How much unspanned volatility can different shocks explain?

Raul Riva

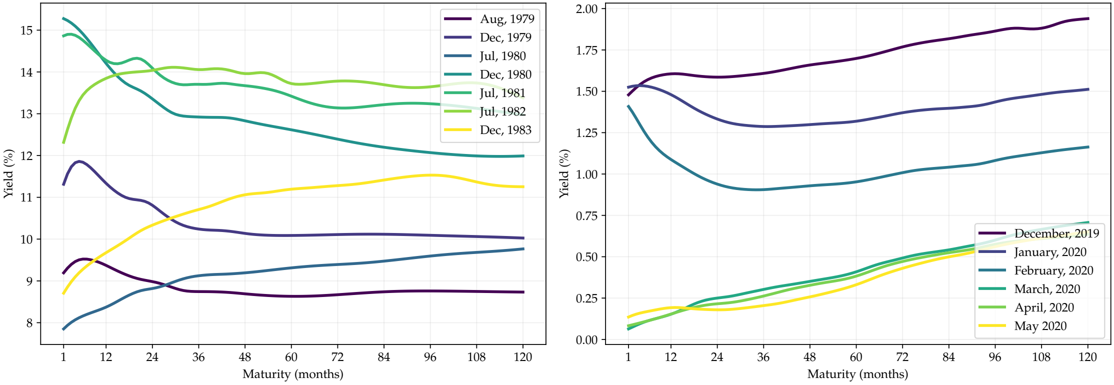
FGV EPGE

August 25th, 2025

Econometrics Reading Group

It likes to move and move!

Figure 1: Salient days for the American Yield Curve



Intro

Why should we care about *volatility* in the nominal US yield curve?

1. Hedging of interest-rate derivatives: huge, liquid market with many players;
2. Tightly linked to volatility of holding returns for bonds: portfolio allocation;
3. Risk management of large bond portfolios from institutional investors;

Intro

Why should we care about *volatility* in the nominal US yield curve?

1. Hedging of interest-rate derivatives: huge, liquid market with many players;
2. Tightly linked to volatility of holding returns for bonds: portfolio allocation;
3. Risk management of large bond portfolios from institutional investors;

Do we have good models for yield curve volatility? Yes and no:

- Workhorse: Dynamic Term Structure Models (very often affine ones);
- Tractable formulas for yields + arbitrage-free framework + convenient for estimation;
- Model-consistent separation between term premia and expected future short rates;

Intro

Why should we care about *volatility* in the nominal US yield curve?

1. Hedging of interest-rate derivatives: huge, liquid market with many players;
2. Tightly linked to volatility of holding returns for bonds: portfolio allocation;
3. Risk management of large bond portfolios from institutional investors;

Do we have good models for yield curve volatility? Yes and no:

- Workhorse: Dynamic Term Structure Models (very often affine ones);
- Tractable formulas for yields + arbitrage-free framework + convenient for estimation;
- Model-consistent separation between term premia and expected future short rates;
- Poor time-series dynamics, sharp restrictions on how yields should behave;
- Important **today**: observed vol should be tightly connected to the cross-section of yields;

Can affine term structure models account for volatility in yields?

Mostly, no. In general, there is more variation than models allow. Some approaches:

- Regress returns from straddles on interest rate changes;
  - Collin-Dufresne and Goldstein (2002) and Li and Zhao (2006)
- Regress changes of implied volatility from options/swaptions on interest rate changes;
  - Filipovic, Larsson, and Trolle (2017) and Backwell (2021)
- Likelihood-ratio tests for conditions that connect yield volatility and in yield levels;
  - Bikbov and Chernov (2009)
- State-price density estimation from options data;
  - Li and Zhao (2009)
- Restrictions from high(er)-frequency data;
  - Andersen and Benzoni (2010)
  - Closest paper to mine, but we deal with jumps and different maturities very differently;

Any room for improvement?

- Jump-diffusion settings are not so common, but jumps are prevalent in bond markets Piazzesi (2010);
- What derivatives to use in an empirical test? Results seem dependent on this choice;
  - Swaptions? Caps and floors? Straddles? At the money? Out of the money?
  - Liquidity and availability of strikes also depend on overall volatility itself...
- Analyses done at the individual maturity level;
  - Too many degrees of freedom;
  - What maturities should we pay attention to?

Any room for improvement?

- Jump-diffusion settings are not so common, but jumps are prevalent in bond markets Piazzesi (2010);
- What derivatives to use in an empirical test? Results seem dependent on this choice;
    - Swaptions? Caps and floors? Straddles? At the money? Out of the money?
    - Liquidity and availability of strikes also depend on overall volatility itself...
- Analyses done at the individual maturity level;
    - Too many degrees of freedom;
    - What maturities should we pay attention to?
- **Crucially**: attempts to tie "excessive" volatility to real-world developments are rare;
    - This is where the money is! Super important for derivative hedging!
    - What can help explain this "unspanned" volatility? Probably not just noise...

This paper: two contributions

New methodology: a new test for excess volatility with a number of advantages;

- Implications for non-parametric measures of yield volatility within the affine framework;
- Only zero-coupon yields needed;
- I don't analyze specific maturities, focus on a decomposition of the whole curve;
- Characterization of an *unspanned volatility* factor: 2/3 of the residual volatility;

This paper: two contributions

New methodology: a new test for excess volatility with a number of advantages;

- Implications for non-parametric measures of yield volatility within the affine framework;
- Only zero-coupon yields needed;
- I don't analyze specific maturities, focus on a decomposition of the whole curve;
- Characterization of an *unspanned volatility* factor: 2/3 of the residual volatility;

New empirical results: what can explain this unspanned volatility factor?

- Focus on shocks from the literature on monetary policy, fiscal policy, and oil shocks;
- Forward-guidance-type shocks, oil, and fiscal policy shocks help driving this factor;
- These shocks explain $\approx 13\%$ of variation. Still a lot to explain (and write about!).

Data

- Yield curve data: daily zero-coupon curve from Liu and Wu (2021), from 1973 to 2022;
- Monetary policy shocks from Swanson (2021) - monthly frequency;
- Oil shocks identified from Känzig (2021) - monthly frequency;
- Fiscal shocks from different sources - quarterly frequency:
  - Defense spending shocks from Ramey (2011) and Ramey and Zubairy (2018);
  - Tax policy shocks from C. D. Romer and D. H. Romer (2010);
  - Stock returns from top US government defense contractors Fisher and Peters (2010);

Quick Flight Plan

1. Basics of an affine term structure model + implications for the quadratic variation process;
2. A decomposition of the yield curve using Nelson-Siegel factors;
3. Data + empirical results: unspanned volatility across the entire maturity spectrum;
   - Characterization of the unspanned volatility factor;
4. How much of unspanned volatility can different shocks explain? Let's project it out!

