Tutoring of Mathematical Analysis I
TEST SIMULATION - 3

1. The domain of the function $f(x) = \sqrt{\log_{1/2}(x-1)+2}$ is:
   (a) $[5, +\infty)$
   (b) $[1, 5]$
   (c) $(1, 5]$
   (d) $\mathbb{R} \setminus \{1\}$
   (e) $(5, +\infty)$

2. Given the sets $A = \{z \in \mathbb{C} : |z - i| < 1\}$ and $B = \{z \in \mathbb{C} : \mathbf{Re}\,(z + i) > 5\}$, it is true that
   (a) $A \cup B$ is the whole complex plane
   (b) $B \setminus A = A \setminus B$
   (c) $A \cap B = \varnothing$
   (d) $A \subseteq B$
   (e) $A = B$

3. $\displaystyle\lim_{x\to-\infty} \frac{e^{2x} + 5x + \cos x}{\sin x - \log|x| - x} =$
   (a) 0
   (b) $-\infty$
   (c) -5
   (d) $+\infty$
   (e) $\nexists$

4. Let $f : \mathbb{R} \to \mathbb{R}$ be such that $\displaystyle\lim_{x\to-\infty} f(x) = -5$. Then we can conclude that
   (a) f is a bounded function
   (b) $\exists M > 0$ such that $f((-\infty, -M))$ is a bounded set
   (c) $\inf\limits_{\mathbb{R}} f = -5$
   (d) $f(x) < 0$ for every $x \in \operatorname{dom} f$
   (e) $\forall M > 0$, $f((-\infty, -M))$ is a bounded set

5. Given the function
$$f(x) = \begin{cases} \dfrac{1-\cosh x}{2x^2} & \text{if } x \neq 0 \\ 5 & \text{if } x = 0 \end{cases}$$
   The limit $\displaystyle\lim_{x\to 0} f(x) =$
   (a) $\dfrac{1}{2}$
   (b) 0
   (c) 5
   (d) $\nexists$
   (e) $-\dfrac{1}{4}$

6. The limit of the sequence $a_n = \left(\dfrac{n+1}{n-1}\right)^n$ equals
   (a) 1
   (b) e
   (c) e^{-1}
   (d) e^2
   (e) $+\infty$

7. Given the monotonic sequence $n \to a_n$, then it is necessarily
   (a) convergent
   (b) not indeterminate
   (c) indeterminate
   (d) divergent
   (e) neiher convergent nor divergent

8. The first derivative of the function $f(x) = \log \dfrac{\sqrt{1+x^2}}{x} - \dfrac{1}{2x^2}$ is
   (a) $f'(x) = \dfrac{1}{x^3\,(x^2+1)}$
   (b) $f'(x) = \dfrac{1}{x^2\,(x^2+1)}$
   (c) $f'(x) = -\dfrac{1}{x^3\,(x^2+1)}$
   (d) $f'(x) = -\dfrac{1}{x^3\,(x^2-1)}$
   (e) none of the previous answers is correct

9. The function $f:\ [-1,3] \longrightarrow \mathbb{R},\ f(x) = x^3 + 2x - 3$
   (a) satisfies the Rolle Theorem hypothesis
   (b) does not satisfy the Lagrange Theorem hypothesis
   (c) has a unique Lagrange point $c = \sqrt{7/3}$
   (d) has two Lagrange points $c_1 = \sqrt{7/3}$ and $c_2 = -\sqrt{7/3}$
   (e) none of the previous answers is correct

10. The function $f:\ [-2,4] \longrightarrow \mathbb{R}, f(x) = \sqrt[3]{x^2}$
    (a) has no global maxima
    (b) has no global minima
    (c) has a global maximum
    (d) has a relative minimum that is not global
    (e) has an inflection point with vertical tangent

11. The function $f\,(x) = x^{2a} \log\left(1 + 2x^a\right)$ is infinitesimal of order 5, for $x \to 0$, if $a =$
    (a) 3
    (b) 5/3
    (c) 5
    (d) 5/2
    (e) $-5/3$

