Slides 9 - The Capital Asset Pricing Model

1. a. Which of the following represent assumptions of the CAPM?

   Check all that apply:

   - [x] All investors have the same holding period.
   - [ ] Investments are limited to stocks and bonds.
   - [x] All investors trade without taxes or transaction costs.
   - [x] All investors have homogeneous expectations about expected returns, variances and covariances.

   The basic CAPM is based on the following assumptions:

   1. All investors have the same one-period holding period.
   2. Possible investments include the whole universe of financial assets, e.g., stocks, bonds, and risk-free assets.
   3. Investors trade without transaction or taxation costs (i.e., all investors face a a tax rate of zero).
   4. All investors use the Markowitz portfolio selection model (they are rational mean-variance optimizers).
   5. All investors have homogeneous expectations - they share the same economic view of the world.
   6. Perfect competition: the financial market is composed of many small investors who are price-takers

2. a. According to the CAPM, the optimal risky portfolio ______.

   - (•) contains all assets in the economy
   - ( ) depends on risk aversion
   - ( ) has the lowest systematic risk
   - ( ) has the highest expected return

   The assumptions of the CAPM imply that all investors hold the same optimal risky portfolio. As a result, the optimal risky portfolio must be the market portfolio, i.e., the portfolio of all assets in the economy with weights equal to their market share.

3. a. Stock A's beta is 1.8 and Stock B's beta is 0.9. If the CAPM holds, which of the following statements is correct?

   - ( ) Held as an individual security, stock A is riskier than B.
   - ( ) Stock B is a better addition to a portfolio than A.
   - (•) The expected return on stock A is greater than that on B.
   - ( ) The expected return on stock B is greater than that on A.

   According to the CAPM, expected return depends only on one characteristic of a stock, its beta. Stocks with higher beta have more systematic risk: to compensate investors for the higher risk, they must offer a higher expected return.

4. a. The graphical representation of the CAPM is called the ______.

   - ( ) CAPM line
   - (•) security market line
   - ( ) security characteristic line
   - ( ) yield curve

   The security market line (SML) shows the relationship between a security's beta and its expected return, as does the CAPM.

5. a. Which of the following statements is true about a stock's alpha?

   Check all that apply:

   - [ ] It represents the difference between a stock's fair expected return and the risk premium on the market portfolio.
   - [ ] Fairly priced stocks have positive alphas.
   - [x] Fairly priced stocks have zero alphas.
   - [ ] Overpriced stocks have positive alphas.
   - [x] It represents the difference between a stock's fair expected return and its actual expected return.

   Alpha represents the difference between a stock's fair expected return and its actual expected return. Fairly priced stocks have zero alphas.

6. Use the expected return-beta equation from the CAPM.

   a. What is the expected return for a stock if the risk-free rate is 3%, beta 0.5 and the expected return for the market portfolio is 6%?

   $E(r) = r_f + \beta (E(r_M) - r_f)$

   $= 0.03 + 0.5 (0.06 - 0.03)$

   = **0.045**

   b. What is the risk-free rate if beta is 1.1, the expected return 6.3% and the expected return for the market portfolio is 6%?

   $E(r) = r_f + \beta (E(r_M) - r_f)$

   $\Leftrightarrow r_f = (E(r) - \beta E(r_M)) / (1-\beta) = (0.063 - 1.1 * 0.06) / (1-1.1) =$ **0.03**

   c. What is beta if the risk-free rate is 3%, the expected return 12% and the expected return for the market is 6%?

   $E(r) = r_f + \beta (E(r_M) - r_f)$

   $\Leftrightarrow \beta = (E(r) - r_f) / (E(r_M) - r_f) = (0.12 - 0.03) / (0.06 - 0.03) =$ **3**

   d. What is the expected return for the market if the risk-free rate is 3%, beta 0.5 and the expected return 12%?

   $E(r) = r_f + \beta (E(r_M) - r_f)$

   $\Leftrightarrow E(r_M) = (E(r) - r_f) / \beta + r_f = (0.12 - 0.03) / 0.5 + 0.03 =$ **0.21**

7. We know the following expected returns for stocks A and B, given different states of the economy:

State (s)	Probability	$E(r_{A,s})$	$E(r_{B,s})$
Recession	0.2	-0.01	0.01
Normal	0.5	0.14	0.04
Expansion	0.3	0.22	0.08

The expected return on the market portfolio is 0.07 and the risk-free rate is 0.02.

   a. What is the standard deviation of returns for stock A?

