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# Research Overview

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# Background

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# The robustness conundrum

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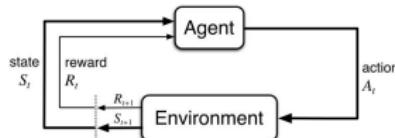
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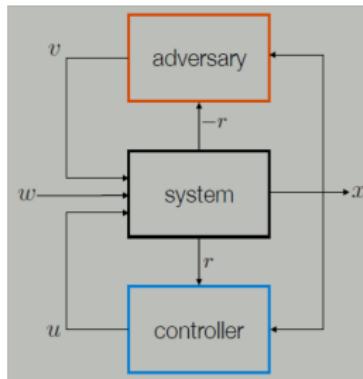
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- How to know *a priori* a policy's robustness limits?



- How to inculcate robustness into multistage decision policies?



# Methods

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- Suppose we have two agents,  $\mathbf{u}$  and  $\mathbf{v}$ , interacting in an environment over an horizon  $T$
- Let their dynamics be described as

$$\mathbf{x}_{t+1} = f_t(\mathbf{x}_t, \mathbf{u}_t, \mathbf{v}_t, \mathbf{w}_t), \quad t = 0, \dots, T-1$$

- where  $\mathbf{x}_t$  and  $\mathbf{u}_t$  are state and control variables
- $\mathbf{v}_t$  and  $\mathbf{w}_t$  are the respective disturbance and stochastic random variables.
- $\mathbf{w} = \{\mathbf{w}_0, \mathbf{w}_1, \dots, \mathbf{w}_{T-1}\}$  has distribution,  
 $\mathcal{P}(\mathbf{w}_t | \mathbf{x}_t, \mathbf{u}_t, \mathbf{v}_t)$ ,  $t = 0, \dots, T-1$ .
- Furthermore,  $\mathbf{u}_t \in \{\pi = \pi_0, \pi_1, \dots, \pi_T\}$ ,  
 $\mathbf{v}_t \in \{\psi_0, \psi_1, \dots, \psi_T\}$ , and  $\mathbf{w}_t \in \mathcal{P}(\mathbf{w}_t | \cdot)$ ,  $i = 0, \dots, T-1$

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- When a policy pair  $(\pi, \psi)$  is adopted, cost of a trajectory with initial condition  $\mathbf{x}_0$  is

$$\mathcal{J}_0(\mathbf{x}_0, \pi, \psi, \mathbf{w}) = \sum_{t=0}^{T-1} \ell_t(\mathbf{x}_t, \mathbf{u}_t, \mathbf{v}_t) + L_T(\mathbf{x}_T) \quad (1)$$

- where  $\ell_t, t = 0, \dots, T - 1$  and  $L_T$  are nonnegative instantaneous costs
- Average cost is thus

$$\tilde{\mathcal{J}}_0(\mathbf{x}_0, \pi, \psi) = \mathbf{E}_{\mathbf{x}_0(\mathbf{w})=\mathbf{x}_0} [\tilde{\mathcal{J}}_0(\mathbf{x}_0(\mathbf{w}), \pi, \psi, \mathbf{w})]$$

- $\mathbf{E}_{\mathbf{x}_0(\mathbf{w})=\mathbf{x}_0} (\cdot)$  is the expectation over random variables  $\mathbf{x}_1(\mathbf{w}), \dots, \mathbf{x}_T(\mathbf{w})$  having value  $\mathbf{x}_0$

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- Define  $\phi(\pi, \psi)$  as

$$\phi(\pi, \psi) = \mathbf{E}[\mathcal{J}_0(\mathbf{x}_0, \pi, \psi)] \quad (2)$$

- where  $\mathbf{E}$  is  $\mathbf{E}_{\mathbf{x}_0(\mathbf{w})=\mathbf{x}_0}$

so that

$$\phi(\pi, \psi) = \mathbf{E}[\tilde{\mathcal{J}}_0(\mathbf{x}_0(\mathbf{w}), \pi, \psi, \mathbf{w})] \quad (3)$$

## Problem Statement

Find an admissible (saddle point equilibrium) policy pair that satisfy,  $\mathcal{J}_0(\mathbf{x}_0, \pi^*, \psi) \leq \mathcal{J}_0(\mathbf{x}_0, \pi^*, \psi^*) \leq \mathcal{J}_0(\mathbf{x}_0, \pi, \psi^*), \forall \pi \in \Pi, \psi \in \Psi$  and  $\mathbf{x}_0$ .

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- $\mathcal{J}_t(\mathbf{x}_t, \pi, \psi)$  would denote the average process cost with initial condition,  $\mathbf{x}_t$  and policy pair,  $(\pi, \psi)$ , i.e.,

$$\mathcal{J}_t(\mathbf{x}_t, \pi, \psi) = \min_{\pi} \max_{\psi} \mathbf{E}_{|\mathbf{x}_t} \tilde{\mathcal{J}}_t(\mathbf{x}_t(\mathbf{w}), \pi, \psi, \mathbf{w})$$

- DP transforms the optimization over whole trajectory to a step-wise optimization over  $(\mathbf{u}_t, \mathbf{v}_t)$  as

$$\mathcal{J}_t(\cdot) = \min_{\mathbf{u}_t \sim \pi} \max_{\mathbf{v}_t \sim \psi} \mathbf{E}_{|\mathbf{x}_t} \left[ \sum_{k=t}^{T-1} \ell_k(\mathbf{x}_k, \pi_k(\mathbf{x}_k), \psi_k(\mathbf{x}_k)) + L(\mathbf{x}_k) \right]$$

with boundary condition  $\mathcal{J}_T(\mathbf{x}_T, \pi, \psi, \mathbf{w}) = L(\mathbf{x}_T)$

- We seek a saddle point equilibrium policy that satisfies

$$\mathcal{J}_t(\mathbf{x}_t, \pi^*, \psi) \leq \mathcal{J}_t(\mathbf{x}_t, \pi^*, \psi^*) \leq \mathcal{J}_t(\mathbf{x}_t, \pi, \psi^*),$$

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- Assume an agent's nominal policy,  $\pi$ , has been found
- Suppose that there is another agent interacting in the nominal agent's environment, so that

$$\begin{aligned} \mathbf{x}_{t+1} &= f_t(\mathbf{x}_t, \mathbf{u}_t, \mathbf{v}_t, \mathbf{w}_t), \quad \mathbf{u}_t \sim \pi_t \\ &:= \tilde{f}_t(\mathbf{x}_t, \mathbf{v}_t), \quad t = 0, \dots, T-1. \end{aligned} \tag{4}$$

- For stage costs of the form,

$$\ell_t(\mathbf{x}_t, \mathbf{u}_t, \mathbf{v}_t) = \left[ \sum_{t=0}^T \underbrace{c(\mathbf{x}_t, \mathbf{u}_t)}_{\text{nominal}} - \gamma \underbrace{g(\mathbf{v}_t)}_{\text{opponent}} \right]$$

- $g_t(\cdot)$  controls the strength of the disturbing agent

# Transition Slide

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# Minimax Iterative Dynamic Game

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- To mitigate lack of robustness, we optimize

$$\mathcal{J}_t(\mathbf{x}_t, \pi, \psi) = \min_{\mathbf{u}_t \sim \pi} \max_{\mathbf{v}_t \sim \psi} \left[ \sum_{t=0}^{T-1} \ell_t(\mathbf{x}_t, \mathbf{u}_t, \mathbf{v}_t) + L_T(\mathbf{x}_T) \right]$$

- seeking a saddle point equilibrium policy that satisfies the following inequality,

$$\mathcal{J}_t(\mathbf{x}_t, \pi^*, \psi) \leq \mathcal{J}_t(\mathbf{x}_t, \pi^*, \psi^*) \leq \mathcal{J}_t(\mathbf{x}_t, \pi, \psi^*),$$

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- Continuously solve an online trajectory optimization problem in a minimax fashion
- Essentially a meta-algorithm that is applicable to e.g. iLQG, DDP, GPS, DQN etc.
- Case study is a two-player iDG which proceeds as follows
  - approximate nonlinear system dynamics,  $\mathbf{x}_{t+1}$ , starting with a schedule of the nominal agent's local controls,  $\{\bar{\mathbf{u}}_t\}$ , and the opposing agent's local controls  $\{\bar{\mathbf{v}}_t\}$  which are assumed to be available,

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- Case study is a two-player iDG which proceeds as follows
  - run passive dynamics with  $\{\bar{\mathbf{u}}\}, \{\bar{\mathbf{v}}\}$  and generate a nominal state trajectory  $\{\bar{\mathbf{x}}_t\}$ , with neighboring trajectories  $\{\mathbf{x}_t\}$
  - we choose a small neighborhood,  $\{\delta\mathbf{x}_t\}$  of  $\{\mathbf{x}_t\}$ , which provides an optimal reduction in cost as the dynamics no longer represent those of  $\{\mathbf{x}_t\}$
  - discretizing time, the new state and control sequence pairs become  $\delta\mathbf{x}_t = \mathbf{x}_t - \bar{\mathbf{x}}_t$ ,  $\delta\mathbf{u}_t = \mathbf{u}_t - \bar{\mathbf{u}}_t$ ,  $\delta\mathbf{v}_t = \mathbf{v}_t - \bar{\mathbf{v}}_t$ .

