Slides 5 - Risk, Return, and the Historical Record

1. a. The nominal interest rate is also called the _____.

   - ○ effective interest rate
   - ○ inflation-adjusted interest rate
   - ◉ quoted interest rate
   - ○ real interest rate

   The nominal interest rate is also called the quoted interest rate. Listed, or quoted, interest rates are always nominal (as opposed to real) rates, unless explicitely noted otherwise.

2. The expected inflation rate is 1.4%.

   a. What is the nominal interest rate if the real rate of interest is 2.1%?

   `R`: nominal interest rate
   `r`: real interest rate
   `i`: expected inflation rate

   According to the Fisher equation:

   `(1+R) = (1+r) (1+i)`

   ⇔ `R = (1+r) (1+i) - 1`

   = `(1+0.021)(1+0.014) - 1`

   = **0.03529**

   b. What is the real interest rate if the nominal rate of interest is 6.6%?

   `(1+R) = (1+r) (1+i)`

   ⇔ `r = (1+R) /(1+i) -1`

   = `(1+0.066) / (1+0.014) - 1`

   = **0.05128**

3. You've invested money at an interest rate of 8%. Your tax rate is 33%.

   a. What is your after-tax interest rate?

   `r_"after tax" = (1-t) * r_"before tax" = (1-0.33) * 0.08 =` **0.0536**

4. a. If interest is compounded quarterly, the ________ expresses the interest rate as if it were compounded annually.

   - ○ APR
   - ◉ EAR
   - ○ period rate
   - ○ quoted rate

   If interest is compounded quarterly, the EAR expresses the interest rate as if it were compounded annually.

b. Given a fixed APR, compounding more frequently leads to

- a lower APR
- a higher APR
- a lower EAR
- a higher EAR

Compounding more frequently leads to a higher EAR, since interest on interest is added more frequently. The APR, on the other hand, is unaffected by the frequency of compounding (which makes it a bad interest rate measure).

5. The quoted interest rate is 3.8% (APR with quarterly compounding).

a. What is the quarterly rate?

$r_q = \frac{APR}{4} = 0.038/4 =$ **0.0095**

b. What is the effective annual rate (EAR)?

$EAR = (1+ r_q)^{\text{#of periods per year}} - 1$

= $(1+ 0.0095)^4 - 1$

= **0.03854**

c. What is the effective rate for 6 months, i.e., the semiannual rate?

Using the quarterly rate from part 1:

$1+r_6 = (1+r_q)^2$

$r_6 = (1+0.0095)^2 -1 = 0.01909$

Alternatively, using the EAR from part 2:

$(1+r_6)^2 = 1+EAR$

⇔ $r_6 = (1+EAR)^{0.5} - 1 = (1+0.03854)^{0.5} - 1 =$ **0.01909**

d. What is the effective daily rate?

The effective daily, r_d, solves the equation:

$(1+r_d)^{365} = 1+EAR$

⇔ $r_d = (1+EAR)^{(1/365)} - 1 = (1+0.03854)^{(1/365)} - 1 =$ **0.00010362**

Make sure to enter the exponent with parentheses on your calculator.

6. You bought a stock for $71.12 and sold it for $74.96 after 6 months. The stock did not pay any dividends.

a. What is the annualized return?

Return over the holding period:

$r = \frac{P_1+D_1}{P_0}-1 = \frac{74.96+0}{71.12}-1=0.05399$

Annualized return:

$EAR =(1+r)^N - 1 = (1+0.05399)^{(12/6)}-1=$ **0.1109**

7. You've recorded the following returns for a stock:

Quarter	Rate of return (%)
1	1.4
2	3.4
3	-8.5
4	4.1

a. What was the realized return over the year?

Annual return:

`1+r_(0,4) = (1+r_1) (1+r_2) (1+r_3) (1+r_4)`

`⇔ r_(0,4) = (1+r_1) (1+r_2) (1+r_3) (1+r_4) - 1`

`= (1+0.014) (1+0.034) (1+ -0.085) (1+0.041) - 1`

`=` **-0.0013109**

8. A security delivered a return of 3% over a holding period of 70 days.

a. What was the return per day?

