Week 6 and Week 7 Theorems on differentiable functions
Study of functions

I'm going to answer these questions!

10 February 2016 - I

1 Consider

$$f(x) = \frac{\log|x|}{\log^2|x| - \log|x| + 1}.$$

(a) Find the domain, possible symmetries and asymptotes. Show that f admits continuous prolongation in $x = 0$.

(b) Denote the prolongation by g, and compute its derivative. Find non differentiable points for g and specify their type.

(c) Find monotonicity intervals for g and minima/maxima, saying if they are local or global.

(d) Draw a qualitative graph for g.

(e) Find the solutions of $g(x) = 1$

31 **January 2018** - I

2 Consider the function

$$f(x) = e^3 - e^{4\sqrt{|x|} - x}.$$

(a) Find domain, symmetry properties, limits at boundary points of the domain and asymptotes, if there are any.

(b) Study differentiability of $f(x)$ on its domain, and establish the nature of its non differentiable points. Compute the derivative $f'(x)$.

(c) Find monotonicity intervals and maximum/minimum points. Say if they are relative or absolute.

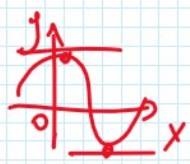
(d) Trace a qualitative graph.

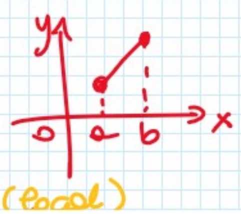
(e) Find the largest interval in the form $(k, +\infty)$, $k \in \mathbb{R}$, such that the restriction of f on such interval, is invertible. Find the domain and study the monotonicity of the inverse function.

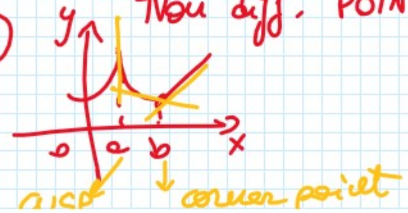
STUDY of FUNCTIONS $y = f(x)$

- domain $\mathrm{Dom}f$, limits at boundary points → discontinuities → asymptotes
- simmetry (even / odd) → $f(-x) = -f(x)$ $\forall x \in \mathrm{Dom}f$
  even: $f(-x) = f(x)$
  (you can focus on $\mathrm{Dom}f \cap \mathbb{R}^+$)
- zeros and study of sign
  $f(x) = 0$ $\qquad$ $f(x) \geq 0$
- monotonicity and MIN/MAX (stationary points $f'(x) = 0$) and NON differentiable points
  → 1 MIN
  → 2 MAX
  → 3 inflection points
  $f'(x) \geq 0$
  (IF $f'(x) > 0 \Rightarrow f$ is STRICTLY increasing)
- convexity and inflection points → $f''(x) = 0$
  $f''(x) \geq 0$
- qualitative graph

Remark Extremum points (MIN/MAX)?
1 stationary points
2 boundary points of Domf
3 Non diff. POINTS

1 

2 

3 

cusp — corner point

Remark MIN/MAX (local) relative OR absolute? (global)

x_0 max IF $f(x) \le f(x_0)$
↳ ABSOLUTE $\forall x \in \mathrm{Dom} f$
↳ RELATIVE $\forall x$ in some Neighborhood of x_0

$k \in \mathbb{Z},\ x = \frac{\pi}{2} + 2k\pi$

$y = \sin x$ $\exists$ infinite absolute max points (x)
(Absolute max VALUE (y) is UNIQUE) ↳ 1

Remark E.G. $\mathrm{Im} f = [0, +\infty)$ $\nexists$ Absolute MAX points

Derivative of Composite function $D(f(g(x))) = f'(g(x)) \cdot g'(x)$

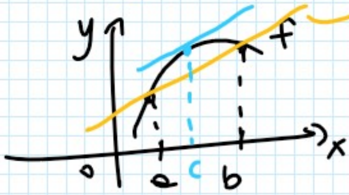
EX 1 (Week 6)

$f(x) = 1 + x + \sqrt{1 - x^2}$ Does Lagrange th. apply?