A Flexible Affine Setup

- There are N latent underlying risk factors X_t. The evolution under Q follows:

$$dX_t = K\left(\Theta - X_t\right) dt + \Sigma\sqrt{S_t} dW_t^Q + Z_t d\mathcal{N}_t^Q \tag{1}$$

- K and Σ are $N \times N$ constant matrices; Θ is an $N \times 1$ vector of long-run means;
- S_t is an $N \times N$ diagonal matrix whose diagonal elements follow:

$$S_{t,[ii]} = s_{0,i} + s_{1,i}' X_t \tag{2}$$

- W_t^Q is a Brownian Motion and $\mathcal{N}_t^Q$ a Poisson process with intensity $\lambda_t = \lambda_0 + \lambda_1' X_t$;
- $Z_t \sim \nu^Q$ represents a jump size, it is independent of both W_t^Q and $\mathcal{N}_t^Q$, with $\mathbb{E}[Z_t Z_t'] = \Omega$;
- The short rate r_t is given by: $r_t = \delta_0 + \delta_1' X_t$;

Bond Prices and Bond Yields

- This setup ensures that zero-coupon yields $y_t^{(\tau)}$ are an affine function of state variables;
- If we trade J fixed maturities $(\tau_1, ..., \tau_J)$ we can write for some vector A and matrix B:

$$Y_t = A + BX_t \tag{3}$$

Bond Prices and Bond Yields

- This setup ensures that zero-coupon yields $y_t^{(\tau)}$ are an affine function of state variables;
- If we trade J fixed maturities $(\tau_1, ..., \tau_J)$ we can write for some vector A and matrix B:

$$Y_t = A + BX_t \tag{3}$$

- If B is full column rank (and it is for the US market - Bauer and Rudebusch (2017)):

$$X_t = (B'B)^{-1}B'(Y_t - A) = \widetilde{A} + \widetilde{B}Y_t \tag{4}$$

- This is a path-by-path condition: movements in yields should reveal movements in X_t;
- It connects the whole distribution of Y_t and X_t;

The Quadratic Variation Process

Definition 1 (Just a fancy variance!)

For a real-valued process M_t, given a partition $\{t_0 = t, t_1, ..., t_{n-1}, t_n = t + h\}$, we define its Quadratic Variation between t and $t_n + h$ as

$$QV_M(t, t+h) \equiv \underset{\delta_n \to 0}{\text{p-lim}} \sum_{k=1} \left(M_{t_k} - M_{t_{k-1}}\right)^2, \quad \delta_n \equiv \sup_{0 \leq k \leq n} \{t_k - t_{k-1}\} \tag{5}$$

The Quadratic Variation Process

Definition 1 (Just a fancy variance!)

For a real-valued process M_t, given a partition $\{t_0 = t, t_1, ..., t_{n-1}, t_n = t + h\}$, we define its Quadratic Variation between t and $t_n + h$ as

$$QV_M(t, t+h) \equiv \underset{\delta_n \to 0}{\text{p-lim}} \sum_{k=1} \left(M_{t_k} - M_{t_{k-1}}\right)^2, \quad \delta_n \equiv \sup_{0 \leq k \leq n} \{t_k - t_{k-1}\} \tag{5}$$

Proposition 1

For any linear combination of yields $L_t = c' Y_t$, its Quadratic Variation between t and $t + h$ is

$$QV_L(t, t+h) = \tilde{\gamma}_0 + \underbrace{\sum_{j=1}^{J} \tilde{\gamma}_{1,j} \cdot \overline{y}^{(\tau_j)}(t, t+h)}_{\textit{Should be spanned by average yields}} + \underbrace{\sum_{k=1}^{\mathcal{N}_{t+h} - \mathcal{N}_t} v' Z_{T_k(t,t+h)} Z'_{T_k(t,t+h)} v}_{\textit{No requirement to span the jump-only part!}} \tag{6}$$

where $\overline{y}^{(\tau_j)}(t, t+h) \equiv \frac{1}{h} \int_t^{t+h} y_s^{(\tau_j)} ds$, $\{\tilde{\gamma}_0, \tilde{\gamma}_1\}$ and v depend on parameters;

Identification

- Measuring the QV of stochastic process is usually easy: Realized Variance!
- Here it would incorporate **both** the diffusive (spanned) part and the jump-driven part;
- Can we tease out the diffusive part from the jumps? Yes: Bipower Variation!

Identification

- Measuring the QV of stochastic process is usually easy: Realized Variance!
- Here it would incorporate **both** the diffusive (spanned) part and the jump-driven part;
- Can we tease out the diffusive part from the jumps? Yes: Bipower Variation!

Definition 2

For a real-valued process M_t, we define the Bipower Variation process over $[t, t+h]$ as:

$$BPV_M(t, t+h) \equiv \operatorname*{p-lim}_{n\to\infty} \sum_{i=2}^{n} \left| M_{t+i\cdot\frac{h}{n}} - M_{t+(i-1)\cdot\frac{h}{n}} \right| \left| M_{t+(i-1)\cdot\frac{h}{n}} - M_{t+(i-2)\cdot\frac{h}{n}} \right| \tag{7}$$

Identification

- Measuring the QV of stochastic process is usually easy: Realized Variance!
- Here it would incorporate **both** the diffusive (spanned) part and the jump-driven part;
- Can we tease out the diffusive part from the jumps? Yes: Bipower Variation!

Definition 2

For a real-valued process M_t, we define the Bipower Variation process over $[t, t+h]$ as:

$$BPV_M(t, t+h) \equiv \underset{n\to\infty}{\text{p-lim}} \sum_{i=2}^{n} \left| M_{t+i\cdot\frac{h}{n}} - M_{t+(i-1)\cdot\frac{h}{n}} \right| \left| M_{t+(i-1)\cdot\frac{h}{n}} - M_{t+(i-2)\cdot\frac{h}{n}} \right| \tag{7}$$

Proposition 2

Under this setup, the Bipower Variation of $L_t = c'Y_t$ identifies the diffusive part of QV_L:

$$BPV_L(t, t+h) = \frac{2}{\pi} \cdot \left[\tilde{\gamma}_0 + \sum_{j=1}^{J} \tilde{\gamma}_{1,j} \cdot \overline{y}^{(\tau_j)}(t, t+h) \right] \tag{8}$$

A Test of Unspanned Volatility

- This condition can be tested:
  - We can approximate both the LHS and RHS;
  - I use daily data to compute these measures at the *monthly* frequency;
- Regressing bipower variation measures on average yields should yield significant coefficients + high R^2;

A Test of Unspanned Volatility

- This condition can be tested:
  - We can approximate both the LHS and RHS;
  - I use daily data to compute these measures at the *monthly* frequency;
- Regressing bipower variation measures on average yields should yield significant coefficients + high R^2;

But why to focus on linear combinations of yields?...

