Draw PV⊥ON

In ΔRVP, $\tan 45° = \frac{PV}{VR} = \frac{PV}{OR+OV} = \frac{\text{breadth}}{12+OR}$

(Since OV = PQ = 12 cm) also, $\text{breadth}^2 + OR^2 = (QR)^2$ In ΔROQ, ⇒ breadth = $x$ + 12 where OR = $x$

$\Rightarrow (x+12)^2 = (4\sqrt{17})^2 - x^2$ (given that QR = $(4\sqrt{17})$

$\Rightarrow 2x^2 + 24x - 128 = 0$

$\Rightarrow x^2 + 12x - 24 = 0$

$\Rightarrow x = 4$ or $-16$

Since $x > 0$, $x = 4$

Breadth = $x$ + 12, that is, 16 cm, and the area of MNOQ = 16 × 18 = 288 sq. cm.

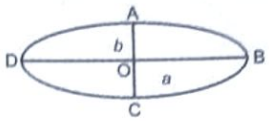
**Example 26** ΔABC has sides AB and AC measuring 2001 and 1002 units, respectively. How many such triangles are possible with all integral sides?

(a) 2001 (b) 1002 (c) 2003 (d) 1004

***Solution*** Value of BC will lie in between 999 and 3003. Hence, 999 < BC < 3003.

So, the total values possible for BC = 2003

**Ellipse** The path of a moving point which moves in such a way that its distance from a fixed point (focus) bears a constant ratio with its distance from a fixed line (directrix)




Given: OB = OD = $a$ (semimajor axis)
OA = OC = $b$ (semi = minor axis)

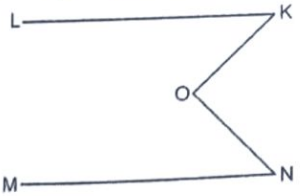
- Area = $\pi ab$
- Perimeter = $\pi(a + b)$

**Maths behind the formula (Area = $\pi ab$)** One way, to see why the formula is true, is to realize that the ellipse is just a unit circle that has been stretched by a factor '$a$' in the $x$-direction and by a factor '$b$' in the $y$-direction. Hence, the area of the ellipse is just $a \times b$ times the area of unit circle.

# Practice Exercises

## Warm Up

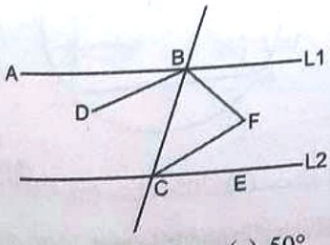
**Q.1** In the given figure, MN and KL are parallel lines.




∠LKO = 70°, ∠KON = 100°
Find ∠MNO.

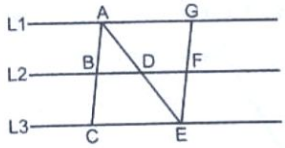
(a) 20° (b) 30° (c) 40° (d) 50°

**Q.2** In the given figure, L1 and L2 are parallel lines. ∠ABC = 80°. If the lines BD and CF are parallel and ∠DBC = 30°, find ∠FCE.




(a) 30° (b) 45° (c) 50° (d) 60°

**Q.3** The lines L1, L2, and L3 are parallel. If AB = 4 cm, BC = 6 cm, AD = 12 cm, and FG = DE, then find EF.

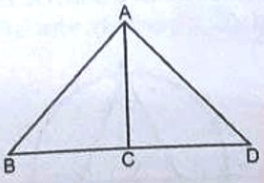



(a) 27 cm (b) 36 cm
(c) 48 cm (d) None of these

**Q.4** The supplement of an angle is five times the angle. What is the measurement of the angle?

(a) 30° (b) 65° (c) 40° (d) 45°

**Q.5** In the given figure, AB = AD. ∠ACB = 95° + ∠BAC and ∠BAD = 150°. Find ∠ACB.




(a) 110° (b) 120° (c) 130° (d) 140°

**Q.6** In the given figure, PS is the altitude drawn to the side QR of the triangle PQR. ∠PRS = ∠QPS − 20°,

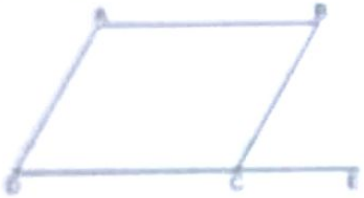
$\angle PQR = 20°$, $\angle PRQ = 50°$. What is the value of $\angle QPR$?

(a) 100° (b) 110° (c) 120° (d) 130°

**Q.7** In $\triangle ABC$, $\angle ACD = 100°$ and $\angle ABC = 40°$. Find $\angle BAC$.

(a) 40° (b) 100° (c) 80° (d) 60°

**Q.8** In the given figure, ABCD is a parallelogram. Find $2\angle ABC - ADC$ if $A = \angle 135°$



(a) 50° (b) 60° (c) 45° (d) 75°

**Q.9** ABCD is an isosceles trapezium with lines AB parallel to CD. If $\angle DCB = 40°$, $\angle BAD$ equals

(a) 40° (b) 80° (c) 100° (d) 140°

**Q.10** If each interior angle of a regular polygon is 135°, find the number of diagonals.

(a) 54 (b) 48
(c) 20 (d) None of these

**Q.11** In the given figure, $\triangle ABC$ is circumscribed by a circle having its centre at O. If OB and OC are the angle bisector of $\angle B$ and $\angle C$, respectively, what is the value of $\angle BAC$?

(a) 45° (b) 60° (c) 75° (d) 50°

**Q.12** In the given figure, AB and CD are two chords of a circle intersecting at O. If $AO = 4$ cm, $OB = 6$ cm, $OC = 3$ cm, then find OD.

(a) 4 cm (b) 6 cm (c) 8 cm (d) 9 cm

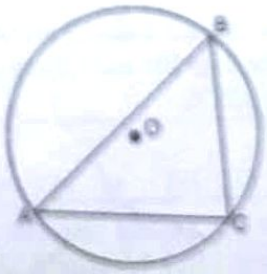
**Q.13** In the given figure, if $2\angle PQR = 3\angle ROS$ and $\angle PQS = 100°$, then find $\angle PQR$.

(a) 20° (b) 30° (c) 45° (d) 60°

**Q.14** In the above figure, O is the centre of the circle, so $\angle AOB = 120°$. Find $\angle ACB$.

(a) 60° (b) 30° (c) 45° (d) 50°

**Q.15** In the given figure, O is the centre of the circle, $\angle AOC = 160°$. If AB = AC, then find $\angle BAC$.

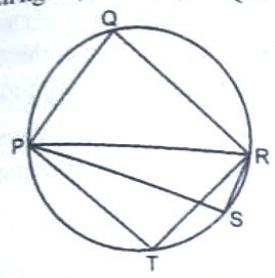


(a) 45° (b) 20° (c) 60° (d) 75°

**Q.16** PQRS is a cyclic quadrilateral. If PQR is an equilateral triangle, find $\angle RSP$.

(a) 120° (b) 60° (c) 30° (d) 150°

Q.17 In the given figure, PT = TR, ∠PQR = 100°. Find ∠PRT.

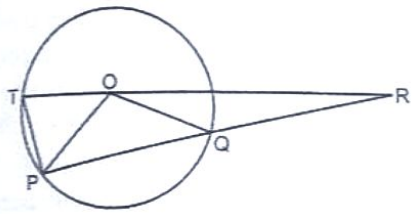



(a) 40° (b) 50° (c) 60° (d) 70°

Q.18 In a triangle ABC, the incentre is at O. If ∠BOC = 100°, find ∠BAC.

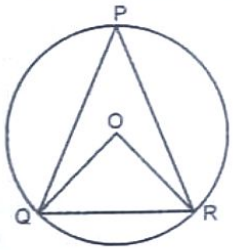
(a) 10° (b) 20° (c) 30° (d) 40°

Q.19 In the given figure, O is the centre of the circle, OQ = QR and ∠QRO = 15°. Find ∠QOP.



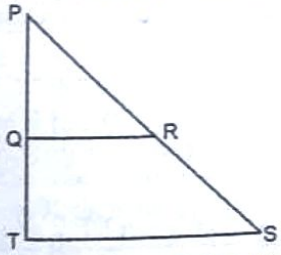

(a) 120° (b) 150° (c) 135° (d) 105°

Q.20 In the given figure, O is the centre of the circle and ∠OQP + ∠ORP = 70°. Find ∠ORQ.



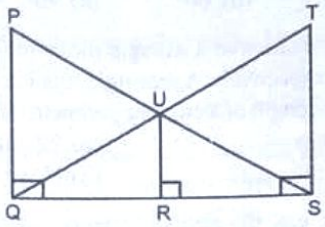

(a) 20° (b) 30° (c) 40° (d) 50°

Q.21 In the given figure PQ × PS = PT × PR. If ∠PQR = ∠PST + 30° and ∠PTS = 100°, then find ∠PRQ.



