Chapter 15 - The Term Structure of Interest Rates

Richard Herron

D'Amore-McKim School of Business, Northeastern University

The yield curve plots yield to maturity as a function of time to maturity

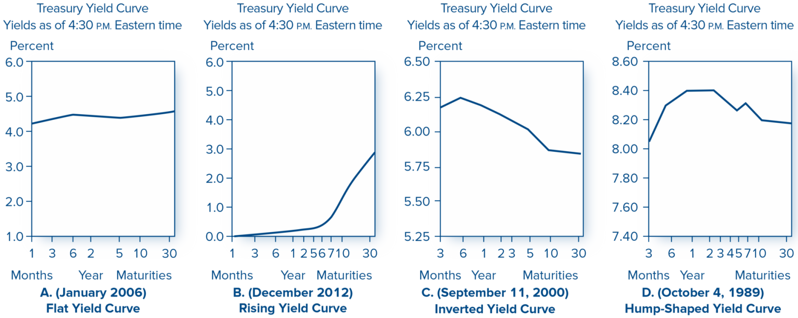

Figure 1: Treasury yield curves (BKM 2023, Figure 15.1)

The yields on different maturity bonds are not equal

- Consider each bond cash flow as a stand-alone zero-coupon bond
- The bond value should be the sum of the values of each bond cash flow
- Bond stripping and bond reconstitution offer arbitrage opportunities

In practice, there are two yield curves

- Pure yield curve: Based on stripped or zero-coupon Treasuries
- On-the-Run yield curve: Based on recently issued coupon bonds selling at or near par value
- These yield curves may differ significantly, and the financial press prefers the on-the-run yield curve because on-the-run Treasuries are more liquid than zero-coupon Treasuries

. Suppose that you are an investment banker, and you just noticed that the price of a Treasury bond is lower than the amount at which the sum of its parts could be sold.

a. Which of the following statements are true?

Check all that apply:

- [ ] This situation creates opportunities for arbitrage.
- [ ] There are no arbitrage opportunities in this situation.
- [ ] You can profit by buying the bond and stripping the bond into several zero-coupon securities and selling them.
- [ ] You can profit by buying the stripped cash flows and reconstituting the bond.

b. Now suppose a situation in which you notice that the price a Treasury bond is higher than the amount at which the sum of its parts could be sold. Which of the following statements are true?

Check all that apply:

- [ ] You can profit by buying the stripped cash flows and reconstituting (reassembling) the bond.
- [ ] You can profit by buying the bond and stripping the bond into several zero-coupon securities and selling them.
- [ ] In this situation, the law of one price is violated.
- [ ] There are no arbitrage opportunities in this situation.

Treasury spot interest rates are as follows:

Maturity (years)	1	2	3	4
Spot rate	1.8%	2.8%	3.7%	4.5%

a. What is the price of a Treasury note with 2 years to maturity, a $1,000 face value, and a coupon rate of 4.8%, paid annually (in $)?

b. What is the yield to maturity?

In a world without uncertainty, all investors know all future interest rates I

- Spot rate
  - The rate that prevails *today* for a period corresponding to the zero's maturity
  - For example, y_2 is the spot rate for the 2-year zero
- Short rate
  - The rate that applies for a given time interval
  - For example, r_1 and r_2 are the short rates that apply to years 1 and 2, respectively
- By the Law of One Price, the cumulative returns on competing bonds must be similar:

$$(1+y_2)^2 = (1+r_1) \times (1+r_2)$$

In a world without uncertainty, all investors know all future interest rates II

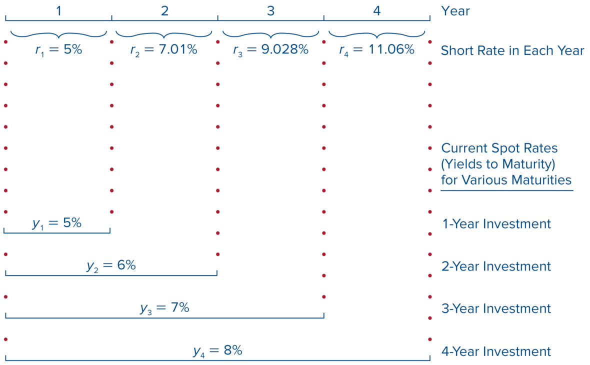

Figure 2: Short rates versus spot rates (BKM 2023, Figure 15.3)

