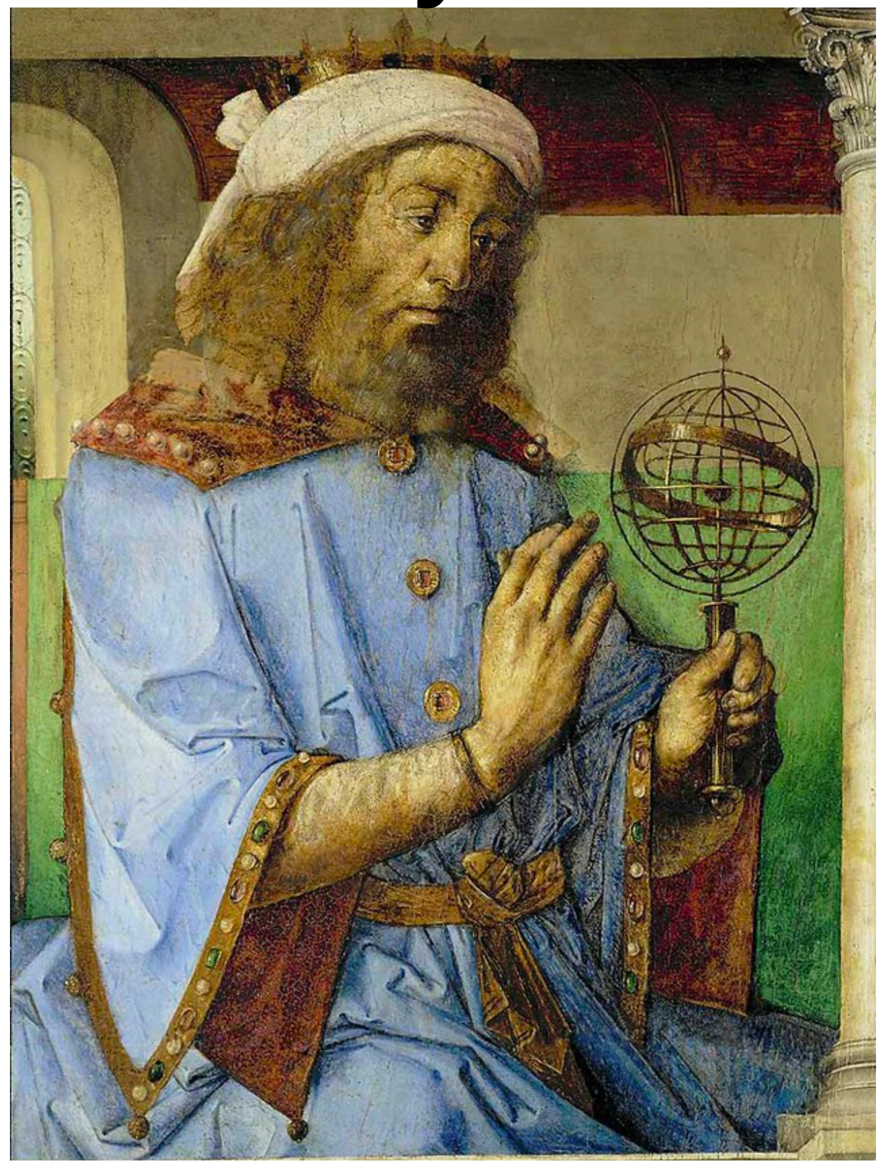