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- Therefore, for

$$\mathcal{J}(\mathbf{x}_t, \boldsymbol{\pi}, \boldsymbol{\psi}) = \min_{\mathbf{u}_t \sim \boldsymbol{\pi}} \max_{\mathbf{v}_t \sim \boldsymbol{\psi}} [\ell(\mathbf{x}_t, \mathbf{u}_t, \mathbf{v}_t) + \mathcal{J}(f(\mathbf{x}_{t+1}, \mathbf{u}_{t+1}, \mathbf{v}_{t+1}))]$$

- Suppose we consider the Hamiltonian as a perturbation around  $\{\mathbf{x}_t, \mathbf{u}_t, \mathbf{v}_t\}$
- Cost over local neighborhood,  $\{\delta\mathbf{x}_t\}$  can be approximated by a 2nd order Taylor expansion,

$$Q(\cdot) \approx \frac{1}{2} \begin{bmatrix} 1 \\ \delta\mathbf{x}_t^T \\ \delta\mathbf{u}_t^T \\ \delta\mathbf{v}_t^T \end{bmatrix}^T \begin{bmatrix} 1 & Q_{xt}^T & Q_{ut}^T & Q_{vt}^T \\ Q_{xt} & Q_{xxt} & Q_{xut} & Q_{xvt} \\ Q_{ut} & Q_{uxt} & Q_{uut} & Q_{uvt} \\ Q_{vt} & Q_{vxr} & Q_{vut} & Q_{vvt} \end{bmatrix} \begin{bmatrix} 1 \\ \delta\mathbf{x}_t \\ \delta\mathbf{u}_t \\ \delta\mathbf{v}_t \end{bmatrix} \quad (5)$$

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■ where

$$Q_{xt} = \ell_{xt} + f_{xt}^T V_{xt+1}, \quad Q_{ut} = \ell_{ut} + f_{ut}^T V_{xt+1}$$

$$Q_{vt} = \ell_{vt} + f_{vt}^T V_{xt+1}, \quad Q_{xxt} = \ell_{xxt} + f_{xt}^T V_{xxt+1} f_{xt}$$

$$Q_{uxt} = \ell_{uxt} + f_{ut}^T V_{xxt+1} f_{xt}, \quad Q_{vxt} = \ell_{vxt} + f_{vt}^T V_{xxt+1} f_{xt}$$

$$Q_{uut} = \ell_{uut} + f_{ut}^T V_{xxt+1} f_{ut}, \quad Q_{vvt} = \ell_{vvt} + f_{vt}^T V_{xxt+1} f_{vt}$$

$$Q_{uvt} = \ell_{uvt} + f_{ut}^T V_{xxt+1} f_{vt}.$$

■ it is expected that 2nd order terms will dominate the higher-order ones, consistent with linearized methods [Polycarpou and Ioannou (1992)].

# Linearization of Dynamics

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- LQR approximation to dynamics becomes

$$\delta \mathbf{x}_{t+1} \approx f_{\mathbf{x}t} \delta \mathbf{x}_t + f_{\mathbf{u}t} \delta \mathbf{u}_t + f_{\mathbf{v}t} \delta \mathbf{v}_t$$
$$\ell(\mathbf{x}_t, \mathbf{u}_t, \mathbf{v}_t) \approx \frac{1}{2} \begin{bmatrix} 1 \\ \delta \mathbf{x}_t^T \\ \delta \mathbf{u}_t^T \\ \delta \mathbf{v}_t^T \end{bmatrix}^T \begin{bmatrix} \ell_{0t} & \ell_{\mathbf{x}t}^T & \ell_{\mathbf{u}t}^T & \ell_{\mathbf{v}t}^T \\ \ell_{\mathbf{x}t} & \ell_{\mathbf{xxt}} & \ell_{\mathbf{uxt}} & \ell_{\mathbf{vxt}} \\ \ell_{\mathbf{u}t} & \ell_{\mathbf{uxt}} & \ell_{\mathbf{uut}} & \ell_{\mathbf{uvt}} \\ \ell_{\mathbf{v}t} & \ell_{\mathbf{vxt}} & \ell_{\mathbf{vut}} & \ell_{\mathbf{vvt}} \end{bmatrix} \begin{bmatrix} 1 \\ \delta \mathbf{x}_t \\ \delta \mathbf{u}_t \\ \delta \mathbf{v}_t \end{bmatrix} \quad (6)$$

- It is easy to verify that the feedback controls are

$$\delta \mathbf{u}_t^* = -Q_{\mathbf{u}ut}^{-1} \left[ Q_{\mathbf{u}t}^T + Q_{\mathbf{u}xt} \delta \mathbf{x}_t + Q_{\mathbf{u}vt} \delta \mathbf{v}_t \right], \quad (7)$$
$$\delta \mathbf{v}_t^* = -Q_{\mathbf{v}vt}^{-1} \left[ Q_{\mathbf{v}t}^T + Q_{\mathbf{v}xt} \delta \mathbf{x}_t + Q_{\mathbf{v}ut} \delta \mathbf{u}_t \right].$$

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- From which we obtain the recursions,

$$\begin{aligned}\Delta V_t &= \mathbf{g}_{\mathbf{u}t} Q_{\mathbf{u}t} + \mathbf{g}_{\mathbf{v}t} Q_{\mathbf{v}t} + \mathbf{g}_{\mathbf{u}t} Q_{\mathbf{u}vt} \mathbf{g}_{\mathbf{v}t} \\ &\quad + \frac{1}{2} (\mathbf{g}_{\mathbf{u}t} Q_{\mathbf{u}ut} \mathbf{g}_{\mathbf{u}t} + \mathbf{g}_{\mathbf{v}t} Q_{\mathbf{v}vt} \mathbf{g}_{\mathbf{v}t}) \\ V_{\mathbf{x}t} &= Q_{\mathbf{x}t} + \mathbf{G}_{\mathbf{u}t}^T Q_{\mathbf{u}t} + \mathbf{G}_{\mathbf{v}t}^T Q_{\mathbf{v}t} + \mathbf{G}_{\mathbf{u}t}^T Q_{\mathbf{u}ut} \mathbf{g}_{\mathbf{u}t} + \mathbf{g}_{\mathbf{u}t} Q_{\mathbf{u}xt} \\ &\quad + \mathbf{g}_{\mathbf{v}t} Q_{\mathbf{v}xt} + \mathbf{G}_{\mathbf{v}t}^T Q_{\mathbf{v}vt} \mathbf{g}_{\mathbf{v}t} + \mathbf{G}_{\mathbf{v}t}^T Q_{\mathbf{u}vt}^T \mathbf{g}_{\mathbf{u}t} + \mathbf{G}_{\mathbf{u}t}^T Q_{\mathbf{u}vt} \mathbf{g}_{\mathbf{v}t} \\ V_{\mathbf{xxt}} &= \frac{1}{2} (Q_{\mathbf{xxt}} + \mathbf{G}_{\mathbf{u}t}^T Q_{\mathbf{u}ut} \mathbf{G}_{\mathbf{u}t} + \mathbf{G}_{\mathbf{v}t}^T Q_{\mathbf{v}vt} \mathbf{G}_{\mathbf{v}t}) + \mathbf{G}_{\mathbf{u}t}^T Q_{\mathbf{u}xt} \\ &\quad + \mathbf{G}_{\mathbf{v}t}^T Q_{\mathbf{v}xt} + \mathbf{G}_{\mathbf{u}t}^T Q_{\mathbf{u}vt} \mathbf{G}_{\mathbf{v}t} \end{aligned} \tag{8}$$

- NB: The gains  $\mathbf{g}_{it}$  and  $\mathbf{G}_{it}$ ,  $i = \mathbf{u}$  or  $\mathbf{v}$  are as defined in ([§II.B]Ogunmolu et al. (2018)).

# Results: Brittleness Quantification

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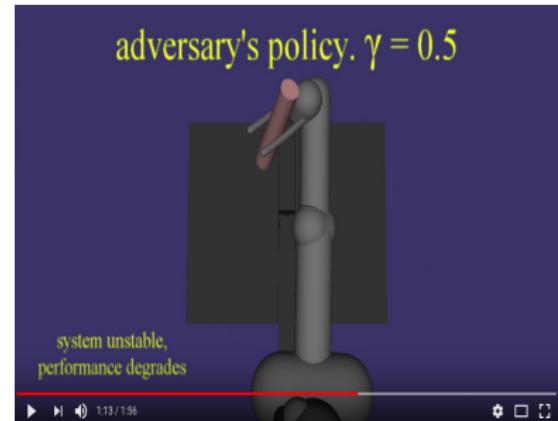
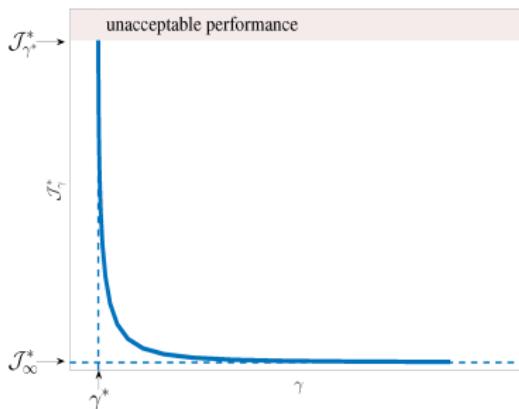
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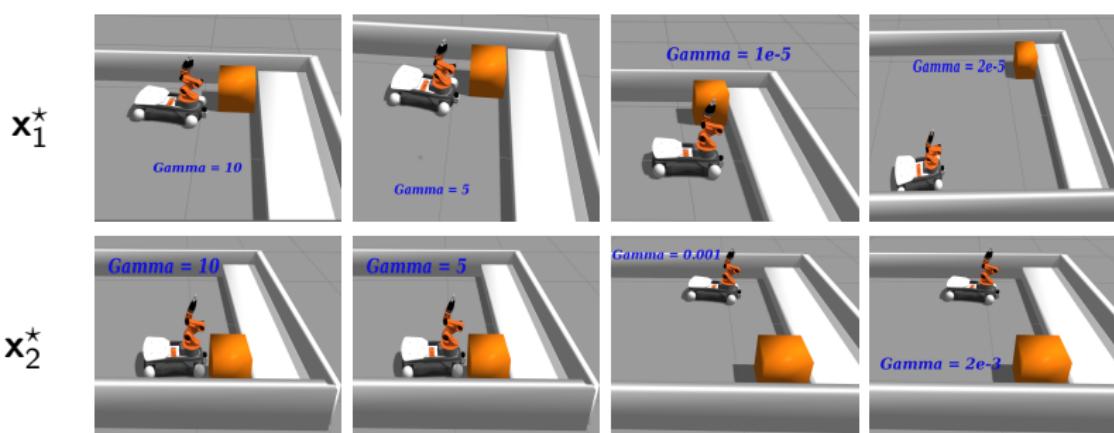
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End pose of the KUKA platform with our iDG formulation given different goal states and  $\gamma$ -values

