Chapter 9 - The Capital Asset Pricing Model

Richard Herron

D'Amore-McKim School of Business, Northeastern University

The capital asset pricing model (CAPM) predicts equilibrium expected returns on risky assets

- Markowitz (1952) proposed modern portfolio theory
- 12 years later, Sharpe (1964), Lintner (1965), and Mossin (1966) proposed the CAPM
- The CAPM does not withstand empirical tests but remains central to modern finance because of its keys insight: *Only systematic risk is rewarded with a risk premium*

The CAPM is based on two sets of assumptions

1. Individual behavior

   a. Investors are rational, mean-variance optimizers
   b. Their common planning horizon is a single period
   c. Investors use identical input lists, an assumption often termed homogeneous expectations. which is consistent with the assumption that all relevant information is publicly available

2. Market structure

   a. All assets are publicly held and trade on public exchanges
   b. Investors can borrow or lend at a common risk-free rate, and they can take short positions on traded securities
   c. No taxes
   d. No trading costs

Suppose that all investors optimize their portfolios using the Markowitz model of efficient diversification

- Each investor uses an input list to draw an efficient frontier and identify a risky portfolio P where the CAL is tangent to the efficient frontier
- Under the CAPM, all investors would find the same efficient frontier, CAL, and risky portfolio P
- The proportion of each stock in P is the market value of each stock divided by the sum of the market values of all stocks
- Therefore
  - The CAL must be the CML
  - The market portfolio M is the optimal tangency portfolio on the efficient frontier
  - Passive investment is efficient

The CAPM predicts the risk premium of the market portfolio I

- Recall that each investor chooses a proportion y to allocate to the optimal portfolio M such that:

$$y = \frac{E(r_M) - r_f}{A\sigma_M^2}$$

- Net borrowing and lending across all investors must be zero
- Therefore, the average position in the risky portfolio is 100% and $\overline{y} = 1$
- Finally, the market risk premium is proportional to its risk and the representative investor's degree of risk aversion, so:

$$E(r_M) - r_f = E(R_M) = \overline{A}\sigma_M^2$$

where $\overline{A}$ is the representative investor's risk aversion

The CAPM predicts the expected return on an asset depends on its contribution to portfolio risk I

- All investors use the same input list, so they all hold the same optimal risky portfolio, and the market portfolio is the optimal risky portfolio
- Consider GE stock, whose contribution to the risk of the market portfolio from the border-multiplied covariance matrix is:

$$w_{GE}[w_1 \text{Cov}(R_1, R_{GE}) + w_2 \text{Cov}(R_2, R_{GE}) + \cdots + w_{GE} \text{Cov}(R_{GE}, R_{GE}) + \cdots + w_n \text{Cov}(R_n, R_{GE})]$$

- The term in the square brackets simplifies to:

$$\sum_{i=1}^{n} w_i \text{Cov}(R_i, R_{GE}) = \sum_{i=1}^{n} \text{Cov}(w_i R_i, R_{GE}) = \text{Cov}(\sum_{i=1}^{n} w_i R_i, R_{GE}) = \text{Cov}(R_M, R_{GE})$$

  because covariances are additive and $\sum_{i=1}^{n} w_i R_i = R_M$
- GE's contribution to the risk of the portfolio is:

$$w_{GE} \text{Cov}(R_M, R_{GE})$$

The CAPM predicts the expected return on an asset depends on its contribution to portfolio risk II

- GE's reward-to-risk ratio is:

$$\frac{w_{GE}E(R_{GE})}{w_{GE}\text{Cov}(R_{GE}, R_M)} = \frac{E(R_{GE})}{\text{Cov}(R_{GE}, R_M)}$$

- All assets must have the same reward-to-risk ratio:

$$\frac{E(R_{GE})}{\text{Cov}(R_{GE}, R_M)} = \frac{E(R_M)}{\text{Cov}(R_M, R_M)}$$

- So:

$$E(R_{GE}) = \frac{\text{Cov}(R_{GE}, R_M)}{\sigma_M^2}E(R_M)$$

which we typically write as:

$$E(r_{GE}) = r_f + \beta_{GE}[E(r_M) - r_f]$$

because $E(R) = E(r) - r_f$ and $\beta_{GE} = \frac{\text{Cov}(R_{GE}, R_M)}{\sigma_M^2}$

The CAPM predicts the expected return on an asset depends on its contribution to portfolio risk III

- So, asset risk premiums depend on:
  - r_f
  - β
  - $E(r_M) - r_f$
- *So, asset risk premiums do not depend on volatility!*
- Note that investors do not see risk premiums and must infer them from market prices and expected cash flows

