

How much unspanned volatility can different shocks explain?

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Intro

Why should we care about *volatility* in the nominal US yield curve?

- 1 Hedging of interest-rate derivatives: huge, liquid market with many players;
- 2 Tightly linked to volatility of holding returns for bonds: portfolio allocation;
- 3 Risk management of large bond portfolios from institutional investors;

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Do we have good models for yield curve volatility? Yes and no:

- Workhorse: Dynamic Term Structure Models (very often affine ones);
- Tractable formulas for yields + arbitrage-free framework + convenient for estimation;
- Model-consistent separation between term premia and expected future short rates;

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- Tractable formulas for yields + arbitrage-free framework + convenient for estimation;
- Model-consistent separation between term premia and expected future short rates;
- Poor time-series dynamics, sharp restrictions on how yields should behave;
- Important **today**: observed vol should be tightly connected to the cross-section of yields;

Can affine term structure models account for volatility in yields?

Mostly, no. In general, there is more variation than models allow. Some approaches:

- Regress returns from straddles on interest rate changes;
 - ▶ Collin-Dufresne & Goldstein (2002); Li & Zhao (2006)
- Regress changes of implied volatility from options/swaptions on interest rate changes;
 - ▶ Filipovic et al. (2017); Backwell (2021)
- Likelihood-ratio tests for conditions that connect yield volatility and in yield levels;
 - ▶ Bikbov & Chernov (2009)
- State-price density estimation from options data;
 - ▶ Li & Zhao (2009)
- Restrictions from high(er)-frequency data;
 - ▶ Andersen & Benzoni (2010)
 - ▶ Closest paper to mine, but we deal with jumps and different maturities very differently;

Any room for improvement?

- Jump-diffusion settings are not so common, but jumps are prevalent in bond markets (Piazzesi, 2010);
- What derivatives to use in an empirical test? Results seem dependent on this choice;
 - ▶ Swaptions? Caps and floors? Straddles? At the money? Out of the money?
 - ▶ Liquidity and availability of strikes also depend on overall volatility itself...
- Analyses done at the individual maturity level
 - ▶ Too many degrees of freedom;
 - ▶ What maturities should we pay attention to?

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- Analyses done at the individual maturity level
 - ▶ Too many degrees of freedom;
 - ▶ What maturities should we pay attention to?
- **Crucially:** attempts to tie “excessive” volatility to real-world developments are rare;
 - ▶ This is where the money is! Super important for derivative hedging!
 - ▶ What can help explain this “unspanned” volatility? Probably not just noise...

This project: two contributions

New methodology: a new test for excess volatility with a number of advantages;

- Implications for non-parametric measures of yield volatility within the affine framework;
- Only zero-coupon yields needed;
- I don't analyze specific maturities, focus on a decomposition of the whole curve;
- Characterization of an *unspanned volatility* factor: $2/3$ of the residual volatility;

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New empirical results: what can explain this unspanned volatility factor?

- Focus on shocks from the literature on monetary policy, fiscal policy, and oil shocks;
- Forward-guidance-type shocks, oil, and fiscal policy shocks help driving this factor;
- These shocks explain $\approx 13\%$ of variation. Still a lot to explain (and write about!).

Data

- Yield curve data: daily zero-coupon curve from [Liu & Wu \(2021\)](#), from 1973 to 2022;
- Monetary policy shocks from [Swanson \(2021\)](#) - monthly frequency;
- Oil shocks identified from [Känzig \(2021\)](#) - monthly frequency;
- Fiscal shocks from different sources - quarterly frequency:
 - ▶ Defense spending shocks from [Ramey \(2011\)](#) and [Ramey & Zubairy \(2018\)](#);
 - ▶ Tax policy shocks from [Romer & Romer \(2010\)](#);
 - ▶ Stock returns from top US government defense contractors [Fisher & Peters \(2010\)](#);

A Flexible Affine Setup

- There are N latent underlying risk factors X_t . The evolution under Q follows:

$$dX_t = K (\Theta - X_t) dt + \Sigma \sqrt{S_t} dW_t^Q + Z_t d\mathcal{N}_t^Q \quad (1)$$

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- K and Σ are $N \times N$ constant matrices; Θ is an $N \times 1$ vector of long-run means;
- S_t is an $N \times N$ diagonal matrix whose diagonal elements follow:

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- $Z_t \sim \nu^Q$ represents a jump size, is independent of both W_t^Q and \mathcal{N}_t^Q , with $\mathbb{E}[Z_t Z'_t] = \Omega$;

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- $Z_t \sim \nu^Q$ represents a jump size, is independent of both W_t^Q and \mathcal{N}_t^Q , with $\mathbb{E}[Z_t Z'_t] = \Omega$;
- The short rate r_t is given by: $r_t = \delta_0 + \delta'_1 X_t$;

Bond Prices and Bond Yields

- This setup ensures that zero-coupon yields $y_t^{(\tau)}$ are an affine function of state variables;
- If we trade J fixed maturities (τ_1, \dots, τ_J) we can write for some vector A and matrix B :

$$Y_t = A + BX_t \tag{3}$$

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- If B is full column rank (and it is for the US market - [Bauer & Rudebusch \(2017\)](#)):

$$X_t = (B'B)^{-1}B'(Y_t - A) = \tilde{A} + \tilde{B}Y_t \quad (4)$$

- This is a path-by-path condition: movements in yields should reveal movements in X_t ;
- It connects the whole distribution of Y_t and X_t ;

The Quadratic Variation Process

Definition 1 (Just a fancy variance!)

