# Diffusion Boundary Determination and Zone Control Via Mobile Actuator-Sensor Networks (MAS-net) – Challenges and Opportunities

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## ABSTRACT

This paper presents challenges and opportunities related to the problem of diffusion boundary determination and zone control via mobile actuator-sensor networks (MAS-net). This research theme is motivated by three example application scenarios: 1) The safe ground boundary determination of the radiation field from multiple radiation sources; 2) The nontoxic reservoir water surface boundary determination and zone control due to a toxic diffusion source; 3) The safe nontoxic 3D boundary determination and zone control of biological or chemical contamination in the air. We focus on the case of 2D diffusion process and on using a team of ground mobile robots to track the diffusion boundary. Moreover, we assume that there are a number of robots that can carry and move networked actuators to release a neutralizing chemical agent so that the shape of the polluted zone can be actively controlled. These two MAS-net applications, i.e., diffusion boundary determination and zone control, are formulated as model-based distributed control tasks. On the technological side, we focus on the node specialization and the power supply problems. On the theoretical side, some recently developed new concepts are introduced, such as the regional/zone observability, regional/zone controllability, regional/zone Luenberger observer etc. We speculate on possible further developments in the theoretical research by noting the combination of diffusion based path planning and regional analysis of the overall MAS-net distributed control system.

**Key Words:** Sensor networks, actuator networks, mobile actuator sensor networks (MAS-net), diffusion process, boundary control, zone control, mobility platform, mobile robots, regional controllability, regional observability.

## 1. INTRODUCTION

In this paper and its companion (in this same meeting[1]), we present some technical challenges and preliminary results, respectively, related to a concept initially proposed in[2] which is based on the original concept of coordination and control of distributed networks of actuated sensors in.[3] In most of the sensor networks research efforts, an important question – "*What is the high-level task to be accomplished using sensor networks*" – seems to be rarely answered explicitly. In this paper and its companion,[1] we answer this question by employing a dynamic system and closed-loop control point of view. Specifically, we consider a broad class of high-level tasks by introducing mobile actuator-sensor networks (MAS-net) as part of a temporal-spatial feedback closed-loop system where the networked sensors can be actuated or moved with high degree of mobility via distributed control commands decided from the distributed sensed environment and a given mission. Two specific high-level tasks, diffusion boundary determination and zone control, are introduced in this paper.

Sensor networks are drawing increased attention from research communities, industry sectors, and government agencies. As stated in,[4] sensor networks will "have significant impact on a broad range of applications relating to national security, health care, the environment, energy, food safety, and manufacturing. The convergence of the Internet, communications, and information technologies with techniques for miniaturization has placed sensor technology at the threshold of a period of major growth." Recent surveys on sensor networks[5–9] also indicate the importance of sensor networks research. Many on-going efforts are focused on various specific

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issues in sensor networks such as sensor structures,[10–12] communication,[6,13] data processing and sensor fusion methods,[5,7,14] sensor deployment and localization,[5,7,15] calibration,[16,17] etc. However, from dynamic systems control point of view, the sensor networks should be part of a complete system with a specific mission defined. Recently, a habitat monitoring task was proposed as an "application driver" for wireless communications technology based sensor networks.[18,19] Clearly, the system in[19] is still an open-loop system simply for monitoring purposes. In,[20] a future research effort was proposed to combine distributed sensing, robotic sampling, and offline analysis for in situ marine monitoring purposes, where the loop is closed as the underwater robots, carrying networked sensors, are to be deployed according to the sensed environment. This system is not real-time feedback-controlled due to the offline analysis. So far, there is no such real-time, closed-loop distributed feedback control system involving networked actuators and sensors.[21,22] However, these are applications that require the use of movable sensors and actuators. These include the following motivating examples.