Recall **Lagrange THEOREM**: $f : [a,b] \to \mathbb{R}$
Hypothesis:
- f continuous on $[a,b]$
- f diff. on (a,b)

$\Longrightarrow \exists\, c \in (a,b) \,/\, \boxed{f'(c)} = \dfrac{f(b) - f(a)}{b - a}$

LAGRANGE POINTS — ITS SLOPE IS



Dom f? $1 - x^2 \ge 0$

Parabola Method
- concavity $a = -1 < 0$
- zeros $1 - x^2 = 0$, $x = \pm 1$

$-1 \le x \le 1$ Dom f $= [-1, 1]$

f contin. on $[-1,1]$ by composition of contin. functions

$f(x) = 1 + x + \sqrt{1 - x^2}$

$f'(x) = 1 + \dfrac{1}{\cancel{2}\sqrt{1 - x^2}} \cdot (-\cancel{2}x)$ Dom f' = ?

To find Dom f' $\begin{cases} 1 - x^2 \ge 0 & \text{(ROOT)} \\ 1 - x^2 \ne 0 & \text{(Denominator)} \end{cases}$ $\to 1 - x^2 > 0$ Parabola Method

$-1 < x < 1$

Dom $f' = (-1, 1)$

$\Longrightarrow$ Lagrange th. applies

Let's compute Lagrange Points:

$\exists\, c \in (-1, 1) \,/\, f'(c) = \dfrac{f(1) - f(-1)}{2}$ $\boxed{\begin{matrix} a = -1 \\ b = 1 \end{matrix}}$

$f(1) = 2$ $f(-1) = 0$

c $\frac{1}{2} - 0$ Solve to find

$f(1)=2 \qquad f(-1)=0$

$$1-\frac{c}{\sqrt{1-c^2}}=\frac{1\cancel{2}-0}{\cancel{2}}$$ Solve to find c

$$\Longleftrightarrow \frac{c}{\sqrt{1-c^2}}=0 \Longleftrightarrow \boxed{c=0}\in(-1,1)$$

EX 3D (Week 6)

De L'Hopital Theorem (...) Hypothesis $\frac{\infty}{\infty}$ or $\frac{0}{0}$

$$\Longrightarrow \lim_{x\to x_0}\frac{f'(x)}{g'(x)}=\lim_{x\to x_0}\frac{f(x)}{g(x)}$$

$$\frac{1}{2}\lim_{x\to 0}\frac{1+6x-\sqrt{(1+4x)^3}}{x\sin x}=\left[\frac{0}{0}\right]\overset{H}{=}\frac{1}{2}\lim_{x\to 0}\frac{6-\frac{3}{\cancel{2}}(1+4x)^{1/2}\cdot\cancel{4}^{2}}{\sin x+x\cos x}=\left[\frac{0}{0}\right]=$$

$(1+4x)^{3/2}$ $\qquad -6(1+4x)^{1/2}$

Recall $D(x^\alpha)=\alpha x^{\alpha-1}$

$D(f\cdot g)=f'g+fg'$

$$\overset{H}{=}\frac{1}{2}\lim_{x\to 0}\frac{-3(1+4x)^{-1/2}\cdot 4}{\cos x+\cos x-x\sin x}=\frac{1}{2}\frac{\frac{-3\cdot 4}{1}}{2}=-3$$

0

EX6 (Week 6)

$$f(x)=\begin{cases}x^3\left(4+\sin\frac{1}{x}\right), & x\neq 0\\ 0, & x=0\end{cases}$$

$x=0$ stationary?