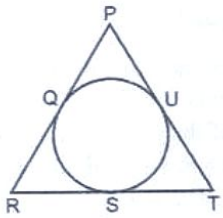

(a) 100° (b) 70° (c) 130° (d) 50°

Q.22 In the above figure, QR = 4 cm and RS = 12 cm. TS = 8 cm and QU is extended to T. Find PQ.



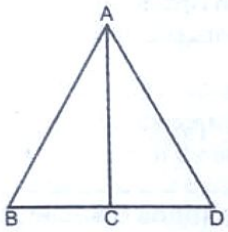

(a) 3 cm (b) 24 cm
(c) 21 cm (d) None of these

Q.23 In the given figure PR, PT and RT are tangents to the circle at Q, U and S, respectively. PR = (RT + 3) cm; PR = (PT + 1) cm. If the perimeter of triangle RPT is 26 cm, what is value of QR + PT?




(a) 12 cm (b) 3 cm
(c) 14 cm (d) 13 cm

Q.24 In the given figure, ∠CAB = 1/2 ∠ACD, ∠BAD = 85° and ∠ADC = 40°. Find ∠ACB.




(a) 45° (b) 55° (c) 70° (d) 60°

Q.25 How many different regular polygons can be formed with the interior angle exceeding the exterior angle and the sum of the interior angles not exceeding 180°?

(a) 7 (b) 8
(c) 9 (d) None of these

## FOUNDATION

Q.1 The ratio of the sides of ΔABC is 1:2:4. What is the ratio of the altitudes drawn onto these sides?

(a) 4:2:1 (b) 1:2.4
(c) 1:4:16 (d) None of these

**Q.2** The largest angle of a triangle of sides 7 cm, 5 cm, and 3 cm is:
(a) 45° (b) 60° (c) 90° (d) 120°

**Q.3** The three sides of a triangle measure 6 cm, 8 cm, and 10 cm, respectively. A rectangle equal in area to the triangle has a length of 8 cm. The perimeter of the rectangle is:
(a) 11 cm (b) 22 cm
(c) 16 cm (d) None of these

**Q.4** X and Y are, respectively, two points on the sides DC and AD of parallelogram ABCD. The area of $\Delta ABX$ is equal to:
(a) $\frac{1}{3}\times$ area of $\Delta BYC$
(b) Area of $\Delta BYC$
(c) $\frac{1}{2}\times$ area of $\Delta BYC$
(d) $2\times$ area of $\Delta BYC$

**Q.5** A rectangular enclosure 40 m × 36 m has a horse tethered to a corner with a rope of 14 m in length. What is the ratio of the respective areas it can graze, if it is outside the enclosure and if it is inside the enclosure?
(a) 2 (b) 2:3 (c) 1:4 (d) 3:1

**Q.6** What is the area of the triangle having the sides 4, 6, and 12 units?
(a) 14.28 (b) 12.6
(c) 16.3 (d) Not possible

**Q.7** Identical spherical balls are spread on a table top so as to form an equilateral triangle. How many balls are needed so that a side of the equilateral triangle contains $n$ balls?
(a) $\frac{n(n+1)}{2}$ (b) $n^2-1$ (c) $n(n-1)$ (d) $n!$

**Direction for Questions 8 and 9: Read the passage below and solve the questions based on it.**

The area of a square is equal to the area of a rectangle. Moreover, the perimeter of the square is also equal to the perimeter of the rectangle.

**Q.8** The side of the square is equal to the:
(a) Length of the rectangle
(b) Breadth of the rectangle
(c) Cannot be determined
(d) Both (a) and (b)

**Q.9** The length of the rectangle is equal to the:
(a) Breadth of the rectangle
(b) Side of the square
(c) Cannot be determined
(d) Both (a) and (b)

**Q.10** A quadrilateral is inscribed in a circle. If an angle is inscribed in each of the segments outside the quadrilateral, then what is the sum of the four angles?
(a) 270° (b) 360° (c) 540° (d) 720°

**Q.11** In a triangle PQR, PQ = 6 cm, QR = 8 cm, and PR = 10 cm. The length of the median bisecting the shortest side is (approx.):
(a) 10 cm (b) 8.5 cm
(c) 9 cm (d) None of these

**Q.12** A square is inscribed in a circle which is inscribed in an equilateral triangle. If one side of the triangle is $a$, find the area of the square.
(a) $\frac{a^2}{3}$ (b) $\frac{a^2}{6}$ (c) $\frac{3a^2}{8}$ (d) $\frac{\pi a^2}{12}$

**Q.13** A cyclic quadrilateral is such that two of its adjacent angles are divisible by 6 and 10, respectively. One of the remaining angles will necessarily be divisible by:
(a) 3 (b) 4
(c) 8 (d) None of these

**Q.14** In the above question, if one of the remaining angles is necessarily divisible by 16, then its measure can be:
(a) 128° (b) 48° (c) 132° (d) 112°

**Q.15** The volume of spheres are proportional to the cubes of their radii. Two spheres of the same material weigh 3.6 kg and 2.7 kg and the radius of the smaller one is 2 cm. If the two were melted down and formed into a single sphere, what would be its radius?
(a) 4 cm (b) 4.3 cm (c) 3 cm (d) 2.6 cm

**Direction for Questions 16 and 17: Read the passage below and solve the questions based on it.**

$f(x)$ is the area of a square where, $x$ is the side of a square. $g(x)$ is the perimeter of square where, $x$ is the side of a square. $h(x, y)$ is the area of a rectangle where $x$ is the length and $y$ is the breadth. $i(x, y)$ is the perimeter of a rectangle where $x$ is the length and $y$ is the breadth.

**Q.16** The value of $g[h\{f\{i(2, 3)\}, 2\}]$ is equal to:
(a) 450 (b) 600 (c) 800 (d) 1000

**Q.17** The value of $\frac{[f(3)-g(2)]}{h(4,2)-i(6,2)}\times\frac{g(6)}{f(1)}$ is equal to:
(a) −3 (b) 2 (c) 1 (d) −4

**Direction for Questions 18 and 19: Read the passage below and solve the questions based on it.**

A 10-m long piece of wire is cut into two pieces, one of which is bent into a circle and the other into the square enclosing it.

Q.18 The area of the square outside the circle is:
(a) 1200 cm² (b) 2400 cm²
(c) 3600 cm² (d) 4200 cm²

Q.19 The ratio of the radius of the circle to the perimeter of the square is:
(a) 8:1 (b) 1:8 (c) 4:1 (d) 1:4

Q.20 There are two regular polygons with the number of sides in the ratio of 4:5 and the interior angles in the ratio of 25:26. The number of sides in the first polygon are:
(a) 8 (b) 10 (c) 12 (d) 15

Q.21 A square is inscribed in a semi-circle of radius 10 cm. What is the area of the inscribed square? (Given that the side of the square is along the diameter of the semicircle.)
(a) 70 cm² (b) 50 cm² (c) 25 cm² (d) 80 cm²

Q.22 ABCD is a parallelogram in which $\angle B = 70^\circ$. Find the number of points X in the plane of the parallelogram such that it is equidistant from its vertices.
(a) zero (b) one (c) two (d) three

Q.23 There is a fan with 3 blades at 120° to each other whose central circular disc has an area of $3\pi$ cm² and a blade is $(20-\sqrt{3})$ cm long. If the tips of the blades are joined so as to form an equilateral triangle, what will be its area?
(a) 900 cm² (b) $300\sqrt{3}$ cm²
(c) $(900+9\pi)$ cm² (d) $(3\pi+300)$ cm²

Q.24 Two circles of an equal radii are drawn, without any overlap, in a semicircle of radius 2 cm. If these are the largest possible circles that the semicircle can accommodate, what is the radius (in cm) of each of the circles?
(a) 0.414 (b) 0.828 (c) 0.172 (d) 0.586

Q.25 The sides of a regular octagon are extended to form a star. Find the measure of the internal angle at each point of the star.
(a) 45° (b) 90° (c) 135° (d) 60°

Q.26 PQRS is trapezium, in which PQ is parallel to RS, and PQ = 3 (RS). The diagonal of the trapezium intersect each other at X, then the ratio of ΔPXQ and ΔRXS is
(a) 6:1 (b) 3:1 (c) 9:1 (d) 7:1

**Direction for Questions 27 and 28: Read the passage below and solve the questions based on it.**

All the angles of a quadrilateral form a GP and its common ratio is a natural number. Exactly two of the angles are obtuse and none of the angles is a right angle.

