

Automating  
Treatment  
Planning in  
Radiation  
Therapy

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Innovation

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Stabilization

Background –  
LINACs RT  
MRI-LINACs  
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Robustness issues

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iDG Results

Future Work

References

# Automating Treatment Planning in Radiation Therapy

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University of Pennsylvania, Philadelphia, PA

Presented by **Lekan Molu** (Lay-con Moh-lu)

March 10, 2021

# Acknowledgments

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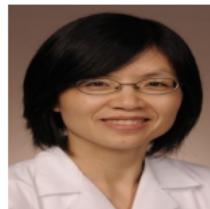
Steve Jiang, UTSW



Nick Gans, UTARI



Xuejun Gu, UTSW



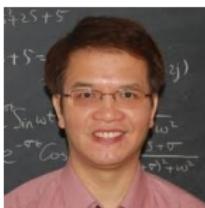
Dan Nguyen, UTSW



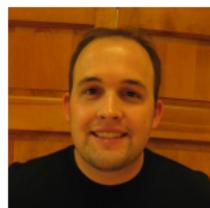
Rod Wiersma, Penn



Xinmin Liu, Penn



Tyler Summers, UTD



Yonas Tadesse, UTD



# Funding Sources

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# Talk Outline

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- Beam Orientation Optimization (BOO)
  - Monte Carlo Tree Search and Neuro-Dynamic Programming for BOO
  - Column Generation as Pretraining for MCTS for BOO
- Patient Head Motion Correction in External Beam Radiation Therapy (RT)
  - Intensity-Modulated RT (IMRT): Earlier PhD Work
  - Magnetic Resonance Imaging and Linear Accelerator Systems (MRI-LINACs)
- Robustness Margins and Robust Deep Policies for Nonlinear Control

# Research Significance

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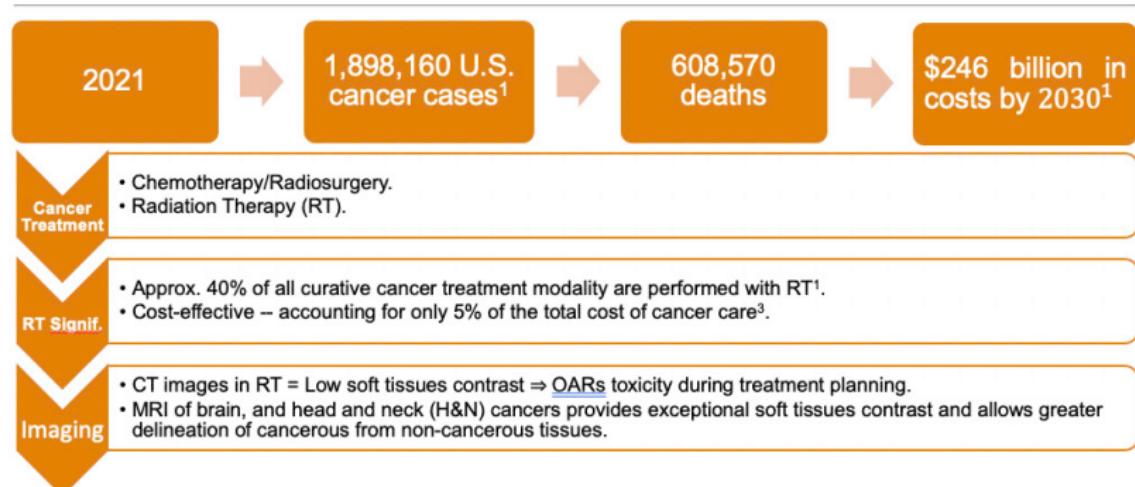
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# IMRT Treatment Planning (Beam Delivery)

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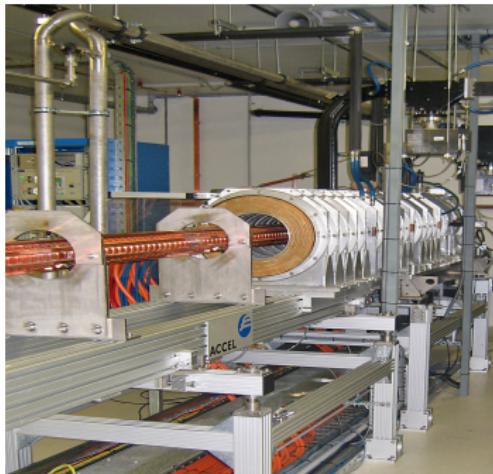
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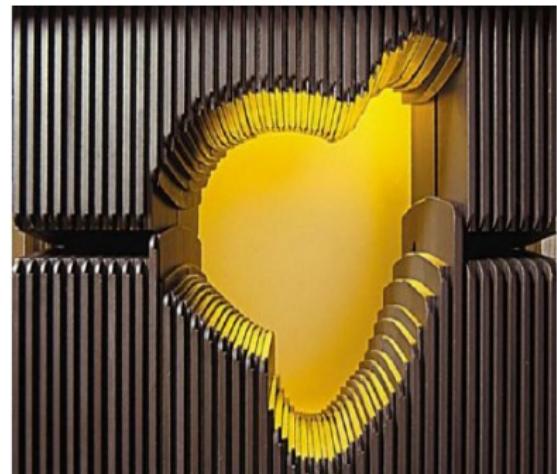
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The Australian Synchrotron.



A Multi-leaf collimator, ©Varian.

# Radiation Delivery Couch and Gantry

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Varian's TrueBeam Radiotherapy System.

# Part I.A: Beam Orientation Optimization (BOO)

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## ■ Beam Orientation Optimization (BOO)

- Monte Carlo Tree Search and Neuro-Dynamic Programming

# Contributions

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## Relevant Publications

Ogunmolu, Olalekan, Michael Folkerts, Dan Nguyen, Nicholas Gans, and Steve Jiang. "Deep BOO! Automating Beam Orientation Optimization in Radiation Therapy." In *Algorithm Foundations of Robotics XIII*, Merida, Mexico. Published in *Springer's Proceedings in Advanced Robotics (SPAR) Book*, 2020.

- A sparse tree lookout strategy for games with large state spaces guides transition between beam angle sets
- Tree lookout strategy guided by a deep neural network policy

# Prostate Cancer Example

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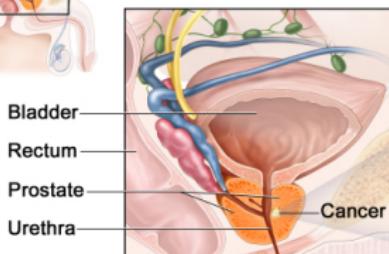
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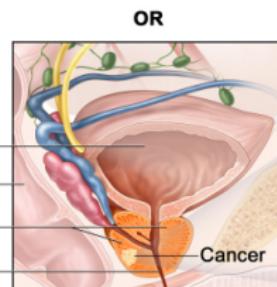
**Stage I Prostate Cancer**



**Found by:** Needle biopsy

**Grade Group:** 1

**PSA level:** Less than 10



**Found by:** Digital rectal exam

**Grade Group:** 1

**PSA level:** Less than 10

**Cancer in:** 1/2 or less of  
one side

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# BOO Process: Fluence Map Optimization

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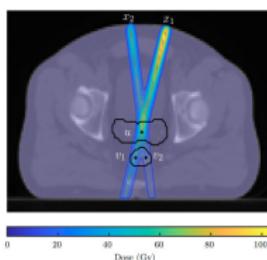
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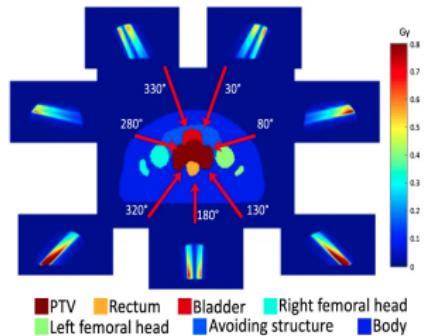
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Prostate CT slice



Prostate before  
BOO



Fluence Map

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## Manually Selection/Protocols Adoption

Laborious process; could take up to 5 days for head and neck cancer treatment.

## Pre-solve Large Sparse Dose Influence Matrix

Takes hours to solve for a single patient. Days/months for multiple patients.

## Solve Fluence Map Optimization

Time-consuming: Often takes minutes.

# Treatment Plan Flowchart

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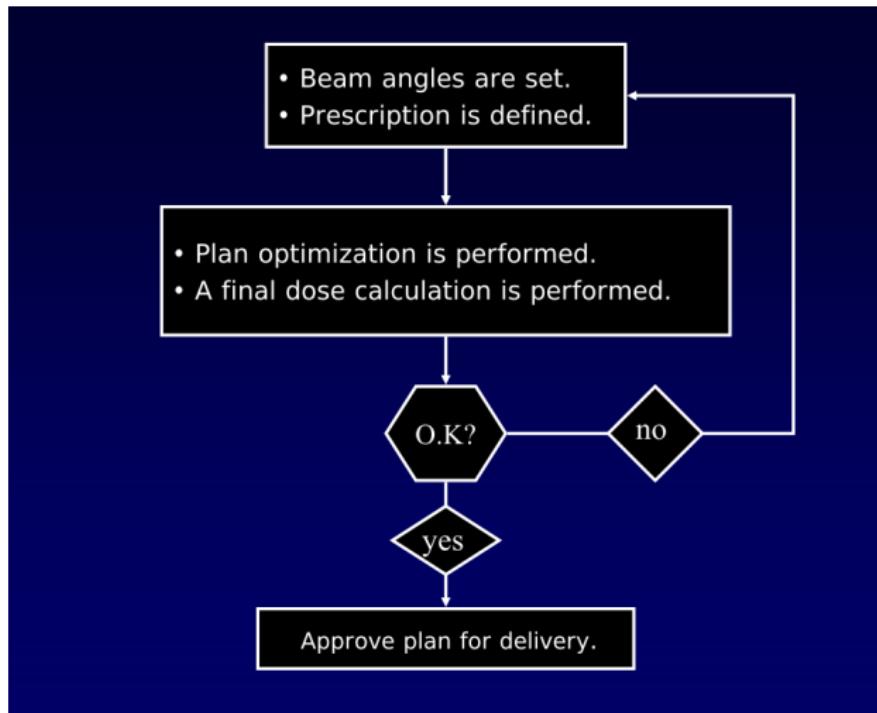
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Reprinted from "IMRT Optimization Algorithms. David Shepard. Swedish Cancer Institute. AAPM 2007."

# Current Approaches and Limitations

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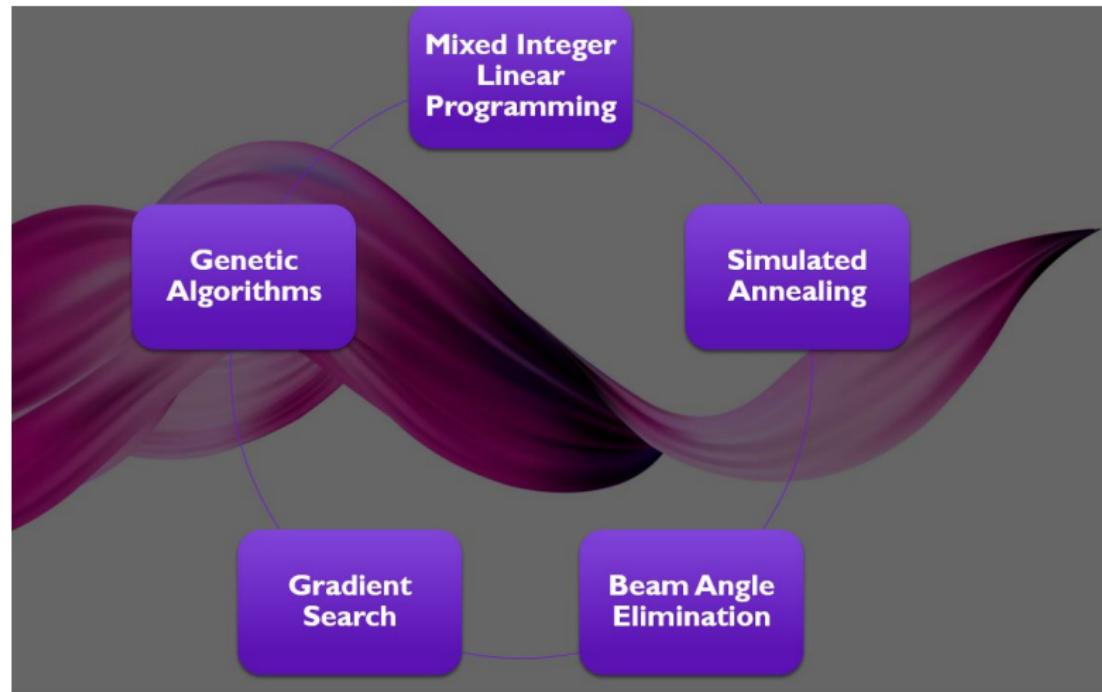
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# Innovation

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- A Tower Neural Network generates a policy that guides MCTS simulations for two players in a zero-sum Markov game
  - Produces a *utility (value) function* & a subjective *probability distribution*
- Each player in a two-player Markov game finds an alternating best response to the current player's average strategy
  - driving the neural network policy's weights toward an approximate **saddle equilibrium** [Heinrich et al. (2015)].
  - aids network in finding an *approximately optimal* beam angle candidate set that meets a dosimetric requirements.

# State Encoding: Prostate Organ Masks

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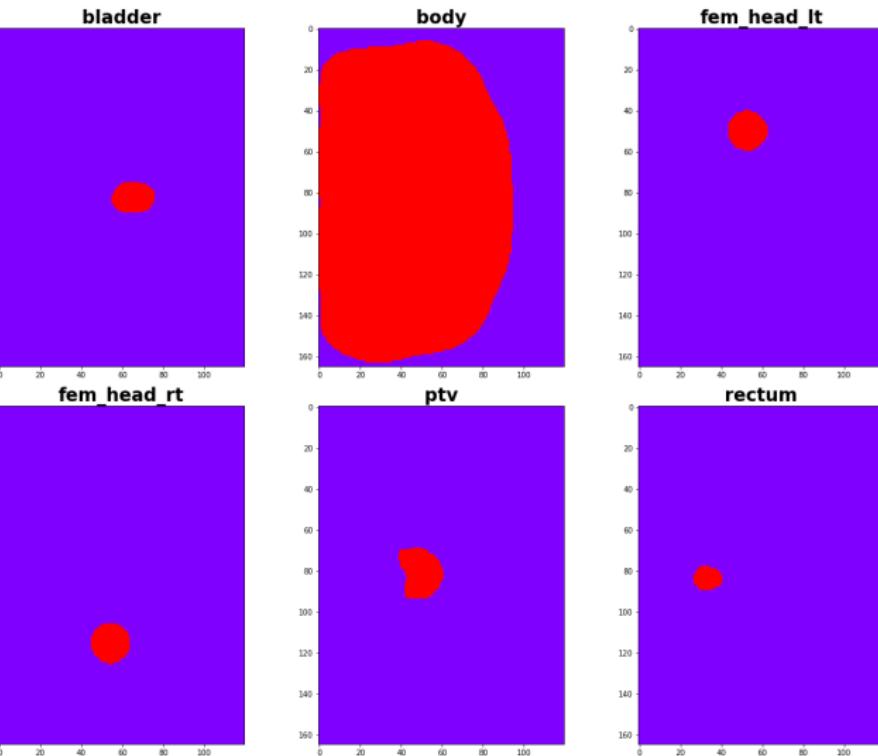
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# State Representation: Beam Angles

