

Slides 24 - Portfolio Performance Evaluation

1. You invested \$16,000 in a mutual fund 19 years ago. Your money has since grown to \$32,499.

a. What was the geometric average return over the 19 years?

$$PV(1+r)^N = FV$$

$$\Leftrightarrow r = ((FV)/(PV))^{(1/N)} - 1$$

$$= (32,499/16,000)^{(1/19)} - 1$$

$$= 0.038$$

Using a financial calculator:

	N	I/Y	PV	PMT	FV
Inputs	19		16,000	0	-32,499
Compute		3.8			

The rate of return was **0.038**.

2. You've recorded the following prices and dividend payments for a stock:

Quarter	Stock price (end of quarter)	Dividend per share (end of quarter)	Shares bought or sold (after dividend payment)	Shares held (before div. payment)
0	75.32		4	0
1	113.25	3.65	1	4
2	97.54	3.65	0	5
3	125.72	3.65	-2	5
4	105.54	3.65	-3	3

a. What was the arithmetic average rate of return?

Quarter	Stock price (end of quarter)	Dividend per share (end of quarter)	Quarterly return
0	75.32		
1	113.25	3.65	0.552
2	97.54	3.65	-0.10649
3	125.72	3.65	0.3263
4	105.54	3.65	-0.13149

$$r_a = (r_1 + r_2 + r_3 + r_4) / 4$$

$$= (0.552 + -0.10649 + 0.3263 + -0.13149) / 4$$

$$= \mathbf{0.1601}$$

b. What was the geometric average rate of return?

$$r_g = [(1+r_1)(1+r_2)(1+r_3)(1+r_4)]^{(1/4)} - 1$$

$$= [(1+0.552)(1+ -0.10649)(1+0.3263)(1+ -0.13149)]^{(1/4)} - 1$$

c. What was the dollar-weighted (money-weighted) rate of return?

	A	B	C	D	E	F
	Quarter	Stock price (end of quarter)	Dividend per share (end of quarter)	Shares bought or sold (after div. payment)	Shares held (before div. payment)	Cash flows
1	0	75.32		4	0	-301.28
2	1	113.25	3.65	1	4	-98.65
3	2	97.54	3.65	0	5	18.25
4	3	125.72	3.65	-2	5	269.69
5	4	105.54	3.65	-3	3	327.57

The dollar-weighted rate of return is the internal rate of return, r , that solves the following equation:

$$0 = C_0 + (CF_1)/(1+r) + (CF_2)/(1+r)^2 + (CF_3)/(1+r)^3 + (CF_4)/(1+r)^4$$

$$0 = -301.28 + -98.65/(1+r) + 18.25/(1+r)^2 + 269.69/(1+r)^3 + 327.57/(1+r)^4$$

Since this equation is non-linear in r , we need to use a financial calculator or Excel's IRR function to solve for r :

=IRR(F1:F5)

=**0.142**

3. a. Which of the following is true about the comparison universe?

- ☐ It is an active management strategy which involves a comparison of different stocks with high beta values.
- ☐ It is used to measure the relative performance of a set of investment managers with different risk characteristics.
- ☒ Portfolio rankings can be misleading because some managers follow a particular portfolio subgroup and therefore portfolios are not truly comparable.
- ☐ The 50th percentile manager has a better investment performance than the 70th percentile manager.

A comparison universe is used to measure the relative performance of a set of investment managers with similar risk characteristics. The portfolio rankings provided by the comparison universe can be misleading because some managers follow a particular portfolio subgroup and therefore portfolios are not truly comparable.

4. a. Which is the right metric when evaluating the performance of a fund manager, given a sequence of returns?

- ☐ The arithmetic average rate of return
- ☐ The money-weighted rate of return
- ☒ The time-weighted rate of return
- ☐ The weighted average rate of return

The time-weighted rate of return is the appropriate measure, since it's unaffected by cash inflows and outflows, which are not under the control of the fund manager. The time-weighted rate of return is also known as the geometric average rate of return.

5. a. Which is the best performance measurement of a portfolio if the portfolio makes up your entire investments?

- ☐ Information ratio
- ☐ Jensen measure
- ☒ Sharpe measure
- ☐ Treynor measure

The Sharpe measure is the most appropriate performance measurement if the portfolio constitutes the entire investments of an investor.

6. a. Which of the following is true about the information ratio?

- ☒ It is a ratio of alpha divided by the standard deviation of the residuals.
- ☐ It is a ratio of portfolio excess return divided by its beta.
- ☐ It is a ratio of portfolio excess return divided by the alpha.
- ☐ It is the most appropriate performance measurement if the portfolio constitutes a small part of an investor's investments.
- ☐ It is the most appropriate performance measurement if the portfolio constitutes the entire investments of an investor.

The information ratio is calculated as alpha divided by the standard deviation of the residuals (the tracking error). It is the most appropriate performance measurement if the investor evaluates a portfolio to be mixed with a benchmark portfolio.

7. a. Which is true about the M-square measure?

- ☐ It is a measure of portfolio excess return divided by its beta.
- ☐ It is a measure of portfolio excess return divided by its standard deviation.
- ☐ It is a ratio of alpha divided by the standard deviation of the diversifiable risk.
- ☒ It is the difference between the rate of return of a portfolio with the right amount of leverage to match the standard deviation of a passive index and the rate of return of that passive index.
- ☐ It is used to measure the portfolio excess return, adjusted for the portfolio delta.

The M-square measure represents a difference between the rate of return of a portfolio with a right amount of leverage to match the standard deviation of a passive index and the rate of return of that passive index. It was introduced by Leah and Franco Modigliani (1997).

