Forecasting Central Bank Liquidity: A Comprehensive Statistical Framework*

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Abstract

Central banks with active liquidity management need to conduct monetary operations to anchor the level of banks' reserves held on their balance sheet (the "*systemic liquidity*"). To this end, central banks need to forecast their autonomous factors, i.e. the the items of a central bank balance sheet that the central bank has no control upon, such as currency in circulation, government deposits and, for some central banks, foreign assets.

This paper presents a comprehensive statistical framework for forecasting the central bank autonomous factors, encompassing models estimation, validation and forecasts reconciliation. The framework relies on a data-driven approach, that estimates a battery of model and combine those with the best out-of-sample performance. The empirical strategy is designed to be flexible and adaptable to new dynamics and structural breaks.

This paper also offers and in-depth user-guide of the companion Python package, deployed to automatize this framework. This toolkit is designed to support central banks to implement modern statistical techniques in their daily liquidity forecasts.

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1 Introduction

The 17 September 2019 money market crash in the US reminded on the importance of liquidity management by the central bank. The conjunction of quarterly corporate tax payments and a large Treasury auctions on September 16 2019 have drained around USD 120 billion of banks' reserves at the central bank over only two business days, drying the interbank money markets liquidity. Consequently, the interbank rates hiked abruptly by more than 250 bps in a single day - against a daily average of 20 bps - pressuring the Fed Funds market.[1]

To implement monetary policy, most central banks have to control the liquidity conditions in the banking system. An important driver of the liquidity conditions in the banking system is the aggregate balance of all the monetary counterparties (commercial banks mostly)[2] that have an account with the central bank. These balances are also called "reserves" or "banks' reserves with the central bank." The sum of banks' reserves is sometimes called the "systemic liquidity".[3]

The aggregate level of reserves is crucial for implementing monetary policy and preserving financial stability. If the aggregate level of banks' reserves is too low, banks can face difficulties to settle their payments on the interbank market. Under low liquidity conditions, the demand for reserves increases, pushing the interbank rate up. Under extreme conditions, some banks might not be able to borrow reserves on the market and, in the absence of a refinancing alternative with the central bank, could default on their payments. On the contrary, under an excessive level of systemic liquidity, most banks have too much reserves, the demand for reserves plunges and the interbank rate converges towards the central bank deposit facility. In this situation, the transmission of monetary policy can be impeded, especially for central banks operating a corridor system.

Importantly, while the distribution of reserves among banks is decided on the intrabank market, with borrowing and lending banks, the aggregate level of reserves is under the control of the central bank. The central bank supplies banks' reserves ()

Central banks forecast liquidity to implement monetary policy. "Liquidity" is defined as on-call account of monetary counterparties (banks) at the central bank, also call "reserves." Liquidity forecasts are an important input to calibrate monetary operations, the objective of which is to align monetary conditions with the announced monetary policy stance. This applies to most monetary policy frameworks and is particularly important when the operational target is short-term interest rates.

Statistical liquidity forecast consists in modelling the behaviors of the central bank non-

[1] For more details, please refer to Anbil et al. (2020)

[2] Although, in a few countries, some non-banks entities - such as central counterparty clearing house - are allowed to maintain an account with the central bank

[3] Gray (2008) defines liquidity as "*the domestic-currency denominated, non-interest-bearing liabilities of the central bank held by the non-state sector*"

monetary counterparties. Those counterparties are not supervised by the central bank and do not have access to the monetary policy operations of the central bank. Central banks can collect information directly from these counterparties. This is the non-statistical component of the liquidity forecast also called “institutional arrangements.” However, central banks also use time-series models to forecast the behaviors of those counterparties.

We focus on modeling the behaviors of three nonmonetary counterparties that can have an influence on liquidity conditions. First, the Government could have an impact on liquidity via the transfers between its account at the central bank and those of the banks due to public expenditures and collection of revenues. Second, the public influences liquidity by demanding banknotes and coins which are issued against banks’ reserves. Third, if the exchange rate is fixed and capital transactions allowed, a broad set of resident and non-resident counterparties could possibly influence liquidity via FX purchases and sales from and to the central bank. The items in the central bank balance sheet under the control of its nonmonetary counterparties are called “autonomous liquidity factors.”

Currency in Circulation follows distinct seasonal patterns due to calendar effects related to the work week as well as public and religious holidays. These effects make it possible to develop point forecasting models for the level of these factors. For this reason, we propose forecasting currency in circulation using models [...]. Forecasting currency in circulation involved specific challenges such as treating mobile holiday (e.g., Ramadan or Chinese New Year) and structural break due to external shocks such as the COVID-19 pandemic.

On the other hand, net foreign assets do not display seasonal patterns, but do exhibit conditional volatility characteristic of financial returns data. For this reason, we propose forecasting net foreign assets, using models for conditional heteroskedasticity. Finally, the state treasury account follows seasonal patterns for some of its determinants but also exhibit conditional volatility characteristics for others. Some items that determined the Treasury account have clear seasonal pattern such as the payment of salaries and pensions as well as the collection of taxes. On the other hand, capital expenditures and tax revenues usually do not follow regular patterns.

In addition to developing forecasting models for three individual autonomous factors, we also consider forecast combination through taking an equally weighted combination either of all forecasts or of a trimmed set of models. The forecast quality is tested based on predictive performance metrics reflecting accuracy, bias, and reliability, which will be explained in this paper. Models are, then, ranked based on their predictive performance to combine them.

Finally, we also apply the method of forecast reconciliation to the liquidity forecasting problem. This involves, first, generating forecasts for the aggregate (or net) liquidity due to net foreign assets, currency in circulation and state account balance, leading to four forecasts (one for each autonomous factor and one of the aggregates). In general, these four forecasts will not be coherent, i.e., the aggregate forecast will not be equal to the aggregate of the three individual factors. Reconciliation adjusts the four forecasts to ensure coherence. This method has been shown to improve forecasts in several contexts and is applied to liquidity forecasting for the first time here.

The remainder of the paper is summarized as follows. Section 2 introduces the data on the autonomous factors, highlighting the main features of each factor. Section 3 introduces the models and methods used both for forecasting and forecast reconciliation. Section 4 presents the results of extensive forecast evaluation and Section 7 concludes .

2 Central Bank Liquidity Management

3 Data

There are several sources of liquidity in a banking system, those that are not under the control of a central bank and therefore represent sources of uncertainty. These are known as *autonomous factors*. The autonomous factors for which forecasting models are built in this study are *currency in circulation*, the *state account balance* and *net foreign assets*. Our data were obtained from the Central Bank of the United Arab Emirates (UAE) and are collected at a daily frequency. We now discuss each of the autonomous factors in turn.

3.1 Currency in Circulation

Currency in Circulation (CIC) is the quantity of money issued by monetary authorities net of currency that has been removed from the money supply.

RF to potentially expand on CIC data, how collected, measured etc.

Currency in Circulation tends to be influenced by calendar effects. In many countries, salaries are paid at the same time each week (or month) leading to a smooth weekly (or monthly pattern) whereby CIC increases after the pay date and slowly declines until just before the next pay date. The weekly pattern in CIC can also be influenced by a tendency of individuals to withdraw cash before the weekend. Public holidays can have a major impact on CIC, it is typical for CIC to increase in the buildup to a public holiday and then decline afterwards. These systematic features make it possible to develop models for forecasting CIC at horizons of 1 to 14 days that outperform naïve forecasting techniques.