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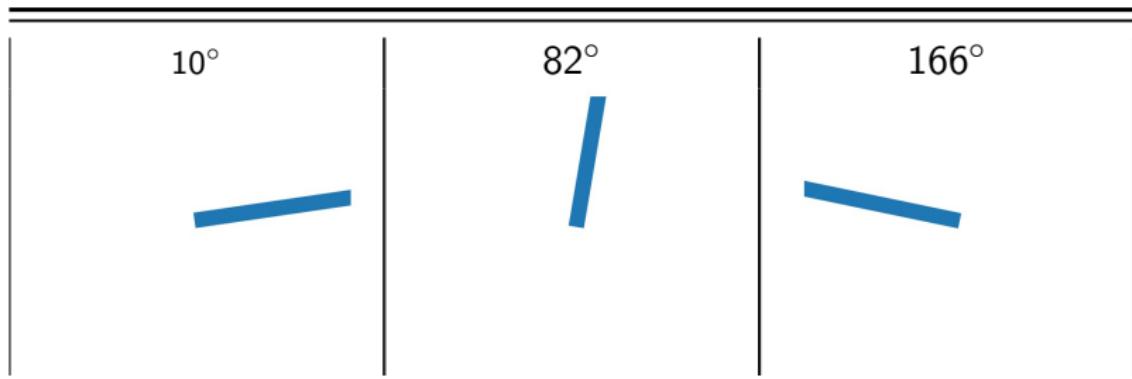
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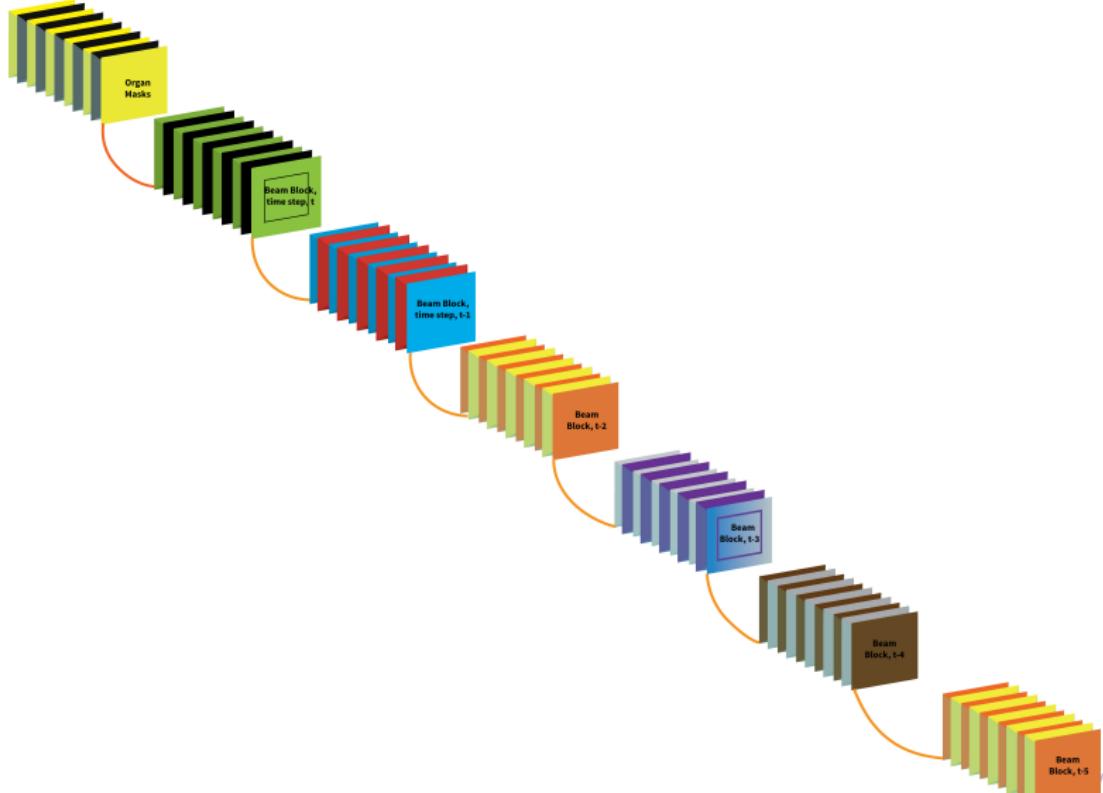
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# State Representation



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# Two-player Fictitious network play with ResNet

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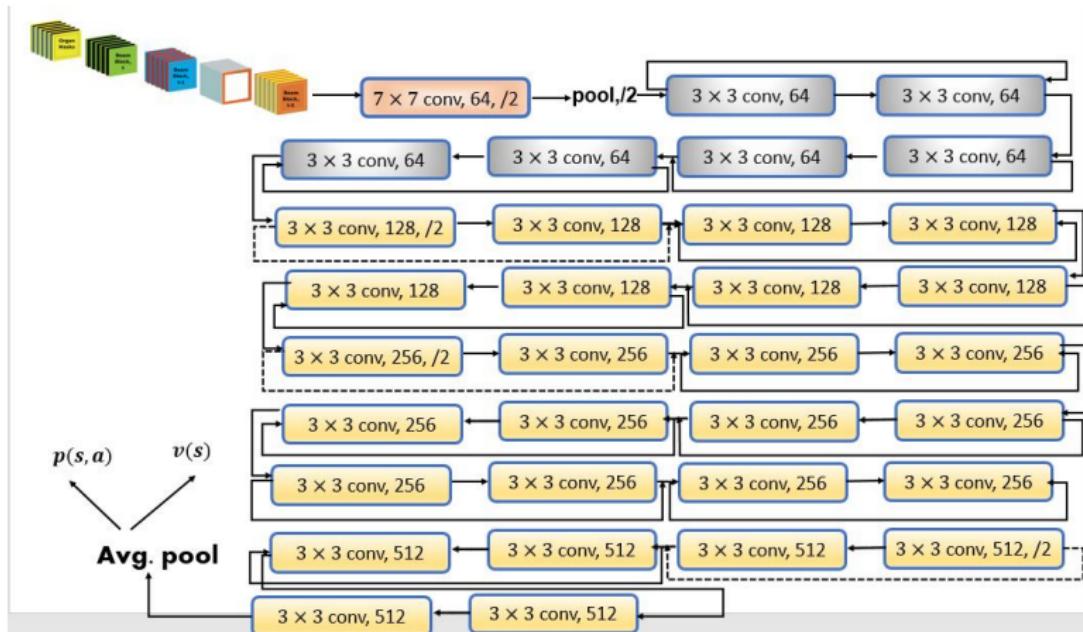
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# Tree Representation and Game Simulation

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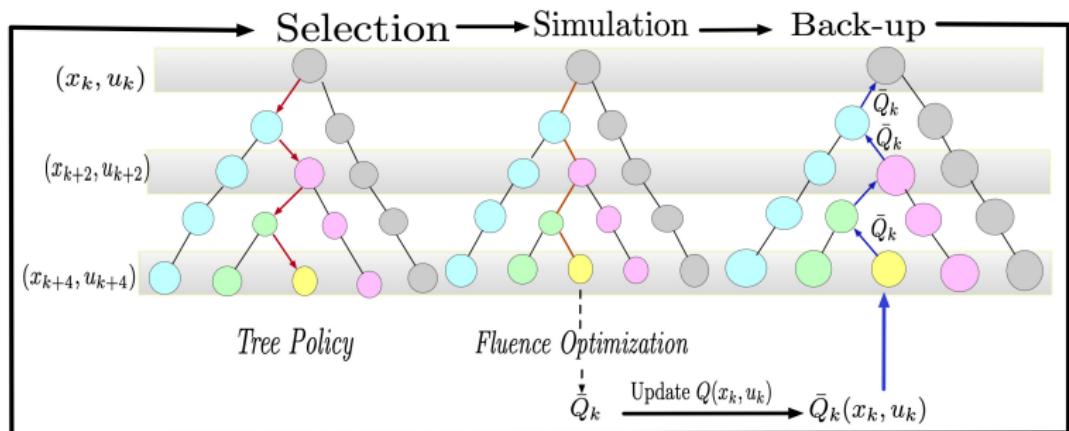
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# Tree Composition

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Every **node** of the tree,  $\mathbf{x}$ , has the following fields:

- a pointer to the parent that led to it,  $\mathbf{x}.p$ ;
- the beamlets,  $\mathbf{x}_b$ , stored at that node;  $b = \{1, \dots, m\}$ ;
- a set of move probabilities prior,  $p(s, a)$ ;
- a pointer  $\mathbf{x}.r$ , to the reward  $r_t$ , for the state  $\mathbf{x}_t$ ;
- a pointer to the state-action value  $Q(s, a)$  and its upper confidence bound  $U(s, a)$ ;
- a visit count  $N(s, a)$ , that indicates the number of times that node was visited; and
- a pointer  $\mathbf{x}.child$ ; to each of its children nodes.

# Saddle Point Strategy Formulation

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- **Saddle point strategies** for optimal control sequence pair  $\{a_t^{p_1^*}, a_t^{p_2^*}\}$  recursively obtained by optimizing,  $V_t(s, a)$

$$V_t^*(s) = Q_t^*(s_t, \pi_t^{p_1}, \pi_t^{p_2}) = \min_{\pi^{p_1} \in \Pi^{p_1}} \max_{\pi^{p_2} \in \Pi^{p_2}} Q_t^*(s_t, \pi^{p_1}, \pi^{p_2})$$
$$\forall s_t \in \mathcal{S}, \pi^{p_1} \in \Pi^{p_1}, \pi^{p_2} \in \Pi^{p_2}.$$

such that

$$v_{p_1}^* \leq v^* \leq v_{p_2}^* \quad \forall \{\pi_t^{p_1}, \pi_t^{p_2}\}_{0 \leq t \leq T}.$$

- $p_1, p_2$  respectively generating a **mixed strategy** via **averaging the outcome** of individual plays.

# Training and Validation Loss

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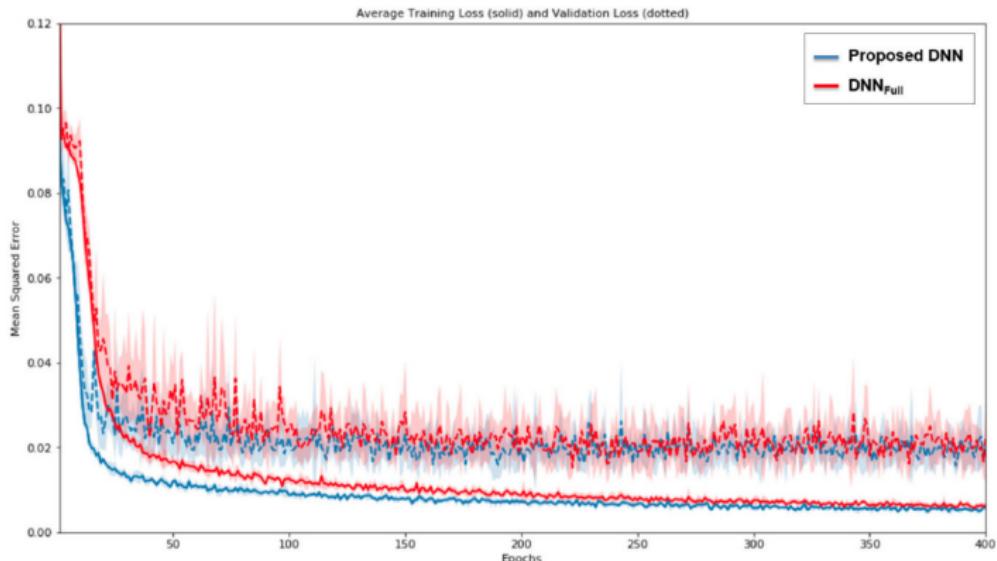
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*Average training (solid) and validation (dotted) loss function (MSE) values across six cross-validation folds for the network (blue) and full network.*

# BOO Results: Testing of self-play network

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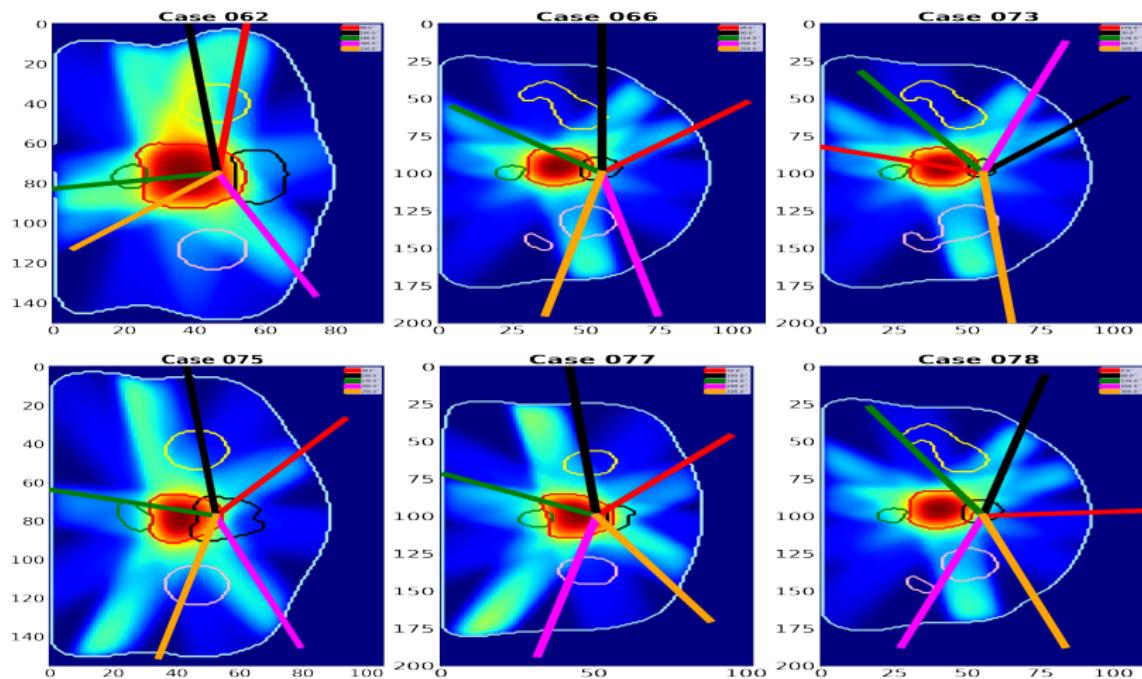
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# Column Generation vs Neural Network

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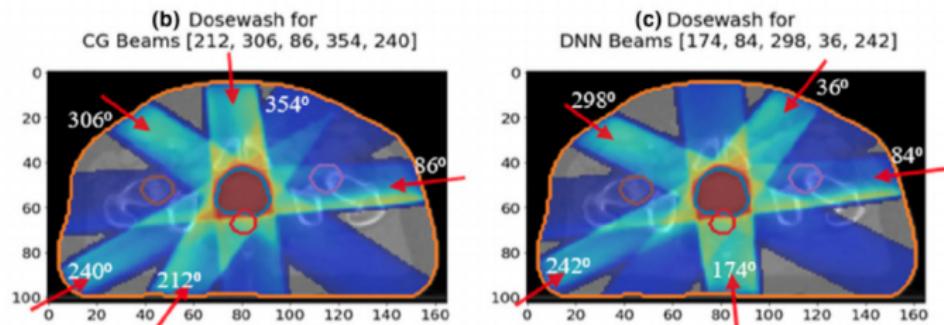
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Dose-Volume Histogram of CG vs DNN architectures [Sadeghnejad Barkousaraie, Azar and Ogunmolu, Olalekan and Jiang, Steve and Nguyen, Dan (2019)].

# Conclusions

- Deep Neural Network optimizes network weights in a separate multiprocessing thread; Network outputs probabilities used to guide search;
- Sparse lookahead search builds tree with nodes labeled by state-action pairs in an alternating manner; sample rewards stored on edges connecting state-action with state nodes;
- Beam angles prediction takes between 2-3 minutes with MCTS vs.  $\sim 60$  seconds with Column Generation Pre-training.