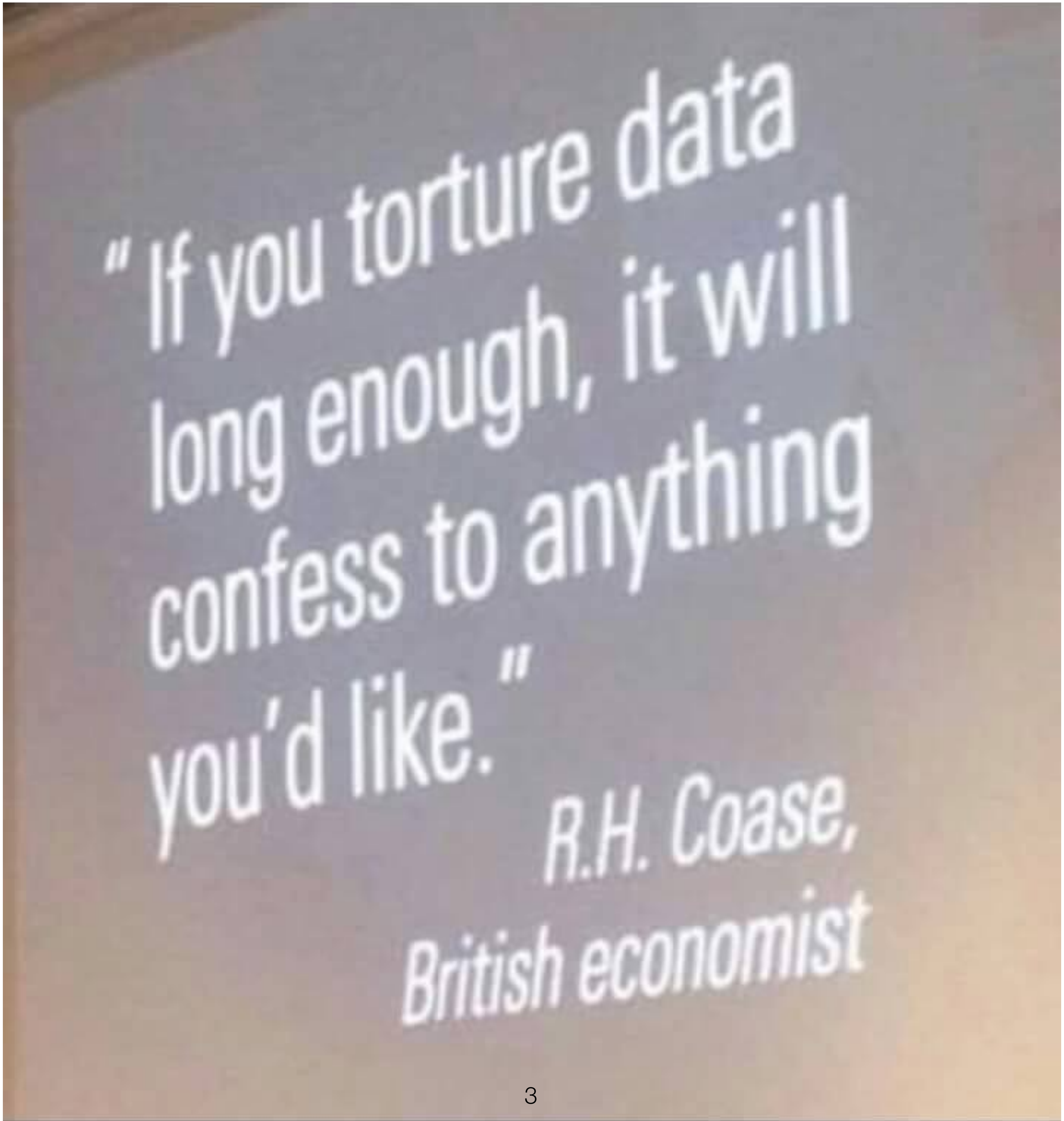
A few words about orbits

