *Freedom*. He has also developed specialized sailboat performance recording equipment and served as a sailing performance test engineer on Jim Kilroy's *Kialoa* maxi-boats.

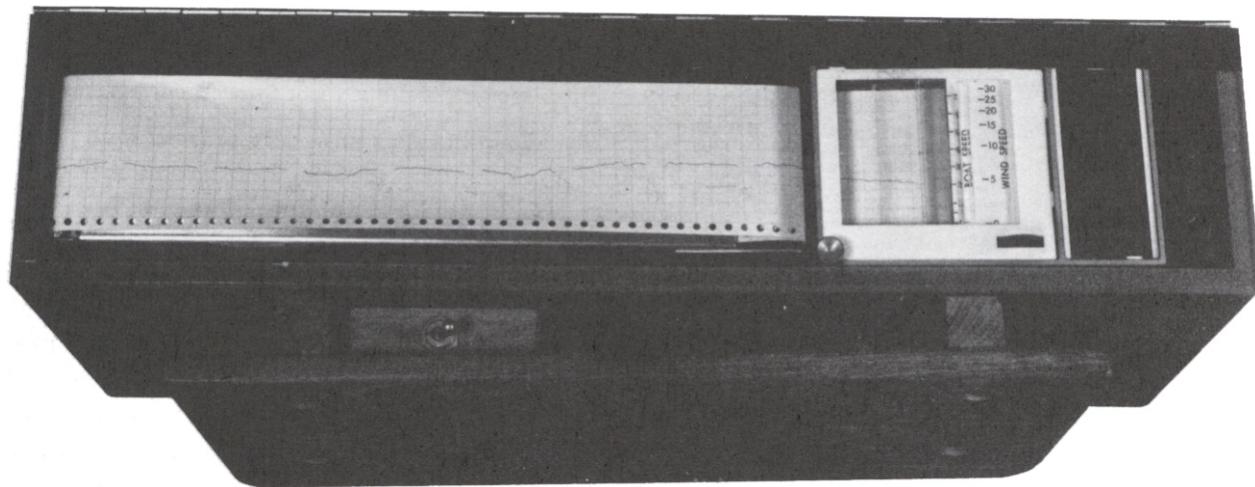


Figure 1 Single channel strip recorder for *Kittiwake* (1972).

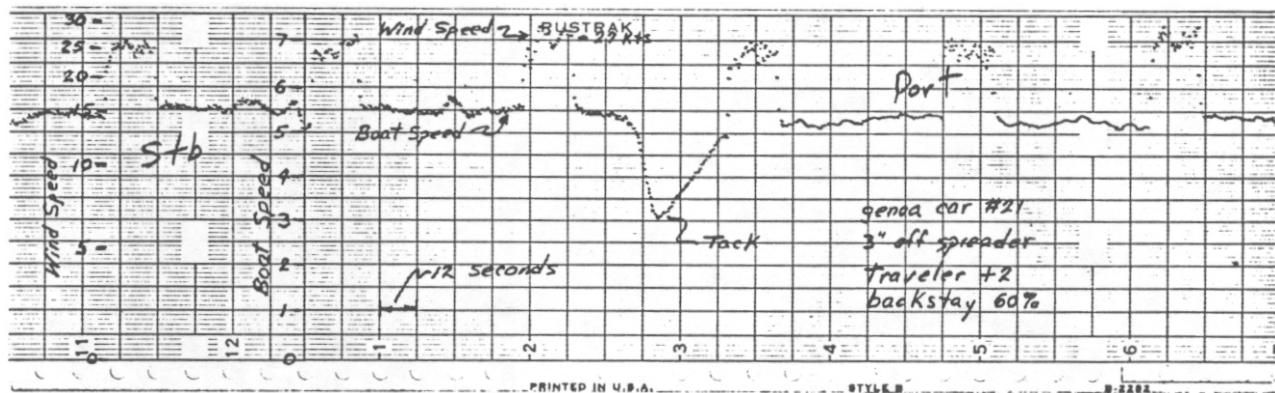


Figure 2. Sample data record from single channel recorder.

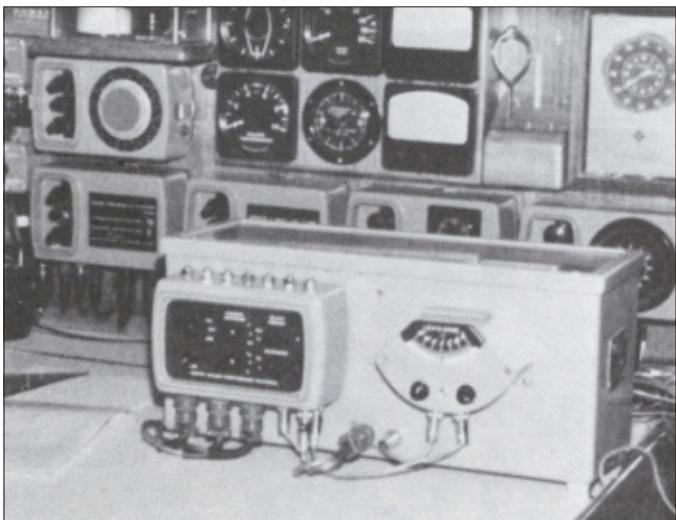


Figure 3. Strip recorder on *Kialoa III* (1975).



Figure 6. Navigation station on *Kialoa IV* (1981)



Figure 4. Performance testing on *Kittiwake* (1976).

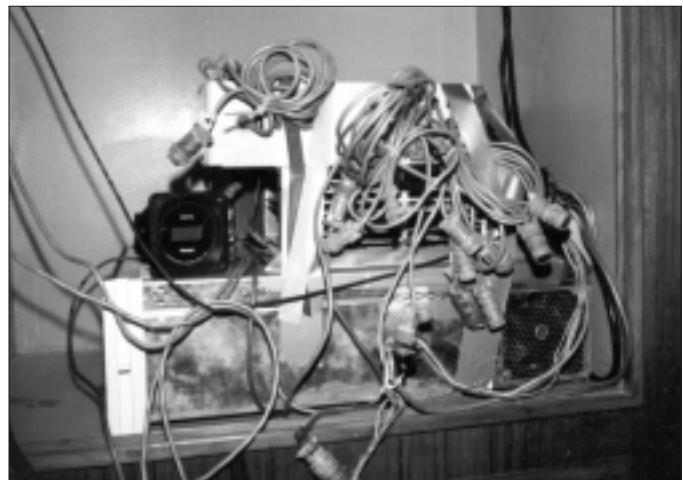


Figure 7. Micro-Nova Computer on *Kialoa IV* (1981).



Figure 5. *Kittiwake* dual channel strip recorder.



Figure 8. Shoreside computer room for *Kialoa IV*.