# Head Stabilization in Radiation Therapy (RT)

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- Head Stabilization in Cancer Radiation Therapy
  - Intensity-Modulated RT (IMRT)

# Robotic Radiosurgery

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## A Patient Head Motion-Correction Mechanism for MRI-LINAC RT

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DEPARTMENT OF RADIATION ONCOLOGY, PENN SCHOOL OF MEDICINE

- Current Collaborators: Rodney Wiersma & Xinmin Liu (UChicago → UPenn)
- Past Collaborators: Steve Jiang, Xuejun Gu, (UT Southwestern); Nick Gans (UT Dallas, UT Arlington)

# Correcting Head Motion: RT and MRI-LINACs

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(a) The BRW SRS Frame [Chelvarajah et al. (2004)]



(b) Thermoplastic masks



(c) Frame With MRI Coils (PSOM)

# 4-D Motion Correction Stage

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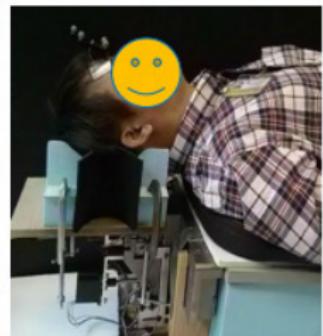
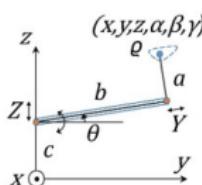
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Liu et al. (2015)

# 4-DOF Motion Controller Block Diagram

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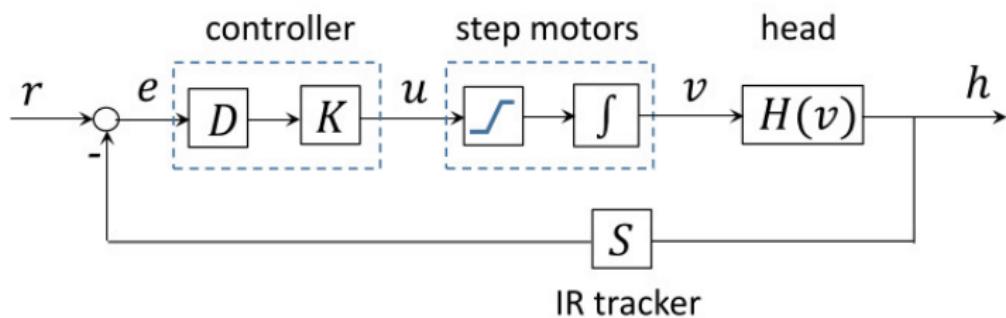
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Liu et al. (2015)

# Phantom Feedback Motion Correction Results

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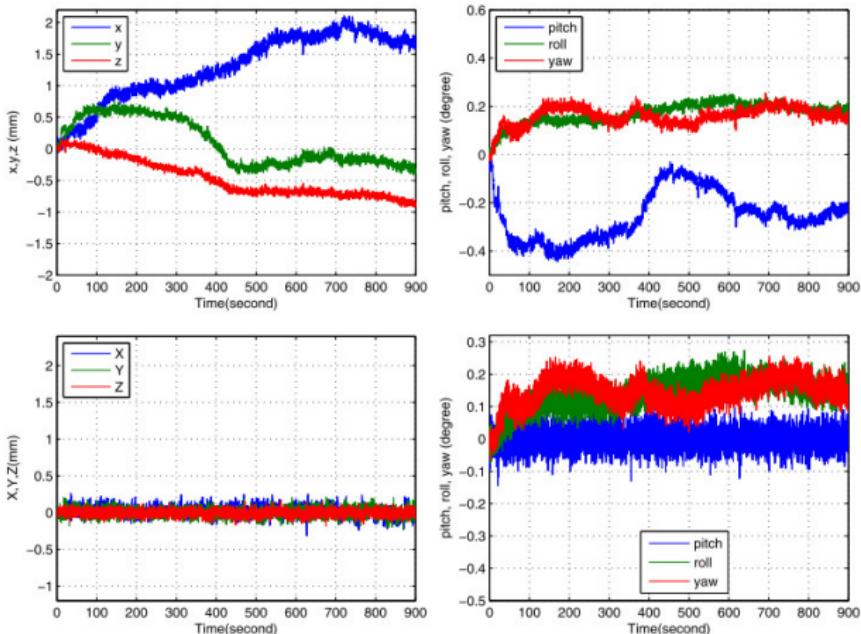
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Time response of feedback control without (left) and with (right) decoupling control [Liu et al. (2015)].

# Human Volunteer Feedback Motion Correction Results

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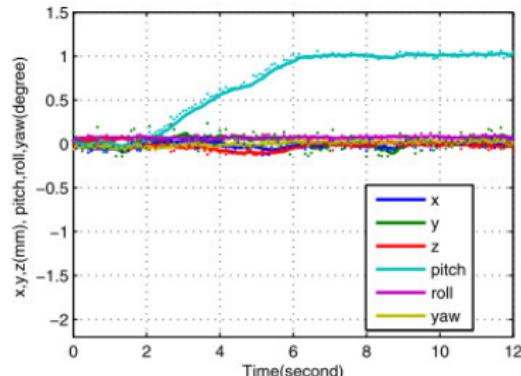
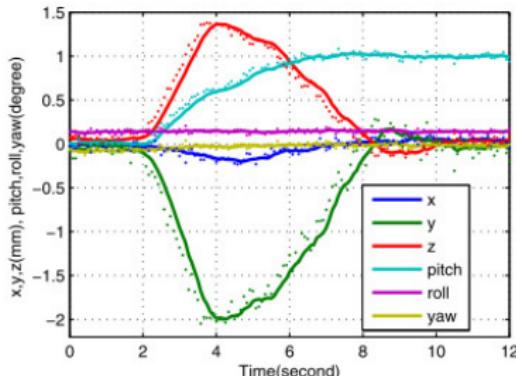
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Head Motion Without and With Motion Correction. Left: Coupled Axes; Right: Decoupled Axes.

# SRS: Wiersma Stewart-Gough Platform

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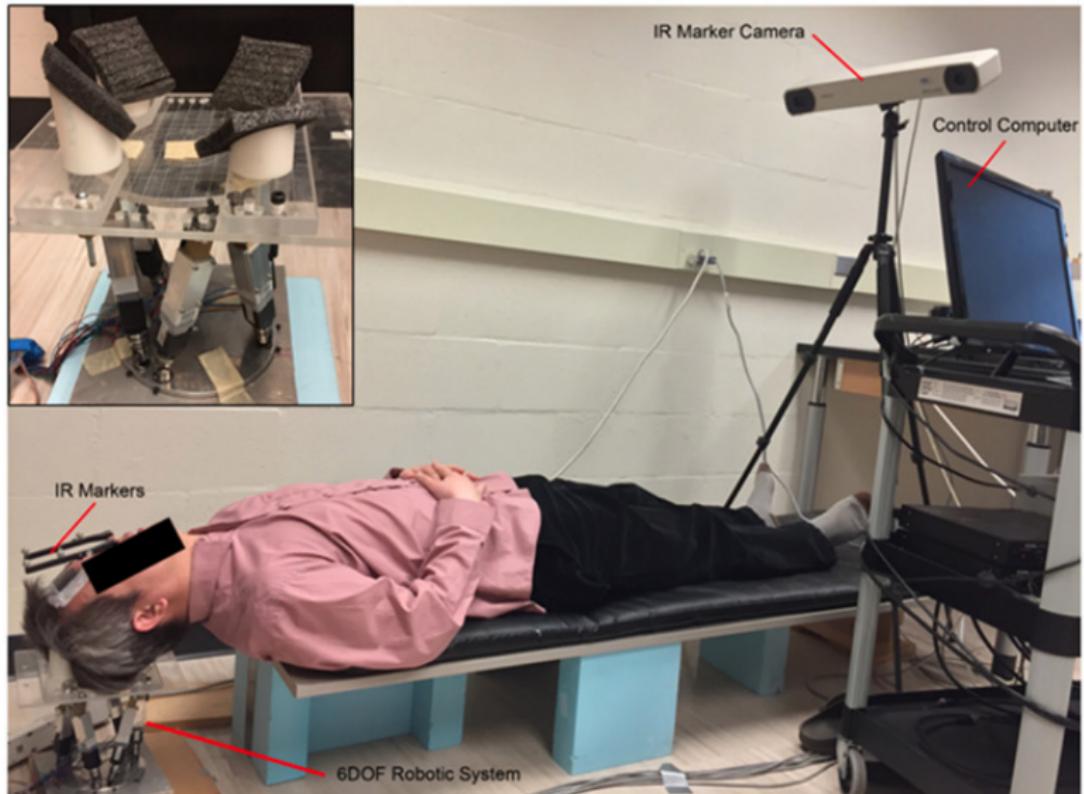
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# 6-DOF Motion Correction Results

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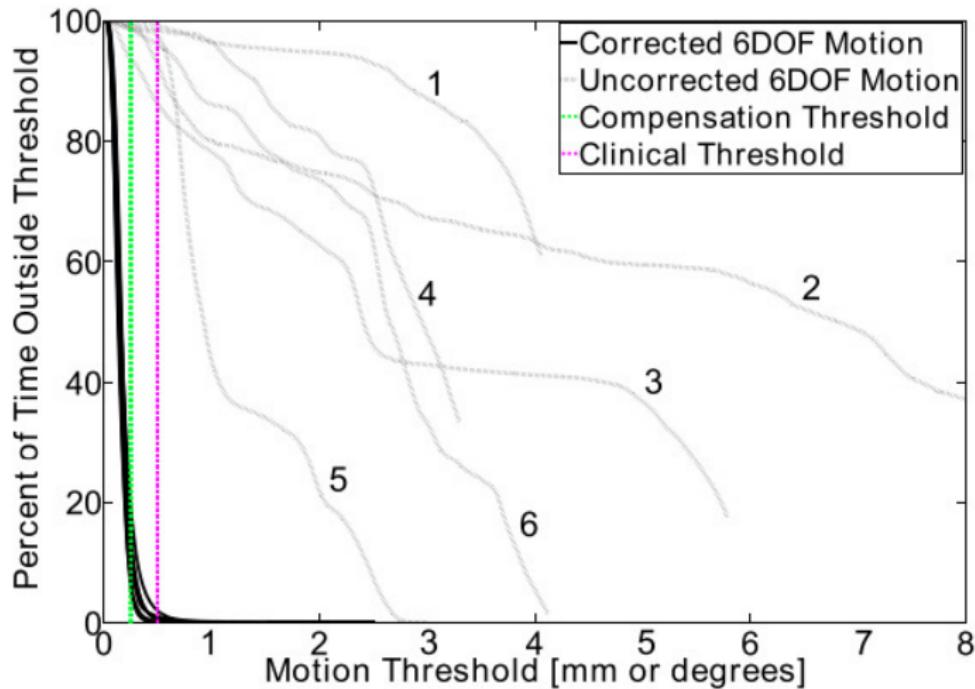
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# Drawbacks of current solutions

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- Rigid patient's body assumption
- Non-compliant immobilization devices
- Invasiveness during radiosurgery/RT
- Attenuation of photon beams

# Radiation Delivery Couch and Gantry

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Varian's TrueBeam Radiotherapy System.

# Next-Gen RT Treatment with MRI-LINACs

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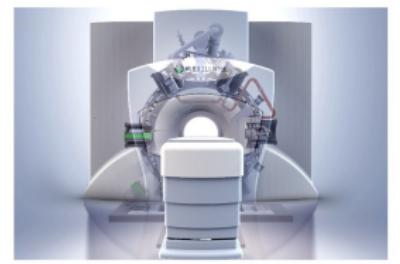
Elekta AB's (Sweden)



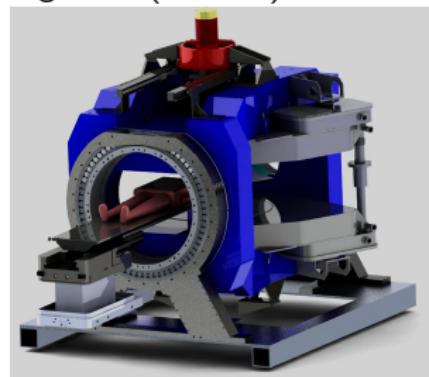
Elekta AB's (Sweden)



ViewRay's MRIdian



MagnetTx (Canada) Aurora RT



# Soft Robots for Head Motion Compensation

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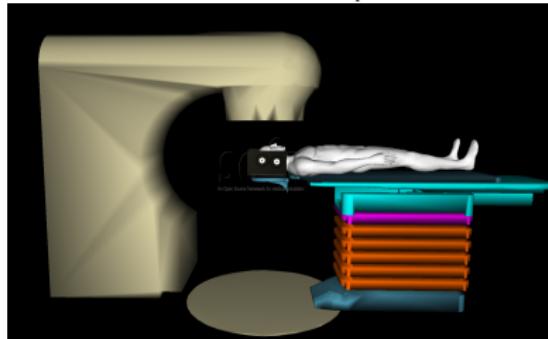
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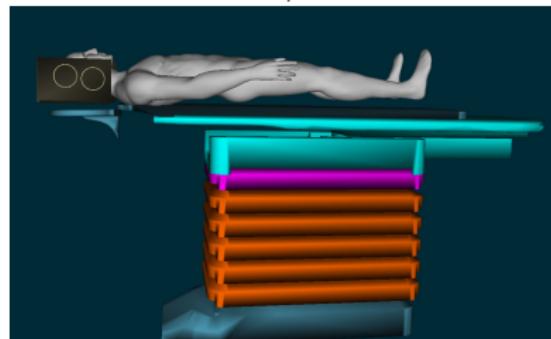
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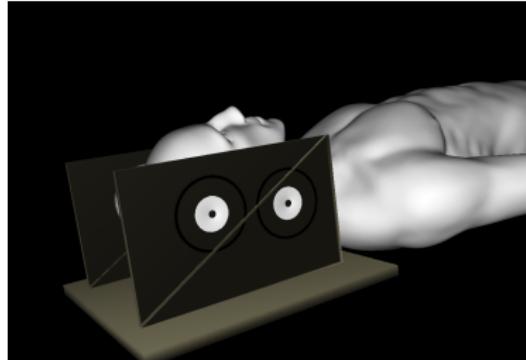
IMRT Setup



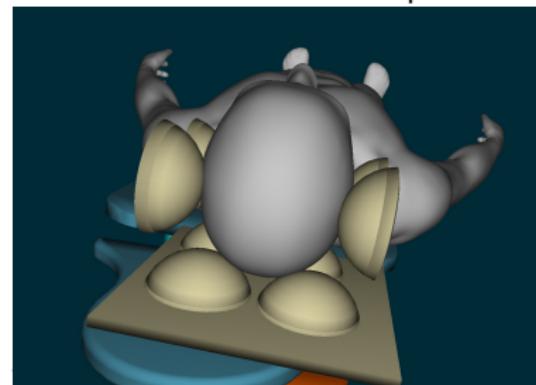
IMRT/MRI



Head and Robot Panel



Head-Robot Closeup



# Morphing in Cephalopods

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Morph Stage 1



Morph Stage 2



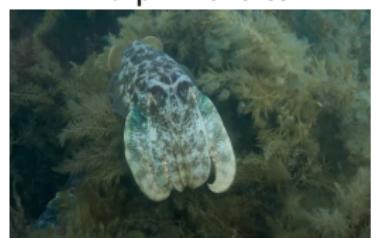
Morph Stage 3



Morph Stage 4



Morph Stage 5



Morph Reversal

©Roger Hanlon, YouTube.

# Cephalopods Neural-Controlled Physical Texture

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Raises Periscope



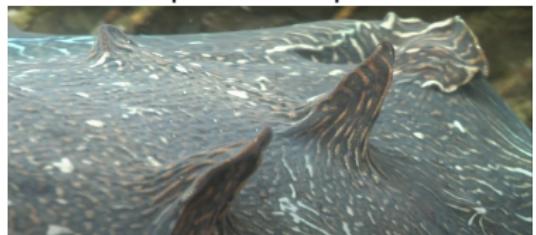
Papillae



Papillae Up



Papillae × Papillae



Cuttlefish' Morphin ©Roger Hanlon, YouTube.

# Cephalopods-inspired Actuator Design

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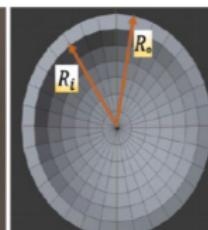
Circumferentially Constrained And Radially Symmetric Elastomers (CCOARSE).



