Verification of complex systems in Stainless

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Abstract

(TODO: Abstract)

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3.1 The Actor Model

(TODO: Actor Model)
[1]

3.1.1 Message

In our framework, messages are modelled as instances of the `Msg` trait.

```
abstract class Msg
```

3.1.2 Actor Reference

Each actor is associated with a unique and persistent reference, modelled as an instance of the `ActorRef` trait.

```
abstract class ActorRef
```

3.1.3 In-flight Messages

In-flight messages are represented as a product of the `ActorRef` of the destination actor, and the message itself.

```
case class Packet(dest: ActorRef, payload: Msg)
```

3.1.4 Actor Context

When a message is delivered to an actor, the latter is provided with a context, which holds a reference to itself, and a mutable list of `Packet`s to send.

```
case class ActorContext(
  self: ActorRef,
  var toSend: List[Packet]
)
```

3.1.5 Behavior

A behavior specifies both the current state of an actor, and how this one should process the next incoming message. In our framework, these are modelled as a subclass of the abstract class `Behavior`, which defines a single abstract method `processMsg`, to be overriden for each defined behavior.

Using the provided `ActorContext`, the implementation of the `processMsg` method can both access its own reference, and register messages to be sent after the execution of the method is complete. It is also required to return a new `Behavior`

```
abstract class Behavior {
  def processMsg(msg: Msg)(implicit ctx: ActorContext): Behavior
}
```

3.1.6 Transition

Whenever a message is processed, we record the transition between the previous state of the system and the one after the message has been been processed. We keep track of this information as an instance of the `Transition` class, which holds: the `Msg` that has been delivered, its sender and receiver, the new behavior of the destination actor, and the list of `Packet`s the destination actor wants to send.

```
case class Transition(
  from: ActorRef,
  to: ActorRef,
  msg: Msg,
  newBehavior: Behavior,
  toSend: List[Packet]
)
```

3.1.7 Actor System

The state of the Actor system at a given point in time is modelled as a case class, holding the behavior associated with each actor reference, the list of in-flight messages between any two actors, as well as a trace of the execution up to that point, modelled as a list of `Transition`s.

```
case class ActorSystem(
  behaviors: CMap[ActorRef, Behavior],
  inboxes: CMap[(ActorRef, ActorRef), List[Msg]],
  trace: List[Transition]
)
```

The `ActorSystem` class is equipped with a `step` method, which takes a pair of `ActorRef` as arguments, and is in charge of delivering the oldest message found in the corresponding inbox, and which returns the new state of the system after the aforementioned message has been processed.

```
def step(from: ActorRef, to: ActorRef): ActorSystem
```

3.2 Operational Semantics

We formulate the small-step operational semantics of our Actor model in Figure 1, where s : `ActorSystem` is an Actor system, m : `Msg` is a message, n, n_{to}, n_{from} : `ActorRef` are references, b, b' : `Behavior` are behaviors, ps : `List[Packet]` a list of packets to send, t : `Transition` is a transition, c : `ActorContext` is a context, and $\emptyset_n$: `ActorContext` is the empty context for an actor whose self-reference is n, defined as $\emptyset_n :=$ `ActorContext`$(n, \texttt{Nil})$.

$$\frac{\nexists m \in s.\texttt{inboxes}(n_{from}, n_{to})}{\langle s.\texttt{step}(n_{from}, n_{to})\rangle \longrightarrow \texttt{s}} \quad \text{(STEP-NOMSG)}$$

$$\frac{\exists m \in s.\texttt{inboxes}(n_{from}, n_{to}) \qquad \langle s.\texttt{deliverMsg}(n_{to}, n_{from}, m)\rangle \Rightarrow (b, ps, t)}{\langle s.\texttt{step}(n_{from}, n_{to})\rangle \longrightarrow s \uplus (n_{to} \mapsto b, \ldots, t)} \quad \text{(STEP)}$$

$$\frac{\langle s.\texttt{behaviors}(n_{to}).\texttt{processMsg}(m, \emptyset_{n_{to}})\rangle \longrightarrow (b, c)}{\langle s.\texttt{deliverMsg}(n_{to}, n_{from}, m)\rangle \longrightarrow (b, c.\texttt{toSend}, t)} \quad \text{(DELIVER-MSG)}$$