- Question from last time: How do we know the masses of the planets?
- The answer is from dynamics. If there’s a satellite (natural or artificial) orbiting an object, you can measure the mass quite precisely just by observing the orbit carefully.
- We’ll need some of these ideas again later when we talk about planets in orbit around other stars.
- The word “planet” means wanderer, because they move relative to the fixed stars.

Claudius Ptolemy

- Lived in Alexandria, in Roman Egypt, 100-170 AD
- Modeled the motion of the planets with an earth-centered system of epicycles
- All motion is circular
- Worked pretty well, and was very influential through the Middle Ages

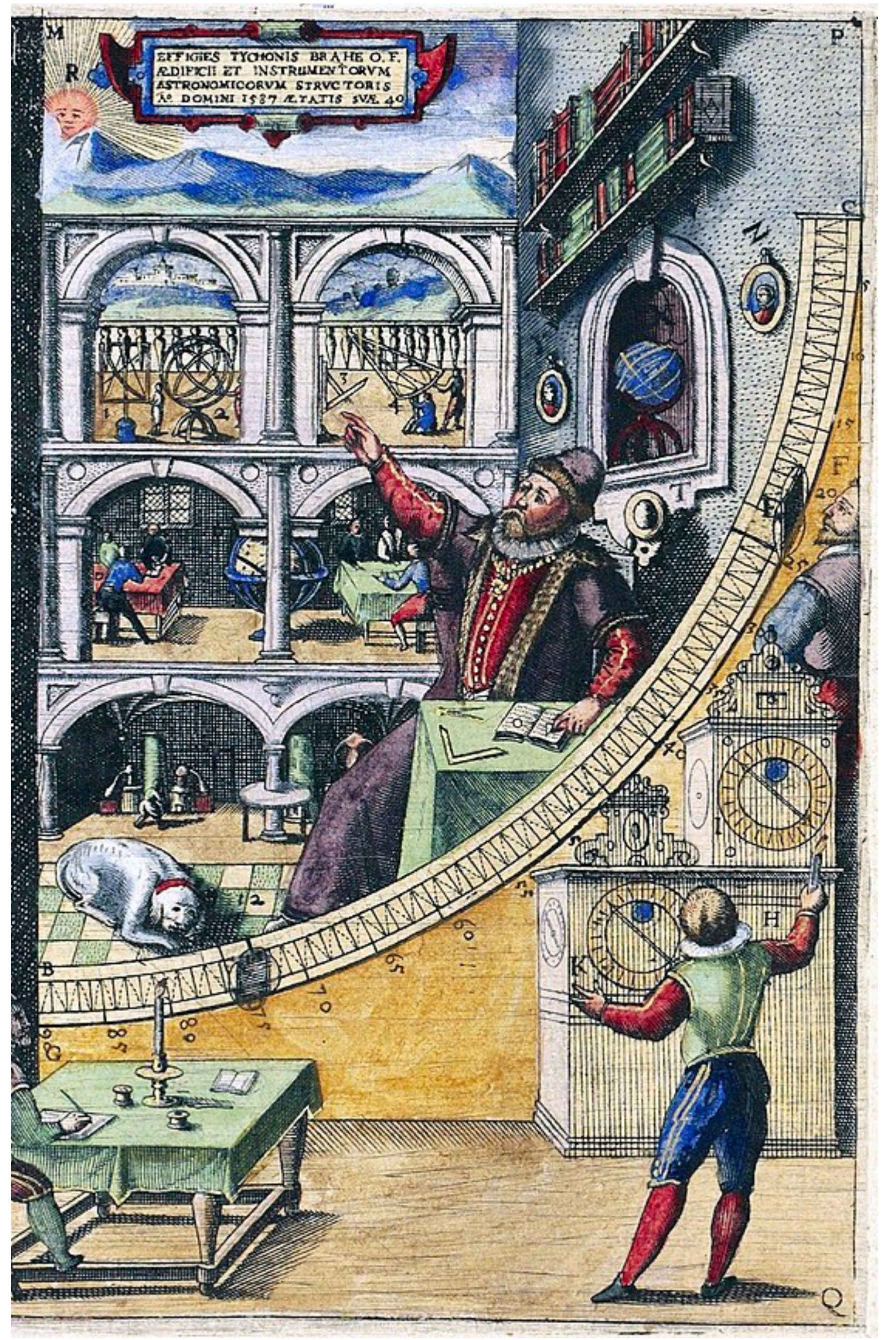



" If you torture data
long enough, it will
confess to anything
you'd like."
R.H. Coase,
British economist

Tycho Brahe

Danish astronomer 1546-1601. Made careful observations of positions of Mars and other planets.

