Tutoring of Mathematical Analysis I
TEST SIMULATION - 2

1. The domain of the function $f(x) = \log \dfrac{(x^2+1)(4-x^2)}{(x^2-2x+1)}$ is:

   (a) $(-2, 2)$
   (b) $(-2, 1) \cup (1, 2)$
   (c) $(-\infty, -2) \cup (2, +\infty)$
   (d) $[-2, 1) \cup (1, 2]$
   (e) $\mathbb{R} \setminus \{1\}$

2. The function $f : \mathbb{R} \to \mathbb{R} : f(x) = \sqrt{|x+1| - 2x}$

   (a) is surjective
   (b) is not invertible
   (c) has domain $(-\infty, 1)$
   (d) $\mathrm{im} f = (-1, +\infty)$
   (e) is injective

3. Given the equation with complex variable $\left|\overline{z} - \dfrac{9}{z}\right| (z^3 - 8i) = 0$, which of the following statements is correct?

   (a) The equation admits only 3 complex solutions
   (b) $z = -2i$ is the only imaginary solution of the equation
   (c) Among the infinite solutions of the equation, only 2 solutions are real
   (d) The solutions, represented in the Gauss plane, are the points with distance 3 from the origin
   (e) the complex numbers $z_{1,2} = \sqrt{3} \pm i$ are solutions of the equation

4. $\displaystyle\lim_{x\to 0^+} \frac{\sqrt{x^4 + x^6} - x^2}{x^4}$ equals

   (a) $+\infty$
   (b) 0
   (c) 1
   (d) 2
   (e) $\dfrac{1}{2}$

5. If $f = o(g)$ e $g = o(x^5)$, for $x \to 0$, then, for $x \to 0$, we can conclude that

   (a) $f(x)g(x) = o(x^{11})$
   (b) $f(x)g(x) \sim x^{10}$
   (c) $f(x) + x^5 = o(g(x))$
   (d) $\dfrac{f(x)}{g(x)} \to 0$
   (e) none of the previous answers

6. The function $f(x) = \log\left(3 + \dfrac{4}{x^2}\right) + 2x$ has the following left oblique asymptote:

   (a) $y = 3x + \log 3$
   (b) $y = 2x + \log 4$
   (c) $y = 5x + \log 4$
   (d) $y = 2x$
   (e) $y = 2x + \log 3$

7. $\lim_{n\to+\infty} \dfrac{\log n^{10} + \sin n}{\sqrt{n} - 3} =$
   (a) 10
   (b) $\nexists$
   (c) $+\infty$
   (d) 0
   (e) 1

8. Given the function $f(x) = log|\sin 2x|$, its first derivative equals
   (a) $f'(x) = \dfrac{\cos 2x}{\sin 2x}$
   (b) $f'(x) = 2\dfrac{\cos x}{\sin x}$
   (c) $f'(x) = \dfrac{1}{|\sin 2x|}$
   (d) $f'(x) = 2\dfrac{\cos 2x}{\sin 2x}$
   (e) $f'(x) = \dfrac{2}{\sin 2x}$

9. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function such that $f(0) = 1, f(4) = e, f'(0) = 2$. Consider the function $h(x) = \log f(x)$; only one sentence is correct:
   (a) There exists at least one point $c \in (0,4)$ such that $h'(c) = \dfrac{e-1}{4}$
   (b) $h'(0) = 2$
   (c) There exists at least one point $c \in (0,4)$ such that $h'(c) = -\dfrac{1}{4}$
   (d) There exists at least one point $c \in (0,4)$ such that $h(c) = 2$
   (e) There exists at least one point $c \in (0,4)$ such that $h'(c) = \dfrac{4}{e-1}$

10. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function such that $f(0) = 0,\ f(3) = 2$. Consider $g(x) = e^{-f(x)}$, then there exists a point $c \in (0,3)$ such that
   (a) $g'(c) = \dfrac{1-e^{-2}}{3}$
   (b) $g'(c) = \dfrac{2}{3}$
   (c) $g'(c) = \dfrac{1-e^{2}}{3}$
   (d) $g'(c) = \dfrac{e^2-1}{3e^2}$
   (e) $g'(c) = \dfrac{e^{-2}-1}{3}$

11. The Mac Laurin expansion of order 4 of the function $f(x) = \sin^2(x - x^2) - x^2 + 2x^3$ is
   (a) $x^4 + o(x^4)$
   (b) $\dfrac{2}{3}x^4$
   (c) $\dfrac{2}{3}x^4 + o(x^4)$
   (d) $4x^3 + \dfrac{2}{3}x^4 + o(x^4)$
   (e) none of the previous answers

12. Let f be a function of class $C^{(3)}(\mathbb{R})$, such that $f(x) = -2 + 2x^2 - 3x^3 + o(x^3)$. It is true that
    (a) $f(1) = -3$
    (b) $f(0) = -2$ e $f'(0) = 0$
    (c) $f(0) = -2$ e $f''(0) = 2$
    (d) The point $x = 0$ is a relative maximum point
    (e) $f'''(0) = -3$

13. The correct statement is:
    (a) if f is defined on $[a, b]$ and $f(a) \cdot f(b) < 0$, then f has at least one zero in (a, b)
    (b) if f is continuous on $[a, b]$ and $f(a) \cdot f(b) < 0$, then f has at least one zero in (a, b)
    (c) if f is continuous on (a, b) and $f(a) \cdot f(b) < 0$, then f has at least one zero in (a, b)
    (d) if f is continuous on $[a, b]$ and $f(a) \cdot f(b) < 0$, then f has at most one zero in (a, b)
    (e) if f is continuous on $[a, b]$ and $f(a) \cdot f(b) < 0$, then $0 \in (a, b)$

