Slides 10 - Arbitrage Pricing Theory and Multifactor Models of Risk and Return

1. a. Which are assumptions of the APT?

   Check all that apply:

   - [x] Security returns can be described by a factor model.
   - [x] There are enough securities to diversify away firm-specific risk.
   - [x] There are no persistent arbitrage opportunities.
   - [ ] There is no non-systematic risk.

   The APT allows for non-systematic risk, but assumes that there are enough securities to create a well-diversified portfolio such that no non-systematic (firm-specific) risk remains in the portfolio.

2. a. An activity that includes an exploitation of price differences between two or more identical or similar securities in order to achieve riskless profits is called _____.

   - (•) arbitrage
   - ( ) diversification
   - ( ) factoring
   - ( ) market timing
   - ( ) optimization

   Arbitrage is the exploitation of price differences between two or more identical or similar securities in order to achieve riskless profits.

   b. Which of the following is true about the arbitrage pricing theory (APT)?

   - ( ) APT assumes that all investors are identical in terms of risk aversion.
   - ( ) APT assumes that all investors are identical in terms of wealth.
   - ( ) APT can be applied to both well-diversified portfolios and individual securities.
   - (•) APT derives the equilibrium rate of return in large capital markets with no arbitrage opportunities.

   The arbitrage pricing theory (APT) was developed by Ross (1976) and derives the equilibrium rate of return in large capital markets with no arbitrage opportunities.

3. a. What does the APT say about alpha?

   - ( ) Alpha equals the risk-free rate.
   - ( ) Alpha is irrelevant.
   - ( ) Alpha is zero for all securities.
   - (•) Alpha is zero for well-diversified portfolios.

   The APT's assumption about no persistent arbitrage opportunities rules out non-negative alphas for well-diversified portfolios.

4. a. Which of the following statements are true about the CAPM and the APT?

   Check all that apply:

   - [x] The APT applies only to well-diversified portfolios.

- [ ] The APT guarantees that in the absence of arbitrage opportunities, the expected return-beta relationship will hold for all assets.
- [x] The CAPM applies to all securities.
- [x] The expected return-beta relationship is the same for both theories.

The APT applies only to well-diversified portfolios and does not guarantee that the expected return-beta relationship will hold for every single asset.

5. a. A well-diversified portfolio is a portfolio that includes a large enough number of _____ securities to make the _____ risk negligible.

   - (•) different; nonsystematic
   - ( ) different; systematic
   - ( ) similar; nonsystematic
   - ( ) similar; systematic

   A well-diversified portfolio is a portfolio that is sufficienty diversified by including a large number of different securities so that the nonsystematic risk becomes negligible.

6. a. Which of the following statements are true about multifactor models?

   Check all that apply:

   - [x] The coefficients on each factor in the multifactor models are called factor sensitivities.
   - [x] The coefficients on each factor in the multifactor models are called factor loadings.
   - [x] The coefficients on each factor in the multifactor models are called factor betas.
   - [ ] The coefficients on each factor in the multifactor models are called factor alphas.
   - [ ] In the multifactor models, the return of the stock responds only to firm-specific influences and not to sources of macro risk.

   In the multifactor models, the return of the stock responds to sources of macro risk and to firm-specific influences. The coefficients on each factor in the multifactor models are called factor sensitivities, factor loadings or factor betas.

7. a. A factor portfolio is a well-diversified portfolio with a/an _____.

   - ( ) alpha of 1 on all of the factors.
   - ( ) alpha of 1 on one of the factors and a beta of 0 on all other factors.
   - ( ) beta of 0 on all of the factors.
   - ( ) beta of 1 on all of the factors.
   - (•) beta of 1 on one of the factors and a beta of 0 on all other factors.

   A factor portfolio is a well-diversified portfolio with a beta of 1 on one of the factors and a beta of 0 on all other factors.

8. An investor holds two well-diversified portfolios on US securities. The expected return on portfolio A is 13% and the expected return on portfolio B is 9%. The investor identified that the US economy has only one factor (industrial production), and $\beta_A = 1$ and $\beta_B = 0.7$.

   a. What should be the risk-free rate according to the APT?

   According to the APT, the expected return on portfolio A is:
   $E(r_A) = r_f + \beta_A * RP$

⇔ $RP = (E(r_A) - r_f) / \beta_A$

Expected return on portfolio B:
$E(r_B) = r_f + \beta_B * RP$
⇔ $RP = (E(r_B) - r_f) / \beta_B$

Setting the two risk premiums equal and solving for r_f:

`(E(r_A) - r_f) / β_A = (E(r_B) - r_f) / β_B`

⇔ `(E(r_A) - r_f) β_B = (E(r_B) - r_f) β_A`

⇔ `r_f * (β_A - β_B) = E(r_B) β_A - E(r_A) β_B`

⇔ `r_f = (E(r_B) β_A - E(r_A) β_B) / (β_A - β_B)`

= `(0.09 * 1 - 0.13 * 0.7) / (1 - 0.7)`

= **-0.003333**

9. A well-diversified portfolio P has an expected return of 5% and a beta of 1.2. The risk-free rate is 3.1% and the expected return on the S&P 500 is 8%.

   a. What is the portfolio's expected alpha?

      Portolio's expected return according to the CAPM:

      `E(r)_"CAPM" = r_f + β (E(r_M) - r_f)`

      = `0.031 + 1.2 * (0.08 - 0.031)`

      = `0.0898`

      Portfolio alpha:

      `α = E(r)_"Security analysis" - E(r)_"CAPM"`

      = `0.05 - 0.0898`

      = **-0.0398**

   b. What should you do?