Figure 1 highlights CIC data for the UAE. Days with missing data include weekends (Friday and Saturday in the UAE) and major holidays. Data on these days are linearly interpolated. Other autonomous factors which may exhibit a different pattern of missingness (for instance data may be available on Fridays for net foreign assets since foreign exchange markets trade on Friday), are interpolated in a similar fashion. As such, interpolating rather than ignoring missing data guarantees a balanced panel of data, which becomes important for reconciliation steps later on. The daily pattern of CIC is apparent from the figure as are spikes around major holidays, in particular Eid al Fitr and Eid al Adha. In early 2020 there is a permanent level shift in CIC associated with measures at the onset of Covid to inject cash into the monetary system.

3.2 Government Deposits

State Account Balance (SAB) is the quantity of money held by the government in its account with the Central Bank.

RF to potentially expand on SAB data, how collected, measured etc.

State Account Balance also tends to be influenced by calendar effects. In particular certain taxes may be due for collection towards the end of the week, month, quarter of financial year which can provide a boost to SAB at these time. Models accounting for these features can also outperform naïve approaches in forecasting SAB.

Figure 1: Currency in Circulation data for the United Arab Emirates measured in million of Dirhams.

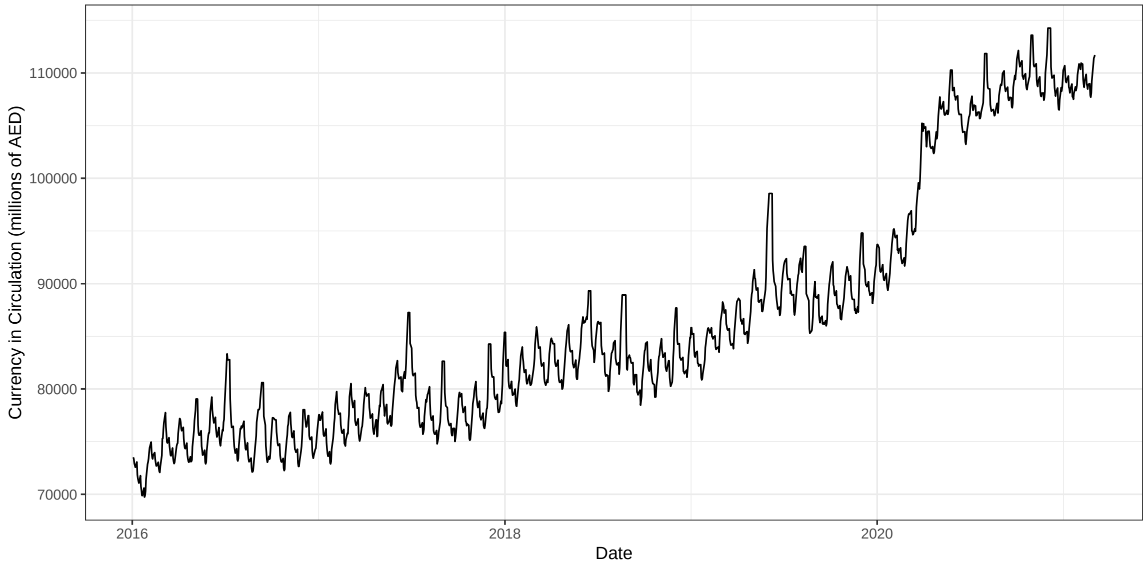


Figure 2 highlights SAB data for the UAE. Apart from spikes associated with seasonal factors, there is also a large one off spike in early 2019 associate with the sale of government assets. In our modelling, such transitory effects can be accounted for via the use of dummy variables. When asset sales are scheduled to occur at some time in the future, this may motivate judgemental adjustments to model forecasts.

3.3 Foreign Currency Assets

Net Foreign Assets (NFA) is the total foreign assets held by a central bank, net of their foreign liabilities

RF to potentially expand on NFA data, how collected, measured etc.

Compared to other autonomous factors, NFA are not influenced by calendar effects and when forecasting the mean, it is often difficult to outperform a naïve forecast of last observed value of NFA. However the daily change in NFA do resemble financial returns in that they often exhibit conditional heteroskedasticity and volatility clustering. This makes the GARCH family of models good candidates for forecasting the entire distribution of NFA data, which can be a critical input into operational decisions of Central Banks in the money market.

Figure 3 shows both the level (top panel) and change (bottom panel) in net foreign assets. The bottom panel in particular shows evidence of conditional heteroskedasticity motivating the use of GARCH models and their extensions

Figure 2: State Account Balance data for the United Arab Emirates measured in million of Dirhams.

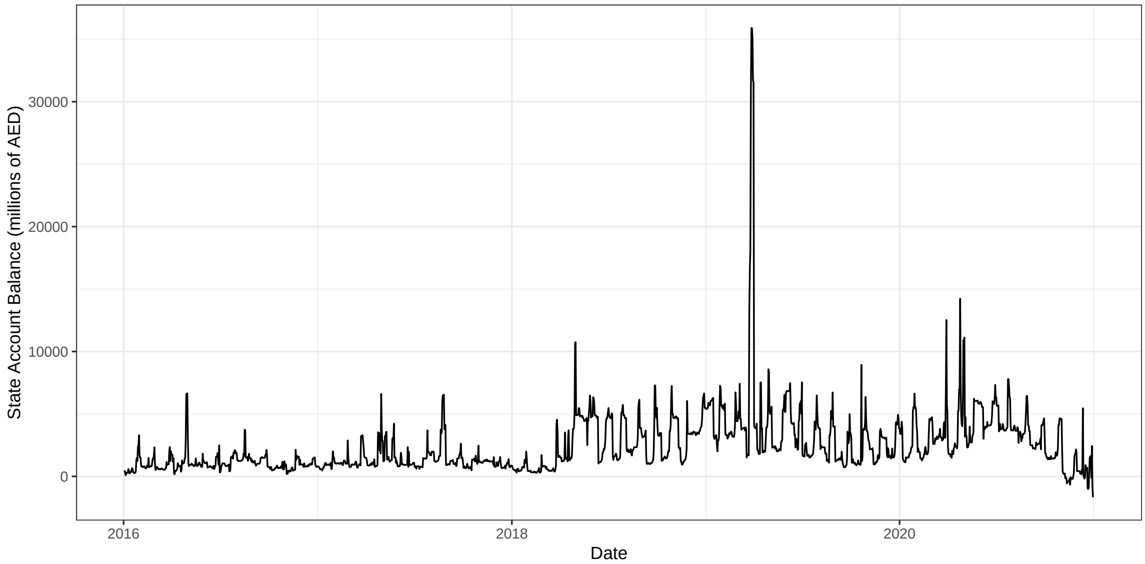
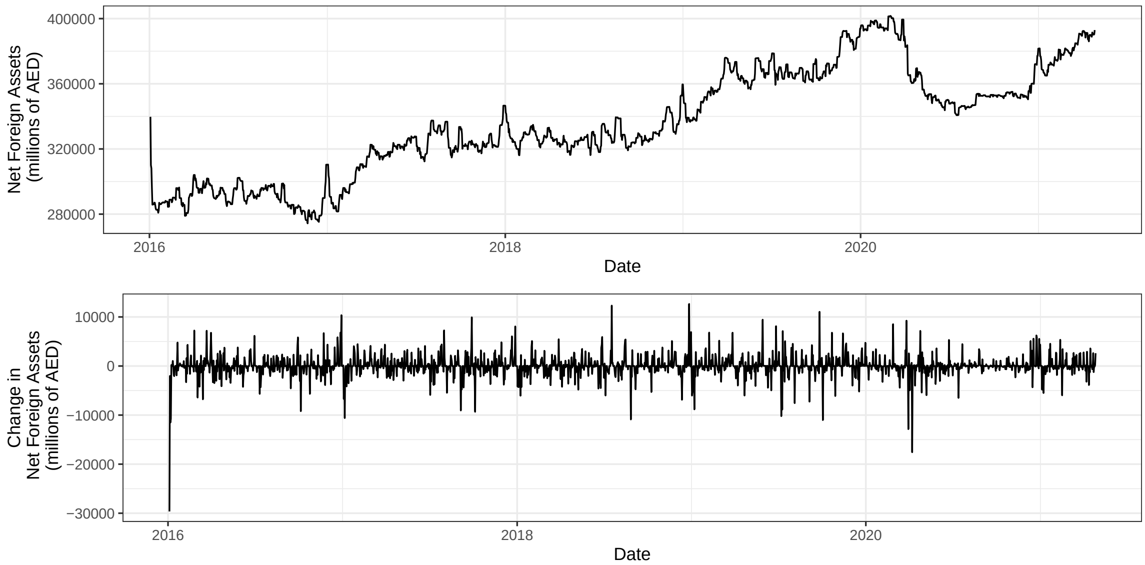


