Second Order Filters

Low-pass filter

$$H(s) = \frac{{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2}$$

High-pass filter

$$H(s) = \frac{s^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2}$$

Band-pass filter

$$H(s) = \frac{2\zeta\omega_n s}{s^2 + 2\zeta\omega_n s + {\omega_n}^2}$$

Band-stop filter

$$H(s) = \frac{s^2 + {\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2}$$

Low-pass derivative

$$H(s) = \frac{s{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2}$$

Tustin Approximation

$$s = \frac{2}{T_s}\frac{z-1}{z+1}$$

Lead-Lag Filters

First Order

$$H(s) = \frac{\alpha s + \omega}{s + \omega}$$

Second Order

$$H(s) = \frac{\alpha\, s^2 + (2\,\alpha\, w)\, s + w^2}{s^2 + (2\, w)\, s + w^2}$$

Lead/Lag

```
close all
clear
clc
syms z alpha w ts s
disp('1st order discrete form:');
```

1st order discrete form:

```
tf = alpha + (1-alpha)/(s/w+1);
disp(simplify(tf));
```

$$\frac{w + \alpha\, s}{s + w}$$

```
s = (2 * (z - 1)) / (ts * (z + 1));
disp(collect(subs(tf), z));
```

$$\frac{(2\,\alpha + \mathrm{ts}\,w)\,z + \mathrm{ts}\,w - 2\,\alpha}{(\mathrm{ts}\,w + 2)\,z + \mathrm{ts}\,w - 2}$$

Second Order Filters

```
close all
clear
clc
syms z ts zeta wn
s = (2 * (z - 1)) / (ts * (z + 1));
den = s^2 + 2 * zeta * wn * s + wn^2;
disp('LPF');
```

LPF

```
tf = wn^2 / den;
disp(collect(tf, z));
```

$$\frac{(\mathrm{ts}^2\,\mathrm{wn}^2)\,z^2 + (2\,\mathrm{ts}^2\,\mathrm{wn}^2)\,z + \mathrm{ts}^2\,\mathrm{wn}^2}{(\mathrm{ts}^2\,\mathrm{wn}^2 + 4\,\zeta\,\mathrm{ts}\,\mathrm{wn} + 4)\,z^2 + (2\,\mathrm{ts}^2\,\mathrm{wn}^2 - 8)\,z + \mathrm{ts}^2\,\mathrm{wn}^2 - 4\,\zeta\,\mathrm{ts}\,\mathrm{wn} + 4}$$

```
disp('HPF');
```

HPF

```
tf = s^2 / den;
disp(collect(tf, z));
```

$$\frac{4\,z^2 - 8\,z + 4}{(\mathrm{ts}^2\,\mathrm{wn}^2 + 4\,\zeta\,\mathrm{ts}\,\mathrm{wn} + 4)\,z^2 + (2\,\mathrm{ts}^2\,\mathrm{wn}^2 - 8)\,z + \mathrm{ts}^2\,\mathrm{wn}^2 - 4\,\zeta\,\mathrm{ts}\,\mathrm{wn} + 4}$$

```
disp('BPF');
```

BPF

```
tf = 2 * zeta * wn * s / den;
disp(collect(tf, z));
```

$$\frac{(4\,\mathrm{ts}\,\mathrm{wn}\,\zeta)\,z^2 - 4\,\mathrm{ts}\,\mathrm{wn}\,\zeta}{(\mathrm{ts}^2\,\mathrm{wn}^2 + 4\,\zeta\,\mathrm{ts}\,\mathrm{wn} + 4)\,z^2 + (2\,\mathrm{ts}^2\,\mathrm{wn}^2 - 8)\,z + \mathrm{ts}^2\,\mathrm{wn}^2 - 4\,\zeta\,\mathrm{ts}\,\mathrm{wn} + 4}$$

```
disp('BSF');
```

BSF

```
tf = (s^2 + wn^2) / den;
disp(collect(tf, z));
```

$$\frac{(\sigma_1 + 4)\, z^2 + (2\,\mathrm{ts}^2\,\mathrm{wn}^2 - 8)\, z + \sigma_1 + 4}{(\sigma_1 + 4\,\zeta\,\mathrm{ts}\,\mathrm{wn} + 4)\, z^2 + (2\,\mathrm{ts}^2\,\mathrm{wn}^2 - 8)\, z + \sigma_1 - 4\,\zeta\,\mathrm{ts}\,\mathrm{wn} + 4}$$

where

$$\sigma_1 = \mathrm{ts}^2\,\mathrm{wn}^2$$

```
disp('LPD');
```

LPD

```
tf = s*wn^2 / den;
disp(collect(tf, z));
```

$$\frac{(2\,\mathrm{ts}\,\mathrm{wn}^2)\, z^2 - 2\,\mathrm{ts}\,\mathrm{wn}^2}{(\mathrm{ts}^2\,\mathrm{wn}^2 + 4\,\zeta\,\mathrm{ts}\,\mathrm{wn} + 4)\, z^2 + (2\,\mathrm{ts}^2\,\mathrm{wn}^2 - 8)\, z + \mathrm{ts}^2\,\mathrm{wn}^2 - 4\,\zeta\,\mathrm{ts}\,\mathrm{wn} + 4}$$

```
disp('HPF + BPF');
```

HPF + BPF

```
tf = (s^2 + 2 * zeta * wn * s) / den;
disp(collect(tf, z));
```

$$\frac{(4\,\mathrm{ts}\,\mathrm{wn}\,\zeta + 4)\, z^2 - 8\, z + 4 - 4\,\mathrm{ts}\,\mathrm{wn}\,\zeta}{(\mathrm{ts}^2\,\mathrm{wn}^2 + 4\,\zeta\,\mathrm{ts}\,\mathrm{wn} + 4)\, z^2 + (2\,\mathrm{ts}^2\,\mathrm{wn}^2 - 8)\, z + \mathrm{ts}^2\,\mathrm{wn}^2 - 4\,\zeta\,\mathrm{ts}\,\mathrm{wn} + 4}$$

```
disp('Lead-Lag');
```

Lead-Lag

```
den = s^2 + 2 * wn * s + wn^2;
tf = zeta + (1 - zeta) * wn^2 / den;
disp(collect(tf, z));
```

$$\frac{(\sigma_1 + 4\,\zeta\,\mathrm{ts}\,\mathrm{wn} + 4\,\zeta)\, z^2 + (2\,\mathrm{ts}^2\,\mathrm{wn}^2 - 8\,\zeta)\, z + \sigma_1 - 4\,\zeta\,\mathrm{ts}\,\mathrm{wn} + 4\,\zeta}{(\sigma_1 + 4\,\mathrm{ts}\,\mathrm{wn} + 4)\, z^2 + (2\,\mathrm{ts}^2\,\mathrm{wn}^2 - 8)\, z + \sigma_1 - 4\,\mathrm{ts}\,\mathrm{wn} + 4}$$

where

$$\sigma_1 = \mathrm{ts}^2\,\mathrm{wn}^2$$