Asymmetric Violations of the Spanning Hypothesis

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Northwestern Job Market Preparation

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Intro

- Yield curve dynamics is of major interest both for policy makers and market participants:
  - Monetary policy transmission and fiscal policy assessment;
  - Risk management and long-term investment decisions;
  - Risk premia measurement and portfolio allocation;

- Workhorse: arbitrage-free Dynamic Term Structure Models;

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- Workhorse: arbitrage-free Dynamic Term Structure Models;

- Pervasive feature: the Spanning Hypothesis;
  - Evolution of yields reveals the evolution of underlying risk factors;
  - Today's yield curve is **all you need** to forecast future yields (and bond returns!);

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- Workhorse: arbitrage-free Dynamic Term Structure Models;
- Pervasive feature: the Spanning Hypothesis;
  - Evolution of yields reveals the evolution of underlying risk factors;
  - Today's yield curve is **all you need** to forecast future yields (and bond returns!);
- Leads to important repercussions:
  - Yield forecasting and portfolio allocation through simple models should be preferred;
  - Hedging interest-rate derivatives with few assets;
  - Model-implied prices of risk are empirically very different;

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  - **Yes**: Cooper and Priestley (2009), Ludvigson and Ng (2009), Joslin et al. (2014), Cieslak and Povala (2015), Greenwood and Vayanos (2014), Fernandes and Vieira (2019), Huang and Shi (2023)
  - **Probably Not**: Duffee (2013), Bauer and Hamilton (2018) $\implies$ inference is usually hard!

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This paper: more nuanced answer - *asymmetric* violations;

- Out-of-sample forecasts + data-rich environment;
- **Stronger** violations at the **shorter end** of the yield curve;
- No evidence of violations at the longer end of the yield curve;
- Violations are economically meaningful for a mean-variance investor;
- Evidence on one possible mechanism: monetary policy;

Flight Plan

1. Literature Review and Data
2. Forecast bond excess returns
   - Can we improve risk premium measurement by incorporating macro data?
3. Decompose bond risk premia and forecast factors
   - Novel decomposition of bond returns using Nelson-Siegel factors;
   - Different machine-learning methods showcase where predictability comes from;
4. How much are we leaving on the table?
   - Macro signals provide asymmetric Sharpe Ratio improvements when trading bonds;
5. Time-varying violations
   - Violations are predicted by deviation from Taylor Rule;

Literature Review and Data

Literature

- Bond return forecasting and testing of different theories about the yield curve:
  - Cochrane and Piazzesi (2005), Cooper and Priestley (2009), Ludvigson and Ng (2009), Joslin et al. (2014), Greenwood and Vayanos (2014), Cieslak and Povala (2015), Bauer and Hamilton (2018), Bianchi et al. (2021), Hoogteijling et al. (2021), van der Wel and Zhang (2021), Borup et al. (2023), Huang and Shi (2023)
- Nelson-Siegel modeling
  - Nelson and Siegel (1987), Diebold and Li (2006), Diebold et al. (2006), Moench (2008), Diebold and Rudebusch (2013), van Dijk et al. (2013), Hännikäinen (2017), Fernandes and Vieira (2019)
- Machine Learning methods for forecasting in Finance and Economics
  - Gu et al. (2020), Medeiros et al. (2021), Bianchi et al. (2021), Giannone et al. (2021), Goulet-Coulombe (2023), Goulet-Coulombe et al. (2023), Filippou et al. (2023), Shen and Xiu (2024)
- **Contribution**: asymmetry in out-of-sample forecasting in data-rich environments

Data

Yield curve data:

- Taken from Liu and Wu (2021) - Full sample: 1973-2021; Out-of-sample: 1990-2021;
- Constructed from CRSP data;
- Provides longer maturities than Fama and Bliss (1987)
- Lower fitting errors than Gurkaynak et al. (2007)

Macroeconomic data:

- FRED-MD, by St. Louis Fed [McCracken and Ng (2016)]
- Monthly frequency, a total of 126 variables covering different groups of variables
- Price indexes, output and unemployment measures, real estate market indicators, exchange rates, monetary aggregates, inventories and investment measures, credit spreads...

Forecasting Bond Returns

Forecasting Excess Bond Returns

- Let $y_t^{(n)}$ be the n-year zero-coupon rate at month t;
- The 1-year excess bond returns for a maturity of n years are given by:

$$xr_{t+12}(n) \equiv n \cdot y_t^{(n)} - (n-1) \cdot y_{t+12}^{(n-1)} - y_t^{(1)} \tag{1}$$

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- Estimate a linear model with an expanding sample forecasting design:

$$xr_{t+12}(n) = \alpha_n + \theta_n' C_t + \gamma_n' PC_t + \epsilon_{t+12,n} \tag{2}$$

- C_t controls for the yield curve using forward rates $f_t(n) = n \cdot y_t^{(n)} - (n-1) \cdot y_t^{(n-1)}$;
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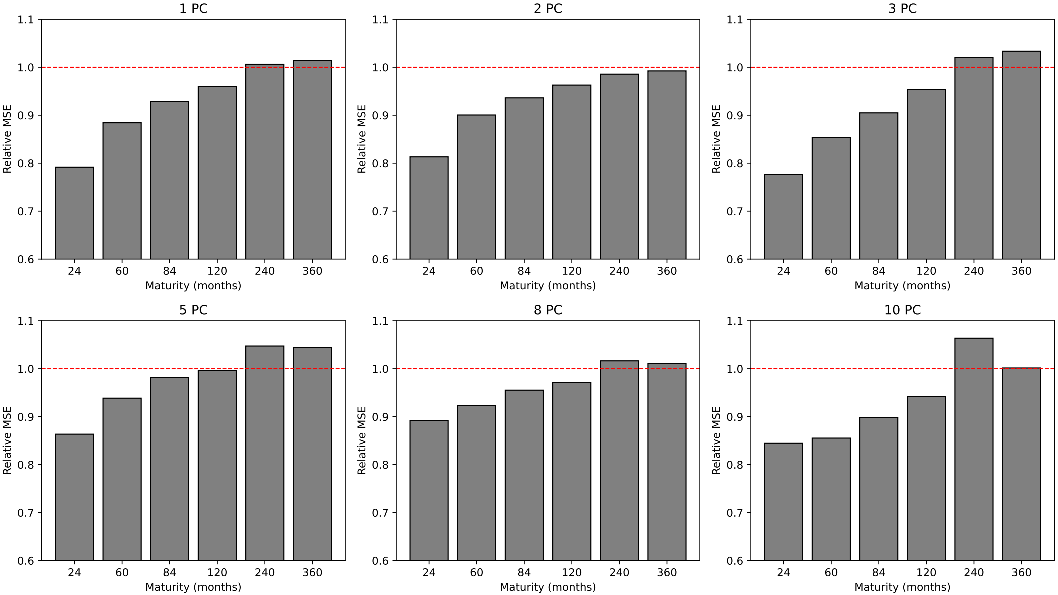
- PC_t are principal components extracted from the FRED-MD data set;
- Spanning hypothesis: allowing for $\gamma_n \neq 0$ should not improve the forecast of $xr_{t+12}(n)$;
- Previous literature focuses on testing $\gamma_n = 0$. We focus directly on $\widehat{xr}_{t+12}(n)$;

MSE With/Without Macro Data



Other Controls | p-values | In-sample | Ragged Edge | GSW Yields

Decomposing Bond Risk Premia

Modeling Yields

- Forecasting returns amounts to forecasting $y_{t+12}^{(n)}$;
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Assume a Nelson-Siegel model for yields as in Diebold and Li (2006):

$$y_t^{(\tau)} = \beta_{1,t} + \beta_{2,t} \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} \right) + \beta_{3,t} \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right) \tag{3}$$

- $\lambda > 0$ is a decay parameter, assumed constant;
- β_1 is a long-run factor: $\lim_{\tau\to\infty} y_t^{(\tau)} = \beta_{1,t}$;
- β_2 is a short-run factor: loading decreases with τ;
- β_3 is a medium-run factor: hump-shaped loadings;