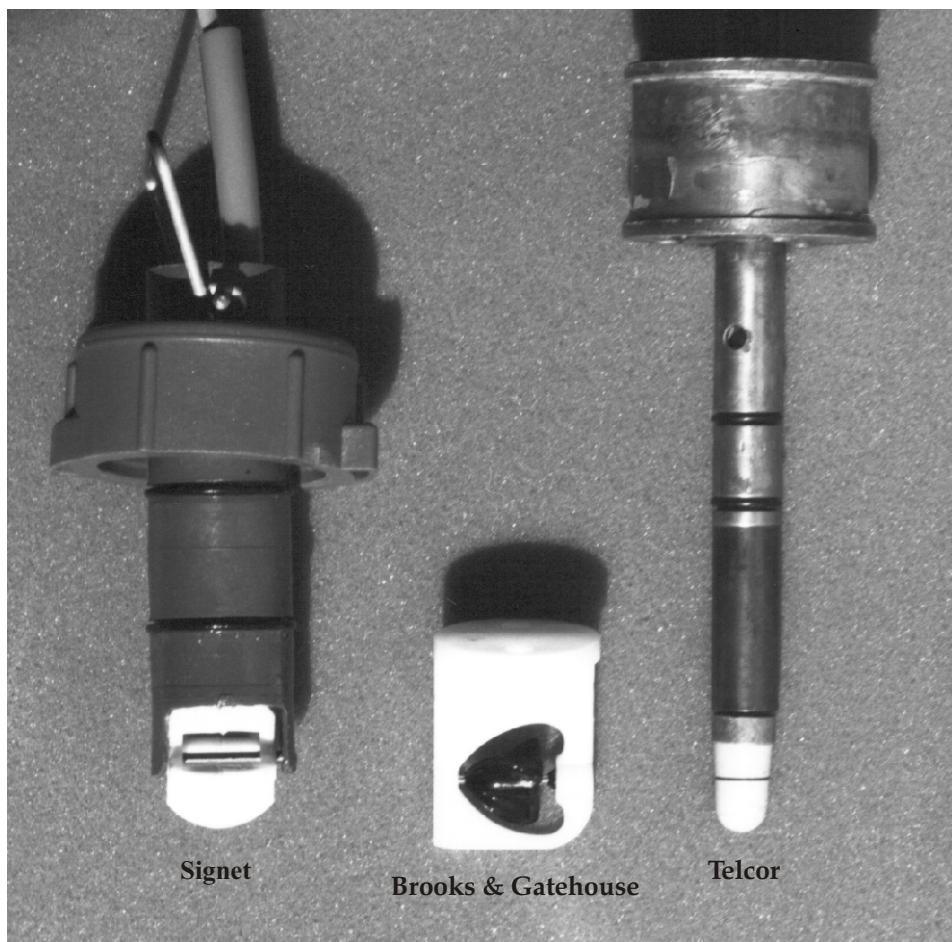


Figure 9. Boatspeed sensors.

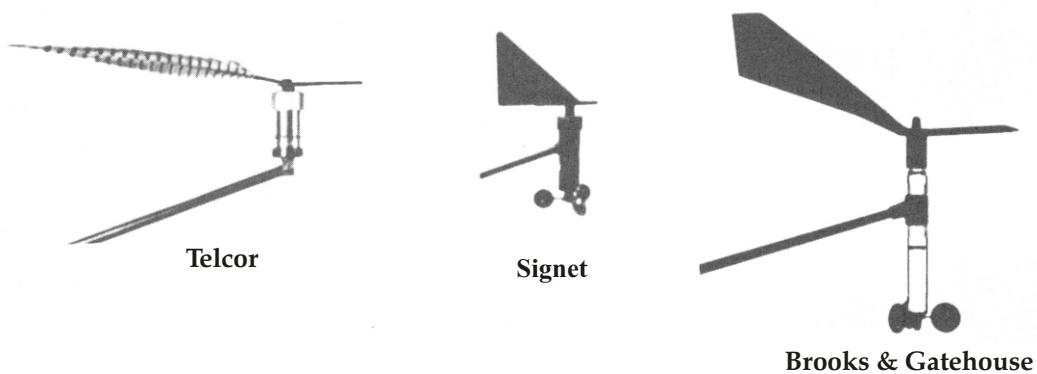


Figure 10. Combination wind speed / wind direction systems.

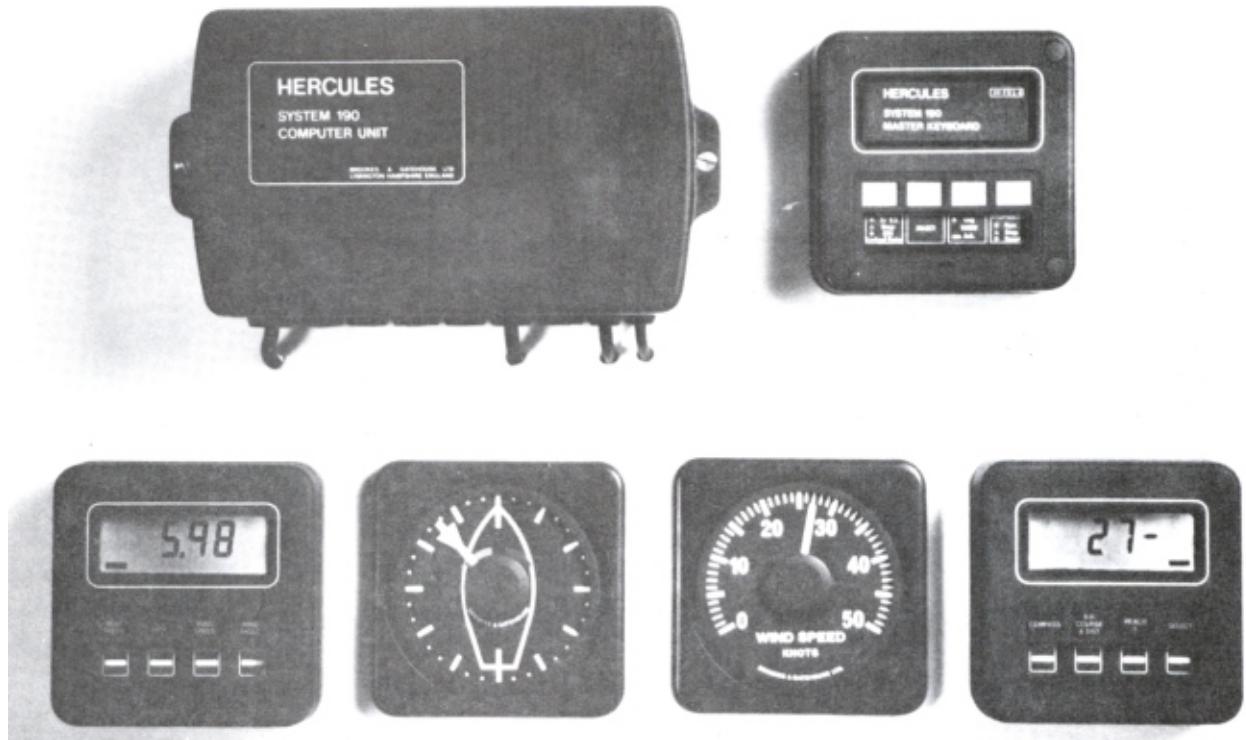


Figure 11. Brooks & Gatehouse Hercules System 190.



Figure 12. Cockpit of New Kialoa IV.

