Chapter 15 - The Term Structure of Interest Rates

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The yield curve plots yield to maturity as a function of time to maturity I

- The yield curve is central to bond valuation
- It allows investors to gauge their expectations for future interest rates against those of the market

The yield curve plots yield to maturity as a function of time to maturity II

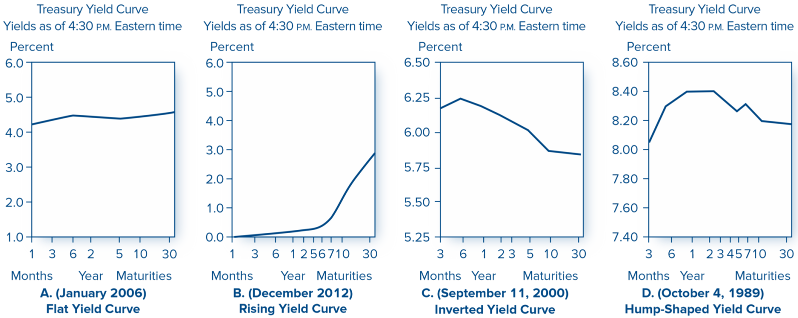

Figure 1: Treasury yield curves (BKM 2023, Figure 15.1)

The yields on different maturity bonds are not equal

- Consider each bond cash flow as a stand-alone zero-coupon bond
- Bond value should be the sum of the values of each bond cash flow
- Bond stripping and bond reconstitution offer arbitrage opportunities

In practice, there are two yield curves

- Pure yield curve: Based on stripped or zero-coupon Treasuries
- On-the-Run yield curve: Based on recently issued coupon bonds selling at or near par value
- These yield curves may differ significantly, and the financial press typically prefers the on-the-run yield curve because on-the-run Treasuries are more liquid than zero-coupon Treasuries

In a world without uncertainty, all investors know all future interest rates I

- Spot rate:
  - The rate that prevails *today* for a period corresponding to the zero's maturity
  - For example, y_2 is the spot rate for the 2-year zero
- Short rate:
  - The rate that applies for a given time interval (e.g., one year)
  - For example, r_1 and r_2 are the short rates that apply to years 1 and 2, respectively
- By the Law of One Price, the multiyear cumulative returns on competing bonds must be similar:

$$(1 + y_2)^2 = (1 + r_1) \times (1 + r_2)$$

In a world without uncertainty, all investors know all future interest rates II

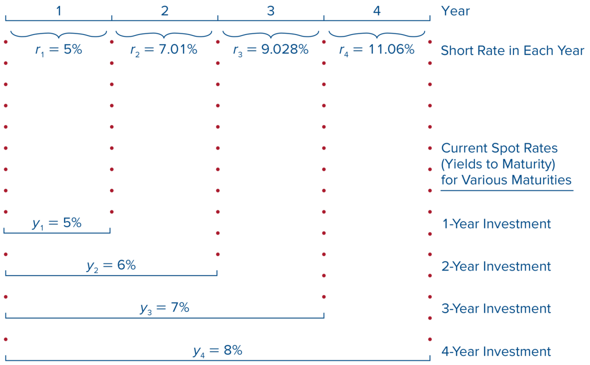

Figure 2: Short rates versus spot rates (BKM 2023, Figure 15.3)

We can generalize the relation between spot and short rates

- For any maturity n, the relation between spot rates for $n-1$ and n and short rates in year n is:

$$(1+y_n)^n = (1+y_{n-1})^{n-1} \times (1+r_n)$$

- Therefore:

$$1+r_n = \frac{(1+y_n)^2}{(1+y_{n-1})^{n-1}}$$

In our world with uncertainty, future short rates are uncertain

- The forward interest rate f_n is a forecast of a future short rate r_n
- Therefore:
$$1 + f_n = \frac{(1 + y_n)^2}{(1 + y_{n-1})^{n-1}}$$
- The forward rate is the "break-even" interest rate that equates the return on an n-period zero-coupon bond to that of an $(n-1)$-period zero-coupon bond rolling over into a 1-year bond in year n
- *When we get to year n, we may find $r_n \neq f_n$*

The term structure is harder to interpret when future interest rates are uncertain

- A one-year investor requires a *liquidity premium* to buy the two-year bond instead of the one-year bond because she faces price risk when she sells the two-year bond in one year
- Conversely, a two-year investor requires a liquidity premium to buy the one-year bond instead of the two-year because she faces rate uncertainty when she rolls over from the first one-year bond to the second one-year bond

Expectations hypothesis theory: Forward rate equals the consensus expectation of the future short rate