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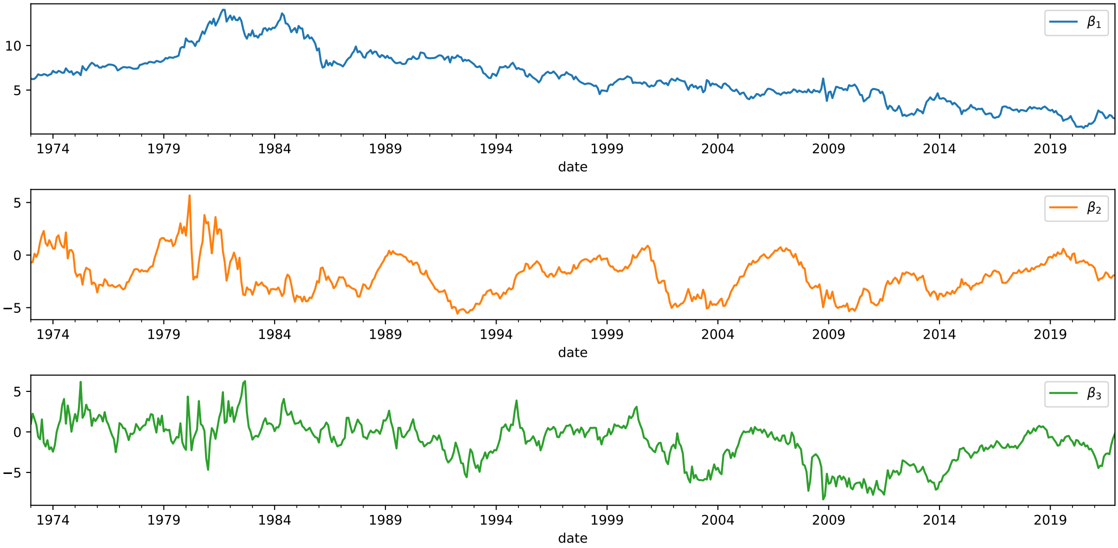
- β_2 is a short-run factor: loading decreases with τ;
- β_3 is a medium-run factor: hump-shaped loadings;
- Set $\lambda = 0.0609$; Estimate using OLS date by date, $1 \leq \tau \leq 120$;
- OLS estimates $\implies$ β's are linear combinations of yields;

Factor Realizations (1973-2021)



▸ Estimation details ▸ Alternative Estimation Procedures ▸ Polynomial Model

Decomposing Returns

Proposition 1

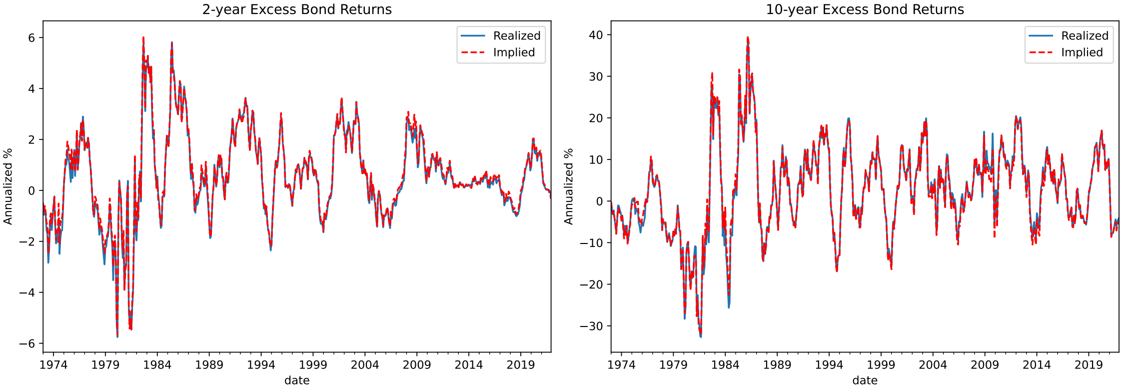
Assume the Nelson-Siegel representation with $\lambda > 0$. Then, $xr_{t+12}(n)$ can be written as:

$$
\begin{aligned}
xr_{t+12}(n) = {} & (n-1)\Big[\beta_{1,t} - \beta_{1,t+12}\Big] \\
& + \left(\frac{1-e^{-\theta(n-1)}}{\theta}\right)\Big[e^{-\theta}\beta_{2,t} - \beta_{2,t+12}\Big] \\
& + \left(\frac{1-e^{-\theta(n-1)}}{\theta} - ne^{-\theta(n-1)} + 1\right)\Big[e^{-\theta}\beta_{3,t} - \beta_{3,t+12}\Big] + \left(1-e^{-\theta(n-1)}\right)\beta_{3,t+12}
\end{aligned}
\tag{4}
$$

where $\theta \equiv 12 \cdot \lambda$.

- Terms in parentheses are **not** time-varying and brackets **do not** depend on the maturity

Is this approximation any good?



- Blue: $xr_t(n)$ observed from data for $n = 2$ and $n = 10$;
- Red: $xr_t(n)$ implied by our estimates of the factors + Proposition 1;
- The Fed actually uses a variant of the NS model to report their yield curve

Forecasting Nelson-Siegel Factors

- OLS factor estimation implies that β's are **linear combinations of yields**;
- Under the spanning hypothesis: macro data should not be helpful to forecast factors:

$$\beta_{i,t+12} = \alpha_i + \theta_i' C_t + \gamma_i' PC_t + \epsilon_{i,t+12}, \quad i \in \{1,2,3\} \tag{5}$$

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- We use the out-of-sample R^2 to measure the forecasting ability:

$$R^2_{oos} = 1 - \frac{\sum_{t=t_0}^{T} \left(\beta_{i,t} - \widehat{\beta}_{i,t}\right)^2}{\sum_{t=t_0}^{T} \left(\beta_{i,t} - \overline{\beta}_{i,t}\right)^2} \tag{6}$$

- $\overline{\beta}_{i,t}$ is a benchmark model: for example a random walk;
- OOS period: 1990-2021, with a recursive forecasting approach (384 total forecasts);
- We use a Diebold-Mariano test to make inference about any forecasting improvement;

Out-of-Sample PCA-based Forecasts

Innovations + other controls | In-sample analysis

Table: R^2 out-of-sample against a random walk and Diebold-Mariano p-values

		Number of Macro PCs					p-values				
Target	No Macro	1	2	3	4	5	1	2	3	4	5
β_1	-0.21	-0.17	-0.19	-0.15	-0.11	-0.09	0.18	0.33	0.13	0.11	0.10
β_2	-0.08	-0.08	**0.17**	**0.22**	**0.21**	**0.23**	0.49	0.01	0.02	0.02	0.02
β_3	-0.12	-0.15	-0.06	-0.07	-0.07	-0.07	0.92	0.07	0.19	0.20	0.21

- Improving over a random walk is hard, but possible for (and *only* for) β_2
- Asymmetry in bond return predictability happens because a single factor can be predicted;
- Result holds if we allow for even more PCs, but we lose statistical power

Leveraging Regularization Methods

- PCA is not "supervised": dimensionality reduction decoupled from prediction;
- Regularization: penalize complex models $\implies$ bias-variance trade-off;
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- Model selection tells exactly what variables were chosen to compose forecasts;

Let $\psi_1, \psi_2 \geq 0$ be scalars and let $||.||_p$ be the L^p norm. Consider the minimization:

$$\min_{\alpha_i, \gamma_i} \left\{ \frac{1}{T - 12 - t_0} \sum_{t=t_0}^{T-12} \left(\beta_{i,t+12} - \alpha_i - \gamma_i' X_t \right)^2 + \underbrace{\psi_1 \cdot ||\gamma_i||_1 + \psi_2 \cdot ||\gamma_i||_2}_{\text{model complexity penalty}} \right\} \tag{7}$$

$$\widehat{\beta}_{i,t+12} = \widehat{\alpha}_i + \widehat{\gamma}_i' X_t \tag{8}$$

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1. $\psi_1 = 0, \psi_2 > 0 \implies$ Ridge
2. $\psi_1 > 0, \psi_2 = 0 \implies$ Lasso
3. $\psi_1, \psi_2 > 0 \implies$ Elastic Net

- Calibrate ψ_1, ψ_2 using a 80-20 split validation set for each date t using grid search.