We can generalize the relation between spot and short rates

For any maturity n, the relation between spot rates for years $n-1$ and n and short rates in year n is:

$$(1+y_n)^n = (1+y_{n-1})^{n-1} \times (1+r_n)$$

so:

$$1+r_n = \frac{(1+y_n)^2}{(1+y_{n-1})^{n-1}}$$

In our world with uncertainty, future short rates are uncertain

- The forward interest rate f_n is a forecast of a future short rate r_n, so:
$$1 + f_n = \frac{(1 + y_n)^2}{(1 + y_{n-1})^{n-1}}$$
- The forward rate is the break-even interest rate that equates the return on an n-period zero-coupon bond to that of an $(n-1)$-period zero-coupon bond rolling over into a 1-year bond in year n
- *When we get to year n, we may find $r_n \neq f_n$*

The yield on 12-month Treasury bills is 1% and the yield on 2-year Treasury notes is 1.9%.

a. What is the 1-year forward rate one year from now?

. The yield on 2-year Treasury notes is 1.7% and the yield on 5-year Treasury notes is 2.5%.

a. What is the implied annual return on a 3-year security two years from now?

The term structure is harder to interpret when future interest rates are uncertain

- A one-year investor requires a *liquidity premium* to buy the two-year bond instead of the one-year bond because she faces price risk when she sells the two-year bond in one year
- Conversely, a two-year investor requires a *liquidity premium* to buy the one-year bond instead of the two-year because she faces rate uncertainty when she rolls over from the first one-year bond to the second one-year bond

Expectations hypothesis theory: Forward rate equals the consensus expectation of the future short rate

- The simplest theory of the term structure
- The forward rate equals the market consensus expectation of future short interest rate, so $f_2 = E(r_2)$ and liquidity premiums are zero
- Therefore, the yields on long-term bonds depend only on expectations of future short rates, so:

$$(1 + y_2)^2 = (1 + r_1) \times [1 + E(r_n)]$$

  and an upward-sloping yield curve indicates that investors expect interest rates to increase

Liquidity preference theory: Short-term investors determine market prices, so the liquidity premium is mostly positive I

- Long-term bonds are more risky than short-term bonds for short-term investors because long-term bonds have short-term price risk
- Therefore, short-term investors require a positive *liquidity premium* to bear this price risk
- The liquidity premium is $f_2 - E(r_2)$, so liquidity preference theory predicts $f_2 > E(r_2)$ and an upward-sloping yield curve indicates the liquidity premium

Liquidity preference theory: Short-term investors determine market prices, so the liquidity premium is mostly positive II

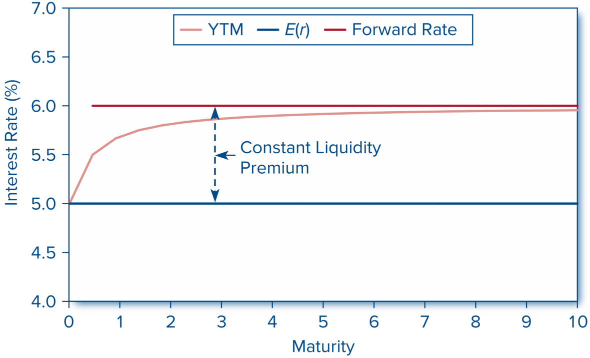

Figure 3: Constant expected short rate. Constant liquidity premium. Result: a rising yield curve. (BKM 2023, Figure 15.4 Panel A)

Liquidity preference theory: Short-term investors determine market prices, so the liquidity premium is mostly positive III

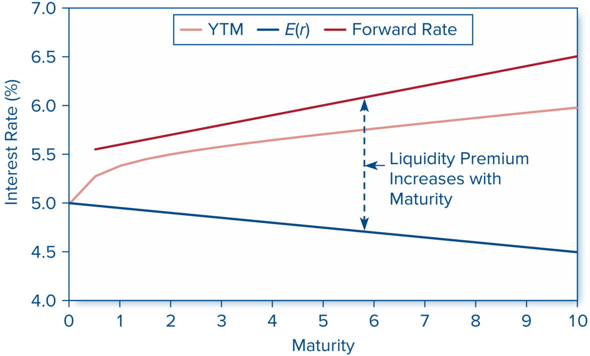

Figure 4: Declining expected short rates. Increasing liquidity premium. Result: a rising yield curve despite falling expected interest rates. (BKM 2023, Figure 15.4 Panel B)

