CALIBRATION OF AN INERTIAL MEASUREMENT UNIT

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***Abstract*—This paper presents a fast and low cost way to calibrate different inertial measurement sensors. In particular the calibration of an accelerometer and a gyroscope using nonlinear least squares is presented. A model of the sensors which includes the main errors that MEMS devices present is used. A calibration method is proposed for estimating the static parameters of the model and a temperature adjustment is proposed and implemented.**

I. INTRODUCTION

The development of the Micro-electromechanical Systems (MEMS) technology has allowed to manufacture many low-cost chip-sensors, such as accelerometers, gyroscopes and magnetometers. Those chips have been adopted in many applications, for instance Inertial Navigation Systems (INS) [1]. However this sensors have many error sources, thus they must be calibrated before being used and they should be re-calibrated periodically for any precision application. The different sources of error are analyzed in more detail in section (II) and the implications of those errors are derived.

The calibration proposed in this work is tested on a 3-axis accelerometer *ADXL345* of *Analog Devices* and a 3-axis gyroscope *ITG-3200* of *InvenSense*, yet the model developed and the methodology can be used in other devices because the model is based on common characteristics of MEMS sensors. The sensors named before are included in a Inertial Measurement Unit (IMU): The Mongoose (figure (1)).

The motivation for this calibration is to have enough precision to estimate the state of an Unmanned Aerial Vehicle (UAV) with quadrotor architecture.

The first step of the calibration is to obtain the static parameters of the devices for the ambiance temperature. This step is based on knowing the exact orientation of the IMU for the calibration of both sensors and the exact angular speed for the gyroscope calibration. Other related works waive this requirement [2], [3] and uses the fact that in any position angular speed (for the gyroscope calibration) and gravity vector (for the accelerometer calibration) are constant.

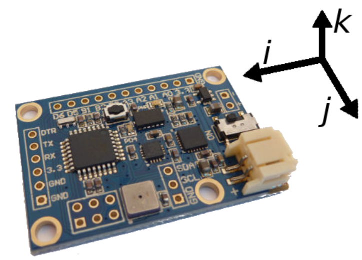


Fig. 1: **Mongoose -** Inertial Measurement Unit used. Black arrows represent the axis platform $S - \{\hat{i}, \hat{j}, \hat{k}\}$.

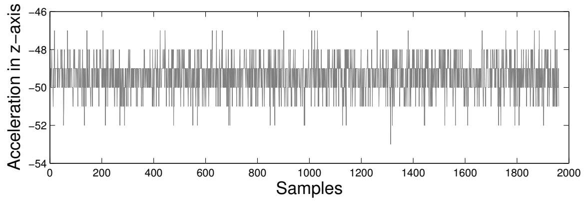
A fact that is not usually taken into account in the literature (see [2], [3], [4]) is taken into consideration: the measures given by this sensors are not independent of the temperature. Previous works [5], considered time varying drifts not depending on temperature but with known dependency on time, and neglecting axis non-orthogonality. Many devices that uses MEMS sensors need to work properly in a wide temperature range. In other cases, the temperature in operation of the system is different (e. g. due to Joule Effect, close drivers, etc.) than the ambiance temperature in which the sensor was calibrated. Therefore a temperature adjust must be made.

II. MODEL OF THE SENSORS

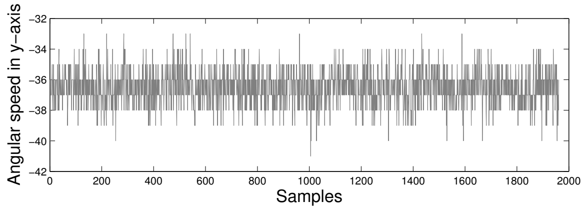
As it was stated before, there exist many error sources in MEMS devices. In [2] two of this sources are mentioned: the nonlinear response of the sensors and the non-orthogonality of the axis of the sensor. In addition, an electric noise (Figure 2) in the measures and a dependence of the temperature can be observed.

According to the datasheets of the devices ([6], [7]), effect of nonlinearities represent the $\pm 0.5.\%$ of the full scale for the accelerometer and the $\pm 0.2\%$ for the gyroscope. Therefore, this effect is not considered and after the calibration, it must be verified.