Regularization Methods - Performance

Table: R^2 out-of-sample of regularized linear models

Target	No Macro Data			All Macro Data			p-value		
	Ridge	Lasso	Elastic Net	Ridge	Lasso	Elastic Net	Ridge	Lasso	Elastic Net
β_1	-4.84	-4.82	-4.69	-4.06	-4.30	-4.18	0.00	0.00	0.00
β_2	-0.08	-0.13	-0.19	**0.07**	**0.07**	**0.06**	0.05	0.00	0.01
β_3	-0.41	-0.59	-0.59	-0.47	-0.46	-0.45	0.78	0.04	0.03
$\Delta\beta_1$	0.12	0.12	0.09	0.01	0.12	0.12	0.96	0.50	0.27
$\Delta\beta_2$	0.01	-0.02	-0.01	**0.15**	**0.22**	**0.19**	0.02	0.00	0.00
$\Delta\beta_3$	0.04	-0.02	-0.03	-0.13	-0.09	-0.08	1.00	0.95	0.95

- No matter the target and the method, predictability through β_2;

▸ Other Controls ▸ Stationarity Matters ▸ Chosen variables

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- Flexibility vs computational cost + low signal-to-noise ratio environment + small sample;
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  - Sequentially pick the variable and splitting point that minimizes the in-sample MSE
  - Procede until each observation belongs to a single final node
  - Each tree is a forecaster with low bias and high variance
- We fit 500 trees for each date and predict using the average - an *ensemble* estimator

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Convenient way of tracking what variables matter: "feature importance":

- For each split, track the reduction in MSE from a naive benchmark (sample mean);
- Compute the fraction of total MSE reduction due to each variable, for each tree and date;
- Average over trees and dates: how important is each variable for total MSE reduction?

What about non-linearities? Random Forests to the rescue!

	Lagged Factors			Forward Rates		
Target	No Macro	All Macro	p-value	No Macro	All Macro	p-value
β_1	-1.48	-1.93	0.87	-0.76	-0.72	0.39
β_2	-0.08	**0.27**	0.01	-0.34	**0.23**	0.00
β_3	-0.41	-0.16	0.02	-0.58	-0.22	0.01
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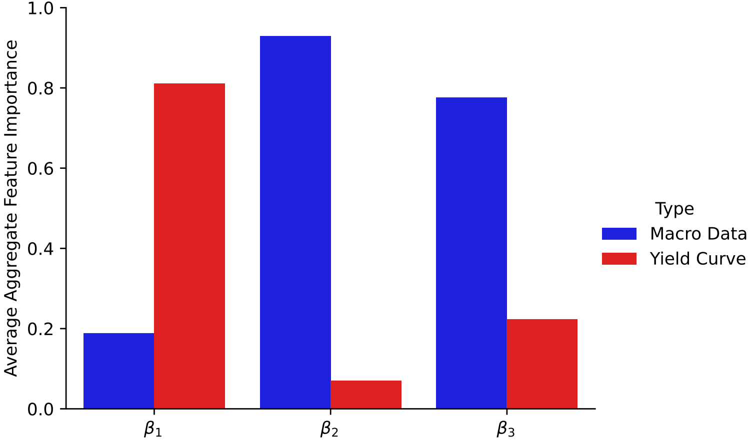
- This is the best method so far with $R^2 > 30\%$ for the first time
- Main result is **not** due to linear forecasting methods
- Forecasting innovations is usually better than forecasting factors directly

Average Feature Importance (Macro Variables vs Yield Curve)



- Heavy lifting is done mainly by the yield curve for β_1;
- Macro signals command almost all of the improvement for β_2;

▸ Individual Feature Importance

Economic Significance

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- $R_{p,t+12}$ is the gross return of her portfolio: $R_{p,t+12} = 1 + y_t^{(1)} + \boldsymbol{w}_t'[\boldsymbol{xr_{t+12}}]$;

$$\max_{\boldsymbol{w}_t} \left\{ \mathbb{E}_t \left[R_{p,t+12}(\boldsymbol{w}_t) \right] - \frac{\gamma}{2} \cdot \mathrm{Var}_t \left[R_{p,t+12}(\boldsymbol{w}_t) \right] \right\}$$

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- $\boldsymbol{\mu}_{t+12|t} \equiv \mathbb{E}_t \left[\boldsymbol{xr}_{t+12} \right]$ and $\Sigma_{t+12|t} \equiv \mathbb{E}_t \left[\left(\boldsymbol{xr}_{t+12} - \boldsymbol{\mu}_{t+12|t} \right) \left(\boldsymbol{xr}_{t+12} - \boldsymbol{\mu}_{t+12|t} \right)' \right]$;
- Optimal solution: $\boldsymbol{w}_t^* = \frac{1}{\gamma} \cdot \Sigma_{t+12|t}^{-1} \boldsymbol{\mu}_{t+12|t}$, and we let $\gamma = 3$;

Conditional Risk Premia and Volatility

- Our methodology delivers estimates of $\boldsymbol{\mu}_{t+12|t}$ with and without macro signals;
- We follow Thornton and Valente (2012) to allow for time-varying volatility:

$$\widehat{\Sigma}_{t+12|t} \equiv \sum_{i=0}^{\infty} \boldsymbol{\epsilon}_{t-i}\boldsymbol{\epsilon}'_{t-i} \odot \Omega_{t-i}, \quad \Omega_{t-i} \equiv \alpha \cdot e^{-\alpha \cdot i} 11'$$

  where $\boldsymbol{\epsilon}_t$ is the 12-month ahead forecasting error; set $\alpha = 0.05$.
- Riva (2024): Spanning Hypothesis also constrains $\Sigma_{t+12|t}$, out of scope here;

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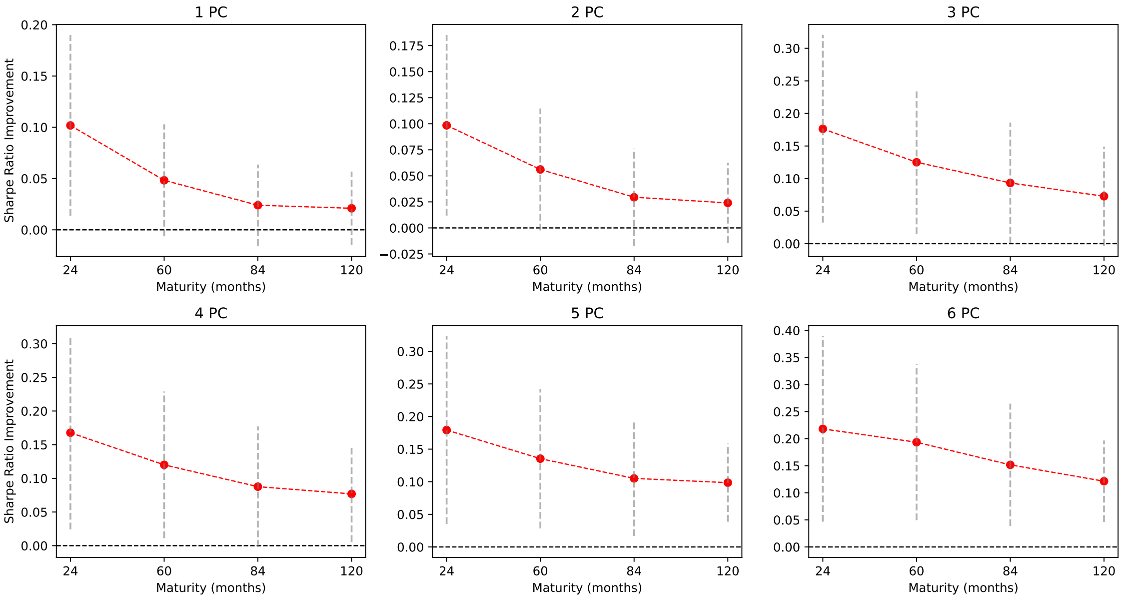
- Riva (2024): Spanning Hypothesis also constrains $\Sigma_{t+12|t}$, out of scope here;
- Leverage? Trading costs? Let $0 \leq w_t^{(n)} \leq 1$ (but also $-1 \leq w_t \leq 2$ in the paper);
- Our metric: Sharpe ratio = average risk premium over its volatility;
- Focus on the Sharpe ratio *improvement*;

Baseline Sharpe Ratio ≈ 0.2 (Constrained Case)

Unconstrained Case



Time-varying violations

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  - δ_0, δ_1 are unknown to everyone;
  - Many reasons to deviate: time-varying loadings, large shocksx, other objectives, etc;
- **Conjecture**: deviations predict future precision for β_2 when we allow for macro signals;

How to test this conjecture?

