An exact nonlinear cylindrical surface wave solution

Cite as: Physics of Fluids B: Plasma Physics 1, 1543 (1989); https://doi.org/10.1063/1.858932
Submitted: 28 February 1989 . Accepted: 30 March 1989 . Published Online: 04 June 1998

L. Stenflo, and M. Y. Yu

This term is negligible for $\nabla \ln \nu \cdot \nabla (1 - 1.46\lambda_m^2 \nabla^2) \ll \nabla^2$. Furthermore, because of the nonconstant value of ν_e, we would also have two additional terms $-(2/3)(D/T_0^{7/2})[\nabla T_1 \cdot \nabla T_0^{5/2} + \frac{5}{2} T_0^{3/2} \nabla T_1 \cdot \nabla T_0)]$—on the left-hand side of (8). Again, these terms can safely be ignored in the local approximation (viz., $\nabla \ln T_0 \cdot \nabla \ll \nabla^2$) for a linear temperature gradient profile $T_0(x) = T_{00}(1 + x/L_t)$. It turns out that the general dispersion relation (9) remains valid in the local approximation even in a collisional plasma.

We would also like to comment on the equilibrium conditions (1) and (2), which are obtained from the zeroth-order momentum and energy equations for the electron fluid assuming fixed ion background. When the ion dynamics is also included in the equilibrium, the gravitational force $n_0(m\mathbf{g}_e + M\mathbf{g}_i)$ equals the pressure gradient force $-\nabla[n_0(T_0 + T_{i0})]$, thereby maintaining the equilibrium. Here, M is the ion mass, $\mathbf{g}_j = -g_j \hat{x}$ is the acceleration of the plasma, and $\nabla(n_0 T_{i0})$ is the equilibrium ion pressure gradient. For $\nabla(n_0 T_{i0}) = 0$, the equilibrium condition reads $n_0(mg_e + Mg_i) = d_x(n_0 T_0)$.

In summary, we have presented a new class of radiative thermal instability in nonuniform unmagnetized electron plasmas. In contrast to previous investigations,[1–5] the present thermal instability involves condensation of magnetic fields of a flutelike electromagnetic radiation. The density condensation associated with the latter is zero. The present study thus suggests that impurity driven radiation losses can generate nonthermal magnetic field fluctuations in an inhomogeneous unmagnetized plasma. Finally, we would like to point out that the instability found here is quite different from that reported by Tidman and Shanny.[13] The latter authors have not considered impurity driven radiation and vortexlike magnetic field perturbations.

ACKNOWLEDGMENT

This research was performed under the auspices of the Deutsche Forschungsgemeinschaft through the Sonderforschungsbereich Plasmaphysik Bochum/Jülich.

[1] E. N. Parker, Astrophys. J. **117**, 431 (1953).

[2] G. B. Field, Astrophys. J. **142**, 531 (1965).

[3] T. E. Stringer, in *Proceedings of the 12th European Conference on Controlled Fusion and Plasma Physics*, Budapest, 1985 (European Physical Society, Budapest, 1985), Part I, p. 86.

[4] J. Neuhauser, W. Schneider, and R. O. Wunderlich, Nucl. Fusion **26**, 1679 (1986).

[5] J. F. Drake, Phys. Fluids **30**, 2429 (1987); J. F. Drake, L. Sparks, and G. Van Hoven, *ibid.* **31**, 813 (1988).

[6] B. Lipschutz, J. Nucl. Mater. **145–147**, 15 (1987).

[7] H. Wolff, Contrib. Plasma Phys. **28**, 131 (1988).

[8] P. C. Liewer, Nucl. Fusion **25**, 543 (1985).

[9] B. A. Al'terkop, E. V. Mishin, and A. A. Rukhadze, Zh. Eksp. Teor. Fiz. Pis'ma. Red. **19**, 291 (1974) [JETP Lett. **19**, 170 (1974)].

[10] S. I. Braginskii, in *Reviews of Plasma Physics*, edited by M. A. Leontovich (Consultants Bureau, New York, 1965), Vol. 1, p. 205.

[11] M. Y. Yu and L. Stenflo, Phys. Fluids **28**, 3447 (1985).

[12] C. Chu, M. S. Chu, and T. Ohkawa, Phys. Rev. Lett. **41**, 653 (1978).