8. A portfolio had an annual return of 12% and an annual standard deviation of 15%. Treasury bills yielded 1% during the same period.

a. What was the Sharpe ratio?

$$S = (r_P - r_f) / \sigma = (0.12 - 0.01) / 0.15 = \mathbf{0.7333}$$

9. Calculate the Sharpe ratios for the following portfolios:

	A	B	C
1	Portfolio	Return	St. dev.
2	1	0.12	0.43
3	2	0.08	0.2
4	3	0.03	0.12
5	4	0.06	0.24
6	5	0.05	0.22
7			
8	Risk-free asset	0.02	

a. What is the highest Sharpe ratio?

	A	B	C	D	E
1	Portfolio	Return	St. dev.	Sharpe ratio	
2	1	0.12	0.43	0.2326	=(B2-B\$8)/C2
3	2	0.08	0.2	0.3	
4	3	0.03	0.12	0.08333	
5	4	0.06	0.24	0.16667	
6	5	0.05	0.22	0.13636	
7					
8	Risk-free asset	0.02			
9	Max. Sharpe ratio			0.3	=MAX(D2:D6)

The portfolio with the highest Sharpe ratio offers the best risk-return trade-off.

10. Return statistics for portfolios A and B and the market portfolio, M, are given below:

	A	B	C	D
1	Risk-free rate	0.03		
2				
3	Portfolio	A	B	M
4	Average return	13%	7%	8%
5	Standard deviation	34%	27%	12%
6	Correlation with M	0.1	0.7	1

a. What is the beta of stock B?

	A	B	C	D	E
1	Risk-free rate	0.03			
2					
3	Portfolio	A	B	M	
4	Average return	13%	7%	8%	
5	Standard deviation	34%	27%	12%	
6	Correlation with M	0.1	0.7	1	
7	Covariance with M	0.00408	0.02268	0.0144	=D6*D5*\$D5
8	Beta	0.2833	1.575	1	=D7/\$D7

b. What is Jensen's alpha for stock B?

	A	B	C	D	E
1	Risk-free rate	0.03			
2					
3	Portfolio	A	B	M	
4	Average return	13%	7%	8%	
5	Standard deviation	34%	27%	12%	
6	Correlation with M	0.1	0.7	1	
7	Covariance with M	0.00408	0.02268	0.0144	=D6*D5*\$D5
8	Beta	0.2833	1.575	1	=D7/\$D7

9	SML return	0.04417	0.10875	0.08	=B1+D8*(\$D4-\$B1)
10	Jensen's alpha	0.08583	-0.03875	0	=D4-D9

11. Return statistics for funds A and B and the market portfolio, M, are given below:

	A	B	C	D
1	Risk-free rate	0.03		
2				
3	Portfolio	A	B	M
4	Average return	10%	7%	8%
5	Standard deviation	34%	24%	12%
6	Correlation with M	0.6	0.7	1

a. What is the beta of fund B?

	A	B	C	D	E
1	Risk-free rate	0.03			
2					
3	Portfolio	A	B	M	
4	Average return	10%	7%	8%	
5	Standard deviation	34%	24%	12%	
6	Correlation with M	0.6	0.7	1	
7	Covariance with M	0.02448	0.02016	0.0144	=D6*D5*\$D5
8	Beta	1.7	1.4	1	=D7/\$D7

b. What is Treynor ratio of fund B?

	A	B	C	D	E
1	Risk-free rate	0.03			
2					
3	Portfolio	A	B	M	
4	Average return	10%	7%	8%	
5	Standard deviation	34%	24%	12%	
6	Correlation with M	0.6	0.7	1	
7	Covariance with M	0.02448	0.02016	0.0144	=D6*D5*\$D5
8	Beta	1.7	1.4	1	=D7/\$D7
9	Treynor ratio	0.04118	0.02857		=(C4-\$B1)/C8

12. You've collected the following information for a stock portfolio, the market portfolio and Treasury bills:

	E(r)	Beta	St.Dev.
Portfolio A	15%	1.3	48%
Market portfolio	8%	1	22%
T-bills	2%	0	0%

a. What is the information ratio for portfolio A?

Jensen's alpha:

$$\begin{aligned}\alpha_A &= r_A - (r_f + \beta_A(r_M - r_f)) \\ &= 0.15 - (0.02 + 1.3 * (0.08 - 0.02)) \\ &= 0.052\end{aligned}$$

Standard deviation of the return residual:

Variance = Systematic risk + firm-specific risk

$$\begin{aligned}\sigma_A^2 &= \beta_A^2 \sigma_M^2 + \sigma^2(e_A) \\ \Leftrightarrow \sigma^2(e_A) &= \sigma_A^2 - \beta_A^2 \sigma_M^2 \\ \Leftrightarrow \sigma(e_A) &= (\sigma_A^2 - \beta_A^2 \sigma_M^2)^{1/2} \\ &= (0.48^2 - 1.3^2 * 0.22^2)^{1/2} \\ &= 0.3855\end{aligned}$$

Information ratio:

$$\text{IR} = \alpha_A / (\sigma(e_A)) = 0.052 / 0.3855 = \mathbf{0.13489}$$

13. You've collected the following historical information:

- Return on risky portfolio: 15%
- Beta of risky portfolio: 1.8
- Standard deviation of returns: 30%
- Residual standard deviation (tracking error): 40%
- Treasury bill rate: 2%
- Return on S&P 500: 4.4%
- Standard deviation of returns on S&P 500: 22%

Your job is to evaluate the performance of the risky portfolio.

a. What is the Sharpe ratio?

$$\text{S}_P = (r_P - r_f) / \sigma_P = (0.15 - 0.02) / 0.3 = \mathbf{0.4333}$$

b. What is the Treynor measure?

$$\text{T}_P = (r_P - r_f) / \beta_P = (0.15 - 0.02) / 1.8 = \mathbf{0.07222}$$

c. What is Jensen's alpha?

$$\begin{aligned}\alpha_P &= r_P - (r_f + \beta_P(r_M - r_f)) \\ &= 0.15 - (0.02 + 1.8 * (0.044 - 0.02)) \\ &= \mathbf{0.0868}\end{aligned}$$

d. What is the information ratio?

$$\text{IR} = \alpha_P / (\sigma(e_P)) = 0.0868 / 0.4 = \mathbf{0.217}$$

e. What is the M² measure?