In Figure (2) can be observed that the electrical noise of the devices is typically 3 LSB (Least Significant Bit) peak to peak, which (according to the datasheets [6], [7]) correspond to $11.7mg$ and $0.21°/s$. The error is not biased, thus, it is



(a) Accelerometer data $\hat{k}$



(b) Gyroscope data $\hat{j}$

Fig. 2: **Noise -** Measures from accelerometer and gyroscope in static equilibrium.

not needed to be considered because many samples are used in the procedure and the effect vanishes.

Based on the assumption that both sensors have a linear response and considering the non-orthogonality of the axis of the devices, a standard model for both sensors is presented (see [2], [3], [4]).

A. Accelerometer

Due to construction issues the sensitivity axis of the device are generally not orthogonal. Lets define a platform system as shown in Figure (1). Let $\mathbf{a}^a$ be the true acceleration (meaning the theoretical acceleration measured in ms^{-2}) expressed in the sensitivity axis of the accelerometer and let $\mathbf{a}^p$ be the theoretical true acceleration (also measured in ms^{-2}) expressed in the platform system. The two acceleration vectors are related according to equation (1):

$$\mathbf{a}^p = \mathbf{T}_a^p \mathbf{a}^a,\ \mathbf{T}_a^p = \begin{pmatrix} 1 & -\alpha_{yz} & \alpha_{zy} \\ \alpha_{xz} & 1 & -\alpha_{zx} \\ -\alpha_{xy} & \alpha_{yx} & 1 \end{pmatrix} \quad (1)$$

In equation (1), scalars α_{ij} represents the rotation of the $i-th$ sensitivity axis of the accelerometer over the $j-th$ axis of the platform system. In Figure (3)[1] this relation can be observed graphically. Since those errors are due to the manufacturing process, they are assumed to remain constant long enough.

As it was stated before, a linear model between the acceleration measured in the sensitivity axis and the true acceleration in the same system is considered. Thus:

$$\tilde{\mathbf{a}}^a = \mathbf{K}_a \mathbf{a}^a + \mathbf{b}_a \quad (2)$$

In equation (2), $\tilde{\mathbf{a}}^a$ is the measure obtained by the accelerometer (measured in bits), $\mathbf{K}_a$ is a diagonal matrix that represents the sensitivity of each axis and $\mathbf{b}_a$ is a vector that represent the bias of each axis. Those parameters may variate

[1]Figure taken form [2]

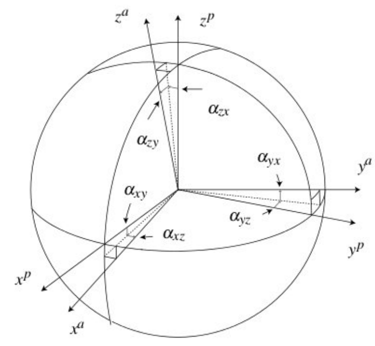


Fig. 3: **Non orthogonality -** Rotations of the sensitivity axis of the accelerometer over the axis of the platform system.

as the temperature changes. However, constant temperature is considered. Using equations (1) and (2) a model of the accelerometer can be established:

$$\tilde{\mathbf{a}}^a = \mathbf{K}_a \mathbf{T}_p^a \mathbf{a}^p + \mathbf{b}_a = \mathbf{K}_a (\mathbf{T}_a^p)^{-1} \mathbf{a}^p + \mathbf{b}_a \quad (3)$$

B. Gyroscope

The error sources considered in this case are the same as those considered for the accelerometer. Thus, the same model is considered:

$$\tilde{\boldsymbol{\omega}}_a = \mathbf{K}_\omega (\mathbf{T}_\omega^p)^{-1} \boldsymbol{\omega}^p + \mathbf{b}_\omega \quad (4)$$

III. Calibration method proposed

The problem of calibration is to establish the values for the unknown parameters of the model that adjust "the better" a certain set of data. This criterion is defined in Section (III-A).

A. Static Parameter Calibration

Let $\boldsymbol{\theta}_s$ (where the subindex s refers to a sensor) be the parameter vector of a certain sensor. This vector can be defined as:

$$\begin{aligned} \boldsymbol{\theta} \quad = \quad & [k_{sx}, k_{sy}, k_{sy}, b_{sx}, b_{sy}, b_{sy}, \\ & \alpha_{sxy}, \alpha_{sxz}, \alpha_{syx}, \alpha_{syz}, \alpha_{szx}, \alpha_{szy}]^T \end{aligned} \quad (5)$$

In equation (6), k_{si}, with $i = 1, 2, 3$ are the diagonal elements of the matrix $\mathbf{K}_s$, b_{si}, with $i = 1, 2, 3$ are the elements of vector $\mathbf{b}_s$ and α_{sij}, with $i = 1, 2, 3,\ j = 1, 2, 3$ and $i \neq j$ are the elements outside the diagonal of the matrix $\mathbf{T}_s^p$.