Mentored Johannes Kepler



Johannes Kepler

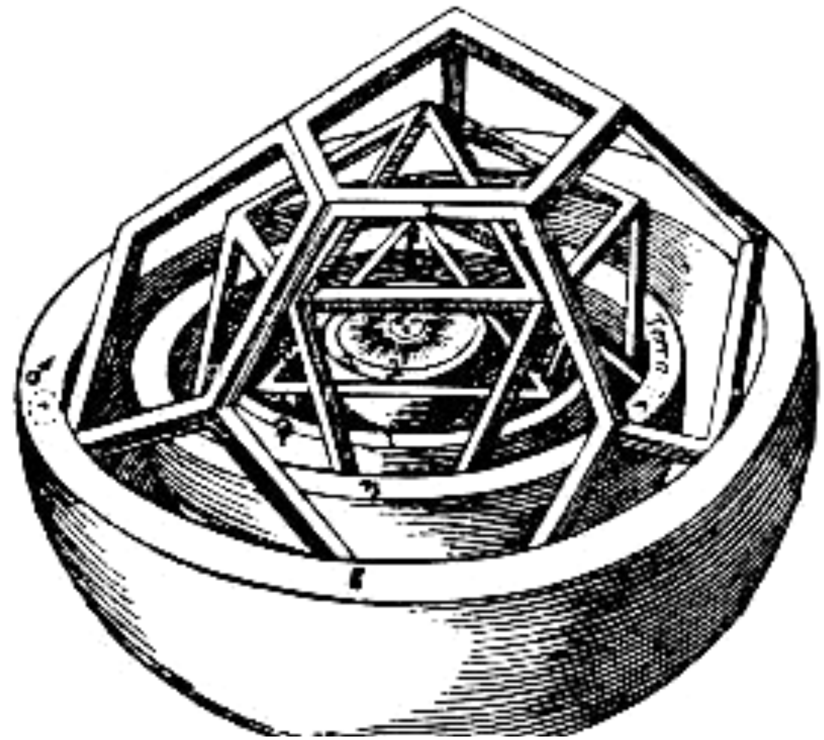
Czech astronomer, 1571-1630

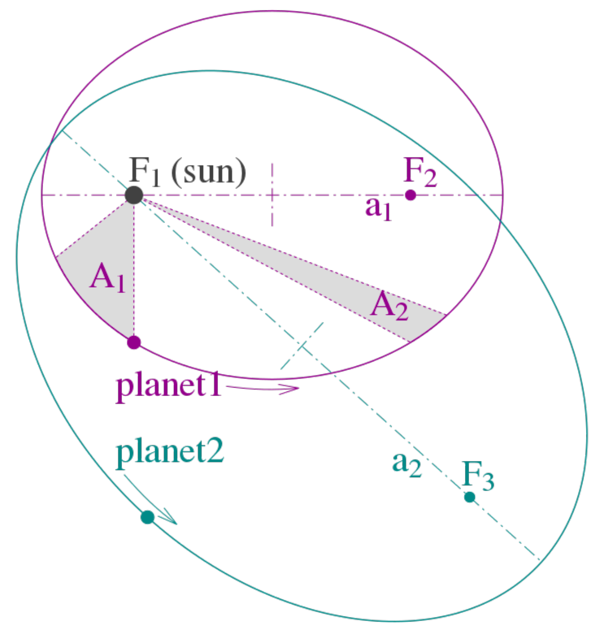
Used Tycho’s data to determine that the shape of the orbit of Mars is an ellipse with the Sun at one focus.