$-1\le\sin\frac{1}{x}\le 1$

Check continuity at $x=0$. Check: $\lim_{x\to 0}f(x)=f(0)=0 \Longleftrightarrow \lim_{x\to 0}x^3\left(4+\sin\frac{1}{x}\right)=0$

0 $\qquad$ bounded in $[3,5]$

Check diff. at $x=0$. Check:

$$f'(0)\overset{def}{=}\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=$$

$$=\lim_{x\to 0}x^2\left(4+\sin\frac{1}{x}\right)=0$$

0 $\qquad$ bounded

Yes: $x=0$ is stationary

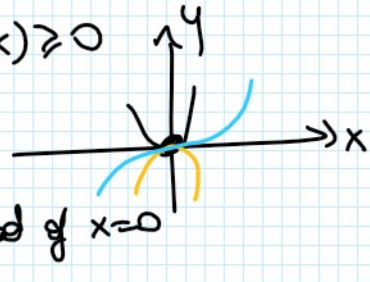
We classify $x=0$: ~~MAX~~ / ~~MIN~~ / INFLECTION P.

1st method: compute $f'(x)$ and study $f'(x)\geq 0$

2nd method: We note that $f(0)=0$

it's sufficient to study $f(x)\geq 0$ in a neighborhood of $x=0$

$\qquad\qquad\qquad\qquad\; f(0)$



• IF $x>0$

$f(0)$

- IF $x>0$
$f(x) = x^3\left(4+\sin\frac{1}{x}\right) > 0$

$\in [3,5]$

- IF $x<0$
$f(x) = \underset{<0}{x^3}\underset{>0}{\left(4+\sin\frac{1}{x}\right)} < 0$

$\Longrightarrow$ f changes SIGN around $x=0$

$\Longrightarrow$ $x=0$ inflection point (with horizontal tangent)

$f(x) = x^3\left(4+\sin\frac{1}{x}\right)$ $\qquad$ $\underline{1^{ST} \text{ method}}$: $f'(x)\geq 0$

$f'(x) = 3x^2\left(4+\sin\frac{1}{x}\right) + x^3\left(\cos\frac{1}{x}\cdot\left(-\frac{1}{x^2}\right)\right) \overset{?}{\geq} 0$

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10 February 2016 - I

Consider

$$f(x) = \frac{\log|x|}{\log^2|x| - \log|x| + 1}.$$

(a) Find the domain, possible symmetries and asymptotes. Show that f admits continuous prolongation in $x = 0$.

(b) Denote the prolongation by g, and compute its derivative. Find non differentiable points for g and specify their type.

(c) Find monotonicity intervals for g and minima/maxima, saying if they are local or global.

(d) Draw a qualitative graph for g.

(e) Find the solutions of $g(x) = 1$

(A) $\begin{cases} \log^2|x| - \log|x| + 1 \neq 0 \\ |x| > 0 \end{cases}$ $\longrightarrow$ $t = \log|x|$, $\quad t^2 - t + 1 \neq 0$ always verified $\quad \Delta < 0$

By def. $|x| \geq 0$, THEN

$|x| > 0 \iff x \neq 0$ $\qquad$ $\mathrm{Dom} f = \mathbb{R}\setminus\{0\}$

$f(-x) = f(x)$ because $|-x| = |x|$ $\Longrightarrow$ even (y AXIS)

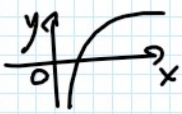
We FOCUS on $(0,+\infty)$ $\leadsto$ By simmetry

$\lim_{x\to+\infty} f(x) = \left[\frac{\infty}{\infty}\right] = \lim_{x\to+\infty} \frac{\cancel{\log|x|}^{\,1} + \cancel{o(\log|x|)}}{\log^{\cancel{2}}|x| + \cancel{o(\log^2|x|)}} = \frac{1}{\infty} = 0$

$\boxed{y=0}$ HORIZ. asymptote (Right and left)

$\lim_{x\to 0^+} \frac{\log x}{\log^2 x - \log x + 1} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to 0^+} \frac{\cancel{\log x}^{\,1}}{\log^{\cancel{2}} x} = \frac{1}{\infty} = 0$

$\hookrightarrow |x| = x$ in a right neigh. of 0



$y = \log x$

$\nexists$ vertical asymptote

f admits contin. prolongation in $x=0$:

$g(x) = \begin{cases} f(x), & x \neq 0 \longrightarrow x > 0 \end{cases}$ $\qquad$ (we focus on $[0,+\infty)$)

$$g(x)=\begin{cases} f(x), & x\neq 0 \to x>0 \\ 0, & x=0\end{cases}$$ (we focus on $[0,+\infty)$)

$\log x = o(\log^2 x)$ as $x\to 0^+$ $\overset{\text{def.}}{\iff}$ $\frac{\cancel{\log x}}{\log^{\cancel{2}} x} \xrightarrow[x\to 0^+]{} 0$

$f(x) \sim_{0^+} \frac{\log x}{\log^2 x}$

$$g'(x) = (\dots) = \frac{-\log^2 x+1}{x(\log^2 x-\log x+1)^2} \quad >0$$

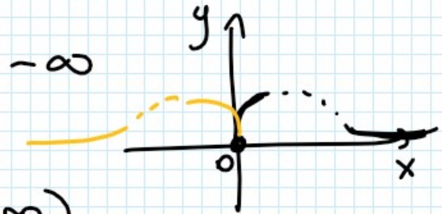
$$g'(0)=\lim_{x\to0^+}\frac{g(x)-g(0)}{x-0} = \lim_{x\to 0^+}\frac{\log x}{x(\log^2 x-\log x+1)} = -\infty$$

$\log x \to -\infty$, $x(\log^2 x-\log x+1) \to 0^+$

change variable $\boxed{\log x = t}$ $\boxed{x=e^t}$

(Recall $\lim_{x\to0^+} x\log x = 0$)

$$\lim_{t\to-\infty}\frac{t}{e^t(t^2-t+1)} = \frac{-\infty}{0^+} = -\infty$$

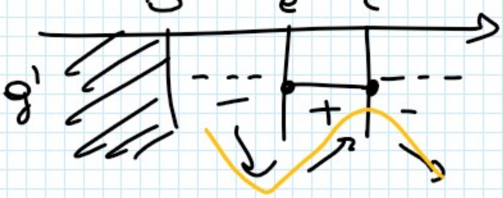


$x=0$ is a cusp

Monotonicity $g'(x)\geq 0$ in $[0,+\infty)$

$-\log^2 x+1\geq 0$ $\quad \log^2 x\leq 1$ $\quad -1\leq \log x\leq 1$

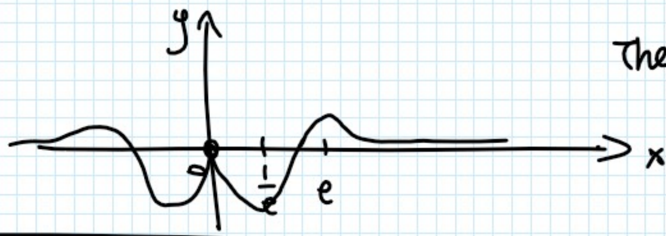
$e^{-1}\leq x\leq e$



$x=\frac{1}{e}$ local MIN (global?)

$x=e$ local MAX

They are GLOBAL extreme



$g(x)=1$ solve $(\dots)$

10 February 2016 - II

Consider

$$f(x) = (\sinh 2x)^2 - 2\sinh 2x - 3 \quad \text{defined for } \boxed{x \geq 0.}$$

(a) Find limits at boundary points of the domain and asymptotes.

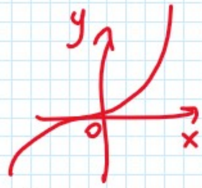
(b) Compute the derivative and show that there exists a unique points $x_0 > 0$ such that $f'(x_0) = 0$ (the value of x_0 is not required).

(c) Find monotonicity intervals and say if there are maximum or minimum points.

(d) Draw a qualitative graph.

(e) Draw $g(x) = f(|x|)$ defined for $x \in R$ and investigate non differentiable points.