$r_{day} = (1+r_{70\ days})^{70\text{-day periods in 1 day}} - 1 = (1+0.03)^{1/70} - 1 =$ **0.0004224**

b. What was the return per month (assume 30 days per month)?

$r_{month} = (1+r_{70\ days})^{70\text{-day periods in 30 days}} - 1 = (1+0.03)^{30/70} - 1 =$ **0.012749**

c. What was the return per quarter (assume 90 days per quarter)?

$r_{quarter} = (1+r_{70\ days})^{70\text{-day periods in 90 days}} - 1 = (1+0.03)^{90/70} - 1 =$ **0.03874**

d. What was the return over a 23-day period?

$r_{23\ days} = (1+r_{70\ days})^{70\text{-day periods in 23 days}} - 1 = (1+0.03)^{23/70} - 1 =$ **0.009759**

9. You invested $16,000 which grew to $20,320 after 14 years.

a. What was the continuously compounded return over the entire period?

There are two ways to solve this question.

1) Solving for the HPR and converting it to a continously compounded rate

`r= (FV)/(PV)-1 = 20,320/16,000-1 = 0.27`

`r_"cc" = ln(1+r) = ln(1+0.27) = 0.239`

2) Calculating the continuously compounded return directly

`r_"cc" = ln((FV)/(PV)) = ln(20,320/16,000) =` **0.239**

10. We know the following expected returns for stocks A and B, given different states of the economy:

State (s)	Probability	$E(r_{A,s})$	$E(r_{B,s})$
Recession	0.2	-0.1	0.01
Normal	0.5	0.08	0.05
Expansion	0.3	0.11	0.07

a. What is the expected return for stock A?

The expected return is the weighted average return across all states of the economy:

`E(r)= sum_s p_sE(r_s)`

For stock A:

`E(r_A) = 0.2 * (-0.1) + 0.5 * 0.08 + 0.3 * 0.11 =`**0.053**

11. We know the following expected returns for stocks A and B, given different states of the economy:

State (s)	Probability	$E(r_{A,s})$	$E(r_{B,s})$
Recession	0.2	-0.03	0.05
Normal	0.5	0.12	0.08
Expansion	0.3	0.2	0.12

a. What is the expected return for stock A?

The expected return is the weighted average return across all states of the economy:

`E(r_A) = sum_s (p_sE(r_(A,s)))`

= 0.2 * (-0.03) + 0.5 * 0.12 + 0.3 * 0.2

= **0.114**

b. What is the expected return for stock B?

`E(r_B) = sum_s (p_sE(r_(B,s)))`

= 0.2 * 0.05 + 0.5 * 0.08 + 0.3 * 0.12

= **0.086**

c. What is the standard deviation of returns for stock A?

Variance:

`sigma^2 = sum_s (p_s (E(r_(A,s)) - E(r_A))^2)`

= 0.2 * $(-0.03-0.114)^2$ + 0.5 * $(0.12-0.114)^2$ + 0.3 * $(0.2-0.114)^2$

= 0.006384

Standard deviation:

`sigma = sqrt(sigma^2)`

= `0.006384^0.5`

= **0.0799**

d. What is the standard deviation of returns for stock B?

Variance:

`sigma^2 = sum_s (p_s (E(r_(B,s)) - E(r_B))^2)`

= 0.2 * $(0.05-0.086)^2$ + 0.5 * $(0.08-0.086)^2$ + 0.3 * $(0.12-0.086)^2$

= 0.000624

Standard deviation:

`sigma = sqrt(sigma^2)`

= `0.000624^0.5`

= **0.02498**

12. a. The risk premium is _____.

Check all that apply:

- [x] the reward for bearing risk
- [ ] normally zero for risky assets
- [x] normally positive for risky assets
- [x] the difference between the expected rate of return on an asset and the risk-free rate

The risk premium compensate investors for bearing risk. An additional return of zero would be no compensation.