Figure 3: Net Foreign Assets data for the United Arab Emirates measured in million of Dirhams. Top panel: level, bottom panel: change.



3.4 Aggregate

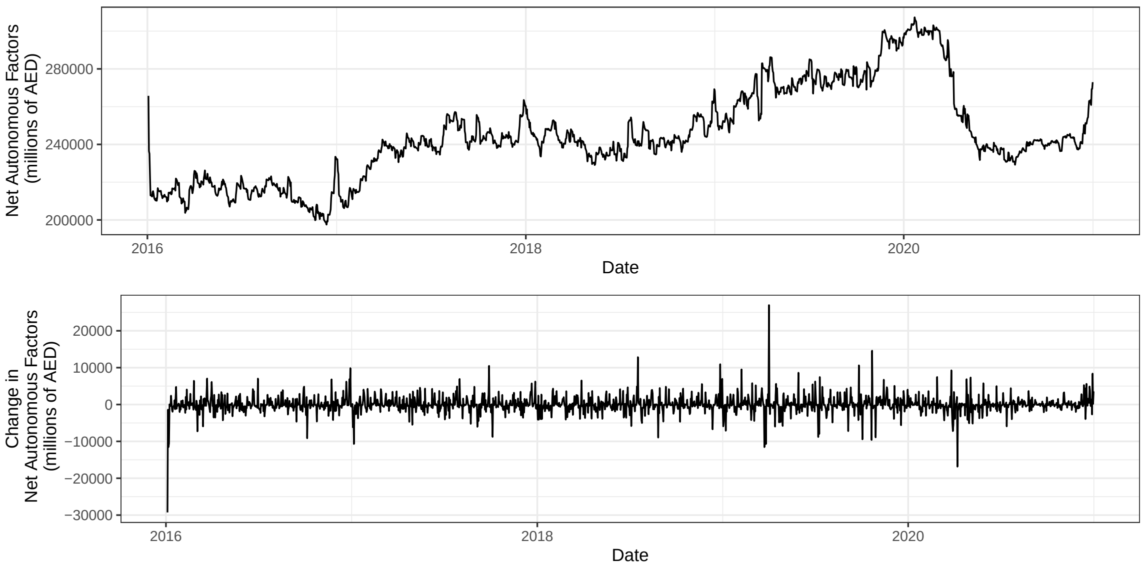
The aggregate (AGG) for the autonomous factors (or net liquidity due to autonomous factors) is given by

$$AGG = NFA - (CIC + SAB)$$

This reflects the fact that while NFA are clearly assets on the central bank balance sheet, CIC and SAB are liabilities for a central bank. This aggregate represents the net liquidity due to autonomous factors. While this aggregate is composed of all autonomous factors, in general the scale of NFA is much higher than CIC and SAB. As a result, the high variability of NFA tends to dominate this time series. Suitable models for forecasting the aggregate series are thus models for conditional volatility. However, information from CIC and SAB can be incorporated into the forecast of the aggregate via forecast reconciliation methodology, discussed in more detail in Section 4.7.

Figure 4 shows the net liquidity due to autonomous factors for the UAE. The top panel shows the level, the bottom panel shows the change. It is clear that variability of the NFA data dominates the influence of CIC and SAB. While the predictable seasonal patterns of CIC and SAB should in principle help with forecasting the aggregate, it will be difficult to model these effects while modelling the aggregate series alone. Later, this motivates forecast reconciliation approaches where separate models are developed for each of the autonomous factors and the aggregate.

Figure 4: Net liquidity due to autonomous factors for the United Arab Emirates measured in million of Dirhams. Top panel: level, bottom panel: change.



4 Empirical Strategy

We now give a brief description to modelling approaches that can be used for forecasting autonomous factors. As a reference to the general model classes described below see Hyndman and Athanasopoulos (2021) and (for conditional volatility models) Tsay (2005). Throughout, y_t denotes the value of an autonomous factor at time t, $\hat{y}_{t+h}$ denotes the h day ahead forecast of an autonomous factor, made at time t, m denotes the seasonal period (in our context $m = 7$ for each day of the week) and k is the integer part of $(h - 1)/m$

4.1 Exponential Smoothing

ETS models are a class of models that employ exponential smoothing with trend and seasonal components (see Hyndman et al., 2008, for details). The level, trend and seasonal components at time t are given by l_t, b_t and s_t respectively. Equations for forecasts and for updating the components are given by

$$\hat{y}_{t+h} = l_t + hb_t + s_{t+h-m(k+1)} \tag{1}$$
$$l_t = \alpha(y_t - s_{t-m}) + (1 - \alpha)(l_{t-1} + b_{t-1}) \tag{2}$$
$$b_t = \beta(l_t - l_{t-1}) + (1 - \beta)b_{t-1} \tag{3}$$
$$s_t = \gamma(y_t - l_{t-1} - b_{t-1}) + (1 - \gamma)s_{t-1}\,, \tag{4}$$

where α, β and γ are parameters that control the smoothness of the level, trend and seasonal components respectively that can be estimated by maximum likelihood. The model can be extended to allow for multiplicative trends (with or without damping) seasonal components, and errors. Depending on the components included and whether they are additive or multiplicative, leads to 30 possible model specifications. All specifications are fit and the model with the lowest corrected Akaike Information Critrion (AICc) given by

$$-2lnL + \frac{2q(q+1)}{T-q-1}$$

where q is the number of model parameters and T is the size of the training sample, is chosen. The `ets` function in the `forecast` package (Hyndman et al., 2022) in the R programming environment (R Core Team, 2020).

4.2 ARIMA

The ARIMA class of models take the form

$$(1 - \phi(L))(1 - L)^d(y_t - \mu) = (1 + \theta(L))\epsilon_t$$

where d is the degree of differencing, L is the lag operator, $\phi(L) = \phi_1 L + \phi_2 L^2 + \cdots + \phi_p L^p$ is a lag polynomial of AR parameters and $\theta(L) = \theta_1 L + \theta_2 L^2 + \cdots + \theta_q L^q$ is a lag polynomial of MA parameters.

The auto arima algorithm (Hyndman and Khandakar, 2008) can be used to select to orders p, q and d. First the degree of differencing d is determined by the sequqntial application of the KPSS test for stationarity. Then using the differenced data, an AR(1), AR(2) and ARMA(2,2) are fit. Starting from the model that minimises AICc, a search amongst neighboring models (i.e. models with p or q that differ by one from the current best model) is used to determing p ad q.

