A Generalized Nash Equilibrium-Seeking Scheme for Trauma Resuscitation

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Abstract: Today, trauma resuscitation in medical emergency procedures is a qualitative procedure that is driven by the experience of hospital worker (HCW) teams. Designing and optimizing quantifiable metrics that accurately capture HCW decisions may augment current resuscitation procedures — which may produce better outcomes in safety-critical emergency environments. In this sentiment, we study socio-technical trauma resuscitation, set in a fabric of distributed generalized Nash equilibrium (GNE)-seeking games. Novel insights (drawn from clinical experience) that model workers' behavior under various resource constraints and team social dynamics are introduced. These inform group dynamics equilibrium that constitute the best possible resuscitation outcome given workers' workloads schedules, resource allocation, transient bottlenecks, worker competencies, and systemic constraints. While our recent efforts explored an empirical deconstruction of the problem as a multi-agent optimization problem with separable objectives, here we explore a holistic GNE treatment within a time-varying graph. We present new results that transform previously disseminated empirical analyses with standardized and systematic treatment, greater simplification, clarity, and improvement. In the future, our proposal may constitute a value chain to current emergency room practices.

Keywords: Cyber-physical and human systems (CPHS); Social computing; Game theories.

1. INTRODUCTION

Trauma resuscitation (TR) in emergency medical environments is a collaborative (often conflicting) coordination objective that must be solved among multiple healthcare workers (HCW). These HCW are usually characterized by significant differential in (i) skillset, (ii) mental and physical alacrity; and (iii) communication skills. Currently, task coordination follows human experience and data trends from screens, verbal and aural cues (Sarcevic et al., 2012) from organization verticals and peer horizontals in the medical environment; and patient vitals monitoring systems by which individual HCW can augment their decision-making prowess. No systematic and quantifiable means exist to integrate all the variables into an algorithmic framework whose performance can be consistently tracked.

Previous efforts have considered the integration of active autonomous systems to support team dynamics (Taylor et al., 2024), deliver medications or supplies (E. Salinas-Avila et al., 2022), or sanitize surgical rooms (Sun and Yeung, 2007). Our recent effort addresses equitable workload allocation in a multi-agent setting by studying the problem in our controlled MARLHospital simulator environment (Ekpo et al., 2025).

In trauma resuscitation procedures with long lead times, individual worker's mental and physical alacrity are prone to degradation over time. Required procedural administration efficacy also reduce as fatigue weighs in, and the capacity for employing all inherent skills optimally do not match demanded requisition as pressure builds up. Lastly, miscommunication or incorrectly parsed communication may inform duplicated task execution, engender redundancy, and result in degraded performance. Burnout and fatigue enervate a positive work environment. All these are factors that can subjugate otherwise potentially good performance in the absence of these drawbacks.

A well-executed TR procedure comprise a well-reasoned, deliberative process so that skill-informed task delegation by clinic leaders, and responsibilities hand-off among peers is executed efficiently despite the constraints of time and mission safety. What (semi-) autonomous decision-making process can we device to (i) augment human healthcare decision systems through automatic workload redistribution among healthcare workers when mental and physical alacrity is in jeopardy, (ii) maximize HCW skill levels and match overall demand whilst counterbalancing the skill levels of other HCWs in the process, (iii) and to enhance communication efficacy — attenuating duplicity of information exchange, redundancy in the performance of tasks all toward better medical outcomes in spite of rapidly changing, high duress, time-intense safety-critical socio-technical environments?

We are concerned with quantifiable algorithms in these socio-technical, life-altering medical settings that can enhance patient outcomes. Such algorithms must be capable of calibrating decisions according to skill levels, alacrity indices, and communication efficacy through organization horizontals and verticals, so that the chaos and time-pressure of emergency procedures do not catalyze human errors that are debilitative in patient outcomes. The tasks delegated to each worker in a hand-off phase must match their abilities, skills, mental alacrity, and suitableness to the task. If done correctly, TR can quickly identify, mitigate, or help manage potential life-threatening injuries.