For a real-valued process M_t , given a partition $\{t_0 = t, t_1, \dots, t_{n-1}, t_n = t + h\}$, we define its Quadratic Variation between t and $t + h$ as

$$QV_M(t, t + h) \equiv \text{p-lim}_{\delta_n \rightarrow 0} \sum_{k=1}^n (M_{t_k} - M_{t_{k-1}})^2, \quad \delta_n \equiv \sup_{0 \leq k \leq n} \{t_k - t_{k-1}\} \quad (5)$$

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Proposition 1

For any linear combination of yields $L_t = c' Y_t$, its Quadratic Variation between t and $t + h$ is

$$QV_L(t, t + h) = \underbrace{\tilde{\gamma}_0 + \sum_{j=1}^J \tilde{\gamma}_{1,j} \cdot \bar{y}^{(\tau_j)}(t, t + h)}_{\text{Should be spanned by average yields}} + \underbrace{\sum_{k=1}^{\mathcal{N}_{t+h} - \mathcal{N}_t} v' Z_{T_k(t, t+h)} Z'_{T_k(t, t+h)} v}_{\text{No requirement to span the jump-only part!}} \quad (6)$$

where $\bar{y}^{(\tau_j)}(t, t + h) \equiv \frac{1}{h} \int_t^{t+h} y_s^{(\tau_j)} ds$, $\{\tilde{\gamma}_0, \tilde{\gamma}_1\}$ and v depend on parameters;

Identification

- Measuring the QV of stochastic process is usually easy: Realized Variance!
- Here it would incorporate **both** the diffusive (spanned) part and the jump-driven part;
- Can we tease out the diffusive part from the jumps? Yes: Bipower Variation!

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Definition 2 (Barndorff-Nielsen & Shephard (2004, 2006))

For a real-valued process M_t , we define the Bipower Variation process over $[t, t + h]$ as:

$$BPV_M(t, t + h) \equiv \text{p-lim}_{n \rightarrow \infty} \sum_{i=2}^n \left| M_{t+i \cdot \frac{h}{n}} - M_{t+(i-1) \cdot \frac{h}{n}} \right| \left| M_{t+(i-1) \cdot \frac{h}{n}} - M_{t+(i-2) \cdot \frac{h}{n}} \right| \quad (7)$$

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Proposition 2

Under this setup, the Bipower Variation of $L_t = c'Y_t$ identifies the diffusive part of QV_L :

$$BPV_L(t, t + h) = \frac{2}{\pi} \cdot \left[\tilde{\gamma}_0 + \sum_{j=1}^J \tilde{\gamma}_{1,j} \cdot \bar{y}^{(\tau_j)}(t, t + h) \right] \quad (8)$$

A Test of Unspanned Volatility

- This condition can be tested:
 - ▶ We can approximate both the LHS and RHS;
 - ▶ I use daily data to compute these measures at the *monthly* frequency;
- Regressing bipower variation measures on average yields should yield significant coefficients + high R^2 ;

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But why to focus on linear combinations of yields?...

- US yield curve admits a low-rank representation (Litterman & Scheinkman (1991));
- A common decomposition is the one from Nelson & Siegel (1987);
- Three factors: a long-end factor β_1 , a short-end factor β_2 , and a medium-end factor β_3 ;
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SPOILER ALERT: yes.

The Nelson-Siegel Representation

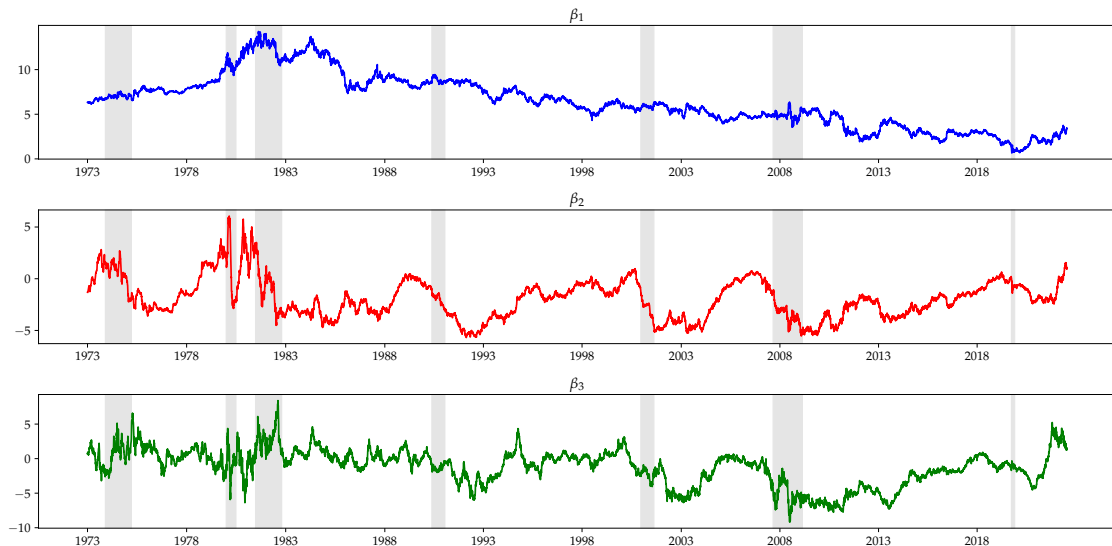
- $y_t^{(\tau)}$: zero-coupon rate at time t and maturity τ ;
- $\psi > 0$: a positive decay parameter;