- Application Scenario 1 (**land**): In this case, each networked sensor is mounted on a ground mobile robot. The mission is to determine the safe radiation boundary of the radiation field from possibly multiple radiation sources. Each robot is actuated according to spatial and temporal sensed information (radiation gradient, spatial position, etc.) from more than one actuated or mobile sensors.
- Application Scenario 2 (**water**): This case will be similar to Application Scenario 1 if the toxic diffusion source is a one-time pouring and the diffusion is in steady state. However, the boundary may be dynamically evolving as the toxic source keeps polluting the reservoir. The actuated or mobile sensors are autonomous boats mounted with toxic chemical concentration sensors. The boats are commanded according to the spatial-temporal sensed information from more than one sensor. Furthermore, assume that some of the boats (not all of the boats) are equipped with relevant neutralizing chemicals to make the water detoxified. By proper design of the distributed sensing and actuation/control strategies, it is possible to control the zone or shape of the toxic region to match the given desirable zone/shape. Now we have a complex distributed feedback control system that is more challenging than the networked actuators and sensors themselves.
- Application Scenario 3 (**air**): This scenario is similar to the above water case, but it is more complicated since 3D space must be explored. Here, the actuated or mobile sensors are unmanned aerial vehicles (UAVs) equipped with concentration detectors and anti-contamination chemical agent(s) distributors. For this case, a more detailed description can be found in.[2]

Motivated by these examples, in this paper, we will first introduce two high-level tasks or missions for MAS-net, i.e., the diffusion boundary determination and zone control via mobile actuator-sensor networks. As a genera case, based on,[2] we summarize the mathematical model-based formulation of the MAS-net for reaction-diffusion processes in Sec 2. Then, in Sec. 3, for the two special cases of high-level tasks or applications, i.e., boundary determination and zone control, the complete distributed control formulation using the specified objective functions corresponding to these applications, based on Sec. 2, will be given. Section 4 presents some technological and theoretical challenges and opportunities for the present distributed control problem. On the technological side, we focus on the node specialization and the power supply problems. On the theoretical side, some recently developed new concepts will be introduced, such as the regional/zone observability, regional/zone controllability, regional/zone Luenberger observer etc. We speculate on some possible further developments in the theoretical research side by noting the combination of diffusion based path planning and regional analysis of the overall MAS-net distributed control system. Finally, conclusions are given in Sec. 5.

## 2. SUMMARY OF THE GENERAL MATHEMATICAL FORMULATION OF MAS-NET FOR REACTION-DIFFUSION PROCESSES[2]

In this section, we summarize the general mathematical formulation of MAS-net for reaction-diffusion processes.[2] The purpose is to introduce the proper notations we used in this paper and the essential components within an MAS-net based system.

- **Plant model.** In this work, we consider a class of reaction-diffusion processes as described by the following general parabolic type partial differential equations:

$$\frac{\partial V(q,t)}{\partial t} + \nabla \cdot (FV(q,t)) = \nabla \cdot (D(q,t)\nabla V(q,t)) + g(q,t) \tag{1}$$

$$V(q_0, t_0) = v_0 \tag{2}$$

where $\nabla^2 = \Delta$ is the Laplace operator; $q = (x, y, z)^T \in \mathbb{R}^3$ is the spatial variable; $V(q,t)$ is the spatial-temporal distribution or concentration of the pollutant or toxic plume; $FV(q,t)$ denotes the effect of external, possibly variable, "inputs" on the plant dynamics (e.g., wind, rain, dust, humidity, etc.); $D(q,t)$ is the diffusion function for the specific problem; $g(q,t)$ reflects the effects of constraints (e.g., gravity), and $V_0, q_0, t_0$ denote the initial conditions.

- **Mobile sensor network.** We begin by assuming that we are given a network of actuated sensors, $NS^A$, made up of a collection of individual sensors that are defined as follows:
  - $S_i^A(q_i)$: an actuated sensor with the following characteristics:
    * Located in space at $q_i(t) = (x_i, y_i, z_i)^T \in \mathbb{R}^3$;
    * Can communicate with all others and with a supervisor.
    * Can generate a measurement of interest to the application, defined by $s_i(q_i, t)$, which is assumed to be a function of both space and time as defined below.
    * Can move freely in three dimensions with dynamics given by

$$\dot{q}_i(t) = f_i(q_i, u_i) \tag{3}$$

where $u_i(t)$ is the motion control input for sensor $S_i^A(q_i)$.

