

Doppler Imaging Fun

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1. INTRODUCTION

Check out [Luger et al. \(2019\)](#) and [Bedell et al. \(2019\)](#) and stuff.

2. THE EQUATION

In the most general form, the Doppler-shifted intensity observed at wavelength λ at position x, y on the surface of the star at time t is

$$\begin{aligned} I(\lambda, \beta, x, y, t) &= I(\lambda, 0, x, y, t) \\ &+ \left. \frac{dI(\lambda, \beta, x, y, t)}{d\beta} \right|_{\beta=0} \Delta\beta(x, y) \\ &+ \left. \frac{d^2I(\lambda, \beta, x, y, t)}{d\beta^2} \right|_{\beta=0} \Delta\beta^2(x, y) \\ &+ \dots \end{aligned} \tag{1}$$

where $\beta \equiv \frac{v}{c}$ is the relativistic parameter for a radial velocity v on the surface.

3. DIFFERENTIATING THE SPECTRUM

The derivatives of the spectrum $I(\lambda)$ with respect to the relativistic parameter β are found by application of Faà di Bruno's formula for taking high order derivatives of the chain rule:

$$\left. \frac{d^n I(\lambda, \beta)}{d\beta^n} \right|_{\beta=0} = \sum_{k=1}^n \left. \frac{d^k I(\lambda_0)}{d\lambda_0^k} \right|_{\lambda_0=\lambda} \lambda^k P_{nk} \tag{2}$$

where

$$P_{nk} \equiv B_{n,k} \left(\left\{ (-1)^j j! \right\}_{j=1}^{n-k+1} \right) \tag{3}$$

and $B_{n,k}$ is the incomplete Bell polynomial. The quantity $\left. \frac{d^k I(\lambda_0)}{d\lambda_0^k} \right|_{\lambda_0=\lambda}$ is just the k^{th} derivative of the spectrum with respect to wavelength in the rest frame, and must either be inferred from the data or computed numerically from the spectrum.

REFERENCES

- Bedell, M., et al. 2019, arXiv e-prints, arXiv:1901.00503
- Luger, R., et al. 2019, AJ, 157, 64