Notes on Physics II (Z2624)

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1 Ideal Gases

Some notions and properties about ideal gases should be stated before diving into thermodynamics and its principles.

1.1 General Gas Equation

$$P\,V = n\,R\,T \quad \text{where } R = 8.314\ \frac{J}{K\ mole} \tag{1}$$

The units used are in this equation are Pa, m^3, $mole$ and K for that value of R. The number of moles can be calculated as:

$$n^\circ\ of\ moles = \frac{m_{gas}}{m_{mole}} \quad \text{where } m_{gas} \text{ is the mass of the gas and } m_{mole} = 1.66 \times 10^-27\ Kg \tag{2}$$

1.2 Specific Volume

$$P\,V = R\,T \quad \text{where } R = 8.314\ \frac{J}{K\ mole} \tag{3}$$

1.3 Internal Energy of an Ideal Gas

The internal energy is defined as the sum of the kinetic and potential energy of each and every particle that forms the gaseous system.

For *monatomic* gases there is no energy associated with bonds between atoms in its molecules and rotational kinetic energy is ignored since the atoms are so small that the moment of inertia can be thought as negligible:

$$U = \frac{3}{2}\,n\,R\,T = \frac{3}{2}\,m_{gas}\,\frac{P}{\delta} \quad \text{where } \delta = \frac{m_{gas}}{V} \tag{4}$$

For ideal gases formed by *diatomic* molecules bonds between atoms and rotational kinetic energy have to be taken into account, therefore U happens to be larger:

$$U = \frac{5}{2}\,n\,R\,T = \frac{5}{2}\,m_{gas}\,\frac{P}{\delta} \quad \text{where } \delta = \frac{m_{gas}}{V} \tag{5}$$

For ideal gases formed by *polyatomic* molecules:

$$U = \frac{6}{2}\ n\ R\ T = \frac{6}{2}\ m_{gas}\ \frac{P}{\delta} \quad \text{where } \delta = \frac{m_{gas}}{V} \tag{6}$$

1.4 Work Done by an Ideal Gas

The amount of work depends on the initial and final state and the path.

$$\int_{x_i}^{x_f} F(x)dx = \int_{x_i}^{x_f} P(x)dv \quad \text{(general formula)} \tag{7}$$

For *isothermal* processes T is constant, therefore $\Delta T = 0$ and $\Delta U = 0$:

$$W_{isothermic} = n\ R\ T\ \ln \frac{V_f}{V_i} \quad \text{(isothermic process)} \tag{8}$$

For *isobaric* processes P is constant:

$$W_{isobaric} = P\ (V_f - V_i) \quad \text{(isobaric process)} \tag{9}$$

For *isochoric* processes V is constant:

$$W_{isochoric} = 0 \quad \text{(isochoric process)} \tag{10}$$

For *adiabatic* processes Q is zero:

$$W_{adiabatic} = \frac{3}{2}\ (P_f V_f - P_i V_i) \quad \text{(adiabatic process)} \tag{11}$$

Note that in this special case equation 1 does not suit. Another identity should be used instead:

$$P_f V_f^\gamma = P_i V_i^\gamma \quad \text{where } \gamma = \frac{5}{3} = \frac{C_P}{C_V} \tag{12}$$

The variables C_P and C_V stand for capacitance at constant pressure and volume respectively, and would be defined in the following section.

2 Thermodynamics

Heat is defined as the thermal energy that flows from two or more systems with different temperatures when they are put in contact. This heat is always transferred from higher temperatures to lower temperatures until an equilibrium is achieved.

2.1 Specific Heat Capacity

$$C = \frac{Q}{\Delta T\ m} \quad \text{measured in } \frac{cal}{^\circ C\ g} \tag{1}$$

Where Q is the flow of heat, m is mass, c is the specific heat (a pseudo thermal inertia), and T is the temperature.