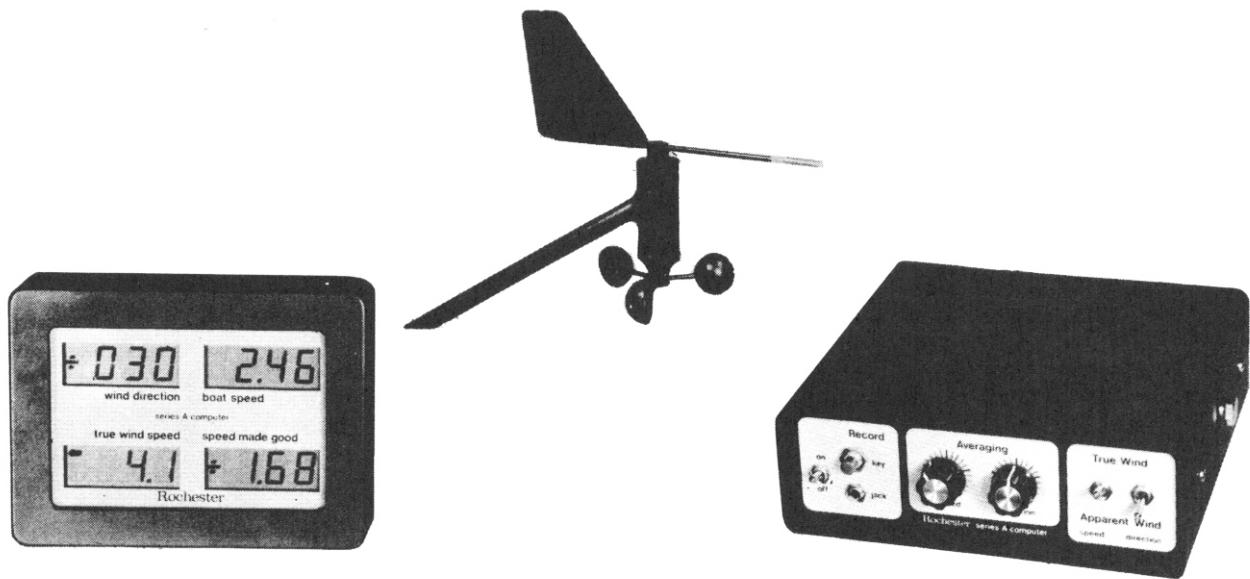


Figure 13. Rochester Microcomputer based instrumentation system.

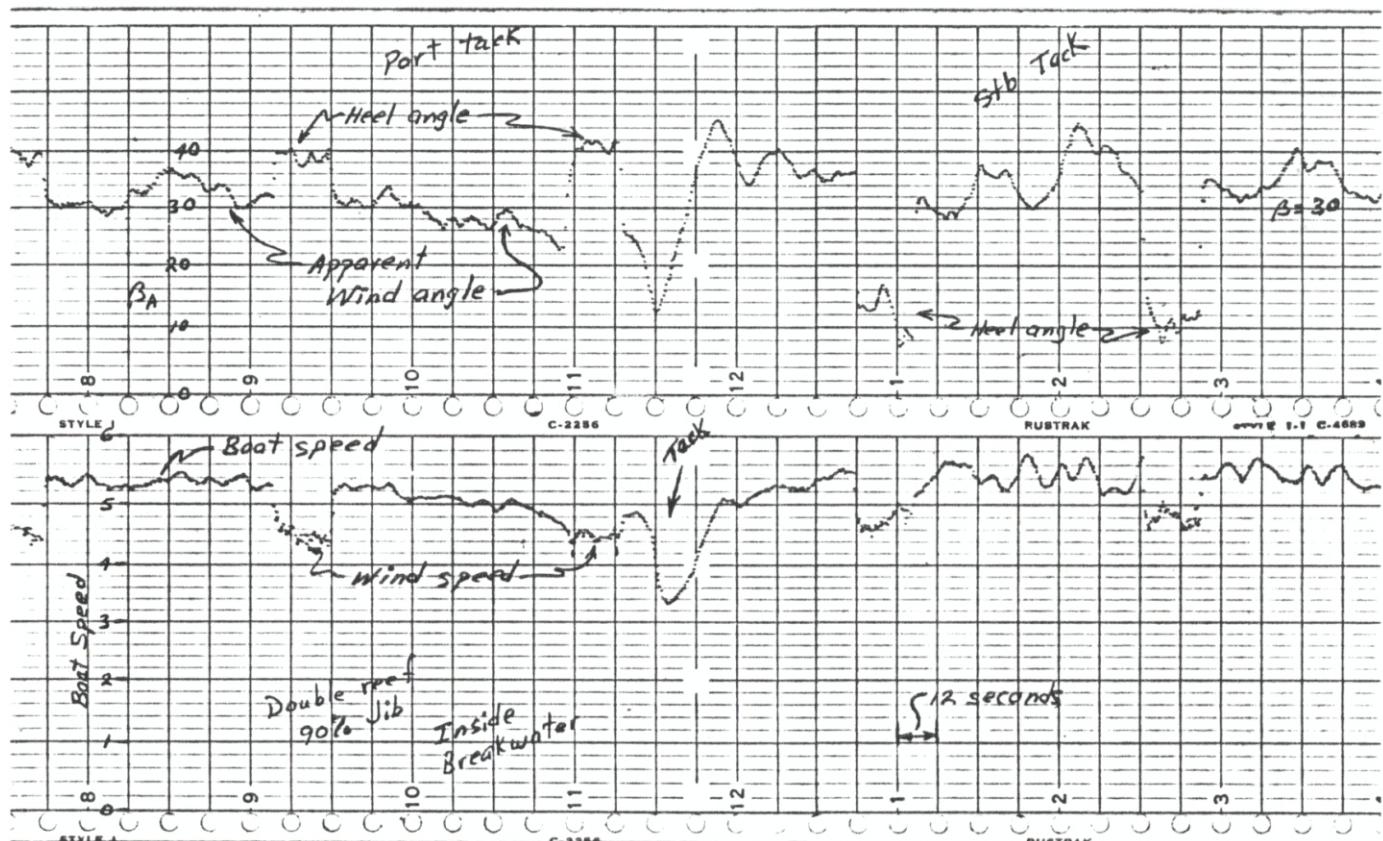


Figure 14. Sample output from *Kialoa III* and *Kittiwake* dual channel strip recorders.

Sheet \_\_\_\_\_ of \_\_\_\_\_

Name of Recorder

Boat Name \_\_\_\_\_  
Date \_\_\_\_\_

Arvel E. Gentry

Leeway Constant =

Spwash Constants:  $S_0 = \frac{S_1}{H}$

Figure 15. Sailboat performance data recording form.

