Tutoring of Mathematical Analysis I
TEST SIMULATION - 4

1. Let $A, B \subset \mathbb{R}$. If $A \subset B$, then
   (a) $\inf A > \inf B$
   (b) $\inf A \leq \inf B$
   (c) $\inf A < \inf B$
   (d) $\inf A = \inf B$
   (e) $\inf A \geq \inf B$

2. The function $f(x) = \cos x + e^x$
   (a) is even
   (b) has infinite zeros
   (c) is injective on $\mathbb{R}$
   (d) is monotonic on $\mathbb{R}$
   (e) is periodic

3. A polynomial with real coefficients has the numbers 0 and $2+i$ as roots, then
   (a) has degree 2
   (b) has degree greater or equal to 3
   (c) has degree strictly less than 3
   (d) has degree 4
   (e) has even degree

4. Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $\lim_{x \to 2} f(x) = 1$. Then
   (a) $f(2) = 1$
   (b) $\forall \delta > 0$, if $|x-2| < \delta$ then $f(x) > 0$
   (c) $\exists \delta > 0$ such that if $|x-2| < \delta$ then $|f(x) - 1| < \delta$
   (d) $\forall \delta > 0$, if $0 < |x-2| < \delta$ then $1 < f(x) < 2$
   (e) $\exists \delta > 0$ such that if $0 < |x-2| < \delta$ then $f(x) > 0$

5. $\lim_{x \to 0} \dfrac{9^x - 1}{\sqrt[4]{1+8x} - 1} =$
   (a) 3
   (b) 2
   (c) $\log 3$
   (d) $\log 2$
   (e) $9/8$

6. The limit $\lim_{x \to +\infty} (3x+2)\sin^2 x$
   (a) equals $+\infty$
   (b) equals 3
   (c) does not exist
   (d) equals 2
   (e) equals 0

7. If the functions $f, g : \mathbb{R} \to \mathbb{R}$ have respectively the straight lines $y = x + 1$, $y = 2x$ as asymptotes as $x \to \infty$, then
   (a) $\lim_{x\to+\infty} (f(x) - g(x)) = -1$
   (b) $\lim_{x\to+\infty} (f(x) - g(x)) = -\infty$
   (c) $\lim_{x\to+\infty} \frac{f(x) - g(x)}{x} = 0$
   (d) $\lim_{x\to+\infty} \frac{f(x) - g(x)}{x} = +\infty$
   (e) $\lim_{x\to+\infty} (f(x) + g(x)) = -\infty$

8. Let $a_n = \frac{3n + (-1)^n}{n + (-2)^n}$. Then
   (a) $\lim_{n\to+\infty} a_n = 0$
   (b) a_n does not admit limit as $n \to +\infty$
   (c) $\lim_{n\to+\infty} a_n = 3$
   (d) $\lim_{n\to+\infty} a_n = +\infty$
   (e) a_n is monotonic

9. Let $f(x) = 2^{x \sin x}$. Then
   (a) $f'(x) = 2^{x \sin x} (\sin x + x \cos x)$
   (b) $f'(x) = 2^{x \sin x}$
   (c) $f'(x) = 2^{x \sin x} x \cos x$
   (d) $f'(x) = 2^{x \sin x} (\sin x + x \cos x) \log 2$
   (e) $f'(x) = 2^{x \sin x} \log 2$

10. If $f(x_0) = 0$ and x_0 is a corner point for f, then $g(x) = (x - x_0) f(x)$
    (a) is differentiable in x_0
    (b) has a cusp in x_0
    (c) has a corner point in x_0
    (d) is not differentiable in x_0
    (e) is discontinuous in x_0

11. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable and invertible function. If $f(2) = 1$ and $f'(2) = 3$ then
    (a) $\left(f^{-1}\right)'(2) = 3$
    (b) $\left(f^{-1}\right)'(3) = 1$
    (c) $\left(f^{-1}\right)'(1) = 1/3$
    (d) $\left(f^{-1}\right)'(1) = 1/2$
    (e) $\left(f^{-1}\right)'(3) = 2$