2.2 Heat Capacity

$$C = \frac{Q}{\Delta T} \quad \text{measured in } \frac{cal}{^\circ C} \tag{2}$$

2.3 Latent Heat

Surplus heat which can cause a phase change, but does not change the system's temperature.

$$L = \frac{Q}{m} \quad \text{measured in } \frac{cal}{g} \tag{3}$$

2.4 First Law of Thermodynamics

This law can be thought of as a conservation of energy equation. It relates changes in the internal energy of a gas to the heat added to a system and the work done by that system.

$$Q = \Delta U + W \quad \text{measured in } \textit{Joules} \text{ (1 cal = 4.1858 J)} \tag{4}$$

2.5 Heat Capacity at Constant Volume $(W = 0)$

$$C_V = \frac{3}{2} n R T \quad \text{tempearute in } K \text{ for the stadard value of } R \tag{5}$$

2.6 Heat Capacity at Constant Pressure

$$C_P = \frac{5}{2} n R T \quad \text{tempearute in } K \text{ for the stadard value of } R \tag{6}$$

Note that **the two formulas stated above (5 and 6) apply only for a monatomic ideal gases**. For all gases, though, this relation holds true:

$$R = C_P - C_V \tag{7}$$

2.7 Second Law of Thermodynamics

No process is possible whose sole result is the absorption of energy in the form of heat (from a thermal reservoir) and the conversion of this heat into an equivalent amount of work.
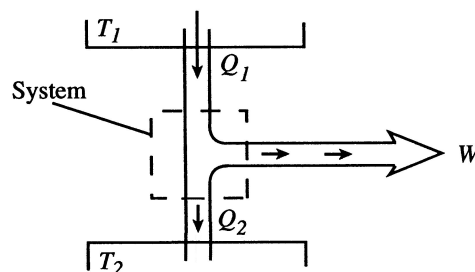

Figure 1: Heat engine

if performance is defined as $\eta = \frac{benefit}{cost}$, then in terms of the second law:

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} \quad \text{where } \eta < 1 \tag{8}$$

2.8 Performance of a Carnot Cycle

$$\eta = 1 - \frac{T_2}{T_1} \quad \text{(best possible performance)} \tag{9}$$

2.9 Benefit of a Reversed Carnot Cycle

Picture a heat engine as shown in figure 1 where the inputs are W and Q_2 and the output is Q_1, then:

$$\varepsilon = \frac{Q_2}{W} = \frac{Q1 - W}{W} = \frac{Q_1}{W} - 1 = \frac{1}{\eta} - 1 \quad \text{(refrigeration cycle)} \tag{10}$$

2.10 Entropy for a Reversible Process

Entropy is a measure of the level of disorder of a system. There is no net change in entropy for reversible processes, therefore:

$$\Delta S = \frac{Q_{(reversible)}}{T} \quad \text{measured in } \frac{J}{K} \tag{11}$$

Since the entropy in the universe is continually increasing, for cyclic reversible processes:

$$\Delta S = \frac{Q_1}{T_1} + \frac{Q_2}{T_2} + \ldots \quad \text{measured in } \frac{J}{K} \tag{12}$$

2.11 Thermal Expansion

If the length of an object is defined as L and the temperature of that object changes by a differential amount dT, then the differential change in the length dL is given by:

$$dL = \alpha \, L \, dT$$

With this notion in mind, it could be stated that:

$$\Delta L \simeq \alpha \, L \, \Delta T \tag{13}$$

Where α stands for the coefficient of thermal expansion of the material that composes the body.

Additionally, If the volume of an object is defined as V and the temperature of that object changes by an amount ΔT, then the change in the volume ΔV of this object is given by:

$$\Delta V \simeq \gamma \, V \, \Delta T \tag{14}$$

Where $\gamma = 3 \, \alpha$ since this expansion is the three-dimensional representation of equation 13.

2.12   Thermal Resistance

Thermal resistance is a heat property and a measurement of a temperature difference by which an object or material resists a heat flow.