Therefore, we take the active step of developing an information-theoretic decision-making mechanism in collaborative trauma resuscitation medical healthcare worker settings, informed by our observations in a representative training environment for healthcare workers at Weill Cornell Emergency Medicine in New York City [1]. The algorithms that will herald the age of high-impact human+embodied decision-making in TR settings may benefit from a systematic mathematical formalization of individual and team behavior, alertness levels, and communication protocols based on a game-theoretic decision-making over multiple healthcare workers within a distributed network. In this sentiment, we formalize the coordination and decision-making workflow in a clinical healthcare setting as a distributed non-cooperative game among various players (healthcare workers). Our objectives are to provide mathematically reasonable equivalences between observed skill levels, tasks coordination models, and provide an algorithmic framework for optimizing trauma resuscitation procedures.

The rest of this paper is structured as follows: related works and our theoretical machinery is set forth in Section 2 for the innovation we bring to bear in Section 3. We conclude our arguments in Section 5.

2. BACKGROUND

This section describes healthcare automation in general multi-agent high-duress environments; which are then contextualized within distributed GNE (Sec. 2.1). Trauma resuscitation as a distributed game is discussed in (Sec. 2.2). The mathematical notations used throughout the rest of the paper are presented (Sec. 2.3) a a games taxonomy is presented in 2.4. Assumptions, definitions, lemmas and theorems used to build our results are given in (Sec. 2.5).

2.1 Multiagent Healthcare Automation

The role of shared mental models and decentralized communication workflows was described in high-duress trauma resuscitation contexts in (Sarcevic et al., 2012; Fornander et al., 2024). Co-design studies have proposed cognitive aids and augmented-reality systems to improve task visibility, information flow, and situational awareness for distributed clinical teams (Taylor et al., 2024; Zhang et al., 2025). Recent frameworks such MATEC (Cho et al., 2025) and TraumaFlow (Neumann et al., 2024) demonstrate that structured software agents can align team actions with clinical protocols, though their coordination logic is often centralized. Whereas other approaches propose distributed agent teams to infer behavioral norms and adapt to emergent clinical patterns (J et al.).

Distributed optimization in games over networks serve as a principled mathematical framework for modeling coordination among HCWs: a generalized Nash equilibrium (GNE) mechanism was proposed for multi-agent social agents with coupling constraints (Dave et al., 2022; Zhao et al., 2025). Operator-theoretic methods, including forward–backward splitting in monotone inclusions, provide scalable distributed algorithms with convergence guarantees (Belgioioso et al., 2021). When agents interact over sparse networks and make decisions based on local observations, these tools yield consensus-seeking strategies with provable stability. Models incorporating agent dynamics, constraints, and communication topology align well with real-world healthcare team behavior, where agents (human or machine) exhibit bounded rationality, partial observability, and asymmetric influence.

2.2 Trauma Resuscitation as a Distributed Game

In trauma resuscitation (TR) socio-technical problems, skill abilities, alacrity (mental alertness) indices, and communication efficacy are all assumed to be given based on past player records. These indices are then used in selecting players (HCWs) appropriate to a TR procedure. Given the selfish objective of each player with respect to their task, and the coupling of each player's action to those of its neighbors, the procedure (or game, Γ) may be considered inherently heterogeneous. A decentralized decision-making mechanism typifies each player's tasks execution, so that the game, Γ, may be characterized as a decentralized decision-making network. Such networks possess a global objective for each trauma procedure. *How do we effectively model subjective human decisions, recommend actions, and provide failure-proof decisions for the end goal of patient resuscitation?* Nash Equilibrium (NE) lends itself well easily to the task at hand. Critically, since the feasible action set of each player in the game is conditioned on those of other players, a generalized Nash equilibrium (GNE) solution concept is most appropriate.

We are therefore concerned with computer-supported cooperative work in TR procedure where γ externalizes critical information arising from player decisions that are relevant to the patient's health outcome. In a control-theoretic sense, this is situated as a scalable GNE-seeking decision-making algorithm over a network of players. As each player does not possess full observability of every other player's action in the system, we cast the game in a partially observable setting where the players are *noncooperative* (that is each player acts selfishly to extremize its given objectives), but collaborative in information sharing with their neighbors so that the imprecision of global information awareness is mitigated.