$$y_t^{(\tau)} = \beta_{1,t} + \beta_{2,t} \left(\frac{1 - e^{-\psi\tau}}{\psi\tau} \right) + \beta_{3,t} \left(\frac{1 - e^{-\psi\tau}}{\psi\tau} - e^{-\psi\tau} \right) \quad (9)$$

- β_1 is a long-run factor: $\lim_{\tau \rightarrow \infty} y_t^{(\tau)} = \beta_{1,t}$;
- β_2 is a short-run factor: its absolute loading decreases with τ ;
- β_3 is a medium-run factor: its loading is hump-shaped;
- **Crucial**: for a fixed ψ , factors = linear combinations of yields;

How to estimate this???

Daily Factors

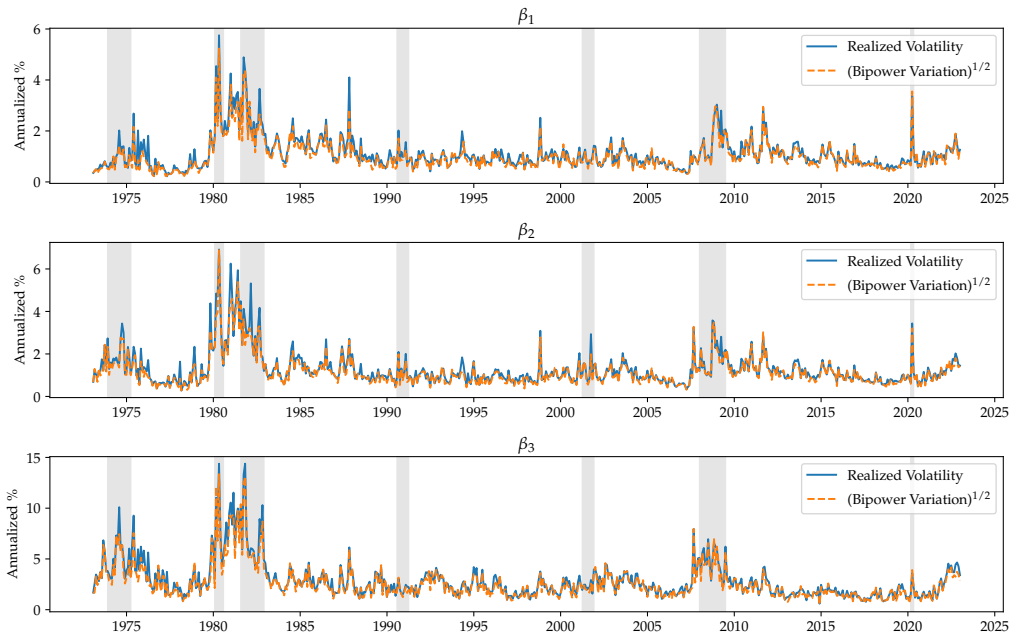


Average daily fitting error over maturities \approx 5bps;

Fitting error

Variation Measures

Covariances



Running the Test

- Recall: diffusive variation should be an affine function of average yields;
- BPV_i : bipower variation of factor $i \in \{1, 2, 3\}$;
- $BPCov_{i,j}$: bipower covariation between factors i and j , using a polarization identity;
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(Don't worry! Robustness checks in the paper!)

Test - Post-Volcker Sample

Table: Post-Volcker Sample (September, 1987 - December, 2022)

	BPV_1	BPV_2	BPV_3	$BPCov_{21}$	$BPCov_{31}$	$BPCov_{32}$
Average β_1	-0.03 (0.04)	-0.01 (0.05)	0.36 (0.30)	0.06 (0.04)	0.02 (0.06)	-0.04 (0.06)
Average β_2	-0.05 (0.07)	-0.03 (0.08)	-0.56 (0.52)	0.08 (0.06)	-0.04 (0.10)	0.00 (0.12)
Average β_3	-0.10 (0.08)	-0.15 (0.09)	-0.21 (0.52)	0.11 (0.07)	0.12 (0.11)	-0.07 (0.08)
N	424	424	424	424	424	424
R^2	0.07	0.08	0.06	0.13	0.02	0.01