This thermal resistance, noted as R_t, depends on the length L (measured in m), the thermal resistivity ρ_t of the material and its cross-sectional area A (measured in m^2) and can be calculated as:

$$R_t = \rho_t \, \frac{L}{A} = \frac{1}{\sigma} \frac{L}{A} \quad \text{measured in } \frac{^\circ C}{Watt} \tag{15}$$

In order to introduce another concept, let $P = \frac{Q}{t}$ (measured in Watts, $\frac{Joules}{sec}$) be the amount of heat transferred per unit of time. Then, in an analogy to Ohm's law a relationship between this variable P, the temperature (measured in $^\circ C$) and the thermal resistance (measured in $\frac{^\circ C}{Watts}$) can be stated:

$$\Delta T = P \, R_t \tag{16}$$

3   Electrostatic

3.1   Charge and Electric Field

Some basic notions to better understand the formulas:

- The net charge of an isolated system is constant (conservative).
- Charges can be positive or negative.
- Like charges repel, unlike charges attract.

3.1.1   Coulomb's Law

Given two charges Q and q (measured in *Coulombs* $\frac{A}{sec}$) separated by a distance r in m then,

$$\vec{F} = K \times \frac{Q \, q}{r^2} \, \hat{r} \quad \text{where } K = \frac{1}{4\pi\varepsilon_0} \approx 9 \times 10^9 \, \frac{N \, m^2}{C^2} \tag{1}$$

In this equation ε_0 stands for permittivity of free space (vacuum permittivity) and its value is approximately $8.85 \times 10^{-12} \, \frac{C^2}{N \, m^2}$.

From this concept we can calculate the permittivity of other mediums (relative epsilon) as $\varepsilon = k\varepsilon_0$.

3.1.2   Electric Field

From the equation 1 enunciated above we can state that given a generic charge q

$$\vec{E} = \frac{\vec{F}}{q} = \frac{K \, Q}{r^2} \, \hat{r} \quad \text{measured in } \frac{N}{C} \tag{2}$$

Note: The electric field is always perpendicular to the surface of the conductor. $\vec{E}$ from a positive charge points away from the charge; $\vec{E}$ from a negative charge points towards the charge.

From this notion, if we know the value of the electric field $\vec{E}$ generated by a charge Q we can calculate the electrical force among this one and another charge q at a point inside the field by simply multiplying this magnitude $\vec{E}$ times the latter charge:

$$||\vec{F}|| = q \, ||\vec{E}|| \tag{3}$$

The equation shows how if q is positive, the force is in the same direction as the field and if q is negative, the force is in the opposite direction as the field.

3.1.3 Electric Flux

Electric flux can be defined a the number of electric field lines that penetrates a given surface S.
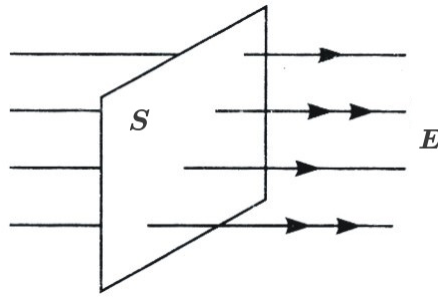
Figure 2: Electric Flux

Consider the surface shown in figure 2 where the normal vector $\vec{n}$ to the surface is parallel to the electric field, this latter one being uniform. In this case the electric field $\vec{E}$ penetrates the whole surface, thus:

$$d\Phi_E = \vec{E} \, d\vec{S}$$

$$\Phi_E = \vec{E} \, \vec{S} \quad \text{(dot product)}$$

if the electric field makes an angle θ with the normal vector $\vec{n}$ to the surface, the electric flux becomes:

$$\Phi_E = \vec{E} \, \vec{S} = ||\vec{E}|| \, ||\vec{S}|| \, cos(\theta) \tag{4}$$

3.1.4 Gauss's Law

The net flux through any closed surface is proportional to the net charge enclosed within that closed surface. This is:

$$\Phi_E = \oint \vec{E} \, d\vec{S} = \frac{\sum Q_{enclosed} - Q_{dielectric}}{\varepsilon_0} \tag{5}$$

In the absence of a dielectric $Q_{dielectric} = 0$.