Liquidity preference theory: Short-term investors determine market prices, so the liquidity premium is mostly positive IV

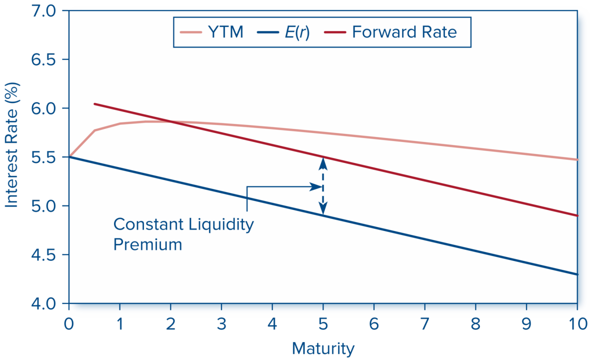

Figure 5: Declining expected short rates. Constant liquidity premium. Result: a hump-shaped yield curve. (BKM 2023, Figure 15.4 Panel C)

Liquidity preference theory: Short-term investors determine market prices, so the liquidity premium is mostly positive V

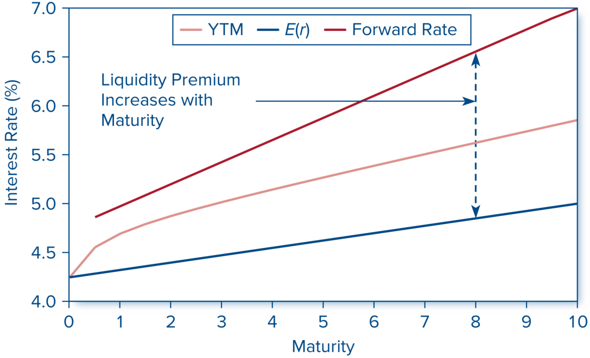

Figure 6: Increasing expected short rates. Increasing liquidity premiums. Result: a sharply rising yield curve. (BKM 2023, Figure 15.4 Panel D)

Market segmentation theory: Long-maturity and short-maturity bonds trade in segmented markets

- Each segmented market finds its equilibrium independently
- These segmented markets determine the yield curve
- This theory is less common today
    - Investors must compare long and short rates as well as expectations of future rates before deciding where to invest
    - If the liquidity premium were too positive or too negative, investors would rush to the same segment

Yield curve reflects expectations of future short rates and other factors, such as liquidity premiums I

- When future rates are uncertain:

$$1 + y_n = \left[(1 + r_1)(1 + f_2)(1 + f_3) \cdots (1 + f_n)\right]^{1/n}$$

  so $f_{n+1} > y_n$ for an upward-sloping yield curve
- An upward-sloping yield curve has two interpretations:
  - Investors expect rates to rise (i.e., $E(r_n)$ is high) ***and/or***
  - Investors require large liquidity premiums to hold long-term bonds (i.e., $f_n = E(r_n) + \underbrace{\text{Liquidity premium}}_{>0}$)
- The yield curve is a good predictor of the business cycle
  - An upward-sloping yield curve may indicate expansion when long-term rates tend to rise
  - An inverted yield curve may indicate recession when interest rates tend to fall

Yield curve reflects expectations of future short rates and other factors, such as liquidity premiums II

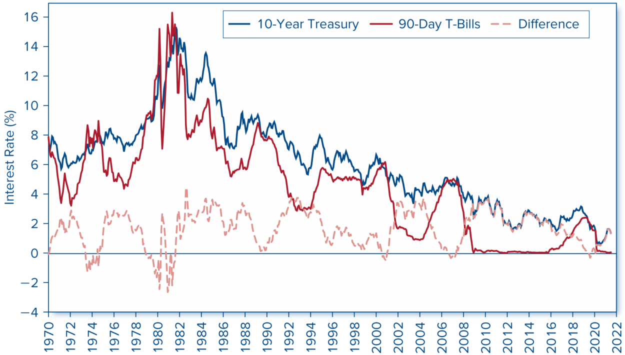

Figure 7: Term spread: Yields on 10-year versus 90-day Treasury securities (BKM 2023, Figure 15.6)

