Elective in Robotics

Quadrotor Modeling

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- Introduction
- Description of the HummingBird
- Modeling
- Control Problems
- Models for control
- Main control approaches

Hummingbird Ascending Technologies GmbH

CEA Quadrotor

DraganFlyer X4 Draganfly Innovations Inc.

Mesicopter Stanford University

applications

- environmental monitoring
- exploration
- communication networks
- search and rescue operations

microdrones GmbH

STARMAC Stanford University

all these activities require

vertical, stationary, slow flight

a quadrotor is characterized by

- high maneuverability
- vertical take-off and landing (VTOL)
- hovering

Pelican

MIT in coll. with Ascending Technologies GmbH

- four motors located at the extremities of a cross-shaped frame
- controlled by varying the angular speed of each rotor

force distribution

$$f_i = b\,\omega_i^2 \qquad i = 1, \ldots, 4$$

$$\tau_{R,i} = d\,\omega_i^2$$

- b thrust factor
- d drag factor

actuation

a low level controller stabilizes the rotational speed of each blade

configuration

- $(x,\, y,\, z)$ position of SR_B origin in SR_I
- $(\varphi,\, \vartheta,\, \psi)$ RPY angles expressing the orientation of SR_B w.r.t. SR_I

$$
{}^{\mathbf{I}}\mathbf{R}_{\mathbf{B}} = \begin{pmatrix} c_\psi c_\vartheta & c_\psi s_\vartheta s_\varphi - s_\psi c_\varphi & c_\psi s_\vartheta c_\varphi + s_\psi s_\varphi \\ s_\psi c_\vartheta & s_\psi s_\vartheta s_\varphi + c_\psi c_\varphi & s_\psi s_\vartheta c_\varphi - s_\varphi c_\psi \\ -s_\vartheta & c_\vartheta s_\varphi & c_\vartheta c_\varphi \end{pmatrix}
$$

equation of motion

dynamics of a rigid body with mass m subject to external forces applied to the CoM according to Newton-Eulero formalism

translational dynamics in SR$_I$ $\boxed{\sum F_I = m\dot{V}_I}$

F_I external force applied to the com and expressed in SR$_I$

$V_I = (v_x,\, v_y,\, v_z)'$ velocity of the com expressed in SR$_I$

rotational dynamics in SR$_B$ $\boxed{\sum M_B = J\dot{\Omega} + \Omega \times J\Omega}$

$M_B,\, J$ external moment around com and inertia tensor expressed in SR$_B$

$\Omega = (\, p,\, q,\, r)'$ rotational velocity expressed in SR$_B$

inertia matrix and rotational velocity

$$J = \begin{pmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{pmatrix} \qquad \Omega = \begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 1 & 0 & -s_\vartheta \\ 0 & c_\varphi & c_\vartheta s_\varphi \\ 0 & -s_\varphi & c_\vartheta c_\varphi \end{pmatrix} \begin{pmatrix} \dot{\varphi} \\ \dot{\vartheta} \\ \dot{\psi} \end{pmatrix}$$

control inputs

$$T = f_1 + f_2 + f_3 + f_4$$

$$\tau_\varphi = l(f_2 - f_4)$$

$$\tau_\vartheta = l(f_1 - f_3)$$

$$\tau_\psi = -\tau_{R,1} + \tau_{R,2} - \tau_{R,3} + \tau_{R,4}$$

applied forces and moments

$$\sum F_I = \begin{pmatrix} 0 \\ 0 \\ mg \end{pmatrix} + {}^{\mathbf{I}}\mathbf{R}_{\mathbf{B}}(\varphi, \vartheta, \psi) \begin{pmatrix} 0 \\ 0 \\ -T \end{pmatrix} + F_A + F_D$$

weight

actuation

aerodynamics

disturbances

$$\sum M_B = \begin{pmatrix} L_\varphi \\ L_\vartheta \\ L_\psi \end{pmatrix} + \begin{pmatrix} \tau_\varphi \\ \tau_\vartheta \\ \tau_\psi \end{pmatrix} + \tau_A + \tau_D$$