(a) Laser-Cut Fiber



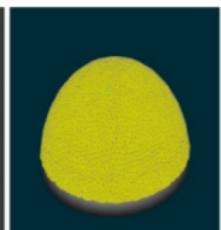
(b) Fiber+Rubber



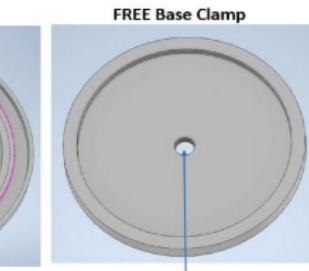
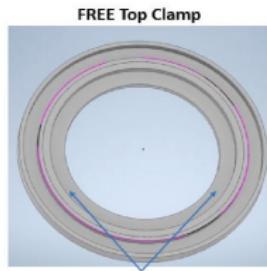
(c) Deformation Model



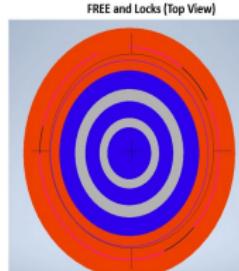
(d) Actuator+PVC Base



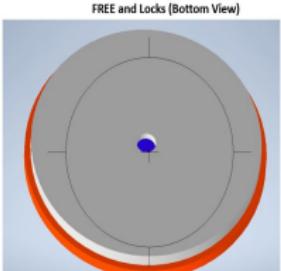
(e) Finite Element Model



(f,g)



FREE and Locks (Top View)



FREE and Locks (Bottom View)

(h)

[Pikul et al. (2019)]

# Nonlinear Elastic Deformation Analysis

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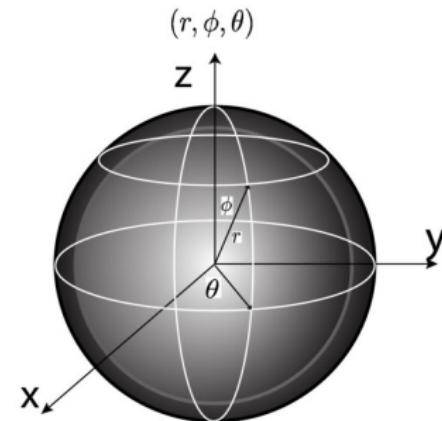
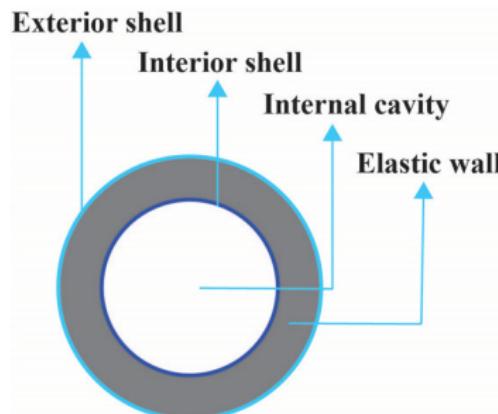
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## IAB SHELLS AND AIR CAVITY/DEFORMATION ANALYSIS



$$r_i \leq r \leq r_o, \quad 0 \leq \theta \leq 2\pi, \quad 0 \leq \phi \leq \pi$$

# Soft IK via Boundary Value Problem

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- With Cauchy's laws of motion, solve boundary value problem of traction, and find that

$$P(r) = \int_{r_i}^{r_0} \left[ 2C_1 \left( \frac{r}{R^2} - \frac{R^4}{r^5} \right) + 2C_2 \left( \frac{r^3}{R^4} - \frac{R^2}{r^3} \right) \right] dr \quad (1)$$

- i.e. Given a prescribed radius, find pressure to deform actuator between configurations

# Volumetric Deformation Results (Simulation)

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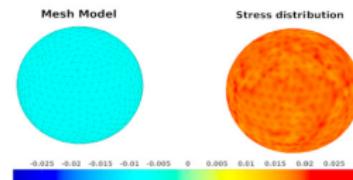
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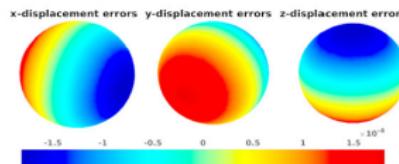
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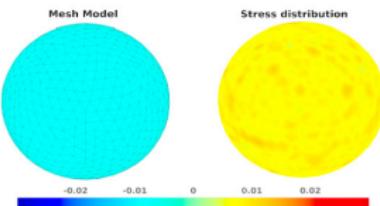
(a) Left: Mesh model. Right: Stress distribution on outer skin.



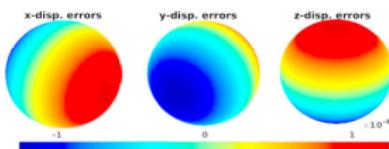
(b) Displacement errors along  $x, y, z$  coordinates.

Inputs				Outputs			
$C_1$	$C_2$	$R_i$	$r_i$	$R_o$	$r_o$	$P$	$\Delta V$
1.1e4	2.2e4	.027	.03	.03	.033	.76	$\approx 0$

Fig. 6: Volumetric Deformation (Expansion).



(a) Left: Mesh model. Right: Stress distribution on outer skin.



(b) Displacement errors along  $x, y, z$  coordinates.

Inputs				Outputs			
$C_1$	$C_2$	$R_i$	$r_i$	$R_o$	$r_o$	$P$	$\Delta V$
1.1e4	2.2e4	.025	.03	.03	.028	-.34	$\approx 0$

Fig. 7: Volumetric Deformation (Compression).

# Pneumatic Control and Deformation Scheme

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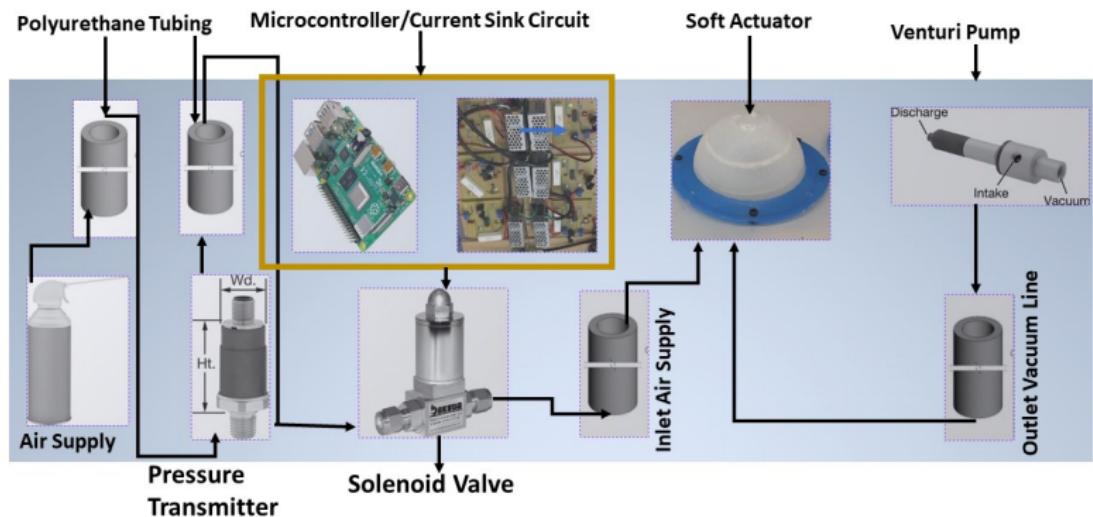
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# Volumetric Deformation Results (Actual)

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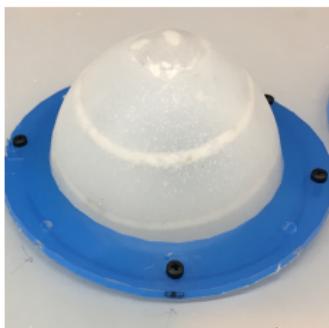
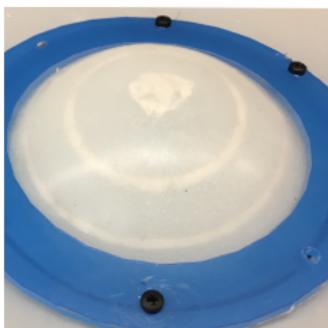
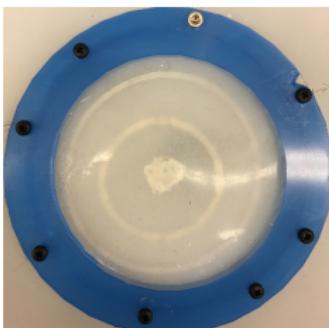
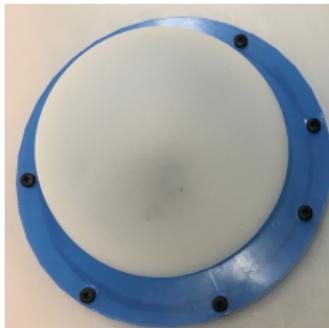
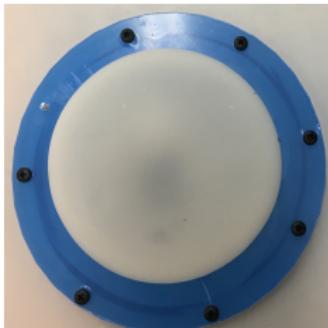
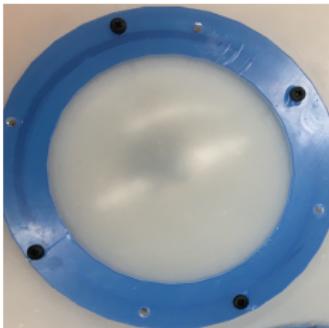
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# Actuator and Overall Mechanism with Phantom

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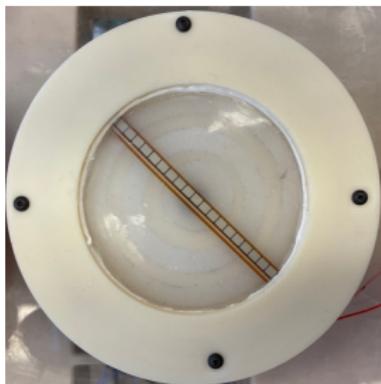
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# Head Motion Open Loop Control

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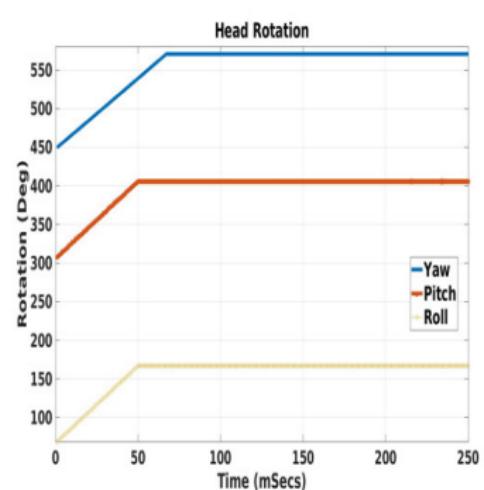
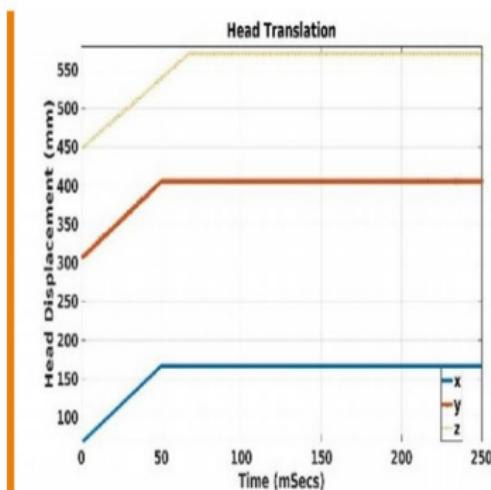
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## Independent Actuation



Head Translation along  $x, y, z$  for a task of raising the head by a certain threshold above the table

Head rotation in Euler angles for a task of tilting the head about the  $x, y, z$  axes on the treatment table.

# Ongoing Work: 6-DOF Closed-loop Control

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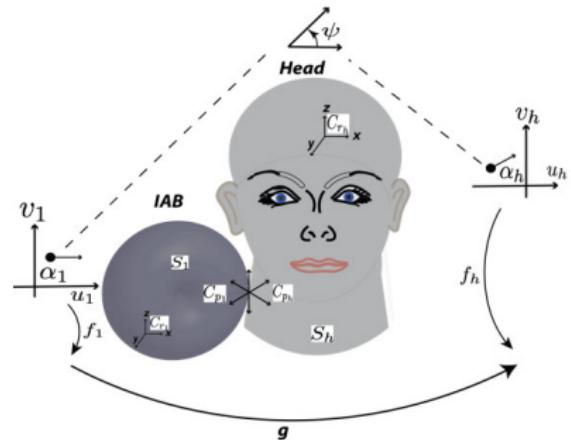
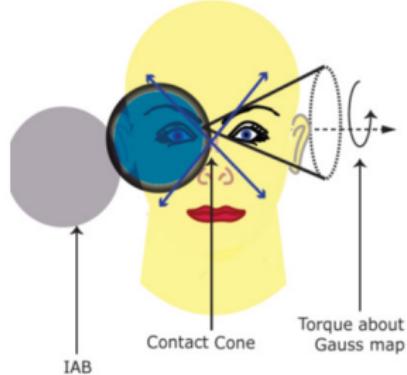
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Continuum Mechanical Model Validation/Differential Geometry/Newton-Euler Dynamics



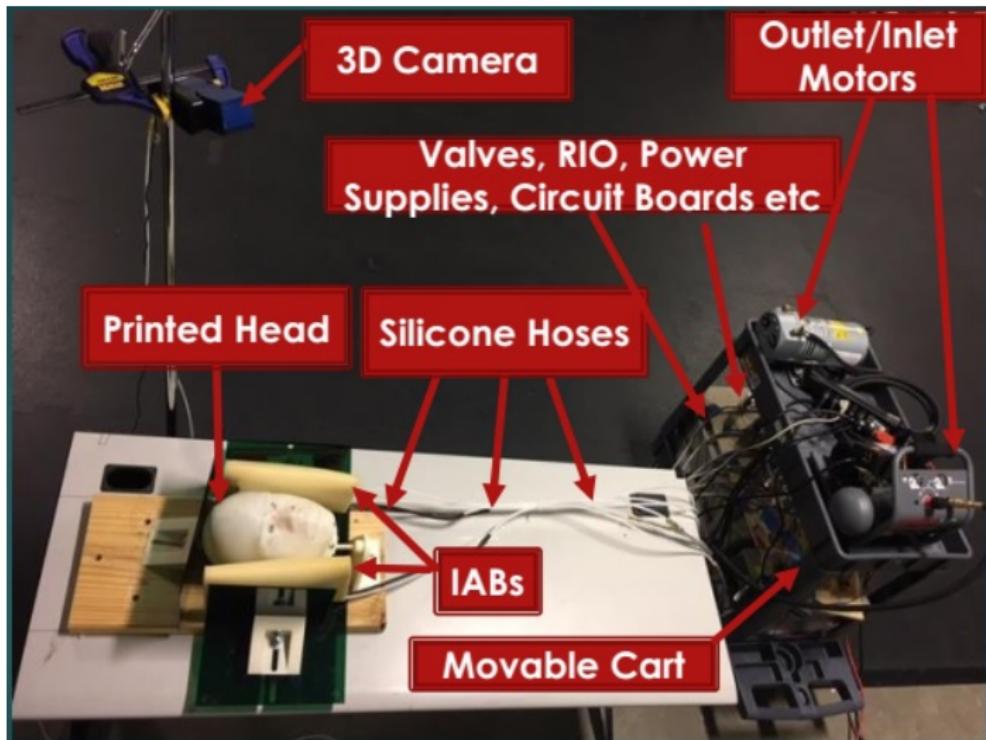
## 3-DOF Simulation Testbed

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# 3-DOF Model Reference Adaptive Control