      ○ Buy the security

      ◉ Short-sell the security

      A negative alpha means that the portfolio is overpriced. We should thus short-sell it.

   c. If you trade $2,000 of the portfolio, how much should you trade of the S&P 500 in order to create a risk-free and costless arbitrage opportunity (in $)? Enter a positive number for buying the S&P 500 and a negative number for short-selling it.

      Since we short-sell the portfolio and thus get a negative beta of -1.2 on this position, we need to create an offsetting position with a positive beta of 1.2 in order to eliminate all systematic risk (non-systematic risk is already eliminated since we're talking about a well-diversified portfolio). This is done by investing 120% in the S&P 500 and -20% in T-bills. Since the S&P 500 has a beta of 1.0 and T-bills have a beta of 0, the resulting beta is 1.2 * 1 + -0.2 * 0 = 1.2.

      Therefore, we need to invest 1.2 * $2,000 = $**2,400** in the S&P 500.

10. Suppose that an investor holds a stock which has a rate of return based on a single-factor model. The model equation can be presented as:

$r_i = E(r_i) + \beta_i F + e_i$

where F is the unanticipated growth in GDP. The market consensus about the GDP growth rate is 4%. The investor currently expects to earn a 10.4% return. The stock's β value is 1.1.

a. Suppose that after some period of time the macroeconomic news suggest that GDP growth will be 5%, which is higher than the market consensus. What will be the revised estimate of the stock's expected rate of return?

Expected rate of return based on single-factor model:
$E(r_{i,1}) = E(r_{i,0}) + \beta_i F$
= 0.104 + 1.1 * (0.05 - 0.04)
= **0.115**

b. What will be the revised estimate of the stock's expected rate of return if the macroeconomic news suggest that GDP will be 2.5%, which is lower than the market expected value?

Expected rate of return based on single-factor model:
$E(r_{i,1}) = E(r_{i,0}) + \beta_i F$
= 0.104 + 1.1 * (0.025 - 0.04)
= **0.0875**

11. An investor analyzed 20 stocks listed on the FTSE and found that half of them have an alpha of 2.3% and the other half an alpha of -2.3%. All stocks listed on the FTSE have the market return as the common factor, and all of them have a beta of 1 on the market index. Also, firm-specific returns have a standard deviation of 35%. The investor decides to invest $5 million in an equally-weighted portfolio of 10 positive alpha stocks, and to sell short $5 million in an equally-weighted portfolio of 10 negative alpha stocks.

a. What would be the expected profit for the investor?

Let R_M be the risk premium on the market portfolio.

Expected profit:
= $\text{Investment}_+ * E(r_+) + \text{Investment}_- * E(r_-)$
= $\text{Investment}_+ * (\alpha_+ + \beta_+ R_M) + \text{Investment}_- * (\alpha_- + \beta_- R_M)$
= 5,000,000 * (0.023 + 1 * R_m) - 5,000,000 * (-0.023 + 1 * R_m)
= 2 * 5,000,000 * 0.023
= **230,000**

b. What is the standard deviation of the profit calculated in the previous part?

Since the long and short positions have the same betas and investment amounts, there is no systematic risk. However, there is firm-specific risk.

Position in each individual stock (long or short):
P_i = 5,000,000 / 10
= 500,000

Variance of dollar returns on portfolio:
$\sigma_P^2 = N * ((P_i * \sigma_i)^2)$
= 20 * ((500,000 * 0.35)^2)
= 612,500,000,000

Standard deviation of dollar returns on portfolio:
$\sigma_P = (\sigma_P^2)^{0.5}$
= $612{,}500{,}000{,}000^{0.5}$
= **782,624**

12. An investor holds two well-diversified portfolios. The risk-free rate is 1.4% and the investor identified two macroeconomic factors: industrial production (X) and consumer confidence (Y). Factor betas and expected rates of return are given below:

Portfolio	Beta on factor X	Beta on factor Y	Expected return
A	0.8	1.1	18%
B	1.1	1.3	25%

a. According to the APT, what should be the expected return for a well-diversified portfolio with β_X of 1.2 and β_Y of 0.4?

We have two equations with two unknows, RP_X and RP_Y.

Expected return of portfolio A:
$E(r_A) = r_f + \beta_{A,X}\, RP_X + \beta_{A,Y}\, RP_Y$

Expected return of portfolio B:
$E(r_B) = r_f + \beta_{B,X}\, RP_X + \beta_{B,Y}\, RP_Y$
$\Leftrightarrow RP_X = (E(r_B) - r_f - \beta_{B,Y} * RP_Y) / \beta_{B,X}$

Plugging the expression for RP_X into the first equation and solving for RP_Y:

`E(r_A) = r_f + β_(A,X) (E(r_B) - r_f - β_(B,Y) * RP_Y) / β_(B,X) + β_(A,Y) RP_Y`

⇔ `RP_Y (β_(A,X) β_(B,Y)/ β_(B,X) - β_(A,Y)) = r_f - E(r_A) + β_(A,X) (E(r_B) - r_f)/ β_(B,X)`

⇔ `RP_Y = (r_f - E(r_A) + β_(A,X) (E(r_B) - r_f)/ β_(B,X)) / (β_(A,X) β_(B,Y)/ β_(B,X) - β_(A,Y))`

= `(0.014 - 0.18 + 0.8 (0.25 - 0.014)/1.1) / (0.8 * 1.3/1.1 - 1.1)`

= -0.03647

Risk premium for factor X:

`RP_X = (E(r_B) - r_f - β_(B,Y) * RP_Y)/β_(B,X)`

= `(0.25 - 0.014 - 1.3 * -0.03647)/1.1`

= 0.2576

Expected return of portfolio C:

`E(r_C) = r_f + β_(C,X) RP_X + β_(C,Y) RP_Y`

= `0.014 + 1.2 * 0.2576 + 0.4 * -0.03647`

= **0.3086**