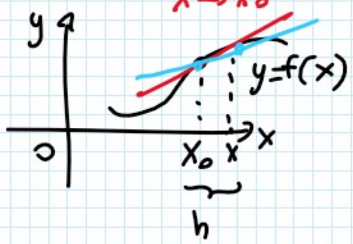
Week 5 **Derivatives – Differentiable functions**

**Recap**

Derivative of $f$ at $x_0$

$$f'(x_0) \overset{def}{=} \lim_{x \to x_0} \frac{f(x)-f(x_0)}{x-x_0} = \lim_{h \to 0} \frac{f(x_0+h)-f(x_0)}{h} \in \mathbb{R}$$

$x = x_0 + h$



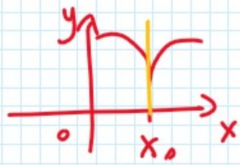

$f'(x_0)$ Slope of the tangent line to the graph of $f$ at $x_0$

$(y = mx + q)$

$m$: slope

Non differentiable points

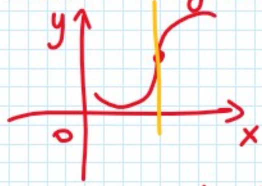
(1) cusp




$$f'_-(x_0) \overset{def}{=} \lim_{x \to x_0^-} \frac{f(x)-f(x_0)}{x-x_0} = -\infty$$

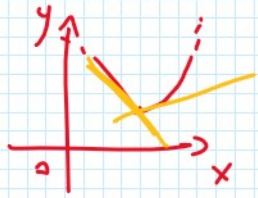
$$f'_+(x_0) \overset{def}{=} \lim_{x \to x_0^+} \frac{f(x)-f(x_0)}{x-x_0} = +\infty$$

(2) inflection point with vertical tangent




$$f'_-(x_0) = f'_+(x_0) = +\infty$$

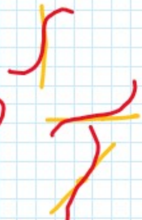
(3) corner point




$f'_-(x_0), f'_+(x_0) \in \mathbb{R}$

$f'_-(x_0) \neq f'_+(x_0)$

Inflection points with tangent line → vertical (1), horizontal (2), oblique (3)



(1) NOT diff.

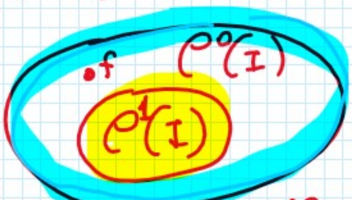
(2) stationary point, diff. $(f'(x_0) = 0)$ → find studying $f'(x) \geq 0$

(3) diff. → find studying $f''(x) \geq 0$

Note that || IF $f$ is differentiable in $I$ (interval) $\Longrightarrow$ $f$ is continuous on $I$

$$C^0(I) = \{ f: I \to \mathbb{R} \ / \ f \text{ is continuous} \}$$

$$C^1(I) = \{ f: I \to \mathbb{R} \ / \ f \text{ is cont. and diff. with } f' \text{ contin.} \}$$

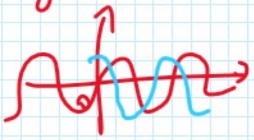



$C^k(I) = \{ f: I \to \mathbb{R} \}$ f contin. and diff. K TIMES

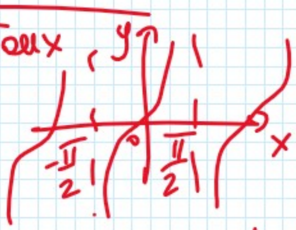
$\mathcal{C}^k(I) = \{ f : I \to \mathbb{R} \mid f$ contin. and diff. $k$ TIMES with $f', f'', \dots, f^{(k)}$ continuous on $I \}$