Thus, searching for a GNE where every respective player's action are optimal with respect to their individual objectives and no single player can improve their objective by unilaterally changing their action seem most appropriate to the task at hand. Such a state of consensus, if it can be achieved in a human-computer interaction manner, would constitute the ideal goal in these trauma resuscitation socio-technical settings. The notations that are used throughout the rest of this paper are next defined.

[1] The base camp for interprofessional pediatric emergency medicine team training in the emergency care of critically ill and injured children was employed (Weill Cornell Medicine, 2025).

2.3 General Notations

The set of real (non-negative) numbers is denoted, $\mathbb{R}\,(\mathbb{R}_+)$, the m-dimensional vector space is $\mathbb{R}^m$, and the $n \times n$ dimensional real matrix is denoted $\mathbb{R}^{n\times n}$. A time-varying variable, x is denoted $x(t)$. For a function f that depends on $x(t)$, we write $f(x;t)$. The absolute value of scalar x is $|x|$. The Euclidean norm of the vector $x \in \mathbb{R}^n$ is $\|x\| := \sqrt{x^\top x}$. The n-dimensional vector of ones (zeroes) is $1_n(0_n)$. For a differentiable function $J(x)$, the first-order derivative of $J(x)$ is denoted $\nabla_x J(x)$. For a set Ω and variable x, the Euclidean projection of x onto Ω is $\mathcal{P}_\Omega(x) = \mathrm{argmin}_{x\in\Omega}\|x - x'\|_2$. The set $\mathcal{S} \subset \mathbb{R}^m$ is a convex set if for $\alpha > 0$ and for every $x, y \in \mathcal{S}$, $\alpha x + (1-\alpha)y \in \mathcal{S}$. The cardinality of a $\mathcal{S}$ is denoted $\lfloor \mathcal{S} \rfloor$. Suppose that $\mathcal{S}$ is a closed convex set, and $\mathcal{P}_\mathcal{S}(x)$ is a projection of x onto the set $\mathcal{S}$, then there is a unique element $\mathcal{P}_\mathcal{S}(x) \in \mathcal{S}$ such that $\|x - \mathcal{P}_\mathcal{S}(x)\| = \inf_{y\in\mathcal{S}} \|x - y\|$. Similarly, the Euclidean projection of x onto the set Ω is denoted $P_\Omega(x)\| = \mathrm{argmin}_{y\in\Omega}\|x - y\|$,

2.4 Games Taxonomy

Let a game be denoted as $\Gamma(\mathcal{V}, \Omega, J)$, where $\mathcal{V}$ are the players $\{1, \ldots, n\}$, each with action x_i belonging in a constraint set $\Omega_i \in \mathbb{R}^m$ and a local cost profile $J_i : \Omega_i \to \mathbb{R}^m$ so that the $J = \{J_1, \cdots, J_n\}$. The action profile of the game is $\Omega := \Omega_1 \times \cdots \times \Omega_n$. The vector formed by all players' actions is $x := (x_i, x_{-i}) \triangleq \{x_i\}_{i=1}^n$, where the vector formed by all the players' actions except those of player i is $x_{-i} = \{x_{i'}\}_{i'=1,\, i'\neq i}^n \in \mathbb{R}^{n-i}$ where $n_{-i} := n - n_i$.

Define $K_i^p := \{x_i \in \Omega_i \,|\, h_i(x_i) = 0\}$ as the set of each individual player's constraint set and $K^s := \{x \in \Omega \,|\, g(x) \geq 0_m\}$ as the set of shared constraints, where $g(x) = c^\top x - d$, for $c = [c_1^\top, \cdots, c_n^\top]^\top$, $c_i \in \mathbb{R}^m$ and $d \in \mathbb{R}$. It follows that player i's action is constrained along two directions: constraints that depend on the action of other players i.e. $g(x) \geq 0$, and individual constraints that depend on player i's action *i.e.* $x_i \in K_i$. Contrary to popular formulations in GNE-seeking literature, we have chosen $g(x) \geq 0$ since the skills, time indices, and other parameters we are optimizing for cannot be negative.

