# How much unspanned volatility can different shocks explain?

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IAAE Conference - Turin, Italy

June 26th, 2025

#### Intro

Why should we care about volatility in the nominal US yield curve?

- Hedging of interest-rate derivatives: huge, liquid market with many players;
- **2** Tightly linked to volatility of holding returns for bonds: portfolio allocation;
- 3 Risk management of large bond portfolios from institutional investors;

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Do we have good models for yield curve volatility? Yes and no:

- Workhorse: Dynamic Term Structure Models (very often affine ones);
- Tractable formulas for yields + arbitrage-free framework + convenient for estimation;
- Model-consistent separation between term premia and expected future short rates;

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- Workhorse: Dynamic Term Structure Models (very often affine ones);
- Tractable formulas for yields + arbitrage-free framework + convenient for estimation;
- Model-consistent separation between term premia and expected future short rates;
- Poor time-series dynamics, sharp restrictions on how yields should behave;
- Important **today**: observed vol should be tightly connected to the cross-section of yields;

# Can affine term structure models account for volatility in yields?

Mostly, no. In general, there is more variation than models allow. Some approaches:

- Regress returns from straddles on interest rate changes;
  - ► Collin-Dufresne & Goldstein (2002); Li & Zhao (2006)
- Regress changes of implied volatility from options/swaptions on interest rate changes;
  - Filipovic et al. (2017); Backwell (2021)
- Likelihood-ratio tests for conditions that connect yield volatility and in yield levels;
  - ► Bikbov & Chernov (2009)
- State-price density estimation from options data;
  - ► Li & Zhao (2009)
- Restrictions from high(er)-frequency data;
  - ► Andersen & Benzoni (2010)
  - ► Closest paper to mine, but we deal with jumps and different maturities very differently;

## Any room for improvement?

- Jump-diffusion settings are not so common, but jumps are prevalent in bond markets (Piazzesi, 2010);
- What derivatives to use in an empirical test? Results seem dependent on this choice;
  - ► Swaptions? Caps and floors? Straddles? At the money? Out of the money?
  - Liquidity and availability of strikes also depend on overall volatility itself...
- Analyses done at the individual maturity level
  - ▶ Too many degrees of freedom;
  - ► What maturities should we pay attention to?

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- Analyses done at the individual maturity level
  - ▶ Too many degrees of freedom;
  - What maturities should we pay attention to?
- **Crucially**: attempts to tie "excessive" volatility to real-world developments are rare;
  - ► This is where the money is! Super important for derivative hedging!
  - ▶ What can help explain this "unspanned" volatility? Probably not just noise...

### This paper: two contributions

New methodology: a new test for excess volatility with a number of advantages;

- Implications for non-parametric measures of yield volatility within the affine framework;
- Only zero-coupon yields needed;
- I don't analyze specific maturities, focus on a decomposition of the whole curve;
- Characterization of an unspanned volatility factor: 2/3 of the residual volatility;

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- Characterization of an unspanned volatility factor: 2/3 of the residual volatility;

New empirical results: what can explain this unspanned volatility factor?

- Focus on shocks from the literature on monetary policy, fiscal policy, and oil shocks;
- Forward-guidance-type shocks, oil, and fiscal policy shocks help driving this factor;
- These shocks explain  $\approx 13\%$  of variation. Still a lot to explain (and write about!).

#### Data

- Yield curve data: daily zero-coupon curve from Liu & Wu (2021), from 1973 to 2022;
- Monetary policy shocks from Swanson (2021) monthly frequency;
- Oil shocks identified from Känzig (2021) monthly frequency;
- Fiscal shocks from different sources quarterly frequency:
  - ▶ Defense spending shocks from Ramey (2011) and Ramey & Zubairy (2018);
  - ► Tax policy shocks from Romer & Romer (2010);
  - ► Stock returns from top US government defense contractors Fisher & Peters (2010);

## A Flexible Affine Setup

• There are N latent underlying risk factors  $X_t$ . The evolution under Q follows:

$$dX_{t} = K(\Theta - X_{t}) dt + \sum \sqrt{S_{t}} dW_{t}^{Q} + Z_{t} d\mathcal{N}_{t}^{Q}$$
(1)

