Slides 15 - The Term Structure of Interest Rates

1. Suppose that you are an investment banker, and you just noticed that the price of a Treasury bond is lower than the amount at which the sum of its parts could be sold.

   a. Which of the following statements are true?

   Check all that apply:

   - [x] This situation creates opportunities for arbitrage.
   - [ ] There are no arbitrage opportunities in this situation.
   - [x] You can profit by buying the bond and stripping the bond into several zero-coupon securities and selling them.
   - [ ] You can profit by buying the stripped cash flows and reconstituting the bond.

   The situation in which the price of a bond is lower than the amount at which the sum of its parts could be sold creates opportunities for arbitrage. In this situation, you can profit by buying the bond and stripping the bond into several zero-coupon securities and selling them.

   b. Now suppose a situation in which you notice that the price a Treasury bond is higher than the amount at which the sum of its parts could be sold. Which of the following statements are true?

   Check all that apply:

   - [x] You can profit by buying the stripped cash flows and reconstituting (reassembling) the bond.
   - [ ] You can profit by buying the bond and stripping the bond into several zero-coupon securities and selling them.
   - [x] In this situation, the law of one price is violated.
   - [ ] There are no arbitrage opportunities in this situation.

   The situation in which the price of a bond is higher than the amount at which the sum of its parts could be sold creates opportunities for arbitrage. In this situation, the law of one price is violated and you can profit by buying the stripped cash flows and reconstituting (reassembling) the bond.

2. Treasury spot interest rates are as follows:

Maturity (years)	1	2	3	4
Spot rate	1.8%	2.8%	3.7%	4.5%

   a. What is the price of a Treasury note with 2 years to maturity, a $1,000 face value, and a coupon rate of 4.8%, paid annually (in $)?

   `P = (Coupon)/(1+r_1) + (Coupon + "Par value")/(1+r_2)^2`

   = `48/(1+0.018) + 1,048/(1+0.028)^2`

   = **1,038.84**

   b. What is the yield to maturity?

   We need to solve for `r` in the following equation:

   `P = (Coupon)/(1+r) + (Coupon + "Par value")/(1+r)^2`

   ⇔ `1,038.84 = 48/(1+r) + 1,048/(1+r)^2`

Using a financial calculator:

	N	I/Y	PV	PMT	FV
Inputs	2		-1,038.84	48	1,000
Compute		2.777			

Using Excel (do not enter the thousands separators):
= RATE(nper, pmt, pv, fv)
= RATE(2, 48, -1,038.84, 1,000)
= **0.02777**

3. a. What are determinants of market interest rates?

Check all that apply:

- [ ] Demand premium
- [x] Inflation premium
- [x] Real rate of interest
- [x] Risk premium

Nominal rate = real rate + inflation premium + risk premium

- Real rate of interest: The interest rate on a risk-free security in a world without inflation (not directly observable)
- Inflation premium: return component to compensate investors for the erosion of purchasing power due to inflation
- Risk premium: return component to compensate investors for various risks, such as default or limited liquidity

4. a. The term structure of interest rates refers to the relationship between _____.

- (•) a bond's time to maturity and its yield
- ( ) a bond's age since issue and its yield
- ( ) a bond's time to maturity and its coupon rate
- ( ) a bond's age since issue and its coupon rate

Typically, bonds with longer times to maturity have higher yields, giving rise to an upward-sloping yield curve. The yield curve is the graphical representation of the term structure of interest rates.

5. a. If term premiums are positive, _____.

- ( ) the spread between yields on long-term and short-term bonds is negative
- (•) the spread between yields on long-term and short-term bonds is positive
- ( ) the yield curve signals an upcoming recession
- ( ) long-term investors dominate the market
- ( ) liquidity premiums are negative

If term premiums are positive than the spread between the yields on long-term and short-term bonds is positive. Term premiums are generally positive.

6. a. The plot of yield that can be represented as a function of maturity for coupon bonds which are issued recently and are selling at or near par value is called the _____.

- (•) on-the-run yield curve

- ○ pure-yield curve
- ○ off-the-run yield curve
- ○ at-par yield curve
- ○ downward-sloping yield curve

The plot of yield that is represented as a function of maturity for coupon bonds which are issued recently and are selling at or near par value is called the on-the-run yield curve.

7. a. A yield curve where yields on longer-term bonds are always higher than yields on shorter-term bonds is called a/an ______ yield curve.

- ○ abnormal
- ○ humped
- ○ inverted
- ◉ normal

A normal yield curve is upward sloping, i.e., yields on longer-term bonds are higher than yields on shorter-term bonds. The reason we typically see normal yield curves is that bonds with longer time to maturity have more interest rate (or price) risk, thus having to compensate investors with higher yields.

8. a. Which of the following statements is true about the expectations hypothesis?

- ○ The theory assumes that long-term investors dominate the market.
- ◉ The theory states that liquidity premiums are zero, because the forward rate equals the market's expectation of future short term interest rates.
- ○ The theory states that liquidity premiums should be positive, because short term investors dominate the market.
- ○ The theory assumes that liquidity premiums should be negative, because forward rates will generally be lower than expected short term rates.

