

will it have, provided the height of the cone was 2 cm and diameter of the base was 4 cm, respectively?

Solution See the figure below.
Assume BPC be the hemisphere and ABC is the cone standing on the base of the hemisphere.

Radius BO of the hemisphere (as well as of the cone)

$$\frac{1}{2} \times 4 \text{ cm} = 2 \text{ cm}$$

Now, let the right circular cylinder EFGH circumscribe the given solid.

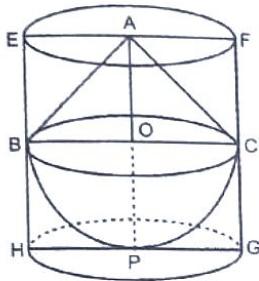
Radius of the base of the right circular cylinder = HP

$$= BO = 2 \text{ cm}$$

Height of the cylinder = AP = AO + OP = 2 cm + 2 cm

$$= 4 \text{ cm}$$

Now, volume of the right circular cylinder - volume of the solid



$$= \left[\pi \times 2^2 \times 4 - \left(\frac{2}{3} \times \pi \times 2^3 + \frac{1}{3} \times \pi \times 2^3 \right) \right] \text{cm}^3$$

$$= (16\pi - 8\pi) \text{ cm}^3$$

$$= 8\pi \text{ cm}^3$$

Hence, the right circular cylinder is having $8\pi \text{ cm}^3$ more space than the solid.

Example 6 A toy consists of a base that is the section of a sphere and a conical top. The volume of the conical top is 30π sq. units and its height is 10 units. The total height of the toy is 19 units. The volume of the sphere (in cubic units), from which the base has been extracted, is:

$$(a) \frac{256}{3}\pi \quad (b) \frac{64}{3}\pi$$

$$(c) \frac{108}{3}\pi \quad (d) \frac{500}{3}\pi$$

Solution Height of the cone = 10 units

Volume of the cone = 30π cubic units = Diameter of the cross-section from where the sphere has been sectioned = 6 units

$$= r + \sqrt{2^2 - 3^2} = 9$$

$$= r = 5 \text{ units}$$

= Volume of the original sphere from which the base has been sectioned = $\frac{4}{3}\pi r^3 = \frac{500}{3}\pi$

Practice Exercises

WARM UP

- Q.1 If the side of a cube is increased by 100%, find by what percentage the surface area of the cube is increased?
(a) 150% (b) 200% (c) 300% (d) 350%
- Q.2 A banquet hall has the dimensions $30 \text{ m} \times 12 \text{ m} \times 6 \text{ m}$. Each person should get 8 m^3 of space. Find the number of persons who can be accommodated in this hall.
(a) 240 (b) 250 (c) 270 (d) 300
- Q.3 Find the height of the cylinder whose volume is 511 cm^3 and the area of the base is 36.5 cm^2 .
(a) 3.5 cm (b) 10.5 cm
(c) 14 cm (d) None of these
- Q.4 The ratio between the curved surface area and the total surface area is 2:3 and the total surface is 924 cm^2 . What is the volume of the cylinder?
(a) 2156 cm^3 (b) 2183 cm^3
(c) 2492 cm^3 (d) None of these

- Q.5 If the curved surface area of a cylinder is 1320 cm^2 and its base radius is 21 cm, then what is its total surface area?
(a) 4092 cm^2 (b) 2084 cm^2
(c) 5104 cm^2 (d) None of these

- Q.6 The radius of an iron rod is decreased to one-fourth of its actual radius. If its volume remains constant, then the length will become:
(a) 2 times (b) 12 times
(c) 8 times (d) 16 times

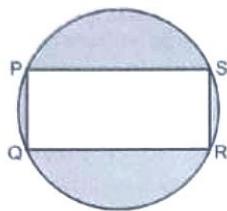
- Q.7 A reservoir is in the shape of a frustum of a right circular cone. It is 8 m wide at the top and 4 m wide at the bottom. If it is 6 m deep, then what is its volume?
(a) 224 m^3 (b) 176 m^3
(c) 204 m^3 (d) None of these

- Q.8 A spherical metal ball of 6 cm radius is melted and recast into three spherical balls. The radii of two of

these balls are 3 cm and 4 cm. What is the radius of the third ball?
 (a) 4.5 cm (b) 5 cm (c) 6 cm (d) 7 cm

Q.9 If the diagonals of a rhombus are 18 cm and 24 cm, respectively, then find its perimeter.
 (a) 15 cm (b) 42 cm (c) 60 cm (d) 68 cm

Q.10 In the figure shown below, PQRS is a rectangle of the dimension 8 cm \times 6 cm and is inscribed in a circle. Find the area of the shaded portion.



- (a) 44 cm² (b) 34.25 cm²
 (c) 32.50 cm² (d) None of these

Q.11 The radius of the base and the volume of the right circular cone are doubled. What is the ratio of the length of the larger cone to that of the smaller cone?
 (a) 1:4 (b) 1:2 (c) 1:3 (d) 4:1

Q.12 A cone and a hemisphere have equal base radius and equal volumes. The ratio of their heights is:
 (a) 3:1 (b) 2:1
 (c) 4:1 (d) None of these

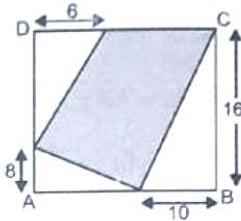
Q.13 If the right circular cone is cut into three solids of volumes V_1 , V_2 , and V_3 by two cuts which are parallel to the base and trisects the altitude, then $V_1:V_2:V_3$ is:
 (a) 1:2:3 (b) 1:4:6
 (c) 1:6:9 (d) None of these

Q.14 Water flows at the rate of 10 m per minute from a cylindrical pipe of radius 2.5 mm. A conical vessel whose diameter is 40 cm and depth 24 cm is filled with water flowing from this pipe. The time taken to fill the conical vessel is
 (a) less than 30 min.
 (b) less than 50 min but more than 30 min.
 (c) more than 50 min but less than 75 min.
 (d) more than 75 min.

