Chapter 14 - Bond Prices and Yields

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A bond is a security with a legally binding borrowing arrangement

- The bond issuer (i.e., the borrower) agrees to make specified payments to the bondholder on specified dates
- Par value (or face value) is the payment to the bondholder on the bond's maturity date
- The coupon rate is the bond's interest payment per year per dollar of par value
  - Firms typically split coupon payments into two equal semiannual payments
  - *Firms typically set coupon rates to induce investors to pay par value for a bond at issue*
- The bond indenture is the contract between the issuer and the bondholder

Treasury notes and bonds

- Maturity
  - Treasury notes: 1 to 10 years
  - Treasury bonds: 10 to 30 years
- Investors may purchase notes and bonds directly from the Treasury
- Denominations are as small as $100, but $1,000 is most common

Investors do not pay the quoted prices for bonds because of accrued interest I

- The quoted prices do not include the interest accrued between coupon payment dates
- If a bond trades between coupon payment dates, the buyer must pay the seller for interest accrued since the last coupon payment:

$$\text{Accrued interest} = \frac{\text{Ann. coupon}}{2} \times \frac{\text{Days since last coupon}}{\text{Days separating coupons}}$$

Investors do not pay the quoted prices for bonds because of accrued interest II

U.S. Treasury Quotes

MATURITY	COUPON	BID	ASK	CHANGE	ASKED YIELD (%)
Aug 15 24	2.375	105.8188	105.8313	0.030	0.417
Aug 15 26	1.500	103.4563	103.4750	0.064	0.790
Aug 15 30	0.625	94.6938	94.7250	0.186	1.246
Aug 15 40	1.125	88.4125	88.4750	1.002	1.847
Aug 15 40	3.875	134.1875	134.2500	1.748	1.749
Nov 15 44	3.000	120.4500	120.5125	1.830	1.904
Feb 15 47	3.000	121.9750	122.0375	1.280	1.905
Aug 15 49	2.250	106.6625	106.7250	1.948	1.938

Figure 1: Prices and Yields of U.S. Treasury Bonds, August 15, 2021 (BKM 2023, Figure 14.1)

Corporations also borrow money by issuing bonds I

- There are many combinations of issuer, maturity, coupon rate, seniority, etc., often leading to thin markets
- Therefore, most corporate bonds trade over the counter instead of on exchanges

Some corporate bonds have additional features I

- Call provisions allow issuers to repurchase bonds at a specified *call price* before maturity
  - Allow firms to call bonds with high coupon rates when interest rates fall
  - The option to call a bond is valuable *to the firm*, so callable bonds typically offer higher coupon rates and promised yields than non-callable bonds
- Convertible bonds give holders the option to exchange each bond for a specified number of shares of the firm's stock
  - The option to convert a bond is valuable *to the bondholder*, so convertible bonds typically offer lower coupon rates and yields to maturity than non-convertible bonds
- Put bonds give the holder the option to exchange the bond for par value at some date or to extend it for a given number of years
  - If a bond's coupon rate is greater than current market yields, the bondholder will extend the bond's life

Some corporate bonds have additional features II

  - If a bond's coupon rate is lower than current market yields, the bondholder will retire the bonds and collect its principal
- Floating-rate bonds pay interest rates that are reset periodically according to a specified market rate

Preferred stock is a hybrid security that is part equity, part bond

- Preferred stock is considered equity but often included in the fixed-income universe
- Like bonds
  - Promises to pay a specified cash flow stream
  - Commonly pays a fixed dividend
  - Rarely gives holders full voting privileges
- Unlike bonds
  - Failure to pay the promised dividend does not result in corporate bankruptcy
  - Dividends owed typically cumulate

International bonds are issued in one country by a borrower from a different country

- Issuers denominate *foreign bonds* in the currency of the country where issued
  - Samurai bonds
  - Bulldog bonds
- Issuers denominate *Eurobonds* in a currency *other than* the country where issued, typically the issuer's home currency
  - Euro-yen bonds
  - Euro-sterling bonds

Bond issuers often develop bonds with unusual features

- Most bonds have maturities of 30 years or less
- However, some have maturities between 50 and 100 years
- Inverse floaters are like floating-rate bonds, except coupon rate *falls* when the general level of interest rates *rises*
- Asset-backed bonds use income from a specified group of assets to service the debt
- Catastrophe bond final payments depend on whether there has been a specified catastrophe, allowing the issuer to transfer catastrophe risk to the market
- Indexed bonds make payments tied to a general price index or the price of a commodity (e.g., Treasury Inflation-Protected Securities or TIPS)

Bond value is the present value of all payments

$$\text{Bond value} = \sum_{t=1}^{T} \frac{\text{Coupon}}{(1+r)^t} + \frac{\text{Par value}}{(1+r)^T}$$

- The first term on the right-hand side of the equation is the present value of an annuity
- The second term is the present value of a single amount, the final payment of the bond's par value