2.5 Games Machinery

Let us set a few definitions and preliminary results in motion. Denote the feasible set of player i' action by $K_i(x_{-i}) := \{x_i \mid (x_i, x_{-i}) \in K\}$, where $K := K^s \cap (K_1 \times \cdots K_n)$. Suppose we define the generalized Nash equilibrium problem (GNEP) as $\Gamma(\mathcal{V}, \Omega, J, K)$, then the individual players in the game "freeze" other players' actions x_{-i} as exogenous variables, to solve

$$\min_{x_i\in\Omega_i} J_i(x_i, x_{-i}), \quad \forall\, i \in \mathcal{V}$$
$$\text{subject to } x_i \in K_i(x_{-i}). \tag{GNEP}$$

***Definition** 1.* (Neighbors of a Player). We define the neighbors $\mathcal{N}_i(t)$ of player i at time t as the set of all agents that lie within a predefined radius, r_i. Formally, $\mathcal{N}_i(t) = \{j \mid (j,i) \in \mathcal{E}\}$.

***Definition** 2.* (Generalized Nash Equilibrium). In problem (GNEP) , the action profile $x^\star := (x_i^\star, x_{-i}^\star)$ is a generalized Nash equilibrium if $J_i(x_i^\star, x_{-i}^\star) \leq J_i(x_i, x_{-i}^\star)$, $\forall\, x_i \in \Omega_i$, $\forall i \in \mathcal{V}$.

Some basic assumptions are stipulated to render the solution of (GNEP) feasible.

***Assumption** 1.* (Interconnectivity). The game occurs over a time-varying *connected* [2] and undirected [3] weighted communication graph $\mathcal{G}(\mathcal{A})(t) : \{\mathcal{V}, \mathcal{E}\}$ with edges $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ and interconnections' adjacency matrix $\mathcal{A} := [a_{ij}] \in \mathbb{R}^{n\times n}$. Player i is connected to (can receive information from) player j if $(i,j) \in \mathcal{E}$ and vice versa. Let $a_{ij} > 0$ if $(i,j) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. For a $D = diag(d_{11}, \cdots, d_{nn}) \in \mathbb{R}^{n\times n}$, where $d_{ii} = \sum_{j=1}^n a_{ij}$ for all $i \in \mathcal{V}$, we have $\mathcal{G}(\mathcal{A})$'s Laplacian as $\mathcal{L} := D - \mathcal{A} \in \mathbb{R}^{n\times n}$

***Assumption** 2.* (Cost Function). For $i \in \mathcal{V}$, $\Omega_i \in \mathbb{R}^m$ is a nonempty, convex, and closed set; and the cost $J_i(x_i, x_{-i})$ is continuously differentiable on Ω and convex in x_i for every fixed x_{-i}.

Define $\nabla J(x) = [\nabla_x J_1^\top(x), \cdots, \nabla_{x_n} J_n^\top(x)]^\top$ as the *single-value* game mapping.

***Assumption** 3.* (Monotonicity and Game-mapping). The single-valued mapping $\mathcal{M} : \mathbb{R}^m \to \mathbb{R}^m$ is strongly monotone on the action constraint set $\Omega \in \mathbb{R}^m$ if there exists a constant $\mu > 0$ for any $x, x' \in \Omega$, where $x \neq x'$ such that $(\nabla J - \nabla J')^\top (x - x') \geq \mu\|x - x'\|^2$ for any $\nabla J \in \mathcal{M}(x)$, $\nabla J' \in \mathcal{M}(x')$.

***Assumption** 4.* (Constraint Function). The inequality constraint function $g(x) \geq 0$ is continuously differentiable and component-wise convex in x. In addition, the feasible action set K is nonempty, convex, and closed — satisfying the Slater's condition *i.e.* , $x^\star \in \mathrm{int}(\Omega)$ [4] such that $g(x^\star) \leq 0_m$.

