Optimization based resource allocation for Software as a Service application in cloud computing

Problem formulation mathematically:

$$U_{\text{saas cloud}} = -k \sum_j \left(\frac{q_{jm}^{cpu} P_j^{cpu}}{C_j^{cpu} u_{jm}^{cpu}} + \frac{q_{jm}^{ram} P_j^{ram}}{C_j^{ram} u_{jm}^{ram}} + \frac{q_{jm}^{net} P_j^{net}}{C_j^{net} u_{jm}^{net}} \right) - \sum_j \left(u_{jm}^{cpu} + u_{jm}^{ram} + u_{jm}^{net} \right)$$

$$+ \sum \left(u_{jm}^{cpu} \log v_{jm}^{cpu} + u_{jm}^{ram} \log v_{jm}^{ram} + u_{jm}^{net} \log v_{jm}^{net} \right) - cost_j$$

$$+ \left\{ \left(B_i - \sum_m \text{SaaSuser-pay}_i^m \right) + \left(T_i - \sum_{m=1}^{N} t_i^m \right) \right\} + \sum_m \text{SaaSuser-pay}_i^m \log x_i^m +$$

$$\max\ U_{\text{saas cloud}} \quad \text{--- (1)}$$

sub to

a) $E_m \geq \sum_j u_{jm}^{cpu} + u_{jm}^{ram} + u_{jm}^{net}$

b) $S_m \geq \sum_i x_i^m$

c) $T_i \geq \sum_m^N t_i^m$

d) $B_i \geq \sum_m \text{SaaSuser-pay}_i^m$

e) $C_j^{cpu} \geq \sum_m v_{jm}^{cpu}$

f) $C_j^{ram} \geq \sum_m v_{jm}^{ram}$

g) $C_j^{net} \geq \sum_m v_{jm}^{net}$

h) $cost_j \leq EC_j$

The SaaS cloud resource allocation presented by (1) is a non-linear optimization problem. To reduce the computational complexity, the SaaS cloud resource allocation is decomposed into sub-problems which are $U_{SaaS\ users}$, $U_{SaaS\ provider}$, $U_{resource\ provider}$

$$U_{SaaS\ cloud} = U_{SaaS\ users} + U_{SaaS\ provider} + U_{resource\ provider}$$

The sub-problems are described as follows:

SaaS user Optimization

SaaS user calculates the optimal payment for the SaaS providers under the constraint of the deadline and the budget to maximize the SaaS user's quality of service.

SaaS user's optimization can be formulated as follows

$$U_{SaaS\ user} = (T_i - \sum_m t_i^m) + (B_i - \sum_m SaaS\,user\text{-}pay_i^m)$$

* since $x_i^m = \dfrac{S_m\, SaaS\,user\text{-}pay_i^m}{Sp_m}$

$$\therefore\ t_i^m = \frac{\cancel{QS_{im}}\ QS_{im}}{x_i^m}$$

$$= \frac{QS_{im}\ Sp_m}{S_m \cdot SaaS\,user\text{-}pay_i^m}$$

$$\therefore\ U_{SaaS\,user} = T_i - \sum_m \frac{QS_{im}\ Sp_m}{S_m \times SaaS\,user\text{-}pay_i^m} + B_i - \sum_m SaaS\,user\text{-}pay_i^m$$

Problem reformulation

Max $U_{SaaS\,user}$

sub to

$$B_i \geq \sum_m SaaS\,user\text{-}pay_i^m$$

$$T_i \geq \sum_m t_i^m$$

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or

$$\text{Min} \sum_m \frac{QSim\ \Delta pm}{Sm * SaaSuser\text{-}pay_i^m} - T_i + \sum_m SaaSuser\text{-}pay_i^m - B_i$$

sub to

a) $$B_i - \sum_m SaaSuser\text{-}pay_i^m \geq 0$$

b) $$T_i - \sum_m t_i^m \geq 0$$

Lagrangian for User's SaaS utility.

$$L_{SaaSuser} = \sum_m \frac{QSim\ \Delta pm}{Sm * SaaSuser\text{-}pay_i^m} - T_i + \sum_m SaaSuser\text{-}pay_i^m - B_i$$

$$- \lambda \left(B_i - \sum_m SaaSuser\text{-}pay_i^m \right) - \gamma \left(T_i - \sum \frac{QSim\ \Delta pm}{Sm * SaaSuser\text{-}pay_i^m} \right)$$