There is an inverse relation between bond values and interest rates I

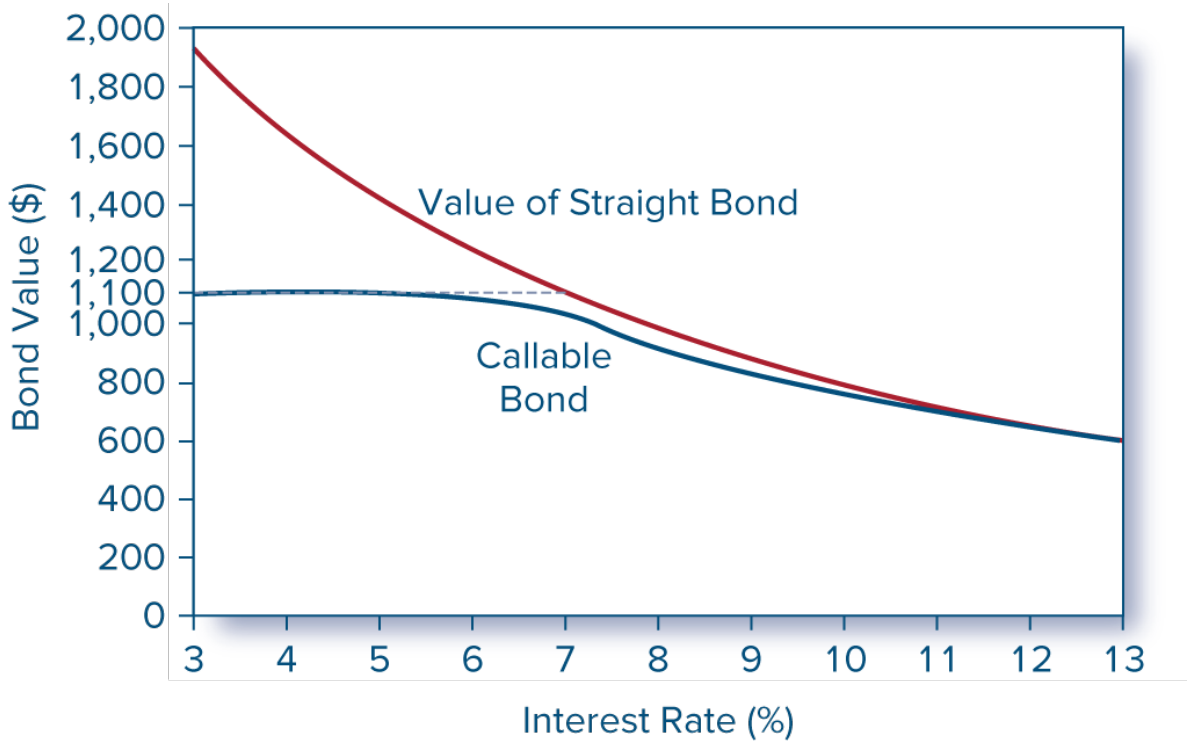

Figure 2: The inverse relationship between bond values and the interest rate. Value of an 8% coupon bond with 30-year maturity making semiannual payments. (BKM 2023, Figure 14.3)

There is an inverse relation between bond values and interest rates II

Time to Maturity	Bond Value at Given Market Interest Rate				
	2%	4%	6%	8%	10%
1 year	1,059.11	1,038.83	1,019.13	1,000.00	981.41
10 years	1,541.37	1,327.03	1,148.77	1,000.00	875.35
20 years	1,985.04	1,547.11	1,231.15	1,000.00	828.41
30 years	2,348.65	1,695.22	1,276.76	1,000.00	810.71

Figure 3: Bond values at different interest rates (8% coupon bond, coupons paid semiannually) (BKM 2023, Table 14.2)

Quoted bond prices do not include interest that accrues between coupon payments

- The buyer pays the seller for accrued interest, which is the prorated share of the upcoming semiannual coupon:

$$\text{Accrued interest} = \frac{\text{Ann. coupon}}{2} \times \frac{\text{Days since last coupon}}{\text{Days separating coupons}}$$

- The sale price (invoice price) of the bond equals the stated price (flat price) plus the accrued interest:

$$\text{Invoice price} = \text{Flat price} + \text{Accrued interest}$$

A corporate bond has a coupon rate of 11% (paid semiannually) and matures on November 15, 2025. Its quoted price is 108. Assume 30 days per month.

a. It is now July 15, 2015. What is the invoice (or dirty) price (in $)?

You purchase the following bond. The bond makes interest payments on May 15 and Nov. 15 of every year:

	A	B
1	Purchase date	4/7/2023
2	Price (% of par)	$89.85
3	Face value	$1,000
4	Coupon rate	8.8%
5	Maturity date	11/15/2033
6	Payment frequency	Semiannual
7	Day count basis	Actual/actual

a. What amount of interest has accrued?

b. What is the invoice price of the bond (in $)?