3.1.5 Electric Field for a Spheric Surface

Picture an empty sphere with a charge Q uniformly distributed along its surface then:

$$\vec{E} = \frac{Q}{4 \pi \varepsilon_0 r^2} \, \hat{r} \tag{6}$$

3.1.6 Electric Field for a Plane Surface (plate)

Suppose an infinitely large plane with charge density $\sigma = \frac{Q}{A}$, then, independently of the distance my test charge is from it the electric field is defined as:

$$\vec{E} = \frac{\sigma}{2 \varepsilon_0} \, \hat{r} \tag{7}$$

What happens with two planes separated by a tiny distance? The electric field outside the planes is equal to zero, whereas inside them $\vec{E}$ is given by:

$$\vec{E} = \frac{\sigma}{\varepsilon_0} \, \hat{r} \tag{8}$$

3.1.7 Electric Field for a Rod (cylinder)

Picture a rod with length L, radius R (much smaller than L) and charge density $\lambda = \frac{Q}{L}$. Then S could be defined as the perimeter of the base times 2π, times the length L. This is: $S = 2 \pi R L$.

Then from Gauss's Law (equation 5):

$$\vec{E} = \frac{\lambda}{2 \pi \varepsilon_0 R} \, \hat{r} \tag{9}$$

3.2 Work and Energy

3.2.1 Electrostatic Potential Energy

The potential energy can be thought of as the work that needs to be done to place a charge inside an electric field.

Picture a static charge Q generating an electric field, and a test charge q placed at a distance r_a from the first one. In order to move this charge from this initial point to a point with distance r_b work has to be done. Denominated with the letter U, the potential energy can be calculated as the work done through the electric field to move q from r_a to r_b:

$$\Delta U = W_{Point} = \int_{r_a}^{r_b} \vec{F_{Point}} \, \vec{dr}$$

$$\Delta U = W_{Point} = - \int_{r_a}^{r_b} \vec{F_{Elec}} \, \vec{dr} \quad \text{general formula} \tag{10}$$

Note that F_{Elec} and F_{Point} are equal in magnitude but have opposite direction thus the negative sign in the expression.

Let an electric field relate two point charges, then:

$$\Delta U = W_{Point} = \frac{Q\,q}{4\,\pi\,\varepsilon_0} \int_{r_a}^{r_b} \frac{\vec{dr}}{r^2}$$

$$\Delta U = W_{Point} = \frac{Q\,q}{4\,\pi\,\varepsilon_0} \left[\frac{1}{r_b} - \frac{1}{r_a} \right] \quad \text{measured in Joules}$$

Note that the work done is independent of the path and that if the initial distance r_a happens to be infinity (or a very large value) then:

$$U_b = W_{Point} = \frac{Q\,q}{4\,\pi\,\varepsilon_0\,r_b}$$

3.2.2 Electric Potential

The electric potential (voltage) of a charge is a measure of the potential energy of it per unit charge.

$$V_{r_b} = \frac{W_{Point}}{q} = \frac{U}{q} = \vec{E}\,r_b$$

This is:

$$\Delta V_{r_a r_b} = V_{r_b} - V_{r_a} = - \int_{r_a}^{r_b} \vec{E}\,dl \tag{11}$$

Suppose now that r_a happens to be infinite. Then this equation could be rewritten as:

$$\Delta V_{r_b r_a} = V_{r_b} - V_\infty$$

$$V_{r_b} - 0 = - \int_{\infty}^{r_b} \vec{E}\,dl$$

$$V_{r_b} = \frac{Q}{4\,\pi\,\varepsilon_0\,r_b} \tag{12}$$

From this equation, this important relationship can be stated:

$$U = \Delta V\,q \tag{13}$$

Suppose that the test charge q is affected by more than one charge. Lets say by a charge Q_1 and a charge Q_2, then:

$$V_{r_b} = V_{r_b Q_1 alone} + V_{r_b Q_2 alone} \tag{14}$$

This same logic applies for three, four, five, infinite charges.