# Video of Results

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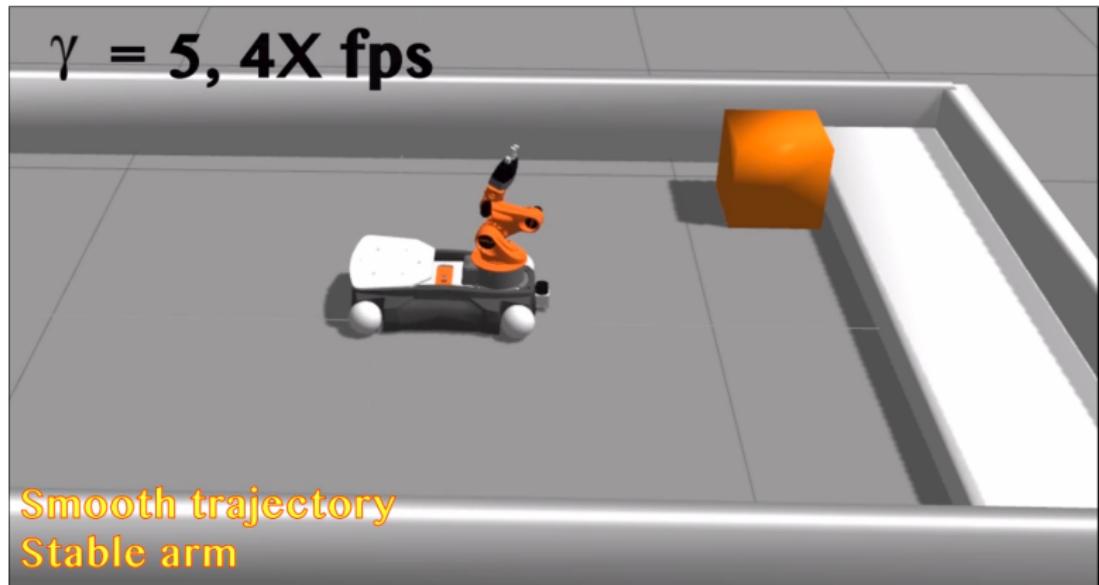
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## Treatment Options

### Surgery

- Oldest technique
- Good for localized cancer cells
- Falls short of being an all-around good option



Source: National Cancer Institute

### Chemotherapy

- Effective for malignant cancer cells
- Highly toxic
- Highly carcinogenic
- Kills healthy cells



Source: Cancer.gov

### Radiosurgery

- Replaces invasive surgery
- Often used alongside surgical tumor removal
- Extremely effective in managing tumors
- Standard care in managing cancer conditions



- Radiation therapy is one of the major cancer therapy modalities
- About 2/3 of U.S. cancer patients in US receive radiation therapy either alone or in conjunction with surgery, chemotherapy, and immunotherapy, etc.

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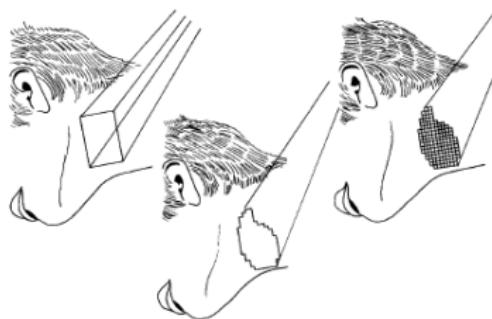
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- Conventional Radiation Therapy
- Conformal Radiation Therapy (CFRT)
- Intensity-Modulated Radiation Therapy (IMRT)



Left: Conventional radiotherapy.

Middle: Conformal radiotherapy (CFRT) without intensity modulation.

Right: CFRT with intensity modulation. Reprinted from Webb (2001).

# Frame-based Radiotherapy Treatment

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- Fractionated radiation dose over many weeks/months



# Frameless (Completely non-invasive) RT

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High-dose volumes with complex shapes [Adler and Cox (1995)]



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# Image Guided Radiotherapy

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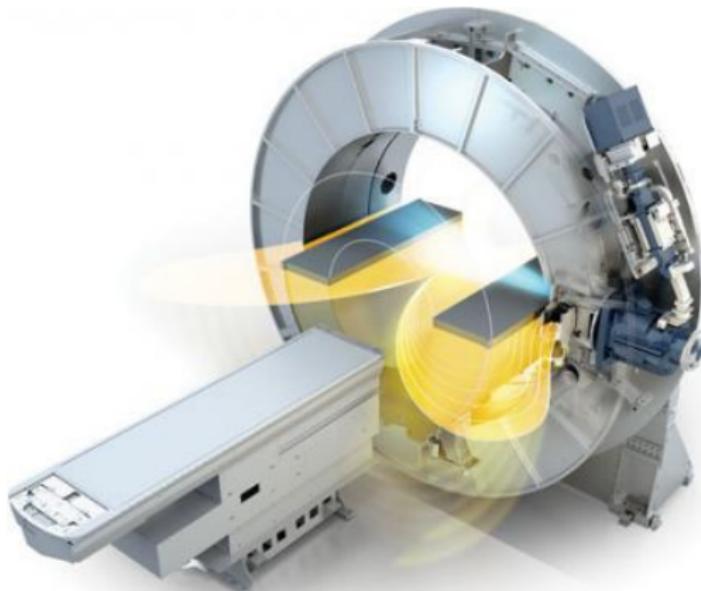
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Reprinted from Imaging Technology News

# Beam Orientation Optimization

- During treatment planning, a **beam orientation optimization** problem (BOO) is separately solved
- Radiation is delivered from  $\approx (5 - 15)$  different beam orientations during IMRT
- BOO determines the best beam angle combinations for delivering radiation.
- Process of determining beamlets' intensities is termed **fluence map optimization** (FMO)
- During clinical treatment planning, beam orientations are still manually chosen or adopted from a standard protocol for clinical use.
- BOO is a field of research and is slowly making way to the clinic.

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Cerviño et al. (2010)



Feasibility evaluation

Liu et al. (2015)



4-D robotic stage couch

Ostyn et al. (2017)



6-DoF robotic couch

# Solution: Soft-Robot Position Correcting Systems

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Olalekan Ogunmolu

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3DOFControl  
Adaptive NeuroControl  
Neural Network Model

- Eliminate rigid frames and metallic rings
- Eliminate attenuation of X-Ray beams
- Control design
  - Feedback control + optimal regulation + robustness to disturbance ✓

# Vision-based 3-DOF Control

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# Simulation Testbed

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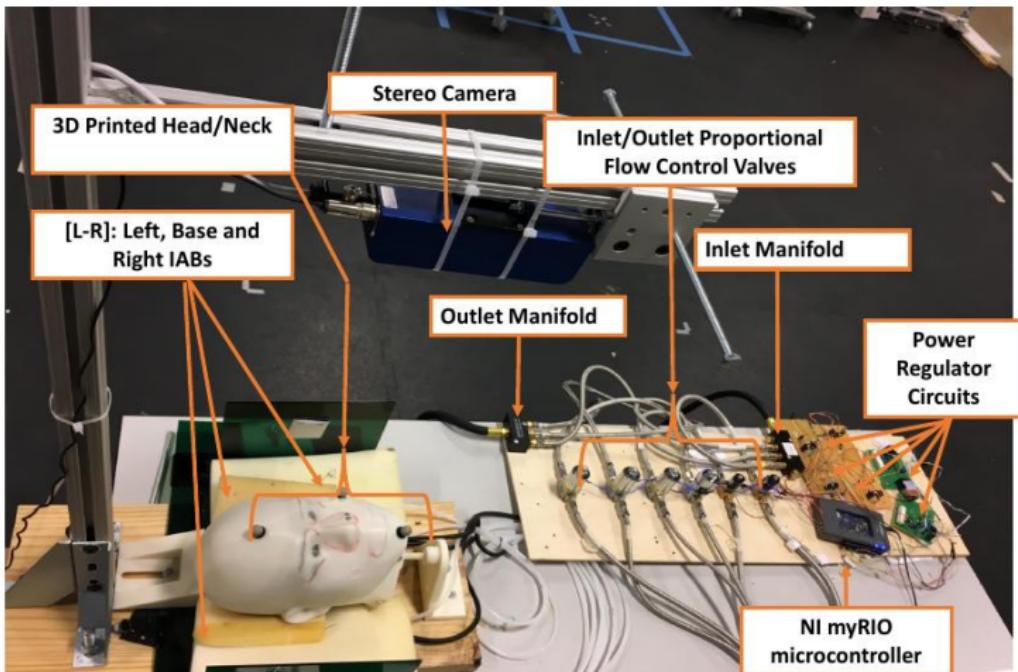
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## Hardware Description