The chosen criterion is to minimize the sum of the squares of the norms of the differences between true acceleration and measured acceleration. This problem can be written as follows:

$$\boldsymbol{\theta} = \arg\min_{\boldsymbol{\theta}} \{ \sum_{i=1}^{M} ||\mathbf{s}_i^p - \mathbf{T}_s^p (\mathbf{K_s})^{-1} (\tilde{\mathbf{s}}_i^s - \mathbf{b}_s)||^2 \} \quad (6)$$

In equation (6), M is the cardinal of the training set, $\mathbf{s}_i^p$ is the true magnitude (acceleration or angular speed) and $\tilde{\mathbf{s}}_i$ are the values given by the sensor.

It is important to consider that the minimize function may have more than one local minimum. To ensure that the solution obtained is the desired one, the seed for the algorithm must be carefully chosen. Datasheet values of the parameters are chosen as seed for the optimization.

1) Accelerometer: Since an accelerometer measures the acceleration in a free fall referential, in static equilibrium it measures an acceleration that has as norm the gravitational constant of the earth ($g = 9.81ms^{-2}$), its direction is collinear with the line defined by the position of the accelerometer and the center of the earth and its sense is from the center of the earth towards the position of the accelerometer, meaning "up". This allows to have an exact expression for the acceleration, the procedure used consists in measuring magnitude with different orientations of the acceleration and then solve the problem stated in equation (6).

The 27 measures made were the following:

- Starting position: x axis pointing "down": 9 measures were made combining two rotations. First, rotating around the z axis $\theta = 0°, 10°, 20°, 30°, 45°$. Then, for each angle defined (except for $\theta = 0°$), two measures were made rotating around the x axis $\psi = 0°, -45°$. This set of measures can be expressed in the following way:

$$\mathbf{a}^p = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & \sin\psi \\ 0 & -\sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -g \\ 0 \\ 0 \end{pmatrix} \tag{7}$$

- Starting position: y axis pointing "up": 9 measures were made combining two rotations. First, rotating around the x axis $\psi = 0°, 10°, 20°, 30°, 45°$. Then, for each angle defined (except for $\theta = 0°$), two measures were made rotating around the y axis $\varphi = 0°, 45°$.

$$\mathbf{a}^p = \begin{pmatrix} \cos\varphi & 0 & \sin\varphi \\ 0 & 1 & 0 \\ -\sin\varphi & 0 & \cos\varphi \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & \sin\psi \\ 0 & -\sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} 0 \\ g \\ 0 \end{pmatrix} \tag{8}$$

- Starting position: z axis pointing "up": 9 measures were made combining two rotations. First, rotating around the y axis: $\phi = 0°, -10°, -20°, -30°, -45°$. Then, for each angle defined (except for $\phi = 0°$), two measures were made rotating around the z axis $\theta = 0°, 45°$.

$$\mathbf{a}^p = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\varphi & 0 & \sin\varphi \\ 0 & 1 & 0 \\ -\sin\varphi & 0 & \cos\varphi \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ g \end{pmatrix} \tag{9}$$

In order to achieve precision, each measure is obtained as the average of 1717 samples (obtained in less than 20 seconds).

The 27 measures are separated in two sets: 14 used for optimization and 13 as test set.

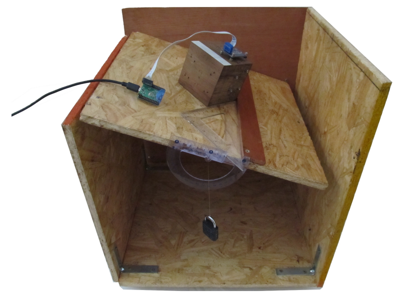


Fig. 4: **Adjustable table**- Used for accelerometer calibration

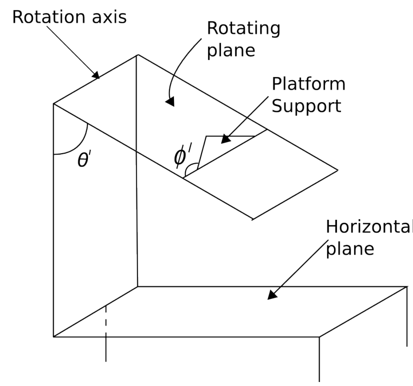


Fig. 5: **Adjustable table**- Scheme

The key issue is knowing the orientation of the platform. Having solved this, it is easy to compute the theoretical acceleration that the device should measure in each axis from equations (7), (8) and (9).