Q.15 If a regular square pyramid has a base of side 8 cm and height of 30 cm, then what is its volume (in cc)?
 (a) 360 (b) 480
 (c) 640 (d) 800

Q.16 A cylinder circumscribes a sphere. The ratio of their volumes is:
 (a) 2:1 (b) 3:2 (c) 4:3 (d) 6:5

Q.17 Find the area of the shaded region in the given figure of square ABCD.



- (a) 128 cm² (b) 184 cm²
 (c) 154 cm² (d) 168 cm²

Q.18 An open box is made of wood 2 cm thick. Its internal dimension is 86 cm \times 46 cm \times 38 cm. What is the cost of painting the outer surface of this box at ₹10 per m²?
 (a) ₹12.35 (b) ₹8.85 (c) ₹15.70 (d) ₹16.50

Q.19 A copper wire when bent in the form of a square encloses an area of 121 m². If the same wire is bent to form a circle, the area enclosed by it would be
 (a) 124 m² (b) 136 m² (c) 154 m² (d) 184 m²

Q.20 A spherical ball was painted black. After getting painted, it was cut into four similar pieces. What is the ratio of the painted area to the non-painted area?
 (a) 1:1 (b) 1:2 (c) 3:1 (d) 3:2

Q.21 Volumes of a sphere, a cube, a tetrahedron, and an octahedron are same. Find for which of the following structures, it will have the maximum surface area?
 (a) Sphere (b) Cube
 (c) Tetrahedron (d) Octahedron

Q.22 The ratio of diagonals of the two cubes is 3:2. What is the ratio of the surface areas of these two cubes, respectively?
 (a) 27:8 (b) 3:2 (c) 9:4 (d) 16:9

FOUNDATION

Q.1 A rectangular classroom has to be painted covering all its surfaces excluding floor. The cost of painting a similar room is ₹1872. But, its length, breadth and height are 10% less, 10% more, and 10% less than the dimensions of the classroom, respectively.

How much will it cost to paint the classroom if its length, breadth, and height are in the ratio of 1:2:3?

- (a) ₹1956 (b) ₹2000
 (c) ₹2100 (d) Cannot be determined

- Q.10** A man, by walking diametrically across a circular grass plot, finds that it has taken 45 s less than if he had kept to the path around the outside. If he walks at the rate of 80 m per minute, the diameter of the grass plot is:
 (a) 35 m (b) 65 m (c) 105 m (d) 145 m

Q.11 Within a rectangular courtyard of length 60 feet, a gravelled path, 3 feet wide, is laid down along all the four sides. The cost of gravelling the path is ₹2 per square feet. If the path had been twice as wide, the gravel would have cost ₹984 more. The width of the courtyard is:
 (a) 24 feet (b) 40 feet
 (c) 45 feet (d) 54 feet

Q.12 A vessel 2 m long, 1 m wide, and 1.5 m deep contains 2 m³ water. How many bricks of 20 cm by 10 cm by 7.5 cm can be put in it so that water does not overflow provided that a brick is supposed to absorb 1/7 of its own volume of water?
 (a) 666 (b) 111
 (c) 555 (d) None of these

Q.13 In Snehans apartment, there is a playground of the dimension $7\frac{1}{2}$ by 7 m. In the centre of the ground, there is a flower-bed cut out of the dimension 3 m by $2\frac{1}{2}$ m. What fraction of the whole ground is occupied by the flower-bed?
 (a) $\frac{3}{7}$ (b) $\frac{3}{5}$ (c) $\frac{1}{7}$ (d) $\frac{1}{11}$

Q.14 The height of a room is $\frac{1}{5}$ th of the sum of its length and breadth. The cost of preparing its wall at ₹4 per m² is ₹640. What is the height of the room?
 (a) 4 m (b) 5 m (c) 6 m (d) 7 m

Q.15 A cylindrical container of 32 cm height and 18 cm radius is filled with sand. Now, all this sand is used to form a conical heap of sand. If the height of the conical heap is 24 cm, what is the radius of its base?
 (a) 12 cm (b) 24 cm (c) 36 cm (d) 48 cm

Q.16 The total area of the four walls of a room is 150 m². If the area of the floor is 50 m² and the width of the floor is 3 m, then find the height of the room.
 (a) 2.6 m (b) 3.8 m (c) 5.42 m (d) 7.32 m

Q.17 Anoop has a wooden box, each sides of which is an integer (in cm) and whose volume (neglecting the thickness of the box) is 216 cm³. What is the minimum possible amount that Anoop will need to spend in order to get the outside of the box painted at the rate of ₹2 per cm²?
 (a) ₹320 (b) ₹225 (c) ₹236 (d) ₹432

Direction for Questions 18 and 19: Read the passage below and solve the questions based on it.

There is a rectangular table of the dimension $2.20\text{ m} \times 1.4\text{ m}$. A rectangular tablecloth of the dimension $2.4\text{ m} \times 0.8\text{ m}$ is spread unfolded on the table so that the longer sides of the cloth and the table are parallel.

Direction for Questions 21 and 22: Read the passage below and solve the questions based on it.

There is an equilateral triangle of side ' a ' units. Now, we join any two sides of the triangle to form a cone.

- Q.21** What is the slant height of the cone formed?
 (a) $\frac{a}{2\pi}$ (b) a (c) $\frac{a}{2}$ (d) $2\pi a$

Q.22 What is the radius of the cone formed?
 (a) a (b) $\frac{a}{4}$
 (c) $\frac{a}{\pi}$ (d) None of these

Q.23 The area of three adjacent faces of a cuboidal box are p , q , and r square cm. What is the volume of this box?
 (a) $(p + q + r)$ cm³ (b) \sqrt{pqr} cm³
 (c) $\sqrt[3]{pqr}$ cm² (d) pqr cm³

Q.24 Three equal cubes of unit side length are placed adjacent to each other in a row. Find the ratio of the total surface area of the new cuboid to that of the sum of the surface areas of all the three cubes.
 (a) 3:5 (b) 4:5 (c) 6:7 (d) 7:9

Q.25 A classroom is to be built to accommodate 70 students. It should be done in such a way that for every student, there is 2.2 m² of floor and 11 m³ of space. If the length of the room is 14 m, then find the breadth and height of the room.