Q.27 What is the measurement of the largest angle?
(a) 124° (b) 156°
(c) 192° (d) None of these

Q.28 What is the measurement of the smallest angle?
(a) 18° (b) 24°
(c) 36° (d) None of these

Q.29 Two circles with centres C1 and C2 and radii 6 cm and 8 cm, respectively, cut each other at right angles. Find the length of the common chord.
(a) 10 cm (b) 4.8 cm (c) 9.6 cm (d) 5 cm

Q.30 Let ABCDEF be a regular hexagon. What is the ratio of the area of the triangle ACE to that of the hexagon ABCDEF?
(a) 1/3 (b) 1/2 (c) 2/3 (d) 5/6

Q.31 The sides of a triangle are in the ratio of $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$.
If the perimeter is 52 cm, then the length of the smallest side is:
(a) 9 cm (b) 10 cm (c) 11 cm (d) 12 cm

Q.32 A pond 100 m in diameter is surrounded by a circular grass walk-way 2 m wide. How many square metres of grass is the on the walk-way?
(a) 98 $\pi$ (b) 100 $\pi$ (c) 204 $\pi$ (d) 202 $\pi$

Q.33 A triangle and a parallelogram are constructed on the same base such that their areas are equal. If the altitude of the parallelogram is 100 m, then the altitude of the triangle is:
(a) 100 m (b) 200 m
(c) $100\sqrt{2}$ m (d) $10\sqrt{2}$ m

Q.34 A rhombus OABC is drawn inside a circle, whose centre is at O, in such a way that the vertices A, B, and C of the rhombus are on the circle. If the area of the rhombus is $32\sqrt{3}$ m², then the radius of the circle is:
(a) 64 m (b) 8 m (c) 32 m (d) 46 m

Q.35 Let A be the area of a square inscribed in a circle of radius '$r$', and let B be the area of a hexagon inscribed in the same circle. Then, B/A equals:
(a) $2\sqrt{3}$ (b) $\frac{3\sqrt{3}}{4}$
(c) $\frac{2\sqrt{3}}{4}$ (d) None of these

Q.36 The dimensions of a rectangular box are in the ratio of 1:2:4 and the difference between the costs of covering it with the cloth and a sheet at the rate of ₹20 and ₹20.5 per sq m, respectively, is ₹126. Find the dimensions of the box.
(a) 2 m, 6 m, 9 m (b) 6 m, 12 m, 24 m
(c) 1 m, 2 m, 4 m (d) None of these

**Q.37** If the sides 50 m and 130 m of the triangular field meet at an angle of 72°, then find the area in which wheat is cultivated. (sin 72° = 0.9510, cos 72° = 0.309)
(a) 100 $m^2$ (b) 125 $m^2$
(c) 160 $m^2$ (d) None of these

**Q.38** Four horses are tied on the four corners of a square field of 14 m length so that each horse can touch just the other two horses. They were able to graze in the area accessible to them for 11 days. For how many days is the ungrazed area sufficient for them?
(a) 3 days (b) 4 days (c) 5 days (d) 2 days

**Q.39** The ratio of the area of a square inscribed in a semi-circle to that of the area of a square inscribed in the circle of the same radius is
(a) 2:1 (b) 2:3 (c) 2:5 (d) 2:7

**Q.40** The area of a rectangle is given by $A = 8x^2 - 2x - 15$. If the length was larger than the breadth, then the length can be
(a) $(3x + 5)$ (b) $(5x + 3)$ (c) $(4x + 5)$ (d) $(2x - 3)$

**Q.41** The ratio of the area of a square to that of the square drawn on the its the diagonal is
(a) 1:4 (b) 2:1 (c) 1:2 (d) 1:3

**Q.42** An equilateral triangle had 7 cm long sides. Equal circles were drawn about the three vertices of the triangle so that each circle touched the other two. Calculate the area of space enclosed between these circles.
(a) 3.934 $cm^2$ (b) 1.967 $cm^2$
(c) 3.9 $cm^2$ (d) 1.95 $cm^2$

**Q.43** What is the area of the triangle in which two of its medians 9 cm and 12 cm long intersect at the right angles?
(a) 72 (b) 60 (c) 56 (d) 48

**Q.44** If one leg of an isosceles right-angled triangle is increased by 6 cm and that of the other leg decreased by 4 cm, then the area of the triangle decreases by 24 sq cm. Find the length of the leg of the original triangle.
(a) 36 cm (b) 30 cm
(c) 24 cm (d) None of these

**Q.45** ABCD is a parallelogram in which $\angle D = 120°$. The bisector of $\angle D$ bisects the side AB. If the length of the bisector is 5 cm, then what is the perimeter of the parallelogram?
(a) 30 cm (b) 25 cm (c) 40 cm (d) 15 cm

**Q.46** Four horses are tethered at four corners of a square plot of side 14 m so that the adjacent horses can just reach one another. There is a small circular pond of area 20 $m^2$ at the centre. Find the ungrazed area.
(a) 22 $m^2$ (b) 42 $m^2$ (c) 84 $m^2$ (d) 168 $m^2$

**Q.47** Euclid has a triangle in mind. Its longest side has length 20 and another of its sides has length 10. Its area is 80. What is the exact length of its third side?
(a) $\sqrt{260}$ (b) $\sqrt{250}$ (c) $\sqrt{240}$ (d) $\sqrt{270}$

**Q.48** AB is the hypotenuse in the right angled triangle ABC. N is the point inside the triangle which divides the triangle in three equal parts ($\Delta ABN$, $\Delta CAN$, $\Delta BCN$). What is the distance between the circumcentre of this triangle from this point N?
(a) $\frac{AB}{4}$ (b) $\frac{AB}{6}$ (c) $\frac{AB}{3}$ (d) $\frac{2}{1+\sqrt{3}}$

**Q.49** Two sides of a triangle are 4 and 5. Then, for the area of the triangle, which one of the following bounds is the sharpest?
(a) <10 (b) ≤10 (c) ≤8 (d) >5

**Q.50** ABC is a triangle and P is a point inside it such that $\angle BPC = \angle CPA = \angle APB$. Then, P is:
(a) The point of intersection of medians
(b) The incentre
(c) The circumcentre
(d) None of these

## MODERATE

**Q.1** The angles of a triangle are in the ratio of 4:1:1. Then, the ratio of the largest side to the perimeter is:
(a) $\frac{2}{3}$ (b) $\frac{1}{2+\sqrt{3}}$ (c) $\frac{\sqrt{3}}{2+\sqrt{3}}$ (d) $\frac{2}{1+\sqrt{3}}$

**Q.2** In $\Delta ABC$, P and Q are mid-points of sides AB and BC, respectively, right angled at B, then:
(a) $AQ^2 + CP^2 = AC^2$
(b) $AQ^2 + CP^2 = \frac{4}{5}AC^2$
(c) $AQ^2 + CP^2 = \frac{3}{4}AC^2$
(d) $AQ^2 + CP^2 = \frac{5}{4}AC^2$

**Q.3** Three identical right angle cones with base radius r are placed on their bases so that each is touching the other two. The radius of the circle drawn through their vertices is:
(a) Smaller than r
(b) Equal to r

(c) Larger than $r$
(d) Depends on the height of the cones

Q.4 The line AB is 6 m in length and is tangent to the inner of the two concentric circles at point C. It is known that the radii of the two circles are integers. The radius of the outer circle is....., where A and B are points on the outer circle.
(a) 5 m (b) 4 m (c) 6 m (d) 3 m

Q.5 Under the usual 2-dimensional coordinate system the equation $|x| + |y| = 1$, where $x$ and $y$ are real numbers, represents:
(a) A rhombus which is not a square.
(b) A parallelogram which is not a rhombus.
(c) A square whose sides are not parallel to the coordinate axis.
(d) A square with sides parallel to the coordinate axis.

Q.6 Two chords of lengths a and b of a circle subtend 60° and 90° angles at the centre, respectively. Which of the following is correct?
(a) $b = \sqrt{2a}$ (b) $b = \sqrt{2b}$ (c) $a = 2b$ (d) $b = 2a$

Q.7 PQR is a right-angled triangle with $\angle Q = 90°$, S is the mid-point of PR, and QS $= \sqrt{117}$ cm. The sum of the length of sides PQ and QR is 30 cm. Area of $\nabla$ PQR is:
(a) 216 cm² (b) 108 cm²
(c) 54 cm² (d) 162 cm²

Q.8 In a trapezium, the diagonals intersect at point O. The ratio of the length of one of the diagonals from one vertex of the trapezium to the point O to its entire length is 2:5. Find the ratio of its parallel sides (smaller side:larger side).
(a) 2:5 (b) 2:3 (c) 2:7 (d) 5:7

Q.9 A chord of length 32 cm is placed inside a circle of radius 20 cm and a point whose distance from the centre of the circle is 13 cm, is marked on the chord. Calculate the lengths of the segment of the chord.
(a) 21 cm and 11 cm (b) 19 cm and 13 cm
(c) 16 cm each (d) 18 cm and 14 cm

Q.10 The sum of the lengths of the hypotenuse and one of the perpendicular sides of a right angled triangle is L. When the area of this triangle is maximum, the angle between these two sides is:
(a) 45° (b) 22.5°
(c) 60° (d) None of these

Q.11 In a triangle, the lengths of the two larger sides are 8 cm and 7 cm. The angles of the triangle are in an arithmetic progression. The length of the remaining side can be:
(a) 3 cm (b) 5 cm
(c) Either 3 cm or 5 cm (d) None of these

Q.12 The biggest possible regular hexagon H is cut out of an equilateral triangle X. The biggest possible equilateral triangle Y is cut out from the hexagon H. What is the ratio of the areas of the equilateral triangles X and Y?
(a) 5:1 (b) 6:1 (c) 8:1 (d) 3:1

Q.13 Nine parallel chords are drawn in a circle of diameter 10 cm. If the distance between any two of the adjacent chords is 1 cm, which of the following statements is always true?
(a) One of the chords is the diameter of the circle.
(b) At least two of the chords must be of an equal length.
(c) The difference between the lengths of any two adjacent chords on the same side of the diameter is greater than 1 cm.
(d) None of these

**Direction for Questions 14 and 15: Read the passage below and solve the questions based on it.**

There are three equal circles of unit radii touching each other.