13. Below are the expected returns for different asset classes for next year:

Asset class	Exp. return
T-bills	1.7%
Corporate bonds	4.7%
Small company stocks	18%
Large company stocks	12.4%

a. What is the risk premium for corporate bonds?

We usually interpret the return on Treasury bills as the risk-free rate and calculate the risk premium as the expected return in excess of the risk-free rate.

Risk premium:
$RP = E(r)_{CB} - r_f$
= 0.047 - 0.017
= **0.03**

b. What is the risk premium for small company stocks?

Risk premium:
$RP = E(r)_S - r_f$
= 0.18 - 0.017
= **0.163**

c. What is the risk premium for large company stocks?

Risk premium:
$RP = E(r)_L - r_f$
= 0.124 - 0.017
= **0.107**

14. Below are the returns for different asset classes for a particular year:

Asset class	Return
T-bills	2.8%
Corporate bonds	4.6%
Small company stocks	15.9%
Large company stocks	8.8%

a. What was the excess return for corporate bonds?

We usually interpret the return on Treasury bills as the risk-free rate and calculate the excess return as the return in excess of the risk-free rate.

Excess return:
$= r_{CB} - r_f$
$= 0.046 - 0.028$
$=$ **0.018**

b. What was the excess return for small company stocks?

$r_S - r_f$
$= 0.159 - 0.028$
$=$ **0.131**

c. What was the excess return for large company stocks?

$r_L - r_f$
$= 0.088 - 0.028$
$=$ **0.06**

15. We know the following historical rates of return for stocks A and B:

Year	Stock A	Stock B
2022	-0.1	0.01
2021	0.08	0.05
2020	0.1	0.07

a. What is the (arithmetic) average annual return for stock A?

`r_A = (r_1 + r_2 + r_3) / 3 = (-0.1 + 0.08 + 0.1) / 3=` **0.02667**

b. What is the (arithmetic) average annual return for stock B?

`r_B = (r_1 + r_2 + r_3) / 3 = (0.01 + 0.05 + 0.07) / 3=` **0.04333**

16. You invested $19,000 in a mutual fund 13 years ago. Your money has since grown to $53,570.

a. What was the geometric average return over the 13 years?

`PV (1+r)^N = FV`

⇔ `r = ((FV)/(PV))^(1/N) - 1`

= `(53,570/19,000)^(1/13)-1`

= 0.083

Using a financial calculator:

	N	I/Y	PV	PMT	FV
Inputs	13		19,000	0	-53,570
Compute		8.3			

The rate of return was **0.083**.

17. You bought Samsung stock for $50 on April 1. The stock paid a dividend of $3 on July 1, and had a price of $53. It is now Oct. 1, and the stock price is $63.

a. What was the arithmetic average quarterly return?

`r_1 = (53 + 3)/50 - 1 = 0.12`

`r_2 = 63/53 - 1 = 0.18868`

`r_a = (r_1 + r_2) / 2= (0.12 + 0.18868) / 2=` **0.15434**

b. What was the geometric average return per quarter?

`r_g = ((1+r_1)(1+r_2))^(1/2) - 1`

= `((1+0.12)(1+0.18868))^(1/2) - 1`

= **0.15383**

18. a. We measure the risk of a single asset or portfolio by measuring the _____.

- skew of its return distribution
- standard deviation of returns
- asset's alpha
- asset's beta

The risk of a single asset, i.e., one not held as part of a portfolio, is measured by the standard deviation of returns. We use standard deviation because

- the risk is to earn a lower return than the expected return,
- asset returns roughly follow a normal distribution,
- the normal distribution is symmetrical,
- standard deviation tells us how dispersed the distribution of returns is around its mean, allowing us to calculate the probability with which a return will be significantly lower (or higher) than the expected return.

19. You've collected the following historical rates of return for stocks A and B:

Year (t)	$r_{A,t}$	$r_{B,t}$
2016	0.02	0.03
2015	0.08	0.05
2014	0.12	0.07

a. What was the average annual return for stock A

`bar r_A = (r_1 + r_2+ r_3) / 3`

= `(0.02 + 0.08 + 0.12) / 3`

= **0.07333**

b. What was the average annual return for stock B?