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- Solve a state feedback and feedforward regulation problem
- An adaptation model based on head pose estimate given past states and past control inputs:
  - $Z^N = \{u(k), u(k-1), \dots, u(k-n_u), y(k), \dots, y(k-n_y)\}$
  - Let a persistently exciting input signal  $u_{ex} \in L_2 \cap L_\infty$  excite the system's nonlinear modes
- Design Goal:
  - Stabilize states,  $\mathbf{y} = [z, \theta, \phi]^T$  out of  $[x, y, z, \theta, \phi, \psi]^T$

# Model Reference Adaptive Control

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- Model head and bladder dynamics as
  - $\dot{\mathbf{y}} = \mathbf{A}\mathbf{y} + \mathbf{B}\Lambda(\mathbf{u} - f(\mathbf{y}, \mathbf{u})) + \mathbf{w}(k)$ 
    - $\mathbf{A}, \Lambda$  unknown,  $\mathbf{B}$ ,  $\text{sgn}\Lambda$  known
    - $f(\mathbf{y}, \mathbf{u}) \triangleq$  nonlinear function to be adapted for
    - $\mathbf{x} \triangleq$  tuple containing past controls and current outputs
- Approximate  $f(\mathbf{y}, \mathbf{u})$  by a neural network with continuous memory states
  - $\hat{f}(\mathbf{y}(k), \mathbf{u}(k-d))$  is realized with a *long-short term memory* cell (Horchreiter and Schmidhuber, '91, '97)
  - **purpose:** remember good adaptation gains

# Assumptions

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- A dynamic RNN with  $N$  neurons,  $\varphi(\mathbf{x})$ , exists
  - maps from a compact input space  $\mathbf{u} \subset \mathbb{U}$  to  $\mathbf{y} \subset \mathbb{Y}$  on the Lebesgue integrable functions within  $[0, T]$  or  $[0, \infty)$
- $f(\mathbf{y}, \mathbf{u})$  is exactly  $\Theta^T \Phi(\mathbf{y})$ 
  - $f$  has coefficients  $\Theta \in R^{N \times m}$  and a Lipschitz-continuous vector of basis functions  $\Phi(\mathbf{y}) \in R^N$
- Inside a ball  $\mathbf{Y}_R$  with known, finite radius  $R$ ,
  - an ideal neural network (NN) approximation  $f(\mathbf{y}) : R^n \rightarrow R^m$ , is realized to a sufficient degree of accuracy,  $\varepsilon_f > 0$ ;
- Outside  $\mathbf{Y}_R$ ,
  - the NN approximation error can be upper-bounded by a known unbounded, scalar function  $\varepsilon_{max}(\mathbf{y})$ ;
  - $\|\varepsilon(\mathbf{y})\| \leq \varepsilon_{max}(\mathbf{y}), \quad \forall \mathbf{y} \in \mathbf{Y}_R$ ;
- There exists an exponentially stable reference model
  - $\dot{\mathbf{y}}_m = \mathbf{A}_m \mathbf{y}_m + \mathbf{B}_m \mathbf{r}$

# Adaptive Neuro-Control Scheme

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- Set control law in terms of parameter estimates from the neural network weights and Lipschitz basis functions
  - $\Phi(\mathbf{y}) = \{\mathbf{y}(k-d), \dots, \mathbf{y}(k-d-4), \mathbf{u}(k-d) \dots \mathbf{u}(k-d-5)\}$
  - i.e. network looks back in time by 5 time steps at every instant, and then makes a prediction
- Derive adaptive adjustment mechanism from Lyapunov analysis for Adaptive Control (Parks, P., 1966)
- $\mathbf{u} = \underbrace{\hat{\mathbf{K}}_y^T \mathbf{y}}_{\text{state feedback}} + \underbrace{\hat{\mathbf{K}}_r^T \mathbf{r}}_{\text{optimal regulator}} + \underbrace{\hat{f}(\mathbf{y}, \mathbf{u})}_{\text{approximator}}$

# Controller formulation

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- $\hat{\mathbf{K}}_y$  and  $\hat{\mathbf{K}}_r$  are adaptive gains to be designed

## Term Contributions

- $\hat{\mathbf{K}}_y^T \mathbf{y}$  term keeps the states of the approximation set  $\mathbf{y} \in \mathbf{B}_R$  stable,
- $\hat{\mathbf{K}}_r^T \mathbf{r}$  term causes the states to follow a given reference trajectory
- Function approximator  $\hat{f}(\mathbf{y}, \mathbf{u})$  ensures states that start outside the approximation set  $\mathbf{y} \in \mathbf{B}_R$  converge to  $\mathbf{B}_R$  in finite time

# Adaptive Control Formulation

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- Assume model matching conditions
  - such that  $\hat{\mathbf{K}}_y = \mathbf{K}_y$ , and  $\hat{\mathbf{K}}_r = \mathbf{K}_r$  (ideally)
- Realize the approximator as  $\hat{f}(\mathbf{y}) = \hat{\Theta}^T \Phi(\mathbf{y}) + \varepsilon_f(\mathbf{y})$ 
  - $\hat{\Theta}^T$  denotes the vectorized weights of the neural network
  - $\Phi(\mathbf{y})$  denotes the vector of lagged inputs and output,
  - $\varepsilon_f(\mathbf{y})$  is the approximation error.
  - $\Phi(\mathbf{y}) = \{\mathbf{y}(k-d) \cdots \mathbf{y}(k-d-4), \mathbf{u}(k-d) \cdots \mathbf{u}(k-d-5)\}$

# Neural Network Model

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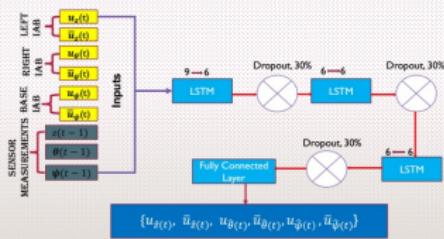
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## Neural Net Architecture



- input: lagged vector of past observations and current control actions
- repeat  $\times 3$ 
  - pass input through an lstm cell
  - followed by 30% dropout
- output will be control predictions directly fed as valve voltages

# Lyapunov Redesign

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## ■ Theorem:

- Given correct choice of adaptive gains  $\hat{\mathbf{K}}_y$  and  $\hat{\mathbf{K}}_r$ , the error state vector,  $\mathbf{e}(k)$  with closed loop time derivative  $\dot{\mathbf{e}}$ , is **uniformly ultimately bounded**, and the state  $\mathbf{y}$  will converge to a neighborhood of  $\mathbf{r}$ .
- Please see proof in Appendix(§7).

# Stability Results

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We find that

$$\begin{aligned}\dot{\mathbf{V}}(\cdot) &= -\mathbf{e}^T \mathbf{Q} \mathbf{e} - 2\mathbf{e}^T \mathbf{P} \mathbf{B} \Lambda \varepsilon_f \\ &\leq -\lambda_{low} \|\mathbf{e}\|^2 + 2\|\mathbf{e}\| \|\mathbf{P} \mathbf{B}\| \lambda_{high}(\Lambda) \varepsilon_{max}\end{aligned}$$

- $\lambda_{low}, \lambda_{high} \equiv$  minimum and maximum characteristic roots of  $Q$  and  $\Lambda$  respectively.
- $\dot{\mathbf{V}}(\cdot)$  is thus negative definite outside the compact set
- $\chi = \left( \mathbf{e} : \|\mathbf{e}\| \leq \frac{2\|\mathbf{P} \mathbf{B}\| \lambda_{high}(\Lambda) \varepsilon_{max}(\mathbf{y})}{\lambda_{low}(Q)} \right)$
- thus, we conclude that the error  $\mathbf{e}$  is uniformly ultimately bounded.
  - i.e.  $\mathbf{y}(t) \rightarrow 0$  as  $t \rightarrow \infty$

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- Solving the general form of the Lyapunov equation, we have

$$\mathbf{P} = \begin{bmatrix} -\frac{170500}{2668} & 0 & 0 \\ 0 & -\frac{170500}{2668} & 0 \\ 0 & 0 & -\frac{170500}{2668} \end{bmatrix}$$

- Solenoid valves operate in pairs

- two valves create a difference in air mass within each IAB at any given time
  - set

$$\mathbf{B} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

- $\mathbf{B}$  maps to the 3-axes controllers

$$[u_z \quad u_\theta \quad u_\psi]^T$$

- non-zero terms are the max. duty-cycle to valves based on the software configuration of the NI RIO PWM generator

# Results

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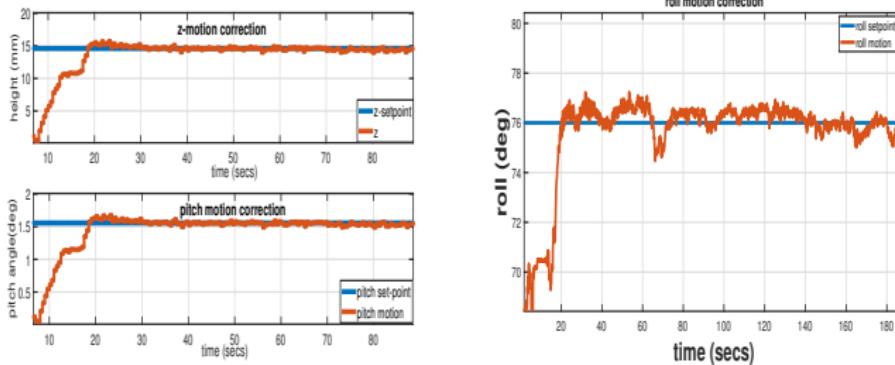
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[Left]: Goal command:  $(z, \theta, \phi) = (2.5\text{mm}, 0.25^\circ, 35^\circ)$  to  $(14\text{mm}, 1.6^\circ, 45^\circ)^T$ . [Right]: Head roll tracking.