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- Model head and bladder dynamics as
  - $\dot{x} = Ax + B\Lambda(u - f(x, u)) + w(k)$
- Approximate  $f(x, u)$  by a neural network with continuous memory states
- Derive adaptive adjustment mechanism from Lyapunov analysis for Adaptive Control Parks (1966)
  - $u = \underbrace{\hat{K}_x^T x}_{\text{state feedback}} + \underbrace{\hat{K}_r^T r}_{\text{optimal regulator}} + \underbrace{\hat{f}(x, u)}_{\text{approximator}}$

# Neural Network Architecture

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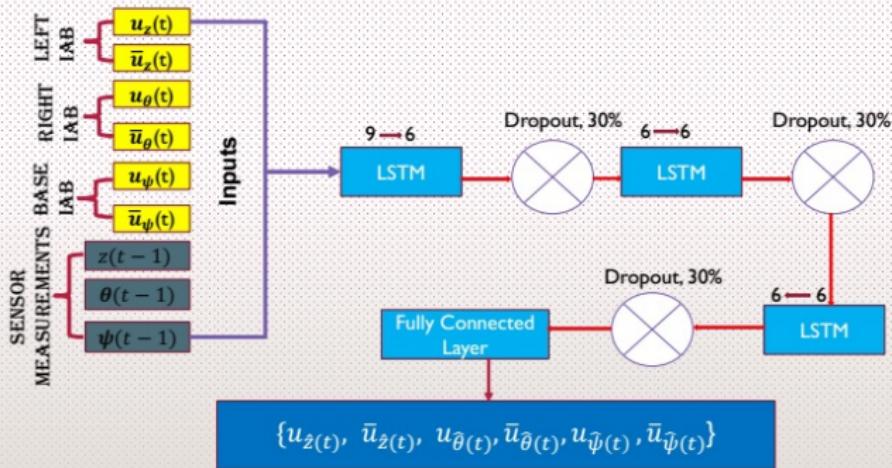
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## Neural Net Architecture



# Lyapunov Redesign: Theorem

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- For correct adaptive gains,  $\hat{\mathbf{K}}_x$  and  $\hat{\mathbf{K}}_r$ ,  $\mathbf{e}(k)$  is ***uniformly ultimately bounded***, and the state  $\mathbf{x}$  converges to a neighborhood of  $\mathbf{r}$ .
- Choose a  $\mathbf{V}$  in terms of  $\mathbf{e}$ ;  $\tilde{\mathbf{K}}_x^T$ ,  $\tilde{\mathbf{K}}_r^T$ ; and parameter error  $\varepsilon_f(\mathbf{x}(k))$  space

$$\mathbf{V}(\mathbf{e}, \tilde{\mathbf{K}}_x, \tilde{\mathbf{K}}_r) = \mathbf{e}^T \mathbf{P} \mathbf{e} + \text{tr}(\tilde{\mathbf{K}}_x^T \Gamma_x^{-1} \tilde{\mathbf{K}}_x | \Lambda |) + \text{tr}(\tilde{\mathbf{K}}_r^T \Gamma_r^{-1} \tilde{\mathbf{K}}_r | \Lambda |)$$

# Results: Z and Pitch Motions

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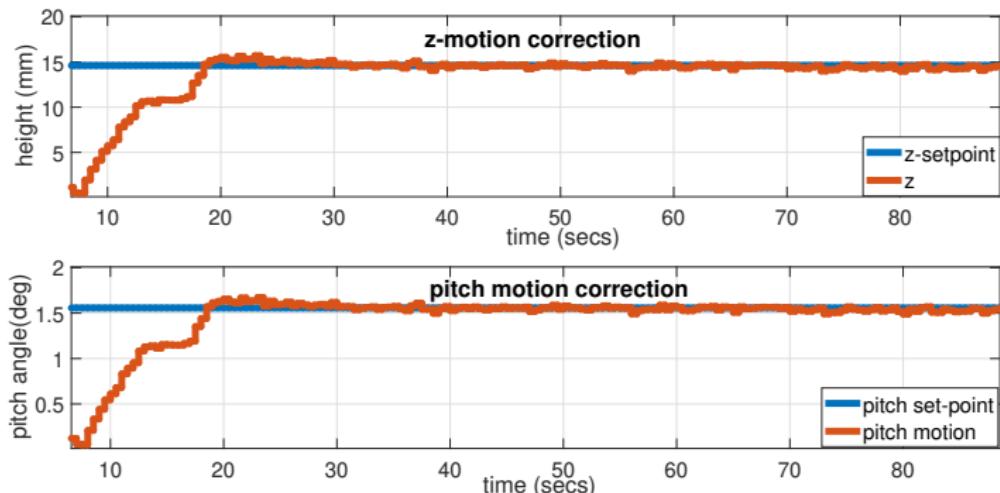
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Goal command:  $(z, \theta, \phi) = (14\text{mm}, 1.6^\circ, 45^\circ)^T$ .

# Results: Roll Motion

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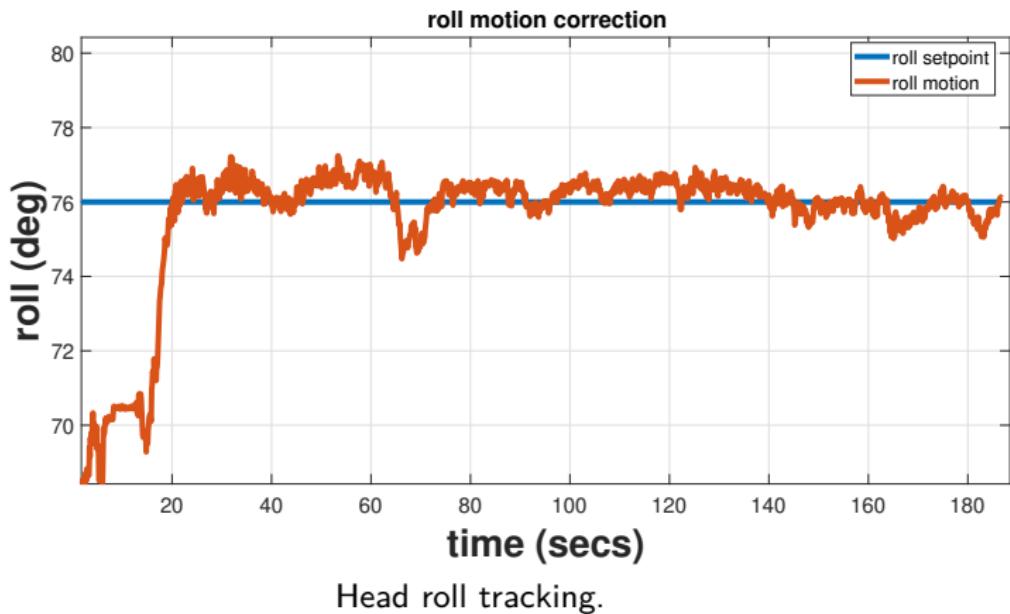
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# Conclusions

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- Non-invasive soft robot for head motion compensation ✓
- Photons-transparent as opposed to rigid/electro-mechanical devices/robots ✓
- Adaptable under MRI coils for newer MRI-LINACs ✓

# Part III: Robustness Margins and Robust Deep Policies

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- Robustness Margins and Robust Deep Policies for Nonlinear Control

# The robustness conundrum

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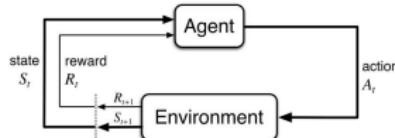
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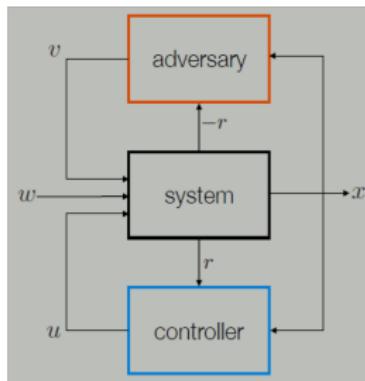
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References

- How to know *a priori* a policy's robustness limits?



- How to inculcate robustness into multistage decision policies?



# Innovation

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- To quantify the brittleness, we optimize the stage cost

$$\max_{\mathbf{v}_t \sim \psi \in \Psi} \left[ \sum_{t=0}^T \underbrace{c(\mathbf{x}_t, \mathbf{u}_t)}_{\text{nominal}} - \gamma \underbrace{g(\mathbf{v}_t)}_{\text{adversarial}} \right]$$

- To mitigate lack of robustness, we optimize the *cost-to-go*

$$\mathcal{J}_t(\mathbf{x}_t, \pi, \psi) = \min_{\mathbf{u}_t \sim \pi} \max_{\mathbf{v}_t \sim \psi} \left( \sum_{t=0}^{T-1} \ell_t(\mathbf{x}_t, \mathbf{u}_t, \mathbf{v}_t) + L_T(\mathbf{x}_T) \right),$$

- and seek a saddle point equilibrium policy that satisfies

$$\mathcal{J}_t(\mathbf{x}_t, \pi^*, \psi) \leq \mathcal{J}_t(\mathbf{x}_t, \pi^*, \psi^*) \leq \mathcal{J}_t(\mathbf{x}_t, \pi, \psi^*),$$

# Results: Brittleness Quantification

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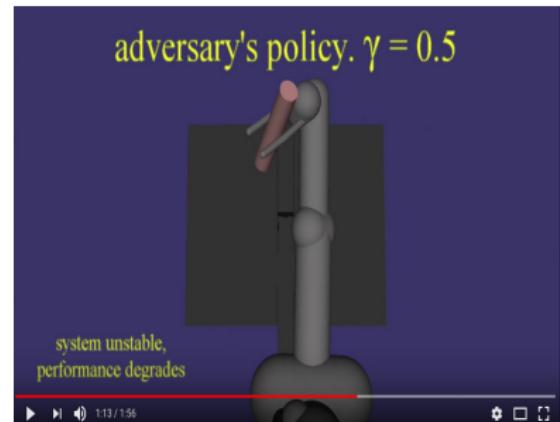
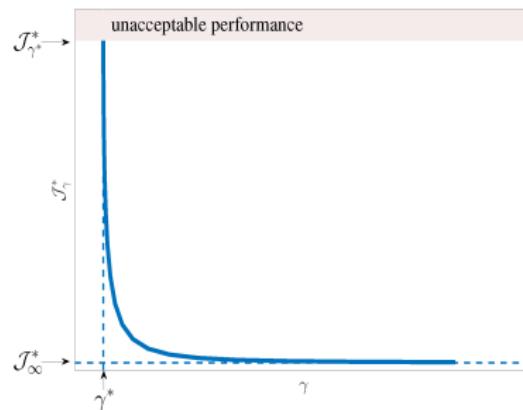
Robustness issues

Innovation

iDG Results

Future Work

References



# Results: Iterative Dynamic Game

Automating Treatment Planning in Radiation Therapy

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MRI-LINACs

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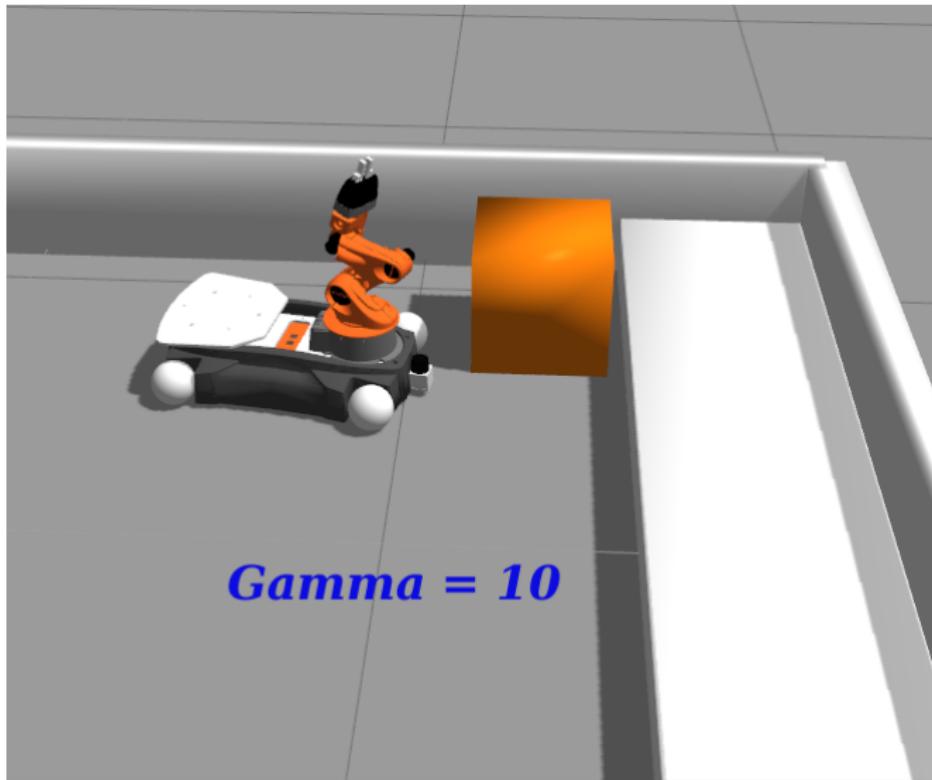
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# Future Work: MRI/RT Immobilization

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References

- Explore multiple parallel robot mechanisms for head motion correction.
- Adopt iterative dynamic game approach [Ogunmolu et al. (2018)] for solving robust controller for head stabilization.
- Build on Freeman and Kokotovic's point-wise min-norm robust control lyapunov function to realize a meaningful value function in deep policies [Freeman and Kokotovic (1996)].

# Conclusions

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References

- Designed a non-invasive soft robot for head motion compensation in IMRT/emerging MRI-LINACs ✓
- Photons-transparent; Adaptable under MRI coils for newer MRI-LINACs ✓
- Fast inference of beam orientations in treatment planning:  
Approx 60 secs beams prediction time✓
- Adapted  $H_{\infty}$  control methods for quantifying the brittleness of deep policies✓
- Devised a min-max-trained deep saddle policy for mitigating model mismatch, transfer errors, and policy sensitivity e.t.c. ✓

# End of Slides/More Acknowledgments

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Future Work

References

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# Publications

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Stabilization

Background –  
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Future Work

References

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# App A: 3-DOF Closed-loop Control

Automating  
Treatment  
Planning in  
Radiation  
Therapy

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## Appendices

App A: 3DOF CL  
Control

App B: 1DOF  
Kalman Filtering

Sensor Fusion

System  
Identification

App C: COARSE  
Actuator

Closed-loop Phantom motion control along 3 DoFs with an adaptive neuro-controller.