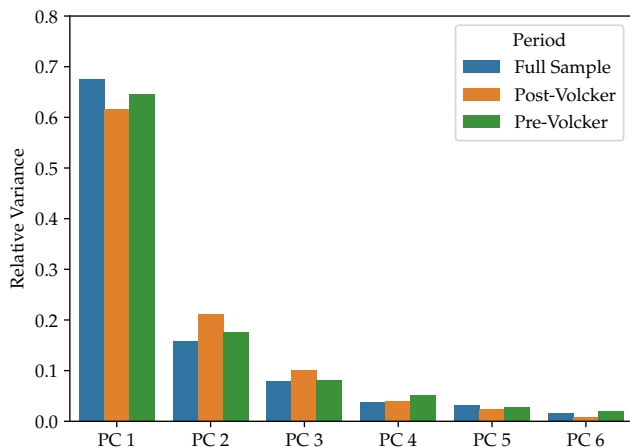
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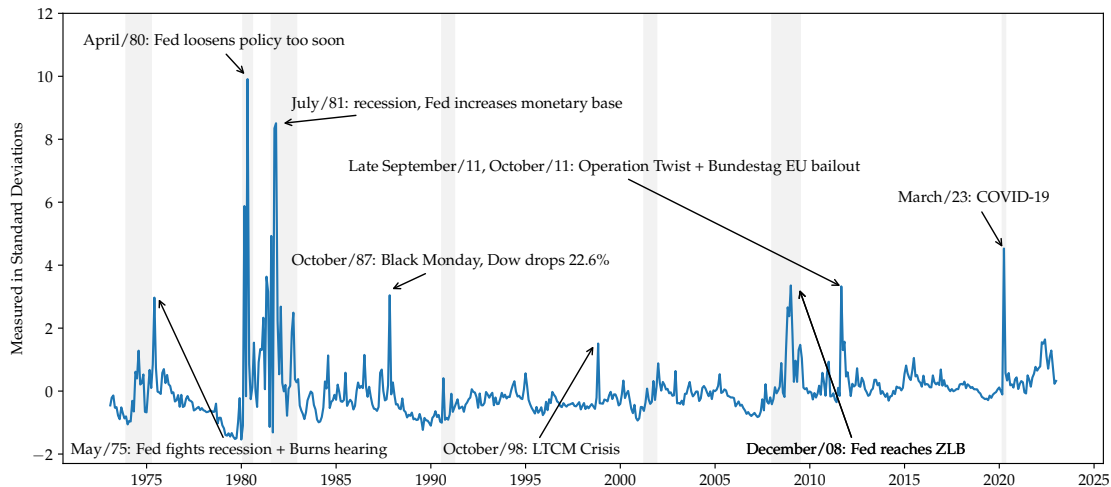
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Figure: Spectral decomposition of residuals



- The first PC of residuals commands 2/3 of the unexplained variation;
- If the failure of the previous tests were due to pure noise, we wouldn't see such a dominant factor;

How does this factor look like?



- Realizations are skewed, spiking up during recessions and major events;
- It's hard to make the case this is pure noise;

What can explain this factor?

- Much of the yield curve volatility is not accounted by affine term structure models;
- Spikes in the unspanned volatility seem related to monetary policy;
- How much of this factor can monetary policy explain?

$$USV_t = \alpha + \theta \cdot |\text{Shock}_t| + u_t \quad (12)$$

- Three types of monetary policy shocks from Swanson (2021);
 - ▶ Pure Fed Funds rate surprise, a forward-guidance shock, and a QE-type shock;
 - ▶ Identified using Fed Funds + Eurodollar futures (1991-2019);

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 - ▶ Identified using Fed Funds + Eurodollar futures (1991-2019);
- What about oil price shocks? \uparrow inflation, \uparrow inflation expectations ([Känzig, 2021](#));
 - ▶ Monthly frequency, identified with daily oil futures prices (1975-2022);
- This is about the US sovereign debt... can fiscal policy help explain volatility? ([Fisher & Peters, 2010](#); [Romer & Romer, 2010](#); [Ramey, 2011](#); [Ramey & Zubairy, 2018](#));

Monetary Policy

	First PC of Residuals			
	(1)	(2)	(3)	(4)
FFR	0.10 (0.10)		0.04 (0.09)	
FG		0.17** (0.08)	0.16** (0.07)	0.30* (0.17)
QE				-0.05 (0.09)
Sample	1991-2019			2009-2016
N	336	336	336	96
R^2	0.01	0.03	0.03	0.08

- 1 sd of FG $\approx \uparrow$ 6 bps on future Fed Funds 1 year ahead; Shocks Time Series
- Back of envelope: 25 bps worth of FG $\approx \uparrow$ 0.64 standard deviations in unspanned vol;

Oil Price Shocks + Monetary Policy

	(1)	(2)	(3)	(4)
Oil Shock	0.42*** (0.15)	0.43** (0.18)	0.43** (0.18)	0.43** (0.18)
FFR		0.05 (0.06)		0.02 (0.05)
FG			0.11** (0.05)	0.10** (0.05)
Sample	1975-2022	1991-2019		
N	576	336	336	336
R^2	0.02	0.09	0.12	0.13

