Slides 6 - Capital Allocation to Risky Assets

1. a. According to the mean-variance criterion, portfolio A is better than portfolio B for a risk-averse investor whenever _____.

   - ○ $E(r_A) \le E(r_B)$ and $\sigma_A \le \sigma_B$
   - ○ $E(r_A) \le E(r_B)$ and $\sigma_A \ge \sigma_B$
   - ◉ $E(r_A) \ge E(r_B)$ and $\sigma_A \le \sigma_B$
   - ○ $E(r_A) \ge E(r_B)$ and $\sigma_A \ge \sigma_B$

   Portfolio A dominates portfolio B if A's expected return is higher and its standard deviation lower than B's, with at least one inequality holding strictly.

2. Infinity Capital fund has an expected return of 11% and a standard deviation of returns of 27%. T-bills have a return of 3%. A client has $8,000 to invest and needs to decide between investing in the Infinity Capital fund and T-bills.

   a. If the client's degree of risk aversion is A = 4, what proportion of his total investment should be invested in the Infinity Capital fund?

   `y =(E(r_P) - r_f) / (A * σ_P^2)= (0.11 - 0.03) / (4 * 0.27^2)=` **0.2743**

   b. What is the standard deviation for the complete portfolio?

   $\sigma_c = y * \sigma_p$
   $= 0.2743 * 0.27$
   = **0.07407**

   c. What is the reward-to-volatility ratio for the complete portfolio?

   Expected rate of return of complete portfolio:
   $E(r_c) = r_f + y * (E(r_p) - r_f)$
   $= 0.03 + 0.2743 * (0.11 - 0.03)$
   $= 0.05195$

   Reward-to-volatility (Sharpe) ratio:

   `S = (E(r_C)-r_f)/ σ_C= (0.05195-0.03) / 0.07407=` **0.2963**

3. A risky portfolio has an expected return of 16% and a standard deviation of 18%. Short-term T-bills have a return of 7%. An investor has $8,000 to invest and needs to decide between investing in the risky portfolio and T-bills. Suppose that the investor has the utility function $U = E(r_p) - (½ A \sigma^2)$ and a risk-aversion parameter of A = 1.8.

   a. What is the utility of the risky portfolio for the investor?

   $U = E(r_p) - (½ A \sigma^2)$
   $= 0.16 - (½ * 1.8 * 0.18^2)$
   = **0.13084**

   b. What is the utility of the T-bills for the investor?

   $U = E(r_p) - (½ A \sigma^2)$
   $= 0.07 - (½ * 1.8 * 0^2)$
   = **0.07**

4. a. What is a portfolio?

- ◯ A case for carrying pictures
- ◉ A group of assets
- ◯ A selection of work created over a period of time
- ◯ A set of pictures

A portfolio is a group of assets owned by an investor.

5. You've invested the following amounts into a three-stock portfolio:

Stock	Investment
1	$2,000
2	$12,000
3	$9,000

a. What is the portfolio weight for stock 1?

Total investment:
$I = I_1 + I_2 + I_3$
= 2,000 + 12,000 + 9,000
= 23,000

Portfolio weight for stock 1:

`w_1=I_1/I=2,000/23,000=` **0.08696**

b. What is the portfolio weight for stock 2?

Portfolio weight for stock 2:

`w_2=I_2/I=12,000/23,000=` **0.5217**

c. What is the portfolio weight for stock 3?

Portfolio weight for stock 3:

`w_3=I_3/I=9,000/23,000=` **0.3913**

You can verify that the portfolio weights sum to 1.

6. An investor is considering an investment in a risky portfolio. Based on his expectations, the end-of-year cash flow derived from the portfolio will be either $70,000 with a probability of 50% or $154,000 with the probability of 50%. T-bills have a yield of 5%. The investor requires a risk premium of 11%.

a. How much is the investor willing to pay for the portfolio, based on his required risk premium?