For seasonal data, a seasonal ARIMA

$$(1 - \phi(L))(1 - \Phi(L^m))(1 - L)^d(1 - L^m)^D(y_t - \mu) = (1 + \theta(L))(1 - \Theta(L^m))\epsilon_t$$

can be used instead, where P, D and Q are the orders of the seasonal AR polynomial, differencing and MA polynomial respectively. The auto arima algorithm can be modified to account for seasonality. This is implemented by the `auto.arima` function of the `forecast` package

4.3 ARIMA with regression

To capture the effects of holidays as well as longer seasonal patterns (e.g. yearly patterns) ARIMA models can be combined with regression. In particular, regression models are fit (possibly after differencing) with the errors then assumed to follow an ARMA process. The following regressors are considered in our modelling framework

4.3.1 Day of Week Dummies

Dummy variables can be used to capture weekly patterns in the data. This is particularly well suited to cases where the weekly pattern remains the same over the entire time series. It is less well suited when the seasonality varies over time, in which case seasonal ARIMA models may be more appropriate.

4.3.2 Fourier Terms

To capture smooth effects that take place over a longer seasonal period, Fourier terms can be used. These come in pairs and take the form

$$x^{(c)}_{j,t} = \cos(2\pi jt/m) \text{ and } x^{(s)}_{j,t} = \sin(2\pi jt/m)$$

for $j = 1, \ldots, J$ and where m is a longer seasonal period such as $m = 365$ (days in the year). For countries with a majority Muslim population $m = 354$ can be used to capture the effect of religious events (e.g. Ramadan) as well as religious holidays (e.g. Eid al Adha, Eid al Fitr) that move relative to the Gregorian calendar. In the case of the UAE, we construct Fourier terms based on four seasonal periods, $m = 30$ roughly corresponding to monthly effects, $m = 90$ roughly corresponding to quarterly effects, and $m = 354$ and $m = 365$ corresponding to the religious and civic year respectively.

4.3.3 Holiday effects

A standard approach to modelling the effects of holidays is to use dummy variables, which would capture a spike in an autonomous factor on a holiday. However, in the case of forecasting CIC in particular, what is more commonly observed is a ramping up of CIC for a week before a holiday, before a ramping down of CIC for a week after the holiday. This is captured by constructing the following covariate

$$x_t^{(h)} = \max\left(\max_{t^* \in \mathcal{T}_{hol}} \left(\frac{t^* - t}{7}\right)^2, 0\right)$$

where $\mathcal{T}_{hol}$ is a set of all t such that day t is a public holiday. If there is a public holiday within 7 days of day t, the term $t^* - t$ measures the amount of days before/after a holiday. This creates a small quadratic hump around each holiday (and bears some similarities with the Prophet model (Taylor and Letham, 2018) which uses a Gaussian density). For days that do not occur within a week of a public holiday $x_t^{(h)} = 0$.

4.3.4 Structural Breaks

Two forms of structural breaks are often observed in autonomous factors. A transitory structural break sees a change in the level of a series for a short period. In this case dummies can be used equal to 1 for the duration of the transitory structural break and 0 otherwise. This is implemented in the case of the State Account Balance for the UAE, with a transitory break starting on March 23, 2020 and ending on April 1, 2020. Alternatively there are permanent structural breaks which see a sustained change in the level of an autonomous factor. These are modelled using dummy variables set to zero before the structural break and one after the structural break. In the case of the UAE, this was implemented for the CIC data from the March 17, 2020.

4.3.5 Predictor Selection in Regression

Since only a small number of groups of regressors are considered, covariates can be chosen by an exhaustive search. To clarify, blocks of day of week dummies, Fourier terms (for each seasonal frequency), holiday dummies and structural break dummies, are either included or excluded all at once, leading to $2^4 = 16$ model specifications. For each both non-seasonal and seasonal ARIMA errors are considered with the auto arima algorithm used to select the ARIMA orders. The model with the lowest AIC overall is then selected. An alternative to this approach would be to carry out a LASSO to conduct variable selection, or, if the blocked structure of predictors is to be preserved, then a blocked LASSO can be used instead. An advantage of an exhaustive search is that it allows the regression specification and ARIMA specification for the errors to be chosen simultaneously - although it should be noted that an exhaustive search can be computationally demanding.

4.4 TBATS

The TBATS model (De Livera et al., 2011) is a form of exponential smoothing model with a number of additional features. Seasonality evolves according to Fourier terms (similar to those used in the regression model), the series is transformed by a Box-Cox transformation (which includes the log transformation as a limiting case) and the errors of the model follow an ARMA model. The main advantage of this model is that multiple seasonal components can be used accounting for seasonality with different periodicities. This is particularly useful in countries where civic and religious calendars may differ for example, in countries with a majority Muslim population. For the UAE, TBATS is implemented with seasonal periods of $m = 7, 30, 90, 354, 365$, roughly corresponding to days in a week, month, quarter, religious year and civic year.

4.5 Conditional Heteroskedasticity models

Net foreign assets and the aggregate series tend to resemble series on asset prices, with conditional heteroskedasticity in the difference of these series a major feature. To capture this we consider models in the GARCH family of distributions. All models for conditional volaitlity were implemented using the `rugarch` package in R (Ghalanos, 2022). The standard GARCH specification (Bollerslev, 1986) is given by

$$u_t = \sigma_t \epsilon_t \tag{5}$$

$$\sigma_t^2 = \omega + \alpha_1 u_{t-1}^2 + \cdots + \alpha_p u_{t-p}^2 + \beta_1 \sigma_{t-1}^2 + \cdots + \beta_q \sigma_{t-q}^2 \tag{6}$$

where u_t are demeaned changes in net foreign assets and ϵ_t is white noise with zero mean and unit variance. In modelling NFA, we consider a GARCH(1,1) specification where $p = q = 1$. This model is estimated by (quasi-)maximum likelihood under the assumption that ϵ_t is normally distributed. Both stationarity and enforcing that variances are positive impose certain constraints on these parameters.

An special case of the GARCH(1,1) model that is non-stationary in given by the EWMA process

$$u_t = \sigma_t \epsilon_t \tag{7}$$

$$\sigma_t^2 = \lambda u_{t-1}^2 (1 - \lambda) \sigma_{t-1}^2 \tag{8}$$

which is a form of simple exponential smoothing for the variance rather than the mean process. An extension of the GARCH(1,1) model is the GJR-GARCH(1,1) model (Glosten et al., 1993)

$$u_t = \sigma_t \epsilon_t \tag{9}$$

$$\sigma_t^2 = \omega + \alpha u_{t-1}^2 + \gamma I(u_t > 0) u_{t-1}^2 \beta \sigma_{t-1}^2 \tag{10}$$

The rationale behind the GJR-GARCH model is that the response of volatility will be stronger following a negative change in the variable rather than a positive change.

The final model we consider for conditional heteroskedasticity is the EGARCH(1,1) model Nelson (1991) given by

$$u_t = \sigma_t \epsilon_t \tag{11}$$

$$\ln(\sigma_t^2) = \omega + \alpha \left(|\epsilon_{t-1}| + \gamma \epsilon_{t-1}\right) + \beta \ln(\sigma_{t-1}^2) \tag{12}$$

An advantage of the EGARCH model is that the parameters do not need to be constrained in a way that ensures positive variances since all modelling of the variance takes place on the log scale.