Yield to maturity (YTM) is the interest rate that equates the present value of bond payments and market price

- We often interpret the YTM as the average rate of return that we earn if we buy the bond and hold it until maturity
- The inverse relation between price and yield is a central feature of fixed-income securities
- Interest rate fluctuations are the main source of risk in the fixed-income market
- The price curve is convex and flattens at higher interest rates
- The longer the maturity of the bond, the more sensitive its price is to changes in market interest rates

YTM and current yield are different

- Yield to maturity (YTM)
  - The bond's internal rate of return
  - The compound rate of return over the life of the bond, assuming the bond holder reinvests all coupon payments at the YTM
  - Proxy for the average return
- Current yield
  - The bond's annual coupon payment divided by its price
  - Premium bonds: Coupon rate $>$ Current yield $>$ YTM
  - Discount bonds: Coupon rate $<$ Current yield $<$ YTM

YTM and yield to call (YTC) are different I

- YTC is similar to YTM except:
  - The time until *call* replaces the time until *maturity*
  - The call price replaces the par value
- At *low* interest rates: The price of the callable bond is flat since the risk of a repurchase or call is high
- At *high* interest rates: The price of the callable bond converges to that of a normal bond since the risk of a call is negligible

YTM and yield to call (YTC) are different II

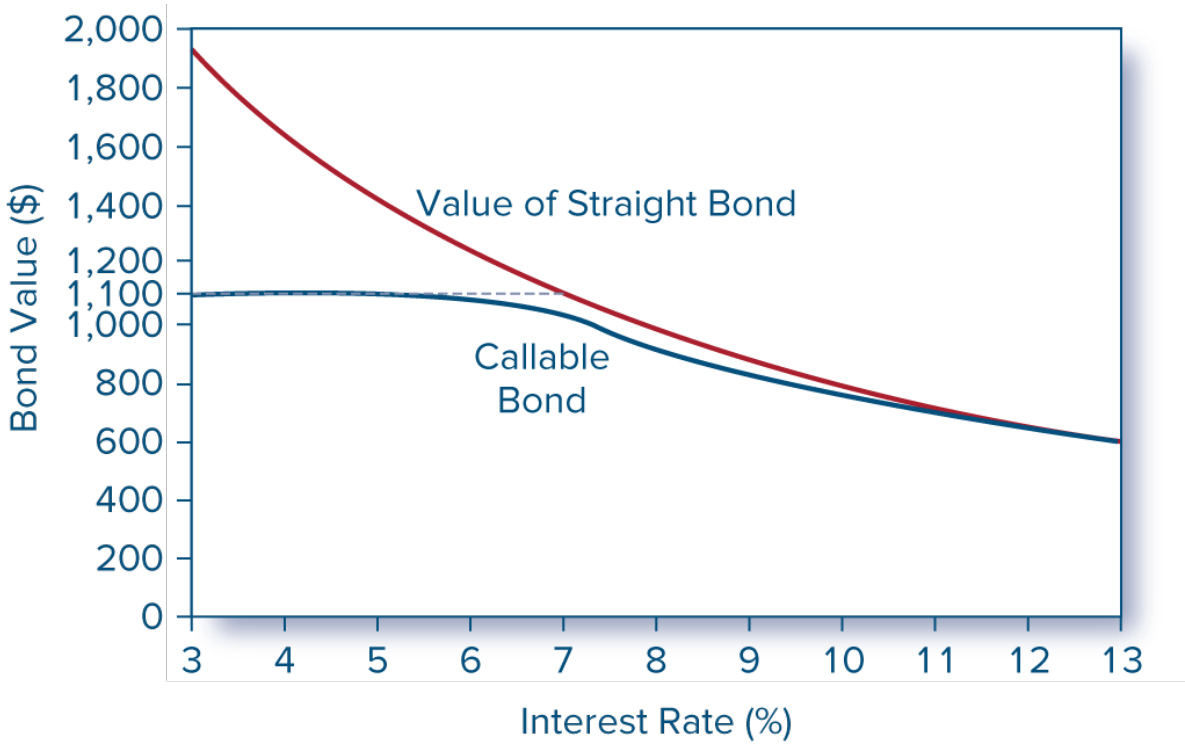

Figure 4: Bond values: Callable and straight debt (coupon = 8%; maturity = 30 years; semiannual payments) (BKM 2023, Figure 14.4)

YTM and realized compound return are different I

- YTM will equal the rate of return realized over the life of the bond if all coupons are reinvested and earn the YTM
- Realized compound return is the compound rate of return assuming that coupon payments are reinvested at market rates until maturity
- Forecasting the realized compound yield over various holding periods or investment horizons is *horizon analysis*
- As interest rates change, bond investors face two offsetting risks
  1. Price risk: When rates rise, bond prices fall
  2. Reinvestment rate risk: When rates rise, reinvestment rates rise
- Chapter 16 discusses how bond investors manage these risks

A year ago, you bought a bond with a coupon rate of 5.2%, just after it paid its semiannual coupon. The bond had 22 years to maturity, a face value of $1,000 and a yield to maturity of 3.3%.

a. Shortly after buying the bond, yields changed to 4.5%. What is your realized yield if you sell the bond now, one year after buying it?