Weight of P in adjusted portfolio P*:

$$\begin{aligned}y \sigma_P &= \sigma_M \\ \Leftrightarrow y &= \sigma_M / \sigma_P \\ &= 0.22 / 0.3 \\ &= \mathbf{0.7333}\end{aligned}$$

Return of adjusted portfolio:

$$\begin{aligned}
 r_{P^*} &= y r_P + (1-y) r_f \\
 &= 0.7333 * 0.15 + (1-0.7333) * 0.02 \\
 &= 0.11533
 \end{aligned}$$

M^2 measure:

$$\begin{aligned}
 M^2 &= r_{P^*} - r_M \\
 &= 0.11533 - 0.044 \\
 &= \mathbf{0.07133}
 \end{aligned}$$

14. The following table shows rates of return for a mutual fund and the market portfolio. The average risk-free rate over that time period was 1%.

	A	B	C
1	Year	Fund	Market
2	1	14%	13%
3	2	-24%	-14%
4	3	-6%	-8%
5	4	5%	28%
6	5	14%	8%
7	6	10%	8%

- a. What is the arithmetic average return for the market portfolio?

	A	B	C	D	E
1	Year	Fund	Market	Risk-free rate	
2	1	14%	13%		
3	2	-24%	-14%		
4	3	-6%	-8%		
5	4	5%	28%		
6	5	14%	8%		
7	6	10%	8%		
8	Average	0.02167	0.05833	0.01	=AVERAGE(C2:C7)

The market portfolio had an average return of **0.05833**.

- b. What is the covariance of returns?

	A	B	C
9	Covariance	0.016443	=COVARIANCE.S(B2:B7,C2:C7)

- c. What is the variance of the market portfolio?

	A	B	C	D
10	Variance	0.02202	0.02274	=VAR.S(C2:C7)

- d. What is the beta of the fund, using the market or industry model (not subtracting the risk-free rate)?

	A	B	C
11	Beta	0.7232	=SLOPE(B2:B7,C2:C7)

- e. What is the Sharpe ratio for the fund?

	A	B	C
12	Sharpe	0.07863	= $(B8-D8)/B10^{0.5}$

f. What is the Treynor ratio for the fund?

	A	B	C
13	Treynor	0.016132	= $(B8-D8)/B11$

g. What is Jensen's alpha for the fund?

	A	B	C
14	Alpha	-0.02329	= $B8-(D8+B11*(C8-D8))$

15. The following table shows rates of return for a mutual fund and the market portfolio, and the risk-free rate.

	A	B	C	D
1	Year	Fund	Market	Risk-free rate
2	1	14%	13%	5%
3	2	-25%	-14%	2%
4	3	-6%	-8%	1%
5	4	5%	28%	2%
6	5	14%	8%	4%
7	6	20%	6%	5%

a. What is the arithmetic average return for the risk-free rate?

	A	B	C	D	E
1	Year	Fund	Market	Risk-free rate	Formula
2	1	14%	13%	5%	
3	2	-25%	-14%	2%	
4	3	-6%	-8%	1%	
5	4	5%	28%	2%	
6	5	14%	8%	4%	
7	6	20%	6%	5%	
8	Average	0.03667	0.055	0.03167	=AVERAGE(D2:D7)

b. What is the covariance of returns?

	A	B	C
9	Covariance	0.01662	=COVARIANCE.S(B2:B7,C2:C7)

c. What is the variance of the market portfolio?

	A	B	C	D
10	Variance	0.02795	0.02263	=VAR.S(C2:C7)

d. What is the beta of the fund, using the industry model (not subtracting the risk-free rate)?

	A	B	C
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11	Beta	0.7344	=SLOPE(B2:B7,C2:C7)
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e. What is the Sharpe ratio for the fund?

	A	B	C
12	Sharpe	0.02991	=(B8-D8)/B10^0.5

f. What is the Treynor ratio for the fund?

	A	B	C
13	Treynor	0.006808	=(B8-D8)/B11

g. What is Jensen's alpha for the fund?

	A	B	C
14	Alpha	-0.012137	=B8-(D8+B11*(C8-D8))

h. What is the information ratio for the fund?

	A	B	C	D	E
1	Year	Fund	Market	Risk-free rate	Difference: B-C
2	1	14%	13%	5%	0.01
3	2	-25%	-14%	2%	-0.11
4	3	-6%	-8%	1%	0.02
5	4	5%	28%	2%	-0.23
6	5	14%	8%	4%	0.06
7	6	20%	6%	5%	0.14
8	Average	0.03667	0.055	0.03167	-0.018333
...					
15	Tracking error	0.13167	=STDEV.S(E2:E7)		
16	IR	-0.09217	=B14/B15		

i. What is the correlation coefficient for the returns on the fund and the market portfolio?

	A	B	C
17	Rho	0.6609	=CORREL(B2:B7,C2:C7)

j. What is the R^2 of the fund?

	A	B	C
18	R^2	0.4368	=B17^2

16. You've collected the following information about two stock portfolios, the market portfolio and Treasury bills:

	E(r)	Alpha	Beta	σ	$\sigma(e)$
Portfolio A	24%	13.6%	1.2	0.29	0.12002
Portfolio B	11%	2.7%	0.9	0.22	0.0959
Market portfolio	9%	0%	1	0.22	0
T-bills	2%	0%	0	0	0

a. If you had to decide to invest your entire money in either portfolio A or portfolio B, which one should you choose?