The CAPM works for portfolios, too

- Portfolio beta β_P is the weighted sum of asset betas β_i:

$$\beta_P = \sum_{i=1}^{n} w_i \beta_i$$

- The market beta (β_M) is one by definition because:

$$\beta_M = \frac{\mathrm{Cov}(R_M, R_M)}{\sigma_M^2} = \frac{\sigma_M^2}{\sigma_M^2} = 1$$

- So we can think of $\beta > 1$ as aggressive and $\beta < 1$ as defensive

The security market line (SML) visualizes the CAPM

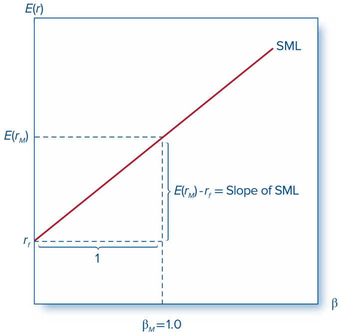

Figure 1: The security market line (BKM 2023, Figure 9.2)

The CML and SML are different concepts

- CML
  - The CML plots the risk premiums of efficient *portfolios* versus their standard deviations
  - Standard deviation is a valid measure of risk for diversified portfolios
- SML
  - The SML plots the risk premiums of *individual assets* versus their systematic risk
  - Asset contribution to portfolio systematic risk is the appropriate measure of risk for assets added to a diversified portfolio
  - *The SML is valid for efficient portfolios and individual assets*

Fairly-priced assets plot on the SML and provide expected returns that compensate for their risk

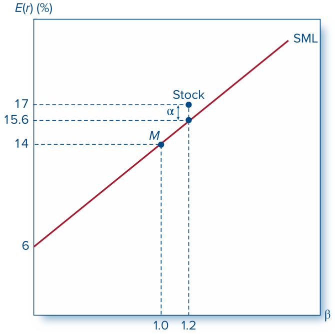

Figure 2: The SML and a positive-alpha stock. The risk-free rate is 6%, and the market's expected return is 14%, implying a market risk premium of 8% (BKM 2023, Figure 9.3)

a. Which of the following represent assumptions of the CAPM?

Check all that apply:

- [ ] All investors have the same holding period.
- [ ] Investments are limited to stocks and bonds.
- [ ] All investors trade without taxes or transaction costs.
- [ ] All investors have homogeneous expectations about expected returns, variances and covariances.

a. Which of the following statements is true about a stock's alpha?

Check all that apply:

- ☐ It represents the difference between a stock's fair expected return and the risk premium on the market portfolio.
- ☐ Fairly priced stocks have positive alphas.
- ☐ Fairly priced stocks have zero alphas.
- ☐ Overpriced stocks have positive alphas.
- ☐ It represents the difference between a stock's fair expected return and its actual expected return.

We know the following expected returns for stocks A and B, given different states of the economy:

State (s)	Probability	$E(r_{A,s})$	$E(r_{B,s})$
Recession	0.2	-0.01	0.01
Normal	0.5	0.14	0.04
Expansion	0.3	0.22	0.08

The expected return on the market portfolio is 0.07 and the risk-free rate is 0.02.

a. What is the standard deviation of returns for stock A?

b. What is the standard deviation of returns for stock B?

c. What is the beta for stock A?

d. What is the beta for stock B?

e. Which stock has more total risk?

- The stock with the lower standard deviation
- The stock with the higher standard deviation
- The stock with the lower beta
- The stock with the higher beta

f. Which stock has more systematic risk?

- The stock with the lower standard deviation
- The stock with the higher standard deviation
- The stock with the lower beta
- The stock with the higher beta

The CAPM is an elegant model built on a set of “uncomfortably restrictive” assumptions

- In this section, we will investigate the implications of relaxing some of these assumptions
- Throughout, we will maintain the fundamental distinction between systematic and diversifiable risk that is the heart of the CAPM

Identical input lists (assumption 1c)

- This assumption is not unreasonable
- When most information is public, it would not be surprising for investors to agree on firm prospects
- Also, wildly differing opinions may offset one another, so prices (and expected risk premiums) reflect consensus expectations
- There are two doubts about the conclusions of the identical input lists assumption
  1. Short-selling frictions limit checks on too-high prices; these short-selling frictions are:
     - Potentially unlimited liabilities on short positions, tying up collateral
     - A limited supply of stocks to be borrowed by would-be short sellers
     - Many investment companies and regulations prohibit short sales
  2. Taxes affect the after-tax returns on the same security