- US yield curve admits a low-rank representation (Litterman and Scheinkman (1991));
- A common decomposition is the one from Nelson and Siegel (1987);
- Three factors: a long-end factor β_1, a short-end factor β_2, and a medium-end factor β_3;
- Time-series characteristics of yields depend on how these factors evolve;

A Test of Unspanned Volatility

- This condition can be tested:
  - We can approximate both the LHS and RHS;
  - I use daily data to compute these measures at the *monthly* frequency;
- Regressing bipower variation measures on average yields should yield significant coefficients + high R^2;

But why to focus on linear combinations of yields?...

- US yield curve admits a low-rank representation (Litterman and Scheinkman (1991));
- A common decomposition is the one from Nelson and Siegel (1987);
- Three factors: a long-end factor β_1, a short-end factor β_2, and a medium-end factor β_3;
- Time-series characteristics of yields depend on how these factors evolve;
- My JMP shows that there is **unspanned risk premium** *only* through β_2;
- What about volatility? Do we have **unspanned volatility** from **every part** of the curve?

A Test of Unspanned Volatility

- This condition can be tested:
  - We can approximate both the LHS and RHS;
  - I use daily data to compute these measures at the *monthly* frequency;
- Regressing bipower variation measures on average yields should yield significant coefficients + high R^2;

But why to focus on linear combinations of yields?...

- US yield curve admits a low-rank representation (Litterman and Scheinkman (1991));
- A common decomposition is the one from Nelson and Siegel (1987);
- Three factors: a long-end factor β_1, a short-end factor β_2, and a medium-end factor β_3;
- Time-series characteristics of yields depend on how these factors evolve;
- My JMP shows that there is **unspanned risk premium** *only* through β_2;
- What about volatility? Do we have **unspanned volatility** from **every part** of the curve?

SPOILER ALERT: yes.

Factor Realizations Fitting Error

The Nelson-Siegel Representation

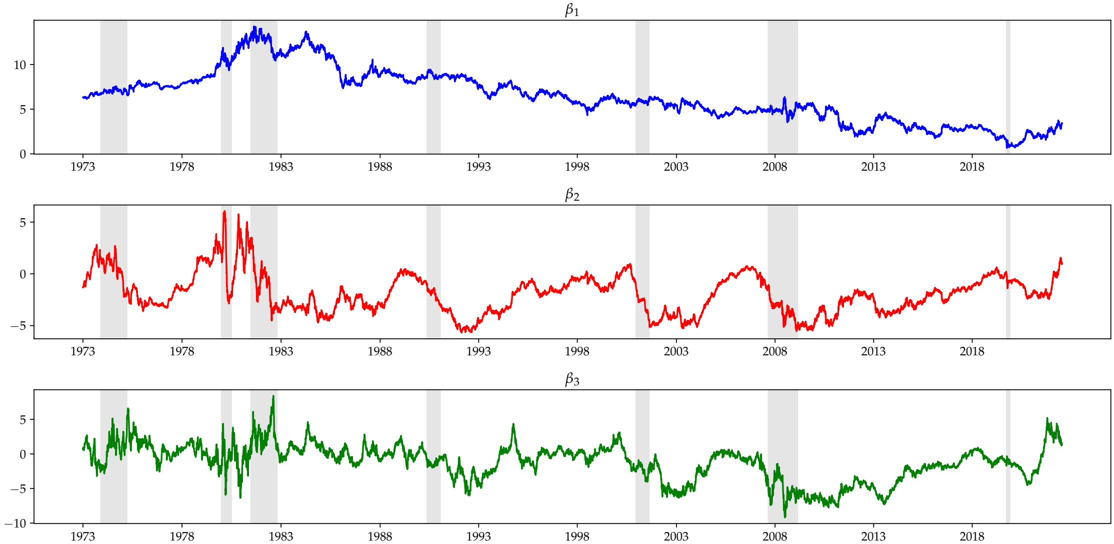
- $y_t^{(\tau)}$: zero-coupon rate at time t and maturity τ;
- $\psi > 0$: a positive decay parameter;

$$y_t^{(\tau)} = \beta_{1,t} + \beta_{2,t}\left(\frac{1-e^{-\psi\tau}}{\psi\tau}\right) + \beta_{3,t}\left(\frac{1-e^{-\psi\tau}}{\psi\tau} - e^{-\psi\tau}\right) \tag{9}$$

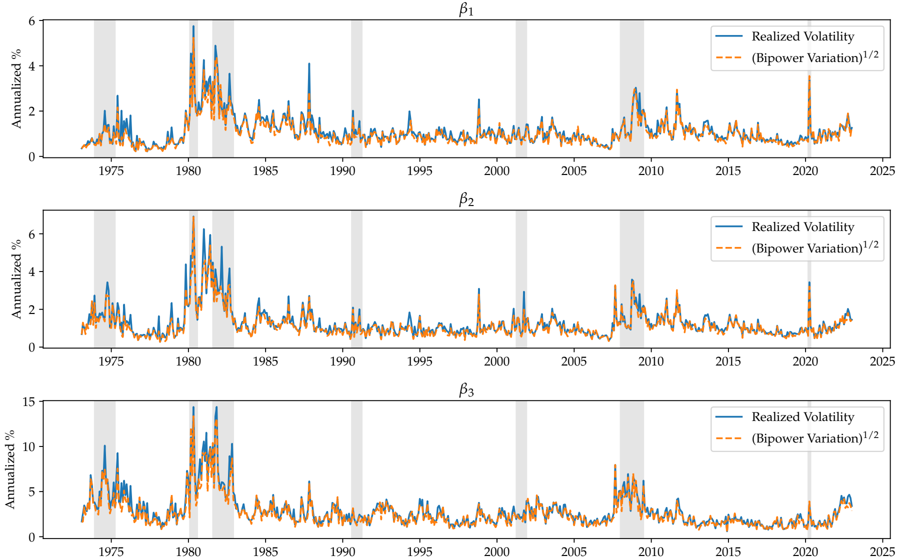
- β_1 is a long-run factor: $\lim_{\tau\to\infty} y_t^{(\tau)} = \beta_{1,t}$;
- β_2 is a short-run factor: its absolute loading decreases with τ;
- β_3 is a medium-run factor: its loading is hump-shaped;
- **Crucial**: for a fixed ψ, factors = linear combinations of yields;

How to estimate this???