Recall $\sinh x = \frac{e^x - e^{-x}}{2}$



(A) Dom $f = \mathbb{R}^+ = [0,+\infty)$

$f(0) = -3$ No vertical asymptotes

$\lim_{x\to+\infty} f(x) = +\infty$ $\quad \sinh x \sim_{+\infty} \frac{e^x}{2}$

$(e^{2x})^2 \quad 2x \quad e^{4x}$

$\lim_{x\to+\infty} f(x) = +\infty$ $\qquad \sinh x \sim_{+\infty} \frac{e^x}{2}$

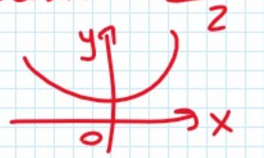
No HORIZ. asymptotes $\qquad f(x) \sim_{+\infty} \left(\frac{e^{2x}}{2}\right)^2 - \cancel{2}\frac{1}{\cancel{2}} e^{2x} \sim_{+\infty} \frac{e^{4x}}{4}$

No oblique asymptotes

(B) $f'(x) = 2\sinh(2x)\cdot\cosh(2x)\cdot 2 - 2\cosh(2x)\cdot 2 =$

$= \underbrace{4\cosh 2x}_{>0}\left(\sinh(2x)-1\right) \geq 0$

$y = \cosh x = \frac{e^x + e^{-x}}{2}$

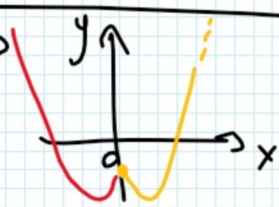


$\iff \sinh(2x) \geq 1 \qquad \frac{e^{+x} - e^{-x}}{2} \geq 1$

$e^x - e^{-x} \geq 2 \qquad \frac{e^{2x} - 1 - 2e^x}{\cancel{e^x}} \geq 0 \qquad e^{2x} - 2e^x - 1 \geq 0$

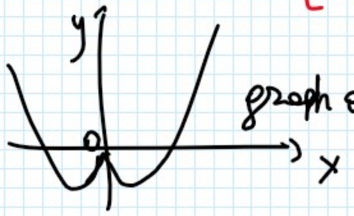
$\boxed{t = e^x} \qquad t^2 - 2t - 1 \geq 0$ Parabola method (...)

(D) graph of $f(x)$ $\longrightarrow$



(E) $\boxed{f(|x|)}$

Recall $f(|x|) \overset{def}{=} \begin{cases} f(x), & x \geq 0 \\ f(-x), & x < 0 \end{cases}$ ↳ SYMMETRIC w.r.t. y-AXIS



graph of $f(|x|)$

$x=0$ is NOT diff.

CHECK: $\lim_{x\to 0^+} f'(x) = -4 \qquad \lim_{x\to 0^-} f'(x) = 4 \implies$ $x=0$ is CORNER POINT

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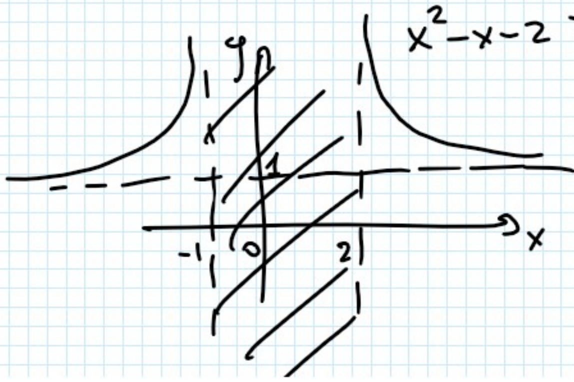
31 January 2018 - II *Consider the function*