- ☐ Portfolio A, because of the higher information ratio
- ☒ Portfolio A, because of the higher Sharpe ratio
- ☐ Portfolio A, because of the higher Treynor measure
- ☐ Portfolio B, because of the higher information ratio
- ☐ Portfolio B, because of the higher Sharpe ratio
- ☐ Portfolio B, because of the higher Treynor measure

The Sharpe ratio is the appropriate performance measure when evaluating entire-wealth portfolios.

Sharpe ratio for portfolio A:

$$S_A = (r_A - r_f) / \sigma_A = (0.24 - 0.02) / 0.29 = 0.7586$$

Sharpe ratio for portfolio B:

$$S_B = (r_B - r_f) / \sigma_B = (0.11 - 0.02) / 0.22 = 0.4091$$

You should choose portfolio A, because of the higher Sharpe ratio.

b. If most of your money were invested in the market portfolio, which portfolio would you add?

- ☒ Portfolio A, because of the higher information ratio
- ☐ Portfolio A, because of the higher Sharpe ratio
- ☐ Portfolio A, because of the higher Treynor measure
- ☐ Portfolio B, because of the higher information ratio
- ☐ Portfolio B, because of the higher Sharpe ratio
- ☐ Portfolio B, because of the higher Treynor measure

The information ratio is the appropriate performance measure when evaluating which portfolio to add to the passive benchmark portfolio.

Information ratio for portfolio A:

$$\alpha_A / (\sigma(e_A)) = 0.136 / 0.12002 = 1.1332$$

Information ratio for portfolio B:

$$\alpha_B / (\sigma(e_B)) = 0.027 / 0.0959 = 0.2816$$

You should choose portfolio A, because of the higher information ratio.

17. Return statistics for AT&T, General motors, the S&P 500 and the risk-free asset, Treasury bills, are given below:

	A	B	C	D	E
1	Year	AT&T	GM	S&P 500	T-bills
2	2015	0.06	0.1	0.13	0.039
3	2016	-0.22	-0.03	-0.14	0.022

4	2017	-0.06	-0.17	-0.08	0.015
5	2018	0.05	0.08	0.28	0.023
6	2019	0.14	0.12	0.08	0.028
7	2020	0.2	0.15	0.04	0.031
8	2021	0.04	0.13	0.05	0.03
9	2022	0.02	0.05	0.07	0.032

- a. Calculate the excess returns for a portfolio composed of 90% AT&T and 10% GM. What is the arithmetic average excess return for the portfolio?

	A	B	C	D
...				
11	Portfolio			
12	Weights	0.9	0.1	
13				
14	Excess returns			
15	Year	Portfolio	S&P 500	
16	2015	0.025		=B\$12*B2+C\$12*C2-E2
17	2016	-0.223		
18	2017	-0.086		
19	2018	0.03		
20	2019	0.11		
21	2020	0.164		
22	2021	0.019		
23	2022	-0.009		
24				
25	Average	0.00375		=AVERAGE(B16:B23)

- b. Regress the excess return on the portfolio on the excess return on the S&P 500. What is the beta of the portfolio?

	A	B	C	D
...				
14	Excess returns			
15	Year	Portfolio	S&P 500	
16	2015	0.025	0.091	=D2-\$E2
17	2016	-0.223	-0.162	
18	2017	-0.086	-0.095	
19	2018	0.03	0.257	
20	2019	0.11	0.052	
21	2020	0.164	0.009	
22	2021	0.019	0.02	
23	2022	-0.009	0.038	
24				
25	Average	0.00375	0.02625	=AVERAGE(C16:C23)

26	Variance	0.013997	0.015531	=VAR.S(C16:C23)
27	St. Dev.	0.11831	0.12462	=STDEV.S(C16:C23)
28	Covar.	0.008675		=COVARIANCE.S(B16:B23,C16:C23)
29	Beta	0.5586		=SLOPE(B16:B23,C16:C23)

c. What is the R^2 of the regression?

	A	B	C	D
...				
14	Excess returns			
15	Year	Portfolio	S&P 500	
16	2015	0.025	0.091	=D2-\$E2
17	2016	-0.223	-0.162	
18	2017	-0.086	-0.095	
19	2018	0.03	0.257	
20	2019	0.11	0.052	
21	2020	0.164	0.009	
22	2021	0.019	0.02	
23	2022	-0.009	0.038	
24				
25	Average	0.00375	0.02625	=AVERAGE(C16:C23)
26	Variance	0.013997	0.015531	=VAR.S(C16:C23)
27	St. Dev.	0.11831	0.12462	=STDEV.S(C16:C23)
28	Covar.	0.008675		=COVARIANCE.S(B16:B23,C16:C23)
29	Beta	0.5586		=SLOPE(B16:B23,C16:C23)
30	Alpha	-0.010913		=INTERCEPT(B16:B23,C16:C23)
31	R^2	0.3462		=RSQ(B16:B23,C16:C23)

d. What is the Jensen's alpha of the portfolio?

	A	B	C	D
...				
25	Average	0.00375	0.02625	=AVERAGE(C16:C23)
26	Variance	0.013997	0.015531	=VAR.S(C16:C23)
27	St. Dev.	0.11831	0.12462	=STDEV.S(C16:C23)
28	Covar.	0.008675		=COVARIANCE.S(B16:B23,C16:C23)
29	Beta	0.5586		=SLOPE(B16:B23,C16:C23)
30	Alpha	-0.010913		=INTERCEPT(B16:B23,C16:C23)
31	R^2	0.3462		=RSQ(B16:B23,C16:C23)
32	J.'s alpha	-0.010913		=B25-B29*C25

e. What is the portfolio's Sharpe ratio?