gyroscopic effects

mathematical model of the system

$$
\begin{aligned}
\dot{x} &= v_x \\
\dot{y} &= v_y \\
\dot{z} &= v_z \\
\dot{v}_x &= F_{A,x} - (\cos(\psi)\sin(\vartheta)\cos(\varphi) + \sin(\psi)\sin(\varphi))\frac{T}{m} \\
\dot{v}_y &= F_{A,y} - (\sin(\psi)\sin(\vartheta)\cos(\varphi) - \sin(\varphi)\cos(\psi))\frac{T}{m} \\
\dot{v}_z &= F_{A,z} + g - \cos(\vartheta)\cos(\varphi)\frac{T}{m} \\
\dot{\varphi} &= p + \sin(\varphi)\tan(\vartheta)q + \cos(\varphi)\tan(\vartheta)r \\
\dot{\vartheta} &= \cos(\varphi)q - \sin(\varphi)r \\
\dot{\psi} &= \sin(\varphi)\sec(\vartheta)q + \cos(\varphi)\sec(\vartheta)r \\
\dot{p} &= \tau_{A,x} + \frac{I_r}{I_x}q\Omega_r + \frac{I_y - I_z}{I_x}qr + \frac{\tau_\varphi}{I_x} \\
\dot{q} &= \tau_{A,y} + \frac{I_r}{I_y}p\Omega_r + \frac{I_z - I_x}{I_y}pr + \frac{\tau_\vartheta}{I_y} \\
\dot{r} &= \tau_{A,z} + \frac{I_x - I_y}{I_z}pq + \frac{\tau_\psi}{I_z}
\end{aligned}
$$

$$\dot{\xi} = f(\xi) + g(\xi)\,u$$

state

$$\xi = (x,\, y,\, z,\, v_x,\, v_y,\, v_z,\, \varphi,\, \vartheta,\, \psi,\, p,\, q,\, r)'$$

inputs

$$u = (T,\, \tau_\varphi,\, \tau_\vartheta,\, \tau_\psi)'$$

control problems

- attitude control
- eight control
- position control
- trajectory tracking
- sensor-based control

simplified model for control design

negligible

- aerodynamics
- gyroscopic effects

assuming

- small φ and $\vartheta \Rightarrow (\dot{\varphi}, \dot{\vartheta}, \dot{\psi}) \simeq (p, q, r)$
- symmetric shape
- negligible disturbances

$$
\begin{aligned}
\ddot{x} &= -(\cos(\psi)\sin(\vartheta)\cos(\varphi) + \sin(\psi)\sin(\varphi))\frac{T}{m} \\
\ddot{y} &= -(\sin(\psi)\sin(\vartheta)\cos(\varphi) - \sin(\varphi)\cos(\psi))\frac{T}{m} \\
\ddot{z} &= -\cos(\vartheta)\cos(\varphi)\frac{T}{m} + g \\
\ddot{\varphi} &= \frac{\tau_\varphi}{I_x} \\
\ddot{\vartheta} &= \frac{\tau_\vartheta}{I_y} \\
\ddot{\psi} &= \frac{\tau_\psi}{I_z}
\end{aligned}
$$

control system

attitude control

determine the torques $\tau_\varphi, \tau_\vartheta, \tau_\psi$ necessary to obtain a stable desired attitude φ_d, ϑ_d, ψ_d

$$\tau_\varphi = [K_{\varphi p}(\varphi_d - \varphi) + K_{\varphi d}(\dot{\varphi}_d - \dot{\varphi})]$$

$$\tau_\vartheta = [K_{\vartheta p}(\vartheta_d - \vartheta) + K_{\vartheta d}(\dot{\vartheta}_d - \dot{\vartheta})]$$

$$\tau_\psi = [K_{\psi p}(\psi_d - \psi) + K_{\psi d}(\dot{\psi}_d - \dot{\psi})]$$

height control

determine the thrust T necessary to bring and keep the quadrotor to a desired height $z_d \Rightarrow$ from the z dynamics:

$$T = \frac{m}{\cos(\vartheta)\cos(\varphi)}[g - K_{zp}(z_d - z) - K_{zd}(\dot{z}_d - \dot{z})]$$

simulation results: error

φ error

ϑ error

ψ error

z error

simulation results: control inputs

T

τ_φ

τ_ϑ

τ_ψ

- advanced nonlinear control techniques guarantee better convergence and robustness performance
- exteroceptive sensors (camera, laser, sonar) allow indoor flight and can be used to obtain an accurate estimation of the system state