- Fit a Taylor Rule with inflation and unemployment:

$$i_t = \delta_0 + \delta_\pi \cdot \text{Inflation}_t + \delta_u \cdot \text{Unemployment}_t$$

- Rolling-window estimation: 60 months;
- Deviation: $\phi_t \equiv i_t - \widehat{i_t}$;

How to test this conjecture?

- Fit a Taylor Rule with inflation and unemployment:

$$i_t = \delta_0 + \delta_\pi \cdot \text{Inflation}_t + \delta_u \cdot \text{Unemployment}_t$$

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- Lower loss $\iff$ more precision;

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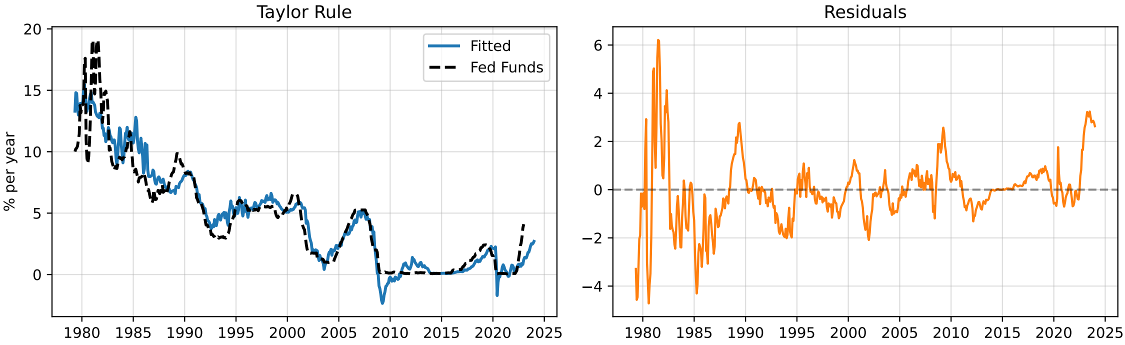
- Loss function: $L_{i,t} \equiv \left(\widehat{\beta}_{i,t} - \beta_{i,t}\right)^2, \quad i = 1, 2, 3;$
- Lower loss $\iff$ more precision;
- Can deviations predict precision?

$$L_{i,t+12} = a + b \cdot |\phi_t| + \eta_{i,t+12}$$

- Any evidence that $b < 0$ for β_2? What about β_1?

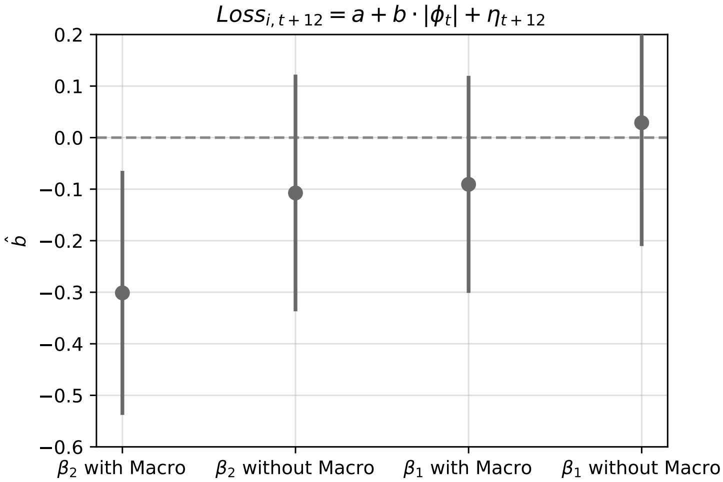
Quality of fit



- Rolling-window Taylor Rule fits well the Fed Funds rate;
- Robustness: headline inflation, monthly inflation;

▸ Other Rules

Improvements in Forecasting



- Deviations $\implies$ better predictions **with** macro signals;
- Only for short-run factor β_2;
- True across Taylor Rules and controls; ▸ Robustness
- In the paper:
  $\uparrow$ inflation $\implies$ $\uparrow$ violations;
  ▸ Extra Results

Wrap-Up

Main takeaways:

- The shorter end of the American nominal yield curve violates the Spanning Hypothesis;
- Extra predictability $\implies$ Sharpe ratio $\uparrow \approx 0.1 - 0.2$;
- Violations connected to monetary policy;

Wrap-Up

Main takeaways:

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And now so what?

- Different frameworks for shorter and longer maturities;
- DTSMs + unknown reaction functions = uncharted territory;
  - Future work!

Wrap-Up

Main takeaways:

- The shorter end of the American nominal yield curve violates the Spanning Hypothesis;
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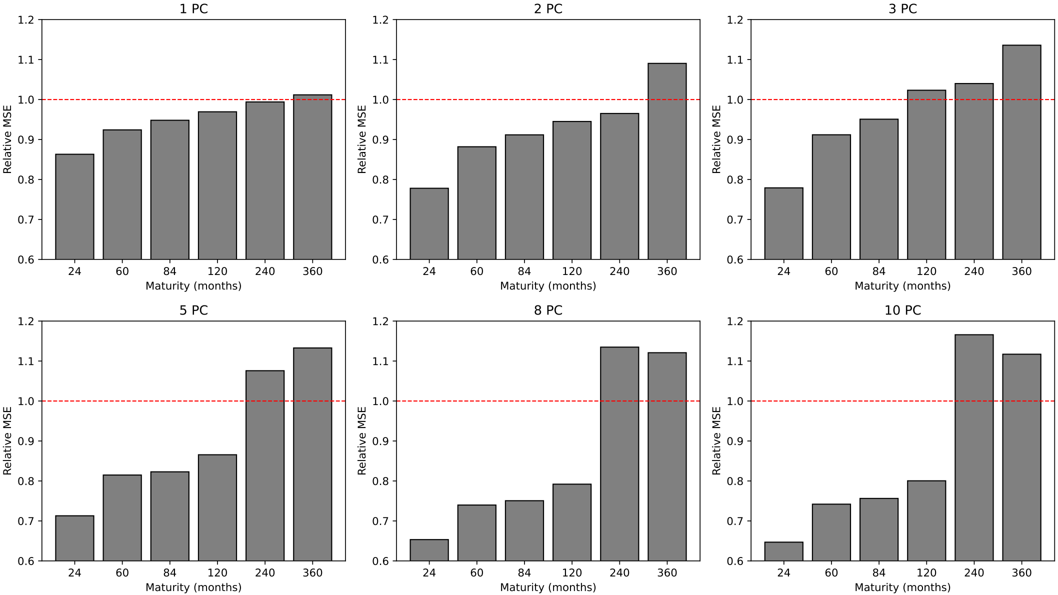
Thank you!

Appendix

(Thank you!)