Risk-free borrowing and the zero-beta model (assumption 2b) I

- Restrictions on borrowing create different tangency portfolios for different investors, creating different optimal risky portfolios
- Merton (1972) and Roll (1977) show the following characteristics of efficient portfolios:
  1. Any combination of tangency portfolios is on the efficient frontier
  2. The market portfolio is an aggregation of efficient portfolios
  3. Every tangency portfolio has a companion zero-beta portfolio on the bottom half of the frontier, so we can rewrite the CAPM as $E(r_i) - E(r_z) = \beta_i[E(r_M) - E(r_z)]$, where $E(r_z)$ is the expected return on the zero-beta portfolio and $E(r_z) > r_f$
- Also, borrowing limits make it difficult to leverage up the tangency portfolio
  - So, risk-tolerant investors tilt their portfolios away from low-beta stocks and toward high-beta stocks
  - This tilt pushes prices up and risk premiums down for high-beta stocks, flattening the SML

Labor income and other nontraded assets (assumption 2a)

- Many assets are not tradeable (e.g., real estate, private businesses, and human capital)
- For example, employee compensation counted for 60% of national income for the 5 years ending 2021
- Therefore, the market portfolio may not be the optimal risky portfolio
- Further, the optimal risky portfolio should differ based on an investor's human capital

Multiperiod model and hedge portfolios

- Investors should be more concerned with the stream of consumption that wealth can buy than with the mean and variance of wealth this period (assumption 1a)
- This assumption rules out concerns about the correlation of asset returns with inflation or prices of important goods and services
- It also rules out demand for assets that could hedge these “extra-market risks” that would increase prices and decrease risk premiums
- Similar extra-market risk factors would arise in a multiperiod model instead of the single-period model (assumption 1b)
- Demand for these hedging assets would violate the CAPM
- The intertemporal CAPM (ICAPM) is a multiperiod model that addresses these concerns and models investors who continually adapt their consumption and investment decisions to changes in wealth, prices, and investments

Consumption-based CAPM (CCAPM)

- The CCAPM centers the CAPM on consumption so that the utility of an additional dollar of consumption today must equal the utility of the expected future consumption financed by investing that marginal dollar today
- Investors value additional income more in states of the world where consumption is low
- Therefore, equilibrium risk premiums will be greater for assets that exhibit higher covariance with consumption growth

Liquidity and the CAPM (assumption 2d)

- Financial costs are non-zero and inhibit trades
  - Under the CAPM, there is little reason to trade because all investors share the same information and demand the same portfolios of risk assets
  - In reality, investors trade because they have different expectations
- Liquidity of an asset is the ease and speed with which investors can sell it at the fair market value
- Illiquidity can be measured by the discount from the fair market value a seller must accept to quickly sell an asset
- Illiquidity can also be measured in part by the spread between the bid and ask prices

Firms with greater liquidity risk have higher average returns

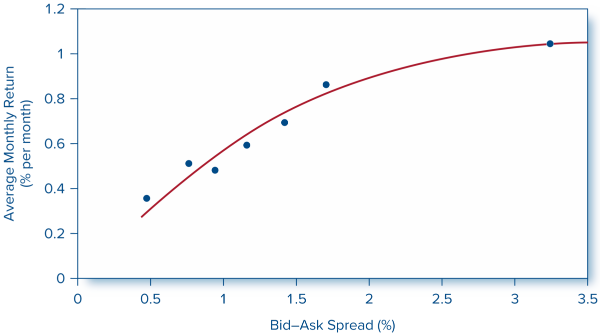

Figure 3: The relationship between illiquidity and average returns (BKM 2023, Figure 9.4)

Assume that the CAPM holds. One stock has an expected return of 7% and a beta of 0.5. Another stock has an expected return of 12% and a beta of 1.5.

a. What is the reward-to-risk ratio?

You've analyzed IBM's stock and expect it to deliver a return of 8% over the next year. The stock has a beta of 0.6. The risk-free rate is 2.5% and the expected market risk premium is 4.5%.

a. What is the security's expected alpha?

b. What is the security's expected alpha in equilibrium according to the CAPM?

Testing the CAPM is surprisingly difficult!

- We cannot observe all tradable assets, so we do not know the right market portfolio
- As a result, tests of the CAPM have a joint hypothesis problem
- If we test and reject the CAPM, do we reject it because it is the wrong model or because we have the wrong market portfolio (or both)?
- Alpha and beta, as well as residual variance, are likely time-varying
- Betas and market risk premiums may not vary *randomly* over time but in response to economic conditions (i.e., similar to the ICAPM and CCAPM)
- Regarding the ICAPM, we have not found the extra-market risk factors that hedge consumption
- *Bottom line: Some researchers consider the CAPM outdated, but it delivers important insights, and there is no clear replacement*

Portfolio theory and the CAPM are accepted tools among practitioners I

- Many professionals think about the distinction between systematic and firm-specific risk and are comfortable with beta as a measure of systematic risk
- In the asset management industry, alpha is regularly computed but rarely used to compensate portfolio managers
- We cannot prove that the market index is the most efficiently diversified portfolio
- But most investors do not beat the market index portfolio
- Average mutual fund alpha is negative most years, and funds with positive alphas one year have negative alphas the following year
- Since only a small fraction of professional managers beat the index over ten-year periods, the market index portfolio is efficient for all practical purposes and is:
  1. A diversification vehicle to mix with an active portfolio from security analysis
  2. A benchmark for performance evaluation
  3. A means to adjudicate lawsuits
  4. A means to determine proper prices in regulated industries, allowing shareholders to earn a fair rate of return, but no more