12. Let $f : [0, 4] \to \mathbb{R}$ be defined as $f(x) = \sqrt{x}$ if $x \neq 1$, and $f(x) = \alpha$ if $x = 1$, with $\alpha \in \mathbb{R}$. Then f takes all the values between 0 and 2 if and only if
    (a) $\alpha = 0$
    (b) $\alpha = 1$
    (c) $\alpha = 2$
    (d) $0 \leq \alpha \leq 4$
    (e) $0 \leq \alpha \leq 2$

13. Let $f : \mathbb{R} \to \mathbb{R}$ be continuous on $\mathbb{R}$ and decreasing in $(-\infty, 0)$ and in $(0, +\infty)$. Then
    (a) $f(\mathbb{R})$ is not an interval
    (b) $f(\mathbb{R}) = \mathbb{R}$
    (c) f is decreasing on $\mathbb{R}$
    (d) $\sup_{\mathbb{R}} f = +\infty$
    (e) f is bounded on $\mathbb{R}$

14. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function such that $f(-1) = 0$ and $f(6) = 3$. Then, in the interval $(-1, 6)$, the derivative $f'(x)$
    (a) takes value $7/3$ in at least one point
    (b) vanishes in at least one point
    (c) takes value $7/3$ in infinite points
    (d) takes value $3/7$ in at least one point
    (e) is never zero

15. For $x \to 0$, it holds
    (a) $\dfrac{1}{x^4 + x^2} \sim \dfrac{1}{x^4}$
    (b) $\dfrac{1}{x^4} + \dfrac{1}{x^2} \sim \dfrac{1}{x^2}$
    (c) $\dfrac{1}{x^4} - \dfrac{1}{x^2} \sim -\dfrac{1}{x^2}$
    (d) $x^4 + x^2 \sim x^2$
    (e) $\dfrac{1}{x^4 - x^2} \sim \dfrac{1}{x^4}$

16. The limit $\lim\limits_{x \to 0} \dfrac{e^{\sqrt[3]{8x}} - 2\sqrt[3]{x} - 1}{\sin \sqrt[7]{x}}$ equals
    (a) 1
    (b) $+\infty$
    (c) 0
    (d) -1
    (e) e

17. The Mac Laurin expansion of order 3 of the function $f(x) = \dfrac{2x}{1+x} - \dfrac{1}{x-1}$ is
    (a) $f(x) = 1 - 3x - x^2 + 3x^3 + o(x^3)$
    (b) $f(x) = 1 + 3x - x^2 - 3x^3 + o(x^3)$
    (c) $f(x) = 1 + 3x - x^2 + 3x^3 + o(x^3)$
    (d) $f(x) = 1 - 3x - x^2 - 3x^3 + o(x^3)$
    (e) $f(x) = 1 + 3x + x^2 + 3x^3 + o(x^3)$

18. Let $F(x)$ be the primitive of $f(x) = \dfrac{e^{\sqrt{x+2}}}{\sqrt{x+2}}$ that vanishes in $x = -1$. Then
    (a) $F(x) = 2\sqrt{x+2}e^{\sqrt{x+2}} - 2e$
    (b) $F(x) = \sqrt{x+2}e^{\sqrt{x+2}} - e$
    (c) $F(x) = e^{\sqrt{x+2}} - e$
    (d) $F(x) = 2e^{\sqrt{x+2}} - 2e$

(e) $F(x) = 2e^{\sqrt{x+2}}$

19. The improper integral $\int_0^{+\infty} \frac{\sqrt{x}}{x+x^2} dx$ is
    (a) convergent
    (b) positively divergent
    (c) negatively divergent
    (d) indeterminate
    (e) 0

20. The differential equation $x'' - 5x' + 6x = e^{2t}$ has a particular solution $x_p(t)$ of the following form
    (a) $x_p(t) = Cte^{3t}$ with $C \in \mathbb{R}$
    (b) $x_p(t) = Ct^2e^{3t}$ with $C \in \mathbb{R}$
    (c) $x_p(t) = Cte^{2t}$ with $C \in \mathbb{R}$
    (d) $x_p(t) = Ct^2e^{2t}$ with $C \in \mathbb{R}$
    (e) $x_p(t) = C$ with $C \in \mathbb{R}$

ANSWERS

Item n.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	e	b	b	e	c	c	b	a	d	a	c	b	c	d	d	c	c	d	a	c