3.2.3 Electric Potential and Electric Field Relationship

Electric field and electric potential have already been defined in this document but when written side by side and compared a pattern can be seen:

$$\vec{E} = \frac{Q}{4\,\pi\,\varepsilon_0\,r^2}\,\hat{r} \qquad\qquad V_{r_b} = \frac{Q}{4\,\pi\,\varepsilon_0\,r}$$

If we take $\frac{dV}{dr}$, then:

$$\frac{dV}{dr} = -\frac{Q}{4\,\pi\,\varepsilon_0\,r^2}$$

Where it can now be seen that if we multiply on both sides by $\hat{r}$ then:

$$\frac{dV}{dr}\,\hat{r} = -\frac{Q}{4\,\pi\,\varepsilon_0\,r^2}\,\hat{r}$$

Demonstrating that the gradient of the electric potential is equal to minus the electric field. Therefore, equipotential surfaces **must** be perpendicular to the field lines.

$$\vec{E} = -\nabla V \quad \text{measured in } \frac{V}{m} \tag{15}$$

$$\vec{E} = -\left(\frac{\partial V}{\partial x}\,\hat{x} + \frac{\partial V}{\partial y}\,\hat{y} + \frac{\partial V}{\partial z}\,\hat{z}\right) = -(\vec{E}_x + \vec{E}_y + \vec{E}_z)$$

4 Capacitors

A capacitor is a device for storing charge. The ability of this device to store charge is quantified in its capacitance and is defined as:

$$C = \frac{Q}{|\Delta V|} \quad \text{measured in Farads } \frac{Coulomb}{Volt} \tag{1}$$

4.1 Parallel-Plate Capacitor

The following equation is used for plane capacitors with and without dielectric:

$$C = \frac{\varepsilon_r\,A}{d} \tag{2}$$

Where d stands for the distance between the plates, A is the area of the plates and ε_r is defined as:

$$\varepsilon_r = K\,\varepsilon_0 \quad \text{in the abscence of a dielectric K = 1}$$

4.2 Spherical Capacitor

The following equation is used for spherical capacitors with and without dielectric:

$$C = 4 \pi \varepsilon_r \frac{r_b \, r_a}{r_b - r_a} \tag{3}$$

Where r_b stands for the external radius and r_a for the internal radius.

It should be noted that if $r_b \gg r_a$ then the capacitance of an spherical capacitor could be calculated as:

$$C \simeq 4 \pi \varepsilon_r \, r_b$$

4.3 Cylindrical Capacitor

The following equation is used for cylindrical capacitors with and without dielectric:

$$C = \frac{2 \pi \varepsilon_r \, L}{ln \left(\frac{r_b}{r_a} \right)} \tag{4}$$

Where L stands for the length of the cylinder, r_b for the external radius and r_a for the internal radius.

4.4 Energy Stored in a Capacitor

The energy stored in a capacitor is the same as the work needed to build up its charge. In the process of charging the capacitor, an electric field is also created and it is here where the energy is stored.

The Energy Stored in a Capacitor:

$$U = \frac{1}{2} C \, |\Delta V|^2 \tag{5}$$

The Energy Density in Electric Field is given by:

$$u_E = \frac{1}{2} \varepsilon_r \, E^2 \tag{6}$$

4.5 Parallel Connection

The following equation works for any number of capacitors connected in parallel:

$$C_{eq} = \sum_{i=1}^{n} C_i = C_1 + C_2 + C_3 + \cdots + C_n \tag{7}$$

In this configuration: $(V_1 = V_2 = \cdots = V_n)$ and $(Q_{eq} = \sum Q_i)$.

4.6 Series Connection

The following equation works for any number of capacitors connected in series:

$$\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n} \tag{8}$$

In this configuration: $(\sum V_i = 0)$ and $(Q_1 = Q_2 = \cdots = Q_n)$.