***Lemma** 1.* (Existence of a GNEP). Let the GNEP of Def. 2 be given and suppose that (i) $\Omega_i \in \mathbb{R}^m$ be a nonempty, convex, and compact such that for $i \in \mathcal{V}$, $K_i(x_{-i})$ is nonempty, convex, closed, and K_i is both upper and lower semi-continuous; and (ii) the local cost $J_i(x_i, x_{-i})$ is quasi-convex on $K_i(x_{-i})$ x for every $i \in \mathcal{V}$. Then, a GNE exists.

Proof. [Existence of GNEP] The Lemma 1 is a statement of Thm 4.1 in (Facchinei and Kanzow, 2010).

***Remark** 1.* When assumptions 2, 3, and 4 are fulfilled, then the GNEP $\Gamma(\mathcal{V}, \Omega, J, K)$ satisfies the existence condition $x^\star$ in Lemma 1.

***Lemma** 2.* (Optimality of the GNE). Suppose that there exists a multiplier $\lambda_i^\star \in \mathbb{R}^m$, $\forall i \in \mathcal{V}$. The optimality condition for each player can be found via the KKT conditions

$$\nabla_{x_i} J_i(x_i^\star, x_{-i}^\star) + \langle \lambda_i^\star, \nabla_x g(x^\star)\rangle = 0_n,\ \forall\, i \in \mathcal{V}, \lambda_i^\star \in \mathcal{V},$$
$$0_m \leq \lambda_i^\star \perp\ g(x_i^\star, x_{-i}^\star) \geq 0_m \tag{1}$$

under the assumption of continuous differentiability. This is a statement of Thm 4.6 in Facchinei and Kanzow (2010).

As a system of equations, (1) can be written as

$$J(x^\star, \lambda^\star) = 0,$$
$$0 \leq \lambda^\star \perp g(x^\star)\ \geq\ 0, \tag{2}$$

[2] That is, any two nodes in $\mathcal{V}$ are connected ny a path.
[3] That is, $(i,j) \in \mathcal{E} \Leftrightarrow (j,i) \in \mathcal{E}$.
[4] The relative interior of the set Ω is denoted $\mathrm{int}(\Omega)$.

Table 1. HCW Skill Attributes Table

Taxonomy	Meaning
CPR	Cardiopulmonary resuscitation
ROSC	Return of spontaneous circulation
IV/IO	Intravenous or intraosseous access
BVM	Bag–valve–mask ventilation
$ETCO_2$	End–tidal CO_2 measurement
ECG	Electrocardiographic rhythm analysis
VF/pVT	Ventricular fibrillation(tachycardia)
PEA	Pulseless electrical activity
Hs/Ts	Reversible arrest causes (hypoxia etc.)
EPI	Epinephrine administration
AMIO	Amiodarone (antiarrhythmic)
LIDO	Lidocaine (antiarrhythmic alternative)
SHOCK	Deliver defibrillation shock
RHYTHM_CHECK	Evaluate rhythm every 2 min
AIRWAY_MGMT	Airway interventions (OPA/NPA/ETT)
VENTILATION	Ventilation cycles
DRUG_PREP	Medication preparation
ACCESS_EST	Establish IV/IO access
TEAM_COMM	Critical team communication
LOG_ACTION	Real-time clinical event logging

for $g(x^\star) = [g^\top(x_1^\star), \cdots, g^\top(x_n^\star)]^\top$, $\lambda = [\lambda_1^{\star\top}, \cdots, \lambda_n^{\star\top}]^\top$, and $J(x^\star, \lambda^\star) = [\nabla_{x_1} J_1^\top(x_1^\star), \cdots, \nabla_{x_n} J_n^\top(x_n^\star)]^\top$.

With the constraint qualification, the $\boldsymbol{x}$ portion of (2) is a first-order necessary condition for the GNEP; and under appropriate convexity assumptions, the $\boldsymbol{x}$ part solves the GNEP so that (2) is also the sufficient condition for the GNEP.

***Remark** 2.* Lemma 2 is the so-called variational inequality problem $\mathrm{VI}(X, \nabla J(x))$ whose solution set is a special class of GNEs with all Lagrange multipliers equal. That is, the solution $x^\star$ to the $VI(X, \nabla J(x))$ under the KKT conditions (1) satisfies the GNE $x^\star$ if $\lambda_1^\star = \cdots = \lambda_n^\star = \lambda^\star$.

