Calibration of an inertial measurement unit

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***Abstract*—This paper presents a way to calibrate different inertial measurement sensors. In particular we present the calibration of an accelerometer and a gyroscope using least square. A model of the sensors is presented based on the main errors that MEMS devices present, a calibration method is proposed for the static parameters of the model. Finally a temperature adjust is made.**

I. Introduction

The development of the Microelectromechanical Systems(MEMS) technology has allowed to manufacture many low-cost chip-sensors, such as accelerometers, gyroscopes and magnetometeres. Those chips have been adopted in many applications, for instance Inertial Navigation Systems (INS) [**?**]. However this sensors have many error sources, thus they must be calibrated before being used and they should be re-calibrated peridoically for any precission application. The different sources of error will be analysed in more detail in section II and the implications of those errors will be derived.

The calibration proposed in this paper is tested on a 3-axis accelerometer ADXL345 of Analog Devices and a 3-axis gyroscope ITG-3200 of InvenSense, yet the model developed and the methodology can be used in other devices because the model is based on comun characteristics of MEMS sensors. The sensors named before are included in a Inertial Measurment Unit(IMU): The Mongoose (figure 1).

The first step of the calibration is to obtain the static parameters of the devices for the ambience temperautre. This step is based on knowing the exact orientation of the IMU for the calibration of both sensors and the exact angular speed for the gyroscope calibration. Other related works waive this requirement [1] and [2] and uses the fact that in any position angular speed(for the gyroscope calibration) and gravity vector (for the accelerometer calibration) are constant.

We take in consideration a fact that is not usually taken into account in the literature (see [1], [2], [3]): the measures given by this sensors are not independant of the temperature. Many devices that uses MEMS sensors need to work properly in a wide temperature range. In other cases, the temperature in operation of the system is different(for instance due to Joule Effect of the wires near the sensors) than the ambiance temperature in which the sensor was calibrated. Therefore a temperature adjust must be made.

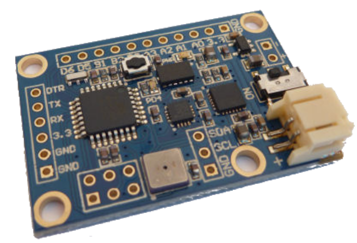


Fig. 1: **Mongoose -** Inertial Measurement Unit used. Red arrows represent the axis plataform $S - \{\hat{x}, \hat{y}, \hat{z}\}$.

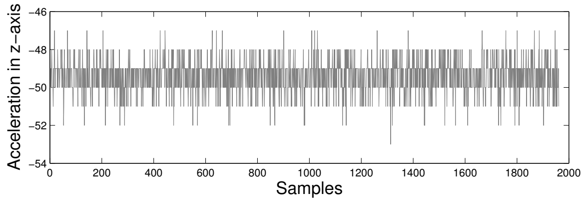
II. Model of the sensors

As it was stated before, there exist many error sources in MEMS devices. In [1] two of this sources are mentioned: the nonlinear response of the sensors and the non-orthogonality of the axis of the sensor. In addition we can observe also that it exist an electric noise in the measures (figure **??**) and a dependance of the temperature.

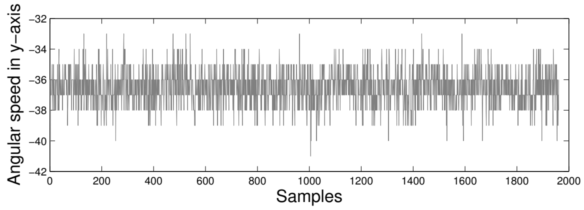
In the datasheets of the considered sensors([4] and [5]) the effect of nonlinearityis negligible since it represents the $\pm 0.5.\%$ of the full scale for the accelerometer and the $\pm 0.2\%$ for the gyroscope. Therefore we will not consider this effect and after the calibration we must verify that the error due to the nonlinear response is negligeable.

In figure 2 we can observe that the electrical noise of the devices is typically 2 bits, this error (according to the datashets [4] and [5]) correspond to $7.8mg$ and $0.14°/s$. The error is not biased, thus we not need to consider it, because several samples are used in any real application and the effect vanishes.

Based on the asumption that both sensors have a linear response and considering the non-orthogonality of the axis of the devices we are going to present a standard model for both sensors(see [1], [2], [3]).



(a) Accelerometer data $\hat{k}$



(b) Gyroscope data $\hat{j}$

Fig. 2: **Noise -** Measures from accelerometer and gyroscope in static equilibrium.

