Tutoring of Mathematical Analysis I

TEST SIMULATION - E

1. The domain of $f(x) = \log(\sqrt{x-2} - 3)$ is:
   (a) $[2, 3]$
   (b) $[11, +\infty)$
   (c) $(5, +\infty)$
   (d) $(11, +\infty)$
   (e) $[3, +\infty)$

2. The function $f(x) = |\log(x - 3)|$
   (a) is invertible in the interval $[3, +\infty)$
   (b) is invertible in the interval $[4, +\infty)$
   (c) is invertible in the interval $(0, +\infty)$
   (d) is not invertible in any interval
   (e) is invertible in the interval $(3, 5)$

3. Let $z = 1 - 2i$. Then $|z^2 + \bar{z}|$ equals
   (a) $2\sqrt{2}$
   (b) 8
   (c) 4
   (d) $4\sqrt{2}$
   (e) 2

4. The limit $\displaystyle\lim_{x\to+\infty} \frac{e^{-2x} - 2x + \cos x}{e^{-x} + 3x - 3\sin x}$ takes value
   (a) -2
   (b) -1
   (c) 2
   (d) 0
   (e) $-\dfrac{2}{3}$

5. For $x \to 0$ $f(x) \sim \left(x^2 \cdot \cos x\right)$ and $g(x) \sim (e^x - 1)$, then:
   (a) $\displaystyle\lim_{x\to 0} \left|\frac{f(x)}{g(x)}\right| = +\infty$
   (b) $\displaystyle\lim_{x\to 0} \frac{f(x)}{g(x)} = \frac{1}{2}$
   (c) $\displaystyle\lim_{x\to 0} \frac{f(x)}{g(x)} = 0$
   (d) $\displaystyle\lim_{x\to 0} \frac{f(x)}{g(x)} = +\infty$
   (e) $\displaystyle\lim_{x\to 0} \frac{f(x)}{g(x)} = 1$

6. The limit $\lim_{x \to +\infty} (3x^3 - 4x^2) \sin x$ takes value
   (a) $-\infty$
   (b) $+\infty$
   (c) 0
   (d) both $+\infty$ and $-\infty$
   (e) does not exist

7. Let a_n be a sequence bounded from below. Then
   (a) $\forall\, k > 0\ \exists \bar{n} \in N$ such that $n > \bar{n} \Rightarrow a_n \geq k$
   (b) for every n, $a_n \geq 0$
   (c) $\exists k < 0$ such that for every n, $a_n < k$
   (d) $\exists \bar{n} \in N$ such that $n > \bar{n} \Rightarrow a_n \geq 0$
   (e) $\exists k < 0$ such that for every n, $a_n > k$

8. Let $f(x) = (3\cos x)^{(4\cos x)}$. Then $f'(0)$ equals
   (a) 1
   (b) 0
   (c) $\frac{3}{4}$
   (d) $\log \frac{3}{4}$
   (e) -1

9. The function $f(x) = \begin{cases} 2x & \text{if } x \leq 0 \\ 2\sqrt{x} & \text{if } x > 0 \end{cases}$
   (a) belongs to C^1
   (b) is differentiable in the origin
   (c) is not continuous in the origin
   (d) is continuous but not differentiable in the origin
   (e) none of previous answers is correct

10. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function such that $f(0) = 4$, $f'(0) = 3$. Given that $h(x) = \dfrac{1}{f(x)}$, it holds
    (a) $h'(0) = -\frac{3}{16}$
    (b) $h'(0) = -\frac{4}{3}$
    (c) $h'(0) = \frac{1}{3}$
    (d) $h'(0) = \frac{3}{16}$
    (e) $h'(0) = -\frac{1}{3}$

11. The McLaurin polynomial of order 6 of the function $f(x) = e^{\cos x^3}$ is
    (a) $1 + \dfrac{1}{2}x^6$
    (b) $e - \dfrac{e}{2}x^6$
    (c) $2 + \dfrac{1}{2}x^6$
    (d) $1 + \dfrac{1}{6}x^6$
    (e) $1 + \dfrac{1}{6}x^6$