The expected cash flow at the end of the year:
E(CF) = p * 70,000 + (1-p) * 154,000
= 0.5 * 70,000 + 0.5 * 154,000
= 112,000

Required rate of return:
r = Risk-free rate + Risk premium
= 0.05 + 0.11
= 0.16

The investor is willing to pay:

`(E(CF)) / (1 +  r)= 112,000 / ( 1 + 0.16)=`**96,552**

7. a. The slope of the capital allocation line is called the _____.

- capital allocation slope
- capital allocation ratio
- Sharpe ratio
- variability-to-reward ratio

The slope of the capital allocation line is the Sharpe ratio, aka the reward-to-volatility ratio.

8. Suppose that you manage a fund with a standard deviation of 35% and an expected return of 21%. Your fund consists of the following investments:

Type	Proportion
Stock A	32%
Stock B	25%
Stock C	15%
Stock D	28%

The Treasury bill rate is 6%. An investor wants to invest a proportion of his investment budget in a T-bill money market fund and the remaining proportion in your fund.

a. What proportion of the investor's money should be invested in your fund in order to achieve an expected return of 12%?

With $E(r_C)$ = 12%:

`E(r_C) = y * E(r_P) + (1 - y) * r_f`

⇔ `y = (E(r_C) - r_f)/(E(r_P)-r_f)`

= `(0.12 - 0.06)/(0.21-0.06)`

= **0.4**

b. What proportion of the investor's complete portfolio will be invested in stock B?

`w_B^"*" = y * w_B`

= `0.4 * 0.25`

= **0.1**

9. You want to invest in either a stock or Treasury bills (the risk-free asset). The stock has an expected return of 7% and a standard deviation of returns of 46%. T-bills have a return of 1%.

a. If you invest 70% in the stock and 30% in T-bills, what is your expected return for the complete portfolio?

$E(r_C) = y\ E(r_S) + (1-y)\ r_f$
= 0.7 * 0.07 + 0.3 * 0.01
= **0.052**

b. What is the standard deviation of returns for such a portfolio?

$\sigma_C = y\ \sigma_S$
= 0.7 * 0.46
= **0.322**

10. You are managing a portfolio with a standard deviation of 40% and an expected return of 22%. The Treasury bill rate is 4%. A client wants to invest 16% of his investment budget in a T-bill money market fund and 84% in your portfolio.

    a. What is the expected rate of return on your client's complete portfolio?

    The proportion invested in your fund:

    $y = 0.84$

    Expected return:

    $E(r_c) = y E(r_p) + (1-y) r_f$

    = $0.84 * 0.22 + 0.16 * 0.04$

    = **0.1912**

    b. What is the standard deviation for your client's complete portfolio?

    Standard deviation:

    $\sigma_c=y*\sigma_p$

    = $0.84 * 0.4$

    = **0.336**

    c. What is the reward-to-volatility (Sharpe) ratio of your client's complete portfolio?

    Reward-to-volatility (Sharpe) ratio:

    $S_C = (E(r_C) - r_f) /\sigma_C$

    = $(0.1912 - 0.04) / 0.336$

    = **0.45**

    d. What is the Sharpe ratio of your portfolio?

    Reward-to-volatility ratio:

    $S_P = (E(r_P) - r_f) /\sigma_P$

    = $(0.22 - 0.04) / 0.4$

    = **0.45**

    Unsurprisingly, the Sharpe ratios are the same, since both portfolios are on the same capital allocation line.

11. Assume that there are only two stocks in the economy, stock A and stock B. The risk-free asset has a return of 3%. The optimal risky portfolio, i.e., the portfolio with the highest Sharpe ratio, is given below:

	A	B	C	D
1		Stock A	Stock B	Risk-free asset
2	Expected return	0.062	0.071	0.03
3	Variance	0.1296	0.0484	
4	Standard deviation	0.36	0.22	
5	Covariance	0.02376		
6				
7	**Optimal risky portfolio**			

8	Weights	0.11206	0.8879	=1-B8
9	Expected return	0.06999		=B8*B2+C8*C2
10	Variance	0.04452		=B8^2*B3+C8^2*C3+2*B8*C8*B5
11	Standard deviation	0.211		=B10^0.5
12	Sharpe ratio	0.18954		=(B9-D2)/B11

a. What is the expected return of a portfolio composed of 50% of the optimal risky portfolio and 50% of the risk-free asset?