$f$ contin. at $x_0 \iff \lim_{x \to x_0} f(x) = f(x_0)$

$y = \sin x$ $\quad$ $y = \cos x$



$y = \tan x$



Dom f $\mathbb{R} \setminus \{ \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \}$

continuous on its domain

ex $\quad$ $C^\infty$ functions? $\quad$ $\left.\begin{array}{l} \text{polynomials} \\ \text{sine} \\ \text{cosine} \\ e^x \end{array}\right\} \mathcal{C}^\infty(\mathbb{R})$

$f(x) = \log x \qquad \text{Dom} f = (0, +\infty)$

$f'(x) = \frac{1}{x} \qquad x \neq 0 \qquad \text{Dom} f' \subseteq \text{Dom} f$

$\boxed{\text{ex 4D}}$ $\quad$ Recall $\quad$ $\boxed{f \text{ diff. on } I \Rightarrow f \text{ contin. on } I}$

$\iff$ $\quad$ $\boxed{f \text{ NOT contin. on } I \Rightarrow f \text{ NOT diff. on } I}$

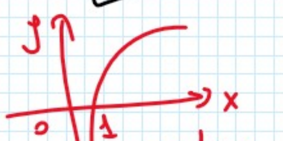
$f(x) = \sqrt{\log(x^2+1)}$ $\qquad$ Dom f : $\begin{cases} \log(x^2+1) \geq 0 & \text{for the square root} \\ x^2+1 > 0 & \to \forall x \in \mathbb{R} \end{cases}$

$0 = \log 1$

$\log(x^2+1) \geq \log 1 \qquad x^2 + 1 \geq 1 \qquad x^2 \geq 0 \quad \forall x \in \mathbb{R}$

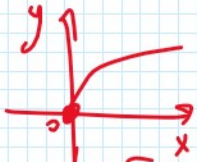
$\boxed{\text{Dom} f = \mathbb{R}}$

$f$ is contin. on $\mathbb{R}$ by composition of contin. functions



$y = \log x$

Dom f $= (0, +\infty)$



$y = \sqrt{x}$

Dom f $= [0, +\infty)$

$f'(x) = \frac{1}{2\sqrt{\log(x^2+1)}} \cdot \frac{1}{x^2+1} \cdot (2x)$

$D(\sqrt{x}) = D(x^{1/2}) = \frac{1}{2\sqrt{x}}$

$D(x^m) = m x^{m-1}$

$D(\log x) = \frac{1}{x}$

COMPOSITE FUNCTION DERIVATIVE : $D(f(g(x))) = f'(g(x)) \cdot g'(x)$

Dom f' $= ?$ $\qquad \begin{cases} \log(x^2+1) > 0 & \text{(root and denominator)} \\ x^2+1 > 0 & \forall x \in \mathbb{R} \\ x^2+1 \neq 0 & \forall x \in \mathbb{R} \end{cases}$

Dom f' $\supseteq \mathbb{R} \setminus \{0\}$

$\left\{ x^2+1 \neq 0 \quad \forall x \in \mathbb{R} \right.$

$x^2+1>1 \qquad x^2>0 \iff x \neq 0 \qquad \mathrm{Dom}\,f' \supseteq \mathbb{R}\setminus\{0\}$

Check if $x_0=0$ is diff. or NOT

Apply def. $\displaystyle\lim_{x\to 0^\pm} \frac{f(x)-\underbrace{f(0)}_{=0}}{x-0} = \lim_{x\to 0^\pm} \frac{\sqrt{\log(x^2+1)}}{x} =$

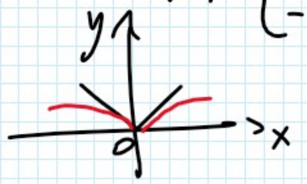
$\log(1+t) \sim_0 t$

$\displaystyle= \lim_{x\to 0^\pm} \frac{\sqrt{x^2}}{x} = \lim_{x\to 0^\pm} \frac{|x|}{x} = \pm 1$

$|x| \overset{\text{def}}{=} \begin{cases} x, & x \geq 0 \\ -x, & x<0 \end{cases}$

$x_0=0$ Corner point (NOT diff.)