Q.14 Find the area of the triangle circumscribing the three circles.
(a) $\sqrt{3}(\sqrt{3}+1)^2$ (b) $(\sqrt{3}+1)^2$
(c) $\sqrt{3}(\sqrt{3}+1)$ (d) None of these

Q.15 What would be the area of the remaining portion if the same three circles are circumscribed by another circle?
(a) $\pi\left(\frac{2}{\sqrt{3}}+1\right)^2$ (b) $\pi\left(\frac{2}{\sqrt{3}}+1\right)^2 - 3\pi$
(c) $\left(\frac{3}{\sqrt{2}}+1\right)^2\pi - 3\pi$ (d) $\left(\frac{1}{\sqrt{3}}+1\right)^2\pi - 3\pi$

Q.16 A circle passes through the vertex A of an equilateral triangle ABC and is tangent to BC at it mid-point. Find the ratio in which the circle divides each of the sides AB and AC.
(a) 1:1 (b) 3:2 (c) 3:1 (d) 2:1

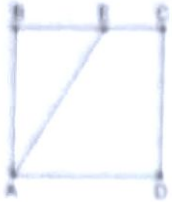
Q.17 In any quadrilateral ABCD, the diagonal AC and BD intersect at a point X. If E, F, G, and H are the mid-points of AX, BX, CX, and DX, respectively, then what is the ratio of (EF + FG + GH + HE) to (AD + DC + CB + BA)?
(a) $\frac{1}{2}$ (b) $\frac{3}{2}$
(c) $\frac{3}{4}$ (d) Data insufficient

**Q.18** Points A and B are on a circle of radius 5 and AB = 6. Point C is the mid-point of the minor arc AB. What is the length of the line segment AC?

(a) $\sqrt{10}$ (b) $\frac{7}{2}$ (c) $\sqrt{14}$ (d) $\sqrt{15}$

**Q.19** Square ABCD has side length 10. Point E is on $\overline{BC}$, and the area of $\triangle ABE$ is 40. What is BE?

(a) 4 (b) 5 (c) 8 (d) 7




**Q.20** A vertical lamp-post OP stands at the centre O of a square ABCD. Let h and b denote the length OP and AB, respectively. Suppose $\angle APB = 60°$ then the relationship between h and b can be expressed as

(a) $2h^2 = b^2$ (b) $2h^2 = b^2$
(c) $3h^2 = 2b^2$ (d) $3h^2 = 2b^2$

**Q.21** Consider two different cloth-cutting processes. In the first one, n circular cloth pieces are cut from a square cloth piece of side a in the following steps: the original square of side a is divided into n smaller squares, not necessarily of the same size, then a circle of the maximum possible area is cut from each of the smaller squares. In the second process, only one circle of the maximum possible area is cut from the square of side a and the process ends there. The cloth pieces remaining after cutting the circles are scraped in both the process. The ratio of the total area of the scrap cloth generated in the former to that in the latter is

(a) 1:1 (b) $\sqrt{2}:1$

(c) $\frac{n(4-\pi)}{4n-\pi}$ (d) $\frac{4n-\pi}{n(4-\pi)}$

**Q.22** Let $S_1$ be a square of side a. Another square $S_2$ is formed by joining the mid-points of the sides of $S_1$. The same process is applied to $S_2$ to form yet another square $S_3$, and so on. If $A_1, A_2, A_3, \ldots$ be the areas and $P_1, P_2, P_3, \ldots$ be the perimeters of $S_1, S_2, S_3, \ldots$, respectively, then the ratio of $\frac{P_1 + P_2 + P_3 + \ldots}{A_1 + A_2 + A_3 + \ldots}$ equals

(a) $\frac{(2+\sqrt{2})}{a}$ (b) $\frac{2(2-\sqrt{2})}{a}$

(c) $\frac{2(2+\sqrt{2})}{a}$ (d) $\frac{2(1+2\sqrt{2})}{a}$

**Q.23** One of the angles of a parallelogram is of 150°. Altitudes are drawn from the vertex of this angle. If these altitudes measure 6 cm and 8 cm, then find the perimeter of the parallelogram.

(a) 28 cm (b) 42 cm (c) 56 cm (d) 64 cm

**Q.24** A square, whose side is 2 m, has its corners cut away so as to form an octagon with all sides equal. Then, the length of the each side of the octagon, in metres is:

(a) $\frac{\sqrt{2}}{\sqrt{2}+1}$ (b) $\frac{\sqrt{2}}{\sqrt{2}-1}$ (c) $\frac{\sqrt{2}}{\sqrt{2}-\frac{1}{2}}$ (d) $\frac{\sqrt{2}}{\sqrt{2}+\frac{1}{2}}$

**Q.25** A certain city has a circular wall around it and this wall has four gates pointing north, south, east, and west. A house stands outside the city 3 km north of the north gate, and it can be seen from a point 9 km east of the south gate. What is the diameter of the wall that surrounds the city?

(a) 6 km (b) 9 km
(c) 12 km (d) None of these

**Q.26** The perimeter of a right-angled triangle is four times the shortest side. The ratio of the other two sides is:

(a) 5:6 (b) 3:4 (c) 4:5 (d) 2:3

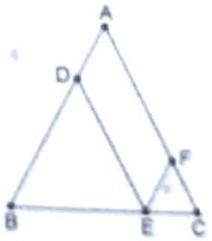
**Q.27** A circle is inscribed in an equilateral triangle and a square is inscribed in the circle. The ratio of the area of the triangle to the area of the square is:

(a) $\sqrt{3}:\sqrt{2}$ (b) $3\sqrt{3}:2$ (c) $3:\sqrt{2}$ (d) $\sqrt{2}:1$

**Q.28** The sides of a triangle are given to be $x^2+x+1$, $2x+1$ and $x^2-1$. Then, the largest of the three angles of the triangle is

(a) 75° (b) $\frac{x}{x+1}\pi$ (c) 120° (d) 135°

**Q.29** In $\triangle ABC$, $AB = AC = 28$ and $BC = 20$. Points D, E, and F are on sides $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$, respectively, such that $\overline{DE}$ and $\overline{EF}$ are parallel to $\overline{AC}$ and $\overline{AB}$, respectively. What is the perimeter of parallelogram ADEF?



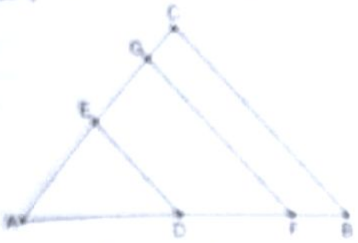

(a) 48 (b) 52 (c) 54 (d) 56

**Q.30** In a triangle ABC, the medians AM and CN to the sides BC and AB, respectively, intersect at the point O. Let P be the mid-point of AC and let MP intersect CN at Q. If the area of the triangle OMQ is s square units, the area of ABC is:

(a) 16s (b) 18s (c) 12s (d) 24s

Q.33 ABCD is a cyclic quadrilateral and the points A, B and C form an equilateral triangle. What is the sum of the lengths of line segments DA and DC?
(a) DB (b) DB/2 (c) $\sqrt{2}$ DB (d) DB/$\sqrt{2}$

Q.34 Triangle ABC is equilateral with AB = 1. Points E and G are on $\overline{AC}$ and points D and F are on $\overline{AB}$ such that both $\overline{EF}$ and $\overline{FG}$ are parallel to $\overline{BC}$. Furthermore, triangle ADE and trapezoids DFGE and FBCG all have the same perimeter. What is DE + FG?