- (a) 12 m, 5.5 m (b) 11 m, 5 m
 (c) 13 m, 6 m (d) 11 m, 4 m

Q.26 Length, breadth, and height of a rectangular parallelopiped are in the ratio of 6:5:4 and its total surface area is $33,300 \text{ m}^2$. Find length, breadth, and height of the parallelopiped (in cm).
 (a) 42, 35, 28 (b) 60, 50, 40
 (c) 102, 85, 68 (d) 90, 75, 60

Q.27 A rectangular tank is of dimension $30 \text{ m} \times 20 \text{ m}$. Water is being flown into it through a square pipe of side length 5 cm. Find the speed of the water if the level of water in the tank rises by 1 m in 8 h?
 (a) 30 km/h (b) 36 km/h
 (c) km/h (d) None of these

Q.28 A cube of side length 3 cm weighs 12 kg. What is the weight of the similar cube of same material whose side length is 12 cm?
 (a) 768 kg (b) 678 kg (c) 964 kg (d) 864 kg

Q.29 A cube of side length 4 cm is cut into cubes of side 1 cm. Find the ratio of the sum of the surface area of all the small cubes to that of the large cube.
 (a) 1:16 (b) 2:3 (c) 4:1 (d) 6:1

Q.30 A cylindrical well of depth 12 m with internal radius of 1.75 m is dug up. The mud, so obtained, is spread evenly to form a platform of dimension $10.5 \text{ m} \times 8.8 \text{ m}$. What is the height of the platform?
 (a) 2.25 m (b) 3.25 m (c) 1.25 m (d) 4.25 m

Q.31 A hollow spherical ball of outer diameter 120 mm is cut into two equal hemispheres. One of the hemispheres is filled with honey and the total surface area of the other hemisphere is $389 \frac{5}{7} \text{ cm}^2$. What is the volume of the honey put in the first hemisphere?
 (a) 128 cm^3 (b) 134 cm^3
 (c) 136 cm^3 (d) 138 cm^3

Q.32 The diameter of a cylindrical vessel is made twice as large. How should the height be changed so that it still contains the same quantity of water?
 (a) $\frac{1}{2} \times \text{Height of the two cylinders}$
 (b) $\frac{1}{4} \times \text{Height of the original cylinder}$
 (c) Let it be the same
 (d) None of these

Q.33 Two shopkeepers are offering Pepsi in two different types of cans. The first shopkeeper offers cylindrical cans with a circular base of radius 5 cm and the second shopkeeper offers the cans having a square base of length 10 cm. If height of both the cans is the same, then what is the difference in their capacities?

- (a) 350 cm^3
 (b) 450 cm^3
 (c) 750 cm^3
 (d) Cannot be determined

Q.34 A bucket is a standard example of frustum of a cone. The circumference of such a bucket of one end is 48 cm and of the other end is 34 cm. If the height of the bucket is 10 cm, find its volume (in cm^3).
 (a) 3400 (b) 1350 (c) 2700 (d) 4050

Q.35 The sum of the length, width, and depth of a cuboid is s and its diagonal is d . What is its surface area?
 (a) s^2 (b) d^2 (c) $s^2 - d^2$ (d) $s^2 + d^2$

Q.36 A solid right circular cylinder with height 10 cm and radius of the base 6 cm is given. A right circular cone of the same height and base is removed from this cylinder. Find the volume (in cm^3) of the remaining solid.
 (a) 855.5 (b) 754.3 (c) 1294 (d) None of these

Q.37 At the Amul co-operative society, right cylindrical vessels were used to store milk. But, after the uncere-monious exit of Mr Durien, it was decided that instead of right cylindrical vessels, right cones having the same diameter and height would be used. What would be the ratio of the number of the right cones to the number of right cylinders needed to hold the same quantity of milk?
 (a) 2:1 (b) 3:1 (c) 4:1 (d) 5:1

Q.38 The radius of a right cylinder is doubled and its height is halved. What is the ratio between the new curved surface area and the previous curved surface area of the cylinder?
 (a) 1:1 (b) 2:1 (c) 3:3 (d) 2:3

Q.39 A solid cone is converted into a solid cylinder of the same radius. If the height of the cylinder is 5 m, what is the height of the cone?
 (a) 25 m (b) 15 m (c) 20 m (d) 10 m

Q.40 The radii of a cylinder and a cone are equal and the height of the cylinder is equal to the slant height of the cone. Find the ratio of the curved surfaces of the cylinder and the cone.
 (a) 1:1 (b) 2:1 (c) 3:1 (d) 4:1

Q.41 A right-circular cone is placed inside a cube such that the edges of the base of the cone are touching the edges of one of the faces of the cube and the vertex is touching the opposite face of the cube. If the volume of the cube is 343 cm^3 , what is the volume of the cone (approximately in cm^3)?
 (a) 90 (b) 60 (c) 80 (d) 85

Q.42 When Natraj circus came to Patna for its show at Sonpur mela last year, I observed quite a few things. Its tent was cylindrical to a height of 3 m and conical above it and its diameter was 105 m and slant height

of the conical portion was 53 m. But, I was not able to calculate the length of the canvas 5 m wide to make the tent. Can you please bail me out by telling this?
 (a) 1857 m (b) 1647 m (c) 1947 m (d) 1847 m

Q.43 A cone and a cylinder have their height in the ratio of 3:2 and the radii of their bases are in the ratio of 4:3. Find the ratio of their volumes
 (a) 9:1 (b) 9:2 (c) 8:9 (d) 3:1

Q.44 A cylindrical structure standing on its base with radius 1.5 m and height 5 m is cut with a saw in such a way that the cutting planes goes through all the points at a distance of 0.625 m from the base. Find the volume of the remaining piece.
 (a) 5.62π (b) 9.24π
 (c) 9.04π (d) None of these

Q.45 The height of a right circular cylinder is 6 m. Three times the sum of the areas of its two circular faces is twice the area of its curved surface. The radius of the base is
 (a) 4 m (b) 2 m (c) 6 m (d) 1.5 m

Q.46 A solid cone kept on its base is cut at 2/3rd of its height along a plane parallel to its circular base. The base radius and the slant height are 14 cm and 50 cm, respectively. What is the ratio of the portion cut-out from the solid to the volume of the remaining solid?
 (a) 1:20 (b) 1:25
 (c) 1:36 (d) None of these

Q.47 A big cube of side 6 cm is formed by putting together 216 small identical cubes each of side 1 cm. Now, if the corner cubes in the topmost layer of the big cube are removed, then what will be the impact on the total surface area of the big cube?
 (a) Will decrease
 (b) Will increase
 (c) Will remain unchanged
 (d) Cannot be determined

Q.48 Mr Sinha and Mr Verma are two brothers. They inherited their paternal land in a rectangular form having the same area but different dimensions. For Mr Sinha, the length of the rectangular plot is increased by 10% and the breadth is decreased by 10%. For Mr Verma's rectangular plot, the length is decreased by 10% and the breadth is increased by 10%. Which of the following is true about the areas of the two rectangular plots now?
 (a) The area of the first rectangle is 1% more than that of the second.
 (b) The area of the second rectangle is 1% more than that of the first.
 (c) The area of the two rectangles is the same.
 (d) We cannot comment unless we know the dimensions of the individual rectangles.