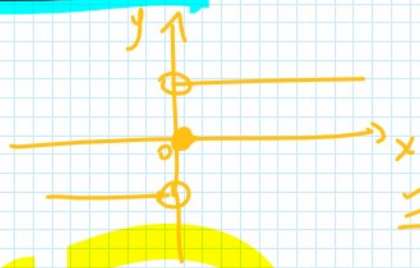
$\Longrightarrow \mathrm{Dom}\,f' = \mathbb{R}\setminus\{0\}$



IF you Check def. $\displaystyle\lim_{x\to x_0^\pm} \frac{f(x)-f(x_0)}{x-x_0} \longrightarrow$ study diff. of $x_0$

IF you check $\displaystyle\boxed{\lim_{x\to x_0^\pm} f'(x)} \longrightarrow$ I'm checking continuity of $f'$

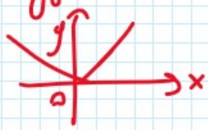
Example



Suppose $f'(x) = \begin{cases} 1, & x>0 \\ 0, & x=0 \\ -1, & x<0 \end{cases} \overset{\text{def}}{=} \mathrm{sign}(x)$

$\exists\, f'(0)=0$

Recall $D(|x|) = \begin{cases} 1, & x>0 \\ -1, & x<0 \end{cases}$ $\qquad x=0$ NOT diff. (corner point)



Domain is $\mathbb{R}\setminus\{0\}$

$\displaystyle= \frac{|x|}{x} = \frac{x}{|x|} = \boxed{\mathrm{sign}(x) \text{ WITH } x \neq 0}$

Because Recall that $|x| = \begin{cases} x & \text{IF } x \geq 0 \\ -x & \text{IF } x<0 \end{cases}$ $\qquad \mathrm{Dom}\,f = \mathbb{R}$

ex $\quad f(x) = \sqrt{|1-(25x)^2|} \qquad \mathrm{Dom}\,f = \mathbb{R}$

$\displaystyle f'(x) = \frac{1}{2\sqrt{|1-(25x)^2|}} \cdot \frac{|1-(25x)^2|}{1-(25x)^2} \cdot \left(-2(25x)\cdot 25\right)$

$\mathrm{Dom}\,f'$? $\qquad 1-(25x)^2 \neq 0 \qquad (25x)^2 \neq 1 \qquad 25x \neq \pm 1$

check which kind of NON diff. points $\longrightarrow \boxed{x \neq \pm 1/25}$

$\pm 1$

check which kind of NON diff. points

**T15** 15. The derivative of $f(x) = \log(x^2 - |x-1| + 3)$ is:

(a) $f'(x) = \dfrac{2x-1}{x^2-x+4}$

(b) $f'(x) = \dfrac{2x-1}{x^2-|x-1|+3}$

(c) $f'(x) = \dfrac{2x-|1|}{x^2-|x-1|+3}$

(d) $f'(x) = \begin{cases} \dfrac{2x+1}{x^2+x+2} & x<1 \\ \dfrac{2x-1}{x^2-x+4} & x \geq 1 \end{cases}$

(e) $f'(x) = \begin{cases} \dfrac{2x+1}{x^2+x+2} & x<1 \\ \dfrac{2x-1}{x^2-x+4} & x>1 \end{cases}$

$$f'(x) = \frac{1}{x^2-|x-1|+3} \cdot \left(2x - \frac{|x-1|}{x-1}\right) = \quad \pm 1$$

$$= \begin{cases} \dfrac{2x-1}{x^2-x+4}, & x>1 \\ \dfrac{2x+1}{x^2+x+2}, & x<1 \end{cases}$$

$$\lim_{x \to 1^+} f'(x) = \frac{1}{4} \neq \lim_{x \to 1^-} f'(x) = \frac{3}{4}$$

$x=1$ corner point

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**ex5A** $f(x) = |x|^x$

- $\exists$ contin. prolongation $\tilde{f}$ on $\mathbb{R}$?
- diff. of $\tilde{f}$?