- Kepler tried a lot of geometrical solutions

- Abandoned sun-centered ellipse because it didn't fit Tycho's data

- Ellipse with the sun at one focus does fit, and does much better than Copernicus or Ptolemy.





Kepler's 3 Laws of planetary motion

1. The orbit of a planet is an ellipse with the Sun at one focus.

2. A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time.

3. The square of the orbital period of a planet is proportional to the cube of the semi-major axis of its orbit.

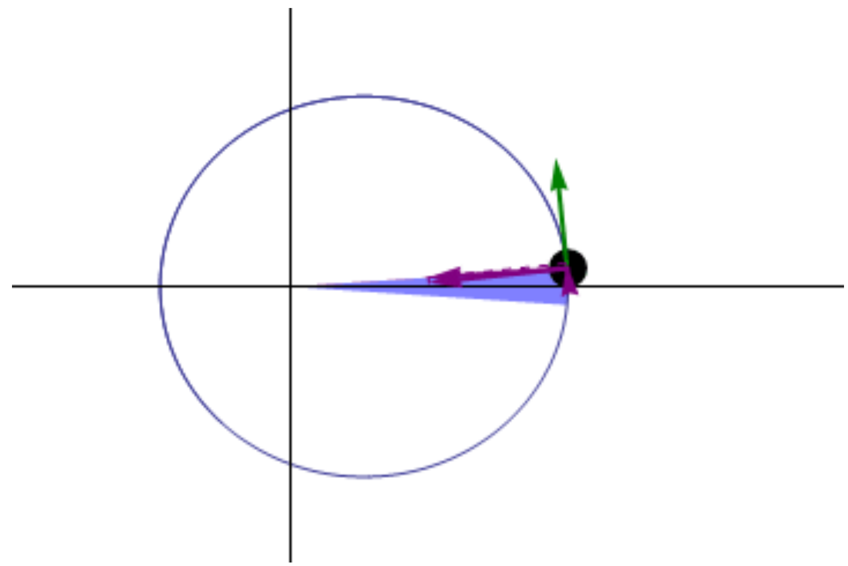
Johannes Kepler's Uphill Battle

Ellipses

- An ellipse is a kind of flattened circle.
- To draw an ellipse, put 2 fixed pushpins through your paper, loop a string around them, put the pen inside the loop, and keeping the string tight, draw.
- each pin is called a “focus”
- The sum of the distance from any point to focus 1, and the distance to focus 2, is constant along the ellipse.
- Kepler tells us the orbits are ellipses with the sun at one focus.

Equal areas per time

- Kepler says the line connecting the sun and a planet sweeps out equal areas per unit time (so many square meters per second, say).
- It turns out this is equivalent to saying the angular momentum of the planet is constant.
- Planets move faster when they're closer to the sun.



Orbital Period vs. Orbit Size

- The *semi-major axis* of an ellipse is half the length (longest dimension). Call it a. It's analogous to the radius of a circle.

- Newton's law of gravity lets us figure out what the constant in the equation is. Objects orbiting a mass M obey this equation:

- $GM / 4 \pi^2 = a^3 / t^2$

- $G = 6.67 \times 10^{-11}$ in metric units.