Date 6/20/1898

Boat Name Spray

Name of Recorder Joshua Slocum

Point No.	Time	P/S	Boat Speed VS	Apparent Speed VA	Heel Angle BA	Heading CH	True Wind Speed VT	Tack/Jibe Y	VMG	Wind Angle Y-λ	Wind Dir.	Up-wash ε	Lee-way λ	Notes		
22	11:11	S	7.10	20	26	25	265	14.47	42.34	74.26	5.25	37.13	302	4.49	5.21	Sea Conditions- 1' chop 55°F
23	11:13	S	7.14	20	27	26	262	14.62	44.34	77.98	5.11	38.99	301	4.46	5.36	#2 genoa, car at #18
24	11:17	S	7.16	20	28	26	260	14.73	45.93	81.01	4.99	40.51	301	4.41	5.33	7K Backstay, Flat reef
25	11:19	S	7.05	19	26	24	262	13.43	42.23	74.23	5.22	37.16	299	4.90	5.07	Good point
26	11:20	S	6.93	19	27	25	259	13.70	44.14	77.34	4.97	38.67	298	4.85	5.47	
27	11:21	S	7.12	20	26	26	260	14.51	42.84	74.92	5.22	37.48	297	4.49	5.39	wind shifting
28	11:22	S	6.83	19	25	24	262	13.51	49.55	70.30	5.19	35.15	297	4.94	5.4	Pinching
															Tack - good	
29	11:28	P	6.93	19	28	27	338	13.96	46.66	81.51	4.76	40.75	297	4.81	5.90	Too wide
30	11:25	P	7.18	19	29	27	340	13.88	48.35	85.70	4.77	42.85	297	4.76	5.50	
31	11:26	P	7.21	20	28	26	337	14.69	45.87	81.24	5.02	40.62	296	4.41	5.25	
32	11:27	P	7.16	20	26	25	333	14.42	42.38	74.51	5.29	32.26	296	4.49	5.12	Good point
33	11:29	P	7.11	20	26	25	333	14.46	42.34	74.30	5.26	37.15	296	4.49	5.19	
34	11:30	P	7.10	19	25	25	331	13.32	44.23	72.05	5.34	36.02	295	4.94	5.21	Halyard tighter (13")
35	11:32	P	7.09	20	26	25	332	14.48	48.33	74.21	5.24	37.11	294	4.49	5.22	
36	11:34	P	7.08	20	25	25	330	14.37	40.82	71.17	5.36	35.59	294	4.53	5.24	
															Lunch Time	
			0.0	13	0	0	287								Head To wind	
37	12:14	P	7.05	6.5	139	5	89	12.74	160.8	40.5	158.8	289	0	1.06	3/4 OZ + Blooper	
38	12:16	P	7.14	6.2	140	4	91	12.58	161.8	38.03	161.0	290	0	0.82		
39	12:17	P	6.85	6.9	142	3	86	13.03	161.2	38.86	160.6	285	0	0.67	Ready to jibe 3/4 OZ	
40	12:18	S	6.98	7.1	141	5	126	13.32	160.8	40.55	158.7	286	0	1.08	complete 3/4 OZ	
															Arvel E. Gentry	

Figure 16. Typical set of hand recorded test data.

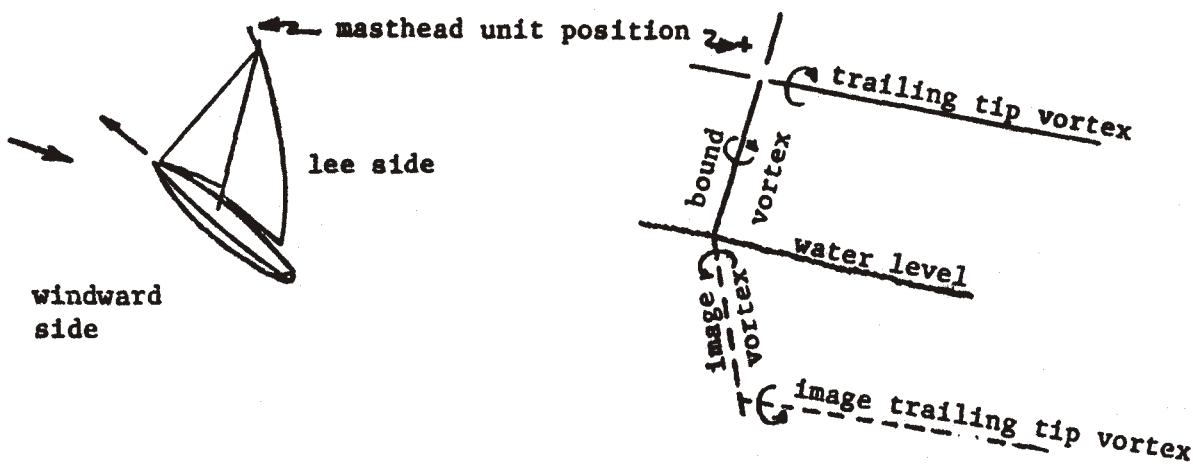


Figure 17. Vortex representation of sail lifting system.

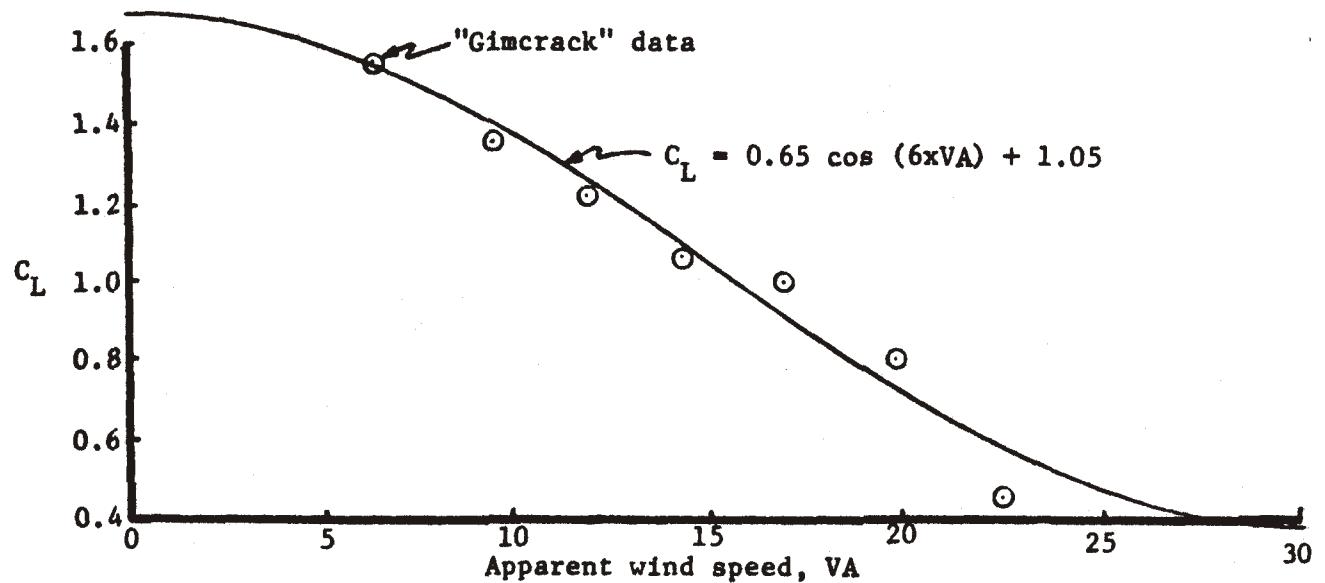


Figure 18. Lift coefficient correlation for *Gimcrack*.