[13] D. Tidman and R. Shanny, Phys. Fluids **17**, 1207 (1974).

An exact nonlinear cylindrical surface wave solution

L. Stenflo
Department of Plasma Physics, Umeå University, S-90187 Umeå, Sweden

M. Y. Yu
Ruhr University, D-4630 Bochum, Federal Republic of Germany

(Received 28 February 1989; accepted 30 March 1989)

An exact solution for strongly nonlinear electrostatic surface waves on a cylindrical boundary of a plasma medium is presented. The effective nonlinear dielectric constant turns out to be dependent upon the geometry as well as the electric field amplitude.

Surface wave phenomena are of fundamental importance in many branches of physics. In particular, the problem of nonlinear surface wave propagation has recently gained much attention in the study of optics, acoustics, condensed matter, and fluids, as well as in laboratory and space plasmas.[1] Common to many areas of research are localized nonlinear waves maintained by electronic motion, such as excitons and polaritons in solids and solitons in optical fibers and plasmas.

Most works on nonlinear surface waves concern weakly nonlinear states and use various *ad hoc* approximation schemes. It is thus of interest to have particular examples of exact solutions. Besides being useful for direct comparison with experiments, such exact solutions are valuable as a guide for evaluating the validity of approximation methods and numerical simulations. Accordingly, we present in this Brief Communication an exact strongly nonlinear surface wave (SNSW) solution for an electronic medium. In particular, we point out that the effective nonlinear dielectric constant ϵ_{nl} can be field as well as geometry dependent. This is in contrast to all the earlier descriptions of surface and guided waves in nonlinear media, where the geometry dependence originates from the linear properties of the materials.

The general equation governing the nonlinear wave motion in a cold electron plasma medium has been derived earlier.[2,3] For an oscillating electrostatic potential

0899-8221/89/071543-02$01.90

$\phi(\mathbf{r},t) = \phi(\mathbf{r})\exp(-i\omega t) + \text{c.c.}$, where ω is the wave frequency, the equation governing the propagation of nonlinear waves is[2]

$$\nabla\cdot\epsilon\nabla\phi = (q^2/m^2\omega^4)\nabla\cdot[(\nabla^2|\nabla\phi|^2)\nabla\phi], \tag{1}$$

where $\epsilon = 1 - \omega_p^2/\omega^2$ is the linear dielectric constant, ω_p is the equilibrium plasma frequency, and q/m is the electron charge to mass ratio. The right-hand side of Eq. (1) originates from the nonlinear contribution to the electron density fluctuations. Equation (1) has been derived by assuming that the ponderomotive force acting on the electrons is balanced by the slowly varying ($\omega_p \gg \partial_t \gg \omega_{pi}$, where ω_{pi} is the equilibrium ion plasma frequency) part of the electrostatic force, while the nonlinear density is obtained from the Poisson equation. The effect of second harmonic generation[3] has been neglected in (1) for simplicity. No assumption is made on the relative magnitudes of ω and ω_p.

Equation (1) is highly nonlinear. For surface wave applications, it is also at least two-dimensional. Approximate small amplitude soliton solutions have been obtained for a semi-infinite plasma.[2] In the following, we present an exact strongly nonlinear surface wave solution of Eq. (1) for the case of a cylindrical electron plasma of radius R and constant density N_0 bounded by vacuum.

It can easily be verified that an *exact* nontrivial solution of Eq. (1) is

$$\phi_{r<R} = \phi_m(r^2/R^2)\cos 2\theta, \tag{2a}$$

while the corresponding vacuum solution outside the cylinder, satisfying the Laplace equation, is

$$\phi_{r>R} = \phi_m(R^2/r^2)\cos 2\theta. \tag{2b}$$

Here, r and θ are the polar coordinates and ϕ_m is the electrostatic potential at the boundary.

The boundary condition to be satisfied may be obtained in the usual manner by integration across the cylindrical surface. We have

$$\epsilon_{\rm nl}\frac{\partial\phi_{r<R}}{\partial r_{r=R}} = \frac{\partial\phi_{r>R}}{\partial r_{r=R}}, \tag{3a}$$

where the nonlinear dielectric constant $\epsilon_{\rm nl}[=1-\omega_p^2/\omega^2-(q/m\,\omega^2)^2\nabla^2|\nabla\phi|^2]$, in view of Eqs. (1) and (2a), can be written as

$$\epsilon_{\rm nl} = 1 - \omega_{p0}^2/\omega^2 - 16q^2\phi_m^2/m^2\omega^4R^4. \tag{3b}$$

Here, we have defined $\omega_{p0}^2 = N_0q^2/\epsilon_0 m$. We note that the nonlinear part of $\epsilon_{\rm nl}$ is dependent on R.