(a) 1 (b) $\frac{3}{2}$ (c) $\frac{21}{13}$ (d) $\frac{13}{8}$ (e) $\frac{5}{3}$

Q.35 The side AB of a rectangle ABCD is a tangent to a circle which passes through the points C and D. The centre of the circle does not lie within the rectangle ABCD. If the length of the rectangle is twice the breadth, then what is the radius of the circle (in terms of breadth of the rectangle)?
(a) Breadth $(2 - \sqrt{2})$ (b) Breadth
(c) 2 × Breadth (d) None of these

Q.36 Let $C_1$ and $C_2$ be the inscribed and circumscribed circles of a triangle with sides 3 cm, 4 cm and 5 cm. The $\frac{\text{area of } C_1}{\text{area of } C_2}$ equals

(a) $\frac{16}{25}$ (b) $\frac{4}{25}$ (c) $\frac{9}{25}$ (d) $\frac{9}{16}$

Q.37 The sides of a triangle are given by: $\sqrt{b^2+c^2}$, $\sqrt{c^2+a^2}$ and $\sqrt{a^2+b^2}$ where $a, b, c$ are positive. Then, the area of the triangle equals

(a) $\frac{1}{2}\sqrt{b^2c^2+c^2a^2+a^2b^2}$ (b) $\frac{1}{2}\sqrt{a^4+b^4+c^4}$

(c) $\frac{\sqrt{3}}{2}\sqrt{b^2c^2+c^2a^2+a^2b^2}$ (d) $\frac{\sqrt{3}}{2}(bc+ca+ab)$

Q.38 Let the bisector of the angle at C of a triangle ABC intersect the side AB in a point D. Then, the geometric mean of CA and CB
(a) is less than CD.
(b) is equal to CD.
(c) is greater than CD.
(d) does not always satisfy any one of the foregoing property.

Q.39 There is a pole at the centre of a stadium which is in the shape of an equilateral triangle. The angle subtended by any side of the stadium at the top of the tower is right-angled. What is the ratio of the height of the pole to the perimeter of the stadium?

(a) $\sqrt{6}$ (b) $\frac{\sqrt{6}}{18}$ (c) $\frac{\sqrt{6}}{3}$ (d) $\frac{\sqrt{6}}{4}$

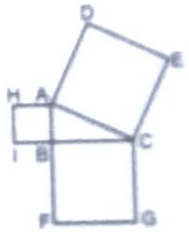
Q.40 In $\triangle ABC$, AB = 96 and AC = 97. A circle with centre A and radius AB intersects $\overline{BC}$ at points B and X. Moreover, BX and CX have integer lengths. What is BC?
(a) 11 (b) 61 (c) 33 (d) 28

Q.41 The length of the common chord of two circles of radii 15 cm and 20 cm, whose centres are 25 cm apart, is (in cm):
(a) 24 (b) 25 (c) 15 (d) 20

Q.42 PQRS is a trapezium with PQ and RS parallel. PQ = 6 cm, QR = 5 cm, RS = 3 cm, PS = 4 cm. The area of PQRS is
(a) 27 cm$^2$ (b) 12 cm$^2$
(c) 18 cm$^2$ (d) Cannot be determined

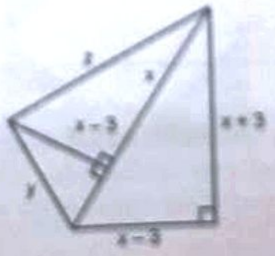
Q.43 If the number of square inches in the area of a square is equal to the number of inches in its circumference, then the diagonal of the square is equal to:
(a) 4 (b) $4\sqrt{2}$ (c) $3\sqrt{2}$ (d) $\sqrt{2}$

Q.44 All the three quadrilaterals ADEC, ABIH and BCGF are squares and ABC = 90°. If the area of ADEC = $x^2$ and the area of AHIB = $y^2$ ($x^2 > y^2$), then the area of BCGF is:



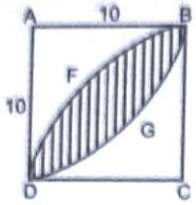

(a) $(x + y)(x - y)$ (b) $(x + y)^2$
(c) $(x - y)^2$ (d) None of these

Q.45 Based on the figure below, what is the value of x, if y = 10.




(a) 10 (b) 11
(c) 12 (d) None of these

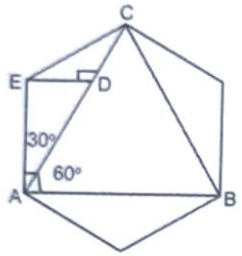
**Q.44** In the figure, ABCD is a square with side 10. BFD is an arc of a circle with centre C. BGD is an arc of a circle with centre A. What is the area of the shaded region?



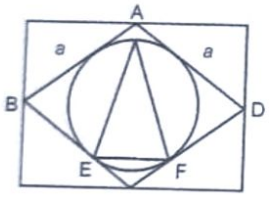

(a) $100\pi - 50$ (b) $100 - 25\pi$
(c) $50\pi - 100$ (d) $25\pi - 10$

**Q.45** In the figure below, if the perimeter of $\nabla ABC$ is $p$, then the perimeter of the regular hexagon is:

(a) $\frac{3p}{\sqrt{2}}$ (b) $\frac{\sqrt{2}p}{3}$ (c) $\frac{\sqrt{3}p}{2}$ (d) $\frac{2p}{\sqrt{3}}$




**Q.46** What is the area of the inner equilateral triangle if the side of the outermost square is 'a'? (ABCD is a square).

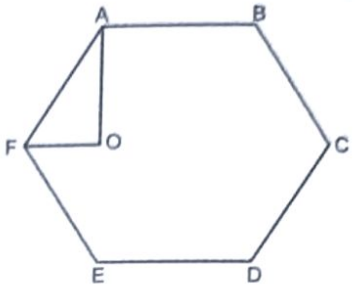



(a) $\frac{3\sqrt{3}a^2}{32}$ (b) $\frac{3\sqrt{3}a^2}{64}$ (c) $\frac{5\sqrt{3}a^2}{32}$ (d) $\frac{\sqrt{3}a^2}{16}$

**Q.47** Each side of a given polygon is parallel to either the X or the Y-axis. A corner of such a polygon is said to be convex if the internal angle is 90° or concave if the internal angle is 270°. If the number of convex corners in such a polygon is 25, then the number of concaves must be:

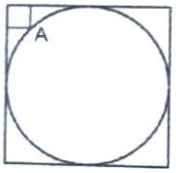
(a) 20 (b) 0 (c) 21 (d) 22

**Q.48** In the figure below, ABCDEF is a regular hexagon and AOF = 90°. FO is parallel to ED. What is the ratio of the area of the triangle AOF to that of the hexagon ABCDEF?




(a) $\frac{1}{12}$ (b) $\frac{1}{6}$ (c) $\frac{1}{24}$ (d) $\frac{1}{18}$

**Q.49** In the figure below, the rectangle at the corner measures 10 cm × 20 cm. The corner A of the rectangle is also a point on the circumference of the circle. What is the radius of the circle in cm?




(a) 10 cm (b) 40 cm
(c) 50 cm (d) None of these

**Q.50** A piece of paper is in the shape of a right-angled triangle and is cut along a line that is parallel to the hypotenuse, leaving a smaller triangle. There was a 35% reduction in the length of the hypotenuse of the triangle. If the area of the original triangle was 34 square inches before the cut, what is the area (in square inches) of the smaller triangle?

(a) 16.665 (b) 16.565 (c) 15.465 (d) 14.365

## ADVANCED

**Q.1** A ball of diameter 15 cm is floating so that the top of the ball is 5 cm above the smooth surface (water) of the pond. What is the circumference in centimetres of the circle formed by the contact of the water surface with the ball?

(a) $10\sqrt{2}\pi$ (b) 50 p (c) 10 p (d) $5\sqrt{2}\pi$

**Q.2** Let S be as an arbitrary point on the side PQ of an acute-angled $\nabla PQR$. Let T be the point of intersection of QR extended with the straight line PT drawn parallel to SR through P. Let U be the point of intersection of PR extended with the straight line QU drawn parallel to SR through Q. If PT = a and QU = b, then the length of SR is:

(a) $\frac{a+b}{ab}$ (b) $\frac{a-b}{ab}$ (c) $\frac{ab}{a+b}$ (d) $\frac{ab}{a-b}$

Q.3 The municipal authorities of three Cities A, B, and C jointly constructed three straight roads connecting these cities. The area enclosed by three roads is $150\sqrt{39}$ km$^2$. The authorities of City B found the length of the road connecting it to the City A is 40 km and that to the City C is 50 km. What is the length of the road connecting the Cities A and C, if it is the longest of the three roads?

(a) $20\sqrt{3}$ km (b) $30\sqrt{3}$ km
(c) $30\sqrt{6}$ km (d) $10\sqrt{55}$ km

Q.4 In an equilateral D, 3 coins of radii 1 unit each are kept in such a way that they touch each other and also the sides of the triangle. What is the area of the triangle (in sq. units)?