- K and  $\Sigma$  are  $N \times N$  constant matrices;  $\Theta$  is an  $N \times 1$  vector of long-run means;
- $S_t$  is an  $N \times N$  diagonal matrix whose diagonal elements follow:

$$S_{t,[ii]} = s_{0,i} + s'_{1,i}X_t \tag{2}$$

- $W_t^Q$  is a Brownian Motion and  $\mathcal{N}_t^Q$  a Poisson process with intensity  $\lambda_t = \lambda_0 + \lambda_1' X_t$ ;
- $Z_t \sim \nu^Q$  represents a jump size, is independent of both  $W_t^Q$  and  $\mathcal{N}_t^Q$ , with  $\mathbb{E}[Z_t Z_t'] = \Omega$ ;
- The short rate  $r_t$  is given by:  $r_t = \delta_0 + \delta_1' X_t$ ;

#### Bond Prices and Bond Yields

- This setup ensures that zero-coupon yields  $y_t^{(\tau)}$  are an affine function of state variables;
- If we trade J fixed maturities  $(\tau_1,...,\tau_J)$  we can write for some vector A and matrix B:

$$Y_t = A + BX_t \tag{3}$$

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• If B is full column rank (and it is for the US market - Bauer & Rudebusch (2017)):

$$X_t = (B'B)^{-1}B'(Y_t - A) = \tilde{A} + \tilde{B}Y_t$$
(4)

- ullet This is a path-by-path condition: movements in yields should reveal movements in  $X_t$ ;
- It connects the whole distribution of  $Y_t$  and  $X_t$ ;

### The Quadratic Variation Process

Definition 1 (Just a fancy variance!)

For a real-valued process  $M_t$ , given a partition  $\{t_0 = t, t_1, ..., t_{n-1}, t_n = t + h\}$ , we define its

Quadratic Variation between t and t + h as

$$QV_{M}(t, t+h) \equiv \operatorname{p-lim}_{\delta_{n} \to 0} \sum_{k=1}^{n} (M_{t_{k}} - M_{t_{k-1}})^{2}, \quad \delta_{n} \equiv \sup_{0 \le k \le n} \{t_{k} - t_{k-1}\}$$
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#### Proposition 1

For any linear combination of yields  $L_t = c' Y_t$ , its Quadratic Variation between t and t + h is

$$QV_{L}(t,t+h) = \tilde{\gamma}_{0} + \sum_{j=1}^{J} \tilde{\gamma}_{1,j} \cdot \overline{y}^{(\tau_{j})}(t,t+h) + \sum_{k=1}^{N_{t+h}-N_{t}} v' Z_{T_{k}(t,t+h)} Z'_{T_{k}(t,t+h)} v$$
 (6)

Should be spanned by average yields

No requirement to span the jump-only part!

where  $\overline{y}^{(\tau_j)}(t,t+h) \equiv \frac{1}{h} \int_t^{t+h} y_s^{(\tau_j)} ds$ ,  $\{\tilde{\gamma}_0,\tilde{\gamma}_1\}$  and v depend on parameters;

#### Identification

- Measuring the QV of stochastic process is usually easy: Realized Variance!
- Here it would incorporate both the diffusive (spanned) part and the jump-driven part;
- Can we tease out the diffusive part from the jumps? Yes: Bipower Variation!

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Definition 2 (Barndorff-Nielsen & Shephard (2004, 2006))

For a real-valued process  $M_t$ , we define the Bipower Variation process over [t, t+h] as:

$$BPV_{M}(t, t+h) \equiv \underset{n \to \infty}{\text{p-lim}} \sum_{i=2}^{n} \left| M_{t+i \cdot \frac{h}{n}} - M_{t+(i-1) \cdot \frac{h}{n}} \right| \left| M_{t+(i-1) \cdot \frac{h}{n}} - M_{t+(i-2) \cdot \frac{h}{n}} \right|$$
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#### Proposition 2

Under this setup, the Bipower Variation of  $L_t = c'Y_t$  identifies the diffusive part of  $QV_L$ :

$$BPV_L(t,t+h) = \frac{2}{\pi} \cdot \left| \tilde{\gamma}_0 + \sum_{j=1}^J \tilde{\gamma}_{1,j} \cdot \overline{y}^{(\tau_j)}(t,t+h) \right|$$
 (8)

- This condition can be tested:
  - ► We can approximate both the LHS and RHS;
  - ▶ I use daily data to compute these measures at the *monthly* frequency;
- Regressing bipower variation measures on average yields should yield significant coefficients + high R<sup>2</sup>;

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But why to focus on linear combinations of yields?...