3. A GNEP TRAUMA RESUSCITATION SCHEME

We now consider the socio-technical trauma resuscitation problem in a typical clinic. Our objective is to translate clinical attributes such as skill proficiency, mental alacrity, physical energy, and communication efficacy—into explicit mathematical quantities that determine each worker's cost, feasible actions, and interaction structure. This provides a concrete model on which the distributed primal–dual algorithm can operate.

3.1 Clinical model

In this section, we present the mathematical transformation of the clinical workers attributes and decision-making modality into quantifiable modeling and decision optimization metrics. Each healthcare worker is considered a player $i \in \mathcal{V}$, with objective J_i (e.g. , start CPR) that is associated with individual constraints K_i^p (e.g. hemorrhage control), and shared constraints with other players K_i^s (e.g. , breathing or pulse checks) in a game $\Gamma(\mathcal{V}, \Omega, \{J_i\}, K)$ where $K_i = K_i^s \cap K_i^p$. Table 1 itemizes typical HCW skill attributes inspired by the ALS code card for adult cardiac arrest (American Red Cross, 2025).

The state, x_i, of each player is characterized as

$$x_i(t) := [\, s_i,\ a_i(t),\ f_i(t),\ c_i(t)\,]^\top \tag{3}$$

with skill proficiency index s_i, and time-dependent parameters: alacrity index $a_i(t)$, workload schedule or fairness index, $f_i(w;t)$, and communication efficiency $c_i(t)$.

The skill proficiency index is a fixed scalar, determined by player i's job complexity and their competence (averaged over past assignments and performance); this skill proficiency index, calibrated between 0 and 1, may be informed by the training a player may have availed themselves including nursing, clinical certifications, physician training etc. A player with a skill proficiency of 1 is very competent and the depravity in skill proficiency reduces based on the qualifications of a worker. Ultimately, this is subjectively assigned based on the institution. In practice, s_i is a calibration of each player's abilities by their employer based on prior records and specialty. ; while c_i measures the communication efficacy between each connected player on the graph.

The action profile of all players in the game is thus contained in the set $\Omega = K_1 \times \cdots \times K_n \subset \mathbb{R}^{4n}$, where K_i is the local action feasibility set for each player.

Players' transient step response is described as a second-order system with transient step response (Nise, 2019)

$$\sigma_i(t) = 1 - \frac{1}{\sqrt{1-\zeta^2 t}} \exp\left(-\zeta\omega_n t\right) \cos(\omega_n \times \sqrt{1-\zeta^2 t} - \phi) \tag{4}$$

where $\phi = \arctan(\zeta/(\sqrt{1-\zeta^2}))$ for a damping ratio ζ and natural frequency ω_n. We are particularly interested in underdamped characterizations of (4) whereupon $\zeta < 1$. This is more suitable to model human energy levels, alertness, and alacrity in general as a function of response to external stimuli. Human players in the game do not make decisions exactly — there is a certain level of oscillations characterized with task executions by human beings that may be more appropriately modeled by underdamped systems. We assume that all HCWs are equipped with pulse or heart rate monitors [5] from which their natural frequency, ω_n, in terms of their resting heart bit per minute (bpm or b_i) can be easily extracted and broadcast to the optimization's computing station.

Alacrity measure: The alacrity index a_i encodes a player's briskness, eagerness, and cheerful readiness to execute tasks that may emerge in the course of the resuscitation and that fall within their skillset. We model a_i as a function of the transient step response (4), lumping the readiness, enthusiasm, and player i's energy characteristics that constitute this index into the *rise time*, t_r — a measure of the player's ability to go from 10% to 90% of the final value of $\sigma(t)$ given a fixed ζ and measured $\omega_n^i := 120\pi\, b_i$. In practice, the normalized rise time, $\omega_n t_r$ is evaluated by fixing $\zeta = k < 1$, choosing $\omega_n^i t$ as the normalized time variable and estimate the values of $\omega_n^i t$ that correspond to the different of $\sigma_i(t) = 0.9$ and $\sigma_i(t) = 0.1$. Then, $a_i(t)$ at time t is found as

$$a_i(t) = \Delta\sigma(t)/\omega_n^i, \quad t \geq 0. \tag{5}$$

Fairness in workload distribution should be encouraged where players share resources with others. Fairness