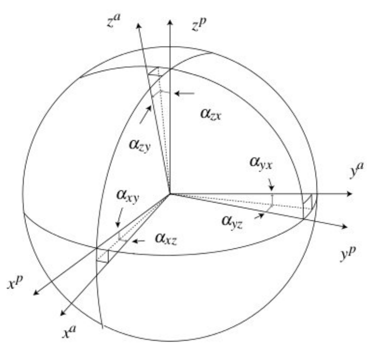


Fig. 3: **Non orthogonality -** Rotations of the sensitivity axis of the accelerometer over the axis of the platform system.

A. Accelerometer

Due to construction issues the sensitivity axis of the device are generaly not orthogonals. Lets define a platform system as shown in figure 1. Let $\mathbf{a}^a$ be the true acceleration expressed in the sensitivity axis of the accelerometer and let $\mathbf{a}^p$ be the true acceleration expressed in the platform system. We model this fact by the following linear transformation between the two acceleration vectors:

$$\mathbf{a}^p = \mathbf{T}_a^p \mathbf{a}^a,\ \mathbf{T}_a^p = \begin{pmatrix} 1 & -\alpha_{yz} & \alpha_{zy} \\ \alpha_{xz} & 1 & -\alpha_{zx} \\ -\alpha_{xy} & \alpha_{yx} & 1 \end{pmatrix} \quad (1)$$

In equation 1, scalars α_{ij} represents the rotation of the $i-th$ sensitivity axis of the acelerometer over the $j-th$ axis of the platform system. In figure 3 this relation can be observed graphically. Since those errors are due to the manufacturing process we will assume that they will remain constant long enough.

As it was stated befor, we are considering a linear model between the accleration measured in the sensitivity axis and the true acceleration in the same system. Thus we have:

$$\tilde{\mathbf{a}}^a = \mathbf{K}_a \mathbf{a}^a + \mathbf{b}_a \quad (2)$$

In equation 2, $\tilde{\mathbf{a}}^a$ is the measure obtained by the accelerometer, $\mathbf{K}_a$ is a diagonal matrix that represents the sensitivity of each axis and $\mathbf{b}_a$ is a vector that represent the bias of each axis. Those parameters may variate as the temperature changes. However, in this stage we are going to consider them constant. Using equations 1 and 2 we can stablish a model of the acelerometer:

$$\tilde{\mathbf{a}}^a = \mathbf{K}_a \mathbf{T}_p^a \mathbf{a}^p + \mathbf{b}_a = \mathbf{K}_a (\mathbf{T}_a^p)^{-1} \mathbf{a}^p + \mathbf{b}_a \quad (3)$$

B. Gyroscope

The error sources consider in the case of the gyroscope are the same that we developed for the acclerometer. Thus, we are going to consider the same model:

$$\tilde{\boldsymbol{\omega}}_a = \mathbf{K}_\omega (\mathbf{T}_\omega^p)^{-1} \boldsymbol{\omega}^p + \mathbf{b}_\omega \quad (4)$$

III. Calibration method proposed

The problem of calibration is to establish the values for the unkwnown parameters of the model that adjust "the better" a certain set of data. This criterium will be defined in section III-A.

A. Static Parameter Calibration

Let $\boldsymbol{\theta}_s$ (where the subindex s refers to a sensor) be the parameter vector of a certain sensor. We can define this vector as:

$$\boldsymbol{\theta} = [k_{sx}, k_{sy}, k_{sz}, b_{sx}, b_{sy}, b_{sz}, \alpha_{sxy}, \alpha_{sxz}, \alpha_{syx}, \alpha_{syz}, \alpha_{szx}, \alpha_{szy}]^T \quad (5)$$

In equation 5, k_{si}, with $i = \{x, y, z\}$ are the diagonal elements of the matrix $\mathbf{K}_s$, b_{si}, with $i = \{x, y, z\}$ are the elements of vector $\mathbf{b}_s$ and α_{sij}, with $i = \{x, y, z\}$, $j = \{x, y, z\}$ and $i \neq j$ are the elements outside the diagonal of the matrix $\mathbf{T}_s^p$.

As adjust criterion we choose to minimize the sum of the squares of the norms of the diferences between true acceleration (or angular speed) and measured acceleration (or angular speed). This problem can be written as follows:

$$\min_{\boldsymbol{\theta}} \sum_{i=1}^{M} ||\mathbf{s}_i^p - \mathbf{T}_s^p (\mathbf{K_s})^{-1} (\tilde{\mathbf{s}}_i^s - \mathbf{b}_s)||^2 \quad (6)$$

In equation 6, M is the cardinal of the training set, $\mathbf{s}_i^p$ is the true magnitude (acceleration or angular speed) and $\tilde{\mathbf{s}}_i$ are the values given by the sensor. This problem can be solved with *MatLab* function *lsqnonlin*.