- US yield curve admits a low-rank representation (Litterman & Scheinkman (1991));
- A common decomposition is the one from Nelson & Siegel (1987);
- Three factors: a long-end factor  $\beta_1$ , a short-end factor  $\beta_2$ , and a medium-end factor  $\beta_3$ ;
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- What about volatility? Do we have unspanned volatility from every part of the curve?

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## The Nelson-Siegel Representation

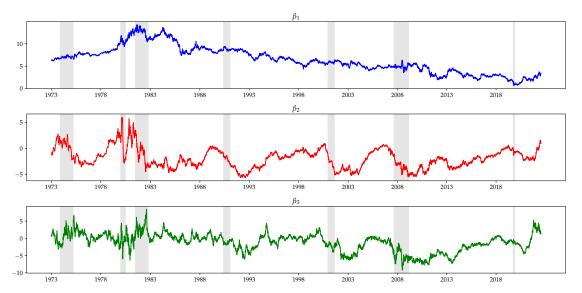
- $y_t^{(\tau)}$ : zero-coupon rate at time t and maturity  $\tau$ ;
- $\psi > 0$ : a positive decay parameter;

$$y_t^{(\tau)} = \beta_{1,t} + \beta_{2,t} \left( \frac{1 - e^{-\psi\tau}}{\psi\tau} \right) + \beta_{3,t} \left( \frac{1 - e^{-\psi\tau}}{\psi\tau} - e^{-\psi\tau} \right)$$
(9)

- $\beta_1$  is a long-run factor:  $\lim_{\tau \to \infty} y_t^{(\tau)} = \beta_{1,t}$ ;
- $\beta_2$  is a short-run factor: its absolute loading decreases with  $\tau$ ;
- $\beta_3$  is a medium-run factor: its loading is hump-shaped;
- **Crucial**: for a fixed  $\psi$ , factors = linear combinations of yields;

How to estimate this???

# Daily Factors

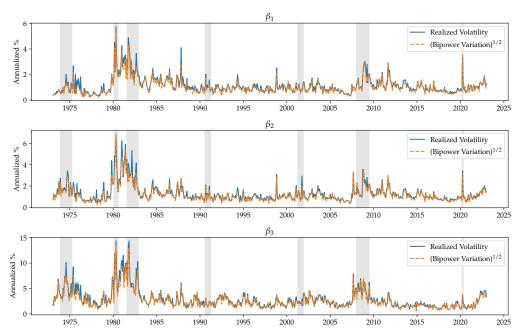


Average daily fitting error over maturities  $\approx$  5bps;



### Variation Measures





- Recall: diffusive variation should be an affine function of average yields;
- $BPV_i$ : bipower variation of factor  $i \in \{1, 2, 3\}$ ;
- $BPCov_{i,j}$ : bipower covariation between factors i and j, using a polarization identity;
- Notation:  $BPCov_{i,i} \equiv BPV_i$ , for any i;

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$$BPCov_{i,j}(t) = \delta_{i,j} + \theta'_{i,j}\overline{Y}_t + \eta_{i,j}(t), \qquad i,j = 1,2,3$$
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$$BPCov_{i,j}(t) = \delta_{i,j} + \theta_{i,j}^{(1)} \overline{\beta}_{1,t} + \theta_{i,j}^{(2)} \overline{\beta}_{2,t} + \theta_{i,j}^{(3)} \overline{\beta}_{3,t} + \eta_{i,j}(t), \qquad i, j = 1, 2, 3$$
 (11)

(Don't worry! Robustness checks in the paper!)