The relative values of coupon rates and YTMs are useful I

- A bond sells at par value when its coupon rate equals its YTM
  - Here, the coupon payments exactly compensate for the time value of money
- A bond sells at *a discount* to par value when its coupon rate is *less than* its YTM
  - Here, the coupon payments do not compensate for the time value of money, so investors require additional capital gains
- A bond sells at *a premium* to par value when its coupon rate is *greater than* its YTM
  - Here, the coupon payments more than compensate for the time value of money, so investors pay more than the par value

The relative values of coupon rates and YTMs are useful II

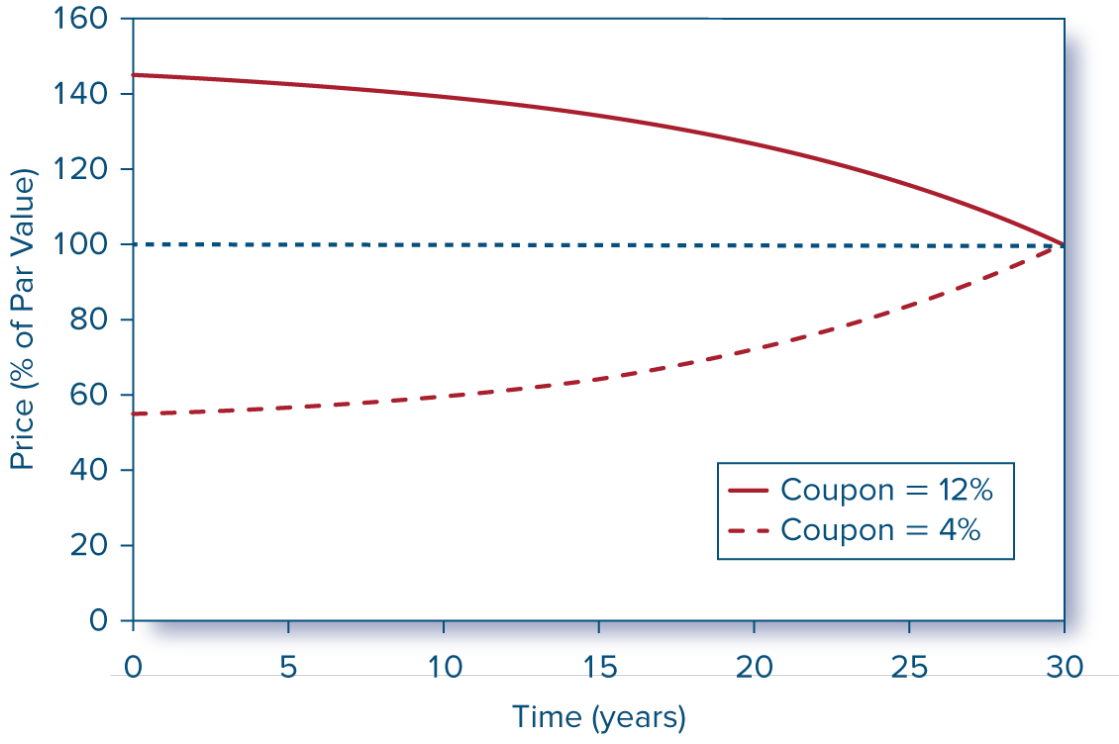

Figure 5: Price path of two 30-year maturity bonds, each selling at a yield to maturity of 8%. Bond price approaches par value as maturity date approaches. (BKM 2023, Figure 14.6)

YTM and holding period return (HPR) are different

- HPR
  - The rate of return over a particular investment period
  - Depends on the bond's price at the end of the holding period, which is an unknown future value
- YTM
  - The average rate of return if the bond is held to maturity
  - Depends on coupon rate, maturity, and par value, which are all known
- When YTM does not change over a period YTM $=$ HPR