Domf = ?

Recall $\boxed{f(x)^{g(x)}} = e^{\log(f(x)^{g(x)})} = \boxed{e^{g(x)\log f(x)}}$

$f(x) = e^{x\log|x|}$ $\quad |x|>0 \iff x \neq 0$

Domf $= \mathbb{R}\setminus\{0\}$

$$\lim_{x\to 0^+} f(x) = e^{0 \cdot \log 0^+} = e^0 = 1$$

Recall $\lim_{t \to 0^+} t \cdot \log t = 0$ $\quad$ $[0 \cdot \infty]$ Indet. form $\quad$ check $\lim_{t\to 0^+} \dfrac{\log t}{\frac{1}{t}} = \dfrac{\infty}{\infty} = 0$

$\to \exists$ contin. prolongation for $f$: $\tilde{f}(x) = \begin{cases} f(x), & x \neq 0 \\ 1, & x=0 \end{cases}$

Check diff. on $\tilde{f}$

$$\tilde{f}'(x) = e^{x\log|x|} \cdot \left(1 \cdot \log|x| + \cancel{x} \cdot \frac{1}{\cancel{|x|}} \cdot \frac{\cancel{|x|}}{\cancel{x}}\right) = |x|^x \cdot (\log|x| + 1)$$

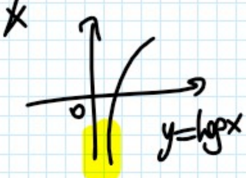
$\forall x \neq 0$ $\quad$ Recall $D(f(x) \cdot g(x)) = f'(x)g(x) + f(x)g'(x)$

Check diff. on $\tilde{f}$ in $x=0$:

$$\lim_{\boxed{x\to 0}} \frac{\tilde{f}(x) - \tilde{f}(0)}{x-0} = \lim_{x\to 0} \frac{e^{x\log|x|} - 1}{x} = \lim_{x \to 0} \frac{\cancel{x}\log|x|}{\cancel{x}} = -\infty$$

$e^t - 1 \sim_0 t$

$x=0$ inflection point with vertical tangent ?




Dom$\tilde{f}' = \mathbb{R}\setminus\{0\}$

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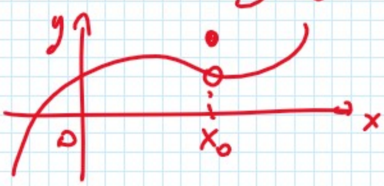
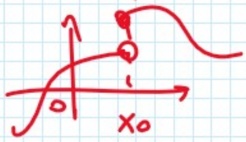
Removable discontinuity (3rd kind) ...

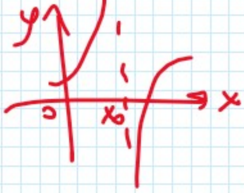
Removable discontinuity ($3^{rd}$ kind)




($1^{ST}$ kind Jump

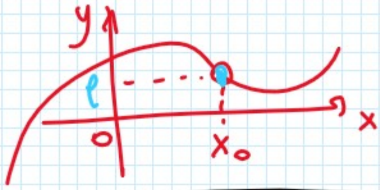



$2^{nd}$ kind)




$\Downarrow$

you can define a continuous prolongation on $\mathbb{R}$




$$\tilde{f}(x) = \begin{cases} f(x) & , x \neq x_0 \\ \ell & , x = x_0 \end{cases} \qquad \mathrm{Dom}\,\tilde{f} = \mathbb{R}$$

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**(T8)** 8. Given the function $f(x) = \frac{x}{2}\ln\frac{x}{2}$, which of the following properties is NOT true?

