NUMBERSYSTEM 11

Number of Factors of a Number

N is a composite number $N = a^p \times b^q \times c^r \times \ldots\ldots$

a, b, c are prime factors of N & p, q, r are positive integers.

Then number of factors of **N = (p + 1) (q + 1) (r + 1) …….**

Example 1 $6 = 2^1 \times 3^1 \rightarrow (1 + 1)(1 + 1) = 4$ factors [1, 2, 3, 6] = 4 factors

Example 2 $140 = 2^2 \times 5^1 \times 7^1 \rightarrow (2 + 1)(1 + 1)(1 + 1) = 12$ factors {including 1 & N}

140 has 10 factors excluding 1 and itself.

Product of factors of composite number N….

Product of factors of N = $N^{n/2}$ where n is total number of factors of N.

Example 3 $N = 24 = 2^3 \times 3^1$ total factors = $(3 + 1)(1 + 1) = 8$, so product = $N^{n/2} = (24)^{8/2} = (24)^4$

Number of ways of expressing a given number as a product of two factors …

Number N can be expressed as the product of two factors in different ways.

Number of ways this can be done (1/2) {(p + 1) (q + 1) (r + 1) ….. },

So 140 can be expressed as a product of two factors in 12/2 = 6 ways.

- If p, q, r.. etc. are all even, then product (p + 1) (q + 1) (r + 1) ….. becomes odd, so we cannot take (1/2) of an odd number to get number of ways.
- If p, q, r … are all even, it mean number N is a perfect square.

Number of ways in which perfect square can be expressed as a product of two different factors.

(a) as a product of two different factors → (1/2) {(p + 1) (q + 1) (r + 1) ….. – 1} ways (excluding $\sqrt{N} \times \sqrt{N}$)

(b) as a product of two factors → (1/2) {(p + 1) (q + 1) (r + 1) ….. +1} ways (including $\sqrt{N} \times \sqrt{N}$)

Ex. 3. Find number of factors of 1225. $1225 = 5^2 \times 7^2 \rightarrow (2 + 1)(2 + 1) = 9$ factors.

Ex. 4. How many divisors excluding 1 and itself does the number 4320 have.

$4320 = 3^3 \times 2^5 \times 5^1 \rightarrow (3 + 1)(5 + 1)(1 + 1) = 48$ factors

Excluding 1 and itself (48 – 2) = 46 factors.

Ex. 5. In how many ways can 3420 be written as a product of two factors.

$3420 = 2^2 \times 3^2 \times 5^1 \times 19^1 \rightarrow (1/2)\{(2 + 1)(2 + 1)(1 + 1)(1 + 1)\} = 18$ ways.

Ex. 6. In how many ways can the number 52900 be written as a product of two different factors.

$52900 = 2^2 \times 5^2 \times 23^2 \rightarrow$ perfect square

Sum of all factors of a Number …

$N = a^p \times b^q \times c^r \times$ sum of all factors of N {including 1 and number itself}

$$= \frac{(a^{p+1}-1)}{(a-1)} \times \frac{(b^{q+1}-1)}{(b-1)} \times \frac{(c^{r+1}-1)}{(c-1)} \ldots\ldots$$

Ex. 7. Find sum of factors of 48.

$$48 = 2^4 \times 3^1$$

Sum $= \dfrac{(2^{4+1}-1)}{(2-1)} \times \dfrac{(3^{1+1}-1)}{(3-1)} = (31 \times 8)/2 = 124$

Factors of 48 = 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

Sum = 1 + 2 + 3 + 4 + 6 + 8 + 12 + 16 + 24 + 48 = 124.

Number of ways of writing a number as product of two coprimes....

$$N = a^p \times b^q \times c^r \times \ldots\ldots$$

Number of ways of writing N as a product of 2 coprimes is 2^{n-1}, where n is number of distinct prime factors of given number N.