- **Sampling action.** It is assumed that the output of the sensor $S_i^A(q_i)$, defined above as $s_i$, is a measurement of the distribution of interest at wherever the sensor is located in space. Thus we can write:

$$s_i(q_i, t) = V(q_i, t). \tag{4}$$

- **Spatial-temporal prediction.** The next step in is to define the diffusion prediction. Of course, if we had perfect knowledge, the problem would be trivial. However, in fact we only have estimates of the initial conditions, of the external inputs, and of the constraints denoted as $\hat{FV}(q,t)$, $\hat{g}(q,t)$, and $\hat{V}_0$, respectively. Then, we can compute the estimated diffusion $\hat{V}(q,t)$ as the solution of

$$\frac{\partial \hat{V}(q,t)}{\partial t} + \nabla \cdot (\hat{FV}(q,t)) = \nabla \cdot (D(q,t)\nabla \hat{V}(q,t)) + \hat{g}(q,t), \tag{5}$$

$$\hat{V}(q_0, t_0) = \hat{V}_0, \tag{6}$$

$$\hat{V}(q_i, t_{s_i}) = s_i(q_i, t_{s_i}) = V(q_i, t_{s_i}), \quad \forall i, \quad \forall t_{s_i}. \tag{7}$$

Notice the introduction of the actual sensor measurements at sample points and sample instants $(q_i, t_{s_i})$ as constraints for the partial differential equation.

- **Motion control of networked mobile sensor nodes.** The next piece added in this framework is the motion control of the mobile sensors. There are various ways to approach this piece. For instance, one could take control actions to be a function of the error between the predicted samples and the actual samples. That is, given a set of samples, make a prediction about the distribution. Then, move to a new point in space and take new samples. The error between what is expected to measure and what is actually measured determines where the system takes its next samples. Therefore, for example, for the diffusion boundary determination task, simply move the sensors such that they are uniformly distributed at the the predicted boundary. In general, we can write

$$q_i^{sp} = H(\hat{V}(q,t)), \tag{8}$$

$$\dot{u}_i(T) = h_i(q_i^{sp} - q_i), \tag{9}$$

where $H$ is chosen based on the cost function the specific problem and $h$ is the control law used to drive the actuated senor to its new setpoint $q_i^{sp}$.

- **Mobile actuator network**: In addition to allowing mobile sensors, we would also like to consider mobile actuators as well. Of course, such actuators might be co-located with the actuated sensors. But, in many applications dispersal or actuating agents will typically be much more expensive than the sensing agents. Thus, it is reasonable to consider them separately. The effect of such mobile actuators can be added to achieve diffusion zone control by noting that the primary effect the actuators introduce is that of modifying the diffusion function in the system dynamics. To proceed, assume that a network of autonomous, mobile actuators is available with the following characterization:

  - $A_j^A(q_j)$: a mobile actuator with the following characteristics:
    * Can be located in space at $q_j(t) = (x_j, y_j, z_j)^T \in \mathbb{R}^3$;
    * Can communicate with all others, with all sensors, and with a supervisor;
    * Can generate an effect of interest to the application, defined by $d_j^a(q_j, t)$, which is assumed to be a function of both space and time.
    * Can move freely in three dimensions with dynamics given by

$$\dot{q}_j = f_j^a(q_j, u_j^a) \tag{10}$$

  where $u_j^a(t)$ is the motion control input for actuator $A_j^A(q_j)$. For this set of actuators, define motion controllers given by

$$\dot{q}_j^{sp} = H^a(V^d(q,t) - \hat{V}(q,t)) \tag{11}$$

$$\dot{u}_j^a = h_j^a(q_j^{sp} - q_j). \tag{12}$$

  It should be pointed out that in the case of a mobile actuator, the function $H^a$ is primarily a comparator and $V^d(q,t)$, the desired distribution over the desired zone $\Omega^d$ where $q \in \Omega^d$, can be ideally taken as zero, i.e., all contaminants are to be detoxified. Note that, however, typically, due to the limit of the total amount of detoxification agent in each mobile actuator, it is expected to pre-specify the desirable distribution in a zone. In this case, $V^d(q,t)$ will not be zero over the given zone $\Omega^d$.