From KKT theorem, we know that the optimal solution is given by $\frac{\partial L_{SaaSuser}}{\partial SaaSuser\text{-}pay_i^m} = 0$ for $\lambda, \gamma > 0$

$$\frac{\partial L_{SaaSuser}}{\partial SaaSuser\text{-}pay_i^m} = 0 = \frac{-QSim\ \Delta pm}{Sm * (SaaSuser\text{-}pay_i^m)^2} + 1 + \lambda - \frac{\gamma\ QSim\ \delta pm}{Sm * (SaaSuser\text{-}pay_i^m)^2}$$

$$(1+\gamma) \frac{QSim\ \Delta pm}{Sm * (SaaSuser\text{-}pay_i^m)^2} = 1 + \lambda$$

$$\therefore SaaSuser\text{-}pay_i^m = \left(\frac{1+\gamma}{1+\lambda} \right)^{\frac{1}{2}} \left(\frac{QSim\ \Delta pm}{Sm} \right)^{\frac{1}{2}} \quad \text{--- (2)}$$

Since $T_i = \sum t_i^m$

$$T_i = \sum \frac{QSim\ Spm}{Sm * SaaSuser\text{-}pay_i^m} \quad \text{--- (3)}$$

putting $SaaSuser\_pay_i^m$ from 2 in 3

$$T_i = \sum_m \frac{QS_{im}\,\delta p_m}{S_m * \left(\frac{1+\gamma}{1+\lambda}\right)^{1/2} \left(\frac{QS_{im}\,\Delta p_m}{S_m}\right)^{1/2}}$$

$$\therefore \quad T_i = \frac{\sum \left(\frac{QS_{im}\,\Delta p_m}{S_m}\right)^{1/2}}{\left(\frac{1+\gamma}{1+\lambda}\right)^{1/2}}$$

$$\left(\frac{1+\gamma}{1+\lambda}\right)^{1/2} = \frac{1}{T_i} \sum \left(\frac{QS_{im}\,\Delta p_m}{S_m}\right)^{1/2} \quad — (4)$$

putting value of $\left(\frac{1+\gamma}{1+\lambda}\right)^{1/2}$ from 4 in 2

$$\therefore \; SaaSuser\_pay_i^m = \left(\frac{QS_{im}\,\Delta p_m}{S_m}\right)^{1/2} \frac{1}{T_i} \sum_m \left(\frac{QS_{im}\,\delta p_m}{S_m}\right)^{1/2}$$

SaaS provider Optimization

The objective of SaaS provider optimization is to maximize $U_{SaaS\,provider}$ under the constraints of the service capacity and the budget

$$U_{SaaS\,provider} = -K \sum_j \left(\frac{q_{m}^{cpu} p_j^{cpu}}{c_j^{cpu} u_{jm}^{cpu}} + \frac{q_{m}^{ram} p_j^{ram}}{c_j^{ram} u_{jm}^{ram}} + \frac{q_{m}^{net} p_j^{net}}{c_j^{net} u_{jm}^{net}} \right)$$

$$- \sum_j \left(u_{jm}^{cpu} + u_{jm}^{ram} + u_{jm}^{net} \right)$$

Problem formulation: Max $U_{saas\,provider}$

sub to a) $E_m \geq \sum_j u_{jm}^{cpu} + u_{jm}^{ram} + u_{jm}^{net}$

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b) $$C_j^{cpu} \geq \sum_m V_{jm}^{cpu}$$

c) $$C_j^{ram} \geq \sum_m V_{jm}^{ram}$$

d) $$C_j^{net} \geq \sum_m V_{jm}^{net}$$

or

$$\min \; K \sum_j \left(\frac{q_m^{cpu} p_j^{cpu}}{C_j^{cpu} U_{jm}^{cpu}} + \frac{q_m^{ram} p_j^{ram}}{C_j^{ram} U_{jm}^{ram}} + \frac{q_m^{net} p_j^{net}}{C_j^{net} U_{jm}^{net}} \right) + \sum_j U_{jm}^{cpu} + U_{jm}^{ram} + U_{jm}^{net}$$

