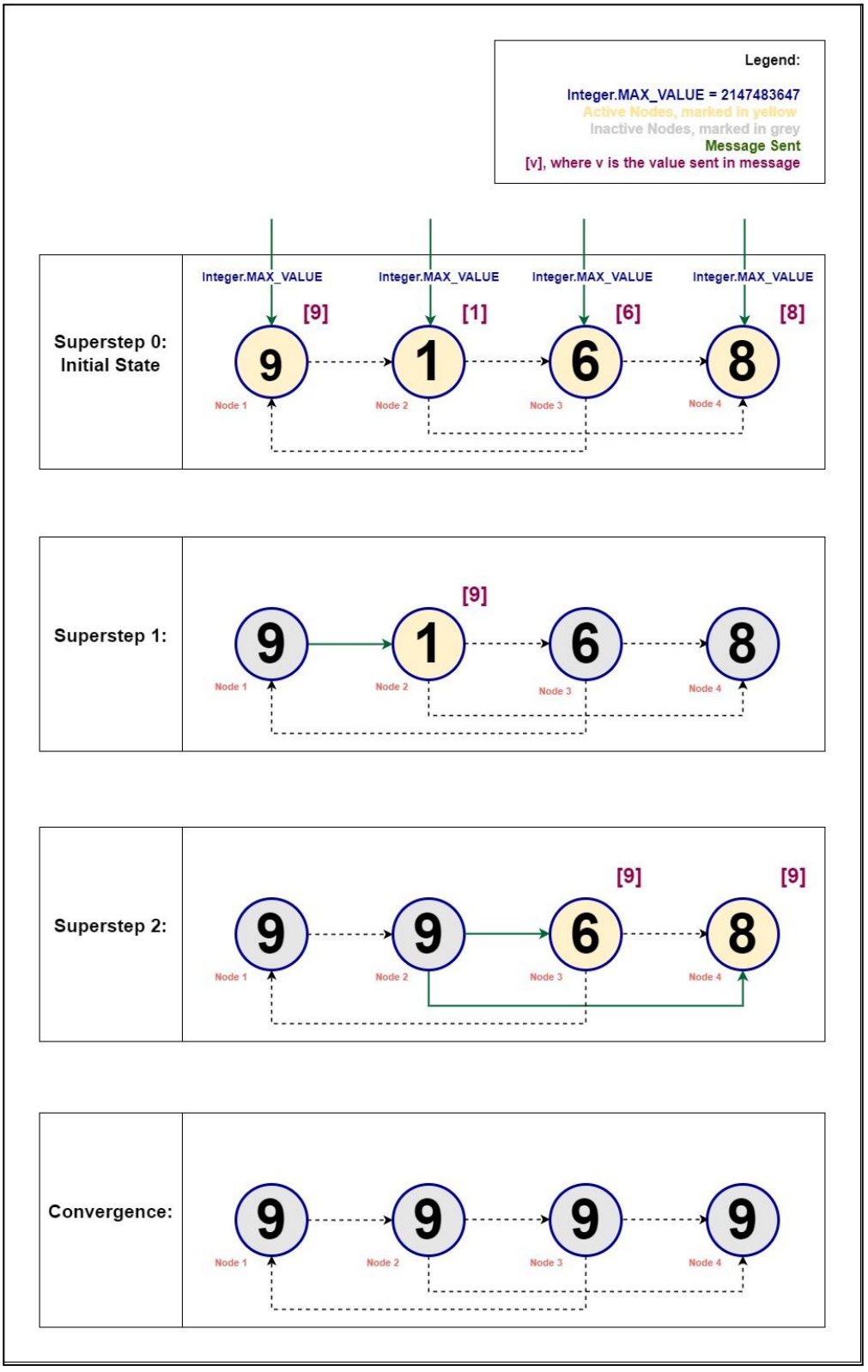
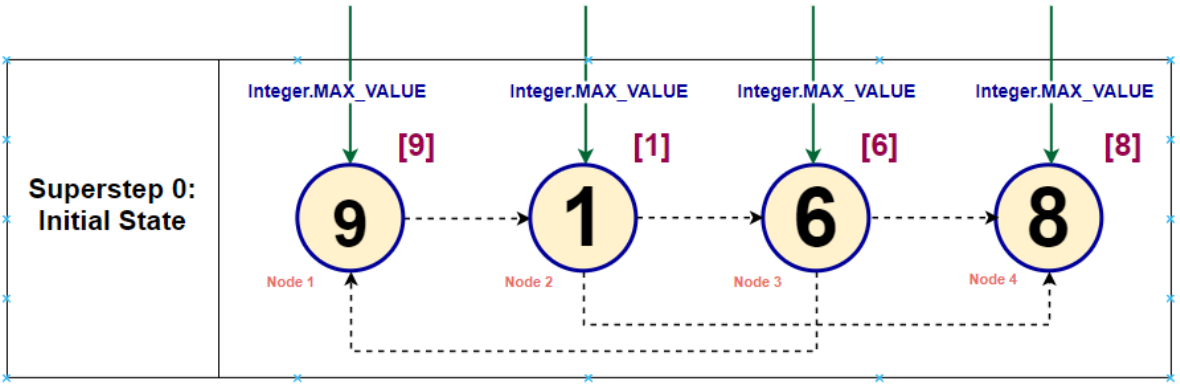
Exercise 1: The following diagram depicts the process to identify the maximum value in the graph. Note that the end state of the superstep is indicated in the diagram. Also, the maximum value of Integer.MAX_VALUE is considered the maximum value of an integer in Java.