Substituting Eqs. (2a) and (2b) into Eq. (3a), we obtain the dispersion relation

$$\epsilon_{\rm nl} = -1, \tag{4}$$

from which one can easily solve for the frequency ω.

We note that Eq. (4) reduces to the linear dispersion relation $\omega = \omega_{p0}/\sqrt{2}$ when $\phi_m \to 0$ or $R \to \infty$ for fixed electric field strength. For weak nonlinearity, one can expand the nonlinear term, and obtain

$$\omega \approx (\omega_{p0}/\sqrt{2})(1 + \alpha q^2\phi_m^2/m^2\omega_{p0}^4R^4), \tag{5}$$

where $\alpha = 16$.

Although Eq. (4) is similar in form to the well-known linear dispersion relation of surface waves, it actually differs significantly from the latter. First, the dielectric constant $\epsilon_{\rm nl}$ given by Eq. (3b) depends not only on the field strength, but also on the geometry. This is in contrast with the usual description[1,4] that the dielectric constant should only depend on the field strength when nonlinearities are included [that is, the last term in Eq. (3b) would be proportional to ϕ_m^2/R^2 rather than the present ϕ_m^2/R^4]. Furthermore, in the case where $\phi_m \gg m\,\omega_{p0}^2R^2/q$ one obtains from (3b) and (4) the frequency

$$\omega \approx 2^{3/4}(q\phi_m/mR^2)^{1/2}, \tag{6}$$

which describes a new type of high-frequency ($\omega \gg \omega_p$) SNSWs. Such SNSWs, or "oscillons," have, to our knowledge, not been discussed previously in plasma or condensed matter physics. It is of interest to note that the frequency given by (6) resembles the bounce frequency of an electron in an oscillating potential.

The present theory may be generalized to include finite length effects by introducing the additional spatial dependence exp (ikz), where z is the direction along the cylinder axis.[5] In this case, no exact solution can be found. One can, nevertheless, obtain a rough qualitative correction of $\epsilon_{\rm nl}$ by multiplying the nonlinear term by the factor $(1 + k^2R^2)$. The effect of second harmonic generation on Eq. (1) has been derived in Ref. 3. It turns out that when such terms are included, the corresponding solution (2a) is unaffected, but the constant α in (5) is changed to -8, whereas our other finding, namely Eq. (6), remains unchanged. Furthermore, it is straightforward to extend our calculations to the case in which the electronic medium is bounded by a linear dielectric rather than vacuum.[5] On the other hand, for more realistic (nonconstant) density profiles or more complicated geometries, numerical work would be required. The temporal evolution and the stability of the present solution could then also be investigated. Finally, we point out that the SNSWs found here may be easily verifiable experimentally because of the strong dependence on the radius.

ACKNOWLEDGMENT

One of us (M.Y.Y.) is partially supported by the Sonderforschungsbereich 162 Plasmaphysik Bochum/Jülich.

[1]See, for example, *Surface Waves in Plasmas and Solids*, edited by S. Vukovic (World Scientific, Singapore, 1986).

[2]O. M. Gradov and L. Stenflo, Phys. Fluids **25**, 983 (1982).

[3]L. Stenflo and O. M. Gradov, IEEE Trans. Plasma Sci. **PS-14**, 554 (1986).

[4]A. Shivarova and M. Y. Yu, Opt. Commun. **57**, 171 (1986); A. D. Boardman, G. S. Cooper, A. A. Maradudin, and T. P Shen, Phys. Rev. B **34**, 8273 (1986).

[5]M. Moisan, A. Shivarova, and A. W. Trivelpiece, Plasma Phys. **24**, 1331 (1982); B. Buti, in *Advances in Space Plasma Physics*, edited by B. Buti (World Scientific, Singapore, 1985), p. 167; I. Zhelyazkov, in *The Physics of Ionized Gases*, edited by J. Purić and D. Belić (World Scientific, Singapore, 1987), p. 459.