sub to

a) $$E_m - \sum_j U_{jm}^{cpu} + U_{jm}^{ram} + U_{jm}^{net} \geq 0$$

b) $$C_j^{cpu} - \sum_m V_{jm}^{cpu} \geq 0$$

c) $$C_j^{ram} - \sum_m V_{jm}^{ram} \geq 0$$

d) $$C_j^{net} - \sum_m V_{jm}^{net} \geq 0$$

Langrangian for SaaS provider's utility is as follows:

$$L_{saas\ provider} = K \sum_j \frac{q_m^{cpu} p_j^{cpu}}{C_j^{cpu} U_{jm}^{cpu}} + \frac{q_m^{ram} p_j^{ram}}{C_j^{ram} U_{jm}^{ram}} + \frac{q_m^{net} p_j^{net}}{C_j^{net} U_{jm}^{net}} + \sum_j U_{jm}^{cpu} + U_{jm}^{ram} + U_{jm}^{net}$$

$$- \lambda \left(E_m - \sum_j U_{jm}^{cpu} + U_{jm}^{Ram} + U_{jm}^{net} \right) - \beta \left(C_j^{cpu} - \sum_m V_{jm}^{cpu} \right) - \gamma \left(C_j^{ram} - \sum_m V_{jm}^{ram} \right)$$

$$- \delta \left(C_j^{net} - \sum_m V_{jm}^{net} \right)$$

From KKT theorem, we know that the optimal solution is given

by $$\frac{\partial L_{saas\ provider}}{\partial U_{jm}^{cpu}} = 0 \quad \text{for } \lambda > 0$$

$$\frac{\partial L_{saas\ provider}}{\partial u_{jm}^{cpu}} = 0 = \frac{-Kq_{m}^{cpu}\, p_j^{cpu}}{c_j^{cpu}\,(u_{jm}^{cpu})^2} + 1 + \lambda$$

similarly

$$\frac{\partial L_{saas\ provider}}{\partial u_{jm}^{ram}} = 0 = \frac{-Kq_{m}^{ram}\, p_j^{ram}}{c_j^{ram}\,(u_{jm}^{ram})^2} + 1 + \lambda$$

$$\frac{\partial L_{saas\ provider}}{\partial u_{jm}^{net}} = 0 = \frac{-Kq_{m}^{net}\, p_j^{net}}{c_j^{net}\,(u_{jm}^{net})^2} + 1 + \lambda$$

$$\therefore \quad u_{jm}^{cpu} = \left(\frac{Kq_m^{cpu}\, p_j^{cpu}}{c_j^{cpu}\,(1+\lambda)}\right)^{1/2} \quad \text{— 1)}$$

$$u_{jm}^{ram} = \left(\frac{Kq_m^{ram}\, p_j^{ram}}{c_j^{ram}\,(1+\lambda)}\right)^{1/2}$$

$$u_{jm}^{net} = \left(\frac{Kq_m^{net}\, p_j^{net}}{c_j^{net}\,(1+\lambda)}\right)^{1/2}$$

since $\lambda > 0$

$$E_m = \sum_j u_{jm}^{cpu} + u_{jm}^{ram} + u_{jm}^{net}$$

$$\therefore \quad E_m = \left(\frac{1}{1+\lambda}\right)^{1/2} \underbrace{\sum \left(\frac{Kq_m^{cpu}\, p_j^{cpu}}{c_j^{cpu}}\right)^{1/2} + \left(\frac{Kq_m^{ram}\, p_j^{ram}}{c_j^{ram}}\right)^{1/2} + \left(\frac{Kq_m^{net}\, p_j^{net}}{c_j^{net}}\right)^{1/2}}_{D}$$

$$\therefore \quad \frac{E_m}{D} = \left(\frac{1}{1+\lambda}\right)^{1/2} \quad \text{— ②}$$

putting 2 in 1

$$u_{jm}^{cpu} = \left(\frac{Kq_{m}^{cpu} p_j^{cpu}}{c_j^{cpu}}\right)^{1/2} \frac{Em}{\sum_j \left(\frac{Kq_{m}^{cpu} p_j^{cpu}}{c_j^{cpu}}\right)^{1/2} + \left(\frac{Kq_{m}^{ram} p_j^{ram}}{c_j^{ram}}\right)^{1/2} + \left(\frac{Kq_{m}^{net} p_j^{net}}{c_j^{net}}\right)^{1/2}}$$

similally we get
optimal value for u_{jm}^{ram} and u_{jm}^{net}

SaaS provider as service supplier:

$$U_{saas\ provider} = \sum_i SaaS\,user\text{-}pay_i^m \log(x_i^m + 1)$$

Max $U_{saas\ provider}$

sub to

a) $S_m \geq \sum_i x_i^m$

or

Min $-\sum_i SaaS\,user\text{-}pay_i^m \log(x_i^m + 1)$

sub to

a) $S_m - \sum_i x_i^m \geq 0$

Langrangean is as follows:

$$L_{saas\ provider} = -\sum SaaS\,user\text{-}pay_i^m \log(x_i^m + 1) - \lambda\left(S_m - \sum_i x_i^m\right)$$