# 3-DOF Simulation Testbed

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Radiation  
Therapy

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## Appendices

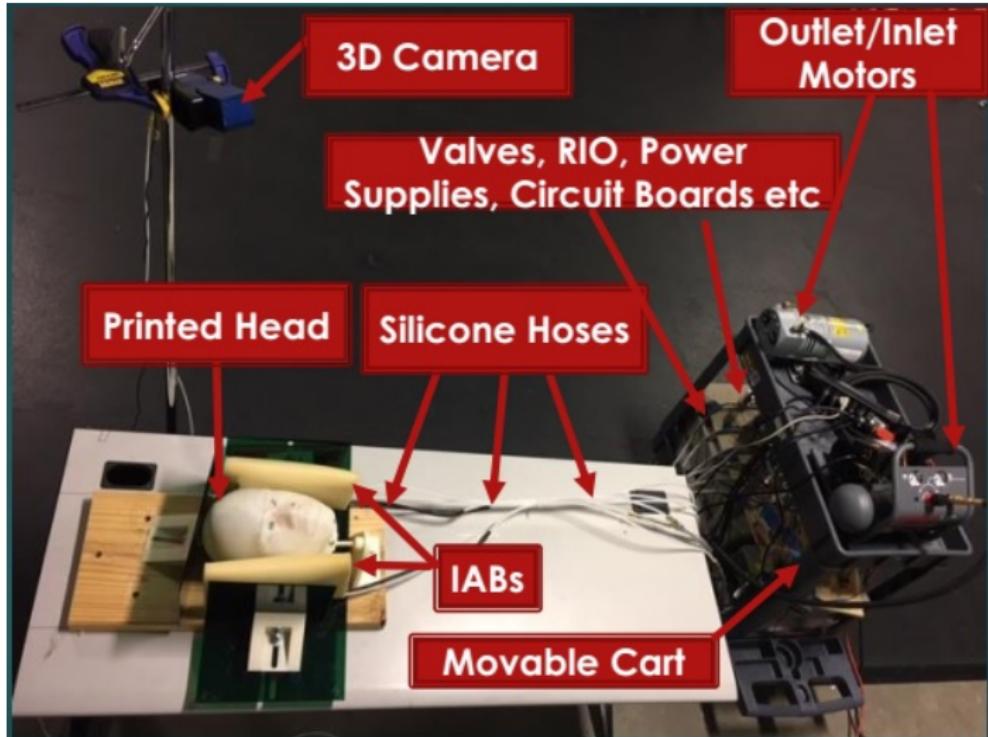
App A: 3DOF CL  
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## Hardware Description

# Control Design Goals

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App A: 3DOF CL  
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Actuator

- Stabilize  $z$ , pitch, and roll states, *i.e.*

$$\mathbf{x} = \begin{pmatrix} z \\ \theta \\ \phi \end{pmatrix}$$

- By solving an adaptive state feedback controller, optimal regulation, and minimize parametric uncertainties

# Control Design Goals

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Treatment  
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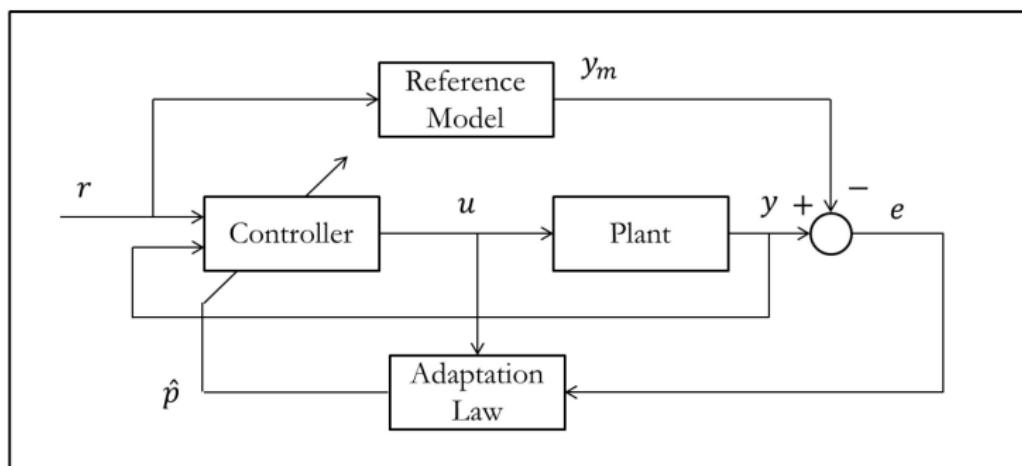
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App C: CCOARSE  
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- Provide closed loop tracking given a desired trajectory,  $r$
- Robustify system to (non-)parametric uncertainties



Indirect MRAC system. (Source mdpi.com)

# Lyapunov Redesign: Theorem

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- For correct adaptive gains,  $\hat{\mathbf{K}}_x$  and  $\hat{\mathbf{K}}_r$ ,  $\mathbf{e}(k)$  is ***uniformly ultimately bounded***, and the state  $\mathbf{x}$  converges to a neighborhood of  $\mathbf{r}$ .
- Choose a  $\mathbf{V}$  in terms of  $\mathbf{e}$ ;  $\tilde{\mathbf{K}}_x^T$ ,  $\tilde{\mathbf{K}}_r^T$ ; and parameter error  $\varepsilon_f(\mathbf{x}(k))$  space

$$\mathbf{V}(\mathbf{e}, \tilde{\mathbf{K}}_x, \tilde{\mathbf{K}}_r^T) = \mathbf{e}^T \mathbf{P} \mathbf{e} + \text{tr}(\tilde{\mathbf{K}}_x^T \Gamma_x^{-1} \tilde{\mathbf{K}}_x^T | \Lambda |) + \text{tr}(\tilde{\mathbf{K}}_r^T \Gamma_r^{-1} \tilde{\mathbf{K}}_r^T | \Lambda |)$$

# Stability proof

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$$\dot{V}(\mathbf{e}, \tilde{\mathbf{K}}_y, \tilde{\mathbf{K}}_r) = \dot{\mathbf{e}}^T \mathbf{P} \mathbf{e} + \mathbf{e}^T \mathbf{P} \dot{\mathbf{e}} + 2\text{tr}(\tilde{\mathbf{K}}_y^T \Gamma_y^{-1} \dot{\tilde{\mathbf{K}}}_y | \Lambda |) \\ + 2\text{tr}(\tilde{\mathbf{K}}_r^T \Gamma_r^{-1} \dot{\tilde{\mathbf{K}}}_r | \Lambda |)$$

$$= [\mathbf{A}_m \mathbf{e} + \mathbf{B} \Lambda [\Delta \hat{\mathbf{K}}_r^T \mathbf{r} + \Delta \hat{\mathbf{K}}_x^T \mathbf{x}]]^T \mathbf{P} \mathbf{e} + \dots$$

$$\mathbf{e}^T \mathbf{P} [\mathbf{A}_m \mathbf{e} + \mathbf{B} \Lambda [\Delta \hat{\mathbf{K}}_r^T \mathbf{r} + \Delta \hat{\mathbf{K}}_x^T \mathbf{x}]] + \dots$$

$$2\text{tr}(\Delta \mathbf{K}_x^T \Gamma_x^{-1} \dot{\hat{\mathbf{K}}}_x | \Lambda |) + 2\text{tr}(\Delta \mathbf{K}_r^T \Gamma_r^{-1} \dot{\hat{\mathbf{K}}}_r | \Lambda |)$$

# Stability Analysis

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$$= \mathbf{e}^T (\mathbf{P} \mathbf{A}_m + \mathbf{A}_m^T \mathbf{P}) \mathbf{e} + 2\mathbf{e}^T \mathbf{P} \mathbf{B} \boldsymbol{\Lambda} \left( \tilde{\mathbf{K}}_y^T \mathbf{y} + \tilde{\mathbf{K}}_r^T \mathbf{r} \right)$$

$$+ 2\mathbf{tr} \left( \tilde{\mathbf{K}}_y^T \boldsymbol{\Gamma}_y^{-1} \dot{\tilde{\mathbf{K}}}_y | \boldsymbol{\Lambda} | \right) + 2\mathbf{tr} \left( \tilde{\mathbf{K}}_r^T \boldsymbol{\Gamma}_r^{-1} \dot{\tilde{\mathbf{K}}}_r | \boldsymbol{\Lambda} | \right)$$

$$= -\mathbf{e}^T \mathbf{Q} \mathbf{e} - 2\mathbf{e}^T \mathbf{P} \mathbf{B} \boldsymbol{\Lambda} \boldsymbol{\varepsilon}_f(\mathbf{y}) + 2\mathbf{e}^T \mathbf{P} \mathbf{B} \boldsymbol{\Lambda} \tilde{\mathbf{K}}_y^T \mathbf{y}$$

$$+ 2\mathbf{tr} \left( \tilde{\mathbf{K}}_y^T \boldsymbol{\Gamma}_y^{-1} \dot{\tilde{\mathbf{K}}}_y \right) + 2\mathbf{e}^T \mathbf{P} \mathbf{B} \boldsymbol{\Lambda} \tilde{\mathbf{K}}_r^T \mathbf{r} + 2\mathbf{tr} \left( \Delta \mathbf{K}_r^T \boldsymbol{\Gamma}_r^{-1} \dot{\tilde{\mathbf{K}}}_r \right)$$

Notice  $x^T y = \mathbf{tr} (y x^T)$  from trace identity

# Stability Analysis Cont'd

Therefore,

$$\begin{aligned}\dot{\mathbf{V}}(\cdot) &= -\mathbf{e}^T \mathbf{Q} \mathbf{e} - 2\mathbf{e}^T \mathbf{P} \mathbf{B} \boldsymbol{\Lambda} \varepsilon_f \\ &\quad + 2 \operatorname{tr} \left( \tilde{\mathbf{K}}_y^T (\boldsymbol{\Gamma}_y^{-1} \dot{\tilde{\mathbf{K}}}_y + \mathbf{y} \mathbf{e}^T \mathbf{P} \mathbf{B} \operatorname{sgn}(\boldsymbol{\Lambda})) \right) |\boldsymbol{\Lambda}| \\ &\quad + 2 \operatorname{tr} \left( \tilde{\mathbf{K}}_r^T (\boldsymbol{\Gamma}_r^{-1} \dot{\tilde{\mathbf{K}}}_r + \mathbf{r} \mathbf{e}^T \mathbf{P} \mathbf{B} \operatorname{sgn}(\boldsymbol{\Lambda})) \right) |\boldsymbol{\Lambda}|\end{aligned}$$

where for a real-valued  $x$ , we have  $x = \operatorname{sgn}(x)|x|$ .

- first two terms will be negative definite for all  $\mathbf{e} \neq 0$ 
  - since  $\mathbf{A}_m$  is Hurwitz
- other terms will be identically null if we choose the adaptation laws

$$\dot{\tilde{\mathbf{K}}}_y = -\boldsymbol{\Gamma}_y \mathbf{y} \mathbf{e}^T \mathbf{P} \mathbf{B} \operatorname{sgn}(\boldsymbol{\Lambda}), \quad \dot{\tilde{\mathbf{K}}}_r = -\boldsymbol{\Gamma}_r \mathbf{r} \mathbf{e}^T \mathbf{P} \mathbf{B} \operatorname{sgn}(\boldsymbol{\Lambda})$$

# Stability Results: Ogunmolu et al. (2017)

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$$\begin{aligned}\dot{\mathbf{V}}(\cdot) &= -\mathbf{e}^T \mathbf{Q} \mathbf{e} - 2\mathbf{e}^T \mathbf{P} \mathbf{B} \boldsymbol{\Lambda} \boldsymbol{\varepsilon}_f \\ &\leq -\lambda_{low} \|\mathbf{e}\|^2 + 2\|\mathbf{e}\| \|\mathbf{P} \mathbf{B}\| \lambda_{high}(\boldsymbol{\Lambda}) \boldsymbol{\varepsilon}_{max}\end{aligned}$$

- $\{\lambda_{low}, \lambda_{high}\} \equiv \min/\max \text{ eigenvalues of } Q \text{ and } \boldsymbol{\Lambda}$ .
- $\dot{\mathbf{V}}(\cdot)$  is thus negative definite outside the compact set:  
$$\chi = \left( \mathbf{e} : \|\mathbf{e}\| \leq \frac{2\|\mathbf{P} \mathbf{B}\| \lambda_{high}(\boldsymbol{\Lambda}) \boldsymbol{\varepsilon}_{max}(\mathbf{y})}{\lambda_{low}(\mathbf{Q})} \right)$$
  - i.e.  $\mathbf{e}$  is uniformly ultimately bounded, or  $\mathbf{y}(t) \rightarrow 0$  as  $t \rightarrow \infty$ .

# Appendix C: 1-DOF Closed-Loop Control

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Treatment  
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# Head Pose Estimation: Sensor Fusion

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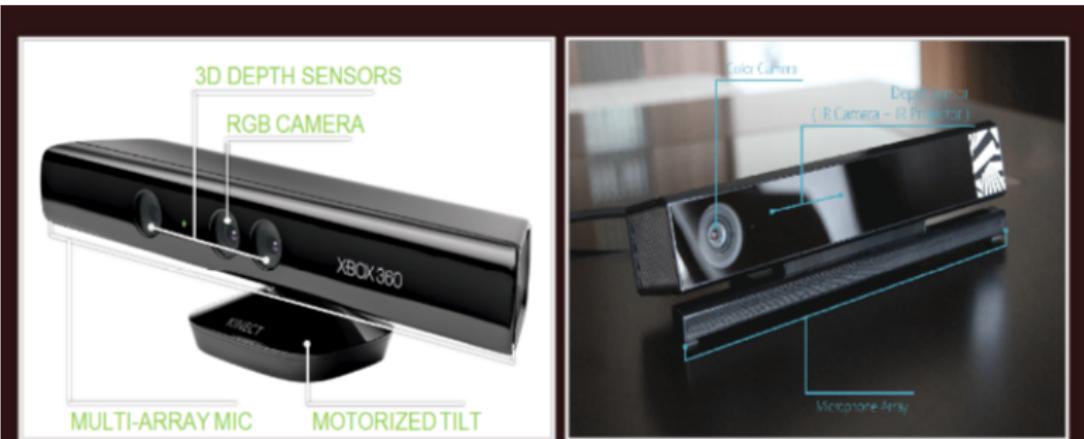
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Kinect Xbox

Kinect v1

# Sensors' Noise Floor

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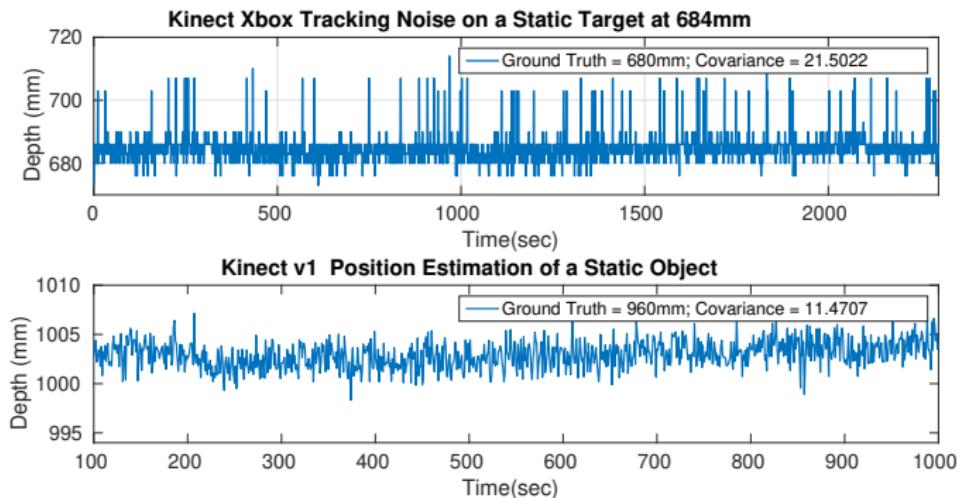
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Case for Sensed Observation Filtering

# Optimal head state estimation and sensor fusion

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- Find the observation estimates  $\hat{\mathbf{x}}(i)$  that minimize the mean-square error from true measurement, i.e.,

$$\hat{\mathbf{x}}(i|j) = \arg \min_{\hat{\mathbf{x}}(i|j) \in \mathbb{R}^n} \mathbb{E}\{(\mathbf{x}(i) - \hat{\mathbf{x}})^T (\mathbf{x}(i) - \hat{\mathbf{x}}) | z(1), \dots, z(j)\}$$

- Define the estimate error's covariance as

$$\mathbf{P}(i|j) \triangleq \mathbb{E}\{(\mathbf{x}(i) - \hat{\mathbf{x}}(i|j))^T (\mathbf{x}(i) - \hat{\mathbf{x}}(i|j)) | Z^j\}. \quad (2)$$

# State estimation with Kalman filters

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- Assume state model,  $\mathbf{F}(k)$
- For a discretized time interval  $\Delta T$  between measurements, we define the state

$$\mathbf{x}(k) = \mathbf{F}(k)\mathbf{x}(k-1) + \mathbf{B}(k)\mathbf{u}_k + \mathbf{G}_k\mathbf{w}_k \quad (3)$$

- with

$$\mathbf{F} = \begin{bmatrix} 1 & \Delta T \\ 0 & 1 \end{bmatrix} \quad (4)$$

# Kalman filters

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- (3)  $\Rightarrow \mathbf{x}_k = \mathbf{F}_k \mathbf{x}_{k-1} + \mathbf{G}_k \mathbf{w}_k$
- $\mathbf{G}_k$  := uncontrolled forces accelerating on head
- Head's acceleration,  $a_k \sim \mathcal{N}(0, \sigma_a)$
- Setting  $\mathbf{G}_k = \mathbf{I}_{2 \times 2}$  and  $\mathbf{w}(k) \sim \mathcal{N}(0, \mathbf{Q}(k))$ 
  - set  $\mathbf{Q}(k)$  to a random walk sequence,  $\mathbf{W}_k = [\frac{\Delta T^2}{2}, \Delta T]^T$

