

Number system 11

[AA]

1. Total number of factors of 540. (a) 20 (b) 24 (c) 30 (d) 48
2. Total divisors of 10500 except 1 and itself is (a) 48 (b) 50 (c) 46 (d) 56
3. Sum of factors of 270. (a) 270 (b) 700 (c) 710 (d) 720
4. Sum of factors of 1520 except unity is (a) 3720 (b) 2730 (c) 2370 (d) 3719
5. Product of factors of 360 is (a) $(360)^{12}$ (b) $(36)^{120}$ (c) $(360)^{22}$ (d) $(6^{24} \times 10^{10})$
6. Find the number of numbers, coprime to 289 and less than it (a) 272 (b) 273 (c) 16 (d) 270
7. Find the sum of the coprimes of 48 less than 48 (a) 360 (b) 136 (c) 388 (d) 384
8. Find the number of factors $2^{10} \times 10^2$ (a) 33 (b) 24 (c) 20 (d) 39

$$N = A^2 \times B^3 \times C^4$$

9. How many factors of N are perfect squares. (a) 6 (b) 12 (c) 8 (d) 10
10. How many factors of N are cube of any natural number. (a) 2 (b) 3 (c) 4 (d) 6

Solution 1-b, 2-c, 3-d, 4-d, 5-a, 6-a, 7-d, 8-d, 9-b, 10-c

[BB]

Consider the number $2^4 \times 3^7 \times 5^2$

1. Number of factors
(a) 110 (b) 120 (c) 240 (d) 12
2. How many are odd
(a) 24 (b) 30 (c) 42 (d) 56
3. How many are even
(a) 60 (b) 75 (c) 96 (d) 105
4. How many are perfect squares
(a) 6 (b) 12 (c) 18 (d) 24
5. How many of them are cubes
(a) none (b) 2 (c) 3 (d) 6
6. How many of them are multiples of $2 \times 3 \times 5$
(a) 24 (b) 32 (c) 40 (d) 56
7. How many of them are perfect squares as well as perfect cube.
(a) none (b) 1 (c) 2 (d) 3

SOLUTION

$$N = (p + 1) (q + 1) (r + 1) \dots$$

$$2^4 \times 3^7 \times 5^2$$

1. $N = (4 + 1) (7 + 1) (2 + 1) = 120$

2. To be odd exponent of 2 should be zero.

3 take values from 3^0 to 3^7 i.e. $\Rightarrow 8$

5 take values from 5^0 to 5^2 i.e. $\Rightarrow 3$

$$\text{Number of odd factors} = 3 \times 8 = 24$$

3. To be even exponent of 2 should be at least 1 i.e. 2^1 to $2^4 \Rightarrow 4$ values

$$3^0 \text{ to } 3^7 \Rightarrow 8 \text{ values}$$

$$5^0 \text{ to } 5^2 \Rightarrow 3 \text{ values}$$

$$\text{Even factors} = 4 \times 8 \times 3 = 96$$

4. For factor to be perfect square exponent of 2, 3 and 5 should be even.

$$2^0 \text{ or } 2^2 \text{ or } 2^4 \times 3^0 \text{ or } 3^2 \text{ or } 3^4 \times 5^0 \text{ or } 5^2$$

$$3 \text{ values} \times 4 \text{ values} \times 2 \text{ values} = 24.$$

5. For factor to be perfect cube, exponent of 2, 3 & 5 should be multiple of 3.

$$2^0 \text{ or } 2^3 \times 3^0 \text{ or } 3^3 \times 5^0$$

$$2 \text{ values} \times 3 \text{ values} \times 1 \text{ value}$$

$$2 \times 3 \times 1 = 6$$

6. To be a multiple of $2 \times 3 \times 5$, there should be atleast one 2, one 3 & 5.

$$2^1 \text{ to } 2^4 \times 3^1 \text{ to } 3^7 \times 5^1 \text{ to } 5^2$$

$$4 \times 7 \times 2 = 56$$

7. To be a perfect square as well as cube, exponent should be a multiple of (2×3) i.e. 6

$$2^0 \times 3^0 \text{ or } 6 \times 5^0$$

$$1 \times 2 \times 1 = 2$$