5 Resistors

The electrical resistance of a conductor is a measure of how difficult it is for the charges to get through it. This resistance depends on the length L (measured in m) and the resistivity ρ (measured in Ωm and with $\rho = \frac{1}{\sigma}$) of the material and its cross-sectional area A (measured in m^2). Therefore we define:

$$R = \rho \, \frac{L}{A} \quad \text{measured in Ohms } (\Omega) \, \frac{Volt}{Ampere} \tag{1}$$

5.1 Parallel Connection

The following equation works for any number of resistors connected in parallel:

$$\frac{1}{R_{eq}} = \sum_{i=1}^{n} \frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n} \tag{2}$$

Note: In this configuration the electric potential (V) is the same along each resistor but the current (I) is different for each ramification of the circuit after the node (for more details view Kirchhoff's first law on page 13).

5.2 Series Connection

The following equation works for any number of resistors connected in series:

$$R_{eq} = \sum_{i=1}^{n} R_i = R_1 + R_2 + R_3 + \cdots + R_n \tag{3}$$

Note: In this configuration the current (I) is the same along the connection but the electric potential (V) is different for each resistor.

5.3 Thermal Expansion for Resistors

Analogous to what has been established in equation 13 of the thermodynamics section, we can calculate the thermal expansion of this devices as follows:

$$R_f = R_i + \alpha \, R_i \, \Delta T \quad \text{measured in } \frac{1}{^\circ C} \tag{4}$$

Where R_i stands for the initial value of the resistance and α for the coefficient of thermal expansion.

This equation shows how as temperature increases so does the resistance. On the other hand, as resistance increases current decreases.

6 Electrodynamics

6.1 Electric Current

The electric current is defined to be the rate at which charges flow across any cross-sectional area. If an amount of charge ΔQ passes through a surface in a time interval Δt, then the average current $I_{average}$ is given by:

$$I_{average} = \frac{\Delta Q}{\Delta t} \quad \text{measured in Amperes } \frac{Coulomb}{sec.} \tag{1}$$

6.2 Current Density

Current density $\vec{J}$ is the amount of charge per unit time that flows through a unit area of a chosen cross section of a conductor.

Let the total current through the conductor be written as:

$$I = \int \int \vec{J} \, dA$$

Then, current density $\vec{J}$ could be defined as:

$$\vec{J} = \frac{I}{A} \quad \text{measured in } \frac{A}{m^2} \tag{2}$$

In a microscopic level if we define q as the charge of each carrier and n as the number of carriers per unit volume, then, the total amount of charge in a cross sectional area of a conductor can be defined as $\Delta Q = q \, n \, A \, \Delta x$.

Let now the charge carriers move with a speed v_d; then the displacement in a time interval Δt will be equal to $\Delta x = v_d \, \Delta t$, which implies:

$$\vec{J} = \frac{I}{A} = n \, q \, v_d \tag{3}$$

6.3 Ohm's Law

A useful linear relationship among the electric current, the potential difference and resistance is described as Ohm's Law:

$$\Delta V = I \, R \tag{4}$$

6.4 Electric Power

Power, or rate at which energy is delivered to a resistor (energy loss because of the resistor) is given by the following relationships:

$$P = I \, \Delta V = I^2 \, R = \frac{(\Delta V)^2}{R} \quad \text{measured in Watts } \frac{Joules}{sec.} \tag{5}$$

As a side note, the power dissipated in a resistor goes into heating the resistor; this is know as Joule heating.

6.5 Kirchhoff's Junction Rule (First Law)

At any point in a circuit where there is a junction between various current carrying branches, by the current conservation principle the sum of the currents into the node must equal the sum of the currents out of the node. This is:

$$\sum I_{in} = \sum I_{out} \tag{6}$$

6.6 Kirchhoff's Loop Rule (Second Law)

The sum of the voltage drops ΔV, across any circuit elements that form a closed circuit is zero:

$$\sum_{closed\ loop} \Delta V = 0 \tag{7}$$

Note that this relationship holds regardless of the direction in which the circuit is traversed.

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The information contained in this document was extracted from my personal notes taken at UTN FRBA (Universidad Tecnológica Nacional Facultad Regional Buenos Aires) for Z2624 Physics II (Z2624) as taught in Spring 2019.

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For more information or additional material visit: https://rnsavinelli.github.io/

Last updated: Monday 20th July, 2020.