It is important to consider that the function that we want to minimize will generally have more than one local minimum. Thus, we can only assure that the optimum found by algorithms is a local optimum. To ensure that the solution obtained is the desired one, we need to choose carefully the seed for the algorithm. Tipically we will choose a vector $\boldsymbol{\theta}_0$ that is close to the values of the unknow parameters. Even if we do not know the exact value of the model's parameters, the datasheet of the different sensors can give us a very good clue about those values.

1) Accelerometer: Since an accelerometer measures the acceleration in a free fall referential, in static equilibrium it will meassure an accleration that has as norm the gravitational constant of the earth ($g = 9.81ms^{-2}$), its direction is colinear with the line defined by the position of the acclerometer and the center of the earth and its sense is from the center of the earth towards the position of the accelerometer, meaning "up". This allows us to have an exact expression for the accleration, the procedure proposed consist in measure this magnitude with different orientations of the acceleration and then solve the problem stated in equation 6.

The 27 measures made were the following:

- Starting position: x axis pointing "down": 10 measures were made combining two rotations. First, rotating arround the z axis $\theta = 0°, 10°, 20°, 30°, 45°$. Then, for each angle defined(except for $\theta = 0°$), two measures were made rotating arround the x axis $\psi = 0°, -45°$. This set of measures can be expressed in the following way:

$$\mathbf{a}^p = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & \sin\psi \\ 0 & -\sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -g \\ 0 \\ 0 \end{pmatrix} \quad (7)$$

- Starting position: y axis pointing "up": 10 measures were made combining two rotations. First, rotating arround the x axis $\psi = 0°, 10°, 20°, 30°, 45°$. Then, for each angle defined(except for $\theta = 0°$), two measures were made rotating arround the y axis $\varphi = 0°, 45°$.

$$\mathbf{a}^p = \begin{pmatrix} \cos\varphi & 0 & \sin\varphi \\ 0 & 1 & 0 \\ -\sin\varphi & 0 & \cos\varphi \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & \sin\psi \\ 0 & -\sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} 0 \\ g \\ 0 \end{pmatrix} \quad (8)$$

- Starting position: z axis pointing "up": 10 measures were combining two rotations. First, rotating arround the y axis: $\phi = 0°, -10°, -20°, -30°, -45°$. Then, for each angle defined(except for $\phi = 0°$), two measures were made rotating arround the z axis $\theta = 0°, 45°$.

$$\mathbf{a}^p = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\varphi & 0 & \sin\varphi \\ 0 & 1 & 0 \\ -\sin\varphi & 0 & \cos\varphi \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ g \end{pmatrix} \quad (9)$$

We are not going to use all the 27 measures to do the calibration, instead we use 14 of this measures as a training set and the other 13 as a test set. The 14 measures of accleration give us 42 different acceleration values (3 for each measure corresponding to each axis), since we want to estimate 12 parameters we consider that is sufficient.

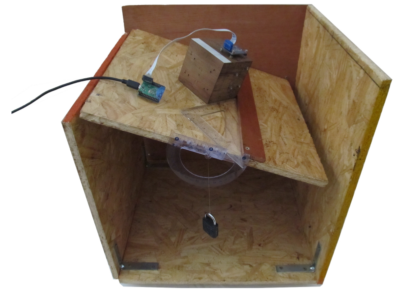


Fig. 4: **Adjustable table**- Used for accelerometer calibration

The delicate issue is knowing the exact orientation of the platform, having solve this is very easy to compute the theoretical acceleration that the device should measure in each axis from equations 7, 8 and 9.

First of all, we atached the platform to a wooden cube. This allows us to perform 90° rotations, thus if we had an horizontal surface we can take measures with the gravity vector aligned to any of three axis. The horizontal surface was assured using a three leg adjustable table (see figure 4). In addition this table allows us to perform rotations of known angles using a scaled semicircle and a pendulum as it is shown also in figure 4. The rotations over the second axis were made using a carpenter's square, therefore we have a proper 45° rotation.