$$\frac{b.\texttt{processMsg}(m, \emptyset_{n_{to}}) = [i_1, \ldots, i_n, b'] \qquad \emptyset_{n_{to}} \vdash \langle [i_1, \ldots, i_n]\rangle \Rightarrow c}{\langle b.\texttt{processMsg}(m, \emptyset_{n_{to}})\rangle \longrightarrow (b', c)} \quad \text{(PROCESS-MSG)}$$

$$\frac{}{\langle \texttt{Nil}, c\rangle \Rightarrow c} \quad \text{(I-NIL)}$$

$$\frac{\langle i, c\rangle \Rightarrow c'}{\langle i \ :: \ is, c\rangle \Rightarrow \langle is, c'\rangle} \quad \text{(I-CONS)}$$

$$\frac{}{\langle n \ ! \ m, c\rangle \Rightarrow (b', \ c.\texttt{copy}(\texttt{toSend} \mapsto (n, m) \ :: \ c.\texttt{toSend}))} \quad \text{(I-SEND)}$$

Figure 1: Operational semantics

3.3 Proving Invariants

After having defined an Actor system with our framework, one might want to verify that this system preserves some invariant between each step of its execution. That is to say, for an `ActorSystem` s, any two `ActorRef` n, m, and an invariant `inv: ActorSystem → Boolean`, if $\texttt{inv}(s)$ holds, then $\texttt{inv}(s.\texttt{step}(n, m))$ should hold as well. We express this property more formally in Figure 2.

$$\forall s : \texttt{ActorSystem}, n : \texttt{ActorRef}, m : \texttt{ActorRef}.\ \texttt{inv}(s) \implies \texttt{inv}(s.\texttt{step}(n, m))$$

Figure 2: Invariant preservation property

This property can be trivially expressed in PureScala as:

```
def inv(s: ActorSystem): Boolean = {
  /* ... */
}

def preserveInv(s: ActorSystem, n: ActorRef, m: ActorRef): Boolean = {
  require(inv(s))
```

```
  inv(s.step(n, m))
} holds
```

When encoutering such a definition, Stainless will generate a verification condition equivalent to Figure 2, which will then be discharged to Inox and the underlying SMT solver.

3.4 Reasoning About Traces

(TODO: Traces)

3.5 Case studies

Replicated Counter (increment)

As a first and very simple case study, we will study an Actor system which models a replicated counter, which can only be incremented by one unit. This system is composed of two actors, a primary counter whose reference is `Primary()`, and a backup counter whose reference is `Backup()`. Each of these reference is associated with a behavior: the primary counter reference with an instance of `PrimaryB`, and the backup counter reference with an instance of `BackupB`, both of which hold a positive integer, representing the value of the counter. Whenever the primary actor receives a message `Inc()`, it forwards that message to the backup actor, and returns a new instance of `PrimaryB` with the counter incremented by one. When the backup actor receives an `Inc()` message, it just returns a new instance of `BackupB` with the counter incremented by one. The corresponding PureScala implementation can be found in Listing 1.

```
case class Primary() extends ActorRef
case class Backup()  extends ActorRef

case class Inc() extends Msg

case class PrimaryB(counter: BigInt) extends Behavior {
  require(counter >= 0)
  def processMsg(msg: Msg)(implicit ctx: ActorContext): Behavior = msg match {
    case Inc() =>
      Backup() ! Inc()
      PrimaryB(counter + 1)
  }
}

case class BackupB(counter: BigInt) extends Behavior {
  require(counter >= 0)
  def processMsg(msg: Msg)(implicit ctx: ActorContext): Behavior = msg match {
    case Inc() => BackupB(counter + 1)
  }
}
```

Listing 1: Replicated counter implementation (increment)

Given such a system, one might want to prove that the following invariant is preserved between each step of its execution:

```
def invariant(s: ActorSystem): Boolean = {
  s.inboxes((Backup(), Backup())).isEmpty && {
    (s.behaviors(Primary()), s.behaviors(Backup())) match {
      case (PrimaryB(p), BackupB(b)) =>
        p.value == b.value + s.inboxes(Primary() -> Backup()).length
      case _ => false
    }
  }
}
```

Listing 2: Replicated counter invariant (increment)

This invariant specifies that the `Backup()` actor does not send itself any messages, that both actors have the proper corresponding behavior, and that, last but not least, the value of the primary counter is equal to the value of the backup counter added to the number of messages that are yet to be delivered to the backup actor.