Zero-coupon bonds sell at a discount and pay interest as capital gains

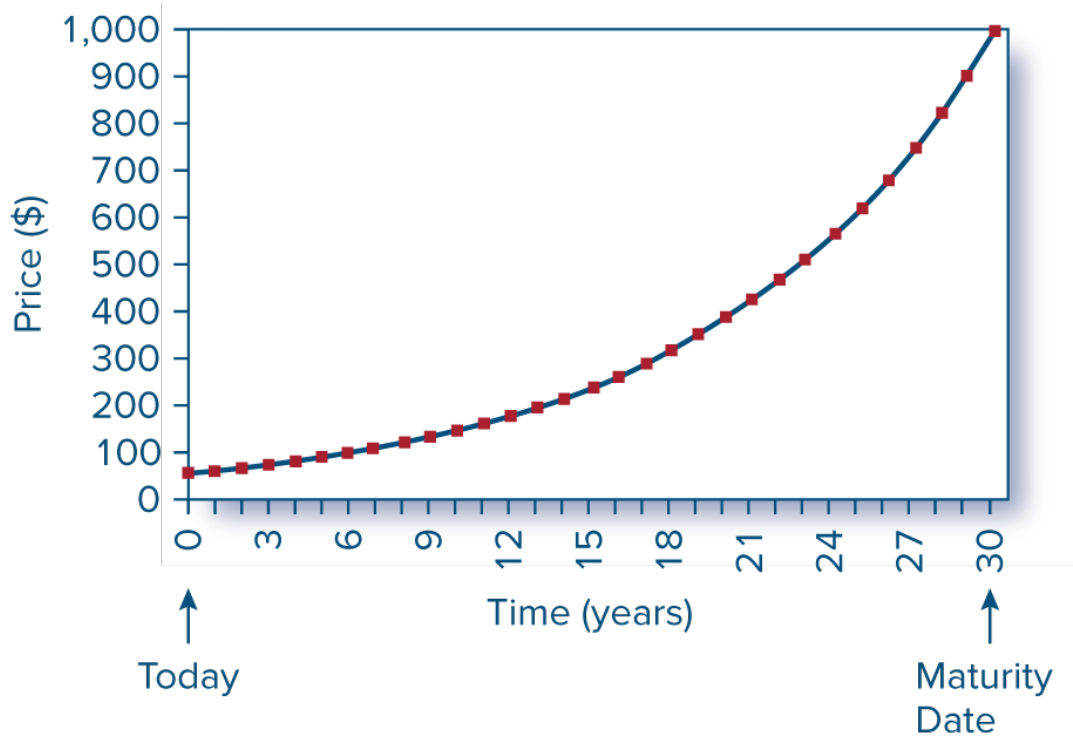

Figure 6: The price of a 30-year zero-coupon bond over time at a yield to maturity of 10%. Price equals $\$1,000/(1.10)^T$, where T is time until maturity. (BKM 2023, Figure 14.7)

A corporate bond with a coupon rate of 8% pays interest semiannually and has a maturity date of May 28, 2030. The trade settles on March 20, 2023. The yield to maturity is 11%.

a. What is the flat (or clean) price of the bond (in percent of par) on the settlement date? Use Excel's PRICE() function. Dates must be entered with Excel's DATE() function.

. A corporate bond with a face value of $1,000 and a coupon rate of 4.4% pays interest semiannually and has a maturity date of May 6, 2027. The trade settles on January 22, 2023. The yield to maturity is 6.1%.

a. How many days have passed since the last coupon payment? Use Excel's COUPDAYBS() function. Dates must be entered with Excel's DATE() function.

b. How many days are in the current coupon period? Use Excel's COUPDAYS() function. Dates must be entered with Excel's DATE() function.

c. What is the accrued interest on the bond (in $)?

d. What is the flat (or clean) price of the bond (in percent of par) on the settlement date? Use Excel's PRICE() function. Dates must be entered with Excel's DATE() function.

e. What is the invoice (or dirty) price of the bond (in $) on the settlement date?

Bonds *promise* a fixed flow of income, which is not guaranteed unless the issuer cannot default

- Credit risk, or default risk, is the risk the bond will not make all promised payments
- Rating companies: Moody's Investor Service, Standard & Poor's, and Fitch Ratings
- Rating categories
  - Highest rating is AAA (or Aaa)
  - Investment grade bonds are rated BBB/Baa or above
  - Speculative-grade or junk bonds are rated below BBB/Baa

Definitions of each bond rating class

Bond Ratings

	Very High Quality		High Quality		Speculative		Very Poor	
Standard & Poor's	AAA	AA	A	BBB	BB	B	CCC	D
Moody's	Aaa	Aa	A	Baa	Ba	B	Caa	C

At times both Moody's and Standard & Poor's have used adjustments to these ratings: S&P uses plus and minus signs: A+ is the strongest A rating and A− the weakest. Moody's uses a 1, 2, or 3 designation, with 1 indicating the strongest.

Moody's	S&P	
Aaa	AAA	Debt rated Aaa and AAA has the highest rating. Capacity to pay interest and principal is extremely strong.
Aa	AA	Debt rated Aa and AA has a very strong capacity to pay interest and repay principal. Together with the highest rating, this group comprises the high-grade bond class.
A	A	Debt rated A has a strong capacity to pay interest and repay principal, although it is somewhat more susceptible to the adverse effects of changes in circumstances and economic conditions than debt in higher-rated categories.
Baa	BBB	Debt rated Baa and BBB is regarded as having an adequate capacity to pay interest and repay principal. Whereas it normally exhibits adequate protection parameters, adverse economic conditions or changing circumstances are more likely to lead to a weakened capacity to pay interest and repay principal for debt in this category than in higher-rated categories. These bonds are medium-grade obligations.
Ba B Caa Ca	BB B CCC CC	Debt rated in these categories is regarded, on balance, as predominantly speculative with respect to capacity to pay interest and repay principal in accordance with the terms of the obligation. BB and Ba indicate the lowest degree of speculation, and CC and Ca the highest degree of speculation. Although such debt will likely have some quality and protective characteristics, these are outweighed by large uncertainties or major risk exposures to adverse conditions. Some issues may be in default.
C	C	This rating is reserved for income bonds on which no interest is being paid.
D	D	Debt rated D is in default, and payment of interest and/or repayment of principal is in arrears.