# Treatment Plan (TP) Optimization

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# IMRT TP Overview

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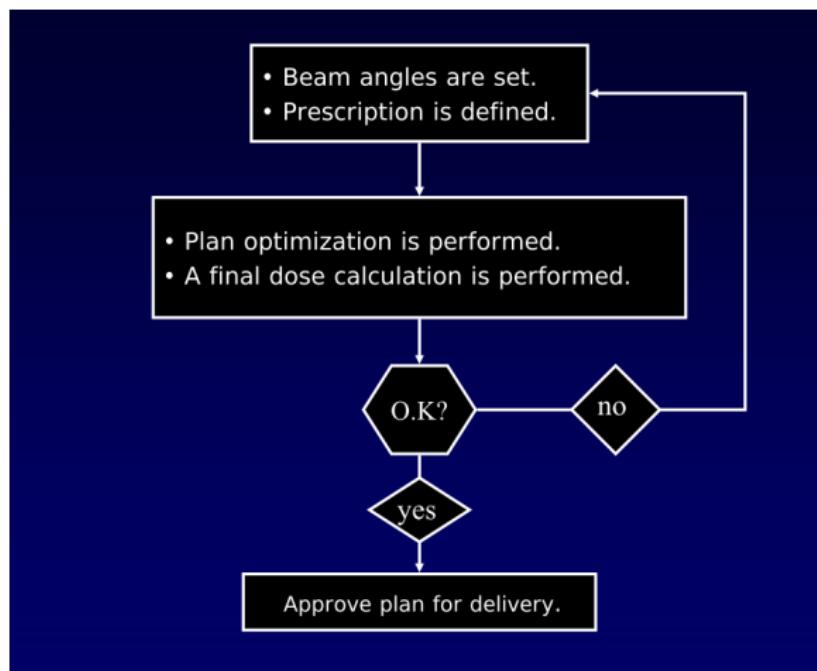
- IMRT delivers geometrically-shaped, high-precision photons to tumors in a beam orientation optimization (BOO) process.
- The **BOO problem** aims to find the right beam angle combinations from which to deliver radiation intensities.
  - essentially a combinatorial optimization problem
  - traditional methods fail at real-time feasible results<sup>1</sup>.
- Afterwards, the intensity of the fluence is modulated in a **fluence map optimization** (FMO) process.

---

<sup>1</sup>Current approaches take too long, and are often not optimal.



# IMRT Treatment Plan Flowchart



Reprinted from "IMRT Optimization Algorithms. David Shepard. Swedish Cancer Institute. AAPM 2007."

# Problem Setup

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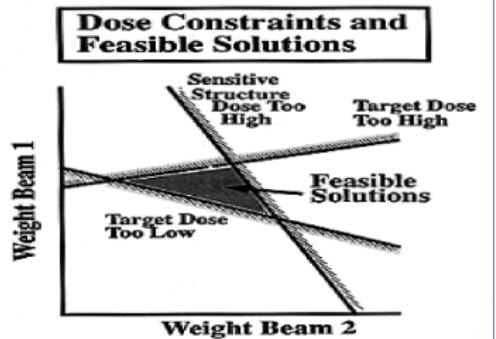
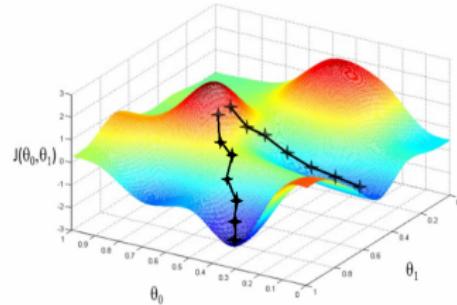
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Neural Network Model

- Given biological statement of prescriptions
  - find a numerical **objective function**
  - accompanied by **constraints**
- Challenge
  - a scalar-valued objective function usually not sufficient



Courtesy of David Shepard

# Common Problem Formulation

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- Maximize weighted least square dose from for PTVs
- Minimize weighted least square dose for OARs
- DVH-based goal treatments

---

- Current Approaches and Limitations

- Stochastic optimization approaches: simulated annealing; genetic algorithms and gradient search, or a combination of genetic and gradient search algorithms
- Mixed-integer programming, branch and cut/bound algorithms, beam angle elimination algorithms
- Commercial planners use some highly non-convex objective (actual function is proprietary and unknown to public).
- General weakness: Feasible solution takes too long to find.

# Our approach

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- Automate the **beam search** problem
- **Ultimate goal** is real-time beam angle prediction given a target volume
- Drawing ideas from
  - **pattern recognition;**
  - **monte carlo evaluations;**
  - **game simulations;** and
  - **approximate dynamic programming**

# What we do

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- Formulate BOO problem into a large game planning strategy
- Neural fictitious self-play [Heinrich et al. (2015)], to refine policy predictions
  - **purpose:** drive policy weights to a **saddle equilibrium**
- a deep neural network models the nonlinear dynamical system (patient's geometry, robot-linac setup)
  - generating a policy that guides MCTS simulations for two players in a zero-sum Markov game

# What we do

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- In each episodic markov decision process (MDP) setting, an MCTS lookout strategy guides transition from one beam angle set to another
- Each player in a two-player Markov game finds a best response strategy to their opponent's average strategy
  - driving the policy weights toward an approximate **saddle equilibrium** Heinrich et al. (2015).

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- Let state of the dynamical system be  $s \in \mathcal{S}$
- To be controlled by a discrete action  $a \in \mathcal{A}$
- States evolve according to an (unknown) dynamics  $p(s_{t+1}|s_t, a_t)$  (to be learned)
- Beam angle combination search task defined by a reward function,  $R_t = \sum_{t=1}^N \gamma^{t-1} r(s_t, a_t)$ 
  - Can be found by recovering a policy,  $p(a_t|s_t; \psi)$
- From now on, we will write  $p(a_t|s_t; \psi)$  as  $\pi_\psi(a_t|s_t)$ .

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- Suppose the first player is  $p_1$ , and the second player is  $p_2$
- $p_1$  chooses its action under a (stochastic) strategy,  
 $\pi^{p_1} = \{\pi_0^{p_1}, \pi_1^{p_1}, \dots, \pi_T^{p_1}\} \subseteq \Pi^{p_1}$ 
  - minimizing the game's outcome  $\zeta$
- $p_2$ 's actions are governed by a policy  
 $\pi^{p_2} = \{\pi_0^{p_2}, \pi_1^{p_2}, \dots, \pi_T^{p_2}\} \subseteq \Pi^{p_2}$ 
  - $p_2$  seeks to maximize  $\zeta$  in order to guarantee an equilibrium solution for a game without saddle point.
- $\Pi^{p_i}$  is the set of all possible nonstationary markovian policies

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- Each player bases its decision on a random event's outcome
  - generating a **mixed strategy** determined by **averaging the outcome** of individual plays.
- Both players constitute a two-player **stochastic action selection strategy**,  $\pi(s, a) = \Pr(a|s) := \{\pi^{P_1}, \pi^{P_2}\}$  that gives the probability of selecting moves in any given state
- Suppose the game simulation starts from an initial condition  $s_0$ .

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- One may write the optimal **reward-to-go** value function for state  $s$  in stage  $t$ , with horizon length  $T$  as

$$V_t^*(s) = \inf_{\pi^{P_1} \in \Pi^{P_1}} \sup_{\pi^{P_2} \in \Pi^{P_2}} \mathbb{E} \left[ \sum_{i=t}^{T-1} V_t(s_0, f(s_t, \pi^{P_1}, \pi^{P_2})) \right],$$
$$s \in S, t = 0, \dots, H-1$$

- where the terminal value  $V_T^*(s) = 0, \forall s \in S$ ;
- $f(\cdot)$  represents the unknown system dynamics
- $\pi^{P_1}$  and  $\pi^{P_2}$  contain the action/control sequences  $\{a_t^{P_1}\}_{0 \leq t \leq T}$  and  $\{a_t^{P_2}\}_{0 \leq t \leq T}$

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- The **saddle point strategies** for an optimal control sequence pair  $\{a_t^{p_1^*}, a_t^{p_2^*}\}$  can be recursively obtained by optimizing a state-action value cost,  $\mathcal{J}_t(s, a)$

$$V_t^*(s) = V_t^*(s_t, \pi_t^{p_1}, \pi_t^{p_2}) = \min_{\pi^{p_1} \in \Pi^{p_1}} \max_{\pi^{p_2} \in \Pi^{p_2}} V_t^*(s_t, \pi^{p_1}, \pi^{p_2})$$
$$\forall s_t \in \mathcal{S}, \pi^{p_1} \in \Pi^{p_1}, \pi^{p_2} \in \Pi^{p_2}.$$

such that

$$V_{p_1}^* \leq V^* \leq V_{p_2}^* \quad \forall \{\pi_t^{p_1}, \pi_t^{p_2}\}_{0 \leq t \leq T}.$$

where  $V_{p_i}^*$  are the respective optimal values for each player.