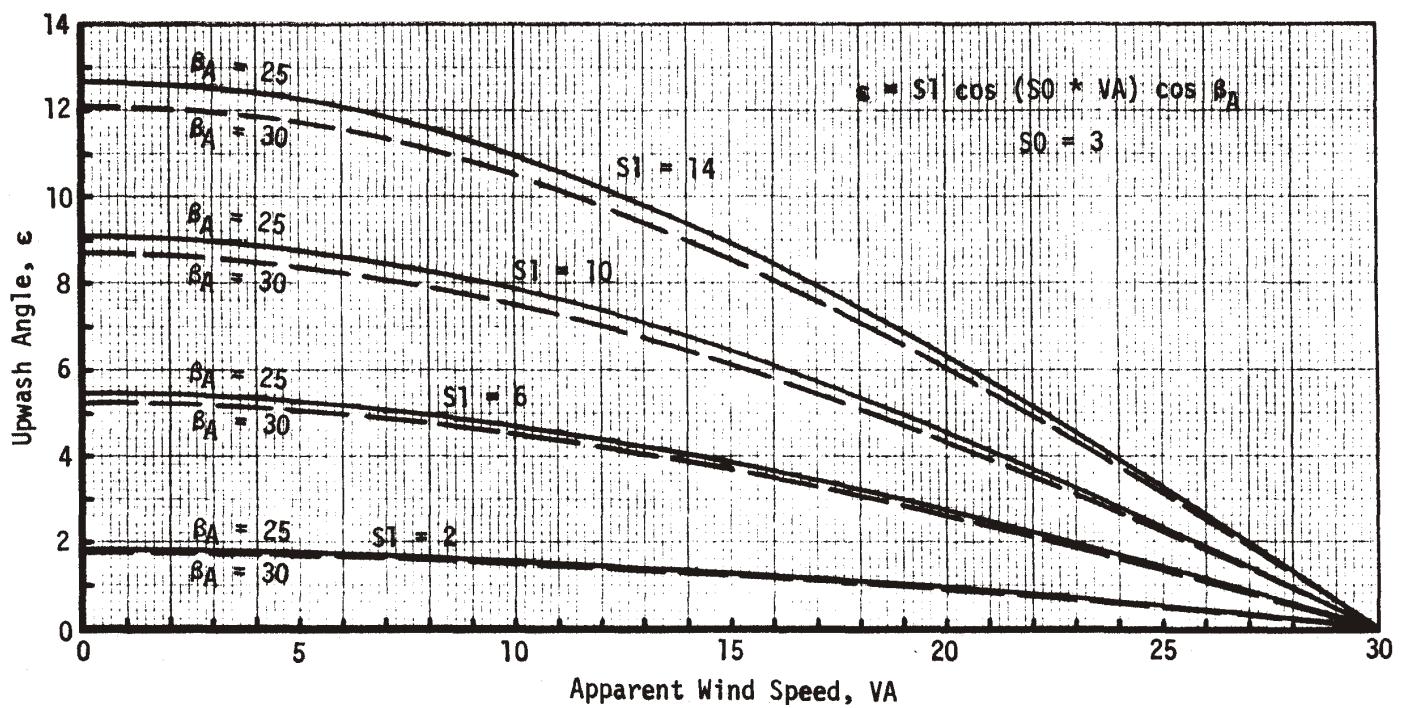
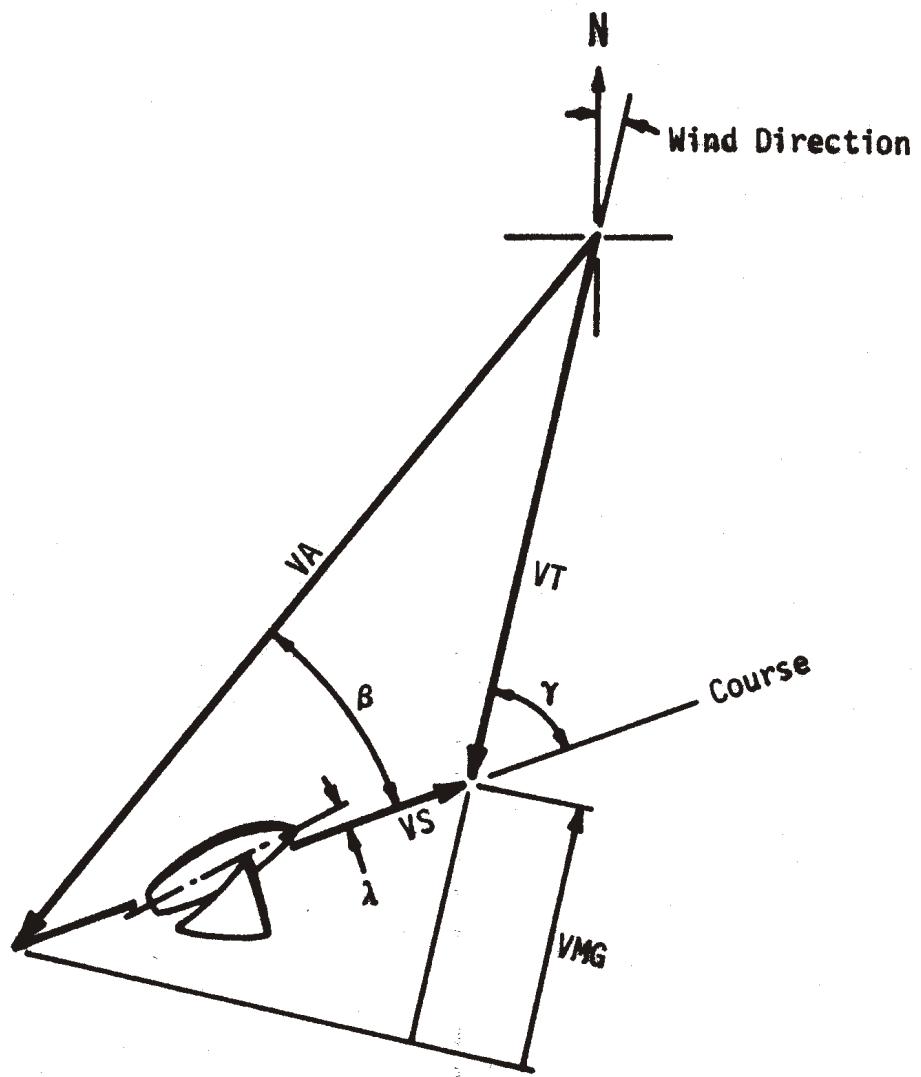


Figure 19. Influence of S1 parameter on upwash angle.



- VT      True wind velocity  
 VS      Boat Speed  
 VA      Apparent wind velocity  
 VMG     Speed made good to windward =  $VS \cos \gamma$   
 $\lambda$      Leeway angle (lambda)  
 $\gamma$      Polar true wind angle (gamma)  
 $\beta$      Apparent wind angle (beta)  
 $\phi$      Heel angle (phi)  
 $\beta_A$     Indicated apparent wind angle (without correction)  
 $\epsilon$      Upwash angle at masthead (epsilon)  
 TA     Tacking angle =  $2 * (\gamma - \lambda)$

Figure 20. Apparent wind triangle parameters.

Figure 21. Performance data reduction program for HP-41C.