Experimental setup: In order to perform the measures, a very simple experimental setup was developed: an adjustable table with a rotating plane (see Figures 4 and 5).

First of all, the platform was attached to a wooden cube. This allows to perform 90° rotations. Having an horizontal surface, measures with the gravity vector aligned to any of three axis, can be easily taken. The horizontal surface was assured using a three leg adjustable table (see Figure (4)). In addition this table allows us to perform rotations of known angles using a scaled semicircle and a pendulum as it is shown also in Figure (4). The rotations over the second axis were made using a carpenter's square, therefore a proper 45° rotation is achieved.

As it was stated in (II) the problem (6) may have more than one local minimum, thus it is needed to assure that the seed of the algorithm is "near" of the values of the parameters. The proposed way to generate such a seed is to use some information of the sensor's datasheet [6]:

- The typical sensitivity for each axis is $256bits/g \approx 26.10bits/m^{s-2}$.
- The typical offset for each axis is $0ms^{-2}$.
- As the axis should be as orthogonal as possible is

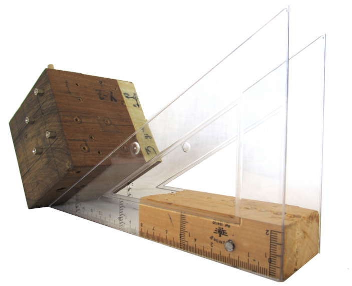


Fig. 6: **Gyroscope setup -** Carpenter's squares were used to assure an exact orientation

reasonable to think that the rotation angles α_{aij} are near zero.

After this considerations the chosen seed is:

$$\theta_0 = [26.10, 26.10, 26.10, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]^T \quad (10)$$

2) Gyroscope: The procedure used to calibrate the gyroscope was very similar to the accelerometer. The main idea is to obtain a constant and known angular speed vector and then change the orientation of the gyroscope to have different measures in each axis.

In this case the constant angular speed was provided with a turntable ($33rev/min$). Its angular speed was measured just to be sure that it was working properly. Different orientations were obtained using the wooden cube mentioned in Section (III-A1),to assure $90°$ rotations and pairs of carpenter's squares to assure $30°$, $45°$ and $60°$ rotations. In Figure (6) is shown the setup that was mounted over the turntable.

Each measure was performed averaging 1934 samples, taken in less than 20 seconds.

According the information in the device's datasheet [7]:

- The typical sensitivity for each axis is $14,375bits/(°s^{-1}) \approx 823.63bits/(rads^{-1})$.
- The typical offset for each axis is $0rads^{-1}$.
- As the axis should be as orthogonal as possible is reasonable to think that the rotation angles $\alpha_{\omega ij}$ are near zero.

After this considerations the chosen seed is:

$$\theta_0 = [823.63, 823.63, 823.63, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]^T \quad (11)$$

B. Temperature adjust

In this work it is also proposed to implement a temperature adjustment. The sensors are exposed to the operating temperature of the system, and it is very common that this temperature is different from the calibration temperature. The thermometer used in this work is the internal thermometer of the IMU, which indicates very precisely the operating temperature of the sensor. Sensors based in MEMS technology are sensitive to such variations. The variation of the measures is critical in some applications. In INS two of the Euler angles are usually obtained from the accelerometer, therefore if the measures are not certain the stability of the system cannot be guaranteed.