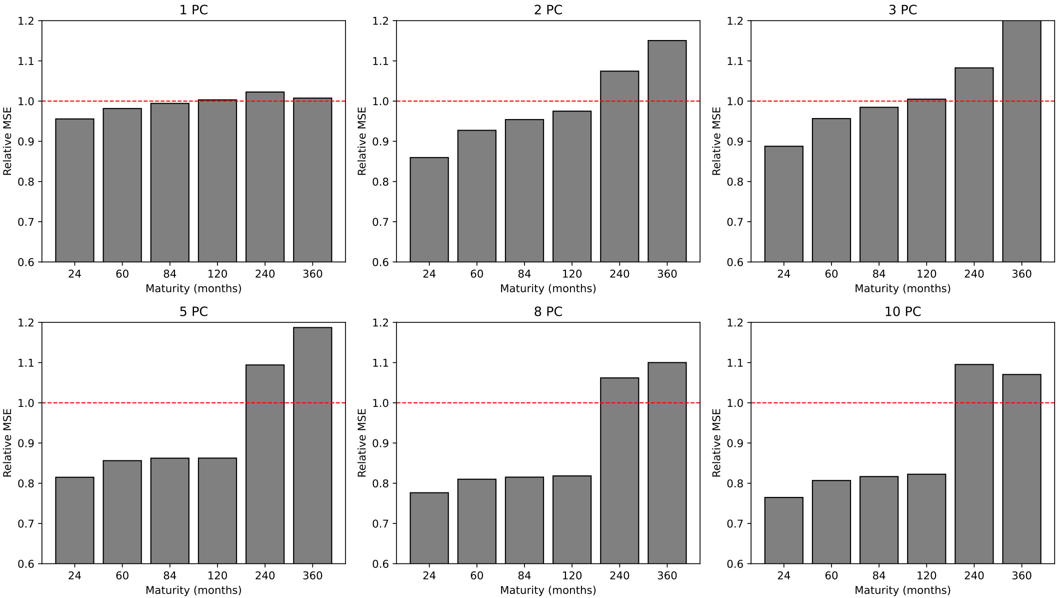
Excess Bond Returns Relative MSE Ratios

Back



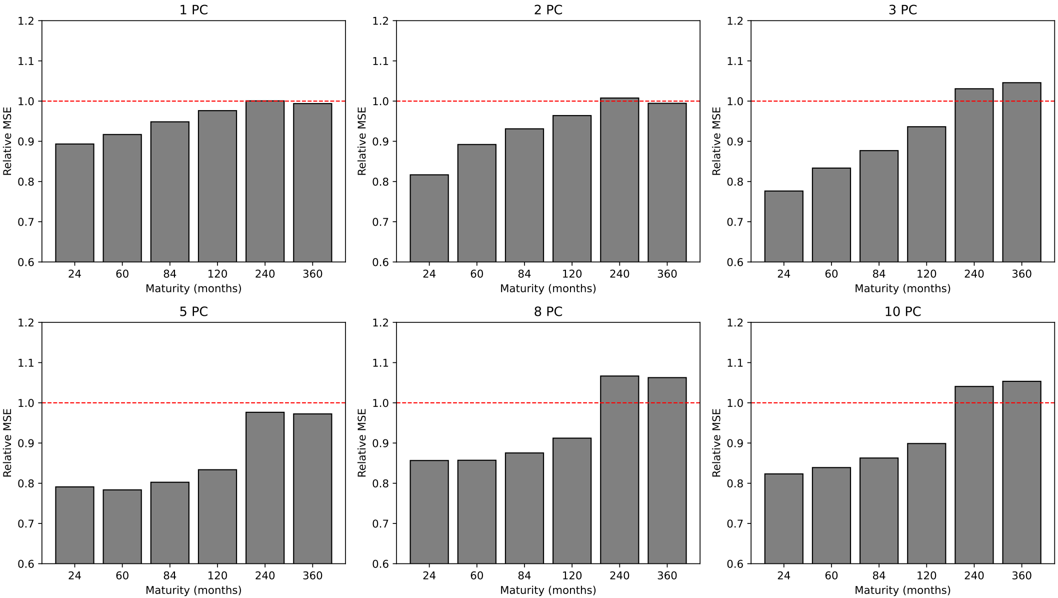
MSE Ratios: GSW Yields (NY Fed)

▸ Back



MSE Ratios: Ragged Edge Case

Back



p-Values for MSE Ratios of Excess Bond Returns

Back

	Maturity in months					
	24	60	84	120	240	360
1 PC	0.00	0.01	0.02	0.05	0.74	0.92
2 PC	0.00	0.01	0.01	0.04	0.16	0.32
3 PC	0.02	0.01	0.04	0.13	0.81	0.96
4 PC	0.04	0.06	0.13	0.24	0.55	0.65
5 PC	0.18	0.28	0.42	0.48	0.80	0.84
6 PC	0.21	0.25	0.35	0.38	0.69	0.66
7 PC	0.16	0.09	0.13	0.16	0.34	0.28
8 PC	0.24	0.23	0.32	0.37	0.59	0.57
9 PC	0.12	0.11	0.19	0.33	0.75	0.80
10 PC	0.15	0.12	0.19	0.28	0.79	0.51

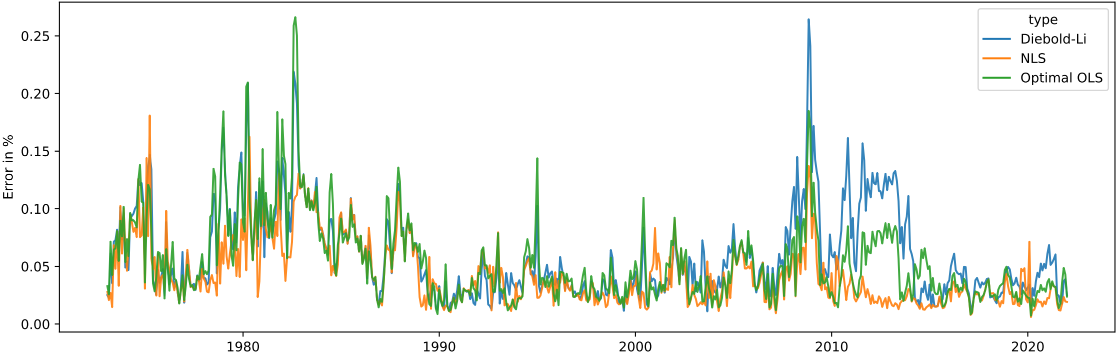
In-Sample Evidence Forecasting Returns

▸ Back

	2-year			10-year			20-year			30-year		
PC 1	0.09***	0.12***	0.13***	0.04**	0.07***	0.07***	-0.01	-0.00	0.00	-0.03	-0.02	-0.03
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)	(0.02)	(0.03)	(0.04)
PC 2		-0.07**	-0.07**		-0.07***	-0.06**		-0.01	0.00		0.00	0.02
		(0.03)	(0.03)		(0.02)	(0.02)		(0.04)	(0.05)		(0.05)	(0.06)
PC 3		0.11***	0.11***		0.08***	0.08***		0.05**	0.05*		0.04	0.03
		(0.03)	(0.02)		(0.03)	(0.02)		(0.03)	(0.03)		(0.03)	(0.03)
PC 4		-0.02	-0.02		-0.05***	-0.06***		-0.06***	-0.06***		-0.09***	-0.08***
		(0.02)	(0.03)		(0.02)	(0.02)		(0.02)	(0.02)		(0.02)	(0.02)
PC 5		-0.04	-0.04		-0.09***	-0.08***		-0.08**	-0.08*		-0.09**	-0.09*
		(0.03)	(0.03)		(0.03)	(0.03)		(0.04)	(0.05)		(0.05)	(0.05)
PC 6			0.03			0.07***			0.04			0.06
			(0.03)			(0.03)			(0.04)			(0.05)
PC 7			0.06*			0.04			0.01			0.01
			(0.03)			(0.03)			(0.03)			(0.03)
PC 8			-0.08***			-0.08***			-0.04			-0.04
			(0.03)			(0.03)			(0.04)			(0.05)
N	588	588	588	588	588	588	422	422	422	422	422	422
R2 Adj.	0.28	0.36	0.40	0.28	0.36	0.40	0.16	0.23	0.24	0.15	0.22	0.23
R2 Adj. (No Macro Data)	0.15	0.15	0.15	0.25	0.25	0.25	0.16	0.16	0.16	0.14	0.14	0.14