01LBL "RDA"	61 GTO "GY"	121 STO 06	181LBL 04
02 "SPRAY"	62 X>Y	122 RCL 00	182 X>?
03 SF 12	63 2	123 COS	183 GTO 05
04 FS? 55	64 *	124 RCL 06	184 RCL 09
05 PRA	65 "T.A. ="	125 COS	185 +
06 CLA	66 XEQ "OUT"	126 /	186LBL 05
07 CF 12	67 XEQ "VMG"	127 RCL 02	187 STO 09
08 "VER 4.3"	68LBL 03	128 *	188 "W DIR ="
09 XEQ "IUPW"	69 RCL 08	129 STO 00	189 XEQ "OUT"
10 XEQ "ILW"	70 "UPWASH="	130 RTN	190 RTN
11LBL "RDB"	71 XEQ "OUT"	131LBL "LEWY"	191LBL "OUT"
12 ADV	72 RCL 05	132 RCL 04	192 TONE 9
13 FIX 2	73 "LEEWAY= "	133 RCL 03	193 ARCL X
14 "VS ?"	74 XEQ "OUT"	134 ENTER	194 AVIEW
15 PROMPT	75 XEQ "WIND"	135 *	195 FS? 55
16 STO 03	76 GTO "RDB"	136 /	196 RTN
17 "VS ="	77LBL "GY"	137 RCL 15	197 PSE
18 XEQ "IO"	78 2	138 *	198 PSE
19 "VA ?"	79 *	139 STO 05	199 RTN
20 PROMPT	80 X>Y	140 RTN	200LBL "IO"
21 STO 02	81 -	141LBL "IUPW"	201 FS? 55
22 "VA ="	82 2	142 RCL 02	202 GTO 02
23 XEQ "IO"	83 *	143 RCL 14	203 RTN
24 "A.W.A.?"	84 "GYBE ="	144 XY?	204LBL 02
25 PROMPT	85 XEQ "OUT"	145 X>Y	205 ARCL X
26 STO 01	86 GTO 03	146 RCL 12	206 AVIEW
27 "A.W.A.?"	87LBL "VMG"	147 *	207 RTN
28 XEQ "IO"	88 RCL 11	148 COS	208LBL "LOCK"
29 "HEEL ?"	89 COS	149 RCL 13	209LBL 00
30 PROMPT	90 RCL 03	150 *	210 SF 11
31 STO 04	91 *	151 RCL 01	211 OFF
32 "HEEL ="	92 "VMG ="	152 COS	212 GTO 00
33 XEQ "IO"	93 XEQ "OUT"	153 *	213 END
34 CLD	94 RTN	154 X<0?	
35LBL "RDC"	95LBL "IUPW"	155 0	USER KEYS:
36 RCL 01	96 3.0 <b>S0</b>	156 STO 08	11 "RDA"
37 STO 06	97 STO 12	157 RCL 06	12 "RDB"
38 XEQ "UPW"	98 10 <b>S1</b>	158 X>Y	13 "RDC"
39 XEQ "HEEL"	99 STO 13	159 -	14 "WIND"
40 XEQ "LEWY"	100 30 <b>S2</b>	160 STO 06	15 "LOCK"
41 RCL 06	101 STO 14	161 RTN	
42 +	102 RTN	162LBL "WIND"	
43 RCL 00	103LBL "ILW"	163 RCL 11	Sample Program
44 P-R	104 10.5 <b>K</b>	164 RCL 05	Output
45 RCL 03	105 STO 15	165 -	
46 -	106 RTN	166 "W ANG? "	SPRAY
47 R-P	107LBL "HEEL"	167 XEQ "OUT"	
48 STO 07	108 RCL 04	168 STOP <del>→ CHS if part</del>	VS =6.20
49 "VT ="	109 RCL 06	169 "HDIRG?"	VA =14.00
50 XEQ "OUT"	110 STO 00	170 PROMPT	A.W.A.=29.00
51 RDM	111 TAN	171 "HEDING= "	HEEL =15.00
52 STO 11	112 X>Y	172 XEQ "IO"	YT =9.02
53 "GAMMA ="	113 COS	173 +	GAMMA =45.68
54 XEQ "OUT"	114 /	174 360	T.A. =83.17
55LBL "TA"	115 RTAN	175 STO 09	VNG =4.33
56 RCL 11	116 X>0?	176 X<Y	UPWASH=6.50
57 RCL 05	117 GTO 01	177 X<Y?	LEEWAY=4.10
58 -	118 180	178 GTO 04	W ANG? 41.58
59 90	119 +	179 X<Y	HEDING=275.00
60 X<=Y?	120LBL 01	180 -	W DIR =233.42

Arvel E. Gentry

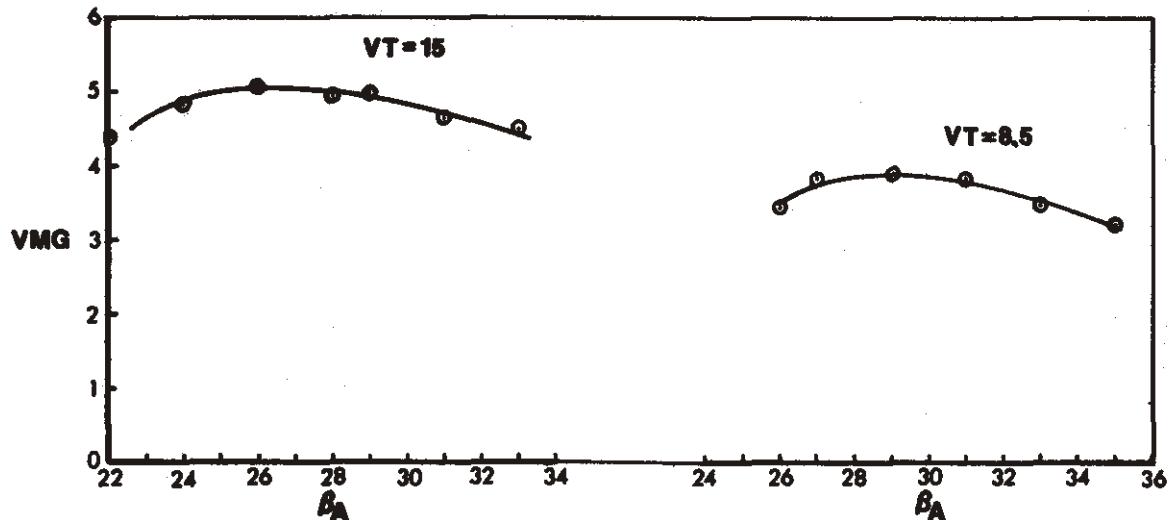


Figure 22. Typical VMG optimization plots.