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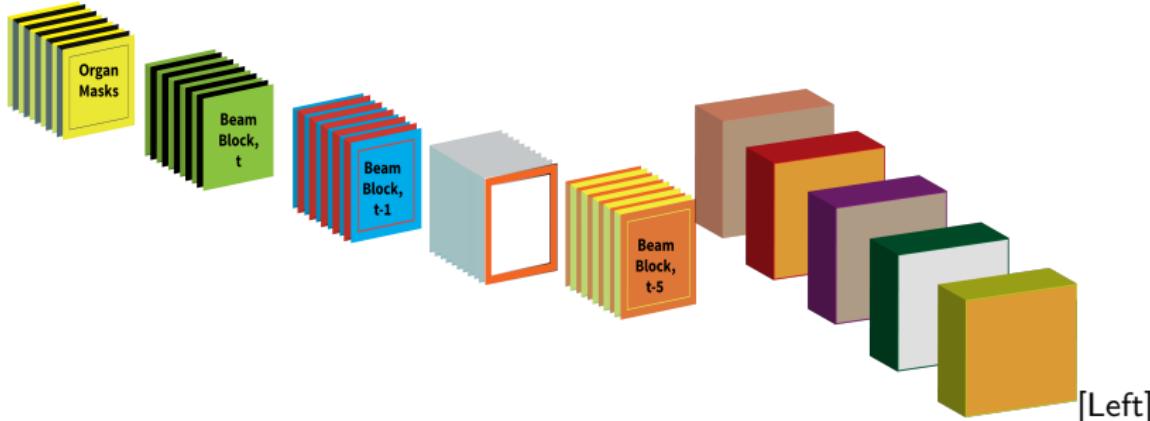
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- Under ideal conditions, we'd like to find the optimal value function under perfect play
- **Caveat:** BOO exhibits Bellman's curse of dimensionality.
- What to do?
  - derive an **approximately optimal** value  $V_\psi^*(s)$
  - by continually estimating the value function  $v_\psi^P(s)$  using e.g. a policy parameterized by a large function approximator

# State Representation



Concatenation of the target volume masks and the beam angles before feeding the input planes to the residual tower neural network. The first six planes (top-most mask of left figure) contain the delineated organs and the PTV. This is concatenated with a block of  $m$  beams from the current time step, regressed to the previous 5 time steps (here, 5 was heuristically chosen). [Right]: Each beam angle in a beam block is represented as shown. Together with the target volume, these form an input plane of size  $36 \times N \times W \times H$  to the policy/value neural network tower of residual blocks.

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- Formulated as a bandit search that imposes a regret term on the  $Q$ -value
- During the planning process, we estimate a *value*,  $v(\mathbf{y}_t)$ , that estimates the optimality of a beam block;
- In parallel, we refine the deep neural network policy by optimizing its weight in a separate thread.
  - This player's weights are continually written to a shared memory such that its values are available to the MCTS search thread.
  - Network parameters updated by a **mixed strategy** which combines its **pure strategy**, which is a best response to the **average pure strategy** of a fictitious opponent.

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- $Q$ -value defined as

$$\bar{Q}(s, a) = Q_j(s, a) + c \sqrt{\frac{2 \ln n(s)}{N(s, a)}}, \quad (9)$$

$$a^* = \arg \max_a \bar{Q}(s, a) \quad (10)$$

---

## Fluence Map Optimization

- Suppose  $\mathcal{X}$  is the total discretized of voxels of interest (*VOI's*) in a target volume
- Suppose  $\mathcal{B}_1 \cup \mathcal{B}_2 \cup \dots \cup \mathcal{B}_n \subseteq \mathcal{B}$  represents the partition subset of a beam  $\mathcal{B}$ ,
  - where  $n$  is the total number of beams from which radiation can be delivered

# Methods: Fluence Map Optimization

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- Suppose further that  $\mathcal{D}_{ij}(\theta_k)$  is the matrix that describes each dose influence,  $d_i$ .
  - delivered to a discretized voxel,  $i$ , in a volume of interest,  $VOI_h$  ( $h = 1, \dots, \mathcal{X}$ ), from a beam angle,  $\theta_k$ ,  
 $k \in \{1, \dots, n\}$
- We compute the matrix  $\mathcal{D}_{ij}(\theta_k)$  by calculating each  $d_i$  for every bixel,  $j$ , at every  $\varphi^\circ$ , resolution, where  $j \in \mathcal{B}_k$ 
  - ending up with an ill-conditioned (*sparse*) matrix,  $\mathcal{D}_{ij}(\theta_k)$ , which consists of the dose to every voxel,  $i$ , incident from a beam angle,  $\theta_k$  at every  $360^\circ/\varphi^\circ$

# Methods: FMO problem definition

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- The pre-calculated dose term is given by  
$$\mathbf{Ax} = \left\{ \sum_s \frac{w_s}{v_s} \mathcal{D}_{ij}^s \mathbf{x}_s \mid \mathcal{D}_{ij} \in \mathbb{R}^{n \times l}, n > l \right\}$$
- Let  $w_s = \{\underline{w}_s, \bar{w}_s\}$  be the respective underdosing and overdosing weights for the OARs and PTVs, and  $v_s$  represents the total number of voxels in each structure.
- We propose the following cost

$$\frac{1}{v_s} \sum_{s \in \text{OARs}} \|(\mathbf{b}_s - \underline{w}_s \mathcal{D}_{ij}^s \mathbf{x}_s)_+\|_2^2 + \frac{1}{v_s} \sum_{s \in \text{PTVs}} \|(\bar{w}_s \mathcal{D}_{ij}^s \mathbf{x}_s - \mathbf{b}_s)_+\|_2^2 \quad (11)$$

# Methods: FMO

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- Rewriting the objective, subject to nonnegative pixel intensity constraints, we have the minimization problem

$$\min \frac{1}{2} \|Ax - b\|_2^2 \quad \text{subject to } x \geq 0.$$

- The Lagrangian becomes

$$L(x, \lambda) = \min \frac{1}{2} \|Ax - b\|_2^2 - \lambda^T x.$$

- Since we are solving a large scale problem, we use the ADMM algorithm
- Introducing an auxiliary variable  $z$ , we have

$$\min_x \frac{1}{2} \|Ax - b\|_2^2, \quad \text{subject to } z = x, \quad z \geq 0,$$

# Methods: FMO

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- Solving either the  $\mathbf{x}$  and  $\mathbf{z}$  sub-problems, we have

$$\mathbf{x}^{k+1} = (\mathbf{A}^T \mathbf{A} + \rho \mathbf{I})^{-1} (\mathbf{A}^T \mathbf{b} + \rho \mathbf{z}^k - \boldsymbol{\lambda}^k). \quad (12)$$

- And using the soft-thresholding operator,  $S_{\boldsymbol{\lambda}/\rho}$ , we find that

$$\mathbf{z}^{k+1} = S_{\boldsymbol{\lambda}/\rho} (\mathbf{x}^{k+1} + \boldsymbol{\lambda}^k), \quad (13)$$

where  $S_{\boldsymbol{\lambda}/\rho}(\tau) = (\mathbf{x} - \boldsymbol{\lambda}/\rho)_+ - (-\tau - \boldsymbol{\lambda}/\rho)_+$ .  $\boldsymbol{\lambda}$  is updated as

$$\boldsymbol{\lambda}^{k+1} = \boldsymbol{\lambda}^k - \gamma (\mathbf{z}^{k+1} - \mathbf{x}^{k+1}), \quad (14)$$

where  $\gamma$  is a parameter that controls the step length.

# Game Tree Simulation

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- For  $b^d$  possible move sequences for a robot-patient setup
  - $b$  = beam angles chosen to construct a fluence
  - $d$  = is the total number of discretized angles.
- Suppose  $b = 180$  and  $d = 5$ , we have  $180^5$  possible search directions
- Exhaustive search becomes real-time infeasible.

# Game Tree Simulation; Approach

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- Simulate a game of **perfect recall**
  - a sequential simulation of different beam angle combinations  $g$
  - guided by probabilities obtained from a two-player zero-sum game of neural FSP
- the probability distribution is over the possible beam angle subsets,  $\theta^j$ , in the beam angle space, ,  $\Theta$
- This strongly discourages classical beam s approaches

# Game Tree Simulation

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- Network roll-out policy then efficiently guides the tree's game,  $\Gamma$  toward a *best-first* set of beam angle candidates
- Best-first leaf node encountered is the child node with the highest reward in the tree
- Essentially, a sampling-based lookout algorithm
  - Focuses learning on regions of the state space that are likely to have a good fluence

# MCTS Simulation

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- After each simulation iteration, a ‘best move’ for the current beam block is selected, as computed by the tree search
- Four steps are applied at each iteration of the simulation for each player in  $\{p_1, p_2\}$ , viz,
  - *Selection*: starting at a root node, we recursively apply a child selection policy to navigate the branches of the tree until an expandable node is encountered.
  - *Expansion*: we iteratively add one or more children to the current node, based on the available move probabilities

## ■ MCTS Steps (cont'd)

- *Simulation:* a simulation from the beam angles in the new node is carried out to determine the optimal fluence objective function
- *Back-up:* from the *leaf node* (encountered during the expansion procedure), the lookout simulation is “backed up” through its direct nodal ancestors – updating each node’s statistics as we traverse the current node up to the root node.