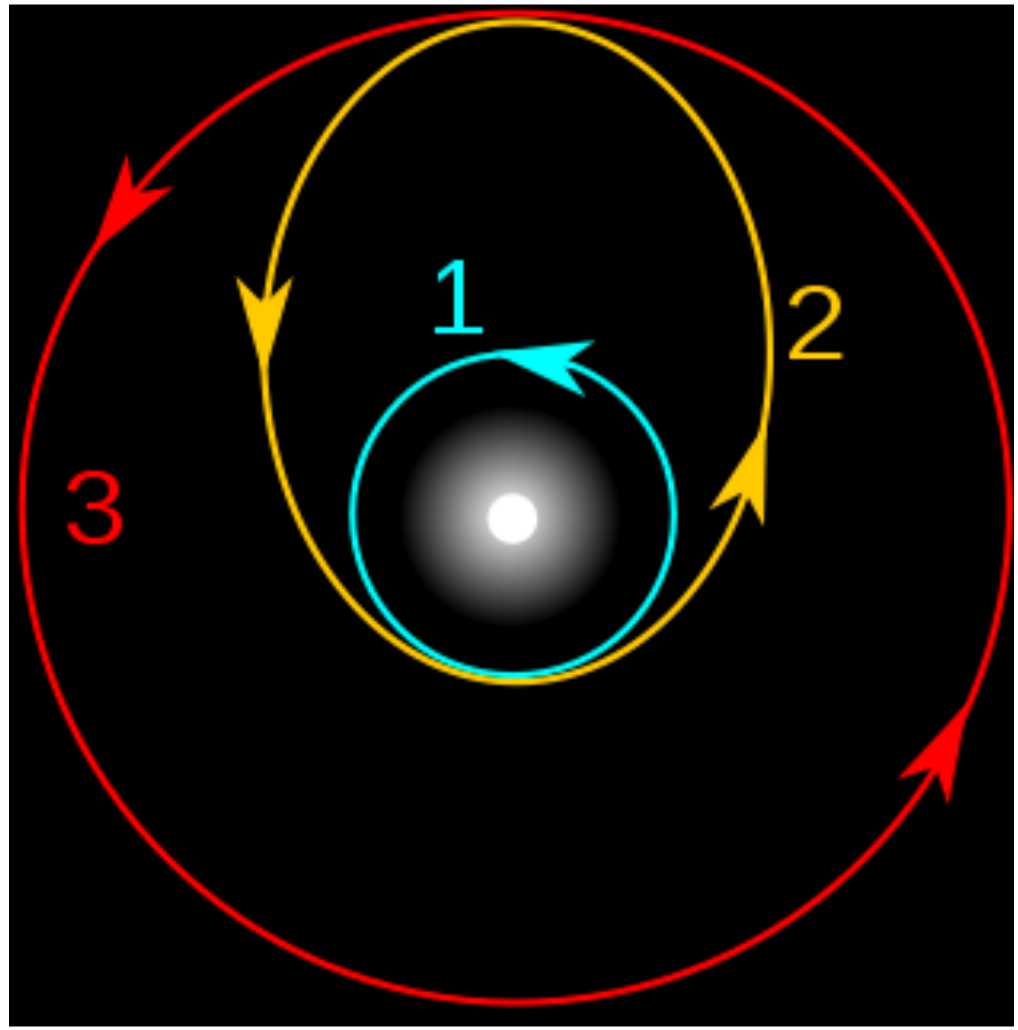
- Note that measuring the orbit sizes and periods for, say, Jupiter's moons, lets us calculate the mass of Jupiter.

- Cool thing: if you measure a in AU, t in years, and M in solar masses, the above equation is just $M = a^3 / t^2$

Bonus: Hohmann Transfer Orbit

- To get from one near-circular orbit to another (e.g. orbit of earth to mars), set up an ellipse [2] that's tangent to circle [1] at one end, and to circle [3] at the other.

- The semi-major axis is then: $a = (r_1 + r_3) / 2$. For earth & mars this is 1.26 AU.



- Kepler's 3rd law then says M = a^3 / t^2, but M=1 (the sun).
- So the orbital period of this ellipse is given by $t^2 = a^3 = 2$ to good accuracy. Therefore it takes half the square root of 2 years to get to Mars, or 0.707 years = 8.5 months.
- There's an opportunity to launch to Mars in Jul 2020, for arrival 8.5 months later in Mar 2021.
- A similar strategy is often used to launch communication satellites into geo-synchronous orbits 22,500 miles up.