- Simplest theory of the term structure
- If the forward rate equals the market consensus expectation of future short interest rate
- Then $f_2 = E(r_2)$ and liquidity premiums are zero
- Therefore, the yields on long-term bonds depend only on expectations of future short rates so

$$(1 + y_2)^2 = (1 + r_1) \times [1 + E(r_n)]$$

and an upward-sloping yield curve indicates that investors expect interest rates to increase

Liquidity preference theory: Short-term investors determine market prices, so the liquidity premium is generally positive I

- Long-term bonds are more risky than short-term bonds for short-term investors because long-term bonds have short-term price risk
- Therefore, short-term investors require a positive *liquidity premium* to bear this price risk
- The liquidity premium is $f_2 - E(r_2)$, so liquidity preference theory predicts $f_2 > E(r_2)$ and an upward-sloping yield curve indicates the liquidity premium

Liquidity preference theory: Short-term investors determine market prices, so the liquidity premium is generally positive II

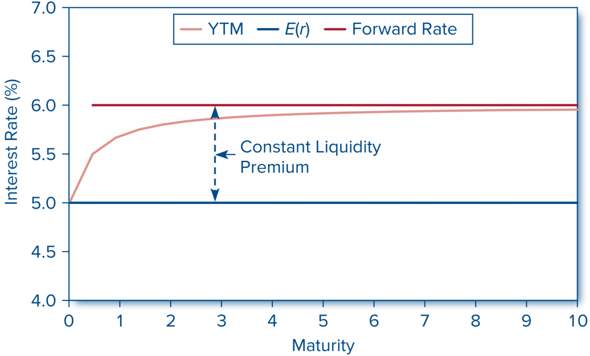

Figure 3: Constant expected short rate. Constant liquidity premium. Result: a rising yield curve. (BKM 2023, Figure 15.4 Panel A)

Liquidity preference theory: Short-term investors determine market prices, so the liquidity premium is generally positive III

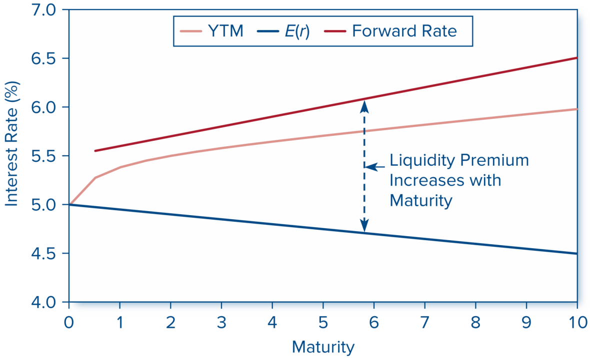

Figure 4: Declining expected short rates. Increasing liquidity premium. Result: a rising yield curve despite falling expected interest rates. (BKM 2023, Figure 15.4 Panel B)

Liquidity preference theory: Short-term investors determine market prices, so the liquidity premium is generally positive IV

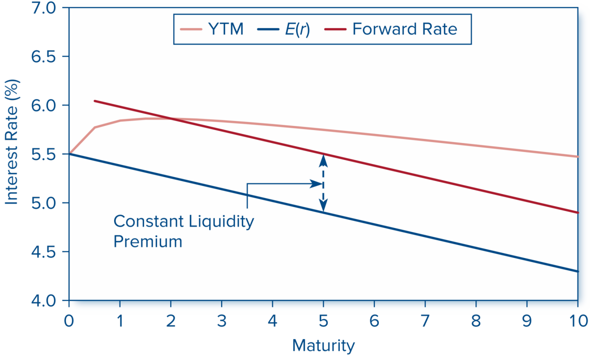

Figure 5: Declining expected short rates. Constant liquidity premium. Result: a hump-shaped yield curve. (BKM 2023, Figure 15.4 Panel C)

Liquidity preference theory: Short-term investors determine market prices, so the liquidity premium is generally positive V

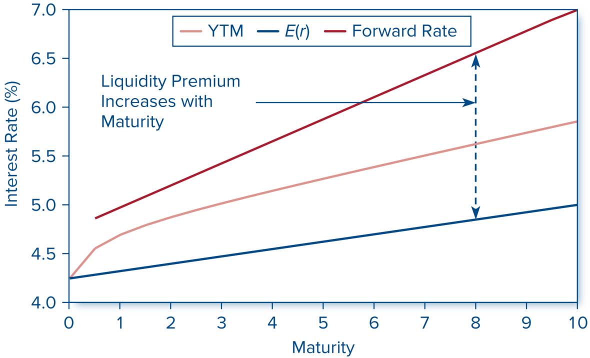

Figure 6: Increasing expected short rates. Increasing liquidity premiums. Result: a sharply rising yield curve. (BKM 2023, Figure 15.4 Panel D)