- Q.49** A big solid sphere of diameter 10 cm is melted and made into solid spheres of diameters 2 cm. What is the percentage increase/decrease in the surface area of smaller spheres over that of the big sphere?
- (a) 200% increase (b) 400% increase
 (c) 400% decrease (d) None of these

- Q.50** A cone is made of a sector of a circle of radius 21 cm and an angle of 90° . What is the total surface area of the cone (in cm^2)?

- (a) 269.5 (b) 308
 (c) 312.5 (d) 231

Moderate

- Q.1** There is a right circular cone with base radius 3 units and height 4 units. The surface of this right circular cone is painted. It is then cut into two parts by a plane parallel to the base so that the volume of the top part (the small cone) divided by the volume of the frustum equals the painted area of the top part divided by the painted area of the bottom part. The height of the small cone is:
- (a) $\frac{7}{3}$ (b) $\frac{5}{4}$
 (c) $\frac{5}{2}$ (d) None of these

Direction for Questions 2 and 3: Read the passage below and solve the questions based on it.

My grandfather owned plenty of land, which easily encompassed an area of $1000 \times 200 \text{ m}^2$. He wanted to give some part of it to his servant Ramu. But, he did not gift it directly. He supplied the material that could form a fence of length 100 m only. Then, he allowed Ramu to take any part with four sides that could be encased with the help of the given fencing material.

- Q.2** What is the maximum possible land Ramu can take away from my grandfather? (All the four sides are fenced.)
 (a) 100 m^2 (b) 2500 m^2 (c) 750 m^2 (d) 625 m^2
- Q.3** Ramu had intimate knowledge of the land. Hence, he selected the site having a natural fencing of rocks on one side, because he could utilize the given material only on three sides of the plot. What is the maximum possible land that Ramu can claim now?
 (a) 125 m^2 (b) 2500 m^2
 (c) 625 m^2 (d) None of these
- Q.4** The diameter of a road-roller is 42 cm and its length is 100 cm. It takes 400 complete revolutions moving once over to level the stretch of the road. If the cost of levelling is ₹100 per m^2 , then the total cost of levelling works out to:
 (a) ₹52,800 (b) ₹5280
 (c) ₹5,28,000 (d) ₹528

- Q.5** Inside a triangular garden, there is a flower-bed in the form of a similar triangle. Around the flower-bed runs a uniform path of such a width that the sides of the garden are double of the corresponding sides of the flower-bed. The areas of the path and the flower-bed are in the ratio
 (a) 1:1 (b) 4:1 (c) 1:3 (d) 3:1

- Q.6** ABCD is a square of side 10 cm. What is the area of the least-sized square that may be inscribed in ABCD with its vertices on the sides of ABCD?

- (a) 0 cm^2 (b) 25 cm^2
 (c) 50 cm^2 (d) 66.66 cm^2

- Q.7** A cylinder of the maximum possible size is made out of a solid wooden cube. How much material is lost in this process (approximately)?
 (a) 20% (b) 22%
 (c) 24% (d) Cannot be determined

- Q.8** Two friends, ND and SD, invited their friends for a party. A hemispherical bowl full of ice-cream of diameter 18 cm was served for dessert. Each of them had three scoops of the ice-cream and it was exactly sufficient for all of them. If the ice-cream was served in a hemispherical scoop of radius 1 cm, how many friends did they have for their party?

- (a) 240 (b) 241 (c) 242 (d) 243

- Q.9** The cost of white washing one m^2 is ₹50. What will be the maximum amount saved in painting the room in the most economical way, if the sum of the length, breadth, and height is 21 m and all the sides are integers (floor is not to be white washed)?

- (a) ₹1,08,050 (b) ₹8400
 (c) ₹9300 (d) ₹8540

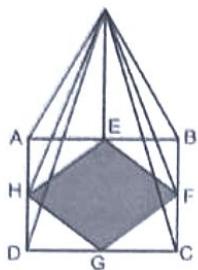
- Q.10** A small bird is taking rest at the centre of the base of a hemispherical cage. Suddenly it stands up, flies to the topmost point in the cage, then in a straight line to the cage door at the intersection of the curved surface and the base. In the process, it covers a total distance of 482 cm. What is the radius of the hemisphere?
 (a) 100 cm (b) 150 cm
 (c) 200 cm (d) 50 cm

- Q.11** A cube is inscribed in a hemisphere of radius R , such that four of its vertices lie on the base of the hemisphere and the other four touch the hemispherical surface of the half-sphere. What is the volume of the cube?

- (a) $0.25R^3$ (b) $0.67\sqrt{\frac{2}{3}}R^3$
 (c) $0.5\sqrt{\frac{2}{3}}R^3$ (d) $0.67R^3$

Direction for Questions 26 and 27: Read the passage below and solve the questions based on it.

In the figure given below, the base of the pyramid is a square ABCD of side 10 cm; another pyramid with base EFGH is formed where the points E, F, G, and H are obtained by joining the mid-points of the sides of the square ABCD. The height of the pyramid is 30 cm.



- Q.26** Find the ratio of the volumes of pyramid with ABCD as base to the pyramid with EFGH as base.

(a) $\sqrt{2}:1$ (b) $2\sqrt{2}:1$
 (c) 4:1 (d) 2:1

- Q.27** Find the volume of the space left.