You've assembled the following portfolio:

Stock	Expected return	Beta	Portfolio weight
1	0.105	1.7	0.2
2	0.075	1.1	0.5
3	0.05	0.6	0.3

a. What is the beta of the portfolio?

b. What is the expected return of your portfolio?

c. If the risk-free rate is 2% and the CAPM holds, what is the expected return for the market portfolio?

You've assembled the following portfolio:

Stock	Beta	Portfolio weight
1	1.6	0.4
2	1.1	0.2
3	0.9	0.4

The expected market return is 7% and the risk-free rate is 2%. Assume that the CAPM holds.

a. What is the beta of the portfolio?

b. What is the expected return of your portfolio?

Summary and Key Equations I

1. The CAPM assumes that investors are single-period planners who agree on a common input list from security analysis and seek mean-variance optimal portfolios. Page 306
2. The CAPM assumes that security markets are ideal in the sense that:
   a. Relevant information about securities is widely and publicly available.
   b. There are no taxes or transaction costs.
   c. All risky assets are publicly traded.
   d. Investors can borrow and lend any amount at a fixed risk-free rate.
3. With these assumptions, all investors hold identical risky portfolios. The CAPM holds that in equilibrium the market portfolio is the unique mean-variance efficient tangency portfolio. Thus, a passive strategy is efficient.
4. The CAPM market portfolio is a value-weighted portfolio. Each security is held in a proportion equal to its market value divided by the total market value of all securities.
5. If the market portfolio is efficient and the average investor neither borrows nor lends, then the risk premium on the market portfolio is proportional to its variance, σ_M^2, as well as the average coefficient of risk aversion across investors, $\overline{A}$:

$$E(r_M) - r_f = \overline{A}\sigma_M^2$$

6. The CAPM implies that the risk premium on any individual asset or portfolio is the product of the risk premium on the market portfolio and the beta coefficient:

$$E(r_i) - r_f = \beta_i \left[E(r_M) - r_f\right]$$

   where the beta coefficient is the covariance of the asset's excess return with that of the market portfolio as a fraction of the variance of the return on the market portfolio:

$$\beta_i = \frac{\text{Cov}\left(r_i, r_M\right)}{\sigma_M^2}$$

Figure 4: Chapter 9 summary (1/2) from BKM (2023)

Summary and Key Equations II

7. When risk-free borrowing is restricted but all other CAPM assumptions hold, then the simple version of the the security market line is replaced by its zero-beta version. Accordingly, the risk-free rate in the expected return–beta relationship is replaced by the zero-beta portfolio's expected rate of return:

$$E(r_i) = E(r_Z) + \beta_i [E(r_M) - E(r_Z)]$$

8. The security market line of the CAPM must be modified to account for labor income and other significant nontraded assets.
9. The simple version of the CAPM assumes that investors have a single-period time horizon. When investors are assumed to be concerned with lifetime consumption and bequest plans, but investors' tastes and security return distributions are stable over time, the market portfolio remains efficient and the simple version of the expected return–beta relationship holds. But if those distributions change unpredictably, or if investors seek to hedge nonmarket sources of risk to their consumption, the simple CAPM gives way to a multifactor version in which the security's exposure to these nonmarket sources of risk command risk premiums.
10. The consumption-based capital asset pricing model (CCAPM) is a single-factor model in which the market portfolio excess return is replaced by that of a consumption-tracking portfolio. By appealing directly to consumption, the model naturally incorporates consumption-hedging considerations and changing investment opportunities within a single-factor framework.
11. Liquidity costs and liquidity risk can affect security pricing. Investors demand compensation for expected costs of illiquidity as well as the risk surrounding those costs.

Figure 5: Chapter 9 summary (2/2) from BKM (2023)

Summary and Key Equations III

Market risk premium: $E(R_M) = \overline{A}\sigma_M^2$

Beta: $\beta_i = \dfrac{\text{Cov}(R_i,\ R_M)}{\sigma_M^2}$

Security market line: $E(r_i) = r_f + \beta_i[E(r_M) - r_f]$

Zero-beta SML: $E(r_i) = E(r_Z) + \beta_i[E(r_M) - E(r_Z)]$

Multifactor SML (in excess returns): $E(R_i) = \beta_{iM}E(R_M) + \sum_{k=1}^{K} \beta_{ik}E(R_k)$

Figure 6: Chapter 9 key equations from BKM (2023)

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