Alternative Estimation Procedures

Back



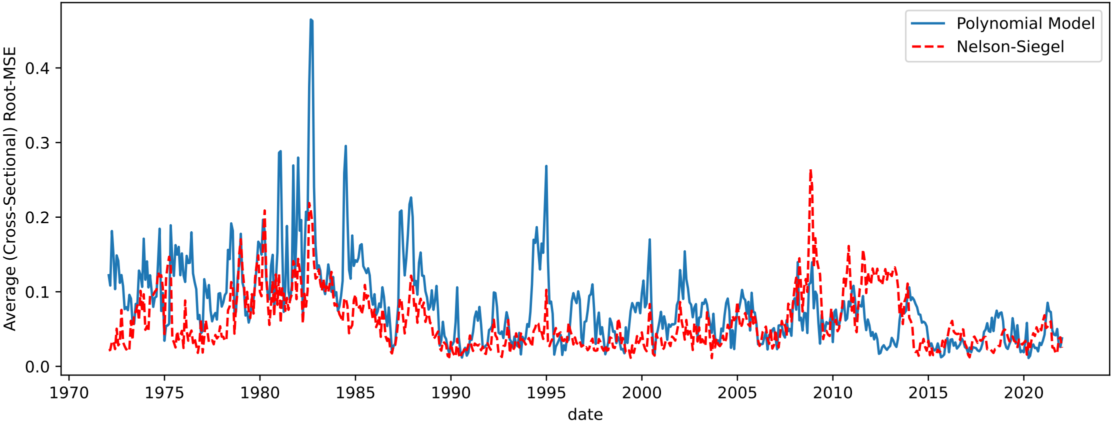
- NLS stands for Non-Linear Least Squares Date by Date
- Optimal OLS is the in-sample best OLS-implied decay fit

Alternative Estimation Procedures

▸ Back

A quadratic polynomial model:

$$y_t^{(\tau)} = c_{1,t} + c_{2,t} \cdot \tau + c_{3,t} \cdot \tau^2 \tag{9}$$



Estimation Details

Back

Define the following matrices for each time t:

$$X \equiv \begin{bmatrix} 1 & \left(\frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1}\right) & \left(\frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} - e^{-\lambda\tau_1}\right) \\ \vdots & \vdots & \vdots \\ 1 & \left(\frac{1-e^{-\lambda\tau_N}}{\lambda\tau_N}\right) & \left(\frac{1-e^{-\lambda\tau_N}}{\lambda\tau_N} - e^{-\lambda\tau_N}\right) \end{bmatrix}, \quad Y_t = \begin{bmatrix} y_t^{(\tau_1)} \\ \vdots \\ y_t^{(\tau_N)} \end{bmatrix} \tag{10}$$

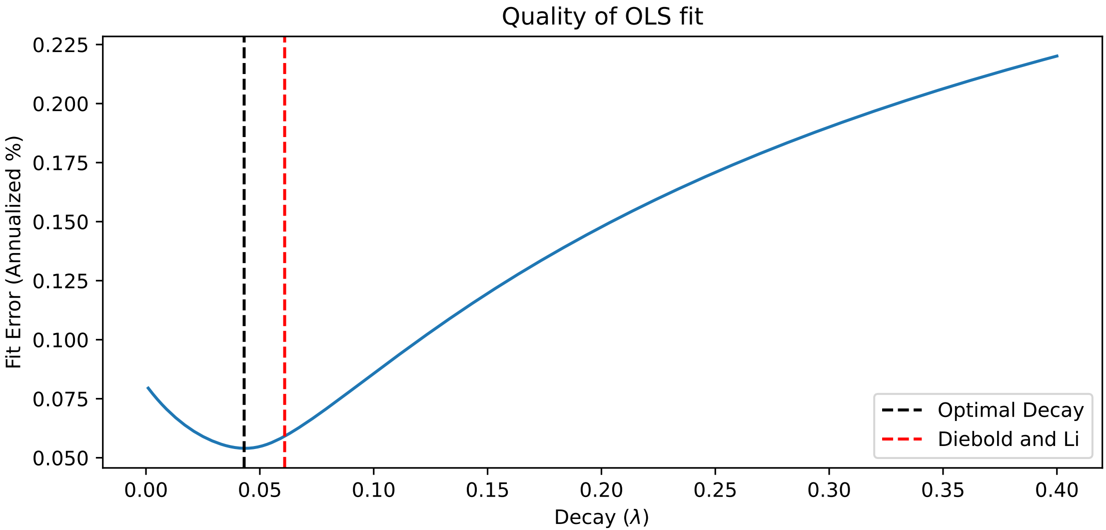
Now estimate betas using OLS:

$$\begin{bmatrix} \beta_{1,t} \\ \beta_{2,t} \\ \beta_{3,t} \end{bmatrix} = \left(X'X\right)^{-1} X' Y_t \tag{11}$$

Notice that X does not depend on t.

Fitting the Decay

Back



- For each λ, fit the model by OLS over the entire sample and compute the average squared fitting error

Out-of-sample PCA-based Forecast of Innovations

▸ Back

Target	No Macro	Number of Macro PCs						p-values					
		1	2	3	4	5	8	1	2	3	4	5	8
Predicting Innovations - Controlling for Forward Rates													
$\Delta\beta_1$	-0.19	-0.15	-0.17	-0.14	-0.10	-0.08	0.05	0.19	0.32	0.17	0.12	0.10	0.01
$\Delta\beta_2$	-0.11	-0.12	0.14	0.18	0.17	0.19	0.18	0.52	0.00	0.02	0.02	0.02	0.05
$\Delta\beta_3$	-0.10	-0.12	-0.06	-0.05	-0.05	-0.06	-0.08	0.93	0.17	0.25	0.26	0.31	0.41
Predicting Factor Levels - Controlling for Lagged Betas													
β_1	-0.10	-0.10	-0.11	-0.14	-0.11	-0.07	0.06	0.51	0.67	0.83	0.56	0.36	0.04
β_2	0.06	0.07	0.21	0.20	0.20	0.20	0.17	0.31	0.01	0.15	0.16	0.18	0.28
β_3	-0.11	-0.14	-0.06	-0.05	-0.05	-0.06	-0.08	0.89	0.16	0.19	0.20	0.23	0.39

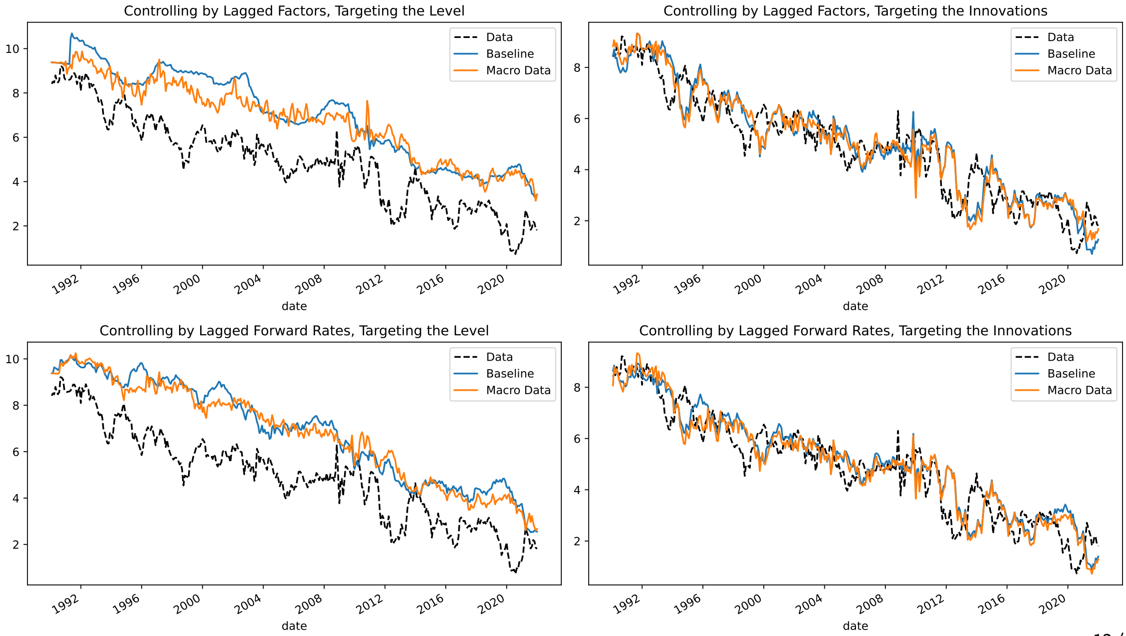
Regularization Methods - Controlling by Lagged Betas