(a) 1500 cm^3 (b) 500 cm^3
 (c) 1000 cm^3 (d) 800 cm^3

- Q.28** All five faces of a regular pyramid with a square base are found to be of the same area. The height of the pyramid is 3 cm. Find the total area of all its surfaces (in cm^2).
 (a) 8 (b) 10 (c) 12 (d) 16

Direction for Questions 29 and 30: Read the passage below and solve the questions based on it.

A right-angled triangle of sides 5 m, 12 m, and 13 m is made to spin about its hypotenuse.

- Q.29** What kind of figure will be formed with this?
 (a) Square (b) Double cone
 (c) Sphere (d) None of these

- Q.30** What is the volume of the figure thus formed?
 (a) 270 m^3 (b) 290 m^3
 (c) 320 m^3 (d) None of these

- Q.31** A solid cube is cut into two halves by a plane passing through exactly two corners of the cube. What is the ratio of the total surface area of both the halves put together and the original total surface area of the cube?
 (a) $\sqrt{3}:\sqrt{3}+\sqrt{2}$ (b) $\sqrt{6}+1:\sqrt{6}$
 (c) $6:6+\sqrt{6}$ (d) $3+\sqrt{2}:3$

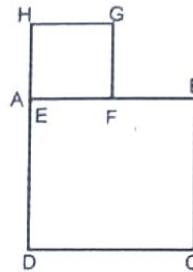
- Q.32** Sanjay wound a piece of string over a cylindrical block around its curved surface and found that the string could

be wound for exactly half a round more than an integral number of rounds. He also figured that if the string had been five times as long, the string would have had wound for exactly 10 cm more than an integral number of the rounds. Find the radius of the cylindrical block. (Assume that all windings are done in the same plane and neglect any increase in the radius due to the winding of the string.)

- (a) $2\frac{6}{22} \text{ cm}$ (b) $3\frac{11}{22} \text{ cm}$
 (c) $3\frac{4}{22} \text{ cm}$ (d) $3\frac{9}{22} \text{ cm}$

Direction for Questions 33 and 34: Go through the figure given below and solve the questions based on it.

Consider the figure given below:



A cube with surface EFGH is put on a cube with a surface ABCD in such a way that vertices E and A of the two blocks coincide and side EF coincides with side AB. EF = 1 unit and AB = 2 units. Now, keeping vertex F fixed, the smaller cube is rotated along the larger cube till G coincides with B once. Then, keeping G fixed, the smaller block is again rotated till GH coincides with BC.

- Q.33** How much is the aerial distance traversed by point E?
 (a) $\sqrt{2\pi}$ units (b) π units

(c) $\sqrt{2\pi} + \frac{1}{2}\pi$ units (d) None of these

- Q.34** In the above question, what is the total distance covered by point E?
 (a) π units (b) $(1 + \pi)$ units
 (c) $(1 + \sqrt{\pi})$ units (d) None of these

- Q.35** How many cuboids of different dimensions can be made assembling 100 identical cubes?
 (a) 9 (b) 8 (c) 12 (d) 10

- Q.36** A cuboid is of the dimension $2 \times 2 \times 3$ units. Of the six surfaces of this cuboid, there are exactly x squares and exactly y rectangles (rectangles which are not squares). Find the product of $x \times y$.
 (a) 8 (b) 12
 (c) 6 (d) None of these

Q.37 A box without a lid has a height of x cm, a square base of side y cm, and a volume of 500 cm^3 . For what value of x (in cm), the least quantity of the material will be needed to construct the box?

- (a) $\sqrt{5}$ (b) $2\sqrt{5}$ (c) 5 (d) $5\sqrt{2}$

Q.38 Under the Indian Posts and Telegraph Act 1885, any package in the form of a right circular cylinder will not be accepted if the sum of its height and the diameter of its base exceeds 10 inches. The height (in inches) of a package of maximum volume that would be accepted is:

- (a) $10/3$ (b) $20/3$ (c) 10 (d) 20

Q.39 A convex pentagon IJKLM is inscribed in a circle. If the angles subtended by the sides IJ, JK, KL, and LM

at the centre are 50° , 80° , 60° , and 60° , respectively, what is the measurement of the smallest interior angle of the pentagon?

- (a) 120° (b) 95°
(c) 85° (d) None of these

Q.40 A right circular cone, with radius to height ratio as $12:5$, is cut parallel to its base to get a smaller cone and a frustum. If the height of the smaller cone to the height of the frustum are in the ratio of $3:1$, by what percentage is the combined total surface area of the smaller cone and frustum will be more with respect to the original cone?

- (a) 22% (b) 32%
(c) 46% (d) None of these

ADVANCED

Q.1 A cuboid of length 20 m, breadth 15 m, and height 12 m is lying on a table. The cuboid is cut into two equal halves by a plane which is perpendicular to the base and passes through a pair of diagonally opposite points of that surface. Then, a second cut is made by a plane which is parallel to the surface of the table again dividing the cuboid into two equal halves. Now, this cuboid is divided into four pieces. Out of these four pieces, one piece is now removed from its place. What is the total surface area of the remaining portion of the cuboid?

- (a) 1290 m^2 (b) 1380 m^2
(c) 1440 m^2 (d) Cannot be determined

Q.2 A metal is made up of a cylindrical base and a conical top with the base of radius 5 cm. The ratio of height of the cone and the cylinder is $2:3$. A cylindrical hole is drilled through the metal solid with height two-third the height of the metal solid. What should be the radius of the hole, so that the volume of the hole is $1/3$ the volume of the metal solid after drilling?

- (a) $\sqrt{45} \text{ cm}$ (b) $\sqrt{\frac{55}{8}} \text{ cm}$
(c) $\sqrt{35} \text{ cm}$ (d) $\sqrt{65} \text{ cm}$

Q.3 Sixteen cylindrical cans of Coke, each with a radius of 1 cm, are placed inside a wooden carton, four in a row. If the cans touch the adjacent cans and/or the walls of the box, then which of the following could be the internal area of the bottom of the carton (in cm^2)?