Daily Factors



Variation Measures



Running the Test

- Recall: diffusive variation should be an affine function of average yields;
- BPV_i: bipower variation of factor $i \in \{1, 2, 3\}$;
- $BPCov_{i,j}$: bipower covariation between factors i and j, using a polarization identity;
- Notation: $BPCov_{i,i} \equiv BPV_i$, for any i;

Running the Test

- Recall: diffusive variation should be an affine function of average yields;
- BPV_i: bipower variation of factor $i \in \{1, 2, 3\}$;
- $BPCov_{i,j}$: bipower covariation between factors i and j, using a polarization identity;
- Notation: $BPCov_{i,i} \equiv BPV_i$, for any i;
- The derivations suggest the following regression:

$$BPCov_{i,j}(t) = \delta_{i,j} + \theta'_{i,j}\overline{Y}_t + \eta_{i,j}(t), \qquad i, j = 1, 2, 3 \tag{10}$$

Running the Test

- Recall: diffusive variation should be an affine function of average yields;
- BPV_i: bipower variation of factor $i \in \{1, 2, 3\}$;
- $BPCov_{i,j}$: bipower covariation between factors i and j, using a polarization identity;
- Notation: $BPCov_{i,i} \equiv BPV_i$, for any i;
- The derivations suggest the following regression:

$$BPCov_{i,j}(t) = \delta_{i,j} + \theta'_{i,j}\overline{Y}_t + \eta_{i,j}(t), \qquad i, j = 1, 2, 3 \tag{10}$$

- But the yield curve has a low-rank factor structure... and what yields to include?
- Why not use the Nelson-Siegel factors directly?

Running the Test

- Recall: diffusive variation should be an affine function of average yields;
- BPV_i: bipower variation of factor $i \in \{1, 2, 3\}$;
- $BPCov_{i,j}$: bipower covariation between factors i and j, using a polarization identity;
- Notation: $BPCov_{i,i} \equiv BPV_i$, for any i;
- The derivations suggest the following regression:

$$BPCov_{i,j}(t) = \delta_{i,j} + \theta'_{i,j}\overline{Y}_t + \eta_{i,j}(t), \qquad i, j = 1, 2, 3 \tag{10}$$

- But the yield curve has a low-rank factor structure... and what yields to include?
- Why not use the Nelson-Siegel factors directly?

$$BPCov_{i,j}(t) = \delta_{i,j} + \theta^{(1)}_{i,j}\overline{\beta}_{1,t} + \theta^{(2)}_{i,j}\overline{\beta}_{2,t} + \theta^{(3)}_{i,j}\overline{\beta}_{3,t} + \eta_{i,j}(t), \qquad i, j = 1, 2, 3 \tag{11}$$

(Don't worry! Robustness checks in the paper!)

Test - Post-Volcker Sample

Table 1: Post-Volcker Sample (September, 1987 - December, 2022)

	BPV_1	BPV_2	BPV_3	$BPCov_{21}$	$BPCov_{31}$	$BPCov_{32}$
Average β_1	-0.03	-0.01	0.36	0.06	0.02	-0.04
	(0.04)	(0.05)	(0.30)	(0.04)	(0.06)	(0.06)
Average β_2	-0.05	-0.03	-0.56	0.08	-0.04	0.00
	(0.07)	(0.08)	(0.52)	(0.06)	(0.10)	(0.12)
Average β_3	-0.10	-0.15	-0.21	0.11	0.12	-0.07
	(0.08)	(0.09)	(0.52)	(0.07)	(0.11)	(0.08)
N	424	424	424	424	424	424
R^2	0.07	0.08	0.06	0.13	0.02	0.01

Full Sample

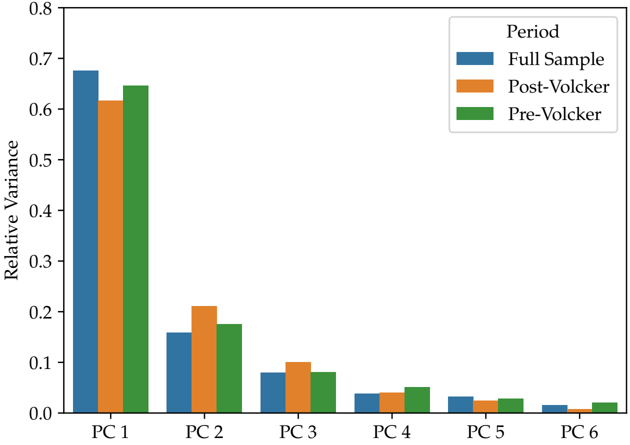
Is everything just noise?

- Each regression delivers a time series of residuals $\implies$ Six residual series in total;

Is everything just noise?

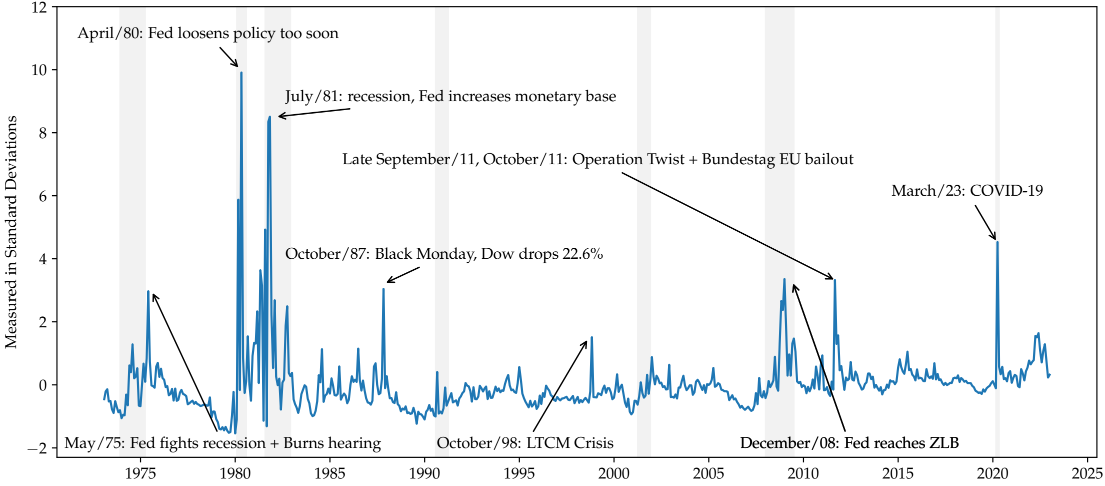
- Each regression delivers a time series of residuals $\Longrightarrow$ Six residual series in total;

Spectral decomposition of residuals



- The first PC of residuals commands 2/3 of the unexplained variation;
- If the failure of the previous tests were due to pure noise, we wouldn't see such a dominant factor;

How does this factor look like?