14. Let f be a continuous function on the interval I. A primitive of f on I is a function F such that:
    (a) $F(x) = \displaystyle\int_{x_0}^{x} f(t)\,\mathrm{d}t,$ with $x_0, x \in I$
    (b) F is defined on I and $F'(x) = f(x), \forall x \in I$
    (c) F is differentiable on I and $F'(x) = f(x), \forall x \in I$
    (d) $f'(x) = F(x), \forall x \in I$
    (e) F is differentiable in $x_0 \in I$ and $F'(x_0) = f(x_0)$

15. Given the function $f(x) = 4\log(\cosh x) - 2x^2 - 3x^3$, its principal part, for $x \to 0$, with respect to the infinitesimal test function $u(x)$, is:
    (a) $-3x^3$
    (b) $-2x^2$
    (c) $-3x^3 + o(x^3)$
    (d) $-2x^2 - 3x^3$
    (e) $-3x^3 - \dfrac{x^4}{3}$

16. The definite integral $\displaystyle\int_2^4 \frac{1}{x(2 - \log x)}\,\mathrm{d}x$ takes value:
    (a) $\log \dfrac{2 - \log 2}{\ln 2 - 1} - \log 2$
    (b) $\log \dfrac{2 - \log 2}{1 - \ln 2}$
    (c) $\log \dfrac{2 - \log 2}{2\ln 2 - 2}$
    (d) $|\log(2 - \log 2) - \log 2|$
    (e) $\log \dfrac{2 - \log 2}{1 - \ln 2} - \log 2$

17. Which is the correct formulation of the Integral Average Value Theorem?

 (a) Let f be an integrable function on $[a, b]$. The integral average value μ of f on $[a, b]$, satisfies the property $\min_{x\in[a,b]}\{f(x)\} \leq \mu = f(c) \leq \max_{x\in[a,b]}\{f(x)\}, \ c \in [a, b]$
 (b) Let f be an integrable function on $[a, b]$. The integral average value μ of f on $[a, b]$ satisfies the property $\min_{x\in[a,b]}\{f(x)\} \leq \mu \leq \max_{x\in[a,b]}\{f(x)\}$
 (c) Let f be an integrable function on $[a, b]$. The integral average value μ of f on $[a, b]$ satisfies the property $\inf_{x\in[a,b]}\{f(x)\} \leq \mu \leq \sup_{x\in[a,b]}\{f(x)\}$
 (d) Let f be a continuous function in $[a, b]$ and integrable in at least one point in (a, b). Let μ be the integral average value of f on $[a, b]$, then μ satisfies the property $\mu = \dfrac{1}{b-a}\displaystyle\int_a^b f'(x)\,\mathrm{d}x$
 (e) Let f be a function differentiable in $[a, b]$. The integral average value μ of f on $[a, b]$ satisfies the property $\min_{x\in[a,b]}\{f(x)\} \leq \mu = f'(c) \leq \max_{x\in[a,b]}\{f(x)\}, \ c \in [a, b]$

18. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function, and $F(x) = \displaystyle\int_x^0 f(t)\,\mathrm{d}t$. If $f \geq 0 \ \forall x \in \mathbb{R}$, then:

 (a) $F(x) \leq 0, \forall x \in \mathbb{R}$
 (b) $F(x) \geq 0, \forall x \in \mathbb{R}$
 (c) $F(x)$ is decreasing in $\mathbb{R}$
 (d) $F(x) \geq 0, \forall x \in (0, +\infty)$
 (e) $F(x)$ is increasing $\forall x \in (-\infty, 0)$

19. Which of the following properties is satisfied by the function $f(x) = \frac{\sin x}{x\sqrt{x}}$?

 (a) $\displaystyle\int_0^{+\infty} f(x)\,\mathrm{d}x$ is convergent
 (b) $\displaystyle\int_0^{\pi} f(x)\,\mathrm{d}x$ is divergent
 (c) $\displaystyle\int_1^{+\infty} f(x)\,\mathrm{d}x$ is not absolutely convergent
 (d) $\displaystyle\int_\pi^{+\infty} |f(x)|\,\mathrm{d}x$ is divergent
 (e) $f(x) \sim \dfrac{1}{\sqrt{x}}$, if $x \to 0$; $f(x) \sim \dfrac{1}{\sqrt{x}}$, if $x \to +\infty$

20. The differential equation $x'(t) = (2 - x)(1 - \sin x)(\pi - t)\sqrt{x}$

 (a) has not separable variables
 (b) is linear
 (c) admits the constant solution $t = \pi$
 (d) admits the constant solutions $x(t) = 2, \ x(t) = 0, \ x(t) = \dfrac{\pi}{2} + 2k\pi, \ k \in \mathbb{N}, \forall t \in \mathbb{R}$
 (e) admits the constant solutions $t = \pi, \ x(t) = 2, \ x(t) = 0, \ x(t) = \dfrac{\pi}{2} + 2k\pi, \ k \in \mathbb{Z}, \forall t \in \mathbb{R}$

ANSWERS

Item n.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	b	e	c	e	d	e	d	d	b	e	c	b	b	c	a	e	c	c	a	d