- (a) 16 (b) 32
(c) 64 (d) None of these

Q.4 The areas of three adjacent faces of a cuboid are a , b , and c . If the volume of the cuboid is N , then N^2 is equal to:

- (a) abc (b) $(ab + bc + ac)$
(c) $1/c$ (d) None of these

Q.5 A square tin sheet of side 12 inches is converted into a box with open top in the following steps: The sheet is placed horizontally. Then, equal-sized squares, each of side x inches, are cut from the four corners of the sheet. Finally, the four resulting sides are bent vertically upwards in the shape of a box. If x is an integer, then what value of x maximizes the volume of the box?

- (a) 3 (b) 4 (c) 1 (d) 2

Q.6 A rectangular piece of cardboard $18 \text{ cm} \times 24 \text{ cm}$ is made into an open box by cutting a square of 5 cm side from each corner and building up the side. What is the volume of the box (in cm^3)?

- (a) 560 (b) 432
(c) 216 (d) None of these

Q.7 Consider the following function:

$$E = f(A) = \sqrt{\frac{A}{7}}$$

where length of the edge E of a Pyramid

and the surface area A of the pyramid are the different terms used. How much longer is the edge of the pyramid with a surface area 3087 square units than the edge of one with a surface area 2023 square units?

- (a) 1064 (b) 152
(c) 4 (d) None of these

Q.8 The ratio of the volumes of the two cylinders are $a:b$, and their heights are in the ratio $c:d$. What is the ratio of their diameters?

- (a) $\frac{ad}{bc}$ (b) $\frac{d^2}{c^2}$
(c) $\sqrt{\frac{ad}{bc}}$ (d) $\sqrt{\frac{a}{b}} \times \frac{c}{d}$

Q.9 From a solid right circular cone made of iron with base of radius 2 cm and height 5 cm, a hemisphere of diameter 2 cm and its centre, coinciding with the centre of the base of the cone, is cut out. The structure so obtained is then dropped in a right circular cylinder whose inner radius is 3 cm and inner height is 10 cm. Water is then poured into the cylinder to fill it up to its brim. What is the volume of the water required to fill it?

- (a) $84\pi \text{ cm}^3$ (b) $\frac{250\pi}{3} \text{ cm}^3$
 (c) $\frac{270\pi}{4} \text{ cm}^3$ (d) None of these

Direction for Questions 10 to 12: Read the passage below and solve the questions based on it.

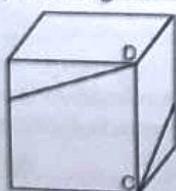
Consider a cylinder of height h cm and radius $r = \frac{2}{\pi}$ cm as shown in the figure (not drawn to scale). A string of a certain length, when wound on its cylindrical surface, starting at a point A and ending at point B, gives a maximum of n turns (in other words, the string length is the minimum length to wind n turns.)

Q.10 What is the vertical spacing in cm between consecutive turns?



- (a) $\frac{h}{n}$
 (b) $\frac{h}{\sqrt{n}}$
 (c) $\frac{h}{n^2}$
 (d) Cannot be determined with the given information.

Q.11 The same string, when wound on the exterior four walls of a cube of side n cm, starting at point C and ending at point D, can give exactly one turn (see figure, not drawn to scale). The length of the string in cm is:



- (a) $\sqrt{2}n$
 (c) n
 (b) $\sqrt{17}n$
 (d) $\sqrt{13}n$

Q.12 In the setup of the previous two questions, how is n related to n ?

- (a) $h = \sqrt{2}n$ (b) $h = \sqrt{17}n$
 (c) $h = n$ (d) $h = \sqrt{13}n$

Q.13 An octagonal prism, of height 20 cm and a side of the base as $\sqrt{4(2 - \sqrt{2})}$ cm, is cut from its base to the top along the edges into two similar parts of equal volume. What is the total surface area of each part?

- (a) $80 + 160\sqrt{2 - \sqrt{2}} + 4\sqrt{2} \text{ cm}^2$
 (b) $80 + 40\sqrt{2 - \sqrt{2}} \text{ cm}^2$
 (c) $80 + 160\sqrt{2 - \sqrt{2}} + 8\sqrt{2} \text{ cm}^2$
 (d) None of these

Direction for Questions 14 to 16: Read the passage below and solve the questions based on it.

There are 300 coins, each coin having radius 2 cm and height 1 cm. The coins are so kept that each coin touches the other two. The base has three coins and the figure is built upon the base.

Q.14 Find the volume of the region enclosed by the coins.

- (a) $600(2\sqrt{3} - \pi) \text{ cm}^3$ (b) $400(2\sqrt{3} - \pi) \text{ cm}^3$
 (c) $100(2\sqrt{3} - \pi) \text{ cm}^3$ (d) $200(2\sqrt{3} - \pi) \text{ cm}^3$

Q.15 Find the number of spheres of the maximum volume that can be accommodated in the above region.

- (a) 324 (b) 323 (c) 162 (d) 161

Q.16 Find the volume of the prism circumscribing the whole structure of the coins.

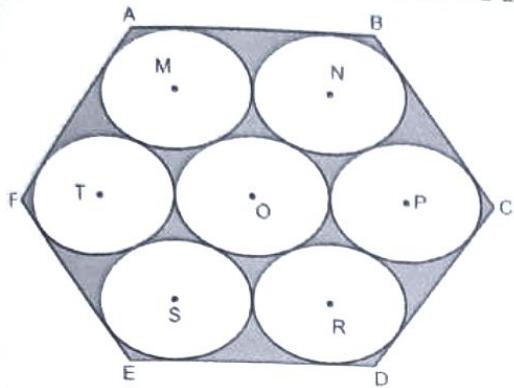
- (a) $100\sqrt{3(7 + 4\sqrt{3})} \text{ cm}^3$
 (b) $2400\sqrt{3(2 + \sqrt{3})} \text{ cm}^3$
 (c) $800\sqrt{3(2 + \sqrt{3})} \text{ cm}^3$
 (d) None of these

Q.17 A cylinder of height ' h ' cm and radius 6π cm is wound around with a string of width ' x ' cm. The string covers the lateral surface area of the cylinder completely without keeping any space between two turns. What is the required length of the string.

- (a) $\frac{12h}{x} \text{ cm}$ (b) $\frac{12x}{h} \text{ cm}$
 (c) $12h \text{ cm}$ (d) $\frac{6h}{x} \text{ cm}$

Direction for Questions 18 to 20: Read the passage below and solve the questions based on it.