- Realizations are skewed, spiking up during recessions and major events;

What can explain this factor?

- Much of the yield curve volatility is not accounted by affine term structure models;
- Spikes in the unspanned volatility seem related to monetary policy;
- How much of this factor can monetary policy explain?

$$USV_t = \alpha + \theta \cdot |\text{Shock}_t| + u_t \tag{12}$$

- Three types of monetary policy shocks from Swanson (2021);
  - Pure Fed Funds rate surprise, a forward-guidance shock, and a QE-type shock;
  - Identified using Fed Funds + Eurodollar futures (1991-2019);

What can explain this factor?

- Much of the yield curve volatility is not accounted by affine term structure models;
- Spikes in the unspanned volatility seem related to monetary policy;
- How much of this factor can monetary policy explain?

$$USV_t = \alpha + \theta \cdot |\text{Shock}_t| + u_t \tag{12}$$

- Three types of monetary policy shocks from Swanson (2021);
  - Pure Fed Funds rate surprise, a forward-guidance shock, and a QE-type shock;
  - Identified using Fed Funds + Eurodollar futures (1991-2019);
- What about oil price shocks? ↑ inflation, ↑ inflation expectations Känzig (2021);
  - Monthly frequency, identified with daily oil futures prices (1975-2022);
- This is about the US sovereign debt... can fiscal policy help explain volatility? (Fisher and Peters 2010; C. D. Romer and D. H. Romer 2010; Ramey 2011; Ramey and Zubairy 2018);

Monetary Policy

	First PC of Residuals			
	(1)	(2)	(3)	(4)
FFR	0.10		0.04	
	(0.10)		(0.09)	
FG		0.17**	0.16**	0.30*
		(0.08)	(0.07)	(0.17)
QE				-0.05
				(0.09)
Sample	1991-2019			2009-2016
N	336	336	336	96
R^2	0.01	0.03	0.03	0.08

- 1 sd of FG $\approx \uparrow$ 6 bps on future Fed Funds 1 year ahead; Shocks Time Series
- Back of envelope: 25 bps worth of FG $\approx \uparrow$ 0.64 standard deviations in unspanned vol;

Oil Price Shocks + Monetary Policy

Dependent Variable: USV_t

	(1)	(2)	(3)	(4)
Oil Shock	0.42***	0.43**	0.43**	0.43**
	(0.15)	(0.18)	(0.18)	(0.18)
FFR		0.05		0.02
		(0.06)		(0.05)
FG			0.11**	0.10**
			(0.05)	(0.05)
Sample	1975-2022	1991-2019		
N	576	336	336	336
R^2	0.02	0.09	0.12	0.13

- 10% oil price increase $\approx \uparrow$ 0.42 standard deviations of USV;
- Oil shock + monetary policy explain at most 13% of the unspanned volatility factor;

Oil Shock Time Series

Fiscal Policy

Projecting Unspanned Vol on Fiscal Policy Shocks

	(1)	(2)	(3)	(4)
Tax Changes	0.67*			0.70*
C. D. Romer and D. H. Romer (2010)	(0.39)			(0.39)
Defense Spending Shocks		0.04		-0.01
Ramey and Zubairy (2018)		(0.07)		(0.08)
Defense Contractors Returns			0.03*	0.02
Fisher and Peters (2010)			(0.02)	(0.02)
End of sample (quarterly data)	2007	2015	2008	2007
N	140	172	144	140
R^2	0.02	0.00	0.02	0.04

- A tax change worth 1% of GDP $\implies \uparrow$ 0.7 standard deviations of USV; Shocks Time Series

Wrap Up

Main takeaways:

- I provide a jump-robust test for the presence of unspanned volatlity;
- I show that there is unspanned volatility steaming from the entire maturity spectrum;
- Unspanned volatility as a single factor, which I formally characterize;
- This factor is *partially* driven by monetary policy, fiscal policy, and oil shocks;

Wrap Up

Main takeaways:

- I provide a jump-robust test for the presence of unspanned volatlity;
- I show that there is unspanned volatility steaming from the entire maturity spectrum;
- Unspanned volatility as a single factor, which I formally characterize;
- This factor is *partially* driven by monetary policy, fiscal policy, and oil shocks;

Going forward:

- Allow for more general dynamics between the unspanned vol factor and shocks? VARs?
- What kind of other sources of variation are interesting here?

Wrap Up

Main takeaways:

- I provide a jump-robust test for the presence of unspanned volatlity;
- I show that there is unspanned volatility steaming from the entire maturity spectrum;
- Unspanned volatility as a single factor, which I formally characterize;
- This factor is *partially* driven by monetary policy, fiscal policy, and oil shocks;

Going forward:

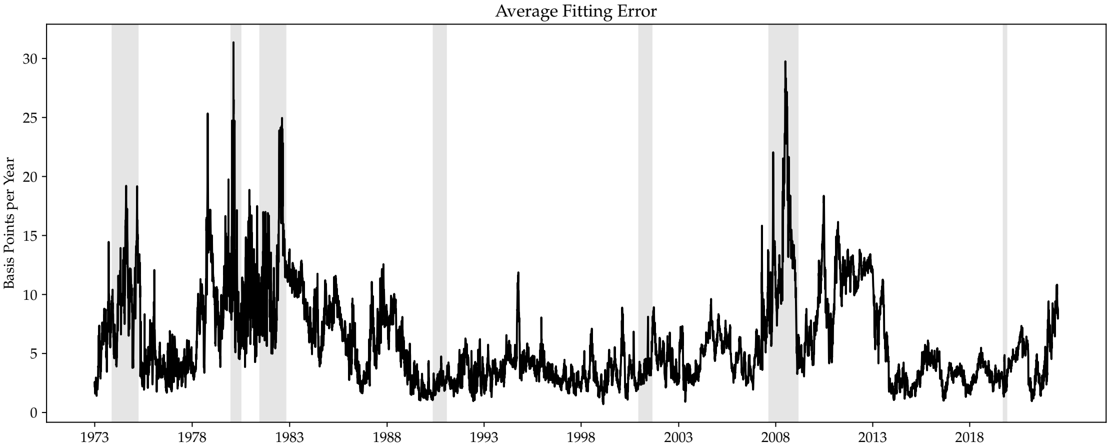
- Allow for more general dynamics between the unspanned vol factor and shocks? VARs?
- What kind of other sources of variation are interesting here?