$$f(x) = \frac{|x|}{\sqrt{x^2 - x - 2}}.$$

$\to$ study of function

Dom $f = (-\infty, -1) \cup (2, +\infty)$

$x^2 - x - 2 > 0 \qquad x < -1 \vee x > 2$



$\lim_{x\to+\infty} f(x) = 1$

(e) *Represent in the plane, the following set*

$$A=\{(x,y)\in\mathbb{R}\times\mathbb{R}:\ y^2(x^2-x-2)-x^2=0\}.$$

Find $y=f(x)$

Note that $(0,0)\in A$

$y^2=\dfrac{x^2}{x^2-x-2}$ IF $x\neq 2$ and $x\neq -1$

$y=\pm\sqrt{\dfrac{x^2}{x^2-x-2}}$ IFF $x^2-x-2>0$

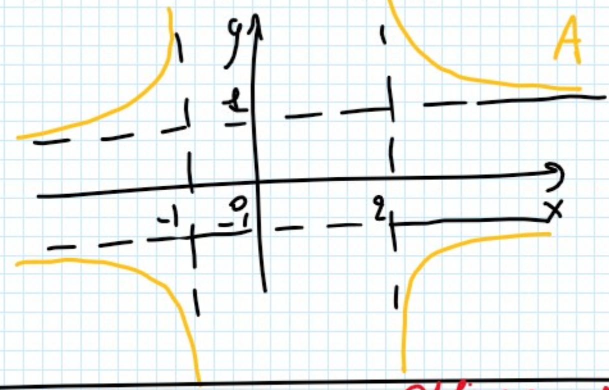
IFF $x\in$ Domf

IFF $x\in(-\infty,-1)\cup(2,+\infty)$

$\sqrt{x^2}=|x|$

$y=\pm\dfrac{|x|}{\sqrt{x^2-x-2}}$

$$\boxed{\begin{array}{l} y=g(x)\\ y=-g(x)\ \text{symm. W.R.T. X Axis}\end{array}}$$



Oblique asymptotes

$y=mx+q$

$f(x)\sim_{+\infty} mx$

$q=\lim_{x\to+\infty}(f(x)-mx)$

9 September 2015 - I

Let

$$f(x)=e^{2(x-3)^3\log|x-3|}.$$

1. Find domain and asymptotes for f. Show that f admits continuous prolongation in $x=3$.
2. Denote the continuous prolongation by f, compute the first derivative and study differentiability in $x=3$.
3. Find monotonicity intervals and maxima/minima for f.
4. Draw a qualitative graph for f.

(1) $|x-3|>0 \iff x\neq 3$

Domf $=\mathbb{R}\setminus\{3\}$

$\lim_{x\to+\infty} f(x)=+\infty$

$$\boxed{\begin{array}{l}\lim_{x\to-\infty} f(x)=0\\ y=0\ \ \text{HORIZ. LEFT AS.}\end{array}}$$

$\lim_{x\to 3^\pm} f(x)=e^0=1$

$\lim_{t\to 0^+} t^\alpha \log t^\beta=0$ $\quad\alpha,\beta>0$

$$f(x)=\begin{cases} e^{2(x-3)^3\log|x-3|}, & x\neq 3\\ 1, & x=3\end{cases}$$

contin. on $\mathbb{R}$

$$f'(x)=\begin{cases} e^{2(x-3)^3\log|x-3|}\cdot\left(6(x-3)^2\log|x-3|+2(x-3)^{3}\dfrac{1}{\cancel{|x-3|}}\cdot\dfrac{\cancel{|x-3|}}{\cancel{x-3}}\right), & x\neq 3\\ \lim_{x\to 3}\dfrac{f(x)-f(3)}{x-3}=\lim_{x\to 3}\dfrac{e^{(\dots)}-1}{x-3} & \text{IF } x=3\end{cases}$$

$$=\begin{cases} e^{2(x-3)^3\log|x-3|}\cdot 2(x-3)^2\left(3\log|x-3|+1\right), & x\neq 3\\ \lim \dfrac{2(x-3)^{3}\log|x-3|}{x-3}=0, & x=3\end{cases}$$

$$= \begin{cases} \lim\limits_{x\to 3} \dfrac{2(x-3)^{1/3}\log|x-3|}{\cancel{x-3}} = 0 & , x=3 \end{cases}$$