It is extremely important to understand which of the parameters are affected based on the information of the sensors' datasheets ([6] and [7]):

- Sensitivity variation of the accelerometer: $\pm 0.01\%/°C$
- Offset variation of the accelerometer: $\pm 1.2mg/°C \approx 0.012ms^{-1}$
- Sensitivity variation of the gyroscope: No information is provided.
- Offset variation of the gyroscope in the whole operating range ($-40°C$ to $85°C$): $\pm 40°s^{-1} \approx 0.70rads^{-1}$

The conclusion of this sum up information is that the variations of the temperature do not produce main changes on the gains of the sensors, yet the offsets are modified. Thus, the following temperature adjust is considered:

$$\mathbf{b}_s(T) = \mathbf{b}_{s0} + \boldsymbol{\alpha}_s(T - T_0) \quad (12)$$

In equation (12) $\mathbf{b}_{s0}$ is the offset vector obtained in the static calibration, T_0 is the ambiance temperature in that calibration, T is the working temperature of the system and $\boldsymbol{\alpha}_s$ is the parameter that is needed to know to complete the process.

The procedure to achieve this adjust was to take several samples of acceleration (or angular speed) in a fixed position (z axis pointing "up" without any rotation), while the sensor was heated up with a hair dryer. The criterion chosen was that $\boldsymbol{\alpha}_s$ must minimize the sum of the squares of the norms of the difference between the true acceleration (or angular speed) and the measured acceleration (or angular speed). Mathematically this means:

$$\boldsymbol{\alpha}_s = \arg\min_{\boldsymbol{\alpha}_s} \{ \sum_{k=1}^{N} ||\mathbf{s}_i^p - \mathbf{T}_{s0}^p (\mathbf{K_{s0}})^{-1} (\tilde{\mathbf{s}}_i^s - \mathbf{b}_s(T))||^2 \} \quad (13)$$

In equation (13), $\mathbf{T}_{s0}^p$, $\mathbf{K_{s0}}$ are the matrix obtained in the static parameter calibration (ambiance temperature), $\mathbf{b}_s(T)$ is the vector defined in (12) and $N = 22740$ is the number of measured samples. There is no need of so many samples, yet all the data of the cooling process was used (see figure 8). Vector $\mathbf{b}_s(T)$ includes the variable to be estimated.

It is important to clarify that to perform a perfect temperature compensation, variations of temperatures should be produced with the sensors in different orientations. However, the most important is that the platform is in horizontal position with the z axis pointing "up". This is because this particular position is the main operating situation in the application for which this calibration method was thougth.

IV. Results and analysis

A. Static parameters

In this section, the parameters that solve problem 6 are presented for both accelerometer and gyroscope. In addition, the statistic over the test set is studied and analyzed.

1) Accelerometer: The parameter vector found in this case is:

$$\begin{aligned}\boldsymbol{\theta}_a \; = \; & [26.70, 27.26, 26.02, 16.05, -1.79, -47.99, \\ & 3,78 \times 10^{-4}, 1.40 \times 10^{-3}, 1.63 \times 10^{-2}, 4.62 \times 10^{-3}, \\ & -2.30 \times 10^{-3}, -6.81 \times 10^{-3}]^T \end{aligned} \tag{14}$$

It is interesting to compare the values obtained with the theoretical values of each parameter. As can be seen, comparing the vector obtained with the seed, the values of the gains and the rotations are similar to what is declared in the datasheets. Furthermore, the offset values are also consistent with the information of the datasheet, yet this is not a straightforward conclusion, thus the following analysis is presented. In [6] is declared that the $0g$ output is $\pm 150mg$ for axes x and y and $\pm 250mg$ for axis z. This correspond to offsets of 39 bits maximum for x and y, and 64 for z. Therefore, can concluded that the results obtained are consistent with what is declared in the datasheets.

The mean error, and the standard deviation founded with the calibration method proposed in the test set are:

$$\mu_a = -0.03ms^{-2}, \; \sigma_a = 0.06ms^{-2} \tag{15}$$

The mean error obtained correspond to less that a bit of error and the standard deviation to a bit and a half. Therefore can concluded that a very good calibration has been achieved for the static parameters of the accelerometer.

2) Gyroscope: For the gyroscope the vector of parameters obtained is the following:

$$\begin{aligned}\boldsymbol{\theta}_\omega \; = \; & [792.44, \; 808.61, \; 799.44, \; -33.52, \; -31.41, \\ & -1.07, \; 0.02, \; 0.06, \; -1.32 \times 10^{-3}, \; 2.59 \times 10^{2}, \\ & -3.25 \times 10^{-2}, \; 6.07 \times 10^{-1}]^T \end{aligned} \tag{16}$$

As can be noticed the gains are very similar to the ones that are declared on the datasheets. The maximum relative error obtained is:

$$\varepsilon_{\omega gain} = \frac{|823.63 - 792.44|}{823.63} = 3.8\% \tag{17}$$

On the other hand, the typical offset error is $\pm 40°s^{-1}$, this correspond to an error of $\pm 574bits$. All the offsets estimated are smaller than this value, representing in the worst case only the 5.8% than the maximum declared in the datasheets.