The expectations hypothesis assumes that the forward rate equals the market's expectation of future short-term interest rates. As a result, liquidity premiums are zero.

9. The yield on 12-month Treasury bills is 1% and the yield on 2-year Treasury notes is 1.9%.

a. What is the 1-year forward rate one year from now?

Based on the pure expectations theory, the expected rate of return from investing in two consecutive one-year investments must equal the return from a two-year investment:

`(1+y_1) (1+f_2) = (1+y_2)^2`

- `y_1`: annual yield on 1-year Treasury bills (0.01)
- `y_2`: annual yield on 2-year Treasury notes (0.019)
- `f_2`: forward rate for a 1-year investment 1 year from now

`f_2 = (1+y_2)^2/(1+y_1) - 1= (1.019)^2/(1.01) - 1=` **0.02808**

10. The yield on 2-year Treasury notes is 1.7% and the yield on 5-year Treasury notes is 2.5%.

a. What is the implied annual return on a 3-year security two years from now?

`(1+y_2)^2 (1+f_(2,3))^3 = (1+y_5)^5`

- `y_2`: annual yield on 2-year Treasury notes (0.017)
- `y_5`: annual yield on 5-year Treasury notes (0.025)

- `f_(2,3)`: annual forward rate for a 3-year investment 2 years from now

`f_(2,3) = [(1+y_5)^5/(1+y_2)^2]^(1/3) - 1= [(1+0.025)^5/(1+0.017)^2]^(1/3) - 1=`**0.03037**

11. a. The liquidity preference theory states that long-term bond rates are usually _____ than short-term bond rates because _____.

Check all that apply:

- [ ] lower; long-term investors dominate the market
- [ ] lower; short-term investors dominate the market
- [x] higher; long-term bonds are riskier
- [ ] higher; long-term bonds are less risky

The liquidity preference theory states that long-term bond rates are usually higher than short-term bond rates. One reason is that long-term bonds are riskier. Another reason is that short-term bonds are more liquid (more easily converted to cash) than long-term bonds, and thus preferred by investors.

12. The prices of some zero-coupon bonds with various maturities and face values of $1,000 are given in the table below. An investor wants to construct a 3-year maturity forward loan commencing in 2 years.

Maturity	Price
1	941
2	862
3	796
4	745
5	675

a. How many 5-year maturity zero-coupon bonds would the investor need to sell to make his initial cash flow equal to zero if we assume that today he buys one 2-year maturity zero-coupon bond?

Number of 5-year zero-coupon bonds that the investor would need to sell:
$N = P_2 / P_5$
= 862 / 675
= **1.277**

13. The Treasury yield curve shows the following yields to maturity for the next 5 years:

Maturity in years	1	2	3	4	5
YTM in %	1.1	2.1	2.7	3.2	3.9

a. What is the 1-year forward rate one year from now?

Based on the pure expectations theory, the expected rate of return from investing in two consecutive one-year investments must equal the return from a two-year investment:

`(1+y_1) (1+f_2) = (1+y_2)^2`

- `y_1`: annual yield on 1-year Treasury bills (0.011)
- `y_2`: annual yield on 2-year Treasury notes (0.021)
- `f_2`: forward rate for a 1-year investment 1 year from now

`f_2 = (1+y_2)^2/(1+y_1) - 1 = (1.021)^2/(1.011) - 1=` **0.0311**

b. What is the 1-year forward rate two years from now?

$f_3 = \frac{(1+y_3)^3}{(1+y_2)^2} - 1 = \frac{(1.027)^3}{(1.021^2)} - 1 =$ **0.03911**

c. What is the annual 2-year forward rate two years from now?

Based on the pure expectations theory, the expected rate of return from investing in two consecutive two-year notes must equal the return from a four-year investment:

$$(1+y_2)^2 (1+f_{(2,2)})^2 = (1+y_4)^4$$

- y_2: annual yield on 2-year Treasury notes
- y_4: annual yield on 4-year Treasury notes
- $f_{(2,2)}$: annual forward rate for a 2-year investment 2 years from now

$f_{(2,2)} = \left[\frac{(1+y_4)^4}{(1+y_2)^2}\right]^{(1/2)} - 1 = \left[\frac{(1+0.032)^4}{(1+0.021)^2}\right]^{(1/2)} - 1 =$ **0.04312**

d. What is the annual 3-year forward rate two years from now?

$$(1+y_2)^2 (1+f_{(2,3)})^3 = (1+y_5)^5$$

- y_2: annual yield on 2-year Treasury notes
- y_5: annual yield on 5-year Treasury notes
- $f_{(2,3)}$: annual forward rate for a 3-year investment 2 years from now

$f_{(2,3)} = \left[\frac{(1+y_5)^5}{(1+y_2)^2}\right]^{(1/3)} - 1 = \left[\frac{(1+0.039)^5}{(1+0.021)^2}\right]^{(1/3)} - 1 =$ **0.05118**

14. There are two bonds that have a face value of $100 and pay annual coupons. Their times to maturity (in years) are: $t_1 = 1$ and $t_2 = 2$. The coupon values are: $c_1 = 4\%$ and $c_2 = 6\%$. Their yields to maturity are: $y_1 = 7\%$ and $y_2 = 8\%$.

a. What is the spot rate for year one?