# Deep BOO MCTS Algorithm

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Algorithm 1 Deep BOO Monte Carlo Tree Search

```
function MCTS( $x_0, c$ )
     $x_0 \leftarrow x_0(s_0)$ 
    while search_time < budget
    do
         $\bar{x} \leftarrow EXPAND\_POLICY(x_0, c)$ 
         $\bar{x}.r \leftarrow FMO\_POLICY(\bar{x})$ 
        BACKUP( $\bar{x}, \bar{x}.r$ )
    end while
    return BEST_CHILD( $x_0$ )
end function

function FULLY_EXPANDED( $x, d$ )
     $d_i \leftarrow pairwise\_distance(x.s)$ 
    min_elem  $\leftarrow \min(d_i)$ 
    if min_elem < d then
        return True
    else
        return False
    end if
end function

function EXPAND( $x, c$ )
     $\tilde{a} = SELECT\_MOVE(x, c)$ 
    sample  $\tilde{f}$  with  $x.p(s, a)$ 
    update  $\tilde{\theta} \leftarrow \tilde{\theta} + \tilde{a}$ 
    with  $\pi_{t-1}$ , create  $\tilde{x}.p(\tilde{x}, \tilde{a})$ 
    while not  $\tilde{x} \in x_0$  do
        add  $\tilde{x}$  to  $x$ 
    end while
    return  $\tilde{x}$ 
end function

function EXPAND_POLICY( $x, c$ )
    while  $x$  nonterminal do
        if  $x$  not f_expanded then
            return EXPAND( $x, c$ )
        else
             $x \leftarrow BEST\_CHILD(x)$ 
        end if
    end while
    return  $x$ 
end function

function BACK_UP( $x, \bar{x}.r$ )
    while  $\bar{x}$  not null do
         $N(\bar{x}) \leftarrow \bar{x} + 1$ 
         $Q(\bar{x}) \leftarrow Q(\bar{x}) + \bar{x}.r$ 
         $\bar{x} = parent of \bar{x}$ 
    end while
end function

function BEST_CHILD( $x_0$ )
    if  $p_1$  to play then
        return  $x_0[\arg\min children of x_0.r]$ 
    else
        return  $x_0[\arg\max children of x_0.r]$ 
    end if
end function
```

where  $K(\bar{x}) = c\sqrt{\frac{2 \ln n(\bar{x}.s)}{N(\bar{x}.s, a)}}$  and  $\bar{x} \in x$  implies  $\bar{x} \in children of x$ .

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- Start by randomly adding five beam blocks to the state queue
- Input planes are passed through the tower residual network, from which probability distributions and a value are predicted
- Add a random walk sequence to the generated pure strategy
- Construct tree with this mixed strategy
- As new beam angle combinations are found according to the MCTS Algorithm, the FIFO queue is updated

# Example Target Volumes

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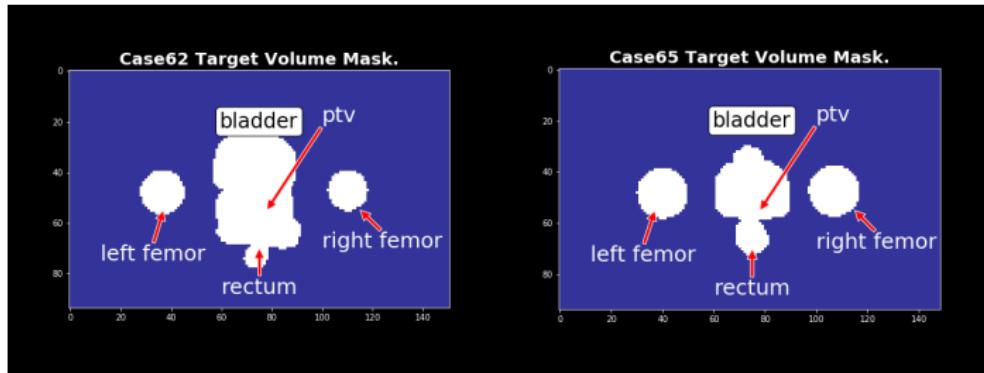
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Example Target Volumes. The PTV is engulfed within the bladder in all cases.

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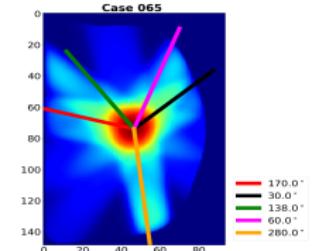
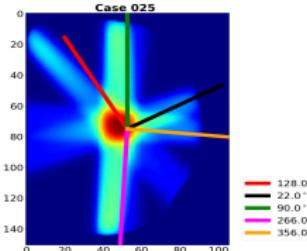
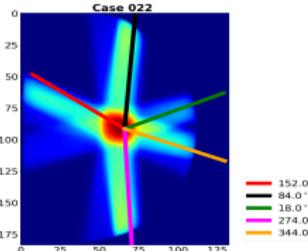
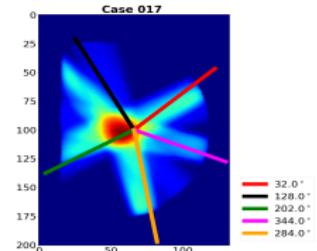
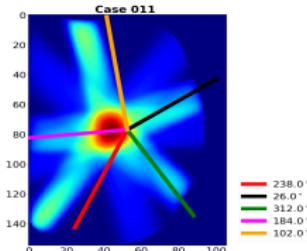
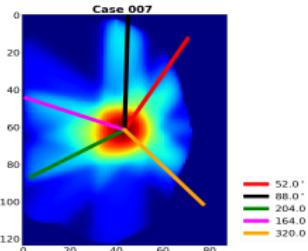
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# Dose washes for select patients during training of the self-play network

## Training Regime



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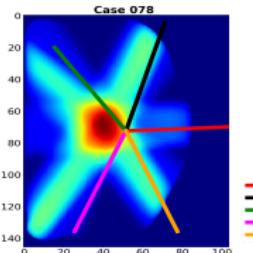
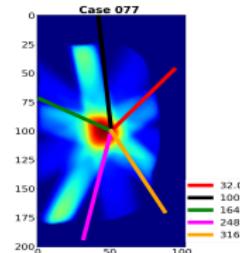
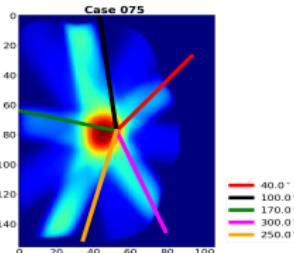
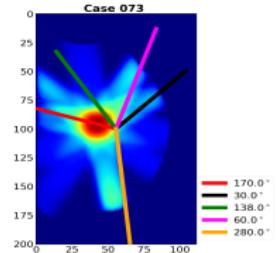
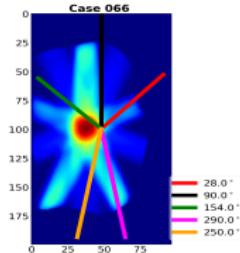
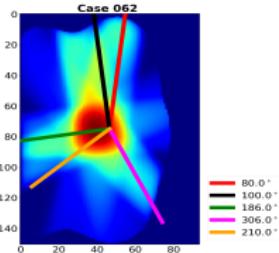
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## Dose washes for select patients during testing of self-play network

### Inference Regime



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- Adopt best practices from one of the SOTA modeling approaches viz
  - *finite element methods* [Coevoet et al. (2017); Bern et al. (2017)],
  - *constant curvature approach* [Godage et al. (2016)],
  - the *continuous Cosserat approach* [Renda et al. (2014)], and
  - the *multi-body hyper-redundant model* [Kang et al. (2012)].

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- Robust Stabilization via inverse optimality
  - Currently working on imbibing Lyapunov stability into traditional optimal controllers
  - Building on Freeman and Kokotovic's point-wise min-norm robust control lyapunov function to realize a meaningful value function (Int. Journal of Optimal Control, 1996)
  -

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## ■ Publications Under Review

- Olalekan Ogunmolu, Xuejun Gu, Steve Jiang, Nicholas Gans. [Nonlinear Systems Identification Using Deep Dynamic Neural Networks](#). Under review at *Physical Review Applied*, American Physical Society, Submitted September 2018.
- Olalekan Ogunmolu, Michael Folkerts, Dan Nguyen, Nicholas Gans, Steve Jiang. [Deep BOO 2.0: End-to-End Training of Beam Orientation Selection Policies for Intensity Modulated Radiation Therapy](#). Under publication invitation review at *International Journal of Robotics Research (IJRR)*, 2018.

## ■ Working Manuscripts

- Olalekan Ogunmolu, Ayaka Kume, Jethro Tan. [A stable Lyapunov approach for designing deep policies for complex robot motion tasks](#). *Robotics and Automation Letters/International Conference on Robotics and Automation (RA-L)*. 2019.
- Olalekan Ogunmolu, Nick Gans, Xuejun Gu, and Steve Jiang. [Simulation and control of a head and neck patient pose correction soft-robot mechanism in intensity modulated radiotherapy](#). *Transactions on Robotics (T-RO)* 2018/2019.

## ■ Accepted Publications

- Olalekan Ogunmolu, Michael Folkerts, Dan Nguyen, Nicholas Gans, and Steve Jiang. [Deep BOO: Automating Beam Orientation Selection in Intensity Modulated Radiation Therapy](#). To appear at *The 13th International Workshop on the Algorithmic Foundations of Robotics (WAFR)*, Mérida, Mexico. December 2018.