# Kalman Filters Design

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- Such that

$$\mathbf{Q} = \mathbf{W}\mathbf{W}^T \sigma_a^2 = \begin{bmatrix} \frac{\Delta T^4}{4} & \frac{\Delta T^3}{2} \\ \frac{\Delta T^3}{2} & \Delta T^2 \end{bmatrix} \sigma_a^2. \quad (5)$$

- Set the transfer matrix from the estimates,  $\mathbf{x}(k)$ , to observations,  $z_1(k)$  and  $z_2(k)$  according to

$$z_s = \mathbf{H}_s(k)\mathbf{x}(k) + v_s(k) \quad s = 1, 2 \quad (6)$$

- where  $\mathbf{H}_s(k) = [1, \ 0]^T$  and  $v_s(k) \sim \mathcal{N}(0, \sigma_{rs}^2)$

# KF Priori and Posteriori State Estimates

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## ■ Prediction Phase:

$$\begin{aligned}\hat{\mathbf{x}}_{k|k-1} &= \mathbf{F}\hat{\mathbf{x}}_{k-1|k-1} + \mathbf{B}_k \mathbf{u}_k \\ \mathbf{P}_{k|k-1} &= \mathbf{F}_k \mathbf{P}_{k-1|k-1} \mathbf{F}_k^T + \mathbf{Q}_k\end{aligned}\quad (7)$$

## ■ Update Phase:

$$\begin{aligned}\mathbf{K}(k) &= \mathbf{P}(k|k-1) \mathbf{H}(k)^T [\mathbf{H}(k) \mathbf{P}(k|k-1) \mathbf{H}(k)^T + \mathbf{R}(k)]^{-1} \\ \hat{\mathbf{x}}(k|k) &= \hat{\mathbf{x}}(k|k-1) + \mathbf{K}(k)(\mathbf{z}(k) - \mathbf{H}(k)\hat{\mathbf{x}}(k|k-1)) \\ \mathbf{P}(k|k) &= (\mathbf{I} - \mathbf{K}(k)\mathbf{H}(k))\mathbf{P}(k|k-1)\end{aligned}\quad (8)$$

# Xbox Filtering Results

Automating  
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App A: 3DOF CL  
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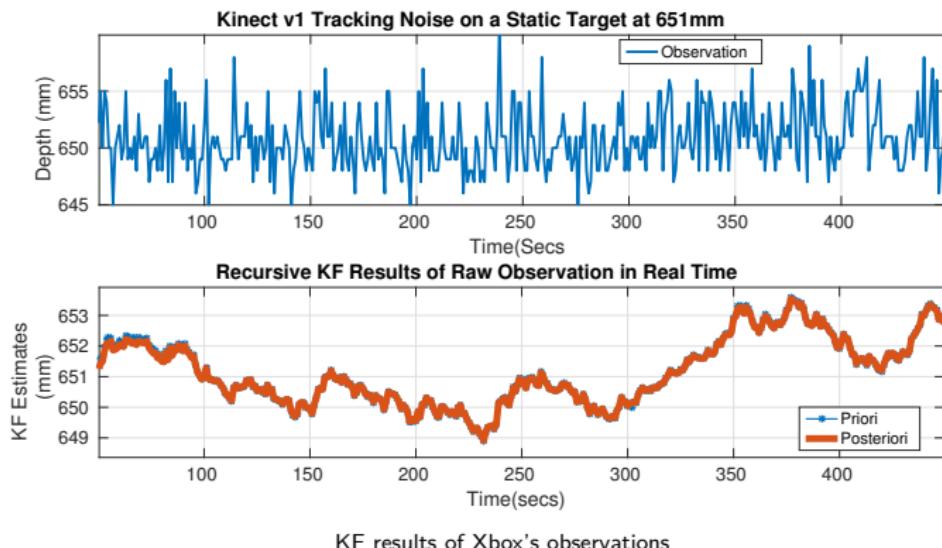
App B: 1DOF

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# Kinect 2 Filtering Results

Automating  
Treatment  
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App A: 3DOF CL  
Control

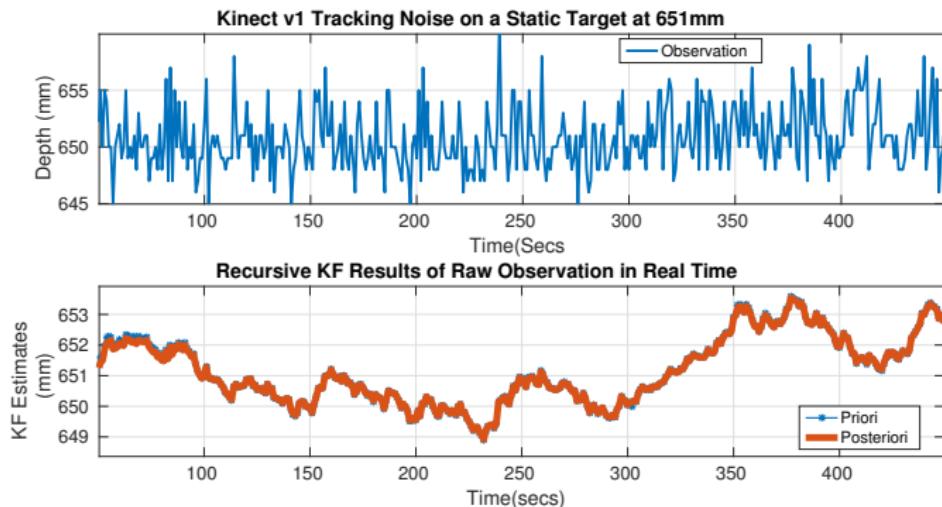
App B: 1DOF

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KF results of Kinect v1's observation

# Filtering Results

Automating  
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App A: 3DOF CL  
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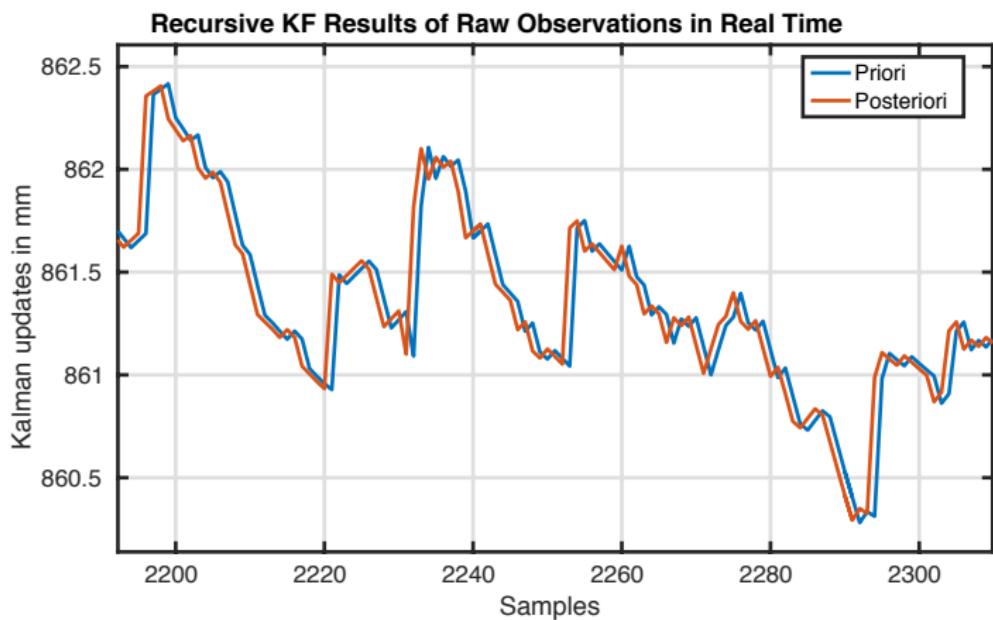
App B: 1DOF

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# Global fusion of local tracks

Automating  
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## Appendices

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### Sensor Fusion

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- Fuse both updates with a variance-weighted average of each local track as follows,

$$\hat{\mathbf{x}}(F)(k|k) = \mathbf{P}(F)(k|k) \sum_{s=1}^N \left[ \mathbf{P}(s)^{-1}(k|k) \hat{\mathbf{x}}(s)(k|k) \right]$$

$$\text{where } \mathbf{P}(F)(k|k) = \left[ \sum_{s=1}^N \mathbf{P}(s)^{-1}(k|k) \right]^{-1}.$$

# Track-to-track Fusion

Automating  
Treatment  
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Control

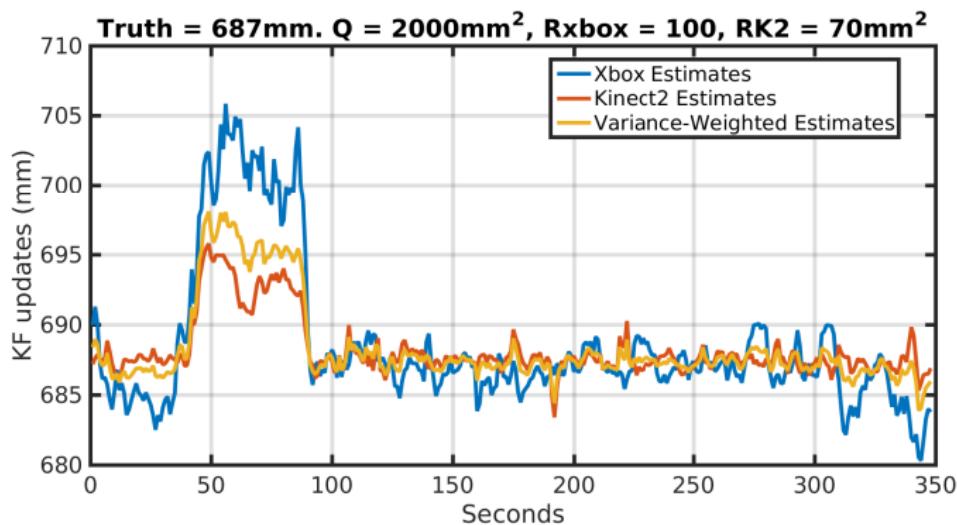
App B: 1DOF

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Track-to-track fusion of both sensors' local track estimates.

# Vision-based Control

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## Appendices

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### Sensor Fusion

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# System Identification

Automating  
Treatment  
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## Appendices

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- From I/O data, estimate a model
- Find set of optimal model parameters via the minimization,

$$G(t) = \arg \min_{\theta} V_N(\theta, Z^N)$$

- where  $V_N(\theta, Z^N) = \sum_{k=1}^K \sum_{i=1}^n \frac{1}{2} (\hat{y}_i(k) - y_i(k))^2$ ,
- and  $Z^N = \{u(1) \cdots u(N), y(1) \cdots y(N)\}$
- After a least squares minimization, we derive a state-space realization,

$$\begin{aligned} \mathbf{x}(k + Ts) &= \mathbf{Ax}(k) + \mathbf{Bu}(k) + \mathbf{Ke}(k) \\ \mathbf{y}(k) &= \mathbf{Cx}(k) + \mathbf{Du}(k) + \mathbf{e}(k) \end{aligned} \quad (9)$$

# PID-PI Cascaded Controller

Automating  
Treatment  
Planning in  
Radiation  
Therapy

Olalekan  
Ogunmolu

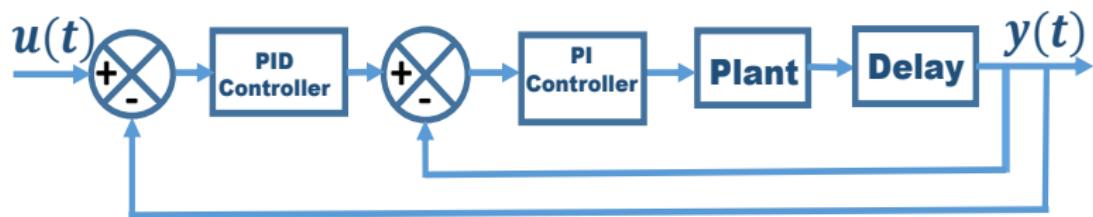
Appendices

App A: 3DOF CL  
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Actuator



# Open Loop Step Response

Automating  
Treatment  
Planning in  
Radiation  
Therapy

Olalekan  
Ogunmolu

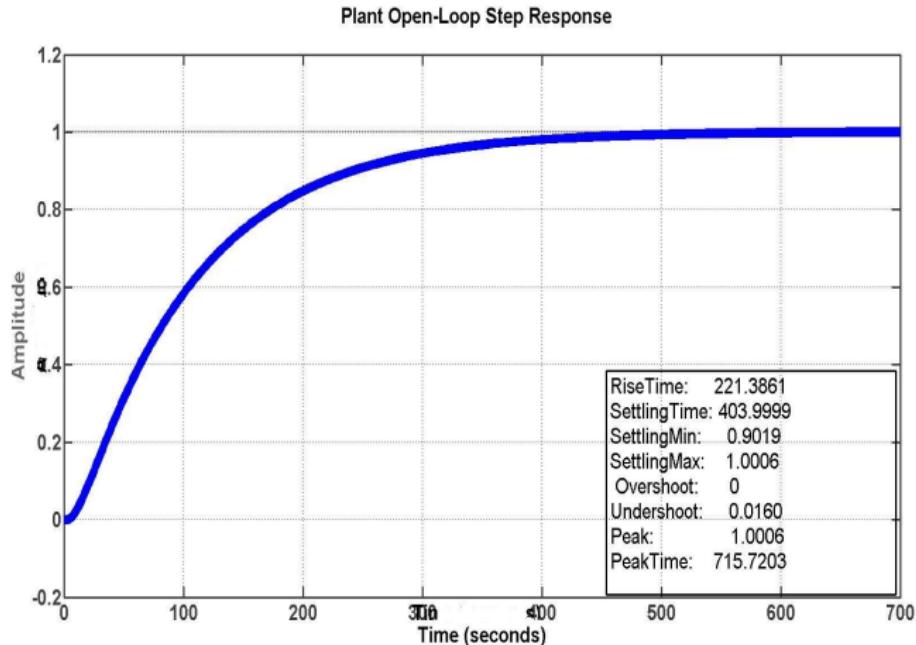
Appendices

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Actuator

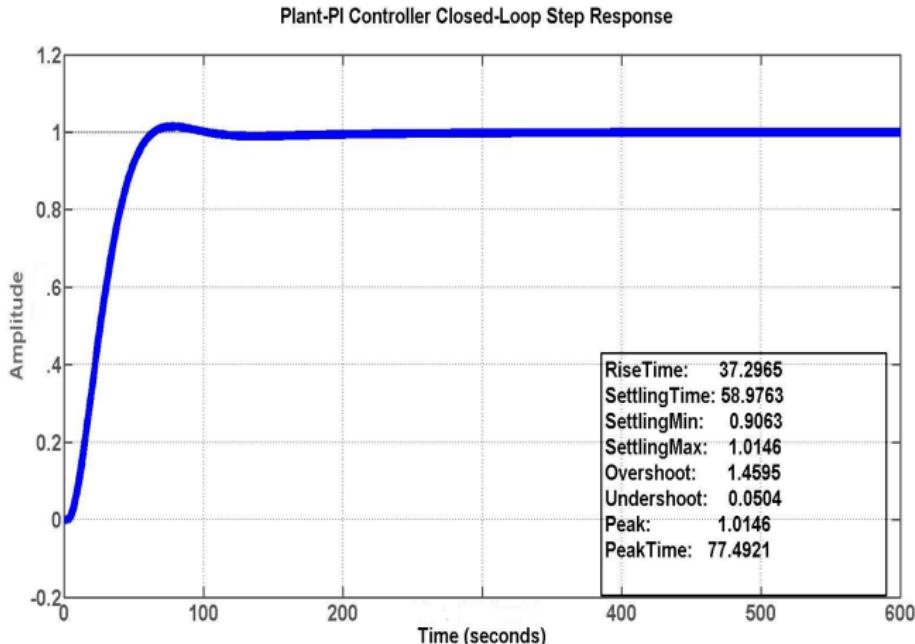


# PI Closed-loop Step Response

Automating  
Treatment  
Planning in  
Radiation  
Therapy

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# Cascaded PID-PI Closed-loop Step Response