	A	B	C	D
...				

25	Average	0.00375	0.02625	=AVERAGE(C16:C23)
26	Variance	0.013997	0.015531	=VAR.S(C16:C23)
27	St. Dev.	0.11831	0.12462	=STDEV.S(C16:C23)
28	Covar.	0.008675		=COVARIANCE.S(B16:B23,C16:C23)
29	Beta	0.5586		=SLOPE(B16:B23,C16:C23)
30	Alpha	-0.010913		=INTERCEPT(B16:B23,C16:C23)
31	R^2	0.3462		=RSQ(B16:B23,C16:C23)
32	J's alpha	-0.010913		=B25-B29*C23
33	Sharpe	0.0317		=B25/B27

f. What is the portfolio's Treynor ratio?

	A	B	C	D
...				
25	Average	0.00375	0.02625	=AVERAGE(C16:C23)
26	Variance	0.013997	0.015531	=VAR.S(C16:C23)
27	St. Dev.	0.11831	0.12462	=STDEV.S(C16:C23)
28	Covar.	0.008675		=COVARIANCE.S(B16:B23,C16:C23)
29	Beta	0.5586		=SLOPE(B16:B23,C16:C23)
30	Alpha	-0.010913		=INTERCEPT(B16:B23,C16:C23)
31	R^2	0.3462		=RSQ(B16:B23,C16:C23)
32	J's alpha	-0.010913		=B25-B29*C23
33	Sharpe	0.0317		=B25/B27
34	Treynor	0.006713		=B25/B29

18. a. Which of the following is true about market timing?

- ☐ It can be viewed as a put option on the market-index portfolio.
☐ It is a strategy in which the investor switches between cash and the risky portfolio based on its risk tolerance.
☒ The value of a perfect-timing ability can be viewed as equivalent to having a call option on the market-index portfolio.
☐ When the return of the risky portfolio is greater than the risk-free rate, the value of the perfect-timing strategy is equal to the risk-free rate.

Market timing is a strategy in which the investor switches between cash and the risky portfolio based on his forecasts of relative performance. The value of a perfect-timing ability can be viewed as equivalent to having a call option on the market-index portfolio.

19. Returns on large cap stocks have a historical standard deviation of 22%. Assume that the risk-free rate is 3% (continuously compounded).

a. What is a fair annual fee to a perfect annual market timer? Enter your answer as a decimal number or with the percentage sign.

Variables:

$$S_0 = 1$$

$$X = e^r = e^{0.03} = 1.0305$$

$$T = 1$$

$$r = 0.03$$

$$\begin{aligned} d_1 &= (\ln(S_0/X) + (r - \delta + \sigma^2/2)T) / (\sigma T^{1/2}) \\ &= (\ln(1/1.0305) + (0.03 - 0 + 0.22^2/2) * 1) / (0.22 * 1^{1/2}) \\ &= 0.11 \end{aligned}$$

$$\begin{aligned} d_2 &= d_1 - \sigma T^{0.5} \\ &= 0.11 - 0.22 * 1^{0.5} \\ &= -0.11 \end{aligned}$$

Using Excel's NORM.S.DIST(d_x , true) function, we find that

$$N(d_1) = N(0.11) = 0.5446$$

$$N(d_2) = N(-0.11) = 0.457$$

$$\begin{aligned} C_0 &= S_0 N(d_1) - X e^{-rT} N(d_2) \\ &= 1 * 0.5446 - 1.0305 e^{-0.03*1} * 0.457 \\ &= 0.08759 \end{aligned}$$

A fair fee would be 8.759% of assets under management, or **0.08759**.

20. a. The return of the benchmark portfolio against which we can compare the performance of the fund manager is called the _____.

- ☐ alpha
- ☒ bogey
- ☐ market-neutral return
- ☐ risk-adjusted return
- ☐ tuple

The bogey is the return of the benchmark portfolio against which we can compare the performance of the fund manager.

21. You want to evaluate the performance of an actively managed portfolio and have gathered the following data:

	Benchmark weights	Benchmark return (%)	Portfolio weights	Portfolio return (%)
Stocks	0.53	7.4	0.61	9.6
Bonds	0.3	3.8	0.25	2.1
Cash	0.17	0.7	0.14	0.8

- a. What was the excess return of the active portfolio compared to the benchmark (in % without the percentage sign)?

Return on active portfolio:

$$\begin{aligned} r_P &= 0.61 * 0.096 + 0.25 * 0.021 + 0.14 * 0.008 \\ &= 0.06493 \end{aligned}$$

Return on benchmark portfolio:

$$\begin{aligned} r_B &= 0.53 * 0.074 + 0.3 * 0.038 + 0.17 * 0.007 \\ &= 0.05181 \end{aligned}$$

Excess return on active portfolio:

$$r_P - r_B = 0.06493 - 0.05181 = 0.01312 = \mathbf{1.312\%}$$

- b. What rate of return was contributed by asset allocation in the active portfolio (in % without the percentage sign)?

	1)	2)	3)	4)	5)=3)-1)	6)=5)*2)
	Benchmark weights	Benchmark return (%)	Portfolio weights	Portfolio return (%)	Excess weight	Contrib. to perfor. (%)
Stocks	0.53	7.4	0.61	9.6	0.08	0.592
Bonds	0.3	3.8	0.25	2.1	-0.05	-0.19
Cash	0.17	0.7	0.14	0.8	-0.03	-0.021
Total						0.381

c. What rate of return was contributed by security selection in the active portfolio (in % without the percentage sign)?