(a) $4+5\sqrt{2}$ (b) $6+4\sqrt{3}$ (c) $4+6\sqrt{3}$ (d) $3+8\sqrt{3}$

Q.5 In a scalene $\Delta$, sum of all the sides can be at most 13 units. How many triangles are possible?

(a) 3 (b) 4 (c) 5 (d) 6

Q.6 Three circles of equal radii have been drawn inside an equilateral triangle, of side a, such that each circle touches the other two circles as well as two sides of the triangles. Then, the radius of each circle is:

(a) $\frac{a}{2(\sqrt{3}+1)}$ (b) $\frac{a}{2(\sqrt{3}-1)}$ (c) $\frac{a}{\sqrt{3}-1}$ (d) $\frac{a}{\sqrt{3}-1}$

Q.7 Consider a circle with unit radius. There are seven adjacent sectors, $S_1, S_2, S_3, \ldots, S_7$, in the circle such that their total area is $\frac{1}{8}$ th of the area of the circle. Further, the area of the jth sector is twice that of the $(j-1)$th sector, for $j = 2, \ldots, 7$. What is the angle, in radians, subtended by the arc of $S_1$ at the centre of the circle?

(a) $\frac{\pi}{508}$ (b) $\frac{\pi}{2040}$ (c) $\frac{\pi}{1016}$ (d) $\frac{\pi}{1524}$

Q.8 In a triangle ABC, let $C_1$ be any point on the side AB other than A or B. Join $CC_1$. The line passing through A and parallel to $CC_1$ intersects the line BC extended at $A_1$. The line passing through B and parallel to $CC_1$ intersects the line AC extended at $B_1$. The lengths $AA_1$, $BB_1$, $CC_1$ are given to be $p$, $q$, $r$ units, respectively. Then:

(a) $r = \frac{pq}{p+q}$ (b) $r = \frac{p+q}{4}$

(c) $r = \frac{\sqrt{pq}}{2}$ (d) None of these

Q.9 Two circles APQC and PBDQ intersect each other at the points P and Q, and APB and CQD are two parallel straight lines. Then, only one of the following statements is always true. Which one is it?

(a) ABDC is a cyclic quadrilateral.
(b) AC is parallel to BD.
(c) ABDC is a rectangle.
(d) $\Delta ACQ$ is a right angle.

Q.10 Let $s = \{(x, y): |x| + |y| = 2\}$. Then, the diameter of S is:

(a) 2 (b) $4\sqrt{2}$ (c) 4 (d) $2\sqrt{2}$

Q.11 Through the centroid of an equilateral triangle, a line parallel to the base is drawn. On this line, an arbitrary point P is taken inside the triangle. Let $h$ denote the distance of P from the base of the triangle. Let $h_1$ and $h_2$ be the distance of P from the other two sides of the triangle. Then:

(a) $h = \frac{h_1 + h_2}{2}$ (b) $h = \sqrt{h_1 h_2}$

(c) $h = \frac{2h_1 h_2}{h_1 + h_2}$ (d) None of these

Q.12 A pyramid has a square base ABCD and vertex E. The area of square ABCD is 196, and the areas of $\Delta ABE$ and $\Delta CDE$ are 105 and 91, respectively. What is the volume of the pyramid?

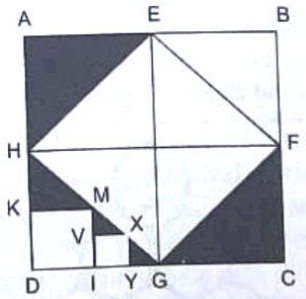
(a) 392 (b) $196\sqrt{6}$ (c) 784 (d) $392\sqrt{3}$

Q.13 Let A $(x_1, y_1)$, B $(x_2, y_2)$, C $(x_3, y_3)$, D $(x_4, y_4)$ be four points such that $x_1, x_2, x_3, x_4$ and $y_1, y_2, y_3, y_4$ are both in an AP. If $\Delta$ denotes the area of the quadrilateral ABCD, then:

(a) $\Delta = 0$
(b) $\Delta = 1$
(c) $\Delta < 1$
(d) $\Delta$ depends on the coordinates A, B, C, and D.

**Direction for Questions 14 to 16: Read the passage below and solve the questions based on it.**

In the figure given below, EFGH is a square formed by joining the mid-points of the sides of the square ABCD.KMID and VXYI are the squares formed inside the right angled triangles HDG and MLG, respectively. The side of the square ABCD is equal to '$a$' cm.




Q.14 What is the ratio of areas of ABCD and KMID?

(a) 4:1 (b) 8:1 (c) 16:1 (d) 12:1

Q.15 What is the ratio of areas of EFGH and VXYI?

(a) 32:1 (b) 64:1 (c) 16:1 (d) 8:1

**Q.16** What is the area of shaded region (in $cm^2$)?

(a) $\frac{5}{16}a^2$ (b) $\frac{19}{64}a^2$ (c) $\frac{7}{16}a^2$ (d) $\frac{27}{64}a^2$

**Q.17** Corners are sliced off a unit cube so that the six faces each become regular octagons. What is the total volume of the removed tetrahedron?

(a) $\frac{5\sqrt{2}-7}{3}$ (b) $\frac{10-4\sqrt{2}}{3}$

(c) $\frac{3-2\sqrt{2}}{3}$ (d) $\frac{8\sqrt{2}-11}{3}$

**Q.18** Three horses are grazing within a semi-circular field. In the diagram given below, AB is the diameter of the semi-circular field with centre at O. Horses are tied up at P, R, and S such that PO and RO are the radii of semi-circles with centres at P and R, respectively, and S is the centre of the circle touching the two semi-circles with diameters AO and OB. The horses tied at P and R can graze within the respective semi-circle and the horse tied at S can graze within the circle centred at S. The percentage of the area of the semi-circles with diameter AB that cannot be grazed by the horses is nearest to:

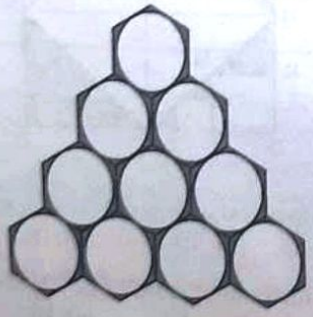

(a) 20 (b) 28 (c) 36 (d) 40

**Q.19** Inradius of a right-angled $\Delta ABC$ is 32 units and the sum of the length of its two sides other than the hypotenuse is 42 units. What is its circumradius?

(a) 26
(b) 52
(c) More than one value possible
(d) Triangle ABC is not possible

**Direction for Questions 20 and 21: Read the passage below and solve the questions based on it.**

In the figure given above, each hexagon is regular and has side measuring '$a$' cm. it is also given that the last row has '$n$' such regular hexagons and all the hexagons are inscribed by circles.



**Q.20** Find the ratio of area of the shaded region to the area of unshaded region.

(a) $\frac{3a^2}{2}[n(n+1)]\left(\sqrt{3}-\frac{\pi}{2}\right)$

(b) $\frac{a^2}{2}[n(n+1)]\left(\frac{3\sqrt{3}}{2}-\pi\right)$

(c) $\frac{3a^2}{2}[n(n+1)]\left(\sqrt{3}-\frac{\pi}{2}\right)$

(d) None of these

**Q.21** Suppose the above figure is a 3-D figure and each hexagon is inscribed by a sphere of maximum volume. The height of the hexagonal figure is just enough to contain the sphere, then find the volume of the shaded region (there are '$n$' hexagonal figures of side '$a$' in the last row).

(a) $\frac{\sqrt{3}}{2}a^3\left(\frac{3\sqrt{3}}{2}-\pi\right)[n(n+1)]a^3$

(b) $\frac{\sqrt{3}}{4}a^3\left(\frac{3\sqrt{3}}{4}-\pi\right)[n(n+1)]$

(c) $\frac{1}{2}\frac{\sqrt{3}}{2}a^3\left(\frac{3\sqrt{3}}{4}-\pi\right)[n(n+1)]$

(d) $\frac{\sqrt{3}}{2}a^3(3\sqrt{3}-\pi)[n(n+1)]$

**Q.22** If a sphere of the maximum volume is placed inside a hollow right circular cone with radius '$r$' and slant height '$\lambda$' such that the base of the cone touches the sphere, then the volume of the sphere is:

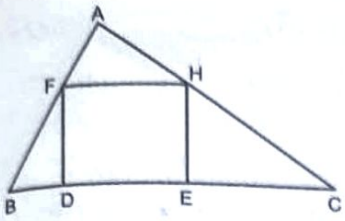
(a) $\frac{4}{3}\pi\left(\frac{\ell+r}{\ell-r}\right)^3$ (b) $\frac{4}{3}\pi r^3\left(\frac{\ell-r}{\ell+r}\right)^{\frac{3}{2}}$

(c) $\frac{4}{3}\pi\left(\frac{\ell-r}{\ell+r}\right)^3$ (d) $\frac{4}{3}\pi r^3\left(\frac{\ell+r}{\ell-r}\right)^{\frac{3}{2}}$

**Q.23** A hollow right circular cylinder of radius r and height $4r$ is standing vertically on a plane. If a solid right circular cone of radius $2r$ and height $6r$ is placed with its vertex down in the cylinder, then the volume of the portion of the cone outside the cylinder is:

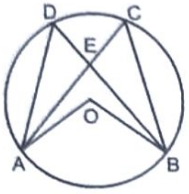
(a) $\frac{8}{3}\pi r^3$ (b) $2\pi r^3$ (c) $\frac{9}{8}\pi r^3$ (d) $7\pi r^3$

**Q.24** Consider the following figure: AB = 10 cm, AC = 17 cm, BC = 21 cm and EHFD is a square. Find the length of the side of square (in cm).