For a specific application, we need to add a properly defined objective function into the above mathematical model-based formulation. In the next section, we will show how to complete the control- theoretical setup for further investigation.

## 3. TWO APPLICATIONS OF MAS-NET FOR DIFFUSION PROCESSES: BOUNDARY DETERMINATION AND ZONE CONTROL

In this section, we consider two applications of MAS-net for diffusion processes: diffusion boundary determination and zone control. First let us define the control objectives in words.

- The *boundary determination* objective is to move and distribute the networked mobile sensors uniformly to approach the diffusion boundary in such a way that beyond the boundary, the concentration of the pollutant is smaller than a pre-specified safe value.

- For the *zone control* problem, the objective is to cooperatively move the networked mobile actuators to release the de-toxic agent, with the networked mobile sensors being activated for spatial-temporal sampling measurements, so that eventually the zone of toxic concentration greater than the pre-specified safe value can be changed to a desired shape. The purpose is to actively restrain the toxic substance within a given zone.

Now, let us give the complete control-theoretical mathematical formulation for these two applications.

- **Goal statement for boundary determination**: For now, only the mobile sensor network is considered. The goal here is actually for the boundary determination, that is $\hat{V}(q_i, t^*) \to V(q_i, t^*)$ for all $i$ where $t^*$ is the moment when $V(q_i, t^*) \leq \varepsilon$ with $\varepsilon$ the pre-specified concentration value. The performance index can be formulated as

$$J^P = \lambda \sum_i (V(q_i, t) - \varepsilon)^2 + \sum_i (V(q_i, t) - \hat{V}(q_i, t))^2, \tag{13}$$

where $\lambda$ is a positive weighting factor.

So, given descriptions of the actuated sensor network and the sampling action, the diffusion system to be characterized, the prediction strategy, and the motion control logic, the complete problem can be stated as follows:

$$\min_{H,h} J^P = \lambda \sum_i (V(q_i, t) - \varepsilon)^2 + \sum_i (V(q_i, t) - \hat{V}(q_i, t))^2 \tag{14}$$

subject to:

  – Diffusion reaction process (1) and (2);
  – Prediction process (5), (6) and (7);
  – Sensor motion dynamics (3) and sampling process (4);
  – Feedforward controller *or* set point maker *or* path planner (8) and feedback control law (8) for actuated sensors.

We comment that the design freedom in the problem is found in the selection of the controller motion functions $H$ and $h_i$. For the most part, the selection of $h_i$ is a straightforward control system design activity that will be specific to the robotic strategy used to actuate the mobile sensors. The selection of $H$ is ultimately the major design effort in the problem. Note that, although this is essentially an open-loop system identification problem, there is in fact a feedback feature to the problem, due to the motion control coupling to the output of the predictor. It should be remarked that in the existing works,[23–27] although the reaction-diffusion model is assumed, there is no explicit use of actuator-sensor networks with a clear mission in mind.

- **Goal statement for zone control**: Without going into the details, the following cost function for the design of the functions $H^a$ and $h_j^a$ to achieve the zone control is proposed:

$$J^C = \lim_{t \to \infty} \int_{\Omega^d} (V^d(q, t) - \hat{V}(q, t))^2 \mathrm{d}q. \tag{15}$$

This cost function seeks to drive the predicted distribution to the final distribution everywhere in the zone $\Omega^d$. At this point, it should be pointed out that arbitrarily given $V^d(q, t)$ and $\Omega^d$ could make the problem unsolvable. Moreover, the zone formation is actually achieved via the boundary discrete control actions of the mobile actuators. There are many interesting distributed decision and control problems for this type of zone control goal.