	1)	2)	3)	4)	5)=4)-2)	6)=5)*3)
	Benchmark weights	Benchmark return (%)	Portfolio weights	Portfolio return (%)	Excess perfor. (%)	Contrib. to perfor. (%)
Stocks	0.53	7.4	0.61	9.6	2.2	1.342
Bonds	0.3	3.8	0.25	2.1	-1.7	-0.425
Cash	0.17	0.7	0.14	0.8	0.1	0.014
Total						0.931

22. You've recorded the following prices (end of quarter), dividend payments (end of quarter) and trades for a stock:

	A	B	C	D	E
1	Quarter	Stock price	Div. per share	Shares held (before div. payment)	Shares bought or sold (after div. payment)
2	0	55.71		0	3
3	1	68.09	2.4	3	1
4	2	56.34	2.4	4	0
5	3	74.52	2.4	4	-2
6	4	77.23	2.4	2	4
7	5	61.1	2.65	6	4
8	6	54.15	2.65	10	2
9	7	56.46	2.65	12	2
10	8	69.58	2.65	14	-14

a. What was the arithmetic average quarterly rate of return?

	A	B	C	F	G
1	Quarter	Stock price	Div. per share	Return	
2	0	55.71			
3	1	68.09	2.4	0.2653	=(B3+C3)/B2-1
4	2	56.34	2.4	-0.13732	
5	3	74.52	2.4	0.3653	
6	4	77.23	2.4	0.06857	
7	5	61.1	2.65	-0.17454	

8	6	54.15	2.65	-0.07038	
9	7	56.46	2.65	0.0916	
10	8	69.58	2.65	0.2793	
11	Avg.			0.08598	=AVERAGE(F3:F10)

b. What was the geometric average quarterly rate of return?

	A	B	C	F	G	H
1	Quarter	Stock price	Div. per share	Return	1+return	
2	0	55.71				
3	1	68.09	2.4	0.2653	1.2653	=1+F3
4	2	56.34	2.4	-0.13732	0.8627	
5	3	74.52	2.4	0.3653	1.3653	
6	4	77.23	2.4	0.06857	1.0686	
7	5	61.1	2.65	-0.17454	0.8255	
8	6	54.15	2.65	-0.07038	0.9296	
9	7	56.46	2.65	0.0916	1.0916	
10	8	69.58	2.65	0.2793	1.2793	
11	Avg.			0.08598		=AVERAGE(F3:F10)
12	Geom.				0.06909	=GEOMEAN(G3:G10)-1

c. What was the dollar-weighted quarterly rate of return?

	A	B	C	D	E	H	I
1	Quarter	Stock price	Div. per share	Shares held (before div. payment)	Shares bought or sold (after div. payment)	Cash flow	
2	0	55.71		0	3	-167.13	=C2*D2-B2*E2
3	1	68.09	2.4	3	1	-60.89	
4	2	56.34	2.4	4	0	9.6	
5	3	74.52	2.4	4	-2	158.64	
6	4	77.23	2.4	2	4	-304.12	
7	5	61.1	2.65	6	4	-228.5	
8	6	54.15	2.65	10	2	-81.8	
9	7	56.46	2.65	12	2	-81.12	
10	8	69.58	2.65	14	-14	1,011.25	
...							
13	IRR					0.071	=IRR(H2:H10)

23. Return statistics for Apple stock, the S&P 500 and the risk-free asset, Treasury bills, are given below:

	A	B	C	D
--	---	---	---	---

1	Year	Apple	S&P 500	T-bill
2	2015	0.06	0.13	0.035
3	2016	-0.26	-0.14	0.022
4	2017	-0.06	-0.08	0.015
5	2018	0.05	0.28	0.017
6	2019	0.14	0.08	0.028
7	2020	0.18	0.03	0.031
8	2021	0.04	0.05	0.035
9	2022	0.02	0.11	0.032

- a. Calculate the excess returns for the stock and the S&P 500. What is the arithmetic average excess return for the S&P 500?

	A	B	C	D
...				
11	Excess returns			
12	Year	Apple	S&P 500	
13	2015	0.025	0.095	=C2-\$D2
14	2016	-0.282	-0.162	
15	2017	-0.075	-0.095	
16	2018	0.033	0.263	
17	2019	0.112	0.052	
18	2020	0.149	-0.001	
19	2021	0.005	0.015	
20	2022	-0.012	0.078	
21				
22	Average	-0.005625	0.03063	=AVERAGE(C13:C20)

- b. Regress the excess return on the stock on the excess return on the S&P 500. What is the R^2 of the regression?

	A	B	C	D
...				
11	Excess returns			
12	Year	Apple	S&P 500	
13	2015	0.025	0.095	=C2-\$D2
14	2016	-0.282	-0.162	
15	2017	-0.075	-0.095	
16	2018	0.033	0.263	
17	2019	0.112	0.052	
18	2020	0.149	-0.001	
19	2021	0.005	0.015	
20	2022	-0.012	0.078	
21				
22	Average	-0.005625	0.03063	=AVERAGE(C13:C20)

23	Variance	0.017361	0.016425	=VAR.S(C13:C20)
24	St. Dev.	0.13176	0.12816	=STDEV.S(C13:C20)
25	Covar.	0.010008		=COVARIANCE.S(B13:B20,C13:C20)
26	Beta	0.6093		=SLOPE(B13:B20,C13:C20)
27	Alpha	-0.02429		=INTERCEPT(B13:B20,C13:C20)
28	R^2	0.3513		=RSQ(B13:B20,C13:C20)

c. What is the stock's Jensen's alpha?