4.6 Model Averaging

It is well established in the forecasting literature that combining forecasts from different classes of models can improve forecast accuracy (Timmermann, 2006). While there is a long literature on finding optimal weights, even weights tend to perform more robustly since there is no additional uncertainty introduced by having to estimate combination weights (Smith and Wallis, 2009). As well as considering an equally weighted combination of all classes of models outlined above, we also consider trimming models by only taking the best K models. For this studey we consider $K = 2$.

4.7 Forecast Reconciliation

When forecasts are generated for the autonomous factors (CIC, SAB and NFA) as well as AGG, there is no guarantee that forecasts respect the aggregation relationship $AGG = NFA - (CIC + SAB)$. To address this issue, different approaches can be used. As a benchmark, consider the case of ignoring that forecasts do not aggregate correctly, this is known as a 'base' or unreconciled forecast. In the following it is convenient to stack these base forecasts in a vector

$$\hat{y}_t = \begin{pmatrix} \mathrm{A\hat{G}G}_t \\ \mathrm{C\hat{I}C}_t \\ \mathrm{S\hat{A}B}_t \\ \mathrm{N\hat{F}A}_t \end{pmatrix}$$

The first approach to achieve coherent forecasts is to ignore the base forecast of the aggregate and instead simply compute an aggreagate forecast as $\hat{\text{NFA}}_t - (\hat{\text{CIC}}_t + \hat{\text{SAB}}_t)$. This is known as a bottom up approach. A more modern approach that does uses forecasts at all levels is forecast reconciliation.

Reconciliation methods require defining a matrix which in the case of autonomous factors is given by

$$\mathbf{S} = \begin{pmatrix} -1 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The first row of this matrix encodes the relationship between net liquidity and the autonomous factors. The second approach to obtain coherent forecasts is the so called OLS method of reconciliation (Hyndman et al., 2011) given by

$$\tilde{\mathbf{y}} = \mathbf{S}\left(\mathbf{S}'\mathbf{S}\right)^{-1}\mathbf{S}'\hat{\mathbf{y}}$$

By construction the reconciled forecasts $\tilde{\mathbf{y}}$ will cohere to the aggregation constraint.

The third way used to obtain coherent forecasts is an improvement on OLS reconciliation, known as the Minimum Trace (MinT) method(Wickramasuriya et al., 2019). This method accounts for covariance in the forecasting errors of each autonomous factor. Reconciled forecasts using the the MinT method are given by

$$\tilde{\mathbf{y}} = \mathbf{S}\left(\mathbf{S}'\boldsymbol{\Sigma}^{-1}\mathbf{S}\right)^{-1}\mathbf{S}'\boldsymbol{\Sigma}^{-1}\hat{\mathbf{y}}$$

The covariance matrix $\boldsymbol{\Sigma}$ is estimated using the residuals from the models for each autonomous factor.

5 Empirical Results

Forecast evaluation is carried out via an expanding window analysis. All data begin on January 3, 2016. For the first evaluation, all models are trained using available data up to and including data on July 3, 2018. One-step to 14-step ahead forecasts are then produced, i.e. from July 4, 2018 to July 17, 2018. The second evaluation rolls the window one week ahead, i.e. using all data up to and including data on July 10, 2018, forecasts for July 11,2018 to July 24, 2018 are produced. The window is expanded one week at a time up to an origin date of December 15, 2020. This gives a total of 129 origin dates (which we denote $\mathcal{T}_{\text{eval}}$, with 14 forecasts at each origin date. The origin dates are all on a Tuesday - a date influence by the operational context in which the Central Bank of the UAE operates.

The h-step ahead forecast made at origin date t are denoted $\hat{y}_{t+h|t}$. These are evaluated using RMSE and MAE given by

$$\text{RMSE}_h = \sqrt{\frac{1}{|\mathcal{T}|}\sum_{\mathcal{T}}(y_t - \hat{y}_{t+h|t})^2} \,\text{and}\, \text{MAE}_h = \sqrt{\frac{1}{|\mathcal{T}|}\sum_{\mathcal{T}}|y_t - \hat{y}_{t+h|t}|}$$

respectively. As a summary measure of forecasting performance over all horizons, squared and absolute errors can be averaged across forecasting horizons as well as forecast origins.

Given the importance of probabilistic forecasting in liquidity management we also require forecast accuracy metrics for probabilistic forecasts. A $100 \times \alpha\%$ prediction intervals $(\hat{l}_{t+h|t}, \hat{u}_{t+h|t})$ can be evaluated by computing the coverage, that is the proportion of times the realised value of a series is does not land within the prediction interval, over the evaluation windows.

To examine whether the differences in forecasting accuracy between different methods are statistically significant we employ the model confidence set of Hansen et al. (2011). This can be interpreted in a similar fashion to a confidence interval, in the same way that a confidence interval will cover the true value of a parameter with a given probability, the model confidence set will cover the best forecasting model with a given level of probability. The model confidence sets are found using the `MCS` package in R (Catania and Bernardi, 2017) with the default settings of a 15% Level of Signficance and 5000 bootstrap replications.

Table 1 shows MAE of currency in circulation for 1-day ahead, 7-day ahead and 14-day ahead forecasts. At a 1-day horizon and 7-day horizon, the best performing methods are the model averages, however, the differences between models are quite small - the model confidence set contains all models. However at a 14-day horizon, the best performing model is the ARIMA with regression. Only this model, and the model average of the two best models are included in the model confidence set. This indicates the importance of modelling seasonality and events allowed in the regression model with ARIMA errors, especially at a two week horizon.

Table 2 shows the results for State Account Balance. For this data the ARIMA models, including ARIMA with regression perform poorly at a 1-day horizon with TBATS and a model average performing best, and exponential smoothing methods within the model confidence set. At longer horizons the TBATS model performs well, although it should be noted that at a 7-day

Table 1: MAE at different forecast horizons (h) for currency in circulation. The methods are Simple Exponential Smoothing (SES), ETS, ARIMA, Seasonal ARIMA (SARIMA), a seasonal ARIMA with regressors for events and trigonometric seasonality (SARIMA Reg.), and two model averages, one an equally weighted average of all models (MA-all), and the other a equally weighted average of the best 2 models (MA-best2). Entries in **bold** indicate all models in the model confidence set.

Method	h=1	h=7	h=14
SES	379	2121	3234
ETS	**363**	**2098**	3188
ARIMA	**359**	**2012**	2813
SARIMA	**378**	**2223**	3891
SARIMA Reg.	**389**	**1890**	**2579**
TBATS	**355**	**2039**	2786
MA-all	**340**	**1865**	2809
MA-best2	**346**	**1854**	**2537**

and 14-day horizon, it is difficult to distringuish between models with only simple exponential smoothing excluded from the model confidence set.

Table 2: MAE at different forecast horizons (h) for state account balance. The methods are Simple Exponential Smoothing (SES), ETS, ARIMA, Seasonal ARIMA (SARIMA), a seasonal ARIMA with regressors for events and trigonometric seasonality (SARIMA Reg.), and two model averages, one an equally weighted average of all models (MA-all), and the other a equally weighted average of the best 2 models (MA-best2). Entries in **bold** indicate all models in the model confidence set.