[5] Such as sports tracking watches etc.

allocation as a performance index is relevant to a holistic player cost. If one worker shares a greater percentage of the workload beyond their capacity, such uneven throughput with respect to bottleneck becomes untenable for an efficient resuscitation outcome. In this sentiment, we think a good fairness metric should be independent of population size since it is an equal thing to seek a GNE in a small or large healthcare workers population. We also think the index should be scale-invariant and bounded to say between 0 and 1 so that it can be expressed as a percentage. Lastly, continuity in the fairness index is crucial so that a minute change in a work allocation variable is reflected in the objective. The Jain fairness index (Jain et al., 1984) is a convex, scale-invariant metric that is most-suited to our requirements. Therefore, we adopt the metric

$$f_i(w;t) = \frac{\left(\sum_{i=1}^{n} w(t)\right)^2}{\sum_{i=1}^{n} w(t)^2}, \quad w(t), t \geq 0 \tag{6}$$

which measures the equity of work w allocated to player i. We see that $f_i(w)$ is 1 if all users get an equitable workload, meaning fairness is 100% and 0 if an allocation procedure favors only a few users.

Communication efficiency In emergency response trauma resuscitation units, it is not enough for the player's transients to be optimal with respect to their individual objective; they must balance this optimization objective against a balanced workload allocation among peer players, efficient communication practices that assure optimal overall outcomes. We define the communication efficacy as a function of the (i) settling time, (ii) engagement level, and (iii) response time.

The **settling time**, t_s^i of $\sigma_i(t)$ *i.e.* the time required for player i's transient $\sigma_i(t)$'s damped oscillations to reach and stay within $\pm 2\%$ of $\sigma_i(t)$'s steady state value after a trigger. Since we model players' behaviors as a second order, underdamped system responding to step function inputs, the settling time may well be approximated as

$$t_s^i = -\ln(\epsilon\sqrt{1-\zeta^2})/\zeta\omega_n \tag{7}$$

where ϵ may be conveniently set to 0.02 *i.e.* , 2% of the final value of $\sigma_i(t)$.

The **response time** is the record time step defined as the difference between reaching steady state and being within 10% of the final value of $\sigma_i(t)$ upon a workload handover by a neighbor j. The timely response, $\bar{t}_i$ for player i is given by $\bar{t}_i = k\Delta t_{ij}$ where Δt_{ij} is the time difference between calls from player i to j for all $j \in x_{-i}$.

The **engagement level** for player i is an average over the edge weights of all i's neighbors *i.e.* $\mathcal{N}_i$, defined as the running mean of all visit counts from neighboring players up to the current (discretized) time $\mathcal{K}$ *i.e.*

$$e_i(t) = \frac{1}{Tn_{-i}} \int_{t_0}^{T} \left(\sum_{k=1}^{\mathcal{K}} \nu_{ij}(t) \right) dt \quad \forall\, j \in x_{-i}.$$

In this sentiment, the **communication efficiency index** is

$$\upsilon_i(t) = \alpha(t_s^i + \bar{t}_i) + (1-\alpha)e_i(t),\ \alpha \in (0,1). \tag{8}$$

3.2 Local costs and local aggregative dynamics

Given the ongoing numerical abstractions, it seems most reasonable to describe the players' dynamics in a resuscitation procedure via an aggregative game, *i.e.* games whereupon the cost function of a player is a function of its own action as well as other players' actions.

***Problem** 1.* Formally, we want a distributed algorithm for a GNEP $\Gamma(\mathcal{V}, \Omega, \nabla J(x), K)$ for the socio-technical aggregative game with coupled constraints, K.