We can derive forward rates from the yield curve

- However:
  - Forward rates will not equal the eventually realized short rate
  - Nor are forward rates even today's expectation of what the short rate will be
- Still, the forward rate is a market rate and an important consideration when making decisions, such as locking in loan rates

. The Treasury yield curve shows the following yields to maturity for the next 5 years:

Maturity in years	1	2	3	4	5
YTM in %	1.1	2.1	2.7	3.2	3.9

a. What is the 1-year forward rate one year from now?

b. What is the 1-year forward rate two years from now?

c. What is the annual 2-year forward rate two years from now?

d. What is the annual 3-year forward rate two years from now?

There are two bonds that have a face value of $100 and pay annual coupons. Their times to maturity (in years) are: $t_1 = 1$ and $t_2 = 2$. The coupon values are: $c_1 = 4\%$ and $c_2 = 6\%$. Their yields to maturity are: $y_1 = 7\%$ and $y_2 = 8\%$.

a. What is the spot rate for year one?

b. What is the spot rate for year two?

There are three bonds that have a face value of $100 and pay annual coupons. Their times to maturity (in years) are: $t_1 = 1$, $t_2 = 2$ and $t_3 = 3$. The coupon values are: $c_1 = 4\%$, $c_2 = 6\%$ and $c_3 = 9\%$. Their yields to maturity are: $y_1 = 6\%$, $y_2 = 8\%$ and $y_3 = 9\%$.

a. What is the spot rate for year one?

b. What is the spot rate for year two?

c. What is the spot rate for year three?

Summary and Key Equations I

Page 500

1. The term structure of interest rates refers to the interest rates for various terms to maturity embodied in the prices of default-free zero-coupon bonds.
2. In a world of certainty, all investments must provide equal total returns for any investment period. Short-term holding-period returns on all bonds would be equal in a risk-free economy; all returns would be equal to the rate available on short-term bonds. Similarly, total returns from rolling over short-term bonds over longer periods would equal the total return available from long-maturity bonds.
3. The forward rate of interest is the break-even future interest rate that would equate the total return from a rollover strategy to that of a longer-term zero-coupon bond. It is defined by the equation

$$(1 + y_{n-1})^{n-1}(1 + f_n) = (1 + y_n)^n$$

   where n is a given number of periods from today. This equation can be used to show that yields to maturity and forward rates are related by the equation

$$(1 + y_n)^n = (1 + r_1)(1 + f_2)(1 + f_3) \cdots (1 + f_n)$$

4. A common version of the expectations hypothesis holds that forward interest rates are unbiased estimates of expected future interest rates. However, there are good reasons to believe that forward rates differ from expected short rates because of a risk premium known as a *liquidity premium*. A positive liquidity premium can cause the yield curve to slope upward even if no increase in short rates is anticipated.
5. The existence of liquidity premiums complicates attempts to infer expected future interest rates from the yield curve. Such an inference would be made easier if we could assume the liquidity premium remained reasonably stable over time. However, both empirical and theoretical considerations cast doubt on the constancy of that premium.
6. Forward rates are market interest rates in the important sense that commitments to forward (i.e., deferred) borrowing or lending arrangements can be made at these rates.

Figure 8: Chapter 15 summary from BKM (2023)

Summary and Key Equations II

Forward rate of interest: $1 + f_n = \dfrac{(1 + y_n)^n}{(1 + y_{n-1})^{n-1}}$

Yield to maturity given sequence of forward rates: $1 + y_n = [(1 + r_1)(1 + f_2)(1 + f_3) \cdots (1 + f_n)]^{1/n}$

Liquidity premium = Forward rate − Expected short rate

Figure 9: Chapter 15 key equations from BKM (2023)

References I

Bodie, Zvi, Alex Kane, and Allan J. Marcus (2023). *Investments*. 13th ed. New York: McGraw Hill.