(a) $\lim_{x\to+\infty} f(2x-4) - f(2) = +\infty$ TRUE

(b) $\lim_{x\to 2} f(2x-4) - f(2) = 0$ TRUE

(c) $f(2x-4) = (x-2)\ln(x-2)$ TRUE

(d) $\lim_{x\to 0} f(x) = -\infty$ False

(e) $f'(1) = \frac{1}{2}(1-\ln 2)$ TRUE

$\mathrm{Dom}\,f = (0,+\infty)$ $\quad \frac{x}{2} > 0$

$$\lim_{x\to+\infty} \frac{2x-4}{2}\log\left(\frac{2x-4}{2}\right) - 0 =$$

$$= \lim_{x\to+\infty} (x-2)\log(x-2) = +\infty$$

$$\lim_{x\to 2^+} \underbrace{(x-2)}_{\to 0}\log(x-2) = 0$$

$$\lim_{x\to 0} \underbrace{\frac{x}{2}}_{\to 0}\log\left(\frac{x}{2}\right) = 0$$

$$f'(x) = \frac{1}{2}\log\frac{x}{2} + \frac{\cancel{x}}{2}\cdot\frac{1}{\cancel{x}/2}\cdot\frac{1}{2}$$

$$f'(1) = \frac{1}{2}\underbrace{\log\frac{1}{2}}_{= -\log 2} + \frac{1}{2} = \frac{1}{2}(1-\log 2)$$

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**(T9)** 9. Let $f : [-2,3) \to \mathbb{R}$, $f(x) = |x^2-1|$ which of the following statements is NOT true?

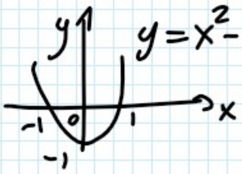
(a) f admits sup, but no absolute maximum TRUE

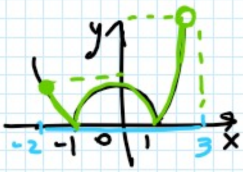
(b) f admits absolute minimum TRUE VALUE = 0

(c) x=1 is an absolute minimum point TRUE

(d) 0 is the absolute minimum of the function TRUE

(e) 8 is the absolute maximum of the function False → VALUE




$f(3) = |3^2-1| = 8$

$f(-2) = |4-1| = 3$

$\mathrm{Im}\,f = [0, 8)$

$\sup_{x\in \mathrm{Dom}f} f \overset{\text{def}}{=} \sup(\mathrm{Im}\,f) = 8 \notin \mathrm{Im}\,f$

$\nexists$ max

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**(T11)** 11. The derivative of the function $f(x) = (x^x)^x$ is:

(a) $f'(x) = x^2 x^{x^2-1}$

(b) $f'(x) = 2x(x^x)^x \log x$

(c) coincides with the derivative of $g(x) = x^{x^x}$

(d) $f'(x) = x^{x^2+1}(2\log x + 1)$

(e) $f'(x) = x^{x^2}(2\log x + 1)$

$$(x^x)^x = x^{x^2} = e^{\log(\ )} = e^{x^2\log x}$$

$\mathrm{Dom}\,f = (0,+\infty)$

$$f'(x) = e^{x^2\log x}\left(2x\log x + x^{\cancel{2}}\cdot\frac{1}{\cancel{x}}\right) =$$

$$= x^{x^2}\cdot(2x\log x + x) =$$

$$= x^{x^2+1}\cdot(2\log x + 1)$$

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**(T12)** 12. Given the function $f(x) = \log(e^x + e^{-x})$; which of the following functions is NOT its first derivative?

$= x^{\cdots} \cdot (2 \log x + 1)$

**(T12)** 12. Given the function $f(x) = \log(e^x + e^{-x})$; which of the following functions is NOT its first derivative?