Market segmentation theory: Long-maturity and short-maturity bonds trade in segmented markets

- Each segmented market finds its equilibrium independently
- These segmented markets determine the yield curve
- This theory is less common today:
    - Investors must compare long and short rates as well as expectations of future rates before deciding where to invest
    - If the liquidity premium were too positive or too negative, investors would rush to the same segment

Yield curve reflects expectations of future short rates, but also other factors such as liquidity premiums I

- When future rates are uncertain:

$$1 + y_n = [(1 + r_1)(1 + f_2)(1 + f_3) \cdots (1 + f_n)]^{1/n}$$

  so $f_{n+1} > y_n$ for an upward-sloping yield curve
- An upward-sloping yield curve has two interpretations:
  - Investors expect rates to rise (i.e., $E(r_n)$ is high)
  - ***and/or***
  - Investors require large liquidity premiums to hold long-term bonds (i.e., $f_n = E(r_n) + \underbrace{\text{Liquidity premium}}_{>0}$)
- The yield curve is a good predictor of the business cycle
  - An upward-sloping yield curve may indicate expansion when long-term rates tend to rise
  - An inverted yield curve may indicate recession when interest rates tend to fall

Yield curve reflects expectations of future short rates, but also other factors such as liquidity premiums II

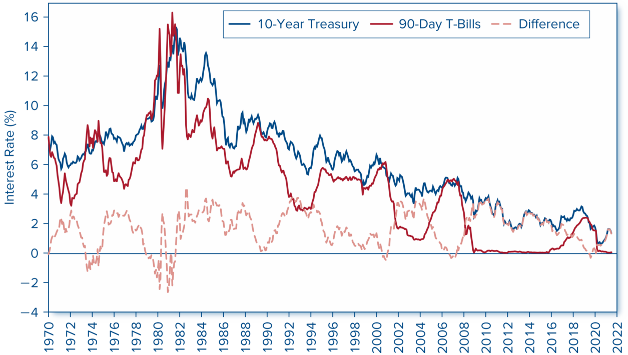

Figure 7: Term spread: Yields on 10-year versus 90-day Treasury securities (BKM 2023, Figure 15.6)

We can derive forward rates from the yield curve

- However:
  - Forward rates will not equal the eventually realized short rate
  - Nor are forward rates even today's expectation of what the short rate will be
- Still, the forward rate is a market rate and an important consideration when making decisions, such as locking in loan rates

Summary from BKM (2023)

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1. The term structure of interest rates refers to the interest rates for various terms to maturity embodied in the prices of default-free zero-coupon bonds.
2. In a world of certainty, all investments must provide equal total returns for any investment period. Short-term holding-period returns on all bonds would be equal in a risk-free economy; all returns would be equal to the rate available on short-term bonds. Similarly, total returns from rolling over short-term bonds over longer periods would equal the total return available from long-maturity bonds.
3. The forward rate of interest is the break-even future interest rate that would equate the total return from a rollover strategy to that of a longer-term zero-coupon bond. It is defined by the equation

$$(1+y_{n-1})^{n-1}(1+f_n) = (1+y_n)^n$$

   where n is a given number of periods from today. This equation can be used to show that yields to maturity and forward rates are related by the equation

$$(1+y_n)^n = (1+r_1)(1+f_2)(1+f_3)\cdots(1+f_n)$$

4. A common version of the expectations hypothesis holds that forward interest rates are unbiased estimates of expected future interest rates. However, there are good reasons to believe that forward rates differ from expected short rates because of a risk premium known as a *liquidity premium.* A positive liquidity premium can cause the yield curve to slope upward even if no increase in short rates is anticipated.
5. The existence of liquidity premiums complicates attempts to infer expected future interest rates from the yield curve. Such an inference would be made easier if we could assume the liquidity premium remained reasonably stable over time. However, both empirical and theoretical considerations cast doubt on the constancy of that premium.
6. Forward rates are market interest rates in the important sense that commitments to forward (i.e., deferred) borrowing or lending arrangements can be made at these rates.

Key equations from BKM (2023)

Forward rate of interest: $1 + f_n = \dfrac{(1 + y_n)^n}{(1 + y_{n-1})^{n-1}}$

Yield to maturity given sequence of forward rates: $1 + y_n = [(1 + r_1)(1 + f_2)(1 + f_3) \cdots (1 + f_n)]^{1/n}$

Liquidity premium = Forward rate − Expected short rate

References I

Bodie, Zvi, Alex Kane, and Allan J. Marcus (2023). *Investments*. 13th ed. New York: McGraw Hill.