Test - Post-Volcker Sample

Table: Post-Volcker Sample (September, 1987 - December, 2022)

	$BPV_1$	$BPV_2$	BPV <sub>3</sub>	BPCov <sub>21</sub>	BPCov <sub>31</sub>	BPCov <sub>32</sub>
Average $\beta_1$	-0.03	-0.01	0.36	0.06	0.02	-0.04
	(0.04)	(0.05)	(0.30)	(0.04)	(0.06)	(0.06)
Average $\beta_2$	-0.05	-0.03	-0.56	0.08	-0.04	0.00
	(0.07)	(80.0)	(0.52)	(0.06)	(0.10)	(0.12)
Average $\beta_3$	-0.10	-0.15	-0.21	0.11	0.12	-0.07
	(80.0)	(0.09)	(0.52)	(0.07)	(0.11)	(80.0)
N	424	424	424	424	424	424
$R^2$	0.07	0.08	0.06	0.13	0.02	0.01

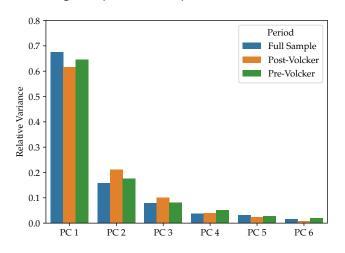
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ullet Each regression delivers a time series of residuals  $\Longrightarrow$  Six residual series in total;

### Is everything just noise?

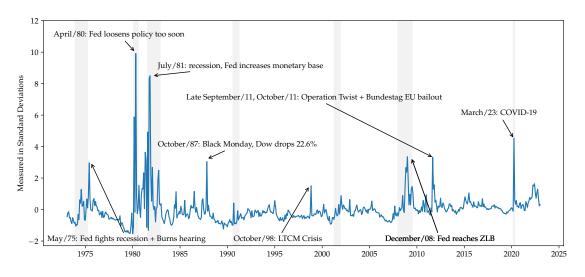
• Each regression delivers a time series of residuals  $\implies$  Six residual series in total;

Figure: Spectral decomposition of residuals



- The first PC of residuals commands 2/3 of the unexplained variation;
- If the failure of the previous tests were due to pure noise, we wouldn't see such a dominant factor;

### How does this factor look like?



- Realizations are skewed, spiking up during recessions and major events;
- It's hard to make the case this is pure noise;

### What can explain this factor?

- Much of the yield curve volatility is not accounted by affine term structure models;
- Spikes in the unspanned volatility seem related to monetary policy;
- How much of this factor can monetary policy explain?

$$USV_t = \alpha + \theta \cdot |\mathsf{Shock}_t| + u_t \tag{12}$$

- Three types of monetary policy shocks from Swanson (2021);
  - Pure Fed Funds rate surprise, a forward-guidance shock, and a QE-type shock;
  - ► Identified using Fed Funds + Eurodollar futures (1991-2019);

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- Three types of monetary policy shocks from Swanson (2021);
  - ▶ Pure Fed Funds rate surprise, a forward-guidance shock, and a QE-type shock;
  - ▶ Identified using Fed Funds + Eurodollar futures (1991-2019);
- What about oil price shocks? ↑ inflation, ↑ inflation expectations (Känzig, 2021);
  - ▶ Monthly frequency, identified with daily oil futures prices (1975-2022);
- This is about the US sovereign debt... can fiscal policy help explain volatility? (Fisher & Peters, 2010; Romer & Romer, 2010; Ramey, 2011; Ramey & Zubairy, 2018);

# Monetary Policy

	First PC of Residuals						
	(1)	(2)	(3)	(4)			
FFR	0.10		0.04				
	(0.10)		(0.09)				
FG		0.17**	0.16**	0.30*			
		(80.0)	(0.07)	(0.17)			
QE				-0.05			
				(0.09)			
Sample		2009-2016					
N	336	336	336	96			
$R^2$	0.01	0.03	0.03	0.08			

- 1 sd of FG  $\approx \uparrow$  6 bps on future Fed Funds 1 year ahead; Shocks Time Series
- Back of envelope: 25 bps worth of FG ≈ ↑ 0.64 standard deviations in unspanned vol;