Method	h=1	h=7	h=14
Simple Exponential Smoothing	**667**	1725	2010
ETS	**687**	**1743**	**2059**
ARIMA	814	**1766**	**2025**
Seasonal ARIMA	803	**1862**	**1961**
SARIMA trig events	829	**1652**	**1807**
TBATS	**731**	**1553**	**1724**
Model Average (all models)	**728**	**1604**	**1845**
Model Average (best 2)	**684**	**1567**	**1801**

Regarding Net Foreign Assets and net liquidity due to autonomous factors, conditional volatility is the dominant feature of these series. Therefore it makes more sense to report the coverage of 95% prediction intervals for these series. Table 3 reports these for net foreign assets. All models achieve coverage close to the desired rate of 95%, with a GARCH model and an model average performing slightly better than the alternatives across all forecast horizons. Table 4 shows the same results for net liquidity due to autonomous factors. Again, all models achieve coverage rates close to 95% with the model averages performing relatively well. It should be noted testing for significant differences in coverage rates will lack power due to the small number of evaluation periods in this empirical study.

In addition to the forecasts from individual models, we also consider forecast reconciliation methods discussed in Section 4.7. For this evaluation we consider the regression model with ARMA errors for CIC, Simple Exponential Smoothing for SAB, an e-GARCH model for NFA and a GJR-GARCH model for AGG. Table 5 summarises the mean absolute errors for each series. The

Table 3: Coverage of 95% prediction intervals at different forecast horizons (h) for net foreign assets. The methods are Exponentially Weighted Moving Average (EWMA), GARCH, GJR-GARCH, E-GARCH (SES), and two model averages, one an equally weighted average of all models (MA-all), and the other a equally weighted average of the best 2 models (MA-best2). Entries in **bold** are those closest to 0.95.

Method	h=1	h=7	h=14
EWMA	0.9250	0.9250	0.9083
GARCH	**0.9500**	**0.9417**	**0.9333**
GJR-GARCH	**0.9500**	0.9333	**0.9333**
E-GARCH	0.9333	**0.9417**	**0.9333**
MA-all	**0.9583**	**0.9417**	**0.9333**
MA-best2	0.9333	0.9333	0.9167

Table 4: Coverage of 95% prediction intervals at different forecast horizons (h) for net liquidity due to autonomous factors. The methods are Exponentially Weighted Moving Average (EWMA), GARCH, GJR-GARCH, E-GARCH (SES), and two model averages, one an equally weighted average of all models (MA-all), and the other a equally weighted average of the best 2 models (MA-best2). Entries in **bold** are those closest to 0.95.

Method	h=1	h=7	h=14
EWMA	0.9302	0.9380	0.9302
GARCH	0.9302	**0.9535**	0.9457
GJR-GARCH	**0.9380**	**0.9535**	0.9457
E-GARCH	0.9147	0.9612	0.9380
MA-all	**0.9380**	0.9612	0.9380
MA-best2	**0.9380**	0.9612	**0.9535**

results are quite mixed, reconciliation does not guarantee an improvement in forecast accuracy for all factors at all horizons. However, there are instances where reconciliation methods lead to improved forecast accuracy. The forecast of the aggregate improves at a short horizon using the MinT method, and at a long horizon using OLS. The currency in circulation forecast can be improved at a short horizon using OLS reconciliation. Net foreign asset forecasts improve across all horizons using the OLS method. In the meantime the state account balance forecasts are not improved by using reconciliation. This result is consistent with theoretical evidence that while reconciliation can improve forecasting accuracy overall, these improvements are not guaranteed to occur for all series (Panagiotelis et al., 2021). Nonetheless, reconciliation does improve forecasts for some series/ forecasting horizons and most importantly ensures that forecasts are coherent, that is they respect the constraint AGG=NFA-(CIC+SAB).

6 Conclusion

Our forecast evaluation provides guide to the econometric and statistical models that can be used for forecasting autonomous liquidity factors measured at a daily frequency. The influence of calendar effects and seasonal patterns that can be captured via regression modelling is shown to be of particular importance for forecasting currency in circulation especially at longer horizons. For state account balance, exponential smoothing techniques, including the TBATS approach

Table 5: MAE for all series at different forecast horizons using different reconciliation methods.

Autonomous Factor	Method	MAE (h=1)	MAE (h=7)	MAE (h=14)
Net Liquidity	Base (Unreconciled)	1760.32	**4301.35**	5724.03
	Bottom Up	1807.91	4451.54	5902.34
	MinT	**1753.49**	4397.29	5767.77
	OLS	1768.10	4306.73	**5715.94**
Currency in Circulation	Base (Unreconciled)	377.69	**1878.46**	**2567.23**
	Bottom Up	377.69	**1878.46**	**2567.23**
	MinT	375.61	1890.23	2583.85
	OLS	**368.58**	1889.95	2584.56
Net Foreign Assets	Base (Unreconciled)	1423.02	4092.57	5643.08
	Bottom Up	1423.02	4092.57	5643.08
	MinT	1441.80	4131.70	5596.20
	OLS	**1404.91**	**3976.77**	**5500.73**
State Account Balance	Base (Unreconciled)	**666.86**	**1724.99**	**2009.55**
	Bottom Up	**666.86**	**1724.99**	**2009.55**
	MinT	742.11	2386.69	3140.85
	OLS	682.95	1907.88	2284.66

that allows for multiple seasonalities, outperform ARMA models. For net foreign assets and the aggregate series, models for conditional heteroskedasticity that allow for an asymmetric response of volatility such as E-GARCH and GJR-GARCH can be used. It should be noted that these conclusions apply to a single country during a single period of time, and that the modelling approaches used should be constantly evaluated and updated. Model combination is an alternative approach to robustify against the choice of model, and is shown here to produce competitive results. Finally, reconciliation methods are shown to improve the forecasts of some series, and guarantee that forecasts of the net liquidity is coherent with forecasts of individual autonomous factors.

Further research could involve expanding the set of models considered for forecasting, including using regularisation approaches for selecting seasonal predictors for CIC and SAB and stochastic volatility models for NFA and net liquidity due to autonomous factors. Since risk management is an important feature of central bank operations, forecasts of quantitites such as Value at Risk and Expected Shortfall could also be considered. This will forecast reconciliation procedures to be extended to probabilistic forecasts with the work of Panagiotelis et al. (2020) providing a promising avenue for doing so.

7 Conclusion

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A Complete results for all forecast horizons.

This appendix contains results for all forecast horizons. Tables 6-9 show the RMSE results for 1 to 14-day ahead forecasts for for each autonomous factor and net liquidity due to autonomous factors. Tables 10-13 show the MAE results for 1 to 14-day ahead forecasts for each autonomous factor and net liquidity due to autonomous factors. Tables 14-17 show the coverage of prediction intervals for 1 to 14-day ahead forecasts for each autonomous factor and net liquidity due to autonomous factors.