- 10% oil price increase $\approx \uparrow 0.42$ standard deviations of USV ;
- Oil shock + monetary policy explain at most 13% of the unspanned volatility factor;

Oil Shock Time Series

Fiscal Policy

Table: Projecting Unspanned Vol on Fiscal Policy Shocks

	(1)	(2)	(3)	(4)
Tax Changes (Romer & Romer, 2010)	0.67* (0.39)			0.70* (0.39)
Defense Spending Shocks (Ramey & Zubairy, 2018)		0.04 (0.07)		-0.01 (0.08)
Defense Contractors Returns (Fisher & Peters, 2010)			0.03* (0.02)	0.02 (0.02)
End of sample (quarterly data)	2007	2015	2008	2007
N	140	172	144	140
R ²	0.02	0.00	0.02	0.04

- A tax change worth 1% of GDP $\implies \uparrow 0.7$ standard deviations of *USV*;

Shocks Time Series

Wrap Up

Main takeaways:

- I provide a jump-robust test for the presence of unspanned volatility;
- I show that there is unspanned volatility steaming from the entire maturity spectrum;
- Unspanned volatility as a single factor, which I formally characterize;
- This factor is *partially* driven by monetary policy, fiscal policy, and oil shocks;

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- What kind of other sources of variation are interesting here?

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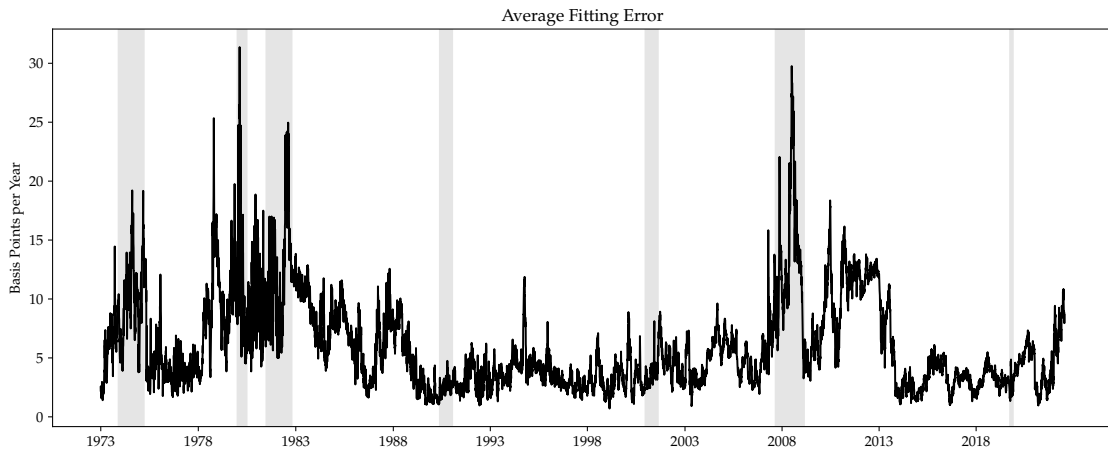
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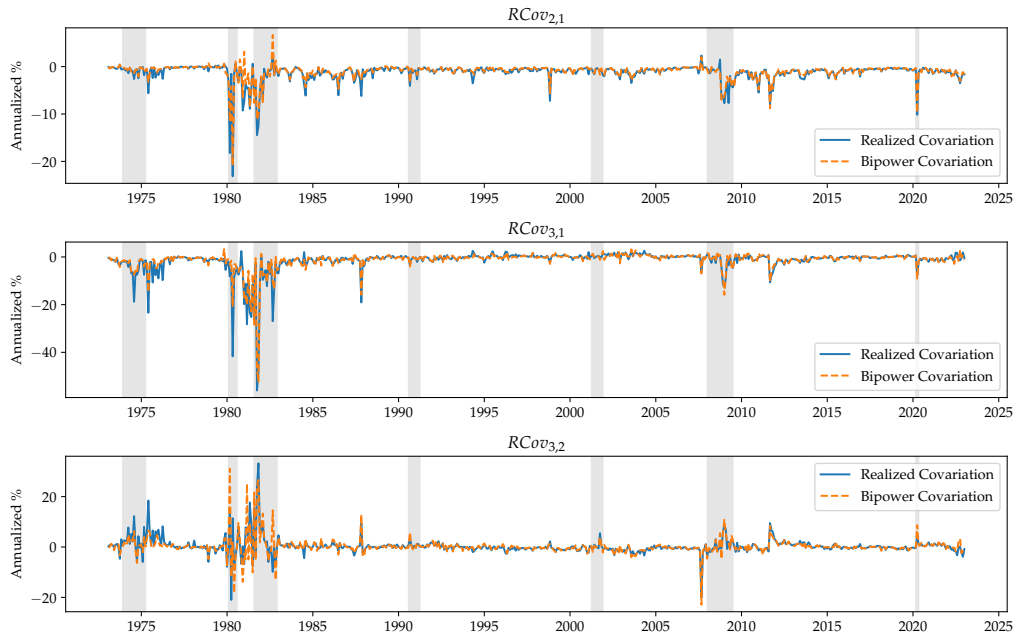
Appendix