The present values of the cash flows from the bond must be equal, whether we discount them by the constant yield to maturity or use the appropriate spot rate for each cash flow.

Equality for the first bond:

$$\frac{C_1 + FV}{1 + y_1} = \frac{C_1 + FV}{1 + s_1}$$

$\Leftrightarrow s_1 = \frac{C_1 + FV}{C_1 + FV}(1 + y_1) - 1 = y_1 =$ **0.07**

b. What is the spot rate for year two?

Equality for the second bond:

$$\frac{C_2}{1 + y_2} + \frac{C_2 + FV}{(1 + y_2)^2} = \frac{C_2}{1 + s_1} + \frac{C_2 + FV}{(1 + s_2)^2}$$

$$\Leftrightarrow s_2 = \left(\frac{C_2 + FV}{\frac{C_2}{1 + y_2} + \frac{C_2 + FV}{(1 + y_2)^2} - \frac{C_2}{1 + s_1}}\right)^{(1/2)} - 1$$

$$= \left(\frac{6 + 100}{\frac{6}{1 + 0.08} + \frac{6 + 100}{(1 + 0.08)^2} - \frac{6}{1 + 0.07}}\right)^{(1/2)} - 1$$

= **0.08031**

15. There are three bonds that have a face value of $100 and pay annual coupons. Their times to maturity (in years) are: $t_1 = 1$, $t_2 = 2$ and $t_3 = 3$. The coupon values are: $c_1 = 4\%$, $c_2 = 6\%$ and $c_3 = 9\%$. Their yields to maturity are: $y_1 = 6\%$, $y_2 = 8\%$ and $y_3 = 9\%$.

a. What is the spot rate for year one?

The bond price is the present value of all cash flows. The cash flows can be discounted with the yield to maturity or the spot rates.

$$P = \frac{C}{1 + y} + \frac{C}{(1 + y)^2} + \ldots + \frac{C + FV}{(1 + y)^n}$$

$$P = \frac{C}{1 + s_1} + \frac{C}{(1 + s_2)^2} + \ldots + \frac{C + FV}{(1 + s_n)^n}$$

Both approaches must yield the same bond price:

$$\frac{C}{1 + y} + \frac{C}{(1 + y)^2} + \dots + \frac{C + FV}{(1 + y)^n} = \frac{C}{1 + s_1} + \frac{C}{(1 + s_2)^2} + \dots + \frac{C + FV}{(1 + s_n)^n}$$

Equality for the first bond:

$$\frac{C_1 + FV}{1 + y_1} = \frac{C_1 + FV}{1 + s_1}$$

⇔ $s_1 = \frac{C_1 + FV}{C_1 + FV}(1 + y_1) - 1 = y_1$ = **0.06**

b. What is the spot rate for year two?

Equality for the second bond:

$$\frac{C_2}{1 + y_2} + \frac{C_2 + FV}{(1 + y_2)^2} = \frac{C_2}{1 + s_1} + \frac{C_2 + FV}{(1 + s_2)^2}$$

⇔ $s_2 = \left(\frac{C_2 + FV}{\frac{C_2}{1 + y_2} + \frac{C_2 + FV}{(1 + y_2)^2} - \frac{C_2}{1 + s_1}}\right)^{1/2} - 1$

= $\left(\frac{6 + 100}{\frac{6}{1 + 0.08} + \frac{6 + 100}{(1 + 0.08)^2} - \frac{6}{1 + 0.06}}\right)^{1/2} - 1$

= **0.08062**

c. What is the spot rate for year three?

Equality for the third bond:

$$\frac{C_3}{1 + y_3} + \frac{C_3}{(1 + y_3)^2} + \frac{C_3 + FV}{(1 + y_3)^3} = \frac{C_3}{1 + s_1} + \frac{C_3}{(1 + s_2)^2} + \frac{C_3 + FV}{(1 + s_3)^3}$$

⇔ $s_3 = \left(\frac{C_3 + FV}{\frac{C_3}{1 + y_3} + \frac{C_3}{(1 + y_3)^2} + \frac{C_3 + FV}{(1 + y_3)^3} - \frac{C_3}{1 + s_1} - \frac{C_3}{(1 + s_2)^2}}\right)^{1/3} - 1$

= $\left(\frac{9 + 100}{\frac{9}{1 + 0.09} + \frac{9}{(1 + 0.09)^2} + \frac{9 + 100}{(1 + 0.09)^3} - \frac{9}{1 + 0.06} - \frac{9}{(1 + 0.08062)^2}}\right)^{1/3} - 1$

= **0.09158**