**Summary of MAS-net based diffusion boundary determination & zone control**[2]:

$$\min_{H,h} J^P = \lambda \sum_i (V(q_i, t) - \varepsilon)^2 + \sum_i (V(q_i, t) - \hat{V}(q_i, t))^2 \tag{16}$$

$$\min_{H^a,h^a} J^C = \lim_{t \to \infty} \int_{\Omega^d} (V^d(q, t) - \hat{V}(q, t))^2 \mathrm{d}q. \tag{17}$$

subject to:

  – Diffusion reaction process (1) and (2);
  – Prediction process (5), (6) and (7). Note that, in (5), $D(q, t)$ should be replaced by a function $w(D(q, t), d_j^a(q_j, t))$ since mobile actuators $A_j^A(q_j)$ are to be applied.

- Mobile sensor motion dynamics (3), mobile actuator motion dynamics (10) and sampling process (4);
- Feedforward controller *or* set point maker *or* path planner (8) and feedback control law (8) for actuated sensors.
- For mobile actuators, feedforward controller *or* set point maker *or* path planner (11) and feedback control law (12) .

Note that here are actually two coupled problems stated. The sensor motion control problem is based on the output of the prediction. But the effect of the mobile actuators is shown in the diffusion function used in the prediction. We denote this as a function $w(D(q,t), d_j^a(q_j,t))$ because in general the effect of a dispersal agent may not necessarily be linear. At this time the effect of this coupling is not clear. One would hope to see the standard separation principle emerge, but that might not be possible. In this case, the research becomes more interesting and challenging with a lot of opportunities for creative investigations. Note that the framework presented here hides the commonly heard terminologies such as cooperative sensing, sensor fusion, cooperative navigation, path planning etc. Within the model based or partial model based approach, the above conventional tasks can be performed in a systematic manner.

The above high-level tasks seem to be ambitious but actually feasible based on the state-of-the-art technologies and the proposed model based approach described in the next section. To verify the proposed framework, a lab test bed distributed system is proposed in.[1] Details of our MAS-net project can be found in its website `http://mas-net.ece.usu.edu`.

The MAS-net involved in this novel distributed control system looks like a swarm-type robot network. There is quite a bit of existing related work on coordination strategies for swarm-type networks,[28,29] with most of the work based on the notion of individual sensors following some type of *a priori* energy potential function or gradient.[30–35] The diffusion processes considered in this project are infinite dimensional systems[36] in nature with some local sensors and actuators.[37–39] There are many relevant issues to be considered such as controllability,[40] observability,[41] and in a more practical sense, regional controllability[42] and regional observability.[43] We mention more about these problems below.

We believe that the above model-based formulation for boundary determination and zone control for a class of diffusive processes using mobile actuator-sensor networks is new, although[37] started as early as 1988 to consider the sensor and actuator be replaceable in the diffusion process control in filament, boundary and zone. With the emerging technologies such as Intel's mote[44] or smart dust,[21] the solution of the problem formulated above will have a significant and broader impact on the development of distributed boundary determination and zone control systems for national security interests, water resource, precision agriculture, ecological and environmental monitoring.

Remark 3.1. *In mechanical systems, such as vibration control of elastic structure using distributed sensing and distributed actuation,[45,46] the dynamics is mainly governed by wave or beam equations.[47] Recently, the time delay effect and its compensation, possibly due to networked sensors and actuators, in the boundary control of wave/beam systems have been investigated in ,[48] based on the newly developed hybrid numerical/symbolic simulation method. It is interesting to note that for mechanical systems, the practical shape control also starts to draw attention from the research community.[49]*

Remark 3.2. *As a side remark, for the diffusive processes under consideration, due to the effect of fluid and pollutant properties, the diffusion equation could be of non integer order[50] where the fractional order modelling and control for robotics is an active recent research topic.[51–53]*

## 4. TECHNOLOGICAL AND THEORETICAL CHALLENGES AND OPPORTUNITIES

The mathematical model-based formulation in Sec. 3 can actually be visually summarized in Fig. 1. Corresponding to Fig. 1, a test bed system has been under construction as described in [1] with some preliminary results.