In the figure given below, a regular hexagon ABCDEF is inscribed by seven circles. The radius of each circle is equal to 6 cm. The centres of the circles touching the sides are M, N, P, R, S, and T, whereas the centre of the central circle is O.



- Q.18** Find the ratio of the perimeter of the hexagon circumscribing the circle with centre O, the perimeter of hexagon formed by joining the points M, N, P, R, S, and T and the perimeter of the hexagon ABCDEF.
 (a) $6:12:(12+2\sqrt{3})$
 (b) $2:2\sqrt{3}:(2\sqrt{3}+1)$
 (c) $1:\sqrt{3}:(\sqrt{3}+1)$
 (d) $1:2:3$
- Q.19** Find the area of the shaded region (in cm^2).
 (a) 136.83
 (b) 129.63
 (c) 139.83
 (d) 138.83
- Q.20** If the above figure is a 3-D figure and seven spherical balls of radius 6 cm each are tightly arranged in a hexagonal box in a single layer, then what will be the volume of the box unoccupied by the balls in cm^3 ?
 (a) $3456\sqrt{3} + 5184 - 252\pi$
 (b) $3456\sqrt{3} + 5184 - 288\pi$
 (c) $576\sqrt{3} + 864 - 288\pi$
 (d) None of these
- Q.21** Four spheres each of radius 10 cm lie on a horizontal table so that the centres of the spheres form a square of side 20 cm. A fifth sphere also of radius 10 cm is placed on them so that it touches each of these spheres without disturbing them. How many cm above the table is the centre of the fifth sphere?
 (a) $10\sqrt{6}$
 (b) $10(1+\sqrt{2})$
 (c) $10(1+\sqrt{3})$
 (d) $10(4-\sqrt{2})$
- Q.22** A cube with an edge 'a' is cut from the angles by planes so that a regular octagon remains from each face, then the volume of the polygon so obtained will be:

(a) $a^3(7\sqrt{2}-9)$

(b) $\frac{7}{3}a^3(\sqrt{2}-1)$

(c) $7a^3(\sqrt{2}-1)$

(d) $\frac{a^3}{3}(7\sqrt{2}-9)$

- Q.23** A square of side x cm is cut from each corner of a rectangular sheet of metal of 10 cm by 14 cm. The resulting projections are folded up and the seams welded to construct an open box. What is the volume of the box thus obtained?
 (a) $4x^3 - 48x^2 + 140x$
 (b) $4x^3 + 48x^2 + 140x$
 (c) $x^3 + 24x^2 + 140x$
 (d) None of these

- Q.24** A sphere of radius 13 cm is cut by a plane whose distance from the centre of the sphere is 5 cm. What is the circumference of the plane circular section?
 (a) $10\pi \text{ cm}$
 (b) $12\pi \text{ cm}$
 (c) $24\pi \text{ cm}$
 (d) $26\pi \text{ cm}$

- Q.25** The square of side 1 cm are cut from four corners of a sheet of tin (having length = l and breadth = b) in order to form an open box. If the whole sheet of tin was rolled along its length to form a cylinder, then the volume of the cylinder is equal to $(343/4) \text{ cm}^3$. Find the volume of the box. (l and b are integers.)
 (a) 154 cm^3
 (b) 100 cm^3
 (c) 126 cm^3
 (d) Insufficient data

- Q.26** Two spheres of radii 6 cm and 1 cm are inscribed in a right circular cone. The bigger sphere touches the smaller one and also the base of the cone. What is the height of the cone?
 (a) 14 cm
 (b) 12 cm
 (c) $85/6$ cm
 (d) $72/5$ cm

- Q.27** If the total length of the diagonals of a cube is 12 cm, then what is the total length of the edges of the cube?
 (a) $6\sqrt{3} \text{ cm}$
 (b) 12 cm
 (c) $12\sqrt{3} \text{ cm}$
 (d) 15 cm

- Q.28** A cylindrical vessel open at the top contains water up to $1/3$ rd of its height. A heavy sphere whose diameter is equal to the height of the cylinder is placed into the vessel touching its curved surface from all sides, then the water
 (a) level will rise to half of its height.
 (b) level will rise to $3/4$ th its height.
 (c) level will rise to its height.
 (d) will overflow.

- Q.29** A sphere of maximum possible volume is to be completely immersed into a cylindrical container of radius 'a' containing water up to a height '2a'. What is the minimum height of the jar so that no water spills out of it?

- (a) $\frac{10a}{3}$ (b) $\frac{11a}{3}$
 (c) $\frac{12a}{3}$ (d) $\frac{13a}{3}$

- Q.30** In a rectangular parallelopiped A, a, b, and c are the lengths of the diagonals on three faces having a corner in common. Let d be the length of the largest diagonal that can be drawn in A, then:
- (a) $2d^2 = a^2 + b^2 + c^2$
 (b) $d^2 = a^2 + b^2 + c^2$
 (c) $d^2 = 2(a^2 + b^2 + c^2)$
 (d) None of these

Direction for Questions 31 to 33: Read the passage below and solve the questions based on it.

A right-angled triangle with base and height measuring 15 cm and 20 cm is rotated along its hypotenuse. The structure so formed is placed by two spheres such that they are just touching the sides of the cone.

- Q.31** Find the volume of the structure formed before the double cone is placed (in cm^3)?

- (a) 1200π (b) 3600π
 (c) 1000π (d) None of these

- Q.32** Find the sum of the volumes of both the sphere (in cm^3).

- (a) $\frac{364}{3}\pi$ (b) $\frac{140}{3}\pi$
 (c) 12π (d) 47π

- Q.33** Find the curved surface area of the figure (in sq. units).

- (a) 840π (b) 400π
 (c) 420π (d) None of these

- Q.34** A square hole of cross-sectional area 4 cm^2 is drilled across a cube with its length parallel to a side of the cube. If an edge of the cube measures 5 cm, what is the total surface area of the body so formed?

- (a) 158 cm^2 (b) 190 cm^2
 (c) 166 cm^2 (d) 182 cm^2

- Q.35** The radius and the height of a right solid circular cone are r and h , respectively. A conical cavity of radius $r/2$ and height $h/2$ is cut out of the cone. What is the whole surface area of the rest of the portion?