`bar r_B = (r_1 + r_2+ r_3) / 3`

= `(0.03 + 0.05 + 0.07) / 3`

= **0.05**

c. What was the standard deviation of returns for stock A?

Variance is defined as the sum of squared deviations from the sample average, divided by N-1. We divide by N-1 instead of N, since statisticians have shown that dividing by N-1 results in a more accurate estimate of the variance of the underlying population. Historical standard deviation is often used as an estimate of future standard deviation, on the assumption that variability doesn't change much from year to year.

Variance:

$\sigma^2 = \frac{1}{N-1} \sum_t (r_t - \bar{r})^2$

= $\frac{1}{3-1} [((0.02-0.07333)^2 + (0.08-0.07333)^2 + (0.12-0.07333)^2)]$

= 0.002533

Standard deviation:

$\sigma = \sqrt{\sigma^2}=0.002533^{0.5}=$ **0.05033**

d. What was the standard deviation of returns for stock B?

Variance:

$\sigma^2 = \frac{1}{N-1} \sum_t (r_t - \bar{r})^2$

= $\frac{1}{3-1} [((0.03-0.05)^2 + (0.05-0.05)^2 + (0.07-0.05)^2)]$

= 0.0004

Standard deviation:

$\sigma = \sqrt{\sigma^2}=0.0004^{0.5}=$ **0.02**

20. A portfolio had an annual return of 6% and an annual standard deviation of 33%. Treasury bills yielded 1% during the same period.

a. What was the Sharpe ratio?

$S = (r_P-r_f)/\sigma = (0.06 - 0.01)/0.33 =$ **0.15152**

21. A stock delivered the following annual returns over the last 4 years:

Year	1	2	3	4
Return	8%	3%	-8%	8%

a. What was the arithmetic average return over the 4 years?

$r_a= (r_1 + r_2 + r_3 + r_4) / 4 = (0.08 + 0.03 + -0.08 + 0.08) / 4=$ **0.0275**

b. What was the geometric average return over the 4 years?

$r_g = [(1+r_1)(1+r_2)(1+r_3)(1+r_4)]^{1/4} - 1$

= $[(1.08)(1.03)(0.92)(1.08)]^{1/4} - 1$

= **0.02534**

c. If you invested $1,000 at the beginning, how much would you have had at the end?

The initial investment of $1,000 would have grown to:

$FV = PV (1+r_g)^T$

= $1{,}000 (1+0.02534)^4$

= **1,105.28**

d. What is the standard deviation of the stock's return based on the historical data?

Variance:

`σ^2 = 1/(N-1) sum_i (r_i - r_a)^2`

= `1/3 [(0.08-0.0275)^2+(0.03-0.0275)^2+(-0.08-0.0275)^2+(0.08-0.0275)^2]`

= 0.005692

Standard deviation:

`σ = sqrt(σ^2) = 0.005692^0.5=` **0.07544**

22. You want to have $500,000 in today's (real) dollars when you retire in 40 years. The expected inflation rate is 2.9% and the nominal return on your investments is 7.6%.

    a. How much money do you have to save now if you can't make any additional deposits?

    According to the Fisher equation:
    (1+r) = (1+R) (1+i)

    r: nominal interest rate
    R: real interest rate
    i: expected inflation rate

    `R = (1+r) / (1+i) - 1= 1.076 / 1.029 - 1= 0.04568`

    The real (inflation-adjusted) return is 4.568%.

    `PV = (FV) / (1+R)^N= 500,000 / 1.0457^40=`**83,771**

23. a. Which statements are correct? The geometric average return _____.

    Check all that apply:

    - [x] takes into account compounding
    - [x] is usually less than the arithmetic mean
    - [x] is better for forecasting returns over many periods
    - [ ] is the simple average of the individual returns

    Since the geometric average return takes into account compounding, it is better for forecasting returns over many periods, when compounding becomes more and more important.