

Topics in Macroeconomics

Unit 3 – Introduction

Richard Foltyn

University of Glasgow

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Outline of unit 3

- 1 Introduction (this mini-lecture)
 - Recursive methods in quantitative macroeconomics
 - Infinite-horizon vs. life-cycle solution methods
- 2 Value function iteration (VFI)
- 3 Endogenous grid-point method (EGM)

Recursive formulation of household problem

Recall that we can write a household problem in two ways:

1 Sequential formulation

$$\begin{aligned} V(a_0, y_0) &= \max_{\{c_t\}_{t=0}^{\infty}, \{a_{t+1}\}_{t=0}^{\infty}} \mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \mid y_0 \right] \\ \text{s.t. } c_t + a_{t+1} &= (1+r)a_t + y_t \quad \forall t \\ c_t \geq 0, a_{t+1} &\geq 0 \quad \forall t \end{aligned}$$

2 Recursive formulation

$$\begin{aligned} V(a, y) &= \max_{c, a'} \left\{ u(c) + \beta \mathbb{E} \left[V(a', y') \mid y \right] \right\} \\ \text{s.t. } c + a' &= (1+r)a + y \\ c \geq 0, a' &\geq 0 \end{aligned}$$

Recursive methods

- The sequential formulation is quite useless for solving heterogeneous-agent models numerically

⇒ We exclusively deal with recursive formulation

- We want to find functions that characterise the solution:

- 1 The value function $V(a, y)$
- 2 The policy functions
 - $c = C(a, y)$ Optimal consumption
 - $a' = A(a, y)$ Optional savings

These functions are defined on discretised grids $a \in \mathcal{G}_a$ and $y \in \mathcal{G}_y$.

Iteration and backwards induction

Two main types of household problems:

Infinite-horizon problems

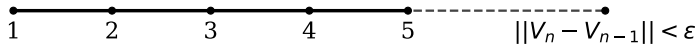
- Need to start with a [guess for the solution](#); often this is just $V_0(a, y) = 0$
- Iterate on some object until consecutive iterations V_n, V_{n+1} are sufficiently close
- We can iterate either on value functions (VFI) or policy functions (PFI, EGM: endogenous grid-point method)

Finite-horizon problems

- Life-cycle and OLG models
- Solve for last period T
- Use backward induction to solve previous periods $T - 1, T - 2, \dots$

Infinite horizon vs. life-cycle

Infinite horizon: iteration



Life-cycle: backward induction

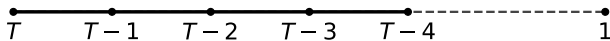


Figure 1: Solving infinite-horizon vs. life-cycle models

Outline of remaining mini-lectures

- We exclusively solve household problems
 - Ignore distribution of households
 - Ignore general equilibrium
- Next mini-lectures:
 - 1 Lecture 1: Value function iteration (VFI)
 - Grid search
 - Interpolation
 - 2 Lecture 2: Endogenous grid-point method (EGM)
- Slides and pre-recorded lectures: general concepts, algorithms, results
- Live sessions: implement examples discussed in slides
- Hands-on approach to complement units 1–2

Overview of unit 3

Source code

- Github repository: <https://github.com/richardfoltyn/mres-macro-topics>
- Python and Matlab source code for examples discussed in lectures / live sessions
- We use Matlab in live sessions