Thank you!

Appendix

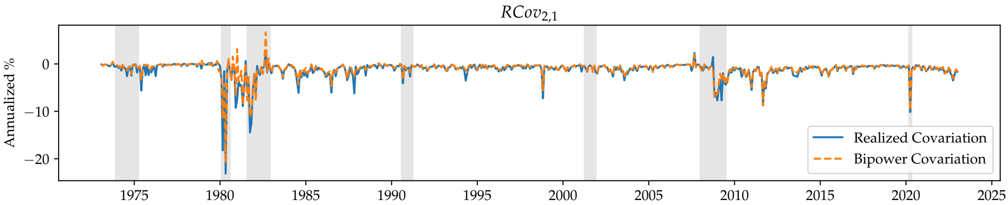
Figures

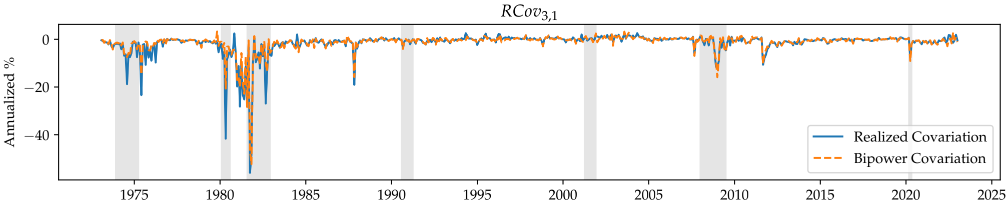
Fitting Error

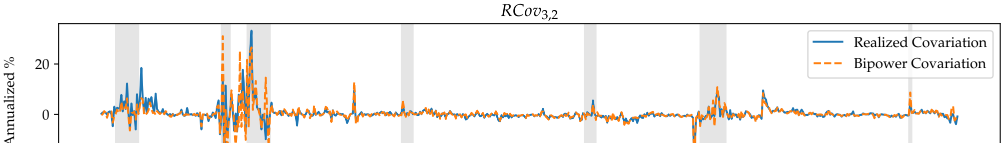


Back

Realized Covariances







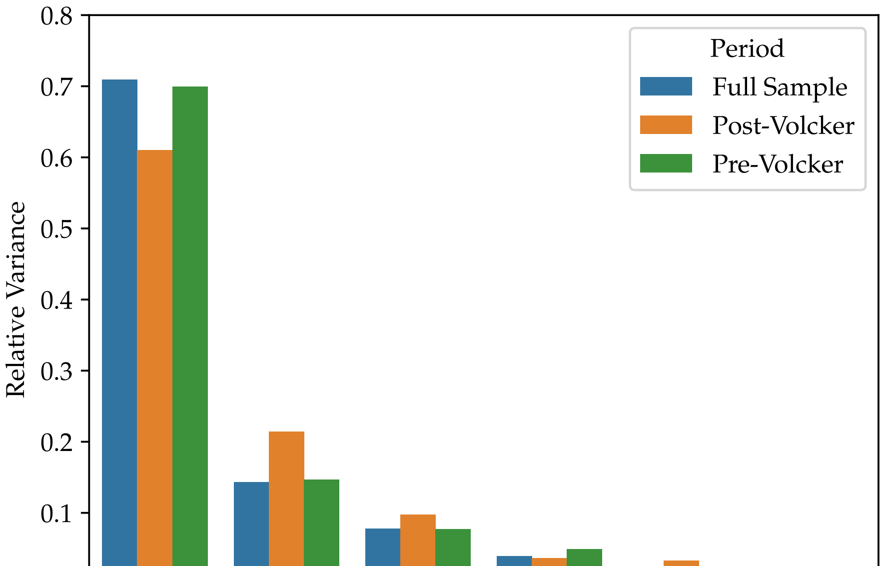
Test - Full Sample (1973-2022)

Table 2: Full Sample (1973-2022)

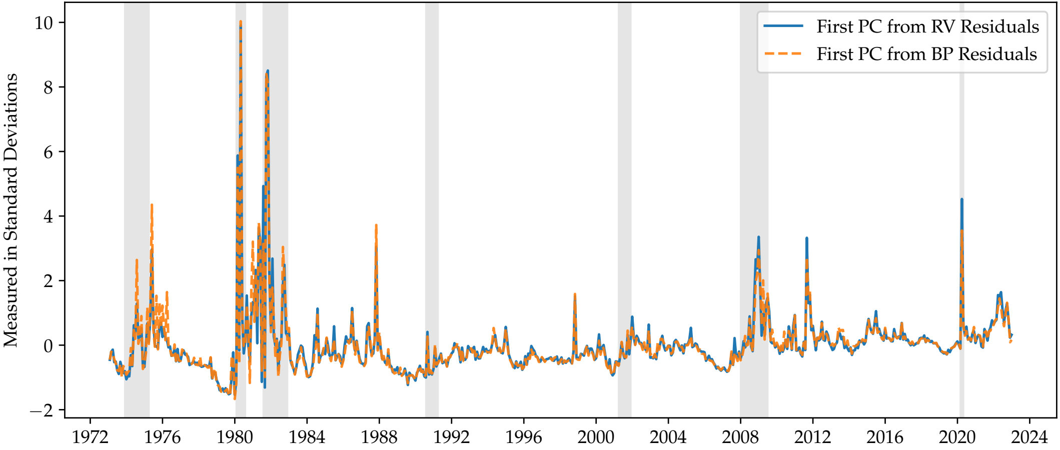
	BPV_1	BPV_2	BPV_3	$BPCov_{21}$	$BPCov_{31}$	$BPCov_{32}$
Average β_1	0.34***	0.56***	2.90***	-0.14**	-0.56**	0.14
	(0.11)	(0.18)	(0.86)	(0.07)	(0.22)	(0.10)
Average β_2	0.34*	0.87***	3.77***	-0.15	-0.64**	0.04
	(0.17)	(0.32)	(1.38)	(0.11)	(0.30)	(0.10)
Average β_3	-0.25**	-0.54***	-1.68*	0.19**	0.25	0.03
	(0.12)	(0.21)	(0.88)	(0.08)	(0.17)	(0.08)
N	600	600	600	600	600	600
R^2	0.18	0.31	0.27	0.08	0.18	0.02