(a) $f'(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$ TRUE

(b) $f'(x) = \dfrac{\sinh x}{\cosh x}$ TRUE

(c) $f'(x) = \tanh x$ TRUE

(d) $f'(x) = \dfrac{e^{2x} - 1}{e^{2x} + 1}$

(e) $f'(x) = \dfrac{1}{e^x + e^{-x}}$

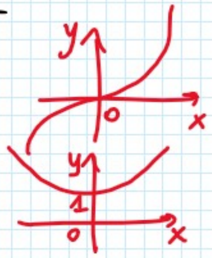
$\sin(0) = 0$
$\sinh(0) = 0$

$\cos(0) = 1$
$\cosh(0) = 1$

Recall

(odd) $\sinh x = \dfrac{e^x - e^{-x}}{2}$

(even) $\cosh x = \dfrac{e^x + e^{-x}}{2}$




$$f'(x) = \frac{1}{e^x + e^{-x}} \cdot \left(e^x + e^{-x} \cdot (-1)\right) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\boxed{e^{-x} = \frac{1}{e^x}}$$

check (D): $f'(x) = \dfrac{e^x - \frac{1}{e^x}}{e^x + \frac{1}{e^x}} = \dfrac{\frac{e^{2x} - 1}{\cancel{e^x}}}{\frac{e^{2x} + 1}{\cancel{e^x}}} = \dfrac{e^{2x} - 1}{e^{2x} + 1}$

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**(ex 6B)**

$$f(x) = \begin{cases} (x - \beta)^2 + 4, & x \geq 0 \\ \alpha \sin x, & x < 0 \end{cases}$$

Find $\alpha, \beta \in \mathbb{R}$ such that $f$ is diff. on $\mathbb{R}$

Dom $f = \mathbb{R}$

Check continuity in $x = 0$: $\boxed{\lim_{x \to 0^{\pm}} f(x) = f(x)}$

IMPOSE

$$\underbrace{\lim_{x \to 0^+} f(x) = \beta^2 + 4}_{= f(0)} \underset{\text{IMPOSE CONTINUITY}}{=} \lim_{x \to 0^-} f(x) = 0$$

$\beta^2 + 4 = 0$

$\nexists \beta \in \mathbb{R}$

$\forall \alpha, \beta$ $f$ NOT contin. in $x = 0$ $1^{ST}$ kind singularity (JUMP)

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**(ex 6C)**

$$f(x) = \begin{cases} (x - \beta)^2 - 4, & x \geq 0 \\ \alpha \sin x, & x < 0 \end{cases}$$

Same

Continuity: $\lim_{x \to 0^+} f(x) = \beta^2 - 4 = \lim_{x \to 0^-} f(x) = 0$ $\quad \beta^2 - 4 = 0$ $\quad \boxed{\beta = \pm 2}$

diff.:

$$f'(x) = \begin{cases} 2(x - \beta), & x > 0 \\ \alpha \cos x, & x < 0 \end{cases}$$

$$\lim_{x \to 0^+} f'(x) = -2\beta = \lim_{x \to 0^-} f'(x) = \alpha \cdot \cos 0 = \alpha$$

$$\begin{cases} \alpha = -2\beta \\ \beta = \pm 2 \end{cases} \qquad \begin{cases} \alpha = \mp 4 \\ \beta = \pm 2 \end{cases}$$

IF $(\alpha, \beta) = (-4, +2)$
OR
$(\alpha, \beta) = (4, -2)$
$\Rightarrow f$ diff. on $\mathbb{R}$

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**(EX 7)** Derivative of the inverse function THEOREM:

Theorem (Hypothesis ...) suppose $y_0 = f(x_0)$

$\boxed{\text{EX 7}}$ Derivative of the inverse function THEOREM:

Theorem (Hypothesis ...) suppose $y_0 = f(x_0)$

$$(f^{-1})'(y_0) = \frac{1}{f'(f^{-1}(y_0))} = \frac{1}{f'(x_0)}$$

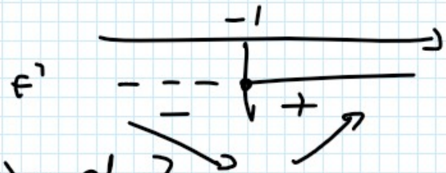
$$x_0 \underset{f^{-1}}{\overset{f}{\rightleftarrows}} y_0$$

$f(x) = xe^x$ Compute $(f^{-1})'(0) = ?$ $(f^{-1})'(e) = ?$

$\mathrm{Dom} f = \mathbb{R}$ $f'(x) = e^x + x \cdot e^x = e^x(x+1) > 0 \iff x+1>0$

study MONOTONICITY to check invertibility $\iff x > -1$

$(e^x > 0 \quad \forall x \in \mathbb{R})$




CAN we find $f^{-1}$?