Figure 7: Definitions of each bond rating class (BKM 2023, Figure 14.8)

Bond rating agencies analyze the levels and trends of issuer financial ratios

	Aaa	Aa	A	Baa	Ba	B	C
EBITA/Assets	12.3%	10.5%	11.2%	9.0%	8.8%	6.6%	4.9%
Operating profit margin	24.8%	18.9%	14.3%	12.9%	11.9%	9.6%	4.6%
EBITA to interest coverage (multiple)	12.0	19.0	12.2	6.5	3.9	2.0	0.8
Debt/EBITDA (multiple)	1.9	1.8	2.1	2.8	3.6	5.1	6.9
Debt/(Debt + Equity)	42.8%	36.6%	39.9%	45.4%	55.7%	65.9%	79.8%
Funds from operations/Total debt	40.4%	44.5%	37.7%	28.2%	21.3%	12.5%	5.7%
Retained Cash Flow / Net Debt	32.3%	29.5%	31.2%	26.6%	21.3%	13.0%	6.0%

Figure 8: Financial ratios by rating class (BKM 2023, Table 14.3)

Financial ratios predict bankruptcy I

- We calculate a Z score based on firm financial characteristics:

$$Z = 3.1\frac{EBIT}{A} + 1.0\frac{Sales}{A} + 0.42\frac{E}{L} + 0.85\frac{RE}{A} + 0.72\frac{WC}{A}$$

- Higher Z scores indicate lower credit risk
  - $Z < 1.23$ indicates vulnerability to bankruptcy
  - $1.23 \leq Z < 2.90$ is a gray area
  - $2.90 \leq Z$ indicates safety

Financial ratios predict bankruptcy II

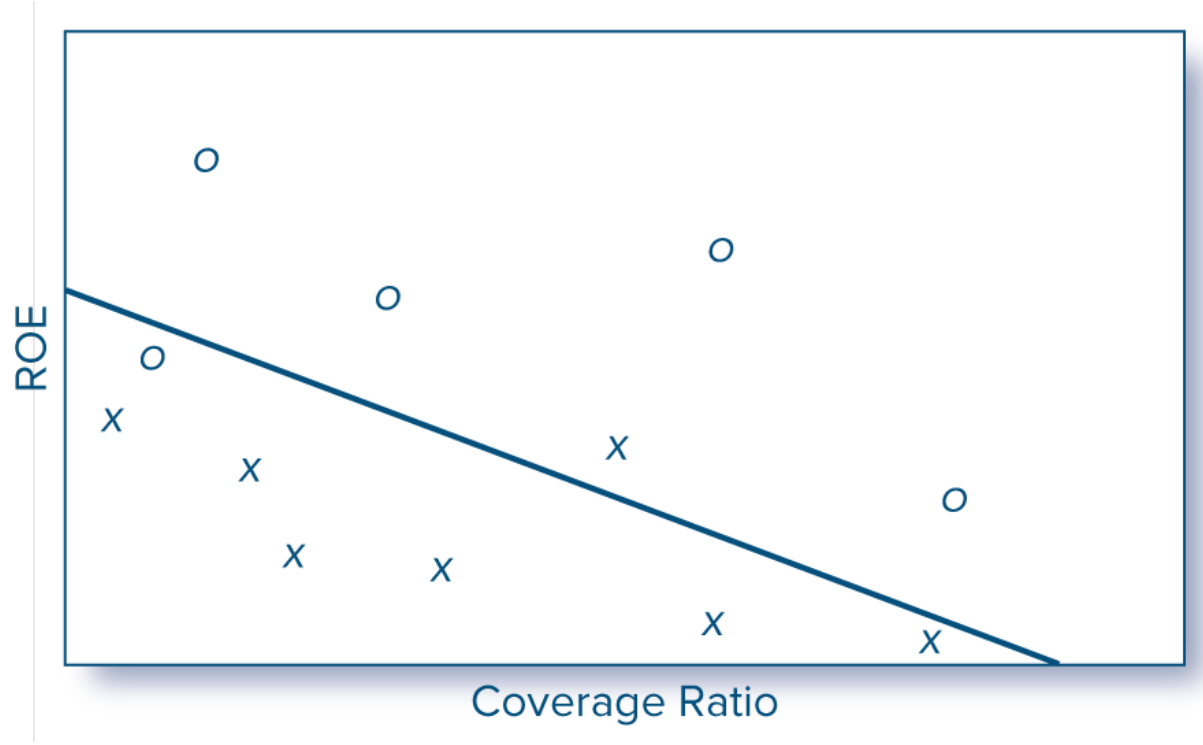

Figure 9: Discriminant analysis (BKM 2023, Figure 14.9)

The bond indenture is the contract between the issuer and bondholders

- Sinking funds call for the issuer to periodically repurchase some proportion of the outstanding bonds before maturity
- The subordination clause restricts the amount of additional borrowing by the firm
- Dividend restrictions limit the payment of dividends by firms (e.g., cumulative dividends cannot exceed cumulative retained earnings)
- Collateral is a particular asset that the bondholders receive if the firm defaults, making collateralized bonds generally safer than debenture bonds