Volcker Test

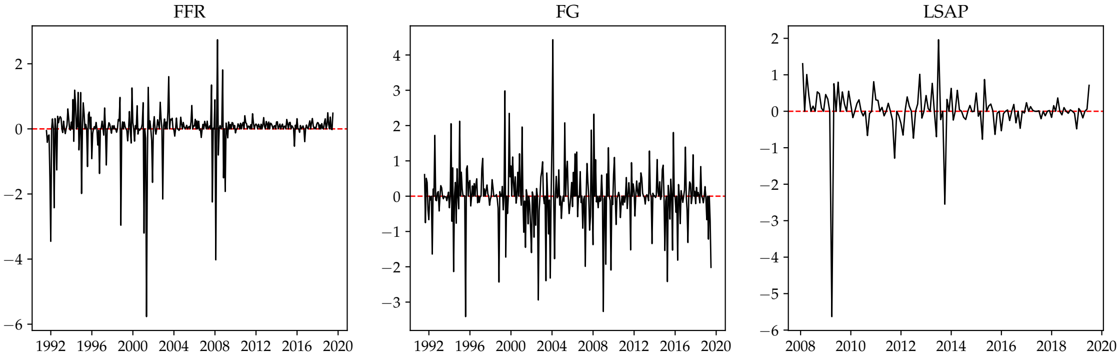
Spectral Decomposition of RV Residuals



Unspanned Factors: RV vs BP

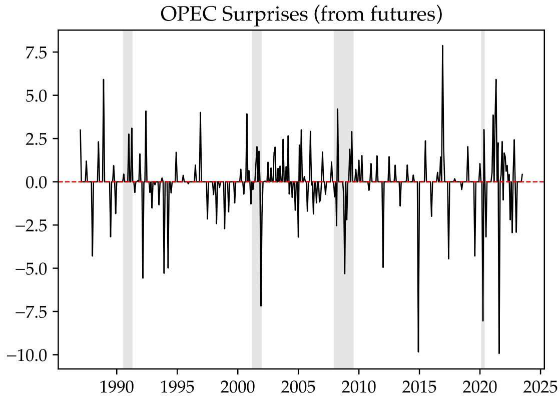


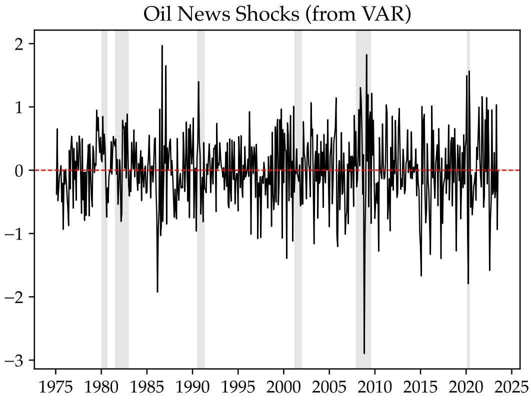
Monetary Policy Shocks from Swanson 2021



Back

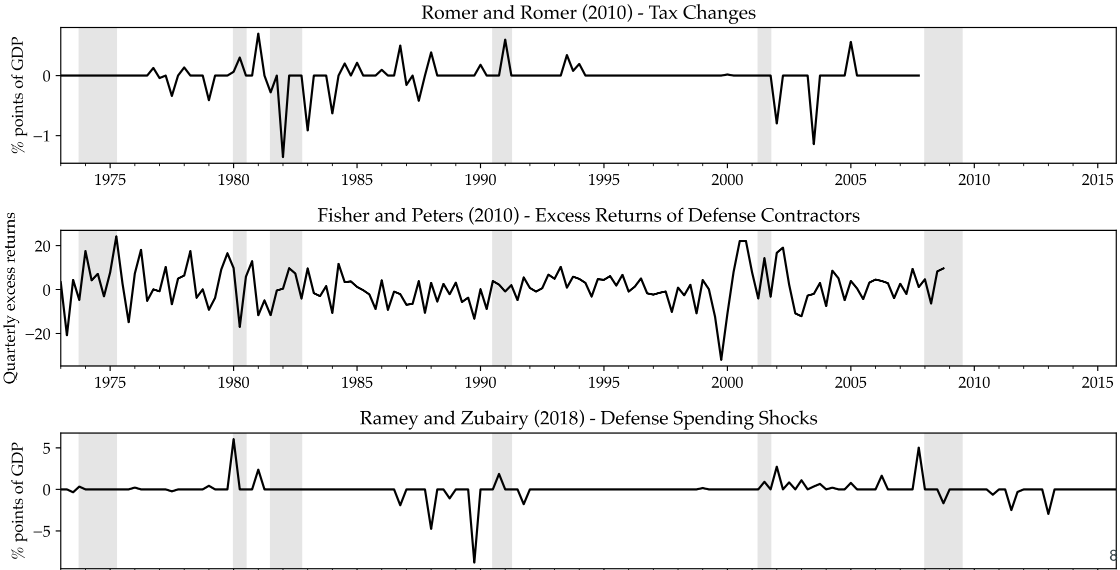
Oil Shocks from Känzig 2021





Back

Fiscal Shocks



Math and Tables

Estimating Nelson-Siegel Factors with OLS

- We estimate the factors using OLS: regress yields on coefficients;
- $\lambda > 0$ is fixed;
- No need of numerical solutions!

$$\begin{bmatrix} \widehat{\beta}_{1,t} \\ \widehat{\beta}_{2,t} \\ \widehat{\beta}_{3,t} \end{bmatrix} = \left(M'M\right)^{-1} M'Y_t, \qquad M \equiv \begin{bmatrix} 1 & \frac{1-e^{-\psi\tau_1}}{\psi\tau_1} & \frac{1-e^{\psi\tau_1}}{\psi\tau_1} - e^{-\lambda\tau_1} \\ 1 & \frac{1-e^{-\psi\tau_2}}{\psi\tau_2} & \frac{1-e^{\psi\tau_2}}{\psi\tau_2} - e^{-\lambda\tau_2} \\ \vdots & \vdots & \vdots \\ 1 & \frac{1-e^{-\psi\tau_J}}{\psi\tau_J} & \frac{1-e^{\psi\tau_J}}{\psi\tau_J} - e^{-\lambda\tau_J} \end{bmatrix}.$$

Back

How Jumpy Are The Factors?

- How much variation is coming from the diffusive part? How much from the jumps?
- Surprisingly stable over factors and over time!

$$JV_i(t) \equiv \max\{RCov_{ii}(t) - BPV_i(t), 0\}, \qquad JR_i(t) \equiv \frac{JV_i(t)}{RCov_{ii}(t)} \tag{13}$$

How Jumpy Are The Factors?