Last but not least, the statistical results over the test set are the following:

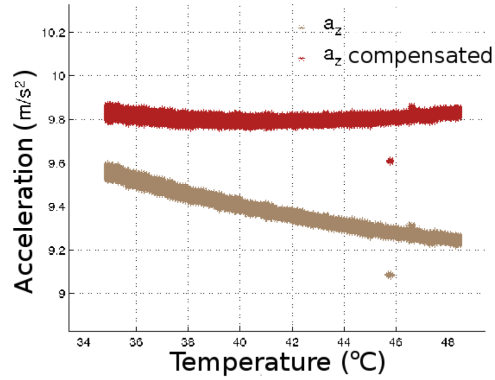


Fig. 7: Accelerometer measurement in z-axis vs. temperature while heating up the sensor.

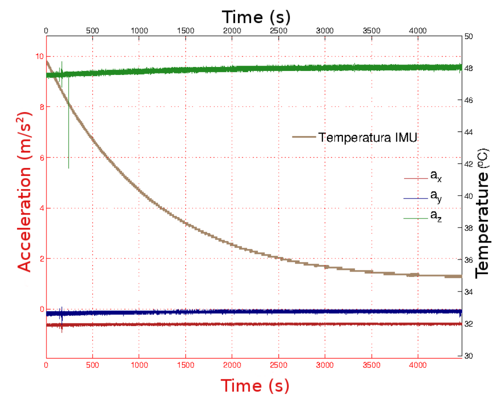


Fig. 8: Accelerometer measurements and temperature (T) vs. time while T varying from 35^oC to 48^oC.

$$\mu_\omega = -3.50 \times 10^{-3} rad/s \, \sigma_\omega = 4.57 \times 10^{-2} rad/s \tag{18}$$

The mean and the standard deviation obtained correspond to an error of approximately 3 and 30 bits respectively.

As can be seen the error obtained, in terms of bits, is bigger for the gyroscope than for the accelerometer. The main reason for this to happen is that in this case the scale of the experiment is far from optimal, yet the only one available in the sensor. The maximal angular speed measured ($198°/s$) is approximately a tenth of the scale of the gyroscope ($2000°/s$) while for the accelerometer the maximal acceleration measured (g) is the half of the scale used ($2g$).

B. Temperature adjust

To evaluate the behavior of the measurement of the accelerometer and compensate the error caused by temperature, the experiment described in Section (III-B) is carried out. It can be divided in two separate parts, the first is heating up the sensor using a hair dryer up to 48^oC and the second part is letting the sensor cool down to normal operating temperature. The two separated parts of the experiment are shown in Figures (7) and (8).

In Figure (7) is shown the behavior of the modulus of the calibrated measurement of the accelerometer in z-axis while the sensor is heated up, both with and without temperature compensation. It is clearly shown that the compensated temperature is much more reliable than the other one, that is varying a 3% of the highest value.

In Figure (8) two different magnitudes are shown at the same time: the three projections of the acceleration and the temperature vs. time, thus two axes are needed to be shown: on left side the acceleration in ms^{-2}, and on right side the temperature in ^oC.

In Figure (7) can be observed that during the experiment, the uncompensated estimation of g is $9.25ms^{-2}$ instead of $9.81ms^{-2}$, which means an error of $0.56ms^{-2}$. Such an error in acceleration measurements can cause severel errors in applications where the operating temperature can differ from the calibration temperature, thus, is vital that temperature compensation is performed.

V. Conclusion

A very easy, fast and low cost procedure for calibrating inertial measurement sensors was presented. It combines non-linear least squares adjustment of sensor parameters and misalignment angles, a very simple and inexpensive experimental setup, and a simple adjustment of temperature bias' drift.

It was successfully implemented and tested to perform the calibration of the sensors unit (IMU) used to estimate the state variables of a quadrotor. Experimental results show a very precise determination of the acceleration and angular velocity. This precision is achieved also during a wide variation of operating temperature of the sensors. This allows satisfactory estimation for flight control in a wide range of operating conditions.

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