In this section, we will briefly summarize the technological and theoretical challenges and opportunities in diffusion boundary determination and zone control problem using MAS-net explained in the previous sections and in the visual summary Fig. 1.

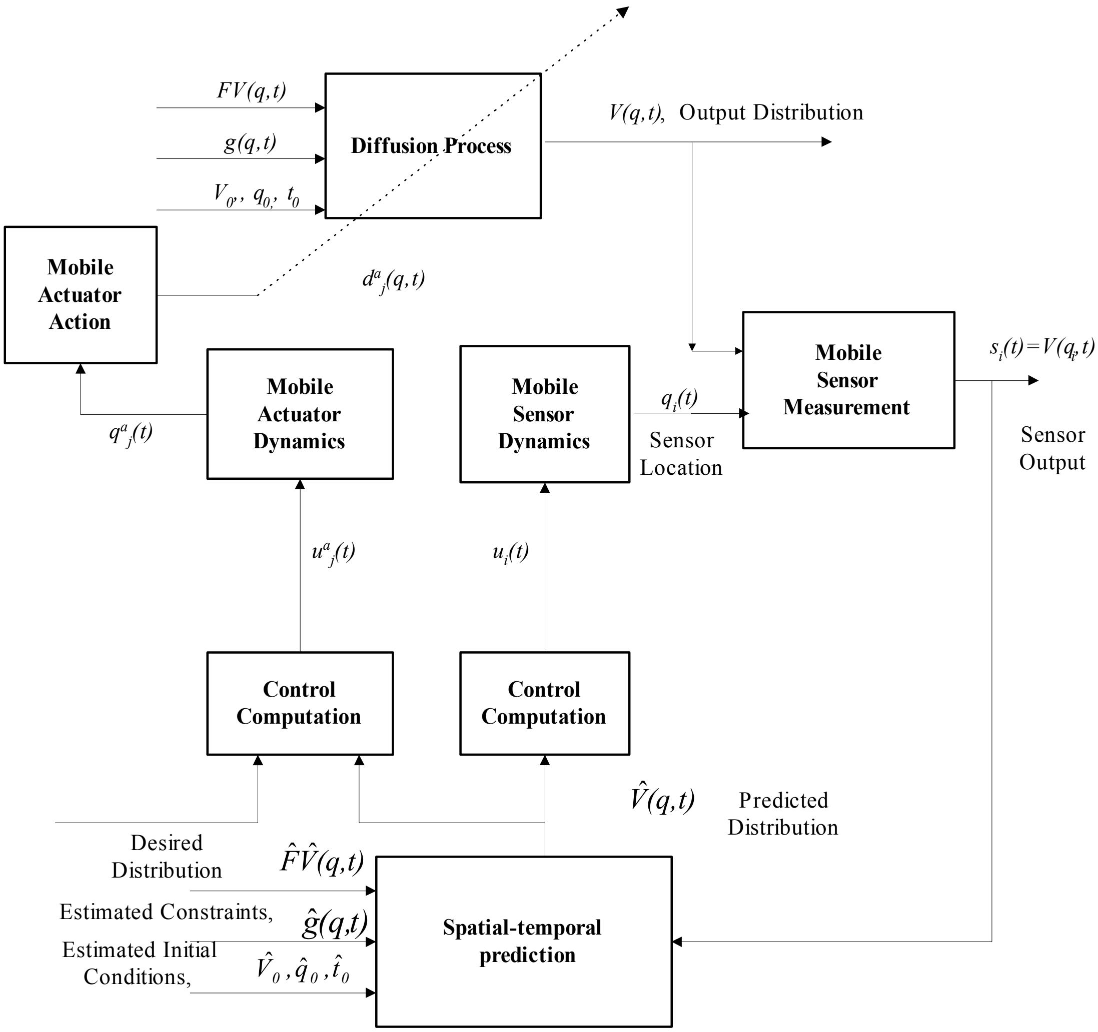


**Figure 1.** MAS-net big picture (adapted from[1,2])

### 4.1. Technological Challenges and Opportunities

A MAS-net based distributed control system should physically include 1) the mobility platform; 2) the wireless mobile sensor networks; 3) the wireless mobile actuator networks. As experienced in our lab implementation of MAS-net, described in the companion paper,[1] it seems that technological advances in the integration and miniaturization of sensors, microprocessors and wireless communication, etc., with ultra low power consumption have been steadily growing (e.g., Intel Motes[44,54] or the so-called "smart dust",[21] etc.) Commercial-off-the-shelf (COTS) mobility platforms, e.g., [55,56] and wireless mobile sensor networks[54] based on TinyOS[57] have already been available. The technological challenges and opportunities are actually *not* at the lower level but at the level of "specialization", a new concept introduced in.[58] Although in this paper we implicitly considered a homogeneous configuration where each MAS-net node, which could be a mobile sensor node, a mobile actuator node, or a mobile actuator-sensor node (a combined node), has the same mobility platform, sensors and/or actuators, in general, however, we should consider that some robots may be able to carry sensors or actuators only, that is, each node may have a different "specialization". Additionally, the mobility platforms could be heterogeneous in both mechanical structure and its environmental sensing capabilities, e.g., encoder, IR, sonar etc. That is, each robot may have a different "specialization", too. Therefore, it is a challenging problem to decide how to assign the "specialization" among nodes and mobility platforms for a specific mission.

Another challenge is the power supply. In sensor networks, power consumption is an important design factor. However, what if we can have some mobile wireless power transmission (WPT)[59–61] nodes within a MAS-net based system? For a MAS-net system, how many mobile WPT stations are needed? For a given number of mobile WPT stations, how should we dispatch mobile WPT nodes to the moving MAS-net nodes in urgent needs? For UAVs (unmanned aerial vehicles), one or more dedicated WPT UAVs to charge other UAVs will be possible in the future. How can we best coordinate and schedule the networked UAVs when WPT nodes are available? These considerations will create many opportunities of research and development.