$\boxed{x \neq 3}$

$f'(x) \geq 0 \iff 3\log|x-3| \geq -1$

Recall $0 = \log 1$, $1 = \log e$

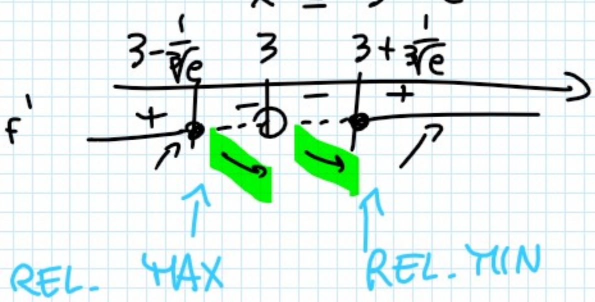
$\log|x-3| \geq -\frac{1}{3}\log e$

$\log|x-3| \geq \log(e^{-1/3})$

$|x-3| \geq e^{-1/3}$

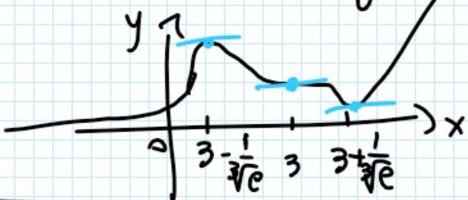
$x-3 \leq -e^{-1/3} \quad \vee \quad x-3 \geq e^{-1/3}$

$x \leq 3-e^{-1/3} \quad \vee \quad x \geq 3+e^{-1/3}$



Note that $x=3$ is stationary!

$x=3$ is inflection point



T3 (week 6)

3. Let $I \subseteq \mathbb{R}$ be a non empty subset ($\neq \mathbb{R}$) and $f: I \to \mathbb{R}$ be a continuous function; let $x_0 \in I$. Then:

- ~~(a)~~ if f is differentiable I and x_0 is a local minimum for f, then $f'(x_0) = 0$ NO x_0 could boundary point of I
- (b) if f is differentiable in x_0 and x_0 is a local minimum for f in the interior of I, then $f'(x_0) = 0$ FERMAT Theorem
- ~~(c)~~ if f is twice differentiable in I and $f'(x_0) = 0$, then $\boxed{f''(x_0) = 0}$ $f(x) = x^2$ $x=0$ MIN
- ~~(d)~~ if x_0 is local minimum for f in the interior I, then $f'(x_0) = 0$ WHAT IF f non diff.?
- ~~(e)~~ if f is differentiable in I and $f'(x_0) = 0$, then x_0 is a maximum or a minimum for f ↳ inflection p.

$I \subseteq \mathbb{R}$

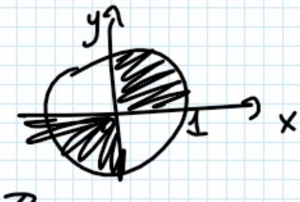
T6 (week 6)

6. Given the function $f(x) = \sin^2 2x$, which of the following is correct?

- ~~(a)~~ $f(x)$ has in $x = 0$ is a relative ~~maximum~~ MIN point
- (b) the points $x = k\frac{\pi}{2}, k \in \mathbb{Z}$ are absolute minimum points for f
- ~~(c)~~ $f(x)$ has minimum period π $\pi/2$
- ~~(d)~~ the points $x = k\frac{\pi}{4}, k \in \mathbb{Z}$ are absolute maxima for f
- ~~(e)~~ for $x \to 0$, $f(x)$ is infinitesimal of order 4

$f'(x) = 2\sin(2x)\cdot\cos(2x)\cdot 2 \geq 0$

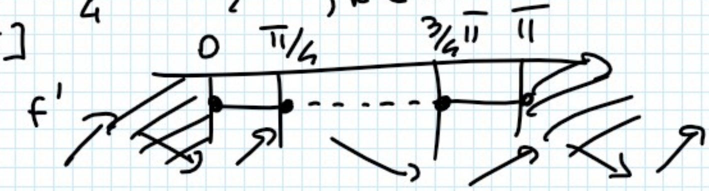
$\iff \sin(2x)$ and $\cos(2x)$ have same sign