	A	B	C	D
...				
22	Average	-0.005625	0.03063	=AVERAGE(C13:C20)
23	Variance	0.017361	0.016425	=VAR.S(C13:C20)
24	St. Dev.	0.13176	0.12816	=STDEV.S(C13:C20)
25	Covar.	0.010008		=COVARIANCE.S(B13:B20,C13:C20)
26	Beta	0.6093		=SLOPE(B13:B20,C13:C20)
27	Alpha	-0.02429		=INTERCEPT(B13:B20,C13:C20)
28	R^2	0.3513		=RSQ(B13:B20,C13:C20)
29	J.'s alpha	-0.02429		=B22-B26*C22

d. What is the stock's Sharpe ratio?

	A	B	C	D
...				
22	Average	-0.005625	0.03063	=AVERAGE(C13:C20)
23	Variance	0.017361	0.016425	=VAR.S(C13:C20)
24	St. Dev.	0.13176	0.12816	=STDEV.S(C13:C20)
25	Covar.	0.010008		=COVARIANCE.S(B13:B20,C13:C20)
26	Beta	0.6093		=SLOPE(B13:B20,C13:C20)
27	Alpha	-0.02429		=INTERCEPT(B13:B20,C13:C20)
28	R^2	0.3513		=RSQ(B13:B20,C13:C20)
29	J.'s alpha	-0.02429		=B22-B26*C22
30	Sharpe	-0.04269		=B22/B24

e. What is the stock's Treynor ratio?

	A	B	C	D
...				
22	Average	-0.005625	0.03063	=AVERAGE(C13:C20)
23	Variance	0.017361	0.016425	=VAR.S(C13:C20)
24	St. Dev.	0.13176	0.12816	=STDEV.S(C13:C20)
25	Covar.	0.010008		=COVARIANCE.S(B13:B20,C13:C20)
26	Beta	0.6093		=SLOPE(B13:B20,C13:C20)
27	Alpha	-0.02429		=INTERCEPT(B13:B20,C13:C20)
28	R^2	0.3513		=RSQ(B13:B20,C13:C20)

29	J's alpha	-0.02429		=B22-B26*C22
30	Sharpe	-0.04269		=B22/B24
31	Treynor	-0.009232		=B22/B26

24. Return statistics for Walmart stock, the S&P 500 and the risk-free asset, r_f , are given below:

	A	B	C	D
1	Year	WMT	S&P 500	r_f
2	2015	0.05	0.15	0.034
3	2016	-0.22	-0.14	0.022
4	2017	-0.06	-0.07	0.015
5	2018	0.05	0.28	0.022
6	2019	0.14	0.08	0.028
7	2020	0.05	0.03	0.031
8	2021	0.04	0.05	0.04
9	2022	0.02	0.11	0.032

- a. Calculate the excess returns for the stock and the S&P 500. What is the arithmetic average excess return for the S&P 500?

	A	B	C	D
...				
11	Excess returns			
12	Year	WMT	S&P 500	
13	2015	0.016	0.116	=C2-\$D2
14	2016	-0.242	-0.162	
15	2017	-0.075	-0.085	
16	2018	0.028	0.258	
17	2019	0.112	0.052	
18	2020	0.019	-0.001	
19	2021	0	0.01	
20	2022	-0.012	0.078	
21				
22	Average	-0.01925	0.03325	=AVERAGE(C13:C20)

- b. Regress the excess return on the stock on the excess return on the S&P 500. What is the R^2 of the regression?

	A	B	C	D
...				
11	Excess returns			
12	Year	WMT	S&P 500	
13	2015	0.016	0.116	=C2-\$D2
14	2016	-0.242	-0.162	
15	2017	-0.075	-0.085	
16	2018	0.028	0.258	

17	2019	0.112	0.052	
18	2020	0.019	-0.001	
19	2021	0	0.01	
20	2022	-0.012	0.078	
21				
22	Average	-0.01925	0.03325	=AVERAGE(C13:C20)
23	Variance	0.010759	0.016219	=VAR.S(C13:C20)
24	St. Dev.	0.10373	0.12735	=STDEV.S(C13:C20)
25	Covar.	0.009236		=COVARIANCE.S(B13:B20,C13:C20)
26	Beta	0.5694		=SLOPE(B13:B20,C13:C20)
27	Alpha	-0.03818		=INTERCEPT(B13:B20,C13:C20)
28	R^2	0.4888		=RSQ(B13:B20,C13:C20)

c. What is the stock's Jensen's alpha?

	A	B	C	D
...				
22	Average	-0.01925	0.03325	=AVERAGE(C13:C20)
23	Variance	0.010759	0.016219	=VAR.S(C13:C20)
24	St. Dev.	0.10373	0.12735	=STDEV.S(C13:C20)
25	Covar.	0.009236		=COVARIANCE.S(B13:B20,C13:C20)
26	Beta	0.5694		=SLOPE(B13:B20,C13:C20)
27	Alpha	-0.03818		=INTERCEPT(B13:B20,C13:C20)
28	R^2	0.4888		=RSQ(B13:B20,C13:C20)
29	J.'s alpha	-0.03818		=B22-B26*C22

d. What is the stock's Sharpe ratio?

	A	B	C	D
...				
22	Average	-0.01925	0.03325	=AVERAGE(C13:C20)
23	Variance	0.010759	0.016219	=VAR.S(C13:C20)
24	St. Dev.	0.10373	0.12735	=STDEV.S(C13:C20)
25	Covar.	0.009236		=COVARIANCE.S(B13:B20,C13:C20)
26	Beta	0.5694		=SLOPE(B13:B20,C13:C20)
27	Alpha	-0.03818		=INTERCEPT(B13:B20,C13:C20)
28	R^2	0.4888		=RSQ(B13:B20,C13:C20)
29	J.'s alpha	-0.03818		=B22-B26*C22
30	Sharpe	-0.18559		=B22/B24

e. What is the stock's Treynor ratio?