Promised YTM and *expected* YTM are different I

- Promised YTM will only be realized if the firm meets the obligations of the bond issue
- Expected YTM must consider the possibility of a default
- Default premium (also known as a credit spread) is the difference between the promised YTM on a corporate bond and the YTM on an otherwise-identical government bond that is default risk-free

Promised YTM and *expected* YTM are different II

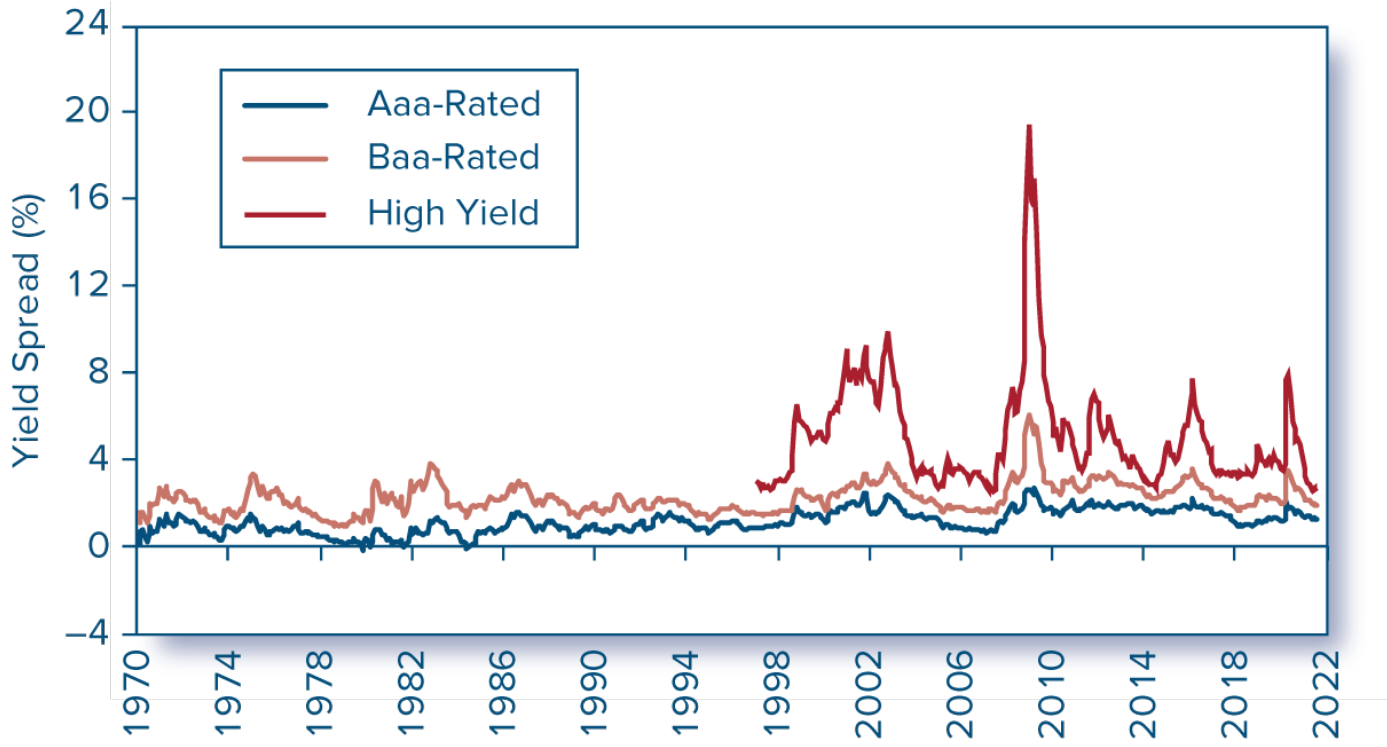

Figure 10: Yield spreads between corporate and 10-year Treasury bonds (BKM 2023, Figure 14.11)

Credit default swaps (CDSs) insure a bond or loan against default

- Allows lenders to insure against default
- The risk structure of interest rates and CDS prices ought to be tightly aligned
- CDS contracts trade on corporate and sovereign debt
- Natural CDS buyers are large bondholders that want to enhance the creditworthiness of their outstanding loans
- CDS buyers can also use them to speculate on the financial health of an issuer

Collateralized debt obligations (CDOs) reallocate credit risk I

- A structured Investment Vehicle (SIV) is a legally distinct entity that buys debt and resells it as a portfolio
- Loans are pooled together and split into *tranches*
- Each tranche is given a different level of seniority to claims on the underlying loan pool
- Mortgage-backed CDOs were an investment disaster during 2007 to 2009

Collateralized debt obligations (CDOs) reallocate credit risk II

		Senior-Subordinated Tranche Structure	Typical Terms
		Senior tranche	70–90% of notional principal, coupon similar to Aa-Aaa rated bonds
		Mezzanine 1	5–15% of principal, investment-grade rating
Bank	Structured investment vehicle, SIV		
		Mezzanine 2	5–15% of principal, higher-quality junk rating
		Equity/first loss/ residual tranche	<2%, unrated, coupon rate with 20% credit spread

Figure 11: Collateralized debt obligations (BKM 2023, Figure 14.13)

. A GM and a Ford bond both have 4 years to maturity, a $1,000 par value, a BB rating and pay interest semiannually. GM has a coupon rate of 6.7%, while Ford has a coupon rate of 5%.

a. The GM bond trades at 92.41 (percent of par). What is the yield to maturity (YTM)?

b. What should be the price of the Ford bond (in $)?