12. If f has Taylor expansion $f(x) = 2 - 4(x+5)^7 + o((x+5)^7)$ for $x \to -5$, then
    (a) f has an inflection point in $x = -5$
    (b) f has a maximum in $x = 0$
    (c) f has a minimum in $x = 0$
    (d) f has a minimum in $x = -5$
    (e) f has a maximum in $x = -5$

13. Let $f : [0,3] \to \mathbb{R}$ be a continuous and decreasing function. Then we can conclude that
    (a) $f([0,3])$ is an open set
    (b) $f((0,3))$ is an open set
    (c) $f((0,3]) = (f(3), f(0)]$
    (d) $f([0,3]) = [f(3), f(0)]$
    (e) $f([0,3])$ contains al least two points

14. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function such that $f(0) = f(1) = 0$. Given $g(x) = f^4(x)$, then
    (a) the derivative $g'(x)$ has at least three zeros
    (b) the derivative $g'(x)$ has exactly two zeros
    (c) the derivative $g'(x)$ has exactly three zeros
    (d) the derivative $g'(x)$ has no zeros
    (e) the derivative $g'(x)$ has at least 4 zeros

15. Let $f(x) = 3x + \sqrt{4x^2 + 2x^3}$. For $x \to 0^+$ its principal part, with respect to $\varphi(x) = x$, is:
    (a) $5x$
    (b) $x^{3/2}$
    (c) $\sqrt{2}x^{3/2}$
    (d) $3x$
    (e) $3x + o(x)$

16. A primitive of the function $f(x) = \dfrac{3x}{2x^2 + 2}$ is:
    (a) $\dfrac{3}{4}\log(x^2 + 1)$
    (b) $\dfrac{3}{4}\arctan x$
    (c) $3\log(x^2 + 1)$
    (d) $\dfrac{3}{4}\arctan(x^2 + 1)$
    (e) none of the previous answers

17. Find which of the following statements is correct.

    (a) if f is differentiable in $[a, b]$, then $\exists c \in [a, b]$ such that $f'(c) = \dfrac{1}{b-a}\displaystyle\int_a^b f(x)dx$

    (b) if f is continuous in $[a, b]$, then $\exists c \in [a, b]$ such that $f'(c) = \dfrac{1}{b-a}\displaystyle\int_a^b f(x)dx$

    (c) if f is continuous in $[a, b]$, then $\exists c \in [a, b]$ such that $f(c) = \dfrac{1}{b-a}\displaystyle\int_a^b f(x)dx$

    (d) se f is continuous in $[a, b]$, then $\exists c \in [a, b]$ such that $f(c) = \displaystyle\int_a^b f(x)dx$

    (e) se f is integrable in $[a, b]$, then $\exists c \in [a, b]$ such that $f(c) = \displaystyle\int_a^b f(x)dx$

18. Let $F(x) = \displaystyle\int_0^x t^2 \cosh(t^2)dt$. Then

    (a) F is increasing on $(0, +\infty)$ and decreasing on $(-\infty, 0)$
    (b) F is increasing on $\mathbb{R}$
    (c) F has a minimum in 0
    (d) F has a maximum in 0
    (e) none of the previous answers is correct

19. Let f be a continuous function on $[0, +\infty)$ and such that $f(x) \leq 0$ for every $x \geq 0$. Then, the improper integral $\displaystyle\int_0^{+\infty} f(x)dx$ is necessarily

    (a) indeterminate
    (b) divergent to $-\infty$
    (c) convergent to a negative number
    (d) convergent, or divergent to $-\infty$
    (e) none of the previous answers

20. The differential equation $y'' - y = 0$

    (a) has at least an unbounded solution on $(0, +\infty)$
    (b) has no bounded solutions on $(0, +\infty)$
    (c) has no unbounded solutions on $(0, +\infty)$
    (d) has at least a solution that changes sign infinite times
    (e) has only positive solutions

ANSWERS

Item n.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	d	b	a	e	c	e	e	b	d	a	b	e	d	a	a	a	c	b	d	a