Back

Target	No Macro Data			All Macro Data			p-value		
	Ridge	Lasso	Elastic Net	Ridge	Lasso	Elastic Net	Ridge	Lasso	Elastic Net
Beta 1	-4.91	-4.73	-4.81	-3.76	-5.08	-4.53	0.00	0.97	0.10
Beta 2	0.00	-0.12	-0.12	0.08	0.07	0.02	0.16	0.00	0.08
Beta 3	-0.41	-0.47	-0.49	-0.45	-0.35	-0.39	0.71	0.04	0.09
Innovation 1	0.12	-0.00	0.11	-0.29	0.04	0.08	1.00	0.30	0.84
Innovation 2	0.10	0.08	0.12	0.18	0.25	0.24	0.11	0.00	0.01
Innovation 3	0.08	0.04	0.02	0.00	0.03	0.07	0.95	0.70	0.02

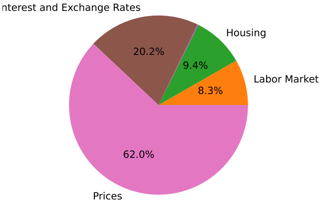
Regularization Failure for β_1

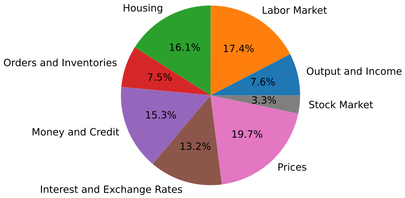
Back

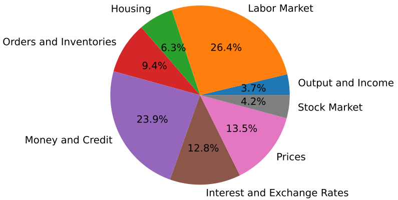


Model Selection - Lasso

How frequently are variables from each group chosen?





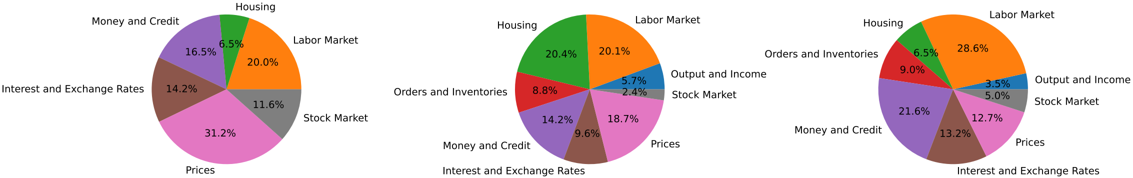


- Typical number of chosen variables is around 10-15
- Price measures are the leading predictor for β_1 - echoes Joslin et al (2014)
- Short and medium run: the "illusion of sparsity" - Giannone et al (2021)

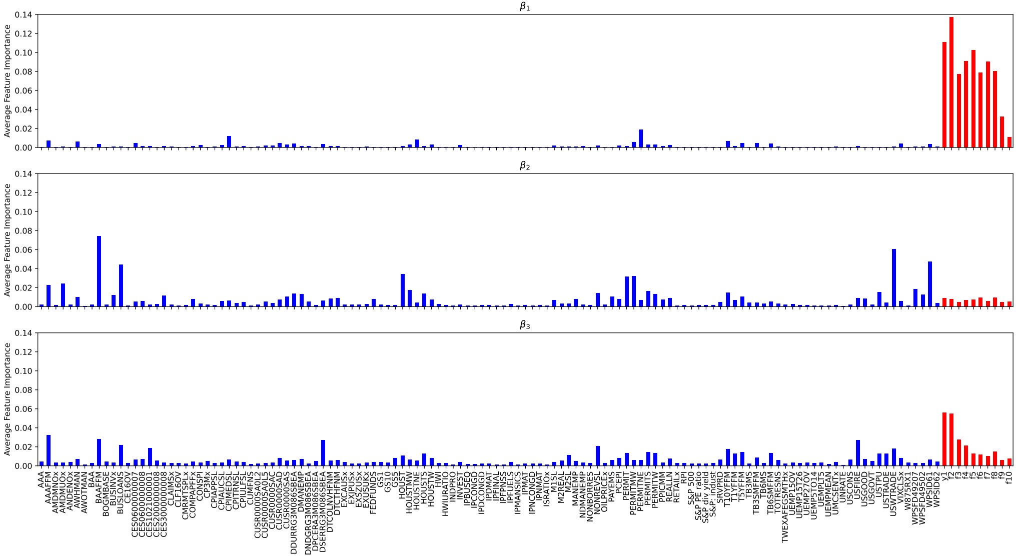
Elastic Net Back

Model Selection - Elastic Net

Back

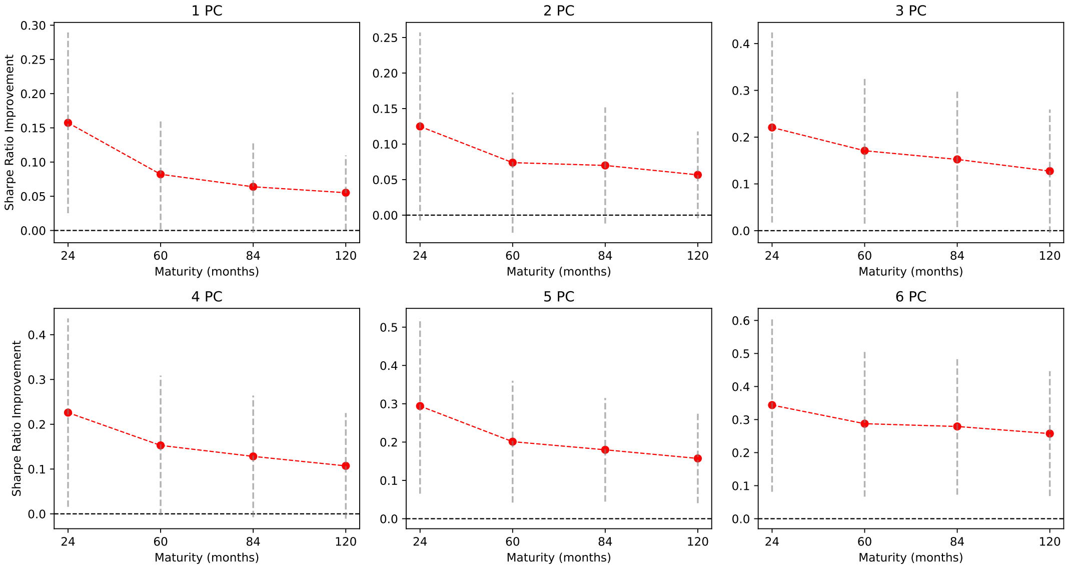


Feature Importance

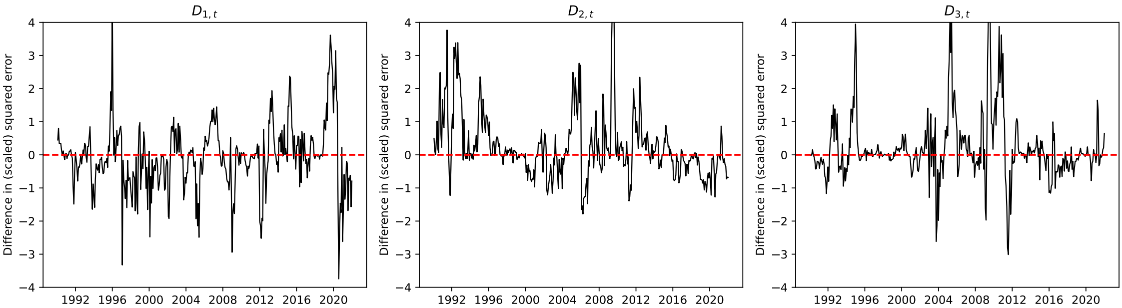


Unconstrained Sharpe Ratio Improvement

Back



Time series of scaled $D_{i,t}$



▸ Back

Random Forrest with Rolling Window (180 months)

	Lagged Factors			Forward Rates		
Target	No Macro	All Macro	p-value	No Macro	All Macro	p-value
β_1	-1.20	-1.32	0.72	-0.63	-0.98	0.98
β_2	-0.07	0.20	0.02	-0.30	0.19	0.00
β_3	-0.47	-0.24	0.04	-0.67	-0.23	0.00