Table 6: RMSE at different forecast horizons (h) for currency in circulation. The methods are Simple Exponential Smoothing (SES), ETS, ARIMA, Seasonal ARIMA (SARIMA), a seasonal ARIMA with regressors for events and trigonometric seasonality (SARIMA Reg.), and two model averages, one an equally weighted average of all models (MA-all), and the other a equally weighted average of the best 2 models (MA-best2).

h	SES	ETS	ARIMA	SARIMA	SARIMA Reg.	TBATS	MA-all	MA-best2
1	603	591	589	591	606	572	572	573
2	1339	1231	1237	1276	1115	1145	1173	1131
3	1644	1428	1493	1507	1250	1354	1347	1296
4	1967	1644	1765	1757	1429	1615	1545	1497
5	2292	1867	2061	2019	1665	1920	1767	1737
6	2606	2544	2394	2677	2129	2385	2274	2181
7	2918	2860	2655	3043	2342	2646	2533	2401
8	3209	3144	2888	3414	2545	2907	2779	2615
9	3502	3362	3100	3788	2666	3014	2978	2750
10	3677	3469	3237	3971	2786	3136	3094	2861
11	3872	3592	3403	4169	2931	3273	3236	2998
12	4085	3729	3592	4380	3098	3417	3402	3158
13	4006	3898	3491	4614	3099	3429	3448	3125
14	4060	3958	3507	4766	3094	3431	3492	3125

Table 7: RMSE at different forecast horizons (h) for state account balance. The methods are Simple Exponential Smoothing (SES), ETS, ARIMA, Seasonal ARIMA (SARIMA), a seasonal ARIMA with regressors for events and trigonometric seasonality (SARIMA Reg.), and two model averages, one an equally weighted average of all models (MA-all), and the other a equally weighted average of the best 2 models (MA-best2).

h	SES	ETS	ARIMA	SARIMA	SARIMA Reg.	TBATS	MA-all	MA-best2
1	1461	1449	1787	1642	1717	1704	1564	1569
2	2111	2096	2801	2589	2724	2497	2331	2274
3	2300	2480	3619	3387	3553	2806	2753	2509
4	2067	2380	3671	3415	3638	2701	2658	2325
5	2341	2316	3636	3326	3552	2789	2660	2480
6	2720	2701	3593	3137	3401	3074	2785	2803
7	2789	2839	2818	3004	2658	2241	2428	2399
8	3564	3623	3344	3406	3080	3038	3159	3202
9	3951	4014	3763	3751	3491	3440	3576	3599
10	4091	4053	4038	4006	3772	3559	3740	3726
11	3914	3840	4054	4061	3881	3376	3614	3536
12	3827	3896	3889	3877	3726	3259	3444	3426
13	3995	4172	4038	4092	3913	3380	3609	3574
14	2965	3075	3352	3313	3093	2364	2651	2503

Table 8: RMSE at different forecast horizons (h) for net foreign assets. The methods are Exponentially weighted moving average (EWMA), GARCH, GJR-GARCH, EGARCH, and two model averages, one an equally weighted average of all models (MA-all), and the other a equally weighted average of the best 2 models (MA-best2).

h	EWMA	GARCH	gjrGARCH	eGARCH	MA-all	MA-best2
1	1988	1996	1992	1981	1987	1983
2	4203	4172	4189	4188	4186	4194
3	4272	4252	4289	4277	4270	4273
4	4265	4245	4298	4259	4264	4260
5	4248	4235	4302	4245	4254	4244
6	5217	5206	5260	5189	5214	5201
7	5753	5750	5807	5727	5755	5738
8	6474	6492	6524	6445	6480	6457
9	7300	7331	7335	7283	7308	7289
10	7361	7386	7416	7348	7373	7352
11	7363	7378	7436	7340	7374	7348
12	7351	7356	7424	7319	7357	7332
13	7713	7763	7799	7730	7745	7718
14	8003	8027	8052	7979	8009	7987

B Software

Table 9: RMSE at different forecast horizons (h) for net liquidity due to all autonomous factors. The methods are Exponentially weighted moving average (EWMA), GARCH, GJR-GARCH, EGARCH, and two model averages, one an equally weighted average of all models (MA-all), and the other a equally weighted average of the best 2 models (MA-best2).

h	EWMA	GARCH	gjrGARCH	eGARCH	MA-all	MA-best2
1	2657	2644	2643	2665	2650	2649
2	4304	4326	4326	4334	4320	4314
3	4328	4345	4338	4343	4336	4331
4	4308	4320	4287	4302	4300	4294
5	4421	4453	4395	4423	4418	4404
6	5388	5427	5352	5409	5389	5366
7	5893	5952	5878	5937	5910	5881
8	6847	6928	6843	6894	6873	6841
9	7214	7299	7207	7248	7236	7206
10	7259	7330	7242	7296	7275	7245
11	7262	7306	7215	7284	7260	7234
12	7276	7300	7235	7277	7265	7250
13	7708	7747	7685	7745	7713	7690
14	7916	7955	7885	7914	7909	7894

Table 10: MAE at different forecast horizons (h) for currency in circulation. The methods are Simple Exponential Smoothing (SES), ETS, ARIMA, Seasonal ARIMA (SARIMA), a seasonal ARIMA with regressors for events and trigonometric seasonality (SARIMA Reg.), and two model averages, one an equally weighted average of all models (MA-all), and the other a equally weighted average of the best 2 models (MA-best2).

h	SES	ETS	ARIMA	SARIMA	SARIMA Reg.	TBATS	MA-all	MA-best2
1	379	363	359	378	389	355	340	346
2	969	827	858	893	793	801	785	780
3	1251	989	1059	1081	935	995	931	932
4	1531	1161	1244	1266	1092	1213	1078	1096
5	1798	1329	1448	1451	1292	1464	1235	1275
6	1890	1881	1807	1947	1741	1837	1687	1702
7	2121	2098	2012	2223	1890	2039	1865	1854
8	2385	2346	2203	2565	2058	2231	2055	2018
9	2647	2552	2353	2926	2188	2307	2222	2109
10	2812	2663	2457	3106	2316	2429	2330	2203
11	2991	2789	2583	3294	2455	2566	2454	2325
12	3172	2919	2732	3485	2593	2703	2595	2464
13	3159	3106	2785	3745	2571	2747	2734	2484
14	3234	3188	2813	3891	2579	2786	2809	2537

Table 11: MAE at different forecast horizons (h) for state account balance. The methods are Simple Exponential Smoothing (SES), ETS, ARIMA, Seasonal ARIMA (SARIMA), a seasonal ARIMA with regressors for events and trigonometric seasonality (SARIMA Reg.), and two model averages, one an equally weighted average of all models (MA-all), and the other a equally weighted average of the best 2 models (MA-best2).

h	SES	ETS	ARIMA	SARIMA	SARIMA Reg.	TBATS	MA-all	MA-best2
1	667	687	814	803	829	731	728	684
2	1004	1034	1215	1204	1194	1063	1062	989
3	1107	1163	1451	1446	1475	1237	1222	1111
4	1051	1204	1444	1490	1555	1251	1218	1083
5	1320	1312	1612	1642	1695	1480	1402	1344
6	1636	1715	1852	1818	1840	1727	1633	1627
7	1725	1743	1766	1862	1652	1553	1604	1567
8	1946	1980	1918	1947	1724	1717	1739	1747
9	2082	2130	1940	1974	1778	1797	1857	1886
10	2181	2121	2110	2126	1946	1879	1952	1969
11	2122	2060	2086	2131	1977	1857	1914	1917
12	2207	2224	2084	2094	1969	1938	1972	2008
13	2291	2423	2212	2204	2083	2035	2104	2093
14	2010	2059	2025	1961	1807	1724	1845	1801

Table 12: MAE at different forecast horizons (h) for net foreign assets. The methods are Exponentially weighted moving average (EWMA), GARCH, GJR-GARCH, EGARCH, and two model averages, one an equally weighted average of all models (MA-all), and the other a equally weighted average of the best 2 models (MA-best2).

h	EWMA	GARCH	gjrGARCH	eGARCH	MA-all	MA-best2
1	1287	1280	1289	1278	1279	1278
2	2940	2912	2938	2923	2927	2932
3	2999	2988	3035	3011	3005	3005
4	2991	2979	3028	2989	2991	2988
5	2970	2973	3024	2985	2981	2975
6	3625	3611	3636	3592	3612	3608
7	3938	3924	3942	3909	3925	3921
8	4581	4585	4575	4559	4570	4567
9	5344	5328	5330	5315	5327	5328
10	5436	5413	5440	5397	5418	5411
11	5427	5414	5461	5399	5422	5410
12	5409	5387	5455	5373	5402	5389
13	5337	5345	5384	5356	5352	5345
14	5405	5373	5460	5369	5395	5384