# Oil Price Shocks + Monetary Policy

	(1)	(2)	(3)	(4)	
Oil Shock	0.42***	0.43**	0.43**	0.43**	
	(0.15)	(0.18)	(0.18)	(0.18)	
FFR		0.05		0.02	
		(0.06)		(0.05)	
FG			0.11**	0.10**	
			(0.05)	(0.05)	
Sample	1975-2022	1991-2019			
N	576	336	336	336	
$R^2$	0.02	0.09	0.12	0.13	

- 10% oil price increase ≈ ↑ 0.42 standard deviations of *USV*;
- Oil shock + monetary policy explain at most 13% of the unspanned volatility factor;

Oil Shock Time Series

## Fiscal Policy

Table: Projecting Unspanned Vol on Fiscal Policy Shocks

	(1)	(2)	(3)	(4)
Tax Changes	0.67*			0.70*
(Romer & Romer, 2010)	(0.39)			(0.39)
Defense Spending Shocks		0.04		-0.01
(Ramey & Zubairy, 2018)		(0.07)		(80.0)
Defense Contractors Returns			0.03*	0.02
(Fisher & Peters, 2010)			(0.02)	(0.02)
End of sample (quarterly data)	2007	2015	2008	2007
N	140	172	144	140
$R^2$	0.02	0.00	0.02	0.04

<sup>•</sup> A tax change worth 1% of GDP  $\implies \uparrow 0.7$  standard deviations of *USV*;

# Wrap Up

#### Main takeaways:

- I provide a jump-robust test for the presence of unspanned volatlity;
- I show that there is unspanned volatility steaming from the entire maturity spectrum;
- Unspanned volatility as a single factor, which I formally characterize;
- This factor is *partially* driven by monetary policy, fiscal policy, and oil shocks;

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- What kind of other sources of variation are interesting here?

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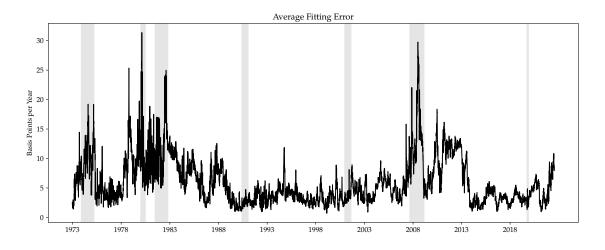
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Thank you!

# ${\sf Appendix}$

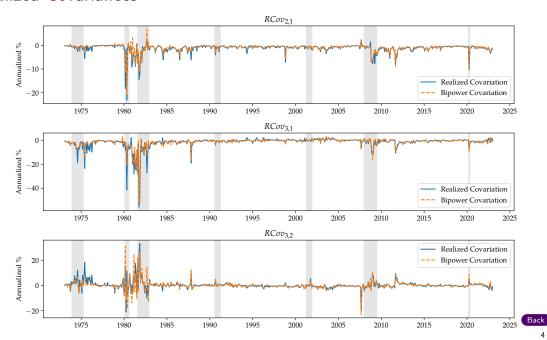
# Figures

# Fitting Error





## Realized Covariances

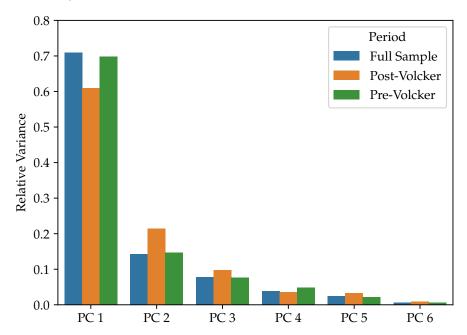


# Test - Full Sample (1973-2022)

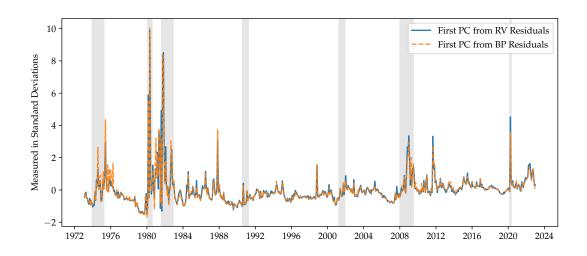
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Average $\beta_1$	0.34***	0.56***	2.90***	-0.14**	-0.56**	0.14
	(0.11)	(0.18)	(0.86)	(0.07)	(0.22)	(0.10)
Average $\beta_2$	0.34*	0.87***	3.77***	-0.15	-0.64**	0.04
	(0.17)	(0.32)	(1.38)	(0.11)	(0.30)	(0.10)
Average $\beta_3$	-0.25**	-0.54***	-1.68*	0.19**	0.25	0.03
	(0.12)	(0.21)	(0.88)	(80.0)	(0.17)	(80.0)
N	600	600	600	600	600	600
$R^2$	0.18	0.31	0.27	0.08	0.18	0.02