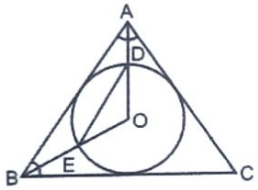

(a) 10.5 (b) 12
(c) 13.5 (d) None of these

Q.25 In the given figure, O is the centre of the circle and $\angle AOB = 120°$. What is the mesure of $\angle AEB$?




(a) 100° (b) 90°
(c) 110° (d) Cannot be determined

Q.26 For the equilateral triangle ABC, D and E are mid-points of AO and OB, and DE = 3 units. The circumference of the circle is equal to:




(a) $4\pi$ units (b) $\pi\sqrt{2}$ units
(c) $2\pi\sqrt{3}$ units (d) None of these

Q.27 The length of the sides CB and CA of a triangle ABC are given by a and b, and the angle C is $2\pi/3$. The line CD bisects the angle C and meets AB at D. Then, the length of CD is:

(a) $\frac{a^2+b^2}{2(a+b)}$ (b) $\frac{ab}{2(a+b)}$ (c) $\frac{1}{a+b}$ (d) $\frac{ab}{a+b}$

Q.28 For a regular octagon inscribed in a circle of radius 1 cm, the product of the distance from a fixed vertex to the other seven vertices is:

(a) 4 (b) 8 (c) 12 (d) 16

**Direction for Questions 29 to 31: Read the passage below and solve the questions based on it.**

In the figure given below, a square ABCD is inscribed in a circle of radius 5 cm. The square ABCD is inscribed by a circle which is inscribed by a triangle EFG. The triangle EFG is inscribed by a circle which is again inscribed by a triangle XYZ.


**Q.29** Find the ratio of the radius of the outermost circle to the radius of the circle inscribed in triangle XYZ.

(a) 4:1 (b) $2\sqrt{2}:1$ (c) $4\sqrt{2}:1$ (d) 8:1

**Q.30** Find the ratio of the area of square ABCD and the area of triangle XYZ.

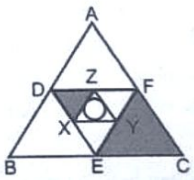
(a) $16:3\sqrt{3}$ (b) $64:3\sqrt{3}$ (c) 16:3 (d) $64:\sqrt{3}$

**Q.31** Find the area of the shaded region (in $cm^2$).

(a) $\frac{500\pi - 800 - 75\sqrt{3}}{16}$ (b) $\frac{900\pi - 1600 - 75\sqrt{3}}{}$

(c) $\frac{400\pi - 800 + 75\sqrt{3}}{32}$ (d) None of these

**Q.32** D, E, and F are the mid-points of the sides AB, BC, and CA, respectively, and X, Y, and Z the are mid-points of DE, EF, and FD, respectively. It is given that the circumradius of triangle ABC is $8\sqrt{3}$ cm and triangle XYZ is inscribed by a circle. (in $cm^2$). What is the ratio of shaded area to that of area of $\Delta ABC$?




(a) $\frac{3}{16}$ (b) $\frac{5}{16}$ (c) $\frac{7}{16}$ (d) $\frac{9}{16}$

**Q.33** The diameter of a right conical tent is 6 m. If a pole of length 2 m can be fixed in the tent at half the distance of the radius from the centre of the base, then the area of the canvas required is (in $m^2$):

(a) $10\pi$ (b) $12\pi$ (c) $15\pi$ (d) $16\pi$

**Q.34** A pyramid has its base as an equilateral triangle, of each side being one m. Its slant edge is 3 m. The whole surface area of the pyramid is equal to:

(a) $\frac{\sqrt{3}+3\sqrt{13}}{4}m^2$ (b) $\frac{\sqrt{3}+2\sqrt{35}}{4}m^2$

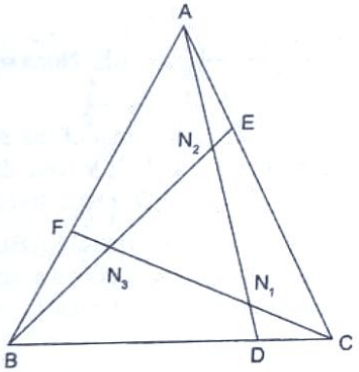
(c) $\frac{\sqrt{3}+2\sqrt{13}}{4}m^2$ (d) $\frac{\sqrt{3}+3\sqrt{35}}{4}m^2$

**Q.35** In the figure given below, if a flower is inscribed in a circle of radius 1 cm, then find the area of the flower.

(a) $\left(\pi-\frac{3\sqrt{3}}{2}\right)\text{cm}^2$ (b) $(2\pi-3\sqrt{3})\text{cm}^2$

(c) $\left(\frac{5\pi}{3}-\frac{5\sqrt{3}}{2}\right)\text{cm}^2$ (d) $(4\pi-3\sqrt{3})\text{cm}^2$

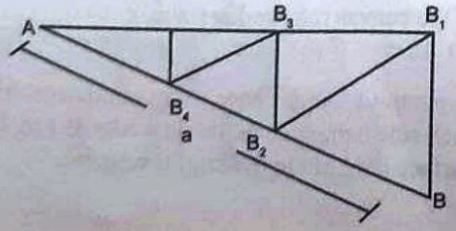
**Q.36** In the figure given below, CD, AE, and BF are one third of their respective sides. It follows that $AN_2 : N_2N_1 : N_1D = 3:3:1$ and similarly, for lines BE and CF. Then, the area of triangle $N_1N_2N_3$ is:




(a) $\frac{1}{7}\Delta ABC$ (b) $\frac{1}{6}\Delta ABC$

(c) $\frac{1}{9}\Delta ABC$ (d) $\frac{1}{10}\Delta ABC$

**Q.37** In the figure given below, two rays are drawn through a point A at an angle of 30°. A point B is taken on one of them at a distance of 'a' from the point A. A perpendicular is drawn from the point B to the other ray, and another perpendicular is drawn from its foot to meet AB at another point from where the similar process is repeated indefinitely. The length of all such lines will be:

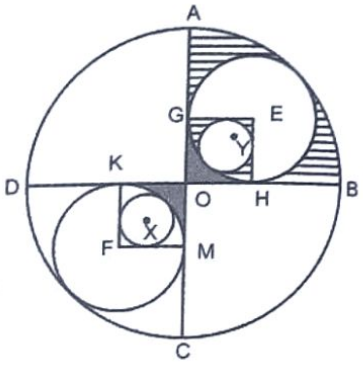



(a) $a(1-\sqrt{3})$ (b) $2a(2+\sqrt{3})$

(c) $\frac{3}{2a}$ (d) $a$

**Direction for Questions 38 to 40: Read the passage below and solve the questions based on it.**

In the figure given below, O is the centre of circles have radius (OB) + $r$. E and F are the centre of circles inscribed in the quarter circles AOB and COD, respectively, whereas X and Y are the centre of circles inscribed in the quarter circle KFM and GEH, respectively.




**Q.38** Find the area of the shaded region (not including the lined region).

(a) $\frac{3}{7}r^2(3-2\sqrt{2})$ (b) $\frac{3}{14}r^2(3-2\sqrt{2})$

(c) $3\frac{3}{14}r^2(3+2\sqrt{2})$ (d) None of these

**Q.39** Find the ratio of the radii of the circle with centre Y, E, and O.

(a) $1:(\sqrt{2}-1):(3+2\sqrt{2})$ (b) $1:(\sqrt{2}+1):(3-2\sqrt{2})$

(c) $1:(\sqrt{2}-1)(3-2\sqrt{2})$ (d) 1:2:4

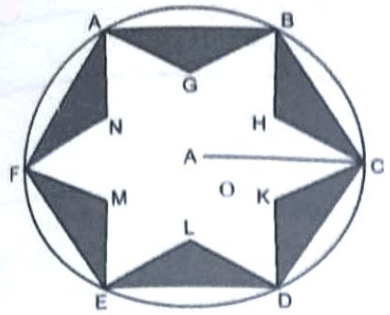
**Q.40** Find the area of the lined region.