### 4.2. Theoretical Challenges and Opportunities

The dynamic system considered in this paper is hybrid continuous-discrete in nature. The diffusion process is continuous. But, with discrete sampling and control action, the overall dynamics will be piecewise continuous. From an application point of view, some plain questions such as "How many sensors and how many actuators are sufficient?", "Whether and how fast the diffusion boundary can be determined by the MAS-net nodes?", "Given the desirable zone shape, is it possible to control the diffusion process within the given zone?", and etc. might be asked, which in fact raises some important theoretical challenges and open new opportunities for further research. Due to the space limit, we briefly introduce the following:

- **Regional stability**. Regional control theory of distributed parameter systems (DPS) was pioneered by A. El Jaï and his co-workers.[37] They are concerned with the analysis and control of a distributed system only within a subregion of the evolution domain.[62] To introduce the concept, define a regular subset $\Omega$ of $\mathbb{R}^n$, $(n = 1, 2, 3)$ and denote $Q$ by $\Omega \times ]0, \infty[$. Consider the evolution equation $\partial z/\partial t = Az$ with $z(\cdot, 0) = z_0 \in L^2(\Omega)$ where $A : \mathcal{D}(A) \subset L^2(\Omega) \to L^2(\Omega)$ is the a strongly continuous semigroup $S(t)$, $t \geq 0$ on $L^2(\Omega)$ and $z(t) = S(t)z_0$ denotes the mild solution of the evolution equation. Let $\omega \subset \Omega$ be open and of positive Lebesgue measure. Denote $\chi_\omega : L^2(\Omega) \to L^2(\Omega)$ the operator restriction within $\omega$. Then, $\chi_\omega^*$ denotes the adjoint operator given by $(\chi_\omega^* z)(x) = z(x)$ if $x \in \omega$ and $(\chi_\omega^* z)(x) = 0$ if $x \notin \omega$. The general abstract evolution equation is *regional weakly stable* (r.w.s) on $\omega$ if $\forall z_0 \in L^2(\Omega)$, the corresponding solution $z(t)$ verifies $\langle \chi_\omega z(t), y \rangle \to 0$ as $t \to \infty$, $\forall y \in L^2(\Omega)$; *regional strongly stable* (r.s.s) on $\omega$ if $\forall z_0 \in L^2(\Omega)$, $\|\chi_\omega z(t)\| \to 0$ as $t \to \infty$; *regional exponentially stable* (r.e.s) on $\omega$ if $\exists M, \alpha > 0$ such that $\|\chi_\omega z(t)\| \leq Me^{-\alpha t}\|z_0\|$. Here $\langle \cdot, \cdot \rangle$ and $\| \cdot \|$ denote the inner product and the corresponding norm in $L^2(\Omega)$. This regional characterization of stability is actually more general since when $\omega = \Omega$, we will recover the conventional stability definitions. Relating to MAS-net, the question is to determine if at particular condition defined by $H$ or $h$ is regionally stable or not.