Figures

Fitting Error



Realized Covariances

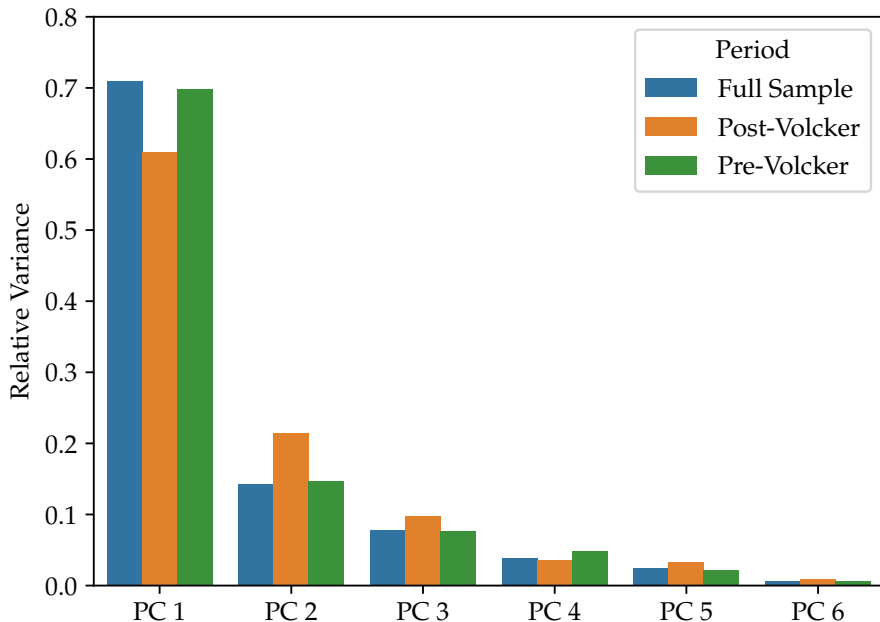


Test - Full Sample (1973-2022)

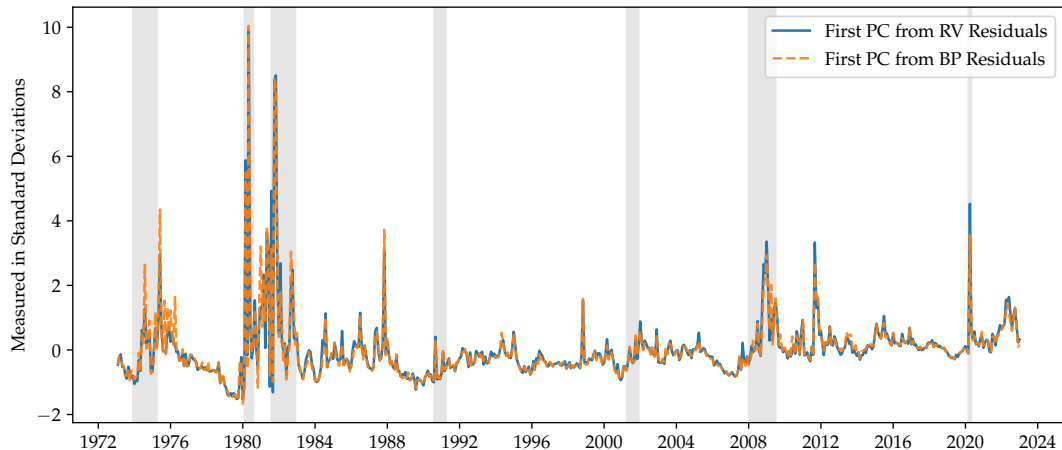
Table: Full Sample (1973-2022)

	BPV_1	BPV_2	BPV_3	$BPCov_{21}$	$BPCov_{31}$	$BPCov_{32}$
Average β_1	0.34*** (0.11)	0.56*** (0.18)	2.90*** (0.86)	-0.14** (0.07)	-0.56** (0.22)	0.14 (0.10)
Average β_2	0.34* (0.17)	0.87*** (0.32)	3.77*** (1.38)	-0.15 (0.11)	-0.64** (0.30)	0.04 (0.10)
Average β_3	-0.25** (0.12)	-0.54*** (0.21)	-1.68* (0.88)	0.19** (0.08)	0.25 (0.17)	0.03 (0.08)
N	600	600	600	600	600	600
R^2	0.18	0.31	0.27	0.08	0.18	0.02