2. A corporate bond with a coupon rate of 7% pays interest semiannually and has a maturity date of May 28, 2030. The trade settles on March 20, 2023. The flat (or clean) price of the bond on the settlement date is 117.21 (in percent of par).

   a. What is the yield to maturity? Use Excel's YIELD() function. Dates must be entered with Excel's DATE() function.

. You purchase the following bond. The bond makes interest payments on April 10 and Oct. 10 of every year:

	A	B
1	Purchase date	8/19/2022
2	Maturity date	10/10/2038
3	First call date	4/10/2030
4	Coupon rate	2.9%
5	Price	87.01
6	Face value	100
7	Payment frequency	2
8	Day count basis	30/360
9	Call price	137.35

a. What is the bond's yield to maturity using Excel's YIELD function?

b. What is the bond's yield to call using Excel's YIELD function?

Summary and Key Equations I

Page 474

1. Fixed-income securities are distinguished by their promise to pay a fixed or specified stream of income to their holders. The coupon bond is a typical fixed-income security.
2. Treasury notes and bonds have original maturities greater than one year. They are issued at or near par value, with their prices quoted net of accrued interest.
3. Callable bonds should offer higher promised yields to maturity to compensate investors for the fact that they will not realize full capital gains should the interest rate fall and the bonds be called away from them at the stipulated call price. Bonds often are issued with a period of call protection. In addition, discount bonds selling significantly below their call price offer implicit call protection.
4. Put bonds give the bondholder rather than the issuer the option to terminate or extend the life of the bond.
5. Convertible bonds may be exchanged, at the bondholder's discretion, for a specified number of shares of stock. Convertible bondholders "pay" for this option by accepting a lower coupon rate on the security.
6. Floating-rate bonds pay a coupon rate at a fixed premium over a reference short-term interest rate. Risk is limited because the rate is tied to current market conditions.
7. The yield to maturity is the single interest rate that equates the present value of a security's cash flows to its price. Bond values and yields are inversely related. For premium bonds, the coupon rate is greater than the current yield, which is greater than the yield to maturity. The order of these inequalities is reversed for discount bonds.
8. The yield to maturity is often interpreted as an estimate of the average rate of return to an investor who purchases a bond and holds it until maturity. However, when future rates are uncertain, actual returns including reinvested coupons may diverge from yield to maturity. Related measures are yield to call, realized compound yield, and expected (versus promised) yield to maturity.
9. Prices of zero-coupon bonds rise exponentially over time, providing a rate of appreciation equal to the interest rate. The IRS treats this built-in price appreciation as imputed taxable interest income to the investor.
10. When bonds are subject to potential default, the stated yield to maturity is the maximum possible yield to maturity that can be realized by the bondholder. In the event of default, however, that promised yield will not be realized. To compensate bond investors for default risk, bonds must offer default premiums, that is, promised yields in excess of those offered by default-free government securities.
11. Bond safety is often measured using financial ratio analysis. Bond indentures are safeguards to protect the claims of bondholders. Common indentures specify sinking fund requirements, collateralization of the loan, dividend restrictions, and subordination of future debt.
12. Credit default swaps provide insurance against the default of a bond or loan. The swap buyer pays an annual premium to the swap seller but collects a payment equal to lost value if the loan later goes into default.
13. Collateralized debt obligations are used to reallocate the credit risk of a pool of loans. The pool is sliced into tranches, with each tranche assigned a different level of seniority in terms of its claims on the cash flows from the underlying loans. High seniority tranches are usually quite safe, with credit risk concentrated on the lower-level tranches. Each tranche can be sold as a stand-alone security.

Figure 12: Chapter 14 summary from BKM (2023)

Summary and Key Equations II

Value of a coupon bond:

$$\begin{aligned}\text{Value} &= \text{Coupon} \times \frac{1}{r}\left[1 - \frac{1}{(1+r)^T}\right] + \text{Par value} \times \frac{1}{(1+r)^T} \\ &= \text{Coupon} \times \text{Annuity factor}(r,\ T) + \text{Par value} \times \text{PV factor}(r,\ T)\end{aligned}$$

Figure 13: Chapter 14 key equations from BKM (2023)

References I

 Bodie, Zvi, Alex Kane, and Allan J. Marcus (2023). *Investments*. 13th ed. New York: McGraw Hill.