We first prescribe the local cost for each player i, which is expressed as the quadratic objective between the objective of player i and those of its neighbors $\mathcal{N}_i$ *i.e.*

$$J_i(x_i, x_{-i}; t) := J_i(x_i; t) + \langle J_i(x_{-i}; t)\rangle_{r_i}, \tag{9}$$

where

$$J_i(x_i;t) = \|s_i\|^2 + \|a_i(t)\|^2 + \|f_i(w;t)\|^2 + \|\upsilon_i(t)\|^2$$

and

$$\langle J_i(x_{-i};t)\rangle_{r_i} = \frac{1}{1+n_i(t)} \left(J_i(x_i;t) + \sum_{j \in \mathcal{N}_i(t)} J_i(x_{-i};t) \right) \tag{10}$$

denotes the cost of neighboring players in the vicinity $r_i > 0$ from player i. We now propose the following GNEP algorithm, omitting the time indices for easy readability

$$\dot{x}_i = P_{\Omega_i}\left(x_i - \nabla_{x_i} J_i(x_i) + \frac{\gamma}{n}\lambda_i \nabla_{x_i} g_i(x_i) \right) - x_i \tag{11a}$$

$$\dot{\lambda}_i = \kappa \sum_{j \in \mathcal{N}_i} \operatorname{sgn}(\lambda_i - \lambda_j) + \lambda_i(g_i(x_i)) \tag{11b}$$

$$\dot{\zeta}_i = \rho \sum_{j \in \mathcal{N}_i} \operatorname{sgn}(\eta_i - \eta_j),\ \dot{\eta}_i = \zeta_i + J_i(x_{-i}) \tag{11c}$$

where κ, ρ, $\gamma > 0$ and (11) has the following for its initial conditions: $x_i(0) \in \Omega_i$, $\lambda_i(0) = g(x_i; 0)$, $\zeta_i(0) = 0_m$. The dynamics in (11c) follow finite-time dynamic averaging consensus, and the discontinuous vector fields therein, being differential equations with discontinuous right-hand-sides possess Filippov solutions.

The proposed distributed generalized Nash equilibrium seeking algorithm (11) is a variant of Liang et al. (2017) but with the inequality constraints $g(x) \geq 0$.

The constants κ and ρ can be obtained as

$$\kappa > (n-1)h_1, \quad \rho > \gamma(n-1)h_2 \tag{12}$$

where

$$h_1 = \sup_{i \in \mathcal{V}} \left(\sup_{x_i \in \Omega_i} \|\nabla_{x_i} J_i(\cdot, x_{-i})\| \sup_{y, y' \in \Omega} \|y - y'\| \right)$$

and

$$h_2 = \sup_{i \in \mathcal{V}} \left(\sup_{x_i \in \Omega_i} \|g_i(x_i)\| \right).$$

And the distributed calculation needed to obtain κ, ρ, γ can be determined by choosing variables a_i with $a_i(0) = \alpha_i$ for $i \in \mathcal{V}$, and update them as $a_i(k+1) = \sup\{a_i(k), a_j(k), j \in \mathcal{N}_i\}$. We thus find $\sup\{\alpha_i, i \in \mathcal{V}\}$ within $N-1$ steps. The convergence follows (Liang et al., 2017)'s analyses.

4. NUMERICAL RESULTS

This section provides some numerical considerations to the solution concepts introduced in (Sec. 3).

The shared constraint set $g(x) \geq 0$ encodes throughput and resource limits and $d \in \mathbb{R}^p$ is a vector of clinical capacity parameters. Typical components of $g(x)$ include:

- upper bounds on combined airway and compression throughput,
- limits on the number of simultaneous invasive procedures,
- caps on aggregate workload in high acuity phases.

We partition C into player specific blocks

$$C = [C_1 \ C_2 \ \cdots \ C_n], \qquad C_i \in \mathbb{R}^{p\times 4}, \tag{13}$$

so that $g(x) = \sum_{i=1}^n C_i x_i - d$. The shared feasible set is

$$K = \{x \in \Omega \,|\, g(x) \leq 0_p\} = \{x \in \Omega \,|\, Cx \leq d\}. \tag{14}$$

5. CONCLUSION

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Appendix A. A SUMMARY OF LATIN GRAMMAR

Appendix B. SOME LATIN VOCABULARY