As it was stated in II the problem 6 may have more than one local minimum, thus we need to assure that the seed of the algorithm is "near" of the values of the parameters. The proposed way to generate such a seed is to use some information of the sensor's datasheet [4]:

- The typical sensitivity for each axis is $256bits/g \approx 26.10ms^{-2}$.
- The typical offset for each axis is $0ms^{-2}$.
- As the axis should be as orthogonal as posible is reasonable to think that the rotation angles α_{aij} are near zero.

After this considerations the choosed seed is:

$$\theta_0 = [26.10, 26.10, 26.10, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]^T \quad (10)$$

2) Gyroscope: The procedure used to calibrate the gyroscope was very similar to the accelerometer. The main idea is to obtain a constant and known angular speed vector and then change the orientation of the gyroscope to have different measures in each axis.

In this case the constant angular speed was provided with a turntable ($33rev/min$). Its angular speed was measured just to be sure that it was working properly. Different orientations were obtained using the wooden cube mentioned in section III-A1,to assure 90° rotations and pairs of carpenter's squares to assure 30°, 45° and 60° rotations. In figure 5 is shown the

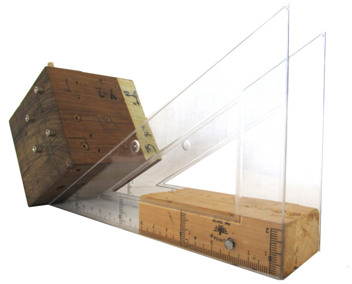


Fig. 5: **Gyroscope setup -** Carpenter's squares were used to assure an exact orientation

setup that was mounted over the turntable.

As we did for the accelerometer, we are going to choose a seed based on the information in the device's datasheet [5]:

- The typical sensitivity for each axis is $14,375bits/(^{\circ}s^{-1}) \approx 823.63bits/(rads^{-1})$.
- The typical offset for each axis is $0rads^{-1}$.
- As the axis should be as orthogonal as posible is reasonable to think that the rotation angles $\alpha_{\omega ij}$ are near zero.

After this considerations the choosed seed is:

$$\theta_0 = [823.63, 823.63, 823.63, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]^T \tag{11}$$

B. Temperature adjust

In this paper we propose also to realize a temperature adjustment. It is commun that the operating temperature of the system in with a sensor used is different from the calibration temperature. Sensors based in MEMS technology are sensitive to such variations. The variation of the measures is critical in some applications (see [**?**] for example). In INS two of the Euler angles are usually obtained from the accelerometer, therefore if the measures are not certain the stability of the system cannot be guaranteed.

It is extremely important to understand which of the parameters are affected based on the information of the sensors' datasheets ([4] and [5]):

- Sensitivity variation of the acclerometer: $\pm 0.01\%/^{\circ}C$
- Offset variation of the accelererometer: $\pm 1.2mg/^{\circ}C \approx 0.012ms^{-1}$
- Sensitivity variation of the gyroscopoe: No information is provided.
- Offset variation of the accelerometer: $\pm 40^{\circ}s^{-1} \approx 0.70rads^{-1}$

The conlusion of this sumed up information is that the variations of the temperature do not produce main changes on the gains of the sensors, yet the offsets are modified. Thus, we are going to consider the following temperature adjust:

$$\mathbf{b}_s(T) = \mathbf{b}_{s0} + \boldsymbol{\alpha}_s(T - T_0) \tag{12}$$

In equation 12 $\mathbf{b}_{s0}$ is the offset vector obtained in the static calibration, T_0 is the ambiance temperature in the calibration, T is the working temperature of the system and $\boldsymbol{\alpha}_s$ is the parameter that we need to know to complete the process.

The procedure to achieve this adjust was take several samples of acceleration (or angular speed) in a fixed position (z axis pointing "up" without any rotation), while the sensor was heat up with a hair dryer. The criterium choosed was that $\boldsymbol{\alpha}_s$ must minimize the sum of the squares of the norms of the difference between the true acceleration (or angular speed) and the measured acceleration (or angular speed). Mathematically this means:

$$\min_{\boldsymbol{\alpha}_s} = \sum_{k=1}^{N} ||\mathbf{s}_i^p - \mathbf{T}_{s0}^p(\mathbf{K_{s0}})^{-1}(\tilde{\mathbf{s}}_i^s - \mathbf{b}_s(T))||^2 \tag{13}$$

In equation 13, N is the number of samples tooked, $\mathbf{T}_{s0}^p$, $\mathbf{K_{s0}}$ are the matrix obtained in the static parameter calibration (ambiance temperature) and $\mathbf{b}_s(T)$ is the vector defined in 12. This last vector includes the variable that we want to estimate.

IV. Results and Analysis

V. Conclusion

References

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