Automating  
Treatment  
Planning in  
Radiation  
Therapy

Olalekan  
Ogunmolu

Appendices

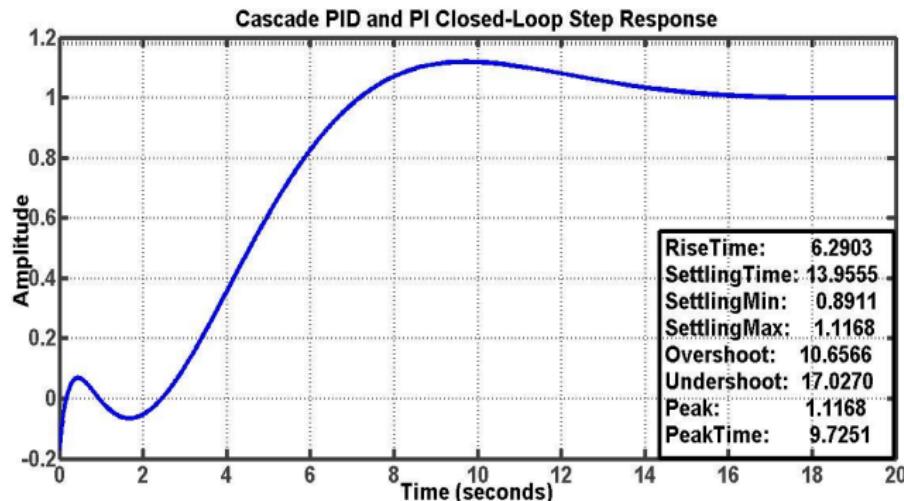
App A: 3DOF CL  
Control

App B: 1DOF

Kalman Filtering  
Sensor Fusion

System  
Identification

App C: CCOARSE  
Actuator

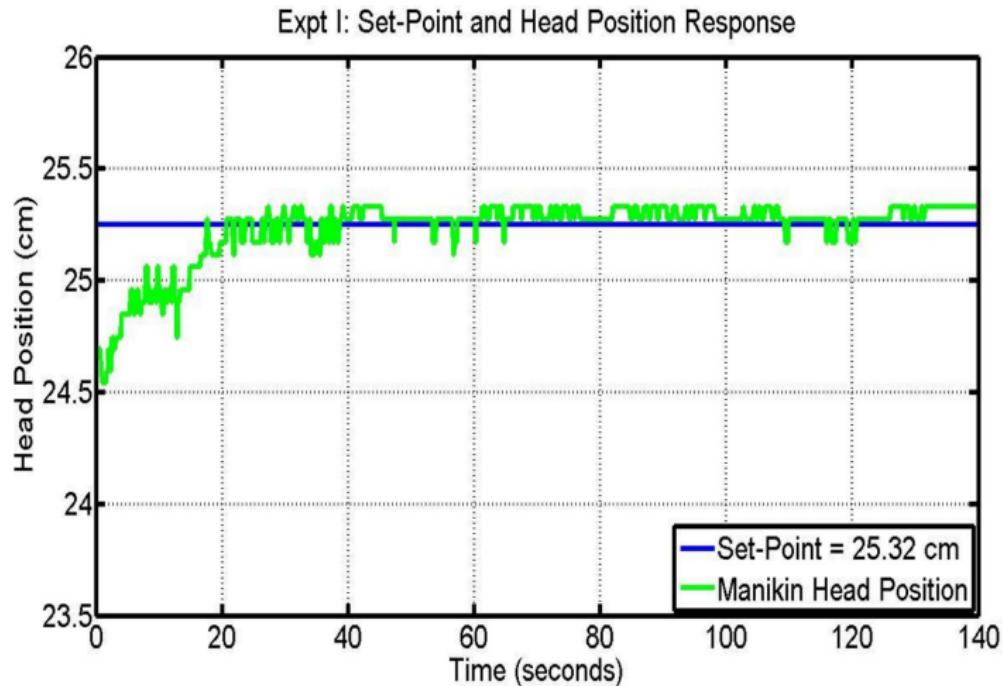


# PI-PD Cascaded Controller Experimental Results

Automating  
Treatment  
Planning in  
Radiation  
Therapy

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Ogunmolu

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App A: 3DOF CL  
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Sensor Fusion  
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Actuator



# Varying Setpoint Simulation

Automating  
Treatment  
Planning in  
Radiation  
Therapy

Olalekan  
Ogunmolu

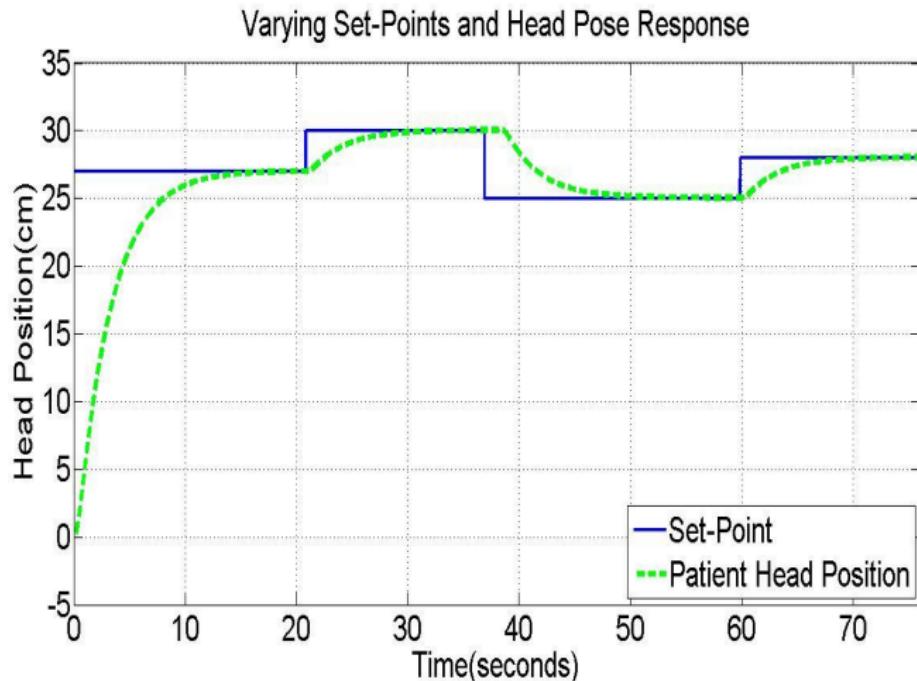
Appendices

App A: 3DOF CL  
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Kalman Filtering  
Sensor Fusion

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Actuator



# Varying Setpoint Experiment

Automating  
Treatment  
Planning in  
Radiation  
Therapy

Olalekan  
Ogunmolu

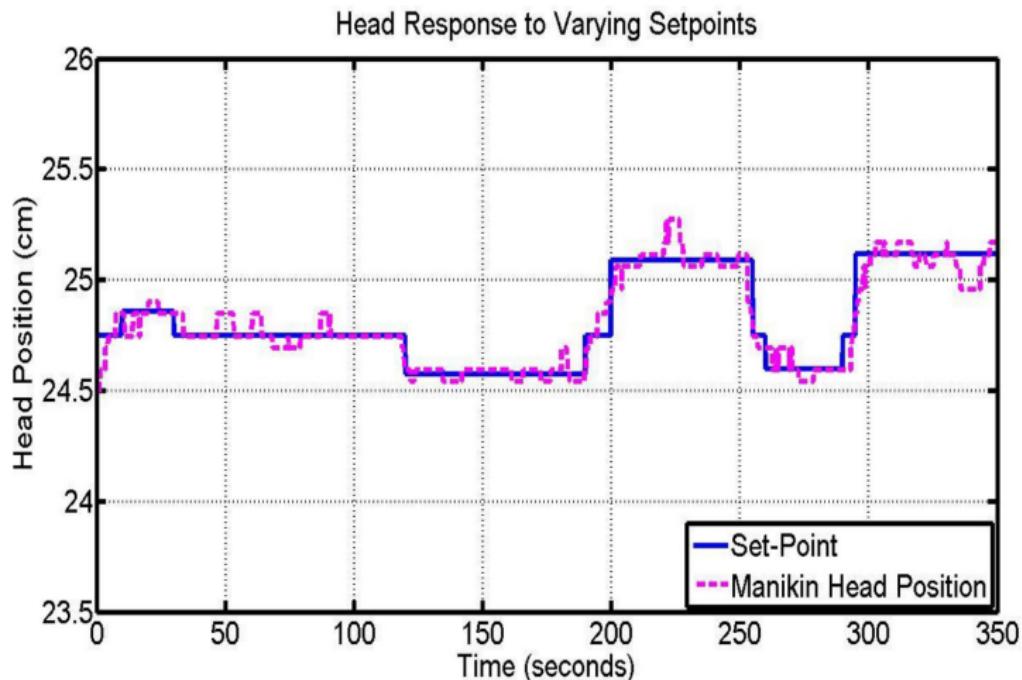
Appendices

App A: 3DOF CL  
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App B: 1DOF  
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Identification

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# 3-DOF Controller Design (IROS 2017)

Automating  
Treatment  
Planning in  
Radiation  
Therapy

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## Appendices

App A: 3DOF CL  
Control

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Identification

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Actuator

- Posing the cost

$$J = \sum_{k=0}^K x(k)^T Q x(k) + u(k)^T R u(k) + 2x(k)^T N u(k)$$

- we can obtain  $u$  as  $\Delta u = \arg \min_{\Delta u} J$
- $\Delta u$  is a future control sequence

# Closed-loop control (Full state observer).

Automating  
Treatment  
Planning in  
Radiation  
Therapy

Olalekan  
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Appendices

App A: 3DOF CL  
Control

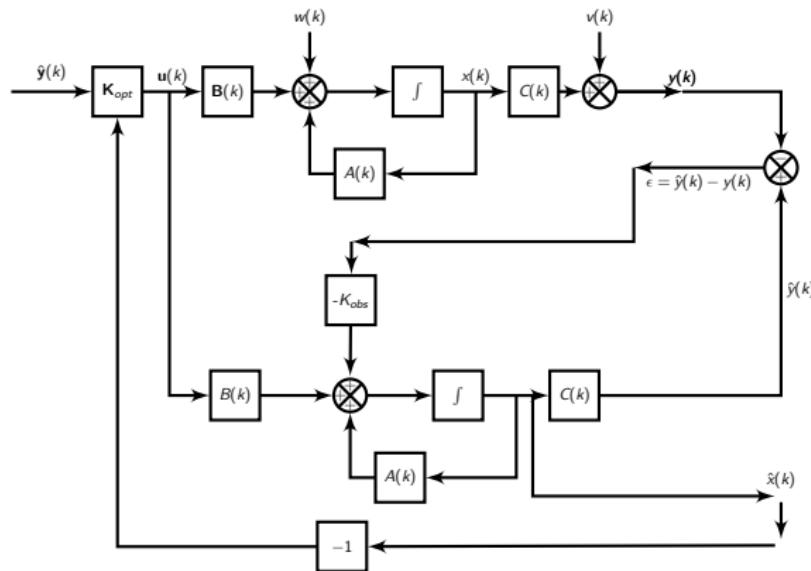
App B: 1DOF

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Actuator



$$\hat{x}(k+1) = A(k)\hat{x}(k) - K_{obs}[C(k)\hat{x}(k) - y(k)] + B(k)u(k).$$

# 1-DOF Control Results

Automating  
Treatment  
Planning in  
Radiation  
Therapy

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Ogunmolu

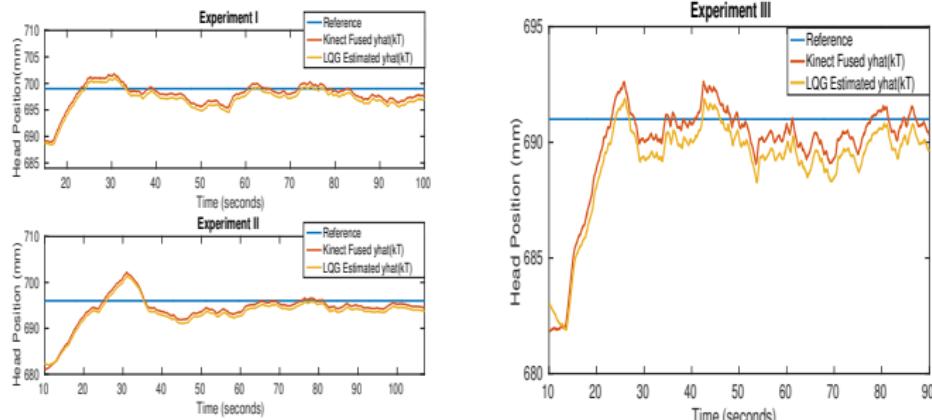
Appendices

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Control

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LQG Controller on mannequine head.

# Appendix D: CCOARSE Actuator Schematic

Automating  
Treatment  
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Radiation  
Therapy

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## Appendices

App A: 3DOF CL  
Control

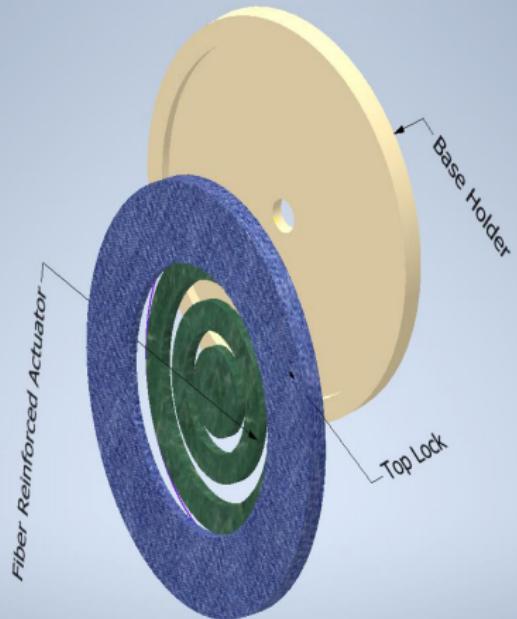
App B: 1DOF

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# Soft IK via Boundary Value Problem

Automating  
Treatment  
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Therapy

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App A: 3DOF CL  
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Actuator

- With Cauchy's laws of motion, solve boundary volume problem of traction
- Using the following boundary conditions for the Cauchy Stress,
  - $\sigma_{rr}|_{R=R_0} = -P_{atm}, \sigma_{rr}|_{R=R_i} = -P_{atm} - P$
  - And together with Cauchy's first law, we find that
    - $\sigma_{rr}(r) = - \int_{r_i}^{r_o} [2C_1\left(\frac{r}{R^2} - \frac{R^4}{r^5}\right) + 2C_2\left(\frac{r^3}{R^4} - \frac{R^2}{r^3}\right)] dr$
    - $\sigma_{rr}(r) = \int_{R_i}^{R_o} [2C_1\left(\frac{1}{r} - \frac{R^6}{r^7}\right) - 2C_2\left(\frac{R^4}{r^5} - \frac{r}{R^2}\right)] dr$
  - With  $\sigma_{rr}|_{R=R_i} = -P_{atm} - P$  and setting  $P_{atm} = 0$ , we find
    - $P(r) = \int_{r_i}^{r_o} [2C_1\left(\frac{r}{R^2} - \frac{R^4}{r^5}\right) + 2C_2\left(\frac{r^3}{R^4} - \frac{R^2}{r^3}\right)] dr$
    - $P(r) = \int_{R_i}^{R_o} [2C_1\left(\frac{1}{r} - \frac{R^6}{r^7}\right) - 2C_2\left(\frac{R^4}{r^5} - \frac{r}{R^2}\right)] dr$