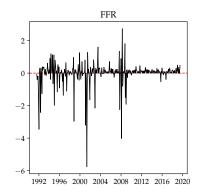
# Spectral Decomposition of RV Residuals

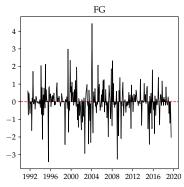


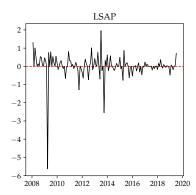
# Unspanned Factors: RV vs BP



# Monetary Policy Shocks from Swanson (2021)

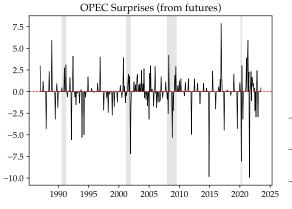


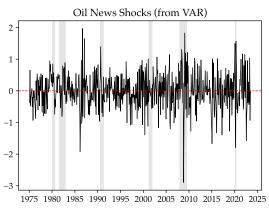






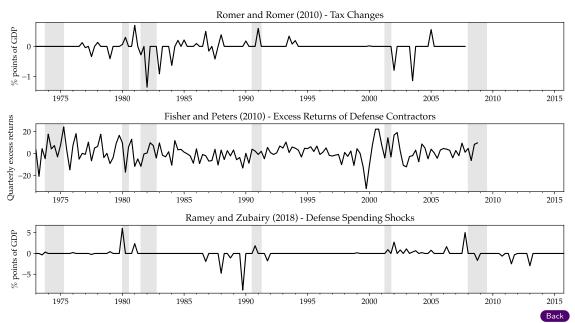
# Oil Shocks from Känzig (2021)







## Fiscal Shocks



Math and Tables

## Estimating Nelson-Siegel Factors with OLS

- We estimate the factors using OLS: regress yields on coefficients;
- $\lambda > 0$  is fixed:
- No need of numerical solutions!

$$\begin{bmatrix} \widehat{\beta}_{1,t} \\ \widehat{\beta}_{2,t} \\ \widehat{\beta}_{3,t} \end{bmatrix} = (M'M)^{-1} M'Y_t, \qquad M \equiv \begin{bmatrix} 1 & \frac{1-e^{-\psi\tau_1}}{\psi\tau_1} & \frac{1-e^{\psi\tau_1}}{\psi\tau_1} - e^{-\lambda\tau_1} \\ 1 & \frac{1-e^{-\psi\tau_2}}{\psi\tau_2} & \frac{1-e^{\psi\tau_2}}{\psi\tau_2} - e^{-\lambda\tau_2} \\ \vdots & \vdots & \vdots \\ 1 & \frac{1-e^{-\psi\tau_J}}{\psi\tau_J} & \frac{1-e^{\psi\tau_J}}{\psi\tau_J} - e^{-\lambda\tau_J} \end{bmatrix}.$$

Back

## How Jumpy Are The Factors?

- How much variation is coming from the diffusive part? How much from the jumps?
- Surprisingly stable over factors and over time!

$$JV_i(t) \equiv \max\{RCov_{ii}(t) - BPV_i(t), 0\}, \qquad JR_i(t) \equiv \frac{JV_i(t)}{RCov_{ii}(t)}$$
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Table: Average jumpiness of Nelson-Siegel factors

	$JR_1$	$JR_2$	$JR_3$
Whole Sample (1973-2022)	0.172	0.158	0.170
Months with MP activity	0.146	0.145	0.161
Months without MP activity	0.150	0.149	0.157
<i>p</i> -value for difference	0.799	0.808	0.812

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