$y = xe^x$ find $x$ as function of $y$? IMPOSSIBLE

We know that $f$ is invertible $\to$ on $(-\infty, -1)$

$\to$ on $(-1, +\infty)$

Apply Previous Theorem:

$$(f^{-1})'(0) = \frac{1}{f'(f^{-1}(0))} = \frac{1}{f'(0)} = \frac{1}{1} = 1$$

$$x = f^{-1}(0) \iff f(x) = 0 \iff xe^x = 0 \iff x = 0$$

$$(f^{-1})'(e) = \frac{1}{f'(f^{-1}(e))} = \frac{1}{f'(1)} = \frac{1}{2e}$$

$$x = f^{-1}(e) \iff f(x) = e \iff xe^x = e \iff x = 1$$

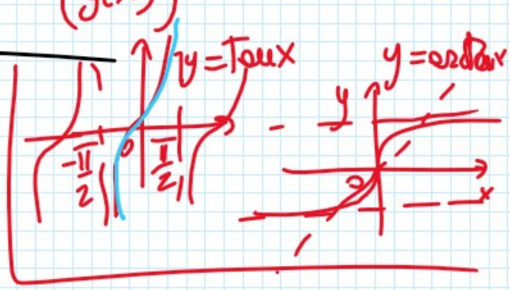
From Past exams

$$f(x) = \arctan\left(\frac{1}{x-1}\right) + \frac{x}{2} + 3$$

$$f'(x) = \frac{1}{\left(\frac{1}{x-1}\right)^2 + 1} \cdot \frac{-1}{(x-1)^2} + \frac{1}{2}$$

$$D(\arctan x) = \frac{1}{x^2+1} \qquad D\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

From Past exams (from 2019)




$$f(x) = \begin{cases} 3\sin^2 x \sin\frac{2}{x} & \text{IF } x \neq 0 \\ 0 & \text{IF } x = 0 \end{cases}$$

check continuity in $x = 0$:

$$\lim_{x \to 0^\pm} \underbrace{3\sin^2 x}_{\downarrow\, 0}\ \underbrace{\sin\frac{2}{x}}_{\downarrow\, \nexists \lim,\ \text{bounded in } [-1,1]} = 0 = f(0) \quad ✓$$

$\sin x \sim x$

$\nexists$ lim, bounded in $[-1,1]$

check diff. in $x=0$ By def. of derivative

$\sin x \sim_0 x$

$$\lim_{x\to 0} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0} \frac{3\sin^2 x \sin\frac{2}{x} - 0}{x-0} = \lim_{x\to 0} \frac{3x^{\cancel{2}}\sin\frac{2}{x}}{\cancel{x}}$$

$$= \lim_{x\to 0} \underbrace{3x}_{0} \underbrace{\sin\frac{2}{x}}_{\text{bounded}} = 0 = f'(0)$$

f diff. at 0

T14

14. The derivative of $f(x) = \log\left|\frac{x^2-3}{x^2-4}\right|$ is:

(a) $f'(x) = \begin{cases} \frac{-2x}{(x^2-3)(x^2-4)} & (-\infty,-2)\cup[-\sqrt{3},\sqrt{3}]\cup(2,+\infty) \\ \frac{-2x}{(x^2-3)(x^2-4)} & (-2,-\sqrt{3})\cup(\sqrt{3},2) \end{cases}$

(b) $f'(x) = \frac{2x}{(3-x^2)(4-x^2)}$

(c) $f'(x) = \frac{2x}{(3-x^2)(x^2-4)}$

(d) $f'(x) = \begin{cases} \frac{2x(x^2-4)}{(x^2-3)^3} & (-\infty,-2)\cup(-\sqrt{3},\sqrt{3})\cup(2,+\infty) \\ \frac{2x(x^2-4)}{(x^2-3)^3} & (-2,-\sqrt{3})\cup(\sqrt{3},2) \end{cases}$