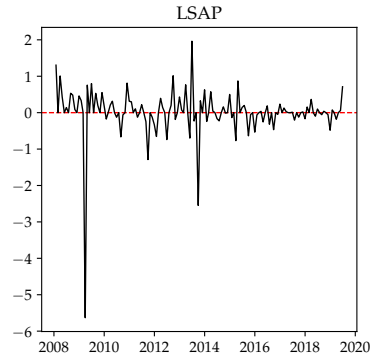
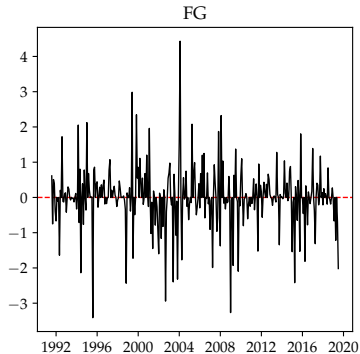
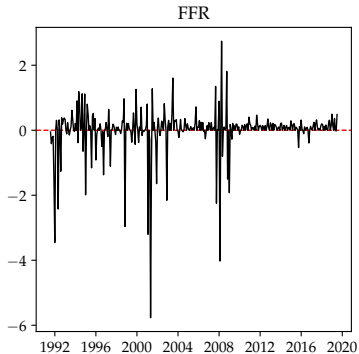
Spectral Decomposition of RV Residuals



Unspanned Factors: RV vs BP

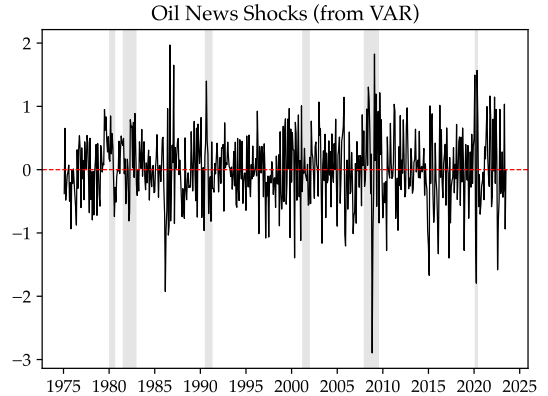
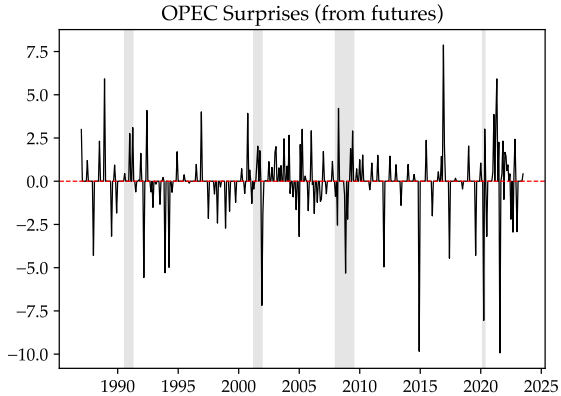


Monetary Policy Shocks from Swanson (2021)



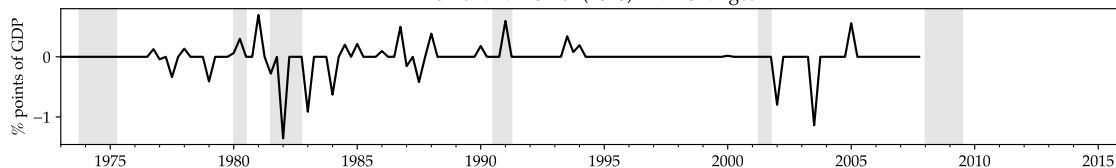
[Back](#)

Oil Shocks from Känzig (2021)

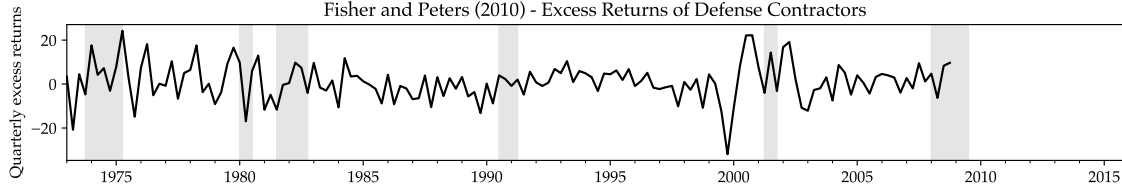


Fiscal Shocks

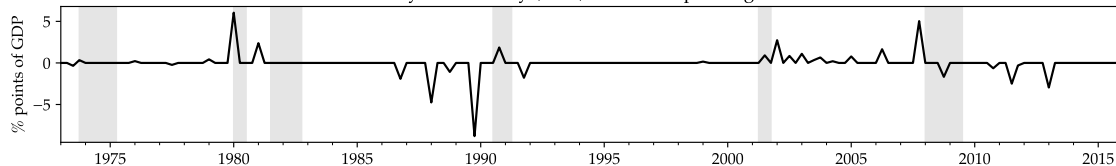
Romer and Romer (2010) - Tax Changes



Fisher and Peters (2010) - Excess Returns of Defense Contractors



Ramey and Zubairy (2018) - Defense Spending Shocks



Math and Tables

Estimating Nelson-Siegel Factors with OLS

- We estimate the factors using OLS: regress yields on coefficients;
- $\lambda > 0$ is fixed;
- No need of numerical solutions!