   Expected return:

   $E(r_A) = \sum_s (p_s E(r_{(A,s)}))$

= `0.2 * (-0.01) + 0.5 * 0.14 + 0.3 * 0.22`

= `0.134`

Variance:

`σ^2 = sum_s (p_s (r_s - E(r_A))^2)`

= `0.2 * (-0.01-0.134)^2 + 0.5 * (0.14-0.134)^2 + 0.3 * (0.22-0.134)^2`

= `0.006384`

Standard deviation:
$\sigma = \sqrt{\sigma^2}$
$= 0.006384^{1/2}$
= **0.0799**

b. What is the standard deviation of returns for stock B?

Expected return:

`E(r_B) = sum_s (p_sE(r_(B,s)))`

= `0.2 * 0.01 + 0.5 * 0.04 + 0.3 * 0.08`

= 0.046

Variance:
`σ^2 = sum_s (p_s (r_s - E(r_B))^2)`

= `0.2 * (0.01-0.046)^2 + 0.5 * (0.04-0.046)^2 + 0.3 * (0.08-0.046)^2`

= `0.000624`

Standard deviation:
$\sigma = \sqrt{\sigma^2}$
$= 0.000624^{1/2}$
= **0.02498**

c. What is the beta for stock A?

`E(r) = r_f + β (E(r_M) - r_f)`

⇔ `β = (E(r) - r_f) /(E(r_M) - r_f)= (0.134 - 0.02) /(0.07 - 0.02)=`**2.28**

d. What is the beta for stock B?

`E(r) = r_f + β (E(r_M) - r_f)`

⇔ `β = (E(r) - r_f) /(E(r_M) - r_f)= (0.046 - 0.02) /(0.07 - 0.02)=`**0.52**

e. Which stock has more total risk?

- ○ The stock with the lower standard deviation
- ◉ The stock with the higher standard deviation
- ○ The stock with the lower beta
- ○ The stock with the higher beta

The stock with the higher standard deviation has more total risk.

f. Which stock has more systematic risk?

- ○ The stock with the lower standard deviation

- ○ The stock with the higher standard deviation
- ○ The stock with the lower beta
- ◉ The stock with the higher beta

The stock with the higher beta has more systematic risk.

8. A stock has a beta of 1.1. The market risk premium is 3% and the risk-free rate is 1%.

   a. What is the expected return for the stock according to the CAPM?

      Using the CAPM:

      $E(r) = r_f + \beta * MRP$
      $= 0.01 + 1.1 * 0.03$
      = **0.043**

9. Assume that the CAPM holds. One stock has an expected return of 7% and a beta of 0.5. Another stock has an expected return of 12% and a beta of 1.5.

   a. What is the reward-to-risk ratio?

      If the CAPM holds, all assets have the same reward-to-risk ratio, i.e., they all plot on the security market line (SML):

      $(E(r_A) - r_f) / \beta_A = (E(r_B) - r_f) / \beta_B$

      $\Leftrightarrow r_f (\beta_A - \beta_B) = E(r_B) \beta_A - E(r_A) \beta_B$

      $\Leftrightarrow r_f = (E(r_B) \beta_A - E(r_A) \beta_B) / (\beta_A - \beta_B) = (0.12 * 0.5 - 0.07 * 1.5) / (0.5 - 1.5) = 0.045$

      Reward-to-risk ratio:

      $S = (E(r_A) - r_f) / \beta_A = (0.07 - 0.045) / 0.5 =$ **0.05**

10. You've analyzed IBM's stock and expect it to deliver a return of 8% over the next year. The stock has a beta of 0.6. The risk-free rate is 2.5% and the expected market risk premium is 4.5%.

    a. What is the security's expected alpha?

       Note that the market risk premium of 4.5% is net of the risk-free rate, by definition of a risk premium:

       $E(r)_{\text{CAPM}} = r_f + \beta (E(r_M) - r_f)$
       $= 0.025 + 0.6 * 0.045$
       $= 0.052$

       $\alpha = E(r)_{\text{Security analysis}} - E(r)_{\text{CAPM}}$
       $= 0.08 - 0.052$
       = **0.028**

    b. What is the security's expected alpha in equilibrium according to the CAPM?

       The CAPM predicts that all expected alphas must be **0** in equilibrium.

       Any stock with a positive expected alpha is undervalued and will be quickly bought by investors, thus driving up its price to its equilibrium level and reducing its expected alpha to zero.

       Any stock with a negative expected alpha is overvalued and will be quickly sold by investors, thus driving down its price to its equilibrium level and eliminating its negative expected alpha.

11. We know the following expected returns for stock A and the market portfolio, given different states of the economy:

State (s)	Probability	$E(r_{A,s})$	$E(r_{M,s})$
Recession	0.3	-0.02	0.05
Normal	0.5	0.13	0.08
Expansion	0.2	0.21	0.12

The risk-free rate is 0.02.

a. Assuming the CAPM holds, what is the beta for stock A?