$0 + k\pi \leq 2x \leq \frac{\pi}{2} + k\pi$

$k\frac{\pi}{2} \leq x \leq \frac{\pi}{4} + k\frac{\pi}{2}, \quad k \in \mathbb{Z}$

Focus on $[0, \pi]$



MIN

$\cancel{e}$ $f(x) \sim_0 (2x)^2$ order 2

MAX MIN

$\sin t \sim_0 t$

$\boxed{\text{Test 9 (week 6)}}$

9. Consider the functions $f_n(x) = \begin{cases} x^n \sin \dfrac{1}{x} & \text{se } x \neq 0 \\ 0 & \text{se } x = 0 \end{cases}$, depending on the parameter $n \in \mathbb{N}$.

- ~~(a)~~ The function $f_1(x)$ is not continuous in $x = 0$
- ~~(b)~~ The function $f_3(x)$ is twice differentiable in $x = 0$
- ~~(c)~~ The function $f_1(x)$ is differentiable in $x = 0$
- ~~(d)~~ The function $f_2'(x)$ is continuous
- (e) The function $f_2(x)$ is differentiable in $x = 0$

f_1 $\boxed{n=1}$

$$f_1(x) = \begin{cases} \underbrace{x}_{\to 0} \underbrace{\sin \frac{1}{x}}_{\text{bounded}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$$\lim_{x \to 0} f_1(x) = 0 \quad \cancel{A}$$

$$\sin\frac{1}{x} \;\; = \;\; f_1'(0) = \lim_{x \to 0} \frac{f_1(x) - f_1(0)}{x - 0} \nexists$$

f_3 $\boxed{n=3}$

$$f_3(x) = \begin{cases} x^3 \sin \frac{1}{x}, & x \neq 0 \\ 0, & x \neq 0 \end{cases}$$

$$f_3'(0) = \lim_{x \to 0} \frac{f_3(x) - f_3(0)}{x - 0} = \lim_{x \to 0} x^2 \sin\frac{1}{x} = 0$$

$\boxed{x \neq 0}$ $f_3'(x) = 3x^2 \sin\frac{1}{x} + \cancel{x^3} \cos\left(\frac{1}{x}\right) \cdot \left(-\frac{1}{\cancel{x^2}}\right)$

$$\boxed{\lim_{x \to 0} f_3'(x)} \stackrel{?}{=} f_3'(0) \quad \text{(continuity of } f_3'\text{)}$$

$$\lim_{x \to 0} 3x^2 \sin\frac{1}{x} - x \cos\frac{1}{x} = 0$$

$$f_3''(0) = \lim_{x \to 0} \frac{f_3'(x) - f_3'(0)}{x - 0} = \lim_{x \to 0} \underbrace{3x \sin\frac{1}{x}}_{\to 0} - \cos\frac{1}{x} = \nexists$$

f_2 $\boxed{n=2}$

$$f_2(x) = \begin{cases} x^2 \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

f_2 is contin. in 0

$$f_2'(0) = \lim_{x \to 0} \frac{f_2(x) - f_2(0)}{x - 0} = \lim_{x \to 0} x \sin\frac{1}{x} = 0 \quad (E)$$

$x \neq 0$

$$f_2'(x) = 2x \sin\frac{1}{x} + \cancel{x^2} \cos\left(\frac{1}{x}\right)\left(-\frac{1}{\cancel{x^2}}\right)$$

Is f_2' contin. at 0? No $\quad \lim_{x \to 0} \underbrace{2x \sin\frac{1}{x}}_{\to 0} + \underbrace{\cos\frac{1}{x}}_{\nexists}$

$\cancel{D}$

$x \to 0$

$\cancel{D}$ $\qquad\qquad\qquad\qquad\qquad\qquad$ 0 $\qquad$ $\nexists$