(a) $\left(\frac{594\sqrt{2}-836}{14}\right)r^2$ (b) $\left(\frac{600\sqrt{2}-836}{14}\right)r^2$

(c) $\left(\frac{594\sqrt{2}-845}{14}\right)r^2$ (d) $\left(\frac{600\sqrt{2}-845}{14}\right)r^2$

**Direction for Questions 41 to 43: Read the passage below and solve the questions based on it.**

O is the centre of a circle having radiua (OC) = r. ABCDEF is a regular hexagon and AGBHCKDLEMFNA is a regular six-pointed star.




Q.41 Find the ratio of the circumference of a circle to the perimeter of the hexagon ABCDEF to the perimeter of the star AGBHCKD LEMFNA.

(a) $\pi:\sqrt{3}$ (b) $\pi:3:2\sqrt{3}$

(c) $2\cdot\pi:6:4\sqrt{2}$ (d) $\pi:2\sqrt{3}$

Q.42 Find the ratio of the area of the shaded region to the area of the star.

(a) 1:1 (b) 1:2 (c) 1:3 (d) 2:3

Q.43 If the triangles ANG, BGH, CHK, DKL, ELM, and FMN and the hexagon NGHKLM are inscribed by circles of the maximum area, then find the sum of the areas of all those circles.

(a) $\frac{\pi}{2}r^2$ (b) $\frac{\pi r^2}{3}$ (c) $\frac{5\pi r^2}{12}$ (d) $\frac{7\pi r^2}{12}$

Q.44 A triangle ABC with an obtuse angle B is inscribed in a circle. The altitude AD of the triangle is tangent to the circle. The side BC has length 12 cm and the segment BD has length 4 cm. Find the area of the triangle ABC (in $cm^2$).

(a) 48 (b) 60 (c) 72 (d) 36

Q.45 If the sum of the squares of the sides of a triangle is denoted by $A$ and its perimeter by $P$, then which of the following is true about $\frac{A^2}{P}$?

(a) $\frac{A^2}{P}<2$ (b) $1\le\frac{A^2}{P}\le 4$

(c) $2\le\frac{A^2}{P}\le 3$ (d) $4\le\frac{A^2}{P}$

Q.46 A ring of diameter 10 cm is suspended from a point 12 cm vertically above the centre by six equal strings. The strings are attached to the circumference of the ring at equal intervals, thus keeping the ring in a horizontal plane. The value of cosine of the angle between the two adjacent strings lies in between:

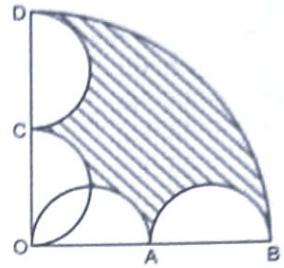
(a) 0 and 1/2 (b) 1/2 and 1

(c) −1/2 to 0 (d) −1 to −1/2

Q.47 A square of side 4 cm is drawn and a circle $C_1$ is inscribed in it. Now, four more circles $C_2$, $C_3$, $C_4$, and $C_5$, each of radius 2 cm, are drawn with centres as A, B, C, and D, respectively. In the resultant figure, what is the total area that is common to at least two of the five circles (in sq. cms)?

(a) $6\pi-16$ (b) $7\pi-16$ (c) $8\pi-16$ (d) $9\pi-16$

Q.48 In the figure, find the area of the shaded portion, that is, the portion of the quadrant DOB which is not included in the 4 semicircles. Given that OA = AB = OC = CD = 2 and $\angle DOB = 90°$.



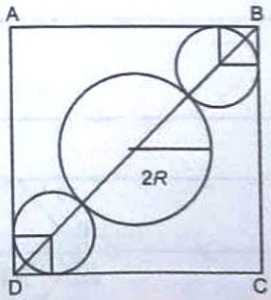

(a) $5\pi-\frac{1}{2}$ (b) $5\pi-1$ (c) $\frac{5\pi}{2}-\frac{1}{2}$ (d) $\frac{5\pi}{2}-1$

Q.49 Rajat cut out two identical triangular pieces of cardboard, each of area 300 sq. cm, and then placed them upon a table, one on top of the other such that the triangles completely coincide with each other. Now, if he rotated one of the two triangles by 180° about a vertical axis passing through its centroid, find the area that is common to both the triangles.

(a) 100 $cm^2$ (b) 150 $cm^2$

(c) 200 $cm^2$ (d) $133\frac{1}{3}$ $cm^2$

Q.50 See the figure given below:
Inside a square ABCD, three circles are drawn touching one another as shown in the figure. Radius of two smaller circles is R units and radius of the bigger circle is 2R units. Diagonal BD of the square passes through the centre of all the circles. What is the ratio of the radius of the smaller circles to the side of the square?




(a) 2:7 (b) 3:8

(c) 1:4 (d) None of these

# Answers

## Warm Up

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. (b) | 2. (c) | 3. (a) | 4. (a) | 5. (c) | 6. (b) | 7. (d) | 8. (c) | 9. (d) | 10. (c) |
| 11. (b) | 12. (c) | 13. (d) | 14. (a) | 15. (b) | 16. (a) | 17. (b) | 18. (b) | 19. (a) | 20. (a) |
| 21. (b) | 22. (d) | 23. (d) | 24. (c) | 25. (b) | | | | | |

## Foundation

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. (d) | 2. (d) | 3. (b) | 4. (b) | 5. (d) | 6. (d) | 7. (d) | 8. (d) | 9. (d) | 10. (c) |
| 11. (b) | 12. (b) | 13. (a) | 14. (b) | 15. (d) | 16. (c) | 17. (a) | 18. (d) | 19. (b) | 20. (c) |
| 21. (d) | 22. (a) | 23. (b) | 24. (b) | 25. (b) | 26. (c) | 27. (c) | 28. (b) | 29. (c) | 30. (b) |
| 31. (d) | 32. (c) | 33. (b) | 34. (b) | 35. (b) | 36. (d) | 37. (d) | 38. (a) | 39. (c) | 40. (c) |
| 41. (c) | 42. (b) | 43. (a) | 44. (d) | 45. (a) | 46. (b) | 47. (a) | 48. (b) | 49. (b) | 50. (d) |

## Moderate

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. (c) | 2. (d) | 3. (c) | 4. (a) | 5. (c) | 6. (a) | 7. (b) | 8. (b) | 9. (a) | 10. (c) |
| 11. (c) | 12. (d) | 13. (d) | 14. (a) | 15. (b) | 16. (c) | 17. (a) | 18. (a) | 19. (c) | 20. (b) |
| 21. (a) | 22. (c) | 23. (c) | 24. (a) | 25. (b) | 26. (c) | 27. (b) | 28. (c) | 29. (d) | 30. (d) |
| 31. (a) | 32. (c) | 33. (b) | 34. (b) | 35. (a) | 36. (c) | 37. (b) | 38. (b) | 39. (a) | 40. (c) |
| 41. (b) | 42. (a) | 43. (d) | 44. (c) | 45. (d) | 46. (d) | 47. (c) | 48. (a) | 49. (c) | 50. (d) |

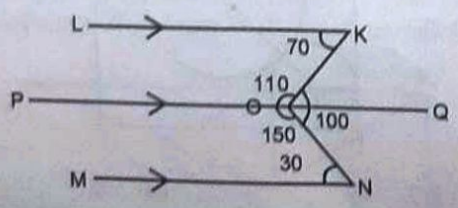
## Advanced

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. (a) | 2. (c) | 3. (b) | 4. (b) | 5. (c) | 6. (a) | 7. (c) | 8. (a) | 9. (b) | 10. (c) |
| 11. (a) | 12. (c) | 13. (a) | 14. (c) | 15. (a) | 16. (b) | 17. (b) | 18. (b) | 19. (d) | 20. (d) |
| 21. (d) | 22. (b) | 23. (d) | 24. (d) | 25. (d) | 26. (c) | 27. (d) | 28. (b) | 29. (b) | 30. (b) |
| 31. (d) | 32. (b) | 33. (c) | 34. (d) | 35. (b) | 36. (a) | 37. (c) | 38. (a) | 39. (c) | 40. (d) |
| 41. (b) | 42. (b) | 43. (c) | 44. (a) | 45. (c) | 46. (b) | 47. (c) | 48. (d) | 49. (c) | 50. (d) |

# Hints and Solutions

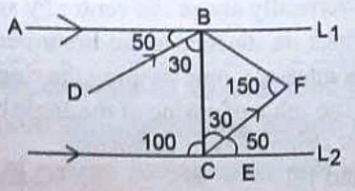
## Warm up

**1.**




Hence, $\angle MNO = 30°$

**2.**




$\angle FCE = 50°$