From KKT theorem, optimal value of x_i^m is for $\frac{\partial L_{saasprovider}}{\partial x_i^m} = 0$

and $\lambda > 0$

$$\frac{\partial L_{saas\ provider}}{\partial x_i^m} = 0 = \frac{-SaaS\,user\text{-}pay_i^m \text{ [redacted]}}{x_i^m + 1} + \lambda$$

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$$\therefore \quad x_i^m + 1 = \frac{SaaSuser\_pay_i^m}{\lambda}$$

$$x_i^m = \frac{SaaSuser\_pay_i^m}{\lambda} - 1 \qquad \text{— 1)}$$

since $\lambda > 0$

$$\therefore \quad S_m = \sum_i x_i^m$$

$$S_m = \sum_i \left(\frac{SaaSuser\_pay_i^m}{\lambda} - 1 \right) \qquad (\text{from } 1)$$

$$S_m = \sum_i \left(\frac{SaaSuser\_pay_i^m - \lambda}{\lambda} \right)$$

$$\lambda S_m = \sum_i SaaSuser\_pay_i^m - \sum_i^m \lambda$$

$$\lambda S_m = \sum_i SaaSuser\_pay_i^m - \lambda m$$

$$\therefore \quad \lambda = \left(\sum_i SaaSuser\_pay_i^m \right) \times \frac{1}{(S_m + m)} \qquad \text{— 2)}$$

putting 2) in 1)

$$\therefore \quad x_i^m = \frac{SaaSuser\_pay_i^m \, (S_m + m)}{\sum_i SaaSuser\_pay_i^m} - 1$$

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SaaS resource provider Optimization

SaaS resource provider maximizes the utility $U_{resource\ provider}$ under the constraint of energy cost limit.

$$U_{resource\ provider} = \sum u_{jm}^{cpu} \log v_{jm}^{cpu} + u_{jm}^{ram} \log v_{jm}^{ram} + u_{jm}^{net} \log v_{jm}^{net}$$

$$- cost_j$$

Problem formulation

$$\text{Max } U_{resource\ provider}$$

sub to

a) $cost_j \le EC_j$

where $cost_j = pe_j \sum_m a_j \left(v_{jm}^{cpu} + v_{jm}^{ram} + v_{jm}^{net} \right)$

or

$$\min -\sum \left(u_{jm}^{cpu} \log v_{jm}^{cpu} + u_{jm}^{ram} \log v_{jm}^{ram} + u_{jm}^{net} \log v_{jm}^{net} \right) \cancel{+ cost_j}$$

$$+ \; pe_j \sum_m a_j \left(v_{jm}^{cpu} + v_{jm}^{ram} + v_{jm}^{net} \right)$$

sub to

$$EC_j - pe_j \sum_m a_j \left(v_{jm}^{cpu} + v_{jm}^{ram} + v_{jm}^{net} \right) \geq 0$$

Lagrangian for SaaS resource provider ~~utility~~ utility is

$$L_{resource\ provider} = -\sum \left(u_{jm}^{cpu} \log v_{jm}^{cpu} + u_{jm}^{ram} \log v_{jm}^{ram} + u_{jm}^{net} \log v_{jm}^{net} \right)$$

$$+ \; pe_j \sum_m a_j \left(v_{jm}^{cpu} + v_{jm}^{ram} + v_{jm}^{net} \right)$$

$$-\lambda \left(EC_j - pe_j \sum_m a_j \left(v_{jm}^{cpu} + v_{jm}^{ram} + v_{jm}^{net} \right) \right)$$

From KKT theorem, we know that optimal solution is given by

$$\frac{\partial L_{resource\ provider}}{\partial v_{jm}^{cpu}} = 0 \quad \text{and } \lambda > 0$$