12. Which of the following statements holds for the function $f\,(x) = (x+2)\sqrt{x}$?
    (a) $\lim\limits_{x \to 0^+} f'(x) = 0$
    (b) $f(x)$ is infinite of order $1/2$ for $x \to +\infty$
    (c) $f(x)$ is continuous and differentiable on its domain
    (d) $f(x)$ is infinitesimal of order $3/2$ for $x \to 0$
    (e) $f(x)$ has a vertical tangent point in $x = 0$

13. Let $f : \mathbb{R} \to \mathbb{R}$ be a function of class C^2 and $x_0 \in \mathbb{R}$. In order to guarantee that x_0 is a maximum point for f, the condition $f'(x_0) = 0$ and $f''(x_0) < 0$ is:
    (a) sufficient but not necessary
    (b) necessary and sufficient
    (c) necessary but not sufficient
    (d) neither necessary, nor sufficient
    (e) necessary if there exists also $f'''(x_0)$

14. The principal part of the function $f(x) = e^{1-\cos x} - x\sin(2x) - 1$ for $x \to 0$ is
    (a) $\frac{8}{3}x^4$
    (b) $-\frac{3}{2}x^2$
    (c) $6x^2$
    (d) $\frac{8}{3}x^4 + o\left(x^4\right)$
    (e) $6x^2 + o\left(x^2\right)$

15. The Mac Laurin expansion of order 3 of the function $f(x) = \sqrt{1+x}$ is
    (a) $f(x) = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3$
    (b) $f(x) = \frac{1}{16}x^3 + o\left(x^3\right)$
    (c) $f(x) = 1 + \frac{1}{2}x + \frac{1}{2}x^2 + \frac{1}{2}x^3 + o\left(x^3\right)$
    (d) $f(x) = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + o\left(x^3\right)$
    (e) $f(x) = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + o\left(x^3\right)$

16. Given the function $f \in C^{(4)}(\mathbb{R})$ and its Mac Laurin expansion $f(x) = 1 - 7x^3 + o\left(x^4\right)$, which of the following is FALSE?
    (a) $f(x)$ has an inflection point in $x = 0$
    (b) $f'(x)$ has a maximum in $x = 0$
    (c) $f''(x)$ is increasing in a neighborhood of $x = 0$
    (d) $f'''(0) = -42$
    (e) $f^{(4)}(0) = 0$

17. $\displaystyle\int_0^4 \frac{1}{1+\sqrt{x}}\,\mathrm{d}x =$
    (a) 0
    (b) $2 - 4\log 3$
    (c) 3
    (d) $4 - 2\log 3$
    (e) $4 - 2\log 3 + c$

18. Suppose the Substitution Theorem applies, by means of the substitution $x = g(t)$, then
    (a) $\int f(x)\,dx = \int f\left(g^{-1}(x)\right)dx$
    (b) $\int f(x)\,dx = \int f(g(t))\,dt$
    (c) $\int f(x)\,dx = \int f(g(t))\,g'(t)\,dt$
    (d) $\int f(x)\,dx = \int f(t)\,g'(t)\,dt$
    (e) $\int f(x)\,dx = \int f(g(t))\,g'(t)\,dx$

19. The improper integral $\int_0^{+\infty} \frac{\arctan x}{x^\alpha} dx$

    (a) converges $\forall \alpha \in \mathbb{R}$
    (b) converges $\forall \alpha < 1$
    (c) converges $\forall \alpha \in (1, 2)$
    (d) converges $\forall \alpha > 1$
    (e) diverges $\forall \alpha \in \mathbb{R}$

20. The differential equation $y'' + y' = 0$

    (a) has infinite constant solutions
    (b) has a unique constant solution
    (c) has no constant solutions
    (d) has $y(x) = e^x$ as particular integral
    (e) has $y(x; c_1, c_2) = c_1 e^{-x} + c_2 e^x$ as general integral

ANSWERS

Item n.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	c	c	c	b	e	d	b	a	c	c	b	e	a	b	e	c	d	c	c	a