	A	B	C	D
1		Stock A	Stock B	Risk-free asset
2	Expected return	0.062	0.071	0.03
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7	**Optimal risky portfolio**			
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11	Standard deviation	0.211		=B10^0.5
12	Sharpe ratio	0.18954		=(B9-D2)/B11
13				
14	**Complete portfolio**			
15	Weight on ORP	0.5		
16	Expected return	**0.05**		=B15*B9+(1-B15)*D2

b. What is the standard deviation of such a portfolio?

	A	B	C	D
1		Stock A	Stock B	Risk-free asset
2	Expected return	0.062	0.071	0.03
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14	**Complete portfolio**			
15	Weight on ORP	0.5		
16	Expected return	0.05		=B15*B9+(1-B15)*D2
17	Standard deviation	**0.10549**		=B15*B11

12. a. The capital market line is the line through the risk-free asset and the ______.

- ○ horizontal axis
- ○ vertical axis
- ◉ market portfolio
- ○ minimum variance portfolio

The capital market line is the line through the risk-free asset and the market portfolio of risky securities. The market portfolio is the portfolio of all risky assets and is the optimal risky portfolio for all investors according to the CAPM.

13. The risk-free rate is 1% and the expected return on the market portfolio is 10%.

a. What is the expected return of an optimal portfolio if you want to expose only 80% of your investment to risk?

Let y be the fraction invested in the market portfolio:

`E(r_P) = r_f + y (E(r_M) - r_f)`

= 0.01 + 0.8 (0.1 - 0.01)

= **0.082**

14. a. Which of the following statements are true about passive vs active investment strategies?

Check all that apply:

- ☑ An investment strategy that avoids any security analysis is called a passive strategy.
- ☐ The capital market line is the capital allocation line that represents an active investment strategy.
- ☑ A passive strategy is an investment in a stock fund that matches a broad market index and an investment in T-bills.
- ☐ A passive strategy is an investment in a portfolio with a reward-to-volatility ratio of 1.
- ☑ The capital market line represents the investment opportunity set of a passive investment strategy.

An investment strategy that avoids any security analysis is called a passive strategy. The passive strategy includes an investment in a stock fund that matches a broad market index and an investment in T-bills. The capital market line represents the investment opportunity set of a passive investment strategy.

15. An investor wants to invest money in Treasury bills and a risky fund managed by Infinity Capital. The investor wants to achieve an expected return of 7% on his complete portfolio. Infinity Capital has an expected return of 15% and a standard deviation of returns of 28%. T-bills have a return of 3%.

a. What proportion of his total investment should he invest in the risky fund in order to achieve the expected return?

Proportion of investment to be invested in the risky portfolio:
$E(r_c) = 0.07 = r_f + y * (E(r_p) - r_f)$
$\Leftrightarrow y = (E(r_c) - r_f) / (E(r_p) - r_f)$
$= (0.07 - 0.03) / (0.15 - 0.03)$

= **0.3333**

b. What is the standard deviation of the complete portfolio?

$\sigma_c = y * \sigma_p = 0.3333 * 0.28 =$ **0.09333**

16. Suppose that you manage a portfolio with a standard deviation of 49% and an expected return of 18%. An investor wants to invest a proportion of his investment budget in your fund. The remaining proportion of his investment budget will be invested in Treasury bills. The T-bill rate is 7%.

a. If the investor wants to maximize their expected return on their complete portfolio subject to the constraint that the complete portfolio's standard deviation will not exceed 18%, what fraction should they invest in your fund?

Standard deviation of complete portfolio:

`sigma_C = y* sigma_P`

⇔ `y = sigma_C/sigma_P = 0.18/0.49 =` **0.3673**

b. Now suppose that the investor has a degree of risk aversion of A = 2.9. What fraction should they invest in your fund?

`y = (E(r_P) - r_f)/(A *σ_P^2)`

= `(0.18 - 0.07)/(2.9 * 0.49^2)`

= **0.15798**