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$$\therefore \quad \frac{\partial L_{resource\ provider}}{\partial v_{jm}^{cpu}} = 0 = -\frac{U_{jm}^{cpu}}{v_{jm}^{cpu}} + pe_j a_j + \lambda pe_j a_j$$

$$\therefore \quad v_{jm}^{cpu} = \frac{U_{jm}^{cpu}}{pe_j a_j (1+\lambda)} \qquad — (1)$$

similarly

$$v_{jm}^{ram} = \frac{U_{jm}^{ram}}{pe_j a_j (1+\lambda)}$$

$$v_{jm}^{net} = \frac{U_{jm}^{net}}{pe_j a_j (1+\lambda)}$$

Now since $\lambda > 0$

$$\therefore \quad EC_j = pe_j \sum_m a_j \left(v_{jm}^{cpu} + v_{jm}^{ram} + v_{jm}^{net} \right)$$

$$EC_j = pe_j a_j \sum \left(\frac{U_{jm}^{cpu}}{pe_j a_j (1+\lambda)} + \frac{U_{jm}^{ram}}{pe_j a_j (1+\lambda)} + \frac{U_{jm}^{net}}{pe_j a_j (1+\lambda)} \right)$$

$$EC_j = \frac{1}{1+\lambda} \sum U_{jm}^{cpu} + U_{jm}^{ram} + U_{jm}^{net}$$

$$\frac{EC_j}{\sum_m \left(U_{jm}^{cpu} + U_{jm}^{ram} + U_{jm}^{net} \right)} = \frac{1}{1+\lambda} \qquad — (2)$$

putting $\frac{1}{1+\lambda}$ from 2 in 1, we get

$$v_{jm}^{cpu\,*} = \frac{U_{jm}^{cpu} \; EC_j}{pe_j a_j \sum_m U_{jm}^{cpu} + U_{jm}^{ram} + U_{jm}^{net}}$$

similarly we get $v_{jm}^{ram\,*}$ and $v_{jm}^{net\,*}$

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Description of Notations

$S_m \rightarrow$ Software service capacity of SaaS service provider m

$C_j^{cpu} \rightarrow$ Cloud provider's CPU capacity

$C_j^{ram} \rightarrow$ Cloud provider's memory capacity

$C_j^{net} \rightarrow$ Cloud provider's bandwidth capacity

$u_{jm}^{cpu} \rightarrow$ CPU payment

$u_{jm}^{ram} \rightarrow$ Memory payment

$u_{jm}^{net} \rightarrow$ Bandwidth payment

$SaaSuser\text{-}pay_i^m \rightarrow$ The payment of SaaS user i

$p_j^{cpu} \rightarrow$ CPU price

$p_j^{ram} \rightarrow$ Memory price

$p_j^{net} \rightarrow$ Bandwidth price

$v_{jm}^{cpu} \rightarrow$ CPU allocated to SaaS provider m

$v_{jm}^{ram} \rightarrow$ Memory allocated to SaaS provider m

$v_{jm}^{net} \rightarrow$ Bandwidth allocated to SaaS provider m

$E_m \rightarrow$ SaaS provider's budget

$EC_j \rightarrow$ Upper limit of energy cost

$Pe_j \rightarrow$ Electricity unit price

$T_i \rightarrow$ SaaS user's deadline

$en_m^j \rightarrow$ Energy consumed by provisioning resource

$q_m^{cpu} \rightarrow$ Computation service requirement

$q_m^{ram} \rightarrow$ Storage service requirement

$q_m^{net} \rightarrow$ transmission service requirement

$U_{resource\ provider}$ ~~$U_{SaaS\ cloud}$~~ → utility function of SaaS cloud system.

$U_{SaaS\ user}$ → utility function of SaaS user.

Here

$$\boxed{x_i^m = S_m * (Saasuser\_pay_i^m / SP_m)}$$

$$\boxed{t_i^n = Q_{s_{in}} SP_m / S_m * Saasuser\_pay_i^m}$$

where,

SP_m → SaaS provider m's service price

x_i^m → fraction of software service leased to SaaS ~~provider m~~ user i by SaaS provider m.

t_i^n → The completion time for i^{th} SaaSuser's n^{th} job.

$U_{saasprovider}$] as consumer or as supplier → utility function of saas provider as a consumer or supplier.

QS_{in} → SaaS service requirement of SaaS user's job.

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