Back

Math Details - Giacomini and White (2006)

Back

- Let x_t be a $q \times 1$ random vector with variables chosen by the econometrician
- Let $\mathrm{z}_{t+h} \equiv \mathrm{x}_t \left(L^{m'}_{t+h} - L^{m}_{t+h} \right)$ for a given forecasting horizon h
- Define

$$\bar{\mathrm{z}}_T \equiv \frac{1}{T-h-t_0} \sum_{t=t_0}^{T-h} \mathrm{z}_{t+h}$$

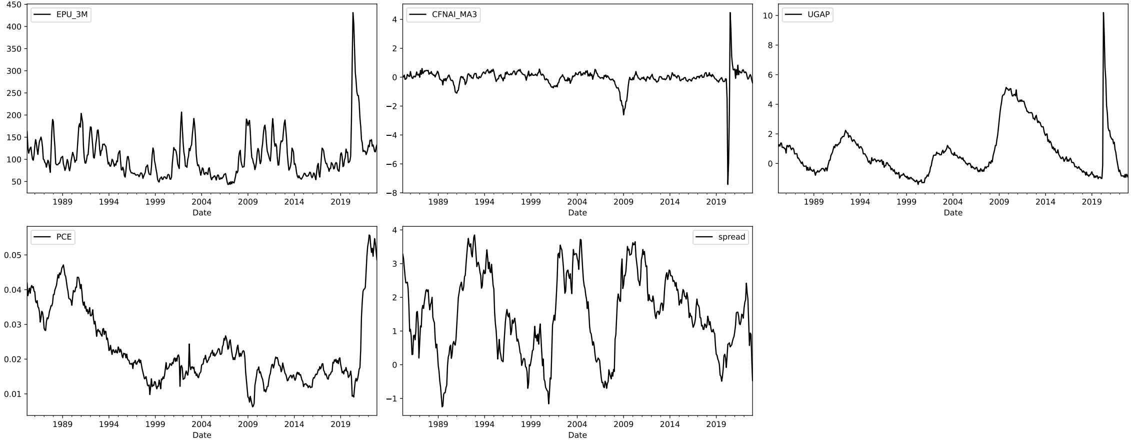
$$\widehat{\Omega}_T \equiv \frac{1}{T-h-t_0} \sum_{t=t_0}^{T-h} \mathrm{z}_{t+h}\mathrm{z}'_{t+h} + \frac{1}{T-h-t_0} \sum_{j=1}^{h-1} w_{j,T} \sum_{t=t_0+j}^{T-h} \left(\mathrm{z}_{t+h-j}\mathrm{z}'_{t+h} + \mathrm{z}_{t+h}\mathrm{z}'_{t+h-j} \right)$$

$$w_{j,T} \to 1, \qquad \text{as } T \to \infty \text{ for each } j \in \{1, ..., h-1\}$$

- Under some regularity conditions, they show that as T diverges to ∞:

$$W \equiv T \cdot \mathrm{z}'_{t+h} \widehat{\Omega}_T^{-1} \mathrm{z}_{t+h} \xrightarrow{d} \chi^2_q \tag{12}$$

Conditioning Variables - Time Series



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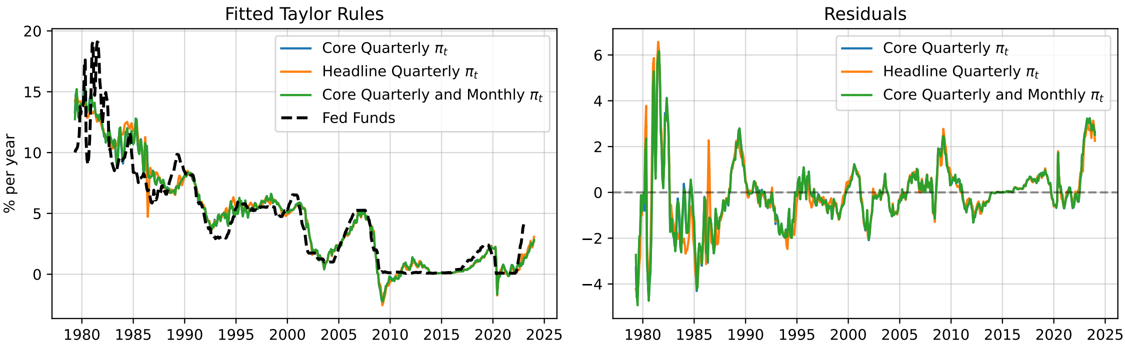
Non-Parametric Evidence on Conditional Predictive Ability

Inflation Tercile	PCE	D_1	D_2	D_3	Control
Low	0.013	-0.152	0.496	2.386	Forward Rates
Medium	0.018	-0.754	0.788	1.923	Forward Rates
High	0.028	0.039	2.430	1.526	Forward Rates
Low	0.013	-0.204	-0.023	0.803	Lagged Factors
Medium	0.018	-0.114	0.120	0.850	Lagged Factors
High	0.028	0.048	1.963	1.492	Lagged Factors

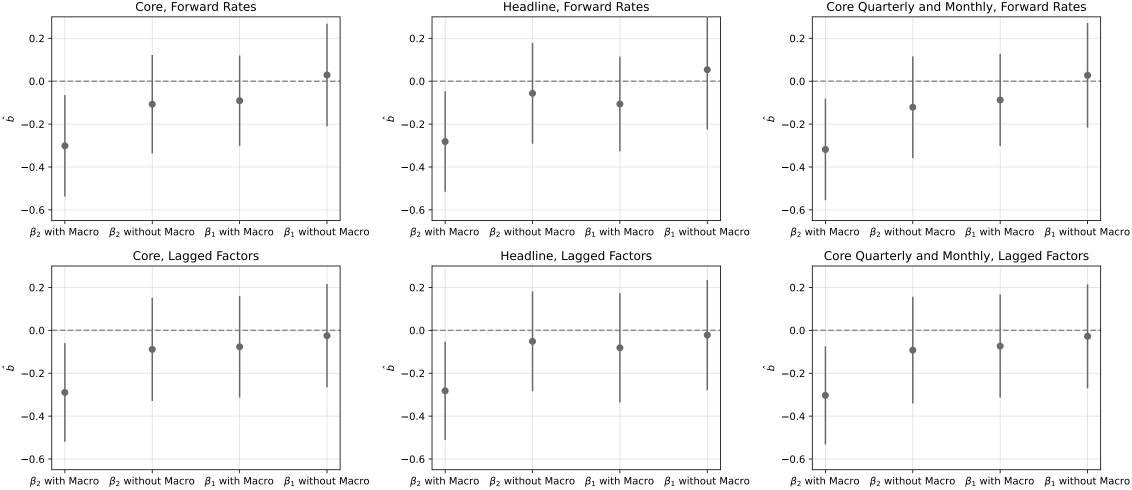
Back

Different Taylor Rules



Robustness on Loss Regression

$$\text{Loss}_{i,t+12} = a + b \cdot |\phi_t| + \eta_{t+12}$$



Conditional Predictive Ability

$$D_{2,t+12} = a + \mathrm{b}'\mathrm{x}_t + u_{t+12}$$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
EPU	-0.08					-0.11				-0.19*
	(0.10)					(0.10)				(0.10)
CFNAI		-0.06				-0.10		-0.09		-0.14
		(0.08)				(0.07)		(0.09)		(0.08)
UGAP			-0.02				0.04	0.01	-0.03	0.05
			(0.10)				(0.09)	(0.09)	(0.08)	(0.13)
PCE				0.30**			0.31**	0.31**	0.30**	0.33***
				(0.12)			(0.12)	(0.12)	(0.12)	(0.11)
Slope					0.09				0.12	0.10
					(0.12)				(0.11)	(0.12)
N	384	384	384	384	384	384	384	384	384	384
R^2	0.01	0.00	0.00	0.09	0.01	0.02	0.09	0.10	0.10	0.13
GW p-values	0.51	0.38	0.84	**0.00**	0.45	0.50	**0.00**	**0.00**	**0.01**	**0.01**

▸ Conditioning Variables ▸ Non-Parametric Evidence

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