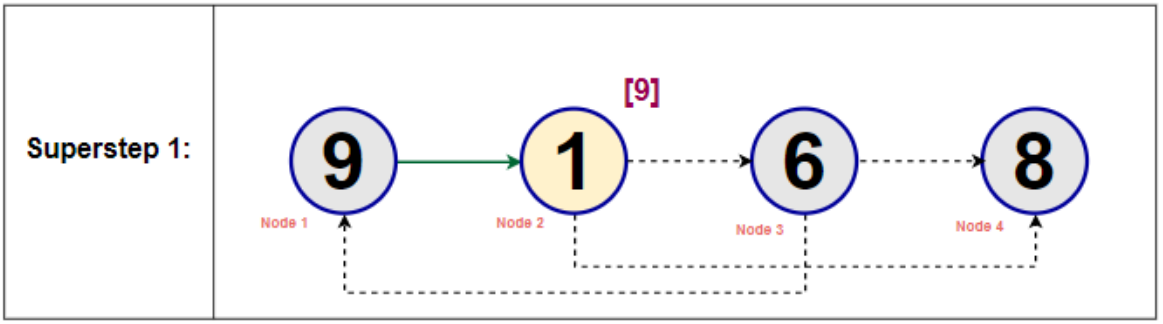
Explanation: The following steps elaborate on the processes that every node goes through in each superstep.

Case - Superstep 0:



- Initial State: All nodes are inactive in this superstep.
- Message Received: All the nodes receive the default Integer.MAX_VALUE in the form of a message in this superstep, initiating the process and activating them.
- Calculations: For all nodes, the message received is the same as the node's value, hence the corresponding node values are sent.
- Result State: All nodes become active at the end of this superstep.
- Message Sent: Corresponding node values are sent. Node 1 - sends 9, Node 2 - sends 1, Node 3 - sends 6 and Node 4 - sends 8.

Case - Superstep 1:

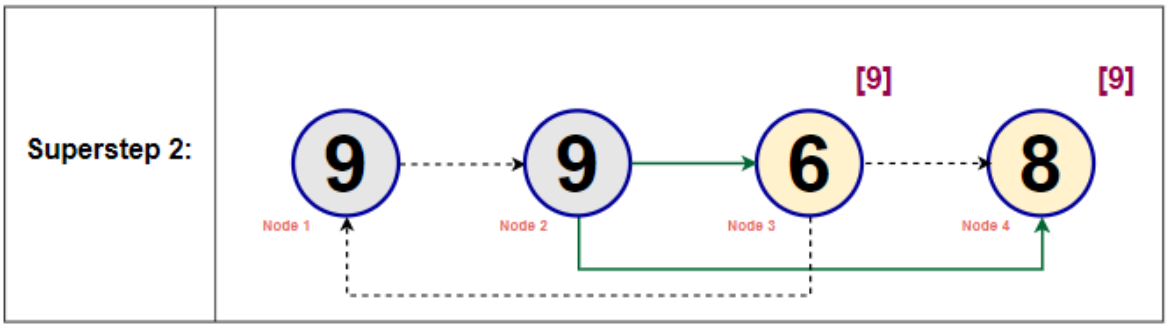


- Initial State: All nodes are active in this superstep.
- Message Received: Node 1 - receives 9, Node 2 - receives 1, Node 3 - receives 6 and Node 4 - receives 8.
- Calculations: Each node cross-checks with the nodes which have an outgoing edge directed towards it from this particular node. If the value of the destination node is lesser than the source node, a message is sent to that node.

- Node 1 - value 9: Node 2 has a value (1), which is lesser than 9, hence the message is sent to Node 2 from Node 1.
- Node 2 - value 1: Since the value is lesser than that of Node 3 (6) and Node 4 (8), no message is sent.
- Node 3 - value 6: Since node 1 has value 9, which is more than its value (6), no message is sent.
- Node 4 - value 8: Since, this node has no outgoing edges, no message is sent from this.

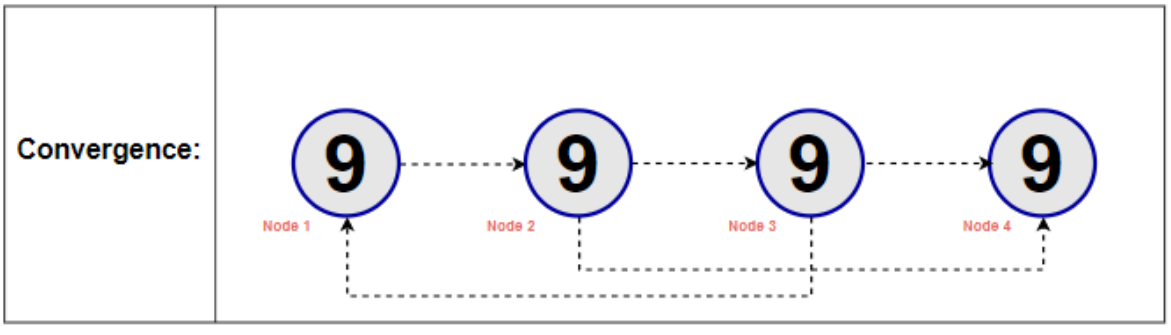
- Result State: Node 2 gets a new value of 9, hence is the only active node at the end of this superstep, all the other nodes are inactive.
- Message Sent: Node 1 - sends 9

Case - Superstep 2:



- Initial State: Only Node 2 is active in this superstep.
- Message Received: Node 2 - receives 9.
- Calculations: Again, each active node - Node 2, in this case, cross-checks with the nodes which have an outgoing edge directed towards it from this particular node. If the value of the destination node is lesser than the source node, a message is sent to that node.
  - Node 2 - value 9: Since Node 3 (6) and Node 4 (8) have a value smaller than Node 2 (9), a message is sent to them.
- Result State: Node 3 and Node 4 get a new value of 9, hence they are the active nodes at the end of this superstep, all the other nodes - Node 1 and Node 2 are inactive.
- Message Sent: Node 2 - sends 9

Case - Superstep 3 (Convergence):



- Initial State: Node 3 and Node 4 are active in this superstep.
- Message Received: Node 3 and Node 4 - receive 9.
- Calculations: Again, each active node - Node 3 and Node 4, in this case, cross-checks with the nodes which have an outgoing edge directed towards it from this particular node. If the value of the destination node is lesser than the source node, a message is sent to that node. Since, all the nodes have a value of 9 at the end of this superstep, none of the nodes need to send a message, thereby no new changes.
- Result State: All the nodes are inactive at the end of this superstep.
- Message Sent: No messages are being sent at the end of this superstep, resulting in convergence.

Exercise 4:

Parameterizing the Page Rank Algorithm: In our solution, the damping factor is chosen as 0.99 (since the reset probability is 0.01) and the maximum number of iterations chosen until the algorithm converges is 5.

Team Members:

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