Ex. 8. $48 = 2^4 \times 3^1$ number of ways $= 2^{n-1} = 2^{2-1} = 2 \rightarrow$ (1 and 48) and (3 and 16)

Number of coprimes to N, that are less than N…

If $N = a^p \times b^q \times c^r \times \ldots\ldots$ then, number of coprimes of N, which are less than N, represented by

ϕ (N) is $= n\left(1-\dfrac{1}{a}\right)\left(1-\dfrac{1}{b}\right)\left(1-\dfrac{1}{c}\right)\ldots$

Ex. 9. $48 = 2^4 \times 3^1$ $\phi(48) = 48\left(1-\dfrac{1}{2}\right)\left(1-\dfrac{1}{3}\right) = 16$

If numbers less than 48 are listed and coprimes to 48 are picked-up, count of coprimes will be 16.

Sum of coprimes to N are less than N….

$$= (N/2) \times \phi(N)$$

Ex. 10. $4^8 = 2^4 \times 3^1$ $\phi(48) = 48\left(1-\dfrac{1}{2}\right)\left(1-\dfrac{1}{3}\right) = 16$

Sum of coprimes of 48, that are less than 48

$$= (N/2) \times \phi(N) = (48 \times 16) / 2 = 384.$$

PERFECT SQUARES ARE SQUARES OF NATURAL NUMBERS

Perfect Squares are squares of natural numbers, for the number, $(p_1^a \times p_2^b \times p_3^c \times \text{-----------})^{1/2}$ to be a natural number, each a, b, c should be divisible by 2 i.e. a,b, c. should be even.

Similarly $(p_1^a \times p_2^b \times p_3^c \times \ldots\ldots\ldots)$ will be a perfect cube if and only if each of a,b,c be divisible by 3 i.e a multiple of 3.

A number $(p_1^a \times p_2^b \times p_3^c \times \ldots\ldots\ldots)$ would be a perfect square as well as perfect cube when each a, b, c is a multiple of 2 as well as of 3 i.e. a multiple of 6.

Example a(a) With what least number should 864 be multiplied so that it is a perfect square as well as a perfect cube.

$$864 = 8 \times 9 \times 12 = 2^3 \times 3^2 \times 2^2 \times 3^1 = 2^5 \times 3^3$$

To be a square and cube, multiple of 6, so it has to be multiplied with $2^1 \times 3^3 = 54$ to be a square and cube.

Example b(b) With what least number should 40500 be divided so that quotient is odd and a perfect cube.

$$40500 = 2^2 \times 3^4 \times 5^3$$

to be odd exponent of 2 is zero, for cube power of 3 should be a multiple of 3. So it should be divided by $2^2 \times 3^1 = 12$.

Example c(c) Find the factors of $2^7 \times 3^4 \times 7^3$ that are even.

A factor of $2^7 \times 3^4 \times 7^3$ is of type $2^{0 \text{ to } 7} \times 3^{0 \text{ to } 4} \times 7^{0 \text{ to } 3}$

For factor to be even, exponent of 2 has to be at least 1, $2^{1 \text{ to } 7} \times 3^{0 \text{ to } 4} \times 7^{0 \text{ to } 3}$

2 can assume 7 distinct values, 3 can assume 5 distinct values, 7 can assume 4 distinct values,

So number of factors that are even $= 7 \times 5 \times 4 = 140$.

Example d(d) Find number of factors of $2^7 \times 3^4 \times 7^3$ that are perfect squares.

A factor of $2^7 \times 3^4 \times 7^3$ is of type $2^{0 \text{ to } 7} \times 3^{0 \text{ to } 4} \times 7^{0 \text{ to } 3}$

For factor to be perfect square, exponent of each 2, 3 and 7 has to be even number.

Thus required factors could now only be of the form

$2^{0 \text{ or } 2 \text{ or } 4 \text{ or } 6} \times 3^{0 \text{ or } 2 \text{ or } 4} \times 7^{0 \text{ or } 2}$

Exponent 2 can assume 4 distinct values, Exponent 3 can assume 3 distinct values

Exponent 7 can assume 2 distinct values, So number of factors that are perfect squares are

$$4 \times 3 \times 2 = 24.$$

Example f(f) Find number of factors of 6912 that are also the factors of 3888.

$$6912 = 2^8 \times 3^3 \text{ and } 3888 = 2^4 \times 3^5$$

A factor of $2^8 \times 3^3$ and $2^4 \times 3^5$ necessarily has to be of the form $2^{0 \text{ to } 4} \times 3^{0 \text{ to } 3}$

Common Factors $= 5 \times 4 = 20$.