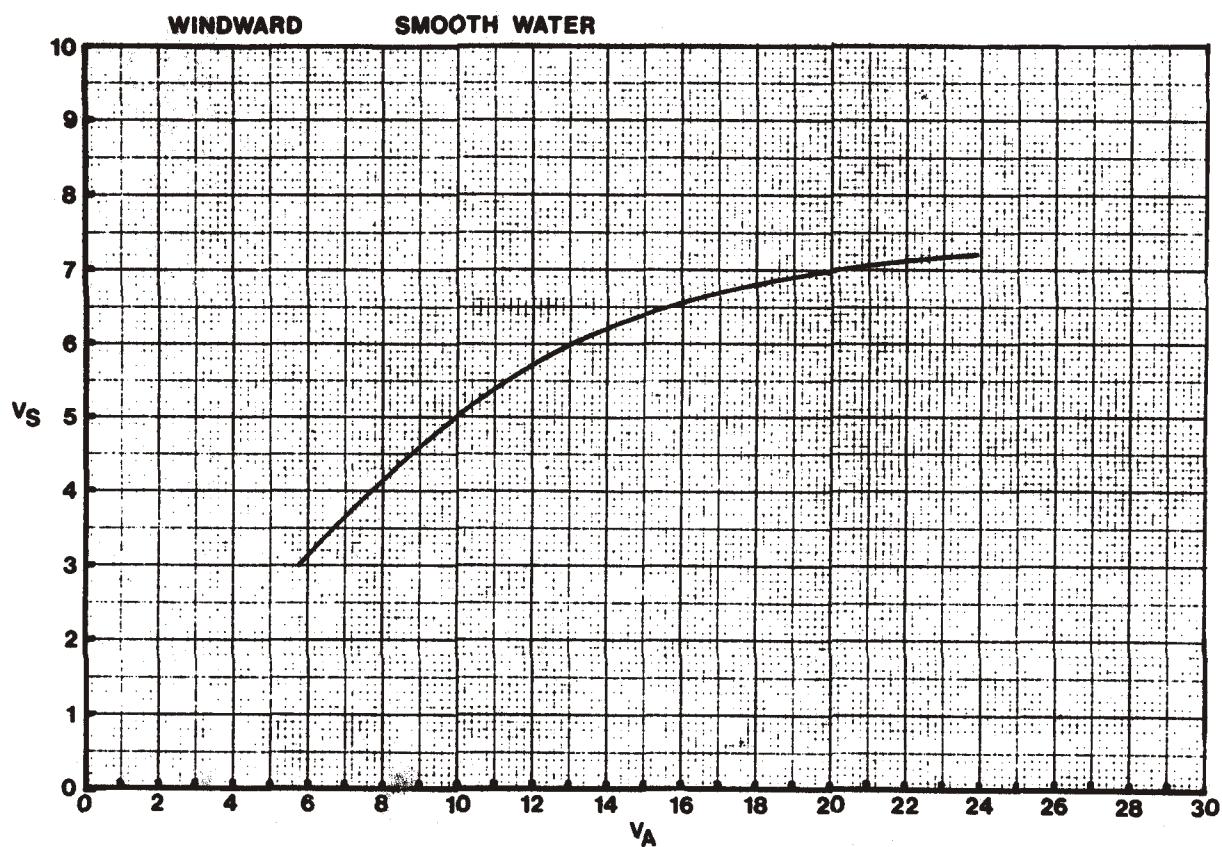


Figure 23. Typical boatspeed plot for optimum VMG conditions.

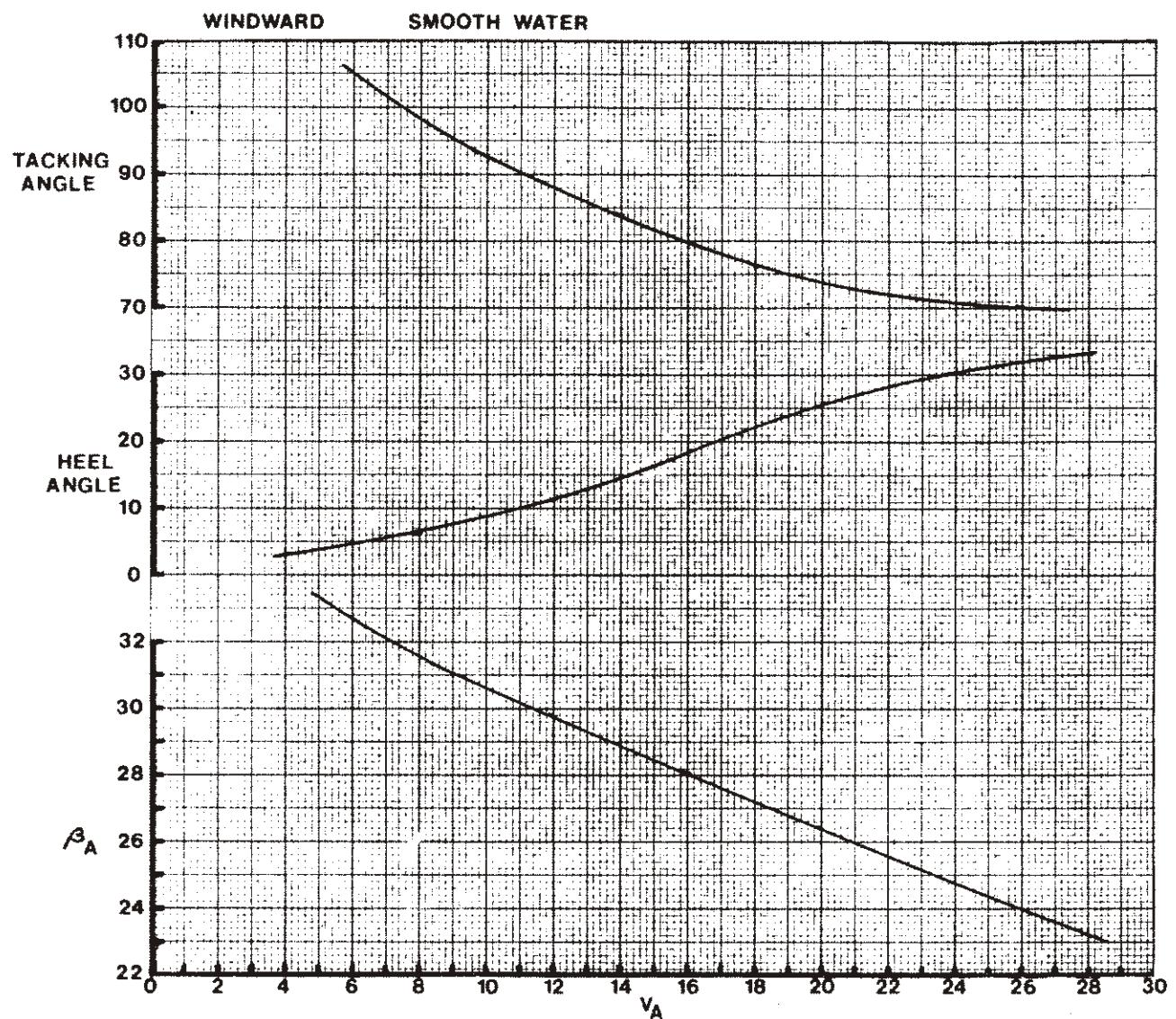


Figure 24. Optimum windward conditions.