- **Regional controllability**. Regional controllability introduced in[37,42] has a similar definition of controllability in usual sense.[40] Similarly, the concept of regional controllability can be generalized to, e.g. *regional boundary controllability*[63] and *regional gradient controllability*,[64] both for parabolic systems.

  From practical point of view, it is very natural to consider the regional controllability problem since in many cases, the evolution system may not be controllable within $\Omega$ but controllable in a subregion $\omega \subset \Omega$. Again, for the MAS-net system, we must be sure that the chosen objective functions admit regional controllability.

- **Regional observability**. The definition can be found in .[37,43] An interesting application example in heat process was presented in.[65] It should be noted that the regional observability property is linked with the sensor structure.[66] Again, when $\omega = \Omega$, it is the usual observability definition.[41] Extension to *regional boundary observability* was discussed in.[67] In parallel, regional detectability and regional state observer design based on Luenberger observer principle have been discussed in.[68–71] Again, when $\omega = \Omega$, the observer will be in its usual form.[72,73] From economic point of view, regional detactability analysis and regional observer design will save unnecessary investment on physical sensors.

Most of the above work was on continuous temporal and spatial domains. In the MAS-net system, as already mentioned, the control problem is even more challenging since the dynamics is hybrid continuous and discrete in nature. It has been already known that when discretizing the continuous time signal, the Shannon Sampling Theorem defines the smallest allowable sampling frequency to ensure the signal reconstruction. A parallel question on the largest possible spatial sampling resolution asks for a similar sampling theorem for spatial signal reconstruction, which is missing from the literature. A joint spatial-temporal sampling theorem would make it possible to answer "How many sensors are sufficient?" for characterizing diffusion processes using MAS-net.

Due to the spatial-temporal sampling and discrete nature of decision and control, the regional analysis will be dominant in the theoretical side of MAS-net research. As we can sense, the MAS-net will create interesting theoretical research topics. The work done by A. El Jaï and his co-workers should be regard as the starting point. As a final remark, the regional analysis should be coupled with the diffusion based path planning method.[1, 33, 34, 74]

## 5. CONCLUDING REMARKS

This paper presented the challenges and opportunities in diffusion boundary determination and zone control via mobile actuator-sensor networks (MAS-net) with a brief summary of the mathematical model-based formulation of the MAS-net for reaction-diffusion processes. For two specific high-level tasks or applications, i.e., boundary determination and zone control, the complete distributed control formulation using specific objective functions corresponding to these applications was given. On the technological side, we focus on the node specialization and the power supply problems. On the theoretical side, some recently developed new concepts were introduced, such as the regional/zone observability, regional/zone controllability, regional/zone Luenberger observer etc. We speculate on possible further developments in the theoretical research side by noting the combination of diffusion based path planning and regional analysis of the overall MAS-net distributed control system. This paper shows that the MAS-net notion opens numerous possibilities for future research.

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