(e) $f'(x) = \frac{2x(x^2-4)}{(x^2-3)^3}$

HOW TO SPLIT $|\cdot|$

$\frac{x^2-3}{x^2-4} \geq 0$

$N \geq 0$ $\quad$ $D > 0$

$x^2-3 \geq 0$ $\quad$ • concavity $>0$ $\cup$ $\quad$ • zeros $x = \pm\sqrt{3}$

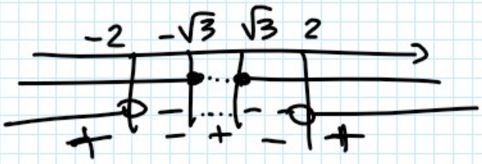
$x \leq -\sqrt{3} \vee x \geq \sqrt{3}$


$x^2-4 > 0$

• concavity $>0$ $\cup$

• $x = \pm 2$


$x < -2 \vee x > 2$




Recall $y = ax^2+bx+c$ parabola equation

$a > 0$ $\cup$

$a < 0$ $\cap$

$x < -2 \vee -\sqrt{3} \leq x \leq \sqrt{3} \vee x > 2$

$(-\infty,-2)\cup[-\sqrt{3},\sqrt{3}]\cup(2,+\infty)$

Domain $\left|\frac{x^2-3}{x^2-4}\right| > 0 \iff \frac{x^2-3}{x^2-4} \neq 0 \iff \boxed{x \neq \pm\sqrt{3}}$

Region in domain where argument of $|\cdot|$ is POSITIVE

$(-\infty,-2)\cup(-\sqrt{3},\sqrt{3})\cup(2,+\infty)$

$$f(x) = \log\left|\frac{x^2-3}{x^2-4}\right|$$

$$f'(x) = \frac{1}{\cancel{\left|\frac{x^2-3}{x^2-4}\right|}} \cdot \frac{\cancel{\left|\frac{x^2-3}{x^2-4}\right|}}{\frac{x^2-3}{x^2-4}} \cdot \frac{2x(x^2-4)-(x^2-3)\cdot 2x}{(x^2-4)^2}$$

$$= \frac{\cancel{x^2-4}}{x^2-3} \cdot \frac{2x(-1)}{(x^2-4)^{\cancel{2}}} = \frac{-2x}{(x^2-3)(x^2-4)} = \frac{2x}{(3-x^2)(x^2-4)}$$

answer C

Week 4 ex 5

$f_1(x) = \sqrt{2x^2+3x+5}$

$f_2(x) = 2x-1$

$x \to +\infty$

compute Principal part and order

$$\lim_{x\to+\infty} f_1(x) = \lim_{x\to+\infty} f_2(x) = +\infty$$

Test function $u(x) = x$

$$f_1(x) \sim_{+\infty} \sqrt{2x^2} = \sqrt{2}\,|x|$$

$3x+5 = o(x^2)$

⚠ $\sqrt{x^2} = |x|$

$f_1(x) \sim_{+\infty} \sqrt{2x^2} = \sqrt{2}\,|x|$ $\qquad 3x+5 = o(x^2)$ $\qquad \boxed{\sqrt{x^2} = |x|}$

$\sim_{+\infty} \sqrt{2}\, x$ order 1

$f_2(x) \sim_{+\infty} 2x$ $\qquad -1 = o(x)$

OR longer method : $f_1(x) = |x| \sqrt{2 + \frac{3}{x} + \frac{5}{x^2}} \sim_{+\infty} |x| \cdot \sqrt{2}$

$\sim_{+\infty} \sqrt{2}\, x$ $\qquad \frac{3}{x} + \frac{5}{x^2} = o(1)$ as $x \longrightarrow +\infty$