- (a) $\frac{\pi r}{4}(5\sqrt{(r^2+h^2)})+3r$ (b) $\frac{5\pi r}{4}\sqrt{r^2+h^2}$
 (c) $\frac{3\pi r}{4}(\sqrt{r^2+h^2}+r)$ (d) $\frac{3\pi r}{7}(\sqrt{r^2+h^2}+r)$

- Q.36** John Nash, an avid mathematician, had his room constructed such that the floor of the room was an equilateral triangle in shape instead of the usual rectangular shape. One day he brought home a bird and tied it to

one end of a string and then tied the other end of the string to one of the corners of his room. The next day, he untied the other end of the string from the corner of the room and tied it to a point exactly at the centre of the floor of the room. Assuming that the dimensions of the room are relatively large compared to the length of the string, find the number of times, by which the maximum possible space in which the bird can fly, increase.

- (a) 4 (b) 5 (c) 6 (d) 7

Direction for Questions 37 to 40: Read the passage below and solve the questions based on it.

Homes of three of my friends—Bango, Mango, and Tatty—are located at the three vertices of the municipal park near my house. This park is triangular in shape. Given below are the directions to reach each of their homes from my home:

- Bango's house is 100 m to the east and 150 m to the north.
- Mango's house is 200 m to the east and 50 m to the north.
- Tatty's house is 50 m to the west and 25 m to the north.
- Neglect dimensions of all the houses while calculating the distances.

- Q.37** What is the area of the municipal park?

- (a) $27,500 \text{ m}^2$ (b) $11,500 \text{ m}^2$
 (c) $13,750 \text{ m}^2$ (d) Cannot be determined

- Q.38** There is a highway (a straight road) passing through the houses of Bango and Mango. Tatty starts from his house and takes the shortest possible route to reach the highway. What is the distance travelled by Tatty?

- (a) $\frac{125}{\sqrt{3}}$ (b) $\frac{275}{\sqrt{2}}$
 (c) $\frac{150}{\sqrt{2}}$ (d) None of these

- Q.39** Travelling along one of the following routes will take me from my house to the point (on the highway) where Tatty reached in the previous question. Which one is it?

- (a) 100 m to the east, 200 m to the north
 (b) 87.5 m to the east and 62.5 m to the north
 (c) 67.5 m to the east and 167.5 m to the north
 (d) None of these

- Q.40** What is the approximate distance between Mango's house and the point Tatty reached in Q.39?

- (a) $\frac{225}{\sqrt{2}} \text{ m}$ (b) $\frac{275}{\sqrt{2}} \text{ m}$
 (c) $\frac{150}{\sqrt{2}} \text{ m}$ (d) $\frac{325}{\sqrt{2}} \text{ m}$

Answers

WARM UP

1. (c)	2. (c)	3. (c)	4. (a)	5. (a)	6. (d)	7. (b)	8. (b)	9. (c)	10. (d)
11. (b)	12. (b)	13. (d)	14. (c)	15. (c)	16. (b)	17. (a)	18. (c)	19. (c)	20. (a)
21. (c)	22. (c)								

FOUNDATION

1. (d)	2. (a)	3. (c)	4. (d)	5. (a)	6. (c)	7. (c)	8. (b)	9. (c)	10. (c)
11. (b)	12. (d)	13. (c)	14. (a)	15. (c)	16. (b)	17. (d)	18. (c)	19. (d)	20. (d)
21. (b)	22. (d)	23. (b)	24. (d)	25. (b)	26. (d)	27. (a)	28. (a)	29. (c)	30. (c)
31. (b)	32. (b)	33. (d)	34. (d)	35. (c)	36. (b)	37. (b)	38. (a)	39. (b)	40. (b)
41. (a)	42. (c)	43. (c)	44. (d)	45. (a)	46. (d)	47. (c)	48. (c)	49. (b)	50. (a)

MODERATE

1. (c)	2. (d)	3. (d)	4. (a)	5. (d)	6. (c)	7. (b)	8. (b)	9. (c)	10. (c)
11. (b)	12. (d)	13. (c)	14. (a)	15. (b)	16. (b)	17. (c)	18. (c)	19. (d)	20. (d)
21. (c)	22. (d)	23. (c)	24. (c)	25. (a)	26. (d)	27. (a)	28. (c)	29. (b)	30. (d)
31. (d)	32. (c)	33. (c)	34. (b)	35. (b)	36. (a)	37. (c)	38. (a)	39. (b)	40. (d)

ADVANCED

1. (d)	2. (b)	3. (c)	4. (a)	5. (d)	6. (a)	7. (c)	8. (c)	9. (a)	10. (a)
11. (b)	12. (c)	13. (d)	14. (d)	15. (d)	16. (c)	17. (a)	18. (c)	19. (d)	20. (b)
21. (b)	22. (b)	23. (a)	24. (c)	25. (b)	26. (d)	27. (c)	28. (c)	29. (a)	30. (a)
31. (a)	32. (b)	33. (c)	34. (d)	35. (a)	36. (b)	37. (c)	38. (b)	39. (b)	40. (a)

Hints and Solutions

WARM UP

1. Let the original side of the cube = a

Then, the surface area = $6a^2$

If side is increased by 100%, then new side = $2a$

and surface area = $6(2a)^2 = 24a^2$

Hence, percentage change in surface area

$$= \frac{\text{change}}{\text{initial value}} \times 100 = \frac{18a^2}{6a^2} \times 100 = 300\%$$

2. Total volume of the hall = $30 \times 12 \times 6 \text{ m}^3$

Given that volume of a man = 8 m^3

Then, the number of people = $\frac{30 \times 12 \times 6}{8} = 270$

3. Height of the cylinder = $\frac{\text{Volume}}{\text{Area of the base}} = \frac{511}{36.5} \text{ m} = 14 \text{ m}$

4. Total surface area = 924 cm^2

Then, the curved surface area

$$= \frac{924}{3} \times 2 = 308 \times 2 = 616 \text{ cm}^2$$

$$\text{Area of the base} = \frac{924 - 616}{2} = 154 \text{ cm}^2$$

Let the radius of base = r and $\pi r^2 = 154 \text{ cm}^2$