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## ■ Accepted Publications (Cont'd)

- **Olalekan Ogunmolu**, Nicholas Gans, Tyler Summers. [Minimax Iterative Dynamic Game: Application to Nonlinear Robot Control Tasks](#). *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, Madrid, Spain. October 2018.
- **Olalekan Ogunmolu**, Dan Nguyen, Xun Jia, Weiguo Lu, Nicholas Gans, and Steve Jiang. [Automating Beam Orientation Optimization for IMRT Treatment Planning: A Deep Reinforcement Learning Approach](#). Selected for Oral Presentation at the *John R. Cameron Young Investigators Symposium – 60th Annual Meeting of the American Association of Physicists in Medicine*, Nashville, TN (AAPM). July 2018.
- Yara Almubarak, Joshi Aniket, **Olalekan Ogunmolu**, Xuejun Gu, Steve Jiang, Nicholas Gans, and Yonas Tadesse, [Design and Development of Soft Robots for Head and Neck Cancer Radiotherapy](#). *SPIE: Smart Structures + Nondestructive Evaluation*, (SPIE), Denver, CO, U.S.A. March 2018.
- **Olalekan Ogunmolu**, Adwait Kulkarni, Yonas Tadesse, Xuejun Gu, Steve Jiang, and Nicholas Gans. [Soft-NeuroAdapt: A 3-DOF Neuro-Adaptive Pose Correction System For Frameless and Maskless Cancer Radiotherapy](#). *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, Vancouver, BC, Canada. September 2017. DOI: 10.1109/IROS.2017.8206211.

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## ■ Accepted Publications (Cont'd)

- Olalekan Ogunmolu, Xuejun Gu, Steve Jiang, and Nicholas Gans. [Vision-based control of a soft-robot for Maskless Cancer Radiotherapy](#). *IEEE Conference on Automation Science and Engineering (CASE)*, Fort-Worth, Texas, August 2016. DOI: 10.1109/CoASE.2016.7743378.
- Olalekan Ogunmolu, Xuejun Gu, Steve Jiang, and Nicholas Gans. [A Real-Time Soft-Robotic Patient Positioning System for Maskless Head-and-Neck Cancer Radiotherapy](#). *IEEE Conference on Automation Science and Engineering (CASE)*, Gothenburg, Sweden, August 2015. DOI: 10.1109/CoASE.2015.7294318.

# Appendix: 3-DOF Lyapunov Stability Proof

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## ■ Proof:

- Choose a Lyapunov function candidate  $\mathbf{V}$  in terms of the generalized error state space  $\mathbf{e}$ , gains,  $\tilde{\mathbf{K}}_y^T$ ,  $\tilde{\mathbf{K}}_r^T$ , and parameter error  $\varepsilon_f(\mathbf{y}(k))$  space

$$\begin{aligned}\mathbf{V}(\mathbf{e}, \tilde{\mathbf{K}}_y, \tilde{\mathbf{K}}_r^T) = & \mathbf{e}^T \mathbf{P} \mathbf{e} + \text{tr}(\tilde{\mathbf{K}}_y^T \Gamma_y^{-1} \tilde{\mathbf{K}}_y^T | \Lambda |) \\ & + \text{tr}(\tilde{\mathbf{K}}_r^T \Gamma_r^{-1} \tilde{\mathbf{K}}_r^T | \Lambda |)\end{aligned}\quad (15)$$

# Appendix: Stability proof

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$$\dot{V}(\mathbf{e}, \tilde{\mathbf{K}}_y, \tilde{\mathbf{K}}_r) = \dot{\mathbf{e}}^T \mathbf{P} \mathbf{e} + \mathbf{e}^T \mathbf{P} \dot{\mathbf{e}} + 2\mathbf{tr}(\tilde{\mathbf{K}}_y^T \Gamma_y^{-1} \dot{\hat{\mathbf{K}}}_y |\Lambda|) \\ + 2\mathbf{tr}(\tilde{\mathbf{K}}_r^T \Gamma_r^{-1} \dot{\hat{\mathbf{K}}}_r |\Lambda|)$$

$$= \left[ \mathbf{A}_m \mathbf{e} + \mathbf{B} \Lambda [\Delta \hat{\mathbf{K}}_r^T \mathbf{r} + \Delta \hat{\mathbf{K}}_x^T \mathbf{x}] \right]^T \mathbf{P} \mathbf{e} + \dots$$

$$\mathbf{e}^T \mathbf{P} \left[ \mathbf{A}_m \mathbf{e} + \mathbf{B} \Lambda [\Delta \hat{\mathbf{K}}_r^T \mathbf{r} + \Delta \hat{\mathbf{K}}_x^T \mathbf{x}] \right] + \dots$$

$$2\mathbf{tr}(\Delta \mathbf{K}_x^T \Gamma_x^{-1} \dot{\hat{\mathbf{K}}}_x |\Lambda|) + 2\mathbf{tr}(\Delta \mathbf{K}_r^T \Gamma_r^{-1} \dot{\hat{\mathbf{K}}}_r |\Lambda|)$$

# Appendix: Stability Analysis

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$$= \mathbf{e}^T (\mathbf{P} \mathbf{A}_m + \mathbf{A}_m^T \mathbf{P}) \mathbf{e} + 2\mathbf{e}^T \mathbf{P} \mathbf{B} \Lambda \left( \tilde{\mathbf{K}}_y^T \mathbf{y} + \tilde{\mathbf{K}}_r^T \mathbf{r} \right) \\ + 2\mathbf{tr} \left( \tilde{\mathbf{K}}_y^T \Gamma_y^{-1} \dot{\tilde{\mathbf{K}}}_y |\Lambda| \right) + 2\mathbf{tr} \left( \tilde{\mathbf{K}}_r^T \Gamma_r^{-1} \dot{\tilde{\mathbf{K}}}_r |\Lambda| \right)$$

$$= -\mathbf{e}^T \mathbf{Q} \mathbf{e} - 2\mathbf{e}^T \mathbf{P} \mathbf{B} \Lambda \varepsilon_f(\mathbf{y}) + 2\mathbf{e}^T \mathbf{P} \mathbf{B} \Lambda \tilde{\mathbf{K}}_y^T \mathbf{y} \\ + 2\mathbf{tr} \left( \tilde{\mathbf{K}}_y^T \Gamma_y^{-1} \dot{\tilde{\mathbf{K}}}_y \right) + 2\mathbf{e}^T \mathbf{P} \mathbf{B} \Lambda \tilde{\mathbf{K}}_r^T \mathbf{r} + 2\mathbf{tr} \left( \Delta \mathbf{K}_r^T \Gamma_r^{-1} \dot{\tilde{\mathbf{K}}}_r \right)$$

Notice  $x^T y = \mathbf{tr} (y x^T)$  from trace identity

# Appendix: Stability Analysis Cont'd

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Therefore,

$$\begin{aligned}\dot{\mathbf{V}}(\cdot) = & -\mathbf{e}^T \mathbf{Q} \mathbf{e} - 2\mathbf{e}^T \mathbf{P} \mathbf{B} \Lambda \varepsilon_f \\ & + 2 \operatorname{tr} \left( \tilde{\mathbf{K}}_y^T (\Gamma_y^{-1} \dot{\tilde{\mathbf{K}}}_y + \mathbf{y} \mathbf{e}^T \mathbf{P} \mathbf{B} \operatorname{sgn}(\Lambda)) \right) |\Lambda| \\ & + 2 \operatorname{tr} \left( \tilde{\mathbf{K}}_r^T (\Gamma_r^{-1} \dot{\tilde{\mathbf{K}}}_r + \mathbf{r} \mathbf{e}^T \mathbf{P} \mathbf{B} \operatorname{sgn}(\Lambda)) \right) |\Lambda|\end{aligned}$$

where for a real-valued  $x$ , we have  $x = \operatorname{sgn}(x)|x|$ .

- first two terms will be negative definite for all  $\mathbf{e} \neq 0$ 
  - since  $\mathbf{A}_m$  is Hurwitz
- other terms will be identically null if we choose the adaptation laws

$$\dot{\tilde{\mathbf{K}}}_y = -\Gamma_y \mathbf{y} \mathbf{e}^T \mathbf{P} \mathbf{B} \operatorname{sgn}(\Lambda), \quad \dot{\tilde{\mathbf{K}}}_r = -\Gamma_r \mathbf{r} \mathbf{e}^T \mathbf{P} \mathbf{B} \operatorname{sgn}(\Lambda)$$

# Appendix: Stability Results

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We find that

$$\begin{aligned}\dot{\mathbf{V}}(\cdot) &= -\mathbf{e}^T \mathbf{Q} \mathbf{e} - 2\mathbf{e}^T \mathbf{P} \mathbf{B} \Lambda \varepsilon_f \\ &\leq -\lambda_{low} \|\mathbf{e}\|^2 + 2\|\mathbf{e}\| \|\mathbf{P} \mathbf{B}\| \lambda_{high}(\Lambda) \varepsilon_{max}\end{aligned}$$

- $\lambda_{low}, \lambda_{high} \equiv$  minimum and maximum characteristic roots of  $\mathbf{Q}$  and  $\Lambda$  respectively.
- $\dot{\mathbf{V}}(\cdot)$  is thus negative definite outside the compact set
- $\chi = \left( \mathbf{e} : \|\mathbf{e}\| \leq \frac{2\|\mathbf{P} \mathbf{B}\| \lambda_{high}(\Lambda) \varepsilon_{max}(\mathbf{y})}{\lambda_{low}(\mathbf{Q})} \right)$
- thus, we conclude that the error  $\mathbf{e}$  is uniformly ultimately bounded.
  - i.e.  $\mathbf{y}(t) \rightarrow 0$  as  $t \rightarrow \infty$

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