Table 13: MAE at different forecast horizons (h) for net liquidity due to all autonomous factors. The methods are Exponentially weighted moving average (EWMA), GARCH, GJR-GARCH, EGARCH, and two model averages, one an equally weighted average of all models (MA-all), and the other a equally weighted average of the best 2 models (MA-best2).

h	EWMA	GARCH	gjrGARCH	eGARCH	MA-all	MA-best2
1	1764	1765	1761	1763	1759	1760
2	3286	3309	3318	3284	3299	3301
3	3279	3300	3295	3263	3281	3283
4	3260	3267	3238	3222	3242	3244
5	3410	3421	3373	3362	3390	3390
6	4015	4091	4025	4023	4036	4018
7	4312	4411	4319	4347	4339	4309
8	5081	5200	5107	5126	5124	5091
9	5442	5520	5468	5484	5474	5449
10	5460	5529	5491	5512	5492	5471
11	5470	5515	5467	5501	5484	5462
12	5443	5485	5448	5473	5455	5444
13	5526	5606	5575	5630	5581	5547
14	5709	5743	5651	5734	5703	5673

Table 14: Coverage of 95% prediction intervals at different forecast horizons (h) for currency in circulation. The methods are Simple Exponential Smoothing (SES), ETS, ARIMA, Seasonal ARIMA (SARIMA), a seasonal ARIMA with regressors for events and trigonometric seasonality (SARIMA Reg.), and two model averages, one an equally weighted average of all models (MA-all), and the other a equally weighted average of the best 2 models (MA-best2).

h	SES	ETS	ARIMA	SARIMA	SARIMA Reg.	TBATS	MA-all	MA-best2
1	0.96	0.94	0.95	0.95	0.92	0.94	0.95	0.94
2	0.93	0.91	0.89	0.90	0.87	0.89	0.92	0.90
3	0.94	0.93	0.93	0.93	0.91	0.93	0.93	0.93
4	0.96	0.94	0.96	0.94	0.94	0.92	0.94	0.94
5	0.96	0.94	0.95	0.94	0.90	0.91	0.95	0.95
6	0.97	0.91	0.93	0.91	0.83	0.88	0.94	0.91
7	0.97	0.91	0.93	0.92	0.86	0.85	0.94	0.90
8	0.96	0.90	0.93	0.92	0.87	0.84	0.94	0.90
9	0.97	0.92	0.92	0.94	0.86	0.81	0.95	0.88
10	0.98	0.93	0.91	0.95	0.86	0.80	0.96	0.88
11	0.99	0.94	0.92	0.95	0.86	0.80	0.97	0.90
12	0.98	0.96	0.92	0.94	0.83	0.78	0.96	0.91
13	0.99	0.96	0.94	0.97	0.87	0.79	0.96	0.92
14	0.99	0.98	0.95	0.98	0.87	0.79	0.98	0.93

Table 15: Coverage of 95% prediction intervals at different forecast horizons (h) for state account balance. The methods are Simple Exponential Smoothing (SES), ETS, ARIMA, Seasonal ARIMA (SARIMA), a seasonal ARIMA with regressors for events and trigonometric seasonality (SARIMA Reg.), and two model averages, one an equally weighted average of all models (MA-all), and the other a equally weighted average of the best 2 models (MA-best2).

h	SES	ETS	ARIMA	SARIMA	SARIMA Reg.	TBATS	MA-all	MA-best2
1	0.98	0.97	0.91	0.90	0.90	0.95	0.94	0.96
2	0.96	0.95	0.90	0.89	0.89	0.92	0.95	0.95
3	0.96	0.94	0.91	0.90	0.91	0.91	0.95	0.95
4	0.99	0.95	0.94	0.94	0.93	0.94	0.95	0.97
5	0.96	0.92	0.94	0.93	0.93	0.91	0.95	0.95
6	0.97	0.96	0.92	0.93	0.92	0.89	0.94	0.95
7	0.97	0.97	0.90	0.89	0.88	0.89	0.97	0.97
8	0.98	0.97	0.91	0.91	0.92	0.85	0.98	0.97
9	0.98	0.98	0.93	0.93	0.95	0.88	0.98	0.98
10	0.98	0.98	0.93	0.93	0.93	0.85	0.97	0.98
11	0.98	0.98	0.93	0.92	0.94	0.86	0.97	0.98
12	0.99	0.98	0.93	0.93	0.91	0.85	0.98	0.98
13	0.98	0.98	0.92	0.91	0.91	0.88	0.96	0.97
14	1.00	1.00	0.93	0.93	0.91	0.89	0.99	0.99

Table 16: Coverage of 95% prediction intervals at different forecast horizons (h) for net foreign assets. The methods are Exponentially weighted moving average (EWMA), GARCH, GJR-GARCH, EGARCH, and two model averages, one an equally weighted average of all models (MA-all), and the other a equally weighted average of the best 2 models (MA-best2).

h	EWMA	GARCH	gjrGARCH	eGARCH	MA-all	MA-best2
1	0.93	0.95	0.95	0.93	0.96	0.93
2	0.85	0.87	0.83	0.83	0.84	0.84
3	0.88	0.89	0.86	0.88	0.88	0.88
4	0.91	0.93	0.93	0.93	0.93	0.92
5	0.94	0.97	0.95	0.96	0.97	0.93
6	0.92	0.95	0.93	0.94	0.93	0.93
7	0.93	0.94	0.93	0.94	0.94	0.93
8	0.88	0.93	0.93	0.91	0.90	0.89
9	0.90	0.93	0.92	0.91	0.92	0.91
10	0.92	0.93	0.93	0.92	0.92	0.92
11	0.93	0.93	0.94	0.93	0.93	0.92
12	0.93	0.93	0.95	0.93	0.93	0.93
13	0.93	0.93	0.94	0.93	0.93	0.92
14	0.91	0.93	0.93	0.93	0.93	0.92

Table 17: Coverage of 95% prediction intervals at different forecast horizons (h) for net liquidity due to all autonomous factors. The methods are Exponentially weighted moving average (EWMA), GARCH, GJR-GARCH, EGARCH, and two model averages, one an equally weighted average of all models (MA-all), and the other a equally weighted average of the best 2 models (MA-best2).

h	EWMA	GARCH	gjrGARCH	eGARCH	MA-all	MA-best2
1	0.93	0.93	0.94	0.91	0.94	0.94
2	0.88	0.83	0.86	0.85	0.85	0.88
3	0.92	0.90	0.91	0.91	0.93	0.93
4	0.97	0.93	0.96	0.93	0.94	0.95
5	0.97	0.96	0.98	0.96	0.97	0.98
6	0.95	0.95	0.97	0.97	0.96	0.96
7	0.94	0.95	0.95	0.96	0.96	0.96
8	0.92	0.92	0.93	0.93	0.92	0.91
9	0.91	0.93	0.92	0.91	0.93	0.91
10	0.91	0.93	0.93	0.92	0.93	0.93
11	0.92	0.93	0.93	0.95	0.93	0.94
12	0.92	0.93	0.95	0.94	0.94	0.95
13	0.91	0.95	0.94	0.92	0.92	0.93
14	0.93	0.95	0.95	0.94	0.94	0.95