$$\begin{bmatrix} \hat{\beta}_{1,t} \\ \hat{\beta}_{2,t} \\ \hat{\beta}_{3,t} \end{bmatrix} = (M' M)^{-1} M' Y_t, \quad M \equiv \begin{bmatrix} 1 & \frac{1-e^{-\psi\tau_1}}{\psi\tau_1} & \frac{1-e^{\psi\tau_1}}{\psi\tau_1} - e^{-\lambda\tau_1} \\ 1 & \frac{1-e^{-\psi\tau_2}}{\psi\tau_2} & \frac{1-e^{\psi\tau_2}}{\psi\tau_2} - e^{-\lambda\tau_2} \\ \vdots & \vdots & \vdots \\ 1 & \frac{1-e^{-\psi\tau_J}}{\psi\tau_J} & \frac{1-e^{\psi\tau_J}}{\psi\tau_J} - e^{-\lambda\tau_J} \end{bmatrix}.$$

How Jumpy Are The Factors?

- How much variation is coming from the diffusive part? How much from the jumps?
- Surprisingly stable over factors and over time!

$$JV_i(t) \equiv \max\{RCov_{ii}(t) - BPV_i(t), 0\}, \quad JR_i(t) \equiv \frac{JV_i(t)}{RCov_{ii}(t)} \quad (13)$$

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Table: Average jumpiness of Nelson-Siegel factors

	JR_1	JR_2	JR_3
Whole Sample (1973-2022)	0.172	0.158	0.170
Months with MP activity	0.146	0.145	0.161
Months without MP activity	0.150	0.149	0.157
<i>p</i> -value for difference	0.799	0.808	0.812

References I

- Andersen, T. G. & Benzoni, L. (2010). Do bonds span volatility risk in the u.s. treasury market? a specification test for affine term structure models. *The Journal of Finance*, 65(2), 603–653.
- Backwell, A. (2021). Unspanned stochastic volatility from an empirical and practical perspective. *Journal of Banking & Finance*, 122, 105993.
- Barndorff-Nielsen, O. & Shephard, N. (2004). Power and bipower variation with stochastic volatility and jumps. *Journal of Financial Econometrics*, 2(1), 1–37.
- Barndorff-Nielsen, O. & Shephard, N. (2006). Econometrics of testing for jumps in financial economics using bipower variation. *Journal of Financial Econometrics*, 4(1), 1–30.
- Bauer, M. & Rudebusch, G. (2017). Resolving the spanning puzzle in macro-finance term structure models. *Review of Finance*, 21(2), 511–553.
- Bikbov, R. & Chernov, M. (2009). Unspanned stochastic volatility in affine models: Evidence from eurodollar futures and options. *Management Science*, 55(8), 1292–1305.
- Collin-Dufresne, P. & Goldstein, R. S. (2002). Do bonds span the fixed income markets? theory and evidence for unspanned stochastic volatility. *The Journal of Finance*, 57(4), 1685–1730.
- Filipovic, D., Larsson, M., & Trolle, A. (2017). Linear-rational term structure models. *The Journal of Finance*, 72(2), 655–704.

References II

- Fisher, J. D. & Peters, R. (2010). Using stock returns to identify government spending shocks. *The Economic Journal*, 120(544), 414–436.
- Känzig, D. R. (2021). The macroeconomic effects of oil supply news: Evidence from opec announcements. *American Economic Review*, 111(4), 1092–1125.
- Li, H. & Zhao, F. (2006). Unspanned stochastic volatility: Evidence from hedging interest rate derivatives. *The Journal of Finance*, 61(1), 341–378.
- Li, H. & Zhao, F. (2009). Nonparametric estimation of state-price densities implicit in interest rate cap prices. *Review of Financial Studies*, 22(11), 4335–4376.
- Litterman, R. B. & Scheinkman, J. (1991). Common factors affecting bond returns. *Journal of Fixed Income*, 1(1), 54–61.
- Liu, Y. & Wu, C. (2021). Reconstructing the yield curve. *Journal of Financial Economics*, 142(3), 1395–1425.
- Nelson, C. & Siegel, A. F. (1987). Parsimonious modeling of yield curves. *Journal of Business*, 60(4), 473–89.
- Piazzesi, M. (2010). *Affine Term Structure Models*, (pp. 691–766). Elsevier.
- Ramey, V. A. (2011). Identifying government spending shocks: It's all in the timing*. *The Quarterly Journal of Economics*, 126(1), 1–50.

References III

- Ramey, V. A. & Zubairy, S. (2018). Government spending multipliers in good times and in bad: Evidence from us historical data. *Journal of Political Economy*, 126(2), 850–901.
- Romer, C. D. & Romer, D. H. (2010). The macroeconomic effects of tax changes: Estimates based on a new measure of fiscal shocks. *American Economic Review*, 100(3), 763–801.
- Swanson, E. T. (2021). Measuring the effects of federal reserve forward guidance and asset purchases on financial markets. *Journal of Monetary Economics*, 118, 32–53.