	A	B	C	D
...				
22	Average	-0.01925	0.03325	=AVERAGE(C13:C20)

23	Variance	0.010759	0.016219	=VAR.S(C13:C20)
24	St. Dev.	0.10373	0.12735	=STDEV.S(C13:C20)
25	Covar.	0.009236		=COVARIANCE.S(B13:B20,C13:C20)
26	Beta	0.5694		=SLOPE(B13:B20,C13:C20)
27	Alpha	-0.03818		=INTERCEPT(B13:B20,C13:C20)
28	R^2	0.4888		=RSQ(B13:B20,C13:C20)
29	J's alpha	-0.03818		=B22-B26*C22
30	Sharpe	-0.18559		=B22/B24
31	Treynor	-0.03381		=B22/B26

25. The following table shows historical end-of-year prices for a stock and the S&P 500, as well as the risk-free rate, r_f .

	A	B	C	D
1	Year	Stock	S&P 500	r_f
2	2015	24.86	1,054.54	0.039
3	2016	21.13	969.78	0.022
4	2017	16.8	944.66	0.015
5	2018	21.4	1,187.85	0.025
6	2019	27.7	1,322.8	0.034
7	2020	37.67	1,387.49	0.035
8	2021	43.66	1,618.81	0.038
9	2022	52.49	1,816.83	0.038
10	2023	57.37	2,000.22	0.035

a. Calculate the annual returns. What was the average risk-free rate?

	A	B	C	D	E
...					
12	Returns				
13	Year	Stock	S&P 500	r_f	
14	2015	-0.15004	-0.08038	0.039	=C3/C2-1
15	2016	-0.2049	-0.0259	0.022	
16	2017	0.2738	0.2574	0.015	
17	2018	0.2944	0.11361	0.025	
18	2019	0.3599	0.0489	0.034	
19	2020	0.15901	0.16672	0.035	
20	2021	0.2022	0.12232	0.038	
21	2022	0.09297	0.10094	0.038	
22					
23	Average	0.12842	0.08796	0.03075	=AVERAGE(D13:D20)

b. Calculate the annual excess returns (returns over the risk-free rate). What was the covariance between the stock excess returns and the S&P 500 excess returns?

	A	B	C	D
--	---	---	---	---

...				
25	Excess returns			
26	Year	Stock	S&P 500	
27	2015	-0.18904	-0.11938	=C14-\$D14
28	2016	-0.2269	-0.0479	
29	2017	0.2588	0.2424	
30	2018	0.2694	0.08861	
31	2019	0.3259	0.014904	
32	2020	0.12401	0.13172	
33	2021	0.16424	0.08432	
34	2022	0.05497	0.06294	
35				
36	Average	0.09767	0.05721	=AVERAGE(C27:C34)
37	Variance	0.04315	0.012234	=VAR.S(C27:C34)
38	St. Dev.	0.2077	0.11061	=STDEV.S(C27:C34)
39	Covar.	0.016265		=COVARIANCE.S(B27:B34,C27:C34)

c. What was the stock's alpha from the regression of the stock's excess return on the S&P 500's excess return?

	A	B	C	D
...				
36	Average	0.09767	0.05721	=AVERAGE(C27:C34)
37	Variance	0.04315	0.012234	=VAR.S(C27:C34)
38	St. Dev.	0.2077	0.11061	=STDEV.S(C27:C34)
39	Covar.	0.016265		=COVARIANCE.S(B27:B34,C27:C34)
40	Beta	1.3295		=SLOPE(B27:B34,\$C27:\$C34)
41	Alpha	0.02162		=INTERCEPT(B27:B34,\$C27:\$C34)

d. What was the stock's Jensen's alpha?

	A	B	C	D
...				
36	Average	0.09767	0.05721	=AVERAGE(C27:C34)
37	Variance	0.04315	0.012234	=VAR.S(C27:C34)
38	St. Dev.	0.2077	0.11061	=STDEV.S(C27:C34)
39	Covar.	0.016265		=COVARIANCE.S(B27:B34,C27:C34)
40	Beta	1.3295	1	=SLOPE(C27:C34,\$C27:\$C34)
41	Alpha	0.02162	0	=INTERCEPT(C27:C34,\$C27:\$C34)
42	J.'s alpha	0.02162	0	=C36-C40*\$C36

Jensen's alpha is the alpha (vertical intercept) of the regression.

e. What was the stock's Sharpe ratio?

	A	B	C	D
...				

36	Average	0.09767	0.05721	=AVERAGE(C27:C34)
37	Variance	0.04315	0.012234	=VAR.S(C27:C34)
38	St. Dev.	0.2077	0.11061	=STDEV.S(C27:C34)
39	Covar.	0.016265		=COVARIANCE.S(B27:B34,C27:C34)
40	Beta	1.3295	1	=SLOPE(C27:C34,\$C27:\$C34)
41	Alpha	0.02162	0	=INTERCEPT(C27:C34,\$C27:\$C34)
42	J's alpha	0.02162	0	=C36-C40*\$C36
43	Sharpe ratio	0.4702		=B36/B38

f. What was the stock's Treynor ratio?

	A	B	C	D
...				
36	Average	0.09767	0.05721	=AVERAGE(C27:C34)
37	Variance	0.04315	0.012234	=VAR.S(C27:C34)
38	St. Dev.	0.2077	0.11061	=STDEV.S(C27:C34)
39	Covar.	0.016265		=COVARIANCE.S(B27:B34,C27:C34)
40	Beta	1.3295	1	=SLOPE(C27:C34,\$C27:\$C34)
41	Alpha	0.02162	0	=INTERCEPT(C27:C34,\$C27:\$C34)
42	J's alpha	0.02162	0	=C36-C40*\$C36
43	Sharpe ratio	0.4702		=B36/B38
44	Treynor ratio	0.07347		=B36/B40