- How much variation is coming from the diffusive part? How much from the jumps?
- Surprisingly stable over factors and over time!

$$JV_i(t) \equiv \max\{RCov_{ii}(t) - BPV_i(t), 0\}, \qquad JR_i(t) \equiv \frac{JV_i(t)}{RCov_{ii}(t)} \tag{13}$$

Table 3: Average jumpiness of Nelson-Siegel factors

	JR_1	JR_2	JR_3
Whole Sample (1973-2022)	0.172	0.158	0.170
Months **with** MP activity	0.146	0.145	0.161
Months **without** MP activity	0.150	0.149	0.157
p-value for difference	0.799	0.808	0.812

References

Andersen, Torben G. and Luca Benzoni (Mar. 2010). “Do Bonds Span Volatility Risk in the U.S. Treasury Market? A Specification Test for Affine Term Structure Models”. In: *The Journal of Finance* 65.2, pp. 603–653. ISSN: 1540-6261. DOI: 10.1111/j.1540-6261.2009.01546.x. URL: http://dx.doi.org/10.1111/j.1540-6261.2009.01546.x.

Backwell, Alex (Jan. 2021). “Unspanned stochastic volatility from an empirical and practical perspective”. In: *Journal of Banking & Finance* 122, p. 105993. ISSN: 0378-4266. DOI: 10.1016/j.jbankfin.2020.105993. URL: http://dx.doi.org/10.1016/j.jbankfin.2020.105993.

Bauer, Michael and Glenn Rudebusch (2017). “Resolving the Spanning Puzzle in Macro-Finance Term Structure Models”. In: *Review of Finance* 21.2, pp. 511–553. URL: https://EconPapers.repec.org/RePEc:oup:revfin:v:21:y:2017:i:2:p:511-553..

Bikbov, Ruslan and Mikhail Chernov (Aug. 2009). “Unspanned Stochastic Volatility in Affine Models: Evidence from Eurodollar Futures and Options”. In: *Management Science* 55.8, pp. 1292–1305. ISSN: 1526-5501. DOI: 10.1287/mnsc.1090.1020. URL: http://dx.doi.org/10.1287/mnsc.1090.1020.

Collin-Dufresne, Pierre and Robert S. Goldstein (Aug. 2002). “Do Bonds Span the Fixed Income Markets? Theory and Evidence for Unspanned Stochastic Volatility”. In: *The Journal of Finance* 57.4, pp. 1685–1730. ISSN: 1540-6261. DOI: 10.1111/1540-6261.00475. URL: http://dx.doi.org/10.1111/1540-6261.00475.

Filipovic, Damir, Martin Larsson, and Anders Trolle (Mar. 2017). “Linear-Rational Term Structure Models”. In: *The Journal of Finance* 72.2, pp. 655–704. ISSN: 1540-6261. DOI: 10.1111/jofi.12488. URL: http://dx.doi.org/10.1111/jofi.12488.

References ii

Fisher, Jonas D.M. and Ryan Peters (May 2010). "Using Stock Returns to Identify Government Spending Shocks". In: *The Economic Journal* 120.544, pp. 414–436. ISSN: 1468-0297. DOI: 10.1111/j.1468-0297.2010.02355.x. URL: http://dx.doi.org/10.1111/j.1468-0297.2010.02355.x.

Känzig, Diego R. (Apr. 2021). "The Macroeconomic Effects of Oil Supply News: Evidence from OPEC Announcements". In: *American Economic Review* 111.4, pp. 1092–1125. DOI: 10.1257/aer.20190964. URL: https://www.aeaweb.org/articles?id=10.1257/aer.20190964.

Li, Haitao and Feng Zhao (2006). "Unspanned Stochastic Volatility: Evidence from Hedging Interest Rate Derivatives". In: *The Journal of Finance* 61.1, pp. 341–378. ISSN: 00221082, 15406261. URL: http://www.jstor.org/stable/3699343 (visited on 06/13/2024).

— (Apr. 2009). "Nonparametric Estimation of State-Price Densities Implicit in Interest Rate Cap Prices". In: *Review of Financial Studies* 22.11, pp. 4335–4376. ISSN: 1465-7368. DOI: 10.1093/rfs/hhp025. URL: http://dx.doi.org/10.1093/rfs/hhp025.

Litterman, Robert B and Josè Scheinkman (June 1991). "Common Factors Affecting Bond Returns". In: *Journal of Fixed Income* 1.1, pp. 54–61. DOI: 10.3905/jfi.1991.692347. URL: https://doi.org/10.3905/jfi.1991.692347.

Liu, Yan and Cynthia Wu (2021). "Reconstructing the yield curve". In: *Journal of Financial Economics* 142.3, pp. 1395–1425. ISSN: 0304-405X. DOI: https://doi.org/10.1016/j.jfineco.2021.05.059. URL: https://www.sciencedirect.com/science/article/pii/S0304405X21002531.

Nelson, Charles and Andrew F Siegel (1987). "Parsimonious Modeling of Yield Curves". In: *Journal of Business* 60.4, pp. 473–89.

Piazzesi, Monika (2010). "Affine Term Structure Models". In: *Handbook of Financial Econometrics: Tools and Techniques*. Elsevier, pp. 691–766. DOI: 10.1016/b978-0-444-50897-3.50015-8. URL: http://dx.doi.org/10.1016/B978-0-444-50897-3.50015-8.

Ramey, Valerie A. (Feb. 2011). "Identifying Government Spending Shocks: It's all in the Timing*". In: *The Quarterly Journal of Economics* 126.1, pp. 1–50. ISSN: 1531-4650. DOI: 10.1093/qje/qjq008. URL: http://dx.doi.org/10.1093/qje/qjq008.

References iii

Ramey, Valerie A. and Sarah Zubairy (Apr. 2018). “Government Spending Multipliers in Good Times and in Bad: Evidence from US Historical Data”. In: *Journal of Political Economy* 126.2, pp. 850–901. ISSN: 1537-534X. DOI: 10.1086/696277. URL: http://dx.doi.org/10.1086/696277.

Romer, Christina D. and David H. Romer (June 2010). “The Macroeconomic Effects of Tax Changes: Estimates Based on a New Measure of Fiscal Shocks”. In: *American Economic Review* 100.3, pp. 763–801. DOI: 10.1257/aer.100.3.763. URL: https://www.aeaweb.org/articles?id=10.1257/aer.100.3.763.

Swanson, Eric T. (Mar. 2021). “Measuring the effects of federal reserve forward guidance and asset purchases on financial markets”. In: *Journal of Monetary Economics* 118, pp. 32–53. ISSN: 0304-3932. DOI: 10.1016/j.jmoneco.2020.09.003. URL: http://dx.doi.org/10.1016/j.jmoneco.2020.09.003.