We can now define the actual theorem we want Stainless to prove for us:

```
def preserveInv(s: ActorSystem, n: ActorRef, m: ActorRef): Boolean = {
  require(invariant(s))
  invariant(s.step(n, m))
} holds
```

Listing 3: Replicated counter theorem (increment)

(TODO: Rep Counter Inc Result)

Replicated Counter (deliver)

Listing 4 shows a variant of the previous case study, where instead of having the primary actor forward the `Inc()` message to the backup actor, it instead delivers it with its new value.

The invariant now reads slightly differently, as can be seen in Listing 5, and can no longer be proven automatically by Stainless on its own. Listing 4 shows the full proof.

(TODO: Rep Counter Del Result)

Leader Election

(TODO: Leader election)

Key-value store

(TODO: KV store)

```
case class Primary() extends ActorRef
case class Backup()  extends ActorRef

case class Inc() extends Msg
case class Deliver(c: BigInt) extends Msg

case class PrimaryB(counter: BigInt) extends Behavior {
  def processMsg(msg: Msg)(implicit ctx: ActorContext): Behavior = msg match {
    case Inc() =>
      Backup() ! Deliver(counter + 1)
      PrimaryB(counter + 1)

    case _ => Behavior.same
  }
}

case class BackupB(counter: BigInt) extends Behavior {
  def processMsg(msg: Msg)(implicit ctx: ActorContext): Behavior = msg match {
    case Deliver(c) => BackupB(c)
    case _          => Behavior.same
  }
}
```

Listing 4: Replicated counter implementation (deliver)

```
def invariant(s: ActorSystem): Boolean = {
  validBehaviors(s)                            &&
  noMsgsToSelf(Primary()).isEmpty              &&
  noMsgsToSelf(Backup()).isEmpty               &&
  noMsgsToSelf(Backup() -> Primary()).isEmpty && {
    val PrimBehav(p) = s.behaviors(Primary())
    val BackBehav(b) = s.behaviors(Backup())
    val bInbox       = s.inboxes(Primary() -> Backup())

    p.value >= b.value && isSorted(bInbox) && bInbox.forall {
      case Deliver(Counter(i)) => p.value >= i
      case _                   => true
    }
  }
}
```

Listing 5: Replicated counter implementation (deliver)

3.6 Spawning Actors

Up until now, our framework has only been able to model Actor systems with a static topology, ie. systems where no new actors besides the ones that are statically defined can be spawned. Let's now attempt to enrich our model to account for dynamic topologies.

To this end, we modify the `ActorRef` definition to include both a name and an optional field holding a reference to its parent `ActorRef` if any. We also add a new constructor of the `ActorRef` data type, which will be assigned to actors spawned from another actor.

```
abstract class ActorRef(
  name: String,
  parent: Option[ActorRef]
)

case class Child(name: String, getParent: ActorRef)
  extends ActorRef(name, Some(getParent))
```

In order for actors to spawn other actors, by specifying their name and associated initial behavior, we modify the `ActorContext` class as follows:

```
case class ActorContext(
  self: ActorRef,
  var toSend: List[Packet],
  var toSpawn: List[(ActorRef, Behavior)]
) {
  def spawn(behavior: Behavior, name: String): ActorRef = {
    val id: ActorRef = Child(name, self)
    toSpawn = toSpawn :+ (id, behavior)
    id
  }
  /* ... */
}
```

As can be seen in the listing above, the context now keeps track of the names and behaviors of the actors to be spawned, and provides a `spawn` method which is in charge of constructing the `ActorRef` of the spawned actor, storing it along with the behavior within the context, and returning the newly generated reference.

Let's now update the case study in Listing 1 to accomodate these changes, while noting that we are not making use of this new feature yet. Only the two `ActorRef` definitions need to be touched, becoming:

```
case class Primary() extends ActorRef("primary", None())
case class Backup()  extends ActorRef("backup", None())
```

Unfortunately, if we now feed the updated benchmark to Stainless, the latter will be unable to prove the very same theorem it previously had no issue whatsoever with.

3.7 Executing an Actor System with Akka

4 Strong Eventual Consistency with CRDTs

(TODO: CRDTs)

5 Biparty Communication Protocols

(TODO: Bipart)

6 Conclusion

(TODO: Conclusion)

7 Future Work

(TODO: Future Work)

A References

[1] S. Yasutake and T. Watanabe, "Actario: A framework for reasoning about actor systems," in *Workshop on Programming based on Actors, Agents, and Decentralized Control*, AGERE 2015, 2015.