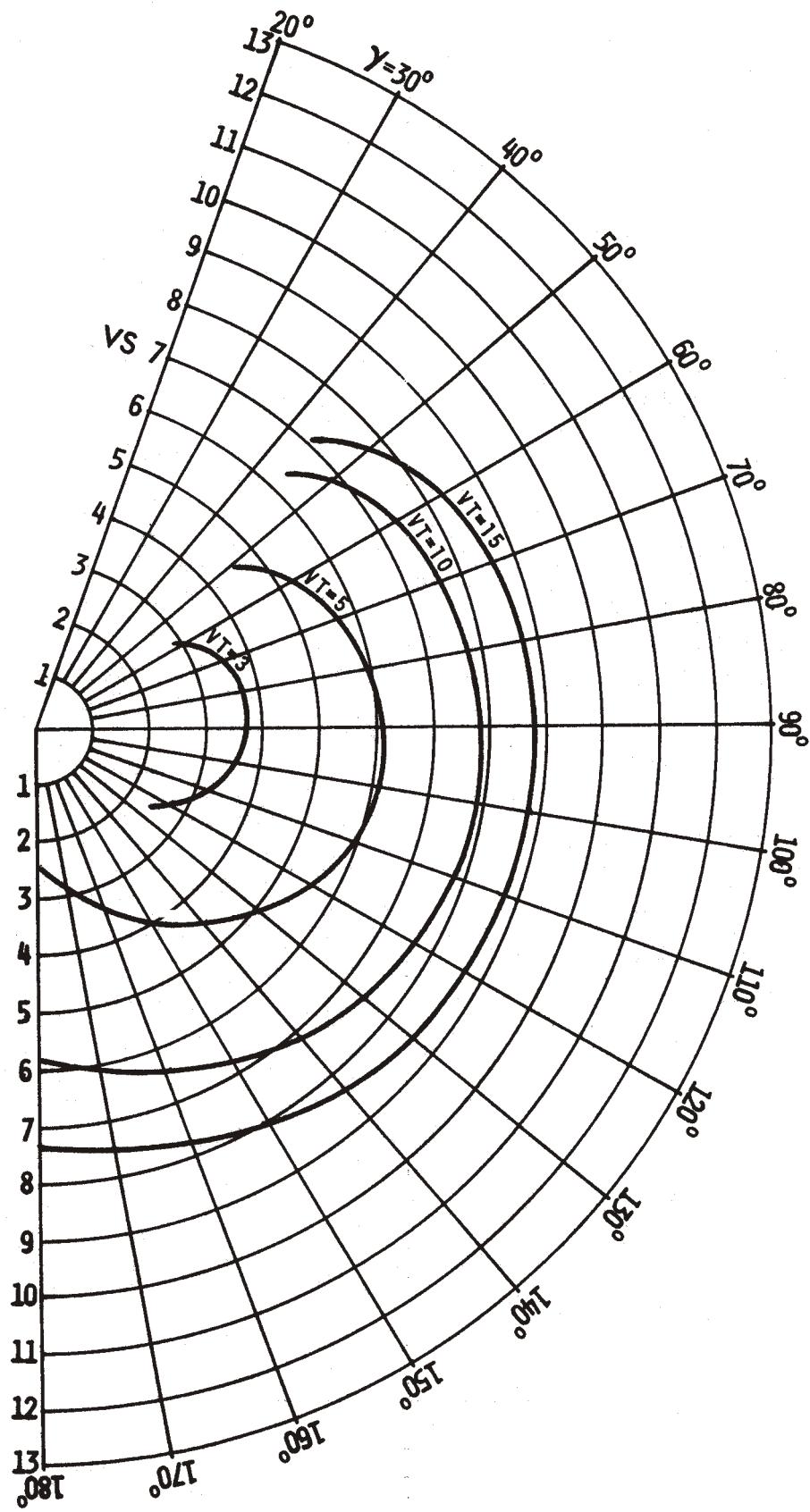


Figure 25. Typical boatspeed polar plot.

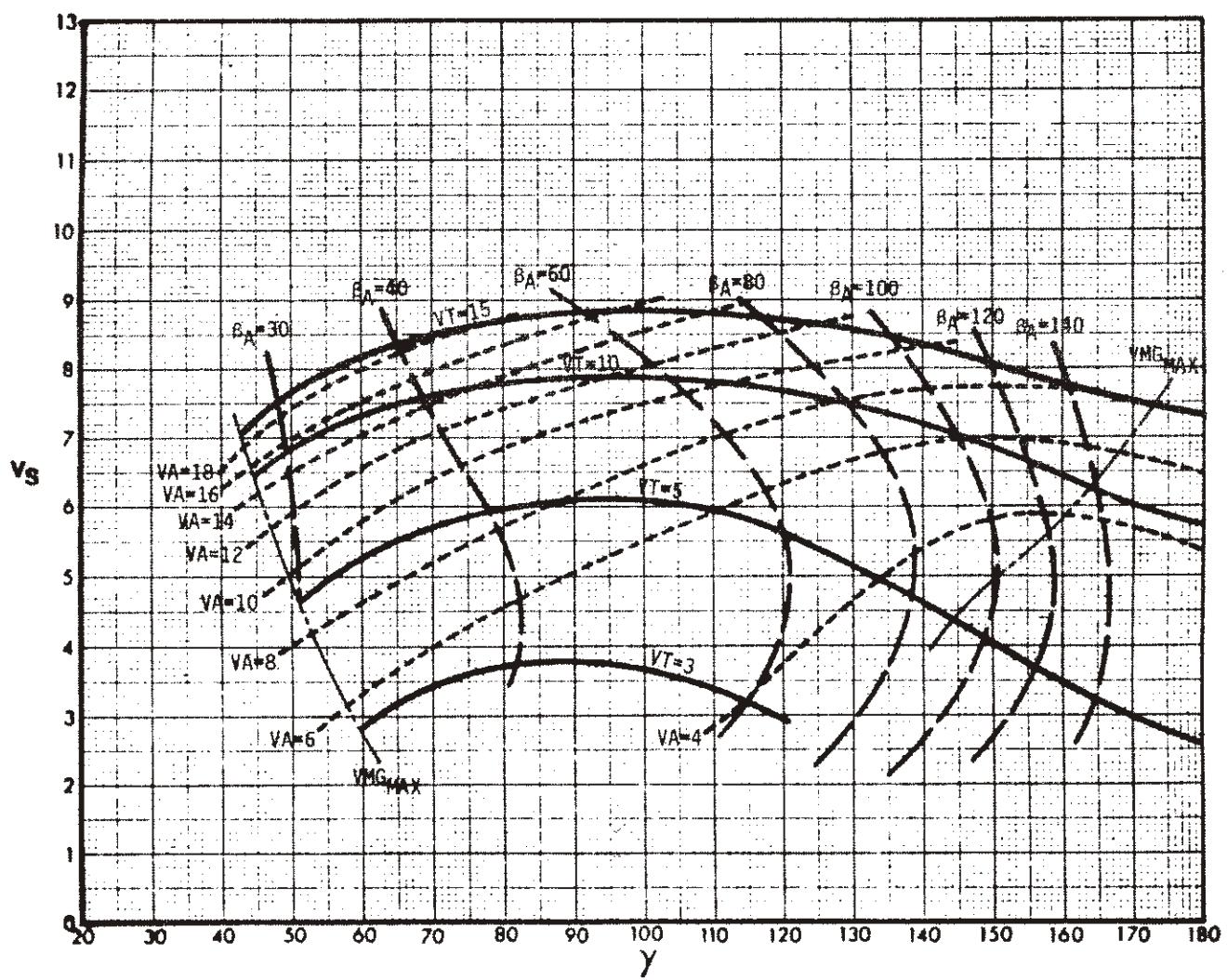


Figure 26. Typical boatspeed polar plot in cartesian coordinate form.