Expected return for the market portfolio:

`E(r_M) = sum_s p_s E(r_(M,s))`

= 0.3 * 0.05 + 0.5 * 0.08 + 0.2 * 0.12

= 0.079

Expected return for stock A:

`E(r_A) = sum_s p_s E(r_(A,s))`

= 0.3 * (-0.02) + 0.5 * 0.13 + 0.2 * 0.21

= 0.101

Beta for stock A:

`E(r) = r_f + β (E(r_M) - r_f)`

⇔ `β = (E(r) - r_f) /(E(r_M) - r_f)= (0.101 - 0.02) /(0.079 - 0.02)=`**1.3729**

12. Assume that the CAPM holds. One stock has an expected return of 10% and a beta of 0.4. Another stock has an expected return of 15% and a beta of 1.5.

a. What is the expected return on the market?

If the CAPM holds, all assets have the same reward-to-risk ratio:

`(E(r_A) - r_f) /β_A = (E(r_B) - r_f) /β_B`

⇔ `r_f  (β_A - β_B) = E(r_B) β_A - E(r_A) β_B`

⇔ `r_f = (E(r_B) β_A - E(r_A) β_B) /(β_A - β_B)= (0.15 * 0.4 - 0.1 * 1.5) / (0.4 - 1.5)= 0.08182`

Expected return on market:

`E(r_A) = r_f + β_A (E(r_M) - r_f)`

⇔ `E(r_M) = (E(r_A) - r_f)/β_A + r_f= (0.1 - 0.08182) / 0.4 + 0.08182=`**0.12727**

13. You've assembled the following portfolio:

Stock	Expected return	Beta	Portfolio weight
1	0.105	1.7	0.2
2	0.075	1.1	0.5
3	0.05	0.6	0.3

a. What is the beta of the portfolio?

`β_P = w_1 β_1 + w_2 β_2 + w_3 β_3`

= `0.2 * 1.7 + 0.5 * 1.1 + 0.3 * 0.6`

= **1.07**

b. What is the expected return of your portfolio?

`E(r_P) = w_1 E(r_1) + w_2 E(r_2) + w_3 E(r_3)`

= `0.2 * 0.105 + 0.5 * 0.075 + 0.3 * 0.05`

= **0.0735**

c. If the risk-free rate is 2% and the CAPM holds, what is the expected return for the market portfolio?

`E(r) = r_f + β (E(r_M) - r_f)`

⇔ `E(r_M) = (E(r) - r_f) /β + r_f= (0.0735 - 0.02) /1.07 + 0.02=`**0.07**

14. You've assembled the following portfolio:

Stock	Beta	Portfolio weight
1	1.6	0.4
2	1.1	0.2
3	0.9	0.4

The expected market return is 7% and the risk-free rate is 2%. Assume that the CAPM holds.

a. What is the beta of the portfolio?

Portfolio beta:
$\beta_P = w_1 \beta_1 + w_2 \beta_2 + w_3 \beta_3$
= 0.4 * 1.6 + 0.2 * 1.1 + 0.4 * 0.9
= **1.22**

b. What is the expected return of your portfolio?

Using the CAPM for stock 1:

`E(r) = r_f + β (E(r_M) - r_f)`

= `0.02 + 1.6 (0.07 - 0.02)`

= `0.1`

Stock	Expected return	Beta	Portfolio weight
1	0.1	1.6	0.4
2	0.075	1.1	0.2
3	0.065	0.9	0.4

Expected portfolio return:

$E(r_P) = w_1 E(r_1) + w_2 E(r_2) + w_3 E(r_3)$

= 0.4 * 0.1 + 0.2 * 0.075 + 0.4 * 0.065

= 0.081

Alternatively, using the CAPM with the portfolio beta:

`E(r_P) = r_f + β_P (E(r_M) - r_f)`

= `0.02 + 1.22 (0.07 - 0.02)`

= **0.081**

15. A stock has a beta of 1.8. The risk-free rate is 3%. Assume that the CAPM holds.

    a. What is the expected return for the stock if the expected return on the market is 11%?

       Using the CAPM:

       $E(r) = r_f + \beta (E(r_M) - r_f)$

       = $0.03 + 1.8 (0.11 - 0.03)$

       = **0.174**

    b. What is the expected return for the stock if the expected market risk premium is 11%?

       Since we are given the market risk *premium*, we must not subtract the risk-free rate:

       $E(r) = r_f + \beta (E(r